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Journal of Banking & Finance 32 (2008) 2057–2063

Journal of BANKING & FINANCE

www.elsevier.com/locate/jbf

Beyond Sharpe ratio: Optimal asset allocation using different performance ratios

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Received 19 December 2006; accepted 17 December 2007
Available online 1 January 2008

Abstract

As the assumption of normality in return distributions is relaxed, classic Sharpe ratio and its descendants become questionable tools for constructing optimal portfolios. In order to overcome the problem, asymmetrical parameter-dependent performance ratios have been recently proposed in the literature. The aim of this note is to develop an integrated decision aid system for asset allocation based on a toolkit of eleven performance ratios. A multi-period portfolio optimization up covering a fixed horizon is set up: at first, bootstrapping of asset return distributions is assessed to recover all ratios calculations; at second, optimal rebalanced-weights are achieved; at third, optimal final wealth is simulated for each ratios. Eventually, we make a robustness test on best performance ratios. Empirical simulations confirm the weakness in forecasting of Sharpe ratio, whereas asymmetrical parameter-dependent ratios, such as the Generalized Rachev, Sortino–Satchell and Farinelli–Tibiletti ratios show satisfactorily robustness.

JEL classification: C8; G0; G1; G8

Keywords: Asset allocation; Performance ratios; One-sided measures; Portfolio optimization

1. Introduction[1,2]

An *evergreen* question in modern asset allocation modelling and managing techniques is how to choose the best fitting performance ratio to use. Starting from the seminal idea of Roy (1952), Sharpe (1966) introduced the well-known Sharpe Ratio for managing mutual funds. Subsequently, Zenios (1993), Zenios and Kang (1993) and Sharpe (1994) improved the ratio suggesting to refer the performance to a benchmark. Although Sharpe ratio and its descendants fully hit the point as the returns are assumed Gaussian distributed, they flag as soon as this property is relaxed. As we tend to align the model to what a vast literature has documented on the asymmetry in stock index distributions (see the recent analysis of Ekholm and Pasternack, 2005 and Leland, 1999 for a sound criticisms to normality assumption) we are bound to skip from the Gaussian world. So a number of different alternatives have been proposed in the literature. Some of these redefine the risk measure such as the Gini ratio (e.g., Shalit and

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[1] The views expressed herein are those of the authors and do not necessarily reflect opinions shared by UBS and Cantonal Bank of Zurich.

[2] This paper was reviewed and accepted while Prof. Giorgio Szego was the Managing Editor of The Journal of Banking and Finance and by the past Editorial Board. The authors also thank two anonymous referees for their constructive comments. All errors are, of course, the authors' sole responsibility.

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doi:10.1016/j.jbankfin.2007.12.026

Yitzhaki, 1984), the Mean Absolute Deviation (MAD) ratio (e.g., Konno and Yamazaki, 1991), the Stable ratio (e.g., Biglova et al., 2004; Huber et al., 2003), the Minimax ratio (e.g., Young, 1998). Others such as the Sortino–Satchell (e.g., Sortino and Satchell, 2001), VaR and CVaR ratios modify the risk measure in order to capture the downside risk (note that MiniMax ratio is a special case of CVaR). However, a drawback continues to exist: all above mentioned ratios attribute a symmetrical weight (even though opposite in sign) to the upside and downside returns. To overcome the question Zenios (1995) and Dembo and Mausser (2000) proposed to use asymmetric weights to upside and downside returns. A recent alternative concerns the use of *parameter-dependent one-sided variability measures*, specifically the truncated unconditional moments (e.g., Farinelli and Tibiletti, 2008 and Menn et al., 2005), and the truncated moments conditioned to tail events (e.g., Biglova et al., 2004).

The aim of the paper is to set a decision aid system up for optimal allocation able to drive step by step the seemly best fitting performance ratio among eleven at disposal. We assume the ultimate agent's objective is maximizing the terminal wealth and we focus on scheduling (at $t = 0$) the best sequence of capital allocations (rolled forward until $t = T$ by an active investment strategy) starting from initial endowment.

