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Multi-objective genetic algorithms for solving portfolio optimization problems in the electricity market

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ABSTRACT

The multi-objective portfolio optimization problem is not easy to solve because of (i) challenges from the complexity that arises due to conflicting objectives, (ii) high occurrence of non-dominance of solutions based on the dominance relation, and (iii) optimization solutions that often result in under-diversification. This paper experiments the use of multi-objective genetic algorithms (MOGAs), namely, the non-dominated sorting genetic algorithm II (NSGA-II), strength Pareto evolutionary algorithm II (SPEA-II) and newly proposed compressed objective genetic algorithm II (COGA-II) for solving the portfolio optimization problem for a power generation company (GenCo) faced with different trading choices. To avoid under-diversification, an additional objective to enhance the diversification benefit is proposed alongside with the three original objectives of the mean–variance–skewness (MVS) portfolio framework. The results show that MOGAs have made possible the inclusion of the fourth objective within the optimization framework that produces Pareto fronts that also cover those based on the traditional MVS framework, thereby offering better trade-off solutions while promoting investment diversification benefits for power generation companies.

1. Introduction

The Markowitz mean–variance (MV) approach [1] is widely regarded as a ground theory in portfolio selection. This framework assumes that investors make an investment decision in asset allocation in order to maximize their utility by maximizing portfolio return and minimizing portfolio risk subject to a given budget constraint. However, assumptions underlying the MV model such as the quadratic utility function and the normal distribution of returns are often violated, both theoretically [2–4] and empirically [5–7].

In addition, the skewness preference theory and its relevance for applications are widely documented [8–11]. The introduction of skewness in portfolio decision-making brings about a new research direction in portfolio selection. In the mean–variance–skewness (MVS) model [12–14], the mean and skewness of portfolio returns are to be maximized and portfolio risk is to be minimized simultaneously. From the viewpoint of optimization, a solution that simultaneously optimizes all objectives does not exist. Nevertheless a set of compromising solutions can be explored. Besides, the MVS portfolio optimization problem is not easy to solve because the objectives compete and conflict with each other. As a result, the optimal Pareto fronts seem to be non-smooth and discontinuous.

In the literature, both the MV [15,16] and MVS frameworks [17] had been used to set up portfolio optimization problems for electricity generation companies. However, given the fact that electricity spot prices are not normally distributed but skewed, asset allocation based on the MVS framework is more suitable than the MV framework for a generation company (GenCo). In spite of the framework, the shape of the Pareto front presented in previous studies [17], for example, does not reflect the nature of a problem that has competing and conflicting objectives.[1] Further, as observed in previous work [20], the number of assets included in most of portfolio optimization solutions was limited, and had greatly reduced the diversification benefit. In view of the weakness, this paper proposes to include a diversification enhancing objective into the MVS

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[1] The Particle Swamp Optimization (PSO) technique was implemented in [17]. In some problems, GAs can provide better optimal solutions. For instance, the optimal solutions of some well-developed GAs were found to perform better than those obtained through PSO [18,19]. It must be noted that, however, no particular class of multi-objective optimization algorithm consistently outperforms the others in all the application problems.

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portfolio model. Therefore, a four-objective portfolio optimization problem (MVS-D) is formulated for a GenCo that produces and trades electricity in a deregulated electricity market. This inclusion adds to the complexity of the optimization problem by increasing the number of objectives from three to four.

In optimization problems, an increase in the number of conflicting objectives significantly raises the difficulty in the use of an algorithm to find the optimal solution [21]. In conventional multi-objective optimization algorithms (MOOAs), when two candidate solutions are compared, solution **a** does not dominate solution **b** unless all objectives from **a** satisfy the domination condition. With a large number of objectives, the chance that no one solution can dominate the other is expectably high. Therefore, in order for algorithms to provide a good approximation of the true Pareto front, a large number of non-dominated solutions have to be screened using suitable techniques [22,23].

During the past decade, genetic algorithms (GAs) had been successfully applied for solving multi-objective portfolio optimization problems (MOPOPs) in finance subject to different constraints [24,25]. Their applications are also common in multi-objective optimization problems in the power systems [26–29] and other resource allocation problems [30,31]. However, the ability of MOGAs for solving MOPOPs with more than three objectives to be optimized has been rarely investigated. Therefore, the first objective of this paper is to explore if GAs can efficiently and reliably solve MOPOPs with a high number of objectives. The second objective is to conduct a cross-algorithm performance comparison. To achieve these objectives, two well established GAs, namely, non-dominated sorting genetic algorithm II (NSGA-II) [32] and strength Pareto evolutionary algorithm II (SPEA-II) [33], and the newly developed compressed objective genetic algorithm II (COGA-II) [34] were utilized and compared in this study.

The paper is organized as follows. The proposed MOPOP is discussed in Section 2. Section 3 explains the portfolio selection problem in electricity market. A description of the three MOGAs together with the performance comparison criteria are given in Section 4. Section 5 exhibits the numerical experiments and parameter setting. The results and discussions are presented in Section 6, while Section 7 states our conclusions.

2. Multi-objective portfolio optimization model

The notion that investors prefer positive skewness in returns is well documented [8–13]. Therefore, in the mean–variance–skewness (MVS) portfolio optimization problem, expected returns and skewness of portfolio will be maximized, meanwhile, variance of portfolio will be minimized at the same time.

We consider a single decision-making period where N assets are available for investment. At the beginning of the decision making process, investors determine the ratio of their initial wealth to be invested in each available asset. The expected portfolio return, denoted by $R(\mathbf{x}) = \sum_{i=1}^{N} x_i R_i$, where $\mathbf{x} = (x_1, x_2, \ldots, x_N)$ is a solution vector, x_i is the ratio of wealth invested in asset i and R_i is the return to asset i which is a random variable that will be realized at the end of the investment period. Portfolio risk is measured by portfolio variance, $V(\mathbf{x})$, while portfolio skewness is denoted by $S(\mathbf{x})$. The MVS portfolio optimization problem can be stated mathematically as follows:

$$\text{Maximize} \quad R(\mathbf{x}) = \sum_{i=1}^{N} x_i R_i \tag{1}$$

$$\text{Minimize} \quad V(\mathbf{x}) = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij} = \sum_{i=1}^{N} x_i^2 \sigma_i^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij} \tag{2}$$

$$\text{Maximize} \quad S(\mathbf{x}) = \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} x_i x_j x_k \gamma_{ijk} = \sum_{i=1}^{N} x_i^3 \gamma_i^3 + 3\sum_{i=1}^{N}\left(\sum_{j=1}^{N} x_i^2 x_j \gamma_{iij} + \sum_{j=1}^{N} x_i x_j^2 \gamma_{iij}\right) \tag{3}$$

$$\text{Subject to} \quad \sum_{i=1}^{N} x_i = 1, \quad x_i \geqslant 0 \tag{4}$$

where σ_{ij} is covariance between asset i and j, and γ_{ijk} represents co-skewness between asset i, j and k. The constraint defining the feasible portfolios implies that all capital must be invested in available assets and short sales are not allowed.

