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PORTFOLIO OPTIMIZATION AND RISK MEASUREMENT BASED ON NON-DOMINATED SORTING GENETIC ALGORITHM

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Abstract. This paper introduces a multi-objective genetic algorithm (MOGA) in regard to the portfolio optimization issue in different risk measures, such as mean-variance, semi-variance, mean-variance-skewness, mean-absolute-deviation and lower-partial-moment to optimize risk of portfolio. This study introduces a PONSGA model by appling the non-dominated sorting genetic algorithm (NSGA-II) to maximize both the expected return and the skewness of portfolio as well as to simultaneously minimize different portfolio risks. The experimental results demonstrated that the PONSGA approach is significantly superior to the GA in all performances, examined such as the coefficient of variation, Sharpe index, Sortino index and portfolio performance index. The statistical significance tests also showed that the NSGA-II models outperformed the GA models in different risk measures.

1. Introduction.

Investment portfolio optimization is the process of optimizing the capital proportion of assets held to fit various constraints; it gives the highest return with the least risk. It plays crucial roles in the theory of portfolio selection that provides financial decision-makers and investors with the ability to efficiently manage investment and monitor risks.

The mean-variance (MV) model originally proposed by Markowitz (1952) is a well-known benchmark for portfolio optimization [20]. The traditional MV equilibrium framework models a return on assets as a normal distribution and defines risk as the standard deviation. Both the mean and the variance of returns are only considered in the original capital asset pricing model (CAPM). Furthermore, the upside and downside risks are considered to be equal risk aversion [19].

The MV model may not be the best choice available to investors in terms of an appropriate risk measure. To improve the limitation of MV model, the alternative risk measures such as semi-variance (SV)[21], mean-variance with skewness (MVS) [23], mean absolute deviation (MAD) [16], lower partial moment (LPM) [11][4], etc, have been proposed. Cui etl. [8] Applied MV, MAD, minimax and conditional value at risk (VaR) models based on minimal transaction and cardinality constraint to

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optimize portfolio. The MV model is a bi-criteria optimization equation, whereby a rational investor's portfolio selection is based on the trade-off between maximum return and minimum risk. Even though the MV model can find an efficient frontier via stochastic programming [27][2], quadratic programming [3][7] and goal programming [6][17],[26], the optimization of asset allocation in a portfolio is still a complex and time-consuming NP-hard problem.

Lin and Ko applied GA to optimize the fitness of a portfolio's asset allocation and VaR. They found that the GA-based portfolio VaR forecasting mechanism could efficiently select more suitable thresholds under the extreme value theory model for each asset in the portfolio [18]. They also used an artificial neural network (ANN) nonlinear, i.e., heuristic search methodologies used to solve portfolio optimization problems in large problem spaces [14]. Chang et al. introduced a GA-based portfolio optimization model with different risk measures, which performed better than an MV model with a small portfolio size [5]. Wilding [28] and Xia, Wang etc. [29] used GA to efficiently construct a portfolio that satisfied multiple objectives. Oh et al. also used GA to measure risks in the portfolio beta instead of the standard deviation in portfolio selection processes [22]. All of the above discussions either used traditional GA to combine multi-objective objectives into a single composite function, or dealt with all objectives individually as the constraint set, because portfolio selection is a multi-objective optimization problem. In practice, it would be difficult to properly select the weight of each objective to characterize the investor's preferences. However, GA could return a single solution rather than a set of solutions. Investors would prefer a set of fitness solutions (frontier efficient) than a single solution when considering a multi-objective portfolio.

Multi-objective optimization problems must satisfy all of the objectives at a reasonable level without being dominated by any other solution. MOGA applies the nonlinear search capability of GA to efficiently achieve an optimal set. Hussein etc. proposed a MOGA-based risk assessment conceptual framework applied to financial derivative hedging, air traffic conflict detection, that uses scenario simulation to analyze the effect of uncertainty on the multiple conflicting objectives [1]. A famous MOGA methodology, the non-dominated sorting genetic algorithm (NSGA-II) [9], [15], [10], specifies the crowding distance operator to select the non-dominated solutions located in a less crowded region. NSGA-II improves the diversity of the population for a better convergence closer to the true Pareto optimal frontier.

