CIS 315, Intermediate Algorithms
Spring 2018

Assignment 5

due Wednesday, May 23, 2018

1. (exercise 15.2-1, p 378 from CLRS) Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is $\langle 5, 10, 3, 12, 5, 50, 6 \rangle$. [**6 points**]

2. For this dynamic programming problem and the next one, be sure to

   (a) describe the subproblem

   (b) give a recurrence for the subproblem

   (c) provide pseudo-code showing how a table for the subproblems is filled

   (d) give the time and space requirements of your method

   The residents of the underground city of Zion defend themselves through a combination of kung-fu, heavy artillery, and efficient algorithms. Recently, they have become interested in automated methods that can help fend off attacks by swarms of robots.

   Heres what one of these robot attacks look like:

   - A swarm of robots arrives over the course of n seconds; in the ith second, xi robots arrive. Based on remote sensing data, you know this sequence $x_1.x_2, \ldots, x_n$ in advance.

   - You have at your disposal an electromagnetic pulse (EMP), which can destroy some of the robots as they arrive; the EMPs power depends on how long its been allowed to charge up. To make this precise, there is a function $f(-)$ so that if j seconds have passed since the EMP was last used, then it is capable of destroying up to $f(j)$ robots.

   - So, specifically, if it is used in the kth second, and it has been j seconds since it was previously used, then it will destroy $\min[x_k, f(j)]$ robots. (After this use, it will be completely drained.)

   - We will also assume that the EMP starts off completely drained, so if it is used for the first time in the jth second, then it is capable of destroying up to $f(j)$ robots.

   Given the data on robot arrivals $X = (x_1, x_2, \ldots, x_n)$, and given the recharging function $f(-)$, the problem is to determine the maximum number of robots that can be destroyed by activating the EMP at certain points in time.

   For example, suppose $n = 4$, $X = (1, 10, 10, 1)$ and $f(-) = (1, 2, 4, 8)$. The best solution would be to activate the EMP in the 3rd and 4th seconds. In the 3rd second, the EMP has gotten to charge for 3 seconds, and so it destroys min(10,4)=4 robots. In the 4th second, the EMP has only gotten to charge for 1 second since its last use, and it destroys min(1,1)=1 robot. This is a total of 5.

   [**8 points**]

3. Solve the PRINTING NEATLY problem, exercise 15-4 (pp 405-406) of CLRS. You do not need to show how the paragraph is printed, just show how to compute the minimum penalty. [**8 points**]

4. Recall the problem of us having coins of denominations $d_1, d_2, \ldots, d_n$ and we want to make change for T cents. We have an unlimited supply of each coin, and want to know the fewest number of coins that add up to T, if possible. In class we looked at the subproblem $C[t]$ as being the minimum number of coins to make change for $t \leq T$ cents ($C[t] = \infty$ means it is not possible). We also derived a recurrence relation for C:

$$C[t] = \begin{cases} \infty & \text{if } t < 0 \\ 0 & \text{if } t = 0 \\ 1 + \min_{j=1}^{n} C[t - d_j] & \text{otherwise.} \end{cases}$$

Give psuedo-code which computes C iteratively. Also, include code that will save some information and then print out the coins used to make change for T. [**8 points**]

Total: 30 points