After maximization we propose an approximating optimization algorithm repeated for an number of unit periods in which $[0, T]$ is split chosen by the management, scheduling a possible portfolio rebalancing at the end of each period. To estimate the model by bootstrapping empirical return distributions we employ historical data of five total return indexes. Since our aim is to suggest optimal portfolio weights generated by the best performance ratio, we simulate optimal path of final wealth at epoch T. Eventually, we carry on robustness tests on best forecasting ability. Since returns are far from being normal, we expect asymmetrical ratios show a more robust outperformance than Sharpe ratio. Eventually, simulations confirm our conjecture.

The paper is organized as follows. In Section 2 the Sharpe ratio and alternative performance ratios are discussed. In Section 3 we set up the optimization model. In Section 4 simulations of optimal asset allocations are carried out. Section 4 ends with back test results. Final remarks and future research lines are discussed in Section 5.

2. Two-sided and one-sided performance ratios

To model future asset values we introduce a probability space $(\Omega, \mathbf{P}, \mathscr{F})$ [3] and assume that total return X belongs to $L^p(\Omega, \mathscr{F}, \mathbf{P})$ for an appropriate $p > 0$; we consider only p-integrable random returns, i.e., $\mathbf{E}[|X|]^p < \infty$ over a fixed time horizon. Typically, investment opportunities are distinguished between those in long position, i.e. the effective managed portfolio with total return X and those in short position usually taken as a benchmark with return B. The latter also belongs to $L^p(\Omega, \mathscr{F}, \mathbf{P})$ and represents a model for life and non-life insurance liabilities (pension fund), market indexes or a reference portfolios (traditional asset managers), or just money market instruments (hedge funds or absolute return products). Note that B could be a degenerate random variable (i.e. risk-free return).

Let $Y := X - B$ be the total *excess* return, a reward-risk performance ratio is defined as a ratio between a reward measure $r(Y)$ and a risk measure $\rho(Y)$. In order the paper is self-contained, we list in Table 1 the ratios used throughout, where $F_Y(y) := \mathbf{P}(Y \leqslant y)$ is the distribution function of the total excess return; $\mathrm{VaR}(Y;\alpha) := -\inf\{y : \mathbf{P}(Y \leqslant y) > \alpha\}$ is the Value-at-Risk of Y at the level $\alpha \in (0,1)$; the positive and negative part of Y are defined as $Y_+ := \max\{Y, 0\}$ and $Y_- := \max\{-Y, 0\}$, respectively; the parameter α in the coefficient

$$A(\alpha; p) := \frac{\sqrt{\pi}\Gamma(1 - p/2)}{2^p\Gamma((1 + p)/2)\Gamma(1 - p/\alpha)}$$

is the index of stability introduced in the context of stable distributions (e.g., Mittnik and Rachev, 2000). Note that the Generalized Rachev ratio in Table 1 has been modified with respect to the original definition (e.g., Biglova et al., 2004 and Menn et al., 2005 p. 210). In fact, to make the ratio homogeneous of order 1, we take the root of order γ and δ of the conditional expectation of reward and risk, respectively.

As known, Sharpe ratio quantifies reward and risk through *two-sided type* measures, consequently positive and negative deviations from the benchmark are weighted in the same manner. Although that may be correct in some circumstances, such as if we are aiming at capturing the "stability" around a "central tendency", that could be misleading if we are interested in keeping under control the overperformance $Y > 0$ and/or the underperformance $Y < 0$. This drawback could get worse if we deal with skewed and fat tailed returns. In fact evidence shows that investors do not share a unilateral risk aversion in rewarding and losing (e.g., empirical investigation stemming from the classic paper of Kahneman and Tversky, 1979; on the Prospect Theory). A way of separately measuring reward and loss is to use of *one-sided type* parameter-dependent measures; that is the case of Farinelli–Tibiletti and Rachev ratios. As suggested in Fishburn (1977), parameters p, q likewise γ, δ reflect different agent's attitude toward overperformance and under-performance, i.e., the higher the order p and q, γ and δ the higher the agent's liking towards (in the case of expected gains) or dislike (in the case of expected losses) of the extrema events. A first empirical evidence of the impact of the aforementioned parameters on

[3] The set Ω represents all a priori possible "futures" or "scenarios", the sigma-algebra $\mathscr{F}$ all *observable* subsets of "futures" or events and the sigma-additive normed measure $\mathbf{P}$ on $\mathscr{F}$ is the so called "physical" or "statistical" probability measure.