Numerous measures of portfolio methods have been proposed to evaluate the ability of managed fund portfolios to outperform benchmarks. Portfolio performance measurement is the process of comparing the return gained on a portfolio during the investment period. The portfolio performance depends on the superior investment analysts' capabilities for asset selection and capital weight allocation. The popular portfolio performance indexes include: Sharpe ratio, Treynor index, Jensen index, Sortino ratio [24], portfolio performance index (PPI) [25] etc. These measures have been highly praised by fund management professionals [23][12].

This study presented a PONSGA model that applies NSGA-II methodology to maximize the expected return while simultaneously minimizing the portfolio risk. The portfolio risk measures of five portfolio optimization models specifically proposed were: the MV, SV, VS, MAD and LPM. The experimental results demonstrated that the PONSGA approach was significantly superior to using GA models in all examined portfolio performance indexes, such as the Sharpe index, Sortino index and PPI.

The remainder of this paper is organized as follows. Section 2 describes the PONSGA model and illustrates the various portfolio risk measures. Section 3 introduces the stock selection process analyzes the characteristics of the sample data. Section 4 shows and discusses the experimental results. Finally, Section 5 presents the conclusions.

2. PONSGA model.

Portfolio optimization (PO) is a process of optimizing the capital weight of assets held to fit various constraints, such as the highest return, the least risk and etc. NSGA is the well-known nonlinear optimization method. NSGA instead of traditional PO method such as quadratic programing is applied to improve the portfolio asset allocation optimization because of its nonlinear search optimization capability. In this study, a PONSGA model was introduced to optimize the asset allocation in a portfolio when considering the maximum return and the minimum risk under different risk measures. The operating procedures of PONSGA, as shown in Figure 1 , are described step by step as follows.

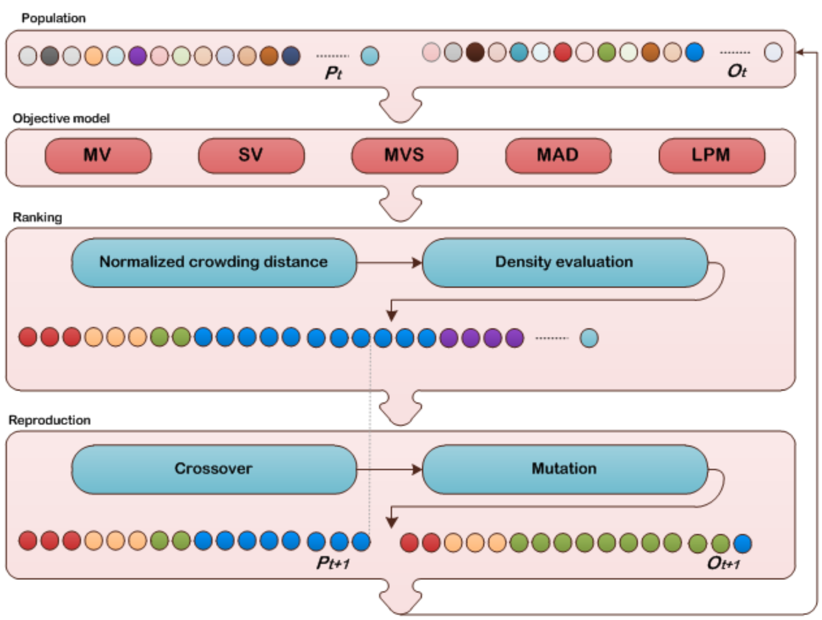

FIGURE 1. Procedure of the PONSGA model

Step 1 Prepare the *t-th* generation population (TP_t), which is composed of the parent population (P_t) and the offspring population (O_t). It means $TP_t = P_t \bigcup O_t$. Each population contains N possible solutions s_i, i=1,2...N, and the sizes of P_t, O_t and TP_t are N, N and $2N$, respectively. Initially, the first generation parent population (P_0) of size N is randomly created. The standard tournament selection, crossover and mutation operations are applied to generate the offspring population (O_0), until size N of O_0 is realized.

