A1

(1) algorithm1 (A, n)

```
if (n ≤ 20)
    return A[n]                 } Θ(1)
x = algorithm1(A, n-5)          T(n-5)
for i=1 to ⌊n/2⌋
    for i=1 to ⌊n/2⌋            } O(n²)
        A[i] = A[i] - A[i]
x = x + algorithm1(A, n-8)      T(n-8)
return x                        Θ(1)
```

Итог: $\begin{cases} \Theta(1), & n \le 20 \\ T(n-5)+T(n-8)+O(n^2), & n>20 \end{cases}$

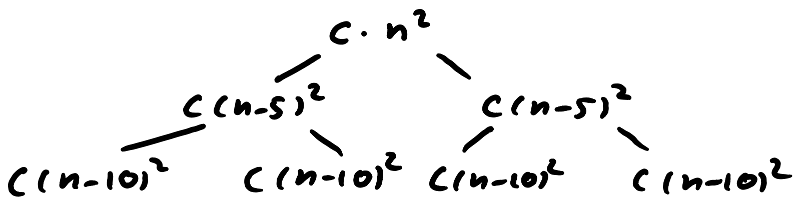
$$T(n) = T(n-5)+T(n-8)+O(n^2)+\Theta(1)+\Theta(1)+\Theta(1) = T(n-5)+T(n-8)+O(n^2)$$

```
algorithm2(A, n)
    if (n ≤ 50)
        return A[n]                 } Θ(1)
    x = algorithm2(A, ⌊n/4⌋)        T(⌊n/4⌋)
    for i=1 to ⌊n/3⌋
        A[i] = A[n-i] - A[i]        } O(n)
    x = x + algorithm2(A, ⌊n/4⌋)    T(⌊n/4⌋)
    return x                        Θ(1)
```

Итог: $\begin{cases} \Theta(1), & n \le 50 \\ 2T(\lfloor n/4 \rfloor)+O(n), & n>50 \end{cases}$

$$T(n) = 2\cdot T(\lfloor n/4 \rfloor)+O(n)+\Theta(1) = 2T(\lfloor n/4 \rfloor)+O(n)$$

(2) $T(n) = T(n-5)+T(n-8)+O(n^2) \le 2\cdot T(n-5)+O(n^2) \le 2T(n-5)+c\cdot n^2$



$\frac{n}{5}$

2^i - подзадач на i-ом уровне

$\sum_{i=0}^{\frac{n}{5}} c(n-i\cdot 5)^2$

$$\sum_{i=0}^{n/5} 2^i\cdot c(n-i5)^2 = c\cdot\sum_{i=0}^{n/5}(n^2-10ni+25i^2) = c\sum_{i=0}^{n/5} n^2\cdot 2^i - 10n\cdot c\cdot\sum_{i=0}^{n/5} 2^i\cdot i + 25\cdot c\cdot\sum_{i=0}^{n/5} 2^i\cdot i^2$$

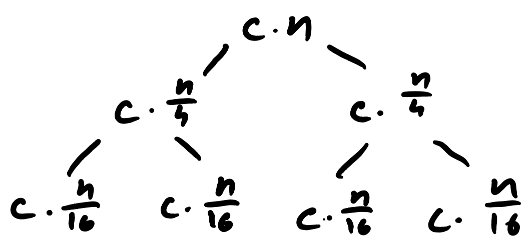
① ② ③

$^*(n-i5)^2 = n^2-10ni+25i^2{}^*$

① $c\cdot\sum_{i=0}^{n/5} n^2\cdot 2^i = c\cdot n^2\cdot\sum_{i=0}^{n/5} 2^i = c\cdot n^2\cdot 2^{n/5+1}-1) = 2cn^2\cdot 2^{n/5}-cn^2$

Algorithm2:

$$T(n) = T(\lfloor n/4 \rfloor)+T(\lfloor n/4 \rfloor)+O(n) \le 2T(n/4)+c\cdot n$$



$\log_4 n$

2^i – подзадач на i-ом уровне

$\sum_{i=0}^{\log_4 n} 2^i\cdot c\frac{n}{4^i}$

$$\sum_{i=0}^{\log_4 n} 2^i \cdot c \cdot \frac{n}{4^i} = c \cdot n \cdot \sum_{i=0}^{\log n} \left(\frac{1}{2}\right)^i = c \cdot n \cdot \frac{\left(1-\frac{1}{2}^{\log_4 n}\right)}{1-\frac{1}{2}} = c \cdot n \cdot 2\left(1-\frac{1}{\frac{1}{2}^{\log_4 n}}\right) =$$

$$= c \cdot n \cdot 2\left(1-\frac{1}{2^{\frac{1}{2}\log_2 n}}\right) = c \cdot n\left(2-\frac{2}{\sqrt{n}}\right) = 2 \cdot c \cdot n - 2\sqrt{n} = O(n)$$

Предположим, $T(n) = \Omega(n)$

Применим метод подстановки

$T(n) = \Omega(n)$

$T(n) \geq cn$

$T(n/4) \geq c \cdot \frac{n}{4}$

$T(n) \geq c \cdot \frac{n}{2} + dn \geq cn$

$\left(\frac{c}{2} + d\right) \geq cn$

$-\frac{c}{2} \geq -d$

$\frac{c}{2} \leq d \Rightarrow$ верно $\forall c \leq 2d \Rightarrow T(n) = \Omega(n)$;

$T(n) = \Omega(n)$

$T(n) = O(n)$