Table 1
Performance ratios, reward and risk measures

Performance	Reward: $r(Y)$	Risk: $\rho(Y)$	Parameters
Sharpe	$\mathbf{E}[Y]$	$\mathbf{E}[(Y-\mathbf{E}[Y])^2]^{\frac{1}{2}}$	–
MiniMax	$\mathbf{E}[Y]$	$\inf_{\omega\in\Omega} Y(\omega)$	–
Stable[a]	$\mathbf{E}[Y]$	$A(p;\alpha)^{\frac{1}{p}}\mathbf{E}[\lvert Y\rvert^p]^{\frac{1}{p}}$	$\alpha\in(0,2)$, $p\in(0,\alpha)$
MAD	$\mathbf{E}[Y]$	$\mathbf{E}[\lvert Y-\mathbf{E}[Y]\rvert]$	–
Gini	$\mathbf{E}[Y]$	$\mathbf{E}[F_Y\circ Y]$	–
CVaR	$\mathbf{E}[Y]$	$\mathbf{E}[-Y\lvert Y\leqslant -\mathrm{VaR}(Y;\alpha)]$	$\alpha\in(0,1)$
Sortino–Satchell	$\mathbf{E}[Y]$	$\mathbf{E}[Y_-^q]^{1/q}$	$q\in(0,\infty)$
VaR	$\mathbf{E}[Y]$	$\mathrm{VaR}(Y-\mathbf{E}[Y];\alpha)$	$\alpha\in(0,1)$
Farinelli–Tibiletti	$\mathbf{E}[Y_+^p]^{1/p}$	$\mathbf{E}[Y_-^q]^{1/q}$	$p,q\in(0,\infty)$
Rachev	$\mathbf{E}[Y\lvert Y\geqslant -\mathrm{VaR}(Y;1-\alpha)]$	$\mathbf{E}[-Y\lvert Y\leqslant -\mathrm{VaR}(Y;\beta)]$	$\alpha,\beta\in(0,1)$
Generalized Rachev	$\mathbf{E}[Y_+^\gamma\lvert Y\geqslant -\mathrm{VaR}(Y;1-\alpha)]^{\frac{1}{\gamma}}$	$\mathbf{E}[Y_-^\delta\lvert Y\leqslant -\mathrm{VaR}(Y;\beta)]^{\frac{1}{\delta}}$	$\alpha,\beta\in(0,1)$, $\gamma,\delta\in(0,\infty)$

risk allocation is given in Farinelli et al. (2007) where simulations prove that as gradually moving from conservative to aggressive risk profile, optimal allocation given through properly fitting Farinelli–Tibiletti and Rachev ratios gradually moves from a low-risk portfolio to a higher risk one.

3. Portfolio optimization

Under mild assumptions reward maximization and risk minimization problems

$$\sup\{r(Y):\rho(Y)\leqslant\rho_{\text{risk}}\}$$
$$\text{and}\quad \inf\{\rho(Y):r(Y)\geqslant r_{\text{return}}\},$$

where $\rho_{\text{risk}}, r_{\text{return}}\in\mathbb{R}^+$ are fixed non-negative level of risk and return, are equivalent. Moreover, assuming a (quasi-)-concave/convex structure of the above problems, the unique optimal tangential portfolio Y^* solves the performance maximization problem

$$\max\{\Phi(Y):Y\in\mathscr{R}\},$$

where we define $\Phi(Y):=\frac{r(Y)}{\rho(Y)}$; $\mathscr{R}$ denotes the set of all possible random excess total returns, given bounds on expected rewards and risks.