Step 2 Check whether the terminated criteria, such as the maximum generation and the variation of fitness value are satisfied. If the terminated criteria are achieved, P_t is returned.

Step 3 Objective models. In the present economy, investors prefer lower risks and higher returns. Five objective models $\mathbb{O}_M$ derived from $M = \{MV, SV, MVS, MAD \text{ and } LPM\}$ are designed in the PONSGA model, respectively. Each objective model is a combinatorial optimization topic of finding optimal value from a set of objective functions $(\mathcal{F}_M)$=$\{max_W R_M(W), min_W V_M(W), max_W S_M(W)\}$, where $R_M(W)$, $V_M(W)$ and $S_M(W)$ are mean, variance and skewness of portfolio return in model M. W represents the vector of asset allocation weight, as shown in Eq.(1). The detailed formulations of $R_M(W)$, $V_M(W)$ and $S_M(W)$ are described in Section 2.2.

$$W = [w_1 \;\; w_2 \;\; ... \;\; w_n]^T \tag{1}$$

The multi-objective equation can be written as Eq. (2):

$$\mathcal{F}_M = \left\{ \underset{W}{max}\, R_M(W), \underset{W}{min}\, V_M(W), \underset{W}{max}\, S_M(W) \right\} \tag{2}$$

s.t.

$$\sum_{i=1}^{n} w_i = 1$$

$$w_i \geq 0, i = 1, 2, ..., n$$

Step 4 All of the solutions s_i^*,$i = 1, 2, ..., 2N$, in the population TP_t are sorted according to each objective function value in ascending order in each model, where the objective functions are denoted as $f(\cdot)$, $f = R, S, V$. The value of $f(\cdot)$ for solution i is z_f^i, and z_f^{max} and z_f^{min} are the maximum and minimum values of the objective function $f(\cdot)$, respectively.

Step 5 Non-dominated ranking. Each solution s_i^* in TP_t is ranked according to the density of s_i^* denoted as $D(s_i^*)$ by applying the Pareto-ranking approach. $D(s_i^*)$ is the sum of $D_f(s_i^*)$, $f = R, S, V$, as $D(s_i^*) = D_R(s_i^*) + D_S(s_i^*) + D_V(s_i^*)$, where $D_f(s_i^*)$, as shown in Eq. (3), is calculated by the normalized crowding distance of two solutions: s_{i-1}^* and s_{i+1}^* on both sides of s_i^*. $D_f(s_1^*)$ and $D_f(s_{2N}^*)$, with the smallest and largest values, are assigned an infinite value:

$$\begin{aligned} D_f(s_i^*) &= \frac{z_f^{i+1} - z_f^{i-1}}{z_f^{max} - z_f^{min}}, i = 2, 3, ..., 2N-1 \\ D_f(s_1^*) &= D_f(s_{2N}^*) = \infty \end{aligned} \tag{3}$$

The Pareto-ranking is based on the *Pareto frontier* (*PF*) which is defined as the set of possible solutions that no further *Pareto improvements* can be made. The *Pareto improvement* represents that a change to a different solution (s_i^*) that makes at least one individual better off without making any other individual worse off. Figure 2 shows an example of Pareto ranking result. The boxed points represent feasible choices (s_i^*) according to different ranking. The Pareto-ranking algorithm is expressed as follows.

function Pareto_ranking (TP_t)
    Let P= TP_t
    Let $rank = 0$
    while $P \neq \emptyset$
        $PO[rank]$ = Find PF set from P
        $P = P - PO$
        $rank$++