The MVS model is a tri-objective optimization problem where objectives are competing and conflicting trade-off exists in different portfolio choices. Procedures and algorithms were introduced in a number of studies for portfolio construction under the MVS framework [12,14,35,36]. These works combined the three objectives to formulate a single objective optimization problem. However, disadvantages of using single objective approach to solve multi-objective optimization problems are widely documented. Since there are three objectives to be optimized, we do not try to find a single optimal solution, but instead a set of optimal solutions, the so called "Pareto-optimal solutions" is searched. According to the Pareto dominance relation, a portfolio $\mathbf{x}^* = (x_1^*, x_2^*, \ldots, x_N^*)$ is said to be an optimal portfolio if there is no other feasible portfolio $\mathbf{x} = (x_1, x_2, \ldots, x_N)$ such that $R(\mathbf{x}^*) \leqslant R(\mathbf{x})$, $V(\mathbf{x}^*) \geqslant V(\mathbf{x})$ and $S(\mathbf{x}^*) \leqslant S(\mathbf{x})$ with at least one strict inequality.

In addition, as observed by prior study [20], most of the portfolio optimization solutions comprised only a limited number of available assets resulting in the reduction of diversification benefit. In order to avoid excessive investment in a small number of assets, some constraints have been proposed, such as cardinality constraint, ceiling limit constraint and class constraint. However, asset allocation ratios tend to be subjectively determined and it is difficult to identify the ratios without knowing the levels of other objectives [37]. Furthermore, in many cases, investor may not arrive at good solutions if constraint values are forced to be identified beforehand. We handle this problem by minimizing the difference between the highest and the lowest ratio of capital investment in $\mathbf{x}$ as an additional objective in order to enhance diversification benefit. Let this difference be denoted as $D(\mathbf{x})$. Our fourth objective can be stated as follows:

$$\text{Minimize} \quad D(\mathbf{x}) = \max\ \mathbf{x} - \min\ \mathbf{x} \tag{5}$$

In this paper, we formulated the multi-objective portfolio optimization problem denoted by MVS-D as follows:

$$\begin{aligned} &\text{Minimize} \quad F(x) = [-R(\mathbf{x}), V(\mathbf{x}), -S(\mathbf{x}), D(\mathbf{x})] \\ &\text{Subject to} \quad \sum_{i=1}^{N} x_i = 1, \quad x_i \geqslant 0 \end{aligned} \tag{6}$$

3. Portfolio optimization in electricity market

3.1. Trading environment in an electricity market

Similar to financial market, the deregulated electricity market facilitates price efficiency and liquidity by offering various contractual instruments for GenCos to trade their generated products in the different types of electricity markets. This paper considers the situation that a GenCo is making decision to create optimal electricity allocation according to the MVS portfolio model when faced with production capacity constraint. Similar to financial investment, a single investment period is assumed which can be a day, a week, a month and so forth. At the beginning of the decision making period, the GenCo of interest determines the ratio of its initial production capacity to be allocated to available trading choices. Therefore, we consider only the physical delivery-based choices, namely, trading in spot market and bilateral forward contracts with customers located in different zones.

For trading in the spot market or the so called "real-time market", market participants can enter to sell their offers or to purchase other participants' bids on real-time basis. A market operator does matching of the bids and offers to determine the market clearing price.

In the case of a bilateral forward contract, the buyer and seller agree in advance on a pre-specified price ($/MW h) for a fixed amount of electricity (MW h) that will be transmitted from the seller to the buyer for a certain period of time. In general, the Locational Marginal Pricing scheme is applied to a physical electricity market. In this pricing scheme, the located zone of buyer and seller may have an influence on the revenue. In the case that electric transmission from a seller located in zone A to a buyer located in zone B confronts with line congestion, thermal limits or voltage constraints, the congestion charge will be applied to both the buyer and seller. The congestion charge is the product of the price difference between zones of the buyer and seller.

The risk characteristic depends on the type of trading. For example, the fluctuation in spot price is a risk associated with trading in the spot market, and the congestion charge is a source of risk from a bilateral forward contract engaging a customer from a different zone. Therefore, the returns and risks vary according to different trading choices.

3.2. Portfolio return, variance and skewness for electricity allocation

Unlike returns from financial assets, the return to a GenCo's trading is calculated from its profit (revenue minus production cost) divide by production cost. Revenue is the product of selling price ($/MW h) and selling quantity (MW h). We conventionally assumed that the production cost of the Genco of interest is a quadratic function of energy output, trading time and energy price. We further assumed that this GenCo trades its electricity in the PJM market where there are N zones. The GenCo is located in zone 1, while, other customers are located in zone 2 to zone N. Since we consider bilateral forward contracts with customers located in other zones and trading in the spot market, there are N-1 bilateral forward contracts and one trading choice in spot market to be considered. Since the objective of this paper is to propose an optimal electricity allocation solution based on the MVS portfolio model, the skewness and co-skewness for the optimization problem is derived from the model of previous studies [15,16].

3.2.1. Portfolio return

We assume that the GenCo has a cost function of $C(P_G,h,\lambda^C)=\left(a+bP_G+cP_G^2\right)h\lambda^C$, where P_G is the output power (MW h), h is the trading time of each trading interval (hour), λ^C is the coal price ($/MBtu), and a, b and c are fuel consumption coefficients. There are T trading intervals in a period that the GenCo has to decide in advance.

The expected return for trading in the spot market is given by:

$$E(R_s)=\frac{\sum_{t=1}^{T}P_Gh\left(E\left(\lambda_{1,t}^S\right)\right)-\sum_{t=1}^{T}\left(a+bP_G+cP_G^2\right)h\lambda_t^C}{\sum_{t=1}^{T}\left(a+bP_G+cP_G^2\right)h\lambda_t^C} \tag{7}$$

$$E(R_s)=K\sum_{t=1}^{T}P_Gh\left(E\left(\lambda_{1,t}^S\right)\right)-1$$

where $K=\frac{1}{\sum_{t=1}^{T}\left(a+bP_G+cP_G^2\right)h\lambda_t^c}$ and $E\left(\lambda_{1,t}^S\right)$ is the expected spot price of zone 1.

The return for a bilateral forward contract signed with a customer in zone i is:

$$E(R_i)=K\sum_{t=1}^{T}P_Gh\left[\lambda_{i,t}^B-\left(E\left(\lambda_{i,t}^S\right)-E\left(\lambda_{1,t}^S\right)\right)\right]-1;\quad i=2,\ldots,N \tag{8}$$

where $\lambda_{i,t}^B$ is the local bilateral contract price ($/MW h) for trading interval t, $E\left(\lambda_{i,t}^S\right)$ is the expected spot price of zone i and $\left(E\left(\lambda_{i,t}^S\right)-E\left(\lambda_{1,t}^S\right)\right)$ represents the expected congestion charge.