Financial institutions tackling the problem to assess a reasonable multiperiod asset allocation plan, usually split the strategic horizon $[0,T]$ into sub-periods of the same length, and try to predict the optimal sequence of portfolio weights maximizing $\Phi(\cdot)$ at the final date T. Let start introducing the N-dimensional stochastic process $X=\{\mathbf{X}(t):t\geqslant 0\}$ where each component $X_i(t)$ describes the return of asset i. Let extend the random variable representing the excess total return by considering the one-dimensional stochastic process $Y=\{Y(t):t\geqslant 0\}$, where $Y(t)=\mathbf{w}(t)\cdot\mathbf{X}(t)-B(t)$ for each t; the N-dimensional vector $\mathbf{w}(t)$ contains the portfolio weights invested at time t; the symbol $\cdot$ denotes the inner product; $B=\{B(t):t\geqslant 0\}$ is the one-dimensional stochastic process for the benchmark. Note that the latter could be a (locally) risk-free return. Formally, we formulate the following stochastic program:

$$\max\quad \Phi(Y(T)) \tag{P1}$$

subject to

$$[1+Y(t)]W(t-1)=W(t)\quad t=1,\dots,T, \tag{3.1}$$
$$\mathbf{w}(t)\cdot\mathbf{e}=1\quad t=1,\dots,T, \tag{3.2}$$
$$\mathbf{w}_{\min}\leqslant\mathbf{A}\cdot\mathbf{w}(t)\leqslant\mathbf{w}_{\max} \tag{3.3}$$

where $\mathbf{e}=(1,\dots,1)$ is the N-dimensional unit vector and $W(t)$ is the total wealth at time t; at the beginning of the planning horizon the investor holds a strictly positive wealth $W(0)$. Notice that the objective in problem (P1) can be written as $\Phi\left(\frac{W(T)}{W(0)\cdot(1+Y(1))\cdots(1+Y(T-1))}-1\right)$, where the stochastic excess return is expressed in term of the final wealth. Constraint (3.1) describes the capital accumulation, *i.e.*, the decision to invest the wealth's fraction $w_i(t)$ in asset i in period t is made after the random asset return $X_i(t)$ and the benchmark $B(t)$ are realized; constraint (3.2) stands for no-leverage. The $N\times N$ matrix $\mathbf{A}$ and the two vectors $\mathbf{w}_{\min}\geqslant 0, \mathbf{w}_{\max}>0$ (which components sum up to 100%) contain deterministic coefficients, hence constraint (3.3) makes possible to fix minimum and maximum stock exposures also for different asset classes. The ultimate decision variables are portfolio weights $w_i(t)$, since in each period the time-t endowment is the sum of market values $w_i(t)W(t)$ invested in each asset. The stochastic optimization problem (P1) is multistage with recourse, but we do not impose neither any given probability distribution on Y nor the finite cardinality of the sample space Ω. In fact, we are not interested in assuming any time series model for the stochastic vector X to simulate the scenario tree in the deterministic equivalent of (P1). Rather, we argue that the performance ratio's maximum in period T should be achieved by solving the sequence $\{\max\Phi(Y(t))\}_{t=1}^T$ of stochastic optimization problems where the ratio is maximized conditional on the realization of $X_i(t)$ and $B(t)$, by using constraints (3.1)–(3.3) for each t. This stochastic approximation reflects the hypothesis of investing (in each unit period $[t-1,t)$) all wealth in the portfolio that maximizes a given performance measure, although this represents a "myopic" strategy (see Kushner and Yin, 2003;

for issues in the basic stochastic approximation algorithms developed in applications and their related error measurements).