end while
end function

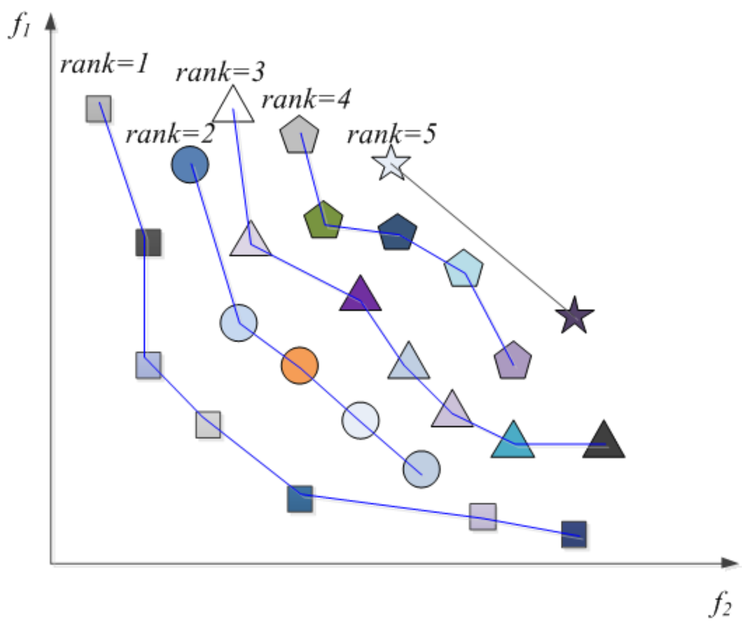

FIGURE 2. Pareto ranking

These are identified as different non-dominated sets nds_i, i=1,2,...,M. The best non-dominated set nds_1 contains the possible best solutions in TP_t. That is, $TP_t = nds_1 \bigcup nds_2 \bigcup ... \bigcup nds_M$. Then, all of the member solutions of the sets nds_i, $i = 1, 2, ..., m$, are chosen to generate the next parent population P_{t+1}, until P_{t+1} is realized so that $|P_{t+1}| \leqq N$. Let m be the index of a non-dominated front nds_m, then $|nds_1 \bigcup nds_2 \bigcup ... \bigcup nds_m| \leqq N$.

Step 6 Reproduction. Use the tournament selection method to eliminate the worst solutions from the nds_i fronts through competition to form the P_{t+1} population for the mating pool. Apply crossover and mutation operators to generate offspring population O_{t+1} in the mating pool.

Step 7 Loop. Go to *Step 1* until the terminated criteria are satisfied.

2.1. **Encoding method.** In order to represent a set of asset allocation weights, an individual in the *t-th* population is divided into n genotypes w_i, $i = 1, ..., n$. Each genotype is a real value encoded to avoid premature convergence and to obtain a higher quality solution with better computation efficiency and robustness. The summation of w_i, $i = 1, ..., n$, is 1.

$$w_1 + w_2 + w_3 + ... + w_n = 1, 0 \leq w_i \leq 1, w_i \in \Re \tag{4}$$

2.2. **Objective models.** In order to find effective portfolio valuation models under maximum return or skewness and minimum risk, the objective models are designed by various portfolio optimization models, such as MV, SV, MVS, MAD and LPM. The models were mathematically represented below.

2.2.1. *Mean-variance model.* The MV model portfolio optimization problem [20] can be stated as the quadratic programming formula as below. The $R_{MV}(W)$ and $V_{MV}(W)$ are two critical objective functions in the portfolio asset allocation problem. First, $R_{MV}(W)$ denotes the expected return of the portfolio, as shown in Eq. (5):

$$R_{MV}(W) = W^T \bar{R} = \sum_{i=1}^{n} w_i \bar{r_i} \tag{5}$$

where $\bar{R}$=$[\bar{r_1}, \bar{r_2}...\bar{r_n}]^T$; $\{\bar{r_i}\}_{i=1,2,\dots,n}$ is the mean of return $r_i.V_{MV}(W)$ and denotes the variance-covariance of portfolio, as shown in Eq. (6):

$$\begin{aligned} V_{MV}(W) &= W^T V W \\ &= \sum_{j=1}^{n} W_i^2 \sigma_i^2 + 2 \sum_{i=1}^{n} \sum_{j=i+1}^{n} w_i w_j \sigma_{ij} (i \neq j), (j > i) \end{aligned} \tag{6}$$

σ_i^2 is the variance of asset i, and σ_{ij} is the covariance between assets i and j. The objective model $\mathbb{O}_{MV}$ is shown as follows:

$$\mathbb{O}_{MV} = \lambda \times V_{MV}(\mathbf{W}) - (1 - \lambda) \times R_{MV}(\mathbf{W}) \tag{7}$$

where λ is weighting parameter $(0 \leq \lambda \leq 1)$, which is the parameter of risk aversion.