3.2.2. Portfolio variance and covariance

The variance of return for trading in the spot market of zone 1 is as follows:

$$V_S=K^2\sum_{t=1}^{T}(P_Gh)^2\left[V\left(\lambda_{1,t}^S\right)\right] \tag{9}$$

The variance of return for trading in a bilateral contract signed with a customer in zone i is given by:

$$V_i=K^2\sum_{t=1}^{T}(P_Gh)^2\left[V\left(\lambda_{1,t}^S\right)+V\left(\lambda_{i,t}^S\right)-2\sigma\left(\lambda_{1,t}^S,\lambda_{i,t}^S\right)\right];\quad i=2,\ldots,N \tag{10}$$

The covariance between the return in the spot market and the return for trading in a bilateral contract signed with a customer in zone i is given by:

$$\sigma_{1,i}=K^2\sum_{t=1}^{T}(P_Gh)^2\left[V\left(\lambda_{1,t}^S\right)-\sigma\left(\lambda_{1,t}^S,\lambda_{i,t}^S\right)\right];\quad i=2,\ldots,N \tag{11}$$

The covariance between the return of a bilateral contract signed with a customer in zone i and that in zone j is given by:

$$\sigma_{i,j}=K^2\sum_{t=1}^{T}(P_Gh)^2\left[V\left(\lambda_{1,t}^S\right)-\sigma\left(\lambda_{1,t}^S,\lambda_{i,t}^S\right)-\sigma\left(\lambda_{1,t}^S,\lambda_{j,t}^S\right)+\sigma\left(\lambda_{i,t}^S,\lambda_{j,t}^S\right)\right];\quad i,j=2,\ldots,N\ \text{and}\ i\neq j \tag{12}$$

3.2.3. Portfolio skewness and co-skewness

The skewness of return for trading in the spot market is stated as follows:

$$S_S=K^3\sum_{t=1}^{T}(P_Gh)^3\left[S\left(\lambda_{1,t}^S\right)\right] \tag{13}$$

The skewness of return for a bilateral contract signed with a customer in zone i is given by:

$$S_i=K^3\sum_{t=1}^{T}(P_Gh)^3\left[S\left(\lambda_{1,t}^S\right)+S\left(\lambda_{i,t}^S\right)-3\gamma\left(\lambda_{1,t}^S,\lambda_{1,t}^S,\lambda_{i,t}^S\right)+3\gamma\left(\lambda_{1,t}^S,\lambda_{i,t}^S,\lambda_{i,t}^S\right)\right];\quad i=2,\ldots,N \tag{14}$$

where $\gamma(\cdot)$ represents the co-skewness of spot prices of different zones. The coefficients of co-skewness of returns for different combinations of trading in the spot market and bilateral contracts signed with customers in different zones are as follows:

$$\gamma_{1,1,i}=K^3\sum_{t=1}^{T}(P_Gh)^3\left[S\left(\lambda_{1,t}^S\right)-\gamma\left(\lambda_{1,t}^S,\lambda_{1,t}^S,\lambda_{i,t}^S\right)\right];\quad i=i=2,\ldots,N \tag{15}$$

$$\gamma_{i,i,1}=K^3\sum_{t=1}^{T}(P_Gh)^3\left[S\left(\lambda_{1,t}^S\right)+\gamma\left(\lambda_{i,t}^S,\lambda_{i,t}^S,\lambda_{1,t}^S\right)-2\left(V(\lambda_{1,t}^S)+\sigma\left(\lambda_{i,t}^S,\lambda_{1,t}^S\right)\right)\right];\quad i=2,\ldots,N \tag{16}$$

$$\begin{aligned}\gamma_{i,i,j}=K^3\sum_{t=1}^{T}(P_Gh)^3\Big[&S\left(\lambda_{1,t}^S\right)-\gamma\left(\lambda_{1,t}^S,\lambda_{1,t}^S,\lambda_{j,t}^S\right)+\gamma\left(\lambda_{i,t}^S,\lambda_{i,t}^S,\lambda_{1,t}^S\right)\\&-\gamma\left(\lambda_{i,t}^S,\lambda_{i,t}^S,\lambda_{j,t}^S\right)-2\left(\gamma\left(\lambda_{1,t}^S,\lambda_{1,t}^S,\lambda_{i,t}^S\right)+\gamma\left(\lambda_{1,t}^S,\lambda_{i,t}^S,\lambda_{j,t}^S\right)\right)\Big];\\&i,j=2,\ldots,N\ \text{and}\ i\neq j\end{aligned} \tag{17}$$

$E\left(\lambda_{i,t}^S\right)$ was forecasted for next T trading intervals and used to calculate the expected return of trading in spot market (7) and expected return of bilateral forward contracts (8).The variables involved in the computation of Eqs. (9)–(17), $V\left(\lambda_{i,t}^S\right)$, $\sigma\left(\lambda_{i,t}^S,\lambda_{j,t}^S\right)$, $S\left(\lambda_{i,t}^S\right)$, $\gamma\left(\lambda_{1,t}^S,\lambda_{1,t}^S,\lambda_{i,t}^S\right)$, $\gamma\left(\lambda_{i,t}^S,\lambda_{i,t}^S,\lambda_{1,t}^S\right)$, $\gamma\left(\lambda_{i,t}^S,\lambda_{i,t}^S,\lambda_{j,t}^S\right)$ and $\gamma\left(\lambda_{1,t}^S,\lambda_{i,t}^S,\lambda_{j,t}^S\right)$ were estimated based on the historical data.

4. Genetic algorithms

Genetic algorithm (GA) is a meta-heuristic search technique in which the search mechanisms are based on the Darwinian concept of survival of the fittest. In this concept, individuals that are better in adapting to the environment will have a better chance of surviving and to reproduce, meanwhile individuals that are less fit will be eliminated. GA is proposed to solve the single-objective optimization problems [38], and it was developed subsequently to solve problems with multiple objectives.

In general, multi-objective genetic algorithms (MOGAs) deal with a set of possible solutions, the so-called population, simultaneously. As a result, many Pareto optimal solutions are found within a single run of the algorithm, as compared to the need to perform a series of separate runs in other classes of algorithms [39]. Additionally, MOGAs can efficiently deal with discontinuous and non-convex Pareto fronts, and also search partially ordered spaces for several alternative trade-offs. These appealing features attracted researchers and practitioners from different fields to use MOGAs for solving complex multi-objective optimization problems. The standard implementation of MOGAs consists of several tasks, namely, population initialization, fitness evaluation, selection, crossover and mutation.

4.1. Implementation of MOGAs

In this study, the optimization problem involves finding the ratios of allocation of energy generated by a GenCo among different trading choices. The electricity allocation ratios are encoded as a solution vector. GA evaluates the fitness of different solutions by computing objective values of each solution vector and comparing them based on the Pareto dominance relation mentioned in Section 2. Solution vectors are developed through crossover and mutation operator.

4.1.1. Population initialization

A solution of an optimization problem is usually encoded as digit string or chromosome. In this paper, the real number encoding is employed. Therefore, a solution vector $\mathbf{x} = (x_1, x_2,...,x_N)$ is directly encoded by assigning real numbers ranging from 0 to 1 to its elements. Chromosome $\mathbf{x}$ contains N bits of which each of the bits x_i represents the allocation ratio for asset i. Nevertheless, by taking the constraint of Eq. (4) into account, the summation of these x_i's may not be equal to 1 in the population initialization step or after the genetic operators are performed. To handle this constraint, x_i in the chromosomes are normalized as follows:

$$x_i' = \frac{x_i}{\sum_{i=1}^{N} x_i} \tag{18}$$

where x_i' is the new ratio allocated to asset i. It should be noted that for the population initialization, x_i in the chromosomes are randomly generated before being normalized according to Eq. (18).

4.1.2. Fitness evaluation

The objective functions are in mathematical form. Therefore, the fitness of each solution vector is directly evaluated from its objective values given in Eqs. (1)–(3) and (5).

4.1.3. GAs operators

The binary selection is performed for choosing good solutions into the mating pool. In the binary selection, two solutions are randomly picked from the population and are compared with each other. The solution with better fitness is selected for the mating pool. The selection is repeated until the mating pool is fully filled. The selected solutions, termed as "parent solutions", are used for creating the next generation solutions, the so-called "offspring solutions".