To estimate $\Phi(\cdot)$ we suggest to bootstrap the empirical distribution of portfolio excess return provided that historical data of daily returns are accessible. More specifically, we assume a monthly unit period $[t-1,t)$ including D trading days.[4] Consider a data set of $M_1 \times (N+1)$ historical quotes (prices) for each asset and the benchmark, with $M_1 > D \times T$, to be used for computing asset returns. For period $[0,1)$ we retain only the first $M_1 - (D \times T)$ days to form the first in-sample from which randomly extracting d_1 times with replacement, a block of d_2 vectors containing the N asset returns plus the benchmark return, each vector being $(X_1^{(k)},\dots,X_N^{(k)},B^{(k)})$, for $k=1,\dots,D$. Notice that $d_1 \times d_2 \approx D$. This procedure is called *moving block bootstrap* which feature is to partly account for autocorrelations in asset return series as well as for cross correlations (e.g., Efron and Tibshirani, 1993; Davison and Hinkley, 1997). We use the bootstrap sample as follow: let $P_i(0), P_{N+1}(0)$ be the initial observed price of asset i and the benchmark, respectively, the recursive scheme

$$P_i(k) = P_i(k-1)(1+X_i^{(k)}), \quad k=1,\dots,D,\ i=1,\dots,N$$
$$P_{N+1}(k) = P_{N+1}(k-1)(1+B^{(k)}) \quad k=1,\dots,D$$

enable us to generate final prices in $t=1$, then arithmetic returns $\widehat{X}_i(s) := \frac{P_i(D)-P_i(0)}{P_i(0)}$ for every asset considered and $\widehat{B}(s) := \frac{P_{N+1}(D)-P_{N+1}(0)}{P_{N+1}(0)}$ for the benchmark. We run the sampling procedure for a total of $S = 10.000$ iterations to get the empirical distributions of portfolio excess return

$$\widehat{F}(x) = \frac{\sum_s \mathbf{I}_{\{Y(s)\leqslant x\}}}{S},$$

where $Y(s)$ is a vector where the s-entry is given by

$$w_1(1)\widehat{X}_1(s) + \cdots + w_N(1)\widehat{X}_N(s) - \widehat{B}(s),$$

for $s=1,\dots,10.000$; $\mathbf{I}_{\{\cdot\}}$ denotes the indicator function. Clearly weights $w_i(1)$ are unknown although we might use $\widehat{F}(x)$ in objective of maximization setup during the period $[0,1)$ for each ratio in Table 1. Finally, we repeat the entire bootstrapping scheme for each month contained in the horizon $[0,T]$ and each stochastic program aforementioned. Note that, we use a rolling window of fixed length $M_1 - (D \times T)$ when advancing through unit periods, so we subtract the first D days of data and add the more recent D ones.

The optimization problem (P1) is not easy to solve, despite it has a polyhedral feasible set, since the objective function should be non-concave. However, when $\Phi(\cdot)$ is the Sharpe ratio one can shows a convex quadratic program equivalent to (P1) exists assuming no-leverage (see constraint (3.1)) and a strictly positive differential $\mathbf{w}(t)\cdot\mathbf{E}[\mathbf{X}(t)] - r_f$ between portfolio return and a benchmark given by the risk-free rate (e.g., Cornuejols and Tütüncü, 2007). Moreover, the Generalized Rachev ratio is a descendant of the CVaR for which Rockafellar and Uryasev (2002) introduced a linear programming formulation that could be extended to solve (P1), speeding the computational time.

4. Empirical results and robustness test on the best ratio

Decision aid system is based on following Algorithms:

Algorithm (1)

```
start
  for t = 1 : T
    with each Φ(·) do
      estimation of F̂(x)
      assignment of parameters' values
      and portfolio constraints
      calculation of Φ(·)
      solution w*(t) of (P1)
    next t
stop
```

Algorithm (2)

```
start
  for t = 1 : T
    load solution vector w*(t)
      W(0) ← value
      W(t) = W(t−1)[1 + X(t) · w*(t) − B(t)]
  next t
  return allocation rule: 'rank the ratios
  according to final wealth and choose
  portfolio weights optimizing the best
  ratio
stop
```

Algorithm (1) is implemented numerically to solve a version of the optimization problem (P1) given by the maximum sequence cited in Section 3. Algorithm (2) provides a numerical backtest of the forecasted weights by using historical data to reconstruct optimal wealth's path within the horizon $[0,T]$. Notice that to estimate the empirical distribution of portfolio excess return $\widehat{F}(x)$ at the end of each month $t=1,\dots,T$, the original dataset with M_1 daily returns is used, but to simulate the optimal wealth's path through the horizon a dataset with $[M_1/D]$ monthly returns is used (symbol $[\cdot]$ denotes the integer part of a number) together with a rolling window based on an in-sample containing $[M_1/D]-T$ monthly returns.

We expect the forecasting ability of all ratios being affected by empirical pattern of asset returns and portfolio constraints. Clearly, also the parameters' choice influences portfolio composition, then final ranking, but in this paper we confine our attention to questioning: *How many times* a performance ratio has been best at the end of the planning

[4] Usually $D \approx 20$ trading days.

horizon? The answer is crucial in evaluating the ratio goodness in driving satisfactorily asset allocation. So, we end our analysis with a *robustness* test on the best outperformance. We suggest to use a different rolling window which in-sample contains $M_2 < [M_1/D] - T$ (with $M_2 > T$) monthly returns for computations pertaining Algorithm (2), and that is repeated for Algorithms (1) and (2) $M_3 = [\{[M_1/D] - M_2\}/T]$ times; a rolling window related to an in-sample of $M_1 - (D \times T \times M_3)$ days is employed to estimate $\widehat{F}(x)$. That enable us to count how many times a ratio was the best and state a ranking based on these frequencies. The following *robustness checking algorithm* is used:

Algorithm (3)

```
start
  for t = 1 : M_3
    with a sample of M_2 data points do
      Algorithms (1) and (2)
  next t
  return allocation rule: 'rank the ratios
  according to final wealth and choose
  portfolio weights optimizing the ratio
  that has performed the best more times
stop
```

4.1. Stock selection

Let now tackle the optimization procedure within real asset data on the market. Let consider the following $N = 5$ total return indexes: ($i = 1$) S&P500; ($i = 2$) Dow-Jones; ($i = 3$) NASDAQ; ($i = 4$) FTSE; ($i = 4$) NIKKEI. We focus on the outperformance with respect to the T-bill rate, taken as a benchmark since it is a proxy of the risk-free rate. Historical data of total returns and T-bill rates cover from April 2, 1984, to October 3, 2005. We have $M_1 = 5160$ daily quotes for each stock but the number D of days is not fixed for every month, so concerning the bootstrapping of empirical distribution $\widehat{F}(x)$ in Algorithm (1) we quote the rolling window's length to be 4979 days, while in Algorithm (2) the backtesting of optimal portfolio wealth is based on 249 months. We look at the final wealth realized over the 9-month period between January 1, 2005 ($t = 0$) and October 3, 2005 ($T = 9$). We know that empirical features of asset return distributions affect the optimization of performance ratios and eventually some of these might provide better results in term of final wealth, hence we check fat-tails and skewness of asset returns. We employ the Jarque–Bera (JB) normality test with each univariate time series, where the null hypothesis is the normal distribution of the data. Results are shown in Table 2. Considering arithmetic stock returns we must reject the null hypothesis for all time series, both at 95% and 99% a confidence level, with the exception of NIKKEI series. In parentheses we indicate BJ statistic's values for log-returns, when the null hypothesis is rejected for all the components of the multivariate time series at the same confidence levels. The test shows evidence of non-normality in data, moreover, skewness and kurtosis coefficients in Table 4 confirm a clear asymmetry in distributions. All five series are negatively skewed and fat-tailed, although asset $i = 5$ has a small excess kurtosis. We also test the multivariate normality of the data using the Mulnor test.[5] Excluding the risk-free rate, we obtained the regression coefficient 0.6235 and a t-ratio 33.0352. Hence, the null hypothesis of jointly Gaussian data is rejected with both 95% and 99% confidence levels. Vice versa, if the risk-free rate is included, the regression coefficient 0.6771 and a t-ratio 26.1305 turned out. That confirms the rejection of the null hypothesis with both confidence levels.

After Algorithms (1) and (2) runnings, we must assign values to parameters showed in Table 1 (see Section 2). We employ the same parameter setting as in Biglova et al. (2004) for Rachev ratio, Generalized Rachev (GR) ratio, and Stable ratio. More specifically, we consider three types of Rachev ratio: $\alpha = \beta = 0.01$; $\alpha = \beta = 0.05$; $\alpha = 0.01$ and $\beta = 0.5$ We always report only the best performed Rachev ratio out of the three. Moreover, the GR-ratio has $\alpha = \beta = 0.5$ and $\gamma = \delta = \alpha/2$.[6] For the Farinelli–Tibiletti (FT) ratio we use two parameter settings different from those suggested by Biglova et al. (2004): $p = 2, q = 0.5$ and $p = 0.5, q = 2$. This could twist the investor's feeling toward overperformance and underperformance according to the definition of FT-ratio. Also, we let $q = 0.5$ for the Sortino–Satchell (SS) ratio.

Finally, since in real asset allocation, investments are bound to respect a set of constrains given by authorities, two options have been considered. Let $\mathbf{A} = \mathrm{diag}(1, \ldots, 1)$, the case of "weak" and "strong" lower and upper bounds:

(i) $\mathbf{w}_{\min} = (0, 0, 0, 0, 0), \mathbf{w}_{\max} = (0.5, 0.5, 0.5, 0.5, 0.5)$;
(ii) $\mathbf{w}_{\min} = (0.1, 0.02, 0.02, 0.1, 0.02), \mathbf{w}_{\max} = (0.5, 0.1, 0.1, 0.5, 0.1)$.

Above values may seem to be an arbitrary assumption, but that is quite common in *practical* asset allocation: if investment management provides information on the realized stock prices under investigation, investors may revise their preferences inquiring alternative portfolio restrictions according to the empirical features of past returns. In order words, the investor prefers those assets with the greater

[5] This test is based on a linear regression of the quantiles' vector of the sample χ^2 distributions against the ordered Mahalanobis distances, related to the data matrix. If the data are jointly Gaussian, then the regression coefficient is expected to be 1, and to test this hypothesis we employ the t-ratio between the estimated slope and the estimated residual sum of squares properly scaled by the degrees of freedom, the latter being equal to the difference between the number of variables and the row number in the estimated vector of the regression's coefficients.

[6] The authors have coded Algorithms (1),(2) and (3) in MATLAB©. The α index of return stability in SR is estimated by using the Stable software by John P. Nolan (1997).

Table 2
Jarque–Bera test for the asset universe

JB $\chi^2(2)$ statistic (1% critical value 9.2103; 5% critical value 5.9915)					
S&P500	DowJones	NASDAQ	FTSE	NIKKEI	T-bill
95.5469	173.3531	143.5600	39.8834	3.4745	191.0930
(214.0823)	(460.5304)	(333.2255)	(105.2271)	(9.4301)	(415.8024)

Table 3
Summary statistics for return series

	Coef. of skewness	Coef. of kurtosis	Mean	Standard deviation
S&P500	−0.7432	5.6328	0.0087	0.0440
Dow Jones	−0.8598	6.6854	0.0070	0.0467
NASDAQ	−0.8209	6.3185	0.0095	0.0448
FTSE	−0.5434	4.6303	0.0105	0.0709
NIKKEI	−0.1234	3.5402	0.0023	0.0604

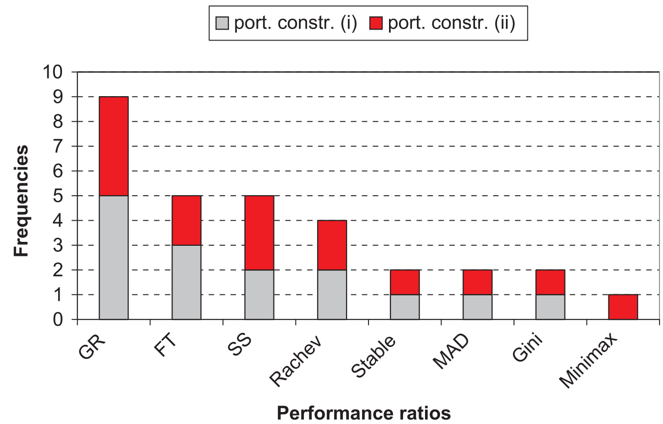

Fig. 1. Frequencies in being the best ratio in 15 iterations of Algorithms (1) and (2).

mean return and a moderate historical volatility, but a limited skewness and kurtosis. From Table 3, FTSE total return index shows the highest historical return with respect to the remaining four indexes, but a greater volatility. However, it displays small skewness and kurtosis. On the other hand, the S&P500 index has the lowest historical volatility, a relevant mean return and a small skewness and kurtosis. Thus, an acceptable trade-off between risk and return seems to be offered by these last indexes. Consequently, S&P500 and FTSE will weight more in the prospective portfolio. Assume the initial unit wealth $W(0) = 1$ is invested in the market portfolio $\mathbf{w}(0)$ at date $t = 0$. Optimal portfolio weights for each month considered, provided by Algorithm (1), are used in Algorithm (2) to compute the final value of wealth, *i.e.* the cumulative portfolio return, by rolling-forward monthly the investment. Results coming from backtesting of all performance ratios through final wealth are listed in Table 4 according to portfolio constraints (i) and (ii). Above simulations seem to confirm the high forecasting ability of asymmetrical performance ratios, in contrast to the low one of Sharpe ratio. Moreover, comparing ranks in Table 4 one should not be agree (ex-post) with the investor's revision of portfolio constraints based on *bad* investment management advises. Results on robustness test are shown in Fig. 1 summing up the cases of restrictions (i) and (ii), with $M_2 = 100$ and $M_3 = 15$. Robustness test results are in Fig. 1, Sharpe ratio is never turned up the best ratio, whereas the all asymmetrical performance ratios, i.e. the Generalized Rachev, Farinelli–Tibiletti and Sortino–Satchell ratios are the leaders. 1. So it should not be advisable to confine the management decisions to a single performance ratio, but it would be better to use more than one.

Table 4
Ranking of performance ratio according to the values of final wealth

Constraints (i)			Constraints (ii)		
Rank	Ratio	Final wealth	Rank	Ratio	Final wealth
(1)	GR	1.0831	(1)	GR	1.0202
(2)	SS	1.0807	(2)	FT	1.0176
(3)	Gini	1.0794	(3)	MAD	1.0144
(4)	FT	1.0712	(4)	Rachev	1.0113
(5)	Rachev	1.0637	(5)	CVaR	1.0107
(6)	Minimax	1.0661	(6)	Minimax	1.0085
(7)	MAD	1.0277	(7)	SS	1.0051
(8)	VaR	1.0148	(8)	VaR	1.0045
(9)	Stable	1.0085	(9)	Stable	1.0040
(10)	CVaR	1.0068	(10)	Gini	1.0033
(11)	Sharpe	1.0039	(11)	Sharpe	1.0028

5. Conclusion and further research

An integrated decision aid system for making portfolio optimization able to select the best fitting ratio among eleven different performance ratios at disposal, is set up. As historical returns are non-gaussian, asymmetrical parameter-dependent ratios show satisfactorily flexibility in taking advantage of the favorable skewness and kurtosis in investment opportunities. A robustness test on being the best ratio shows the Sharpe ratio weakness, in fact it has never outperformed the best, whereas robust ability in outperforming of asymmetrical ratios, i.e., Sortino–Satchell, Generalized Rachev and Farinelli–Tibiletti ratios, is empirically proved. Nevertheless, further tests are to be stressed in order to investigate on the influence of asset class constrains and choice of personalized parameters in final results (e.g., Farinelli et al., 2007, for a first attempt of making investor tailored-made asset allocation by means of proper parameters balancing).

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