2.2.2. *Semi-variance model.* Markowitz proposed the SV model instead of variance-covariance matrix of the quadratic objective function to improve the assumption of an asymmetric return distribution of the MV model [21]. The SV model also considers two objective functions: $R_{SV}(W)$ and $V_{SV}(W)$, where $R_{SV}(W)$ is the same as $R_{MV}(W)$and $V_{SV}(W)$ is shown below:

$$V_{SV}(W) = \frac{1}{T} \times \sum_{t=1}^{T} (r_{p,t} - \bar{r_p})^2 \tag{8}$$

where the $r_{p,t}$ is the portfolio return over the time period t = 1, 2, ... T; $\bar{r_p}$ is the mean of $r_{p,t}$.The following equation shows the objective model $\mathbb{O}_{SV}$:

$$\mathbb{O}_{SV} = \lambda \times V_{SV}(W) - (1 - \lambda) \times R_{SV}(W) \tag{9}$$

2.2.3. *Mean-variance with skewness model.* Maximizing the skewness of return could efficiently improve the performance of the traditional MV model that has been widely discussed in numerous studies [2][5][28][23]. The MVS model considers $R_{MSV}(W)$, $V_{MSV}(W)$ and $S_{MVS}(W)$, simultaneously. $R_{MSV}(W)$ and $V_{MSV}(W)$ are the same as $R_{MV}(W)$ and $V_{MV}(W)$. The $S_{MVS}(W)$ represents the skewness of the expected return shown in Eq. (10):

$$\begin{aligned} S_{MVS}(W) &= E([W^T(R - \bar{R})]^3) \\ &= \sum_{i=1}^{n} w_i^3 S_i^3 + 3 \sum_{i=1}^{n} \left[\sum_{j=i+1}^{n} w_i^2 w_j S_{iij} + \sum_{j=t=1}^{n} w_i w_j^2 S_{ijj} \right] \\ &+ 6 \sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=j+1}^{n} w_i w_j w_k S_{ijk} \end{aligned} \tag{10}$$

where S_i is the skewness of the expected return of assets I; S_{iij}, S_{ijj}, and S_{ijk} are the co-skewness of the expected return of assets i, j, or k(i=1,2, ..., n; $j = i + 1$, $i + 2$, ..., $i + n$; $k = j + 1$, $j + 2$, ...$j + n$). $\mathbb{O}_{MVS}$ is shown as follows:

$$\mathbb{O}_{MVS} = \lambda \times V_{MVS}(W) - (1 - \lambda - \theta) \times R_{MVS}(W) - \theta \times S_{MVS}(W) \tag{11}$$

where λ and θ are the weighting parameters($0 \leq \lambda \leq 1$, $0 \leq \theta \leq 1$).

2.2.4. *Mean absolute deviation model.* The *MAD* model, proposed by Konno and Yamazaki in 1991[16] is widely used by practitioners because it can reduce time-consumption to solve a large scale portfolio optimization problem with linear programming rather than with quadratic programming as in the case of *MV* model. The objective model $\mathbb{O}_{MAD}$ considers $R_{MAD}(W)$ and $V_{MAD}(W)$. $V_{MAD}(W)$ is shown in Eq.(12):

$$V_{MAD}(W) = \frac{1}{T} \times \sum_{t=1}^{T} |r_{p,t} - \bar{r_p}| \tag{12}$$

$\mathbb{O}_{MAD}$ is shown below.

$$\mathbb{O}_{MAD} = \lambda \times V_{MAD}(W) - (1 - \lambda) \times R_{MAD}(W) \tag{13}$$