Next, crossover is performed to the parent solutions in order to reproduce the offspring solutions. In this process, the elements of an offspring's chromosome are created from mixed elements between two parent solutions to preserve their characteristics. We employed the simulated binary crossover (SBX) [40] which adapts the one-point crossover on binary strings for the real number encoded chromosomes.

Then, the Variable-Wise Polynomial mutation [40] is used to transform an offspring individual into a new individual. Mutation is used to maintain the diversity of individuals in a population to prevent premature convergence of solutions. A crossover operation creates new individuals which are of some distances away in the search space from their parent individuals. Therefore, the mutation operation can be thought as a small perturbation on the chromosomes of an individual. In other words, mutation leads to a search at a neighboring point from the original search point of the offspring as dictated by the structure of its chromosome.

4.2. Choice of MOGAs

As stated earlier, one of our objectives is to investigate and evaluate the ability of MOGAs for solving MOPOPs of a GenCo. The NSGA-II [32] and the SPEA-II [33] are chosen according to the fact that they are the most cited and well established MOGAs. Meanwhile, COGA-II [34] is proposed since it has shown outstanding performance when dealing with standard problems of more than three optimization objectives.

4.2.1. The non-dominated sorting genetic algorithm II (NSGA-II)

The fast elitist non-dominated sorting genetic algorithm (NSGA-II) introduces an improved technique for maintaining diversity of solutions by proposing a crowding distance sorting method. However, the ranking technique of its previous version remained unchanged in its procedure.

In NSGA-II, all individuals in combined populations (parents and offsprings) are ranked on the basis of Pareto dominance relation, and then classified into several layers based on which front an individual is located. In each front, individuals are arranged in a descending order of magnitude of the crowding distance value. In the binary tournament selection process, the algorithm firstly selects an individual lying on a better non-dominated front. In cases where individuals with an identical front are compared, the crowded tournament selection operator chooses the winner based on the crowing distance value. The individual with a higher crowding distance value, i.e. residing in the less crowed area, has a priority to survive.

The elitist strategy of NSGA-II considers all non-dominated solutions of the combined populations as the solution candidates for the next generation. If the number of the non-dominated solutions is less than the population size, all of them are retained as the next-generation solutions. Otherwise, the candidates for the next generation are selected through the crowding distance criterion. This criterion has the advantage of maintaining diversity of solutions in the population in order to prevent premature convergence.

4.2.2. Improved strength Pareto evolutionary algorithm (SPEA-II)

The improved strength Pareto evolutionary algorithm (SPEA-II) is a revised version of the strength Pareto evolutionary algorithm (SPEA) [41]. This new version has three major differences from the previous version. Firstly, for each individual, a fine-grained fitness assignment strategy is incorporated to take into account the number of individuals that dominate it and the number of individuals that it dominates. The second improvement is the use of a nearest neighbor density estimation technique for guiding more efficient search. Third is to ensure the preservation of boundary solutions by using an enhanced archive truncation method. Unlike the previous version, the number of individuals in an archive, i.e. the archive size, of SPEA-II is constant in every generation.

In SPEA-II, the strength value of a non-dominated individual is computed by taking into account both dominating and dominated solutions. Then the binary tournament selection is performed by choosing the stronger individual. For individuals sharing an identical strength value, the diversity value of the individual is used for discrimination. The diversity value of an individual is evaluated from the density estimation technique, which is an adaptation of the kth nearest neighbor method [42], whereby the density of a point i is inversely related to its kth nearest distance or d_i^k, where k is equal to square root of the data size, i.e. population size plus archive size. In SPEA-II, the density of an individual i is simply taken to be the inverse of d_i^k. An individual with a large density has many neighboring points. It should be noted that if this individual is selected, it contributes little diversity to the mating pool. Thus, in this algorithm, the individual with the lower nearest neighbor density value is selected into the mating pool.

4.2.3. Compressed objectives genetic algorithms II (COGA-II)

The compressed objective genetic algorithm II (COGA-II), an improvement of its previous version [43], is intentionally designed to tackle optimization problems with a large number of objectives. The COGA-II employs a rank assignment as used in NSGA-II for screening of non-dominated solutions that best approximate the Pareto front. Then, COGA-II transforms a number of optimized objectives to one preference objective, named as "winning score", during the selection process. The winning score takes into account the number of superior objectives of individual i compared to the corresponding objectives of individual j. Therefore, it helps improve the ability of an algorithm to pick out a good solution from a vast number of non-dominated solutions. The external archive technique is used in COGA-II for sustaining the extreme solutions.

4.3. Performance evaluation criteria

This paper utilized two standard algorithm performance comparison methods as follows.

4.3.1. Average distance to true Pareto-optimal front (M_1)

The average distance to true Pareto-optimal solutions (M_1) [44] can be evaluated in the solution space or objective space. In this paper, the metric M_1 is measured, in the objective space, by a distance of a solution i to the true Pareto-optimal front, di, which is the Euclidean distance of the solution i to its nearest solution j on the true Pareto-optimal front. The Euclidean distance is given by:

$$d_i = \sqrt{\sum_{k=1}^{m}\left(\frac{f_{ik} - f_{jk}}{(f_k)_{\max} - (f_k)_{\min}}\right)^2} \tag{19}$$

where f_{ik} and f_{jk} are the values of objective k for solutions i and j respectively, while $(f_k)_{\min}$ and $(f_k)_{\max}$ are the minimum and maximum values of objective k for the true Pareto-optimal solutions. Since the true Pareto-optimal front of a tested problem is not known, the artificial true Pareto-optimal front which is obtained from the merged non-dominated individuals from all runs of the three MOGAs was used instead in the evaluation of M_1. The distance, d_i, of a solution i was estimated as the Euclidean distance of the solution i to its nearest solution j on the artificial true Pareto-optimal front. M_1 is the average of d_i for all individuals in a set of non-dominated solutions. However, it should be noted that M_1 obtained from the artificial true Pareto-optimal front is not the exact value of that obtained from the true Pareto-optimal front. It can only be used to compare the closeness of solutions from the employed MOGAs to the Pareto-optimal front.

4.3.2. Hypervolume (HV)

The hypervolume, a maximum criterion, was originally proposed and employed in [41]. It measures not only the closeness to the true Pareto-optimal front but also the diversity of solutions. The *HV* refers to area (two objectives), volume (three objectives) or hypervolume (four or more objectives) between a given reference point and a non-dominated front to be evaluated. The *HV* is a popular criterion especially for a problem with unknown true Pareto solutions.

5. Numerical experiments

Before solving the MVS-D optimization problem in (6) formulated in Section 2, all the variables including expected return, variance, covariance, skewness and co-skewness of the available trading choices for the GenCo of interest given in (7)–(17) must be computed. The parameter settings are described as below.

5.1. Parameter setting for MVS-D optimization problem

In this study, nine trading choices in PJM market that consists of trading in spot market (denoted by z_1) and eight bilateral forward contracts with customers located in other zones (denoted by $z_2,\ldots,z_9$) were considered. We assume that the GenCo of interest is located in PEPCO zone. Suppose that this GenCo has a 350-MW fossil generator. Therefore, its unit characteristics are: a = 647.0865 MBtu/h, b = 14.8661 MBtu/MW h and c = 0.0065 MBtu/MW^2 h [45]. The cost of coal (energy) is assumed constant at 1.29 $/MBtu [46] during the period of analysis. Different parameters were considered under two case studies. The located zones of customers together with the bilateral forward contract prices of these two case studies are listed in Table 1.

Consider that the GenCo is making decisions on an optimal electricity allocation solution for the period of August 2006. Therefore, the expected spot prices from 1 to 31 August 2006 of nine locations need to be forecasted. Given the properties of electricity spot price, accurate forecasting is a crucial task and a big challenge in this field. The forecast technique applied in a previous study [47] was utilized in this paper. Then the expected returns for the considered trading choices were calculated according to (7) and (8) and the results are exhibited in Table 2. In the next step, variance, covariance, skewness and coskewness were estimated using the historical data. The electricity spot prices on 24-hourly basis between June 1998 and July 2006 were collected from the PJM market [48] for this purpose.

To ensure that the MVS portfolio model is suitable for this electricity allocation problem, the test for normality of the returns for the nine trading choices was performed. If the test results suggest non-normal distributions, an assumption of the MV model is violated and therefore MVS is more suitable. We conducted the test using Jarque–Bera statistic [49]. Table 2 presents the summary statistics for the expected returns of the considered trading choices and the Jarque–Bera statistics.

Table 1
Bilateral forward contract prices with customers located in different zones.

Bilateral forward contract price ($/MW h)	AEGO (z_2)	BGE (z_3)	DPL (z_4)	METED (z_5)	PECO (z_6)	PENELEC (z_7)	PPL (z_8)	PSEG (z_9)
Case study 1	40.9	40.2	40.7	39.0	39.6	37.0	38.3	41.4
Case study 2	40.5	40.5	40.5	40.5	40.5	40.5	40.5	40.5

Table 2
Expected returns of considered trading choices.

	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9
Case study 1									
Mean	0.684	0.002	0.286	0.339	0.156	0.086	0.333	0.407	0.018
Std. dev.	0.166	0.231	0.127	0.154	0.143	0.193	0.129	0.051	0.185
Jarque–Bera	9.884	13.527	14.162	15.325	12.116	12.596	19.615	25.933	15.215
Case study 2									
Mean	0.684	−0.013	0.299	0.331	0.216	0.124	0.475	0.497	−0.020
Std. dev.	0.166	0.231	0.127	0.154	0.143	0.193	0.129	0.051	0.185
Jarque–Bera	9.884	13.527	14.162	15.325	12.116	12.596	19.615	25.933	15.215

Table 3
Parameter setting of COGA-II, NSGA-II and SPEA-II.

Parameter	Setting and values
Objectives	MVS and MVS-D
Chromosome coding	Real-value chromosome with 9 decision variables
Crossover method	SBX crossover [40] with probability = 1.0
Mutation method	Variable-wise polynomial mutation [40] with probability = 1/chromosome length
Population size	100
Archive size (except NSGA-II)	100
Number of generations	600
Number of repeated runs	30

The Jarque–Bera statistic, exhibited in Table 2, follows a chi-square distribution with two degrees of freedom for the test of normality. Under the null hypothesis of a normal distribution, if the Jarque–Bera statistic is higher than 5.99, the normality assumption is rejected at 5% level of significance. The results from Table 2 indicate that the expected returns of all the trading choices are not normally distributed. Therefore, the MVS portfolio model is more suitable than the MV model for the electricity allocation problem in this study.

5.2. Parameter setting for MOGAs

The parameters for the COGA-II, NSGA-II and SPEA-II used in the implementation are illustrated in Table 3. The algorithms were evaluated for the MVS-D optimization problem as well as for the original MVS framework with three objectives. A solution to the problem is encoded into real-value chromosome with the length of 9, following the number of available allocation choices. The crossover probability was set as 1 to allow the crossover operator to be performed on all parent individuals. The mutation probability is equal to one divide by the chromosome length to so that, on average, one bit of a solution will be mutated. The population and archive sizes were both set to 100, while the number of generations was fixed at 600 so the performance of different algorithms is distinguishable for comparison. Since the number of data points generated should be sufficient for statistical computations, the number of repeated runs was set at 30.

6. Results and discussions

The averages and standard deviations of the M_1 and HV for the three algorithms considered are exhibited in Tables 4 and 5 respectively. The statistics were computed from 60,000 solutions generated from 30 repeated runs.

The results of the performance comparison from a pair of MOGAs can be compared statistically using the paired t-test. According to the M_1 criterion, COGA-II performs significantly better than NSGA-II and SPEA-II (p-value < 0.01 for almost all pairs of MOGAs). Similar results hold for the HV criterion, where COGA-II statically outperforms NSGA-II and SPEA-II with p-value < 0.001. Furthermore, we found that the superiority of COGA-II performance, regardless of the performance criteria, is strengthened when dealing with the optimization problem with 4 objectives. This superiority can be visualized in Figs. 1 and 2 that plot the averages of M_1 and HV versus number of generated solutions of MVS-D problem from all 30 repeated runs. The figures show that the solutions of all the three MOGAs rapidly achieved convergence within about 10,000 generated solutions which is equivalent to only 100 generations, i.e. number of generated solutions = population size $\times$ number of generations.

Due to the outstanding performance of COGA-II, the Pareto fronts of MVS and MVS-D problem are obtained from COGA-II and presented in Fig. 3. The results support our hypothesis that Pareto fronts are discontinuous in shape that logically represents the nature of problem having completing and conflicting objectives. The fourth objective of minimizing $D(\mathbf{x})$ that we propose to add to the MVS framework is to distribute x_i in order to increase investment diversification. Fig. 3(b) reveals that MVS-D non-dominated solutions achieve in enhancing the diversification benefit since a mass of non-dominated solutions reside in the low-standard deviation space. It implies, for GenCo point of view, that allocating electricity to many trading choices help reduce trading risk measured by standard deviation. In fact, this additional objective poses more challenge to the use of MOGAs for the optimization problem as the increased objective dimension could cause deterioration to the performance of the algorithm. However, the results of the M_1 and HV criteria suggest that the superiority of the COGA-II performance over the other two algorithms is even higher for the MVS-D problem compared to the MVS problem. To support this finding further, the solutions from using COGA-II for the optimization of the MVS and MVS-D problems are also investigated in the

Table 4
Average (Avg) and standard deviation (SD) of M_1.

Problems		COGA-II		NSGA-II		SPEA-II	
Obj.	Case	Avg	SD	Avg	SD	Avg	SD
MVS	1	0.008645	0.002505	0.012545	0.003550	0.016276	0.002911
MVS	2	0.014418	0.005326	0.017278	0.003404	0.022765	0.005879
MVS-D	1	0.007901	0.001661	0.022788	0.006305	0.016384	0.002741
MVS-D	2	0.009622	0.001773	0.025145	0.006171	0.020339	0.003728

Table 5
Average (Avg) and standard deviation (SD) of HV.

Problems		COGA-II		NSGA-II		SPEA-II	
Obj.	Case	Avg	SD	Avg	SD	Avg	SD
MVS	1	51.02478	0.148316	50.06765	0.245983	50.68226	0.150190
MVS	2	56.43273	0.201165	54.91343	0.334048	56.03167	0.212187
MVS-D	1	21.36057	0.181966	20.62899	0.316303	21.05846	0.175315
MVS-D	2	22.87187	0.190710	21.65327	0.397034	22.48699	0.192791

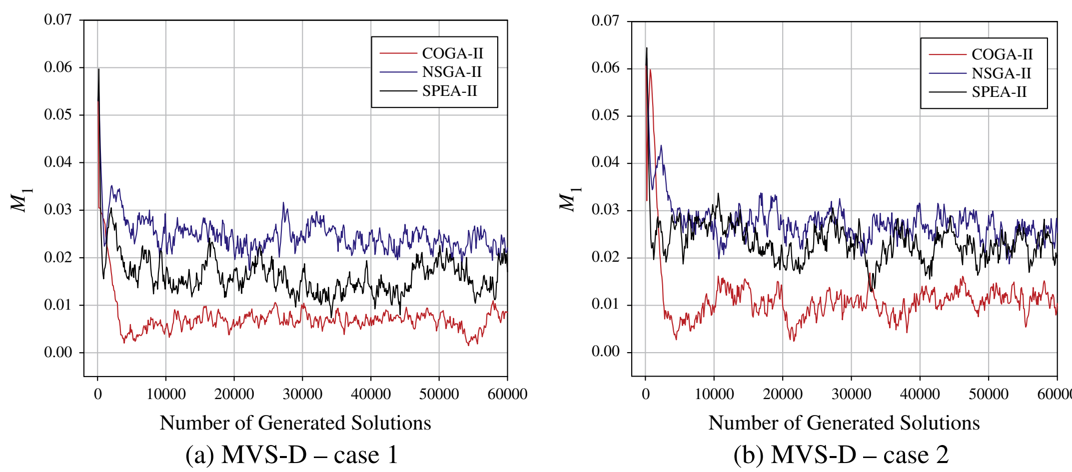


Fig. 1. Average M_1 versus number of generated solutions: cases 1&2 of the MVS-D problem.

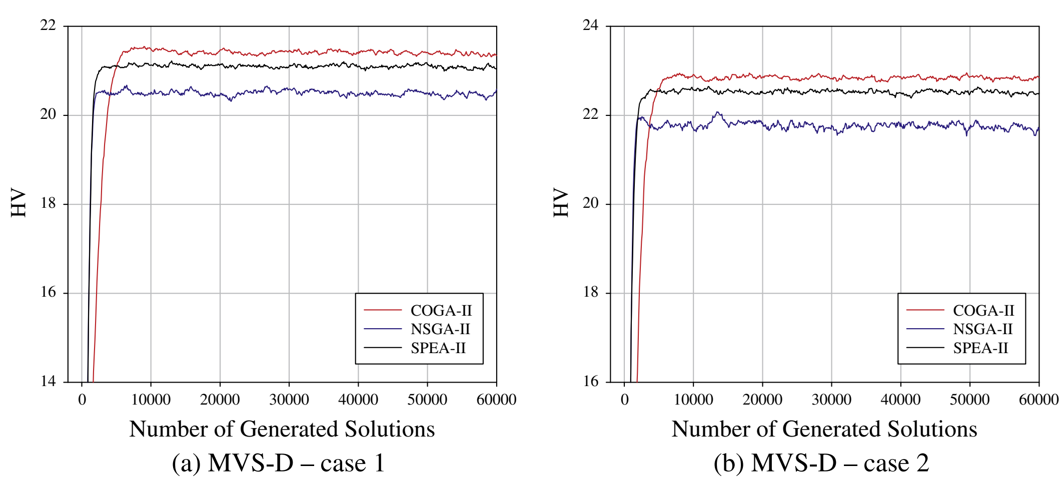
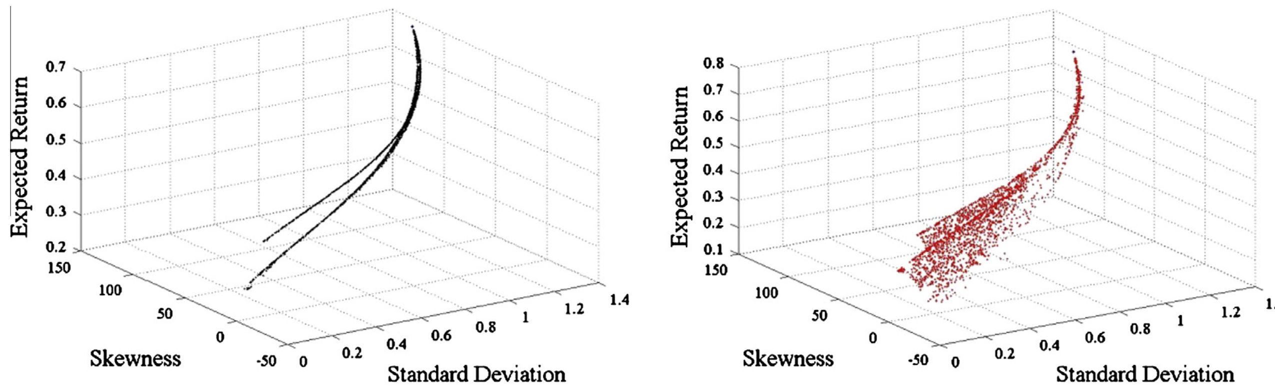


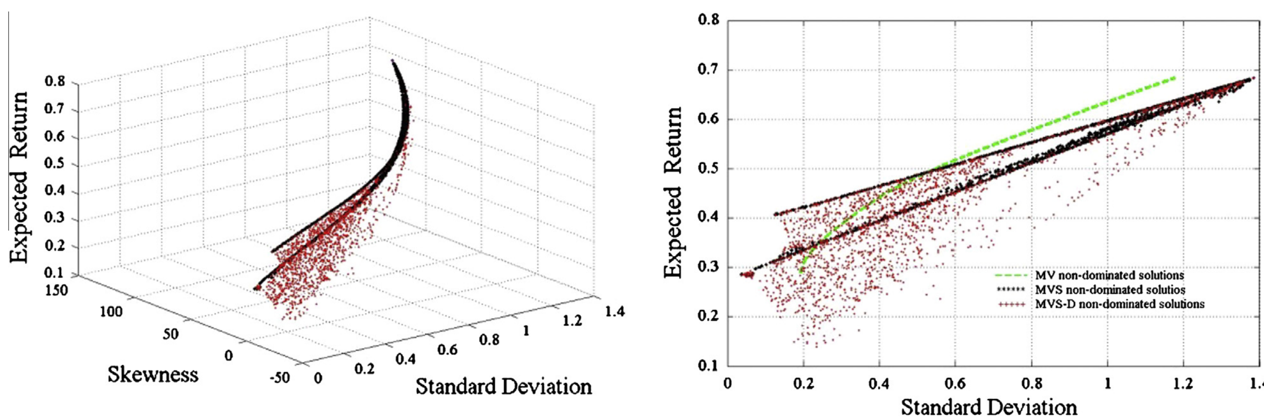
Fig. 2. Average HV versus number of generated solutions: cases 1&2 of the MVS-D problem.

following manner. This investigation is performed by comparing non-dominated fronts, the so called "Pareto fronts", from case study 1 in the MVS and the MVS-D optimization problems respectively. Fig. 4 illustrates the comparison of Pareto fronts plotted in the MVS and MV objective space respectively. Fig. 4(a) shows that the Pareto fronts of COGA-II for the MVS-D problem fully envelopes those for the MVS problem. This suggests that the performance of COGA-II does not deteriorate when the number of objectives is in-



(a) MVS Pareto fronts (b) MVS-D Pareto fronts

Fig. 3. Pareto fronts of case 1 using COGA-II for optimization of MVS (a) and MVS-D (b) problem.



(a) Comparison between MVS and MVS-D (b) Comparison between MV, MVS and MVS-D

Fig. 4. Pareto fronts comparison: (a) MVS and MVS-D in MVS space, and (b) MV, MVS and MVS-D in MV space.

Table 6
Expected returns of considered trading choices.

Problem	Optimal electricity allocation solutions (%)									Expected return	Standard deviation	Skewness
	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9			
MVS	31.19	0.00	0.00	0.00	0.00	0.00	0.00	68.81	0.00	0.49	0.53	12.39
	43.63	0.00	0.00	56.37	0.00	0.00	0.00	0.00	0.00	0.49	0.68	18.75
	49.64	0.00	47.04	3.32	0.00	0.00	0.00	0.00	0.00	0.49	0.71	22.12
	49.63	0.00	40.99	0.00	0.00	0.00	0.00	9.37	0.00	0.49	0.71	20.79
MVS-D	32.01	0.00	8.04	0.00	0.00	0.00	0.33	59.62	0.00	0.49	0.53	10.89
	44.62	0.00	20.57	0.00	0.08	0.00	0.00	32.04	2.69	0.49	0.68	15.18
	47.45	0.18	17.85	33.66	0.00	0.14	0.61	0.12	0.00	0.49	0.71	17.72

creased because the algorithm can still retain the non-dominated front of the solutions for the problem with a lesser number of objectives. In addition, we solved the Pareto front under the traditional MV framework and plotted, in Fig. 4(b), with the Pareto fronts of MVS and MVS-D problem on the two dimensions MV space. The results illustrate that the Pareto fronts of MVS and MVS-D problem dominate Pareto front of traditional MV framework when expected return less than 49.4% and standard deviation below about 0.53 are taken into consideration. Meanwhile, if the range of expected return above 49.4% and standard deviation more than about 0.53 is considered, portfolio optimization under traditional MV framework seems to offer the better solution.

Table 6 gives an example of optimal electricity allocation solutions of MVS and MVS-D problem. The chosen optimal solution of MVS problem depends on GenCo's trade-off between return, risk and skewness, while number of chosen trading choices is included in the case of MVS-D problem. Suppose that GenCo's return target is 49%, as a result, the optimal solutions can be screened from a vast number of non-dominated solutions showed in Fig. 3. Table 6 reveals that, at the same level of expected return and standard deviation, a solution of MVS-D problem allocates electricity to more trading choice compared to that of MVS problem. However, a MVS's solution offers the higher level of skewness than that of MVS-D problem. This finding is consistence with evidence in finan-

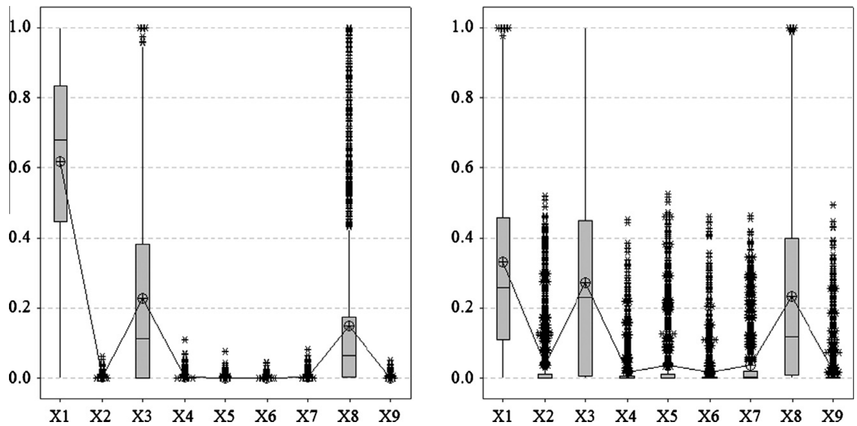


Fig. 5. Box plot of x_i of case 1 by using COGA-II for optimization of MVS (left panel) and MVS-D (right panel) problems.

cial literature stated that portfolio skewness decreases with an increasing in number of asset in portfolio [4].

Fig. 5 presents the box plots of x_i for case study 1 by using COGA-II for solving the MVS and MVS-D problems. The symbol '$\oplus$' represents the mean of x_i. The box in the plot contains 50% of the data points from the 25th to 75th percentile, and the line drawn across the box is the median of x_i. Observations outside the interquartile range are plotted on the whiskers of the box. The left panel of Fig. 5 shows that for the MVS problem, the optimal solutions by COGA-II allocate an average of 62% of the electricity to trading in the spot market (x_1), 23% to the bilateral contract with the customer located in AEGO zone (x_2), and 15% to the bilateral contract with the customer located in PPL zone (x_8). It should be noted that the optimal energy allocation solution in this three-objective model excessively allocates electricity to a particular trading choice. The right panel of Fig. 5 presents the optimal electricity allocation solutions of the MVS-D problem. The optimal solutions exhibit a reduction in the electricity allocation to the trading in spot market (x_1) to an average of 33%, while the allocation to the bilateral contract with the customers located in AEGO zone (x_2) increased to an average of 27% and the allocation to the bilateral contract with the customers located in PPL zone (x_8) was raised to 23%. The remaining 17% is allocated to bilateral contracts with customers located in other zones. The inclusion of the fourth objective has not only allocated the generated electricity more uniformly, but it has also spread out the investment to more trading choices that will increase the diversification benefit.

7. Conclusions

This paper proposes the use of MOGAs for solving multi-objective optimization problems in the MVS framework widely applied in finance. The problem is extended to the trading of electricity in a deregulated market by a power generation company. To overcome the potential weakness of the MVS framework where optimized solutions have a tendency to limit the scope of investment, an additional objective is proposed to increase diversification benefits. The shift from the three-objective problem to one with four objectives increased the complexity of the optimization process. To deal with this, we suggest the use of the newly proposed COGA-II, of which the results were compared to the widely accepted algorithms of NSGA-II and SPEA-II. The results suggest the superiority of performance in COGA-II in dealing with high dimensional multi-objective optimization problems in terms of not only proximity to the Pareto-optimal solutions but also diversity of its solution. In addition, COGA-II also produces solutions where the non-dominated fronts of a problem with more objectives envelope those of a problem with a lesser number of objectives.

In the context of the application, solving the electricity allocation problem based on the MVS framework together with the proposed additional objective (MVS-D) is particularly useful because COGA-II can provide solutions with Pareto fronts that also cover those based on the traditional MVS framework. As a result, the approach avoided over-concentration of investment in a few trading choices. The electricity was more uniformly allocated among a larger number of trading choices that promote diversification benefit for the power generation companies.

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References

[1] Markowitz H. Portfolio selection. J Financ 1952;7:77–91.
[2] Levy H. The rationale of the mean-standard deviation analysis: comment. Am Econ Rev 1974;64:434–41.
[3] Blume ME, Friend I. The asset structure of individual portfolios and some implications for utility functions. J Financ 1975;30:585–603.
[4] Simkowitz MA, Beedles WL. Diversification in a three-moment world. J Financ Quant Anal 1978;13:927–41.
[5] Christie AA. The stochastic behavior of common stock variances: value, leverage and interest rate effects. J Financ Econ 1982;10:407–32.
[6] Singleton JC, Wingender J. Skewness persistence in common stock returns. J Financ Quant Anal 1986;21:335–41.
[7] Campbell JY, Hentschel L. No news is good news: an asymmetric model of changing volatility in stock returns. J Financ Econ 1992;31:281–318.
[8] Arditti FD. Risk and the required return on equity. J Financ 1967;22:19–36.
[9] Levy H. A utility function depending on the first three moments. J Financ 1969;24:715–9.
[10] Samuelson PA. The fundamental approximation theorem of portfolio analysis in terms of means, variances and higher moments. Rev Econ Stud 1970;37:537–42.
[11] Kraus A, Litzenberger RH. Skewness preference and the valuation of risk assets. J Financ 1976;31:1085–100.
[12] Konno H, Suzuki K-i. A mean–variance–skewness portfolio optimization model. J Oper Res Soc Jpn 1995;38:173–87.
[13] Chunhachinda P, Dandapani K, Hamid S, Prakash AJ. Portfolio selection and skewness: evidence from international stock markets. J Bank Financ 1997;21:143–67.
[14] Liu S, Wang SY, Qiu W. Mean–variance–skewness model for portfolio selection with transaction costs. Int J Syst Sci 2003;34:255.
[15] Liu M, Wu FF. Portfolio optimization in electricity markets. Electr Power Syst Res 2007;77:1000–9.
[16] Liu M, Wu FF. Risk management in a competitive electricity market. Int J Electr Power 2007;29:690–7.

[17] Pindoriya NM, Singh SN, Singh SK. Multi-objective mean–variance–skewness model for generation portfolio allocation in electricity markets. Electr Power Syst Res 2010;80:1314–21.

[18] Kumari MS, Maheswarapu S. Enhanced genetic algorithm based computation technique for multi-objective optimal power flow solution. Int J Electr Power 2010;32:736–42.

[19] Jeyadevi S, Baskar S, Babulal CK, Willjuice Iruthayarajan M. Solving multiobjective optimal reactive power dispatch using modified NSGA-II. Int J Electr Power 2011;33:219–28.

[20] Streichert F, Ulmer H, Zell A. Evolutionary algorithms and the cardinality constrained portfolio optimization problem. In: Ahr D, Fahrion R, Reinelt G, editors. Operations research proceedings 2003: selected papers of the international conference on operations research (OR 2003). Heidleberg: Springer; 2004. p. 253–60.

[21] Deb K, Thiele L, Laumanns M, Zitzler E. Scalable test problems for evolutionary multiobjective optimization. In: Abraham A, Jain L, Goldberg R, editors. Evolutionary multiobjective optimization. Berlin Heidelberg: Springer; 2005. p. 105–45.

[22] Pierro FD, Soon-Thiam K, Savic DA. An investigation on preference order ranking scheme for multiobjective evolutionary optimization. IEEE Trans Evol Comput 2007;11:17–45.

[23] Purshouse RC, Fleming PJ. On the evolutionary optimization of many conflicting objectives. IEEE Trans Evol Comput 2007;11:770–84.

[24] Chang T-J, Yang S-C, Chang K-J. Portfolio optimization problems in different risk measures using genetic algorithm. Exp Syst Appl 2009;36:10529–37.

[25] Anagnostopoulos KP, Mamanis G. The mean–variance cardinality constrained portfolio optimization problem: an experimental evaluation of five multiobjective evolutionary algorithms. Exp Syst Appl 2011;38:14208–17.

[26] Pires DF, Antunes CH, Martins AG. NSGA-II with local search for a multi-objective reactive power compensation problem. Int J Electr Power 2012;43:313–24.

[27] Szuvovivski I, Fernandes TSP, Aoki AR. Simultaneous allocation of capacitors and voltage regulators at distribution networks using genetic algorithms and optimal power flow. Int J Electr Power 2012;40:62–9.

[28] Torres J, Guardado JL, Rivas-Dávalos F, Maximov S, Melgoza E. A genetic algorithm based on the edge window decoder technique to optimize power distribution systems reconfiguration. Int J Electr Power 2013;45:28–34.

[29] Hassan LH, Moghavvemi M, Almurib HAF, Muttaqi KM, Ganapathy VG. Optimization of power system stabilizers using participation factor and genetic algorithm. Int J Electr Power 2014;55:668–79.

[30] Hou J, Mi W, Sun J. Optimal spatial allocation of water resources based on Pareto ant colony algorithm. Int J Geogr Inf Sci 2013;28:213–33.

[31] Vazhayil JP, Balasubramanian R. Optimization of India's electricity generation portfolio using intelligent Pareto-search genetic algorithm. Int J Electr Power 2014;55:13–20.

[32] Deb K, Pratap A, Agarwal S, Meyarivan T. A fast and elitist multiobjective genetic algorithm: NSGA-II. IEEE Trans Evol Comput 2002;6:182–97.

[33] Zitzler E, Laumanns M, Thiele L. SPEA-II: improving the strength Pareto evolutionary algorithm. Zurich, Switzerland: Swiss Federal Institute of Technology (ETH); 2002.

[34] Boonlong K, Chaiyaratana N, Maneeratana K. Improved compressed objective genetic algorithm: COGA-II. In: The international conference on evolutionary computation. Valencia, Spain; 2010. p. 95–103.

[35] Jean WH. The extension of portfolio analysis to three or more parameters. J Financ Quant Anal 1971;6:505–15.

[36] Lai T-Y. Portfolio selection with skewness: a multiple-objective approach. Rev Quant Financ Acc 1991;1:293–305.

[37] Steuer R, Qi Y, Hirschberger M. Suitable-portfolio investors, nondominated frontier sensitivity, and the effect of multiple objectives on standard portfolio selection. Ann Oper Res 2007;152:297–317.

[38] Goldberg DE. Genetic algorithms in search, optimization, and machine learning. Reading, MA: Addison-Wesley; 1989.

[39] Coello CAC. A comprehensive survey of evolutionary-based multiobjective optimization techniques. Knowl Inf Syst 1999;1:129–56.

[40] Deb K. Multi-objective optimization using evolutionary algorithms. New York: John Wiley & Sons, Inc.; 2001.

[41] Zitzler E, Thiele L. Multiobjective evolutionary algorithms: a comparative case study and the strength Pareto approach. IEEE Trans Evol Comput 1999;3:257–71.

[42] Silverman BW. Density estimation for statistics and data analysis. London: Chapman and Hall/CRC Press; 1986.

[43] Maneeratana K, Boonlong K, Chaiyaratana N. Compressed-objective genetic algorithm. In: Runarsson T, Beyer H-G, Burke E, Merelo-Guervós J, Whitley L, Yao X, editors. Lecture notes computer science. Berlin/Heidelberg: Springer; 2006. p. 473–82.

[44] Zitzler E, Deb K, Thiele L. Comparison of multiobjective evolutionary algorithms: empirical results. Evol Comput 2000;8:173–95.

[45] Grigg C, Wong P, Albrecht P, Allan R, Bhavaraju M, Billinton R, et al. The IEEE reliability test system-1996. A report prepared by the reliability test system task force of the application of probability methods subcommittee. IEEE Trans Power Syst 1999;14:1010–20.

[46] Annual energy outlook 2011 with projections to 2035. U.S. Energy Information Administration (EIA).

[47] Suksonghong K, Goh KL. Forecasting electricity spot prices: GARCH with seasonality. In: Chunli H, editor. IEEE international conference on engineering technology and economic management. Institute of Electrical and Electronics Engineers, Inc.; 2012. p. 520–4.

[48] PJM Interconnection. <http://www.pjm.com>.

[49] White KJ. Shazam econometrics computer program. User's reference manual version 7.0. Vancouver, Canada: McGraw Hill; 1993.