2.2.5. *Lower partial moment model.* Fishburn [11] and Bawa et al. [4] proposed an n-degree lower partial moment to evaluate risk measures characterized by a utility function of portfolio in contrast to the *MV* model; the higher the degree of LPM measures, the greater the risk aversion of the investor. The $V_{LPM}(W)$ and the objective model $\mathbb{O}_{MAD}$ are shown in Eqs. (14) - (15).

$$V_{LPM}(W) = \frac{1}{T} \sum_{i=1}^{T} [Max(0, \tau - r_{p,t})]^2 \tag{14}$$

$$\mathbb{O}_{PLM} = \lambda \times V_{LPM}(W) - (1 - \lambda) \times R_{LPM}(W) \tag{15}$$

where $V_{LPM}(W)$ is the lower partial moment of degree 2; τ is the investor-target rate of return; $r_{p,t}$ is the portfolio return at t period.

3. **Data analysis.** The data analysis contains data sampling, data exploration, stock selection, descriptive statistics and normality test to illustrate the characteristics of data source.

- Data sampling

  There are 782 companies currently listed in the *Taiwan Stock Exchange Corporation (TSE)*. Among them, 571 companies have been listed for 10 years or more in the *Taiwan Economic Journal (TEJ)* data bank. These companies were extracted as our experimental targets. The weekly stock return was used to avoid the 7% daily price upward/downward limit in the *TSE*. There were 570 observations in the data set of each company from January 2000 to December 2010.
- Data exploration

  Exploration of data is generally a time-consuming process. The data collected is not often presented in a ready state that can be transferred into a database table. Sometimes, the sample data collected contains missing data or an unnumbered data set. Missing data is usually expressed as a blank or as *NULL* data. Unnumbered data normally is described as *NAN* or *N.A.* (not available). Both the *NULL* data and *NAN/N.A* data were further eliminated from our experimental target set. Finally, 324 companies were selected as candidates for the next selection process.

- Stock selection

The objective during the selection stage of this study was to find an efficient stock portfolio for the next modeling process. The selected portfolio maximizes the investment return and minimizes the risk of investment. In addition, it also needs to maintain an appropriate fitness of portfolio diversification. However, the stock selection process is time consuming and complicated. It requires extensive knowledge to obtain a superior investment return. Most investors are defensive investors [13] who do not possess the required skills. This study adopted Graham's [13] value-investing rules to select a set of worthy stocks. The idea of value investing is that any stock investment should be worth more than what an investor invests in it. A well-designed stock selection system can assist investors in choosing a diversified portfolio to produce better investment performance. The simple and mechanistic rules are described as follows.

1. **Financial condition**: (1) The current ratio is larger than 200%, which means the company's current assets should be at least twice its current liabilities. (2) Long term debt should not exceed the net current assets.
2. **Earning stability**: Companies have positive earnings in each of the past three years.
3. **Dividend record**: Companies have an uninterrupted dividend payment record for at least the past five years.
4. **Earning growth**: There is a minimum increase of at least one-third in per-share earnings in the past five years.
5. **Earnings ratio**: Price to earnings ratio (PER) is lower than 15. Stock market price should not be larger than 15 times the average earnings per share (EPS) of the past three years.

The stock selection system extracted a total of 17 companies scattered in 9 industries, as shown in Table 1, to be the experimental targets.

- Normality test

The weekly return (R_{it}) was defined as the continuously compounded return, represented by:

$$R_{it} = log(\frac{P_{it}}{P_{it} - 1}) \tag{16}$$

where P_{it} is the $i\text{-}th$ asset price at time t.

The distributional assumption of MV is that the stock returns follow normal distributions. The descriptive statistics and normality testing are presented as Table 2. The mean weekly returns of stocks are near to zero; that is consistent with most studies where the long term return is close to zero. The average skewness was 0.4287. The positive skew indicated that the sample data are concentrated on the left side. All of the kurtosis coefficients are larger than 3. It means that the return distribution exhibited the leptokurtic distribution phenomenon. The Jarque-Bera (JB) and Lilliefors (LF) tests were used to check if the data followed a normal law.

The mean-variance model assumes that the return follows a normal distribution. It was important to test whether the distribution of the return was a normal distribution in the sample companies. If either the JB or LF test is

TABLE 1. Sample companies

Industries	Stock ID	Business Name
Food	1232	TTET Union Corp.
Textile and fiber	1308	Asia Polymer Corp.
	1326	Formosa Chemicals & Fiber Corp.
Electrical machinery	1540	Roundtop Machinery Industries Co., Ltd.
Iron and steel	2010	Chun Yuan Steel Industry Co., Ltd.
	2029	Sheng Yu Steel Co., Ltd.
Rubber	2103	TSRC Corp.
	2108	Nantex Industry Co., Ltd.
	2104	China Synthetic Rubber Corp.
Construction	2534	Hung Sheng Construction Ltd.
	2536	Hung Poo Real Estate Development Co., Lt
Chemical	1726	Yung Chi Paint & Varnish Mfg. Co., Ltd.
Electronics	2417	AverMedia Technologies Inc.
	2428	Thinking Electronic Industrial Co., Ltd.
	2468	Fortune Information Systems Corp.
	2489	Amtran Technology Co., Ltd.
Transportation	2617	Taiwan Navigation Co., Ltd.

TABLE 2. Return distribution tests results

Stock ID	Mean	Std	Min	Max	Skewness	Kurtosis	JB	LF
1232	0.0048	0.0366	-0.1517	0.1586	0.2238	7.2086	425.4337	0.1239
1308	0.0040	0.0669	-0.2414	0.3000	0.2559	4.9465	96.2052	0.0789
1326	0.0046	0.0475	-0.1745	0.2299	0.3217	5.2188	126.7504	0.0771
1540	0.0053	0.0629	-0.2000	0.3857	0.9280	8.2001	724.0547	0.1084
2010	0.0018	0.0380	-0.1592	0.1722	0.3204	5.9657	218.6480	0.0858
2029	0.0029	0.0449	-0.1448	0.2171	0.4374	5.1301	125.9344	0.0886
2103	0.0022	0.0471	-0.1397	0.2609	0.4858	5.7184	197.9235	0.0742
2108	0.0065	0.0636	-0.2367	0.2734	0.2082	5.3160	131.5140	0.0616
2104	0.0034	0.0687	-0.2376	0.3963	0.5854	6.7423	365.1678	0.0918
2534	0.0037	0.0503	-0.1852	0.2151	0.3900	5.4977	162.6175	0.1008
2536	0.0052	0.0733	-0.2732	0.3495	0.8056	6.6938	385.7008	0.0869
1726	0.0048	0.0670	-0.3025	0.2895	0.4138	5.7337	193.7525	0.0875
2417	0.0006	0.0683	-0.3342	0.3622	0.2531	6.5081	298.3677	0.0914
2428	0.0043	0.0770	-0.2160	0.3970	0.3570	4.6969	80.4913	0.0576
2468	0.0039	0.0792	-0.2955	0.3810	0.6620	5.4235	181.1260	0.0908
2489	0.0075	0.0864	-0.2909	0.3724	0.4166	4.7399	88.3837	0.0727
2617	0.0058	0.0649	-0.2541	0.3031	0.2228	6.0724	228.9097	0.0981
average	0.0042	0.0613	-0.2257	0.2979	0.4287	5.8713	237.1165	0.0868

less than 0.05, the null hypothesis cannot be rejected at the 5% level (specified) of significance. Table 2 shows that all of the samples' returns reject the null hypothesis in the JB and LF tests. Since the statistic value is significantly higher than the critical value of JB (5.8746) and LF (0.0377), the null hypothesis of normality can be rejected.

4. **Experimental result and analysis.** In this section, the simulation environment, experimental modeling and experimental results are described and analyzed.

The PONSGA approach combined with different risk measure programs was written using Java packages. Finally, the experimental results were analyzed to show the overall portfolio performance.

4.1. **Modeling.** To test the versatility and robustness of the PONSGA portfolio risk optimization approach, five risk measure techniques were compared with GA to serve as the performance benchmarks. The capital weights of stocks are learned by GA and PONSGA, respectively, to calculate the portfolio return and risk.

1. GAMV & PONSGAMV: GA and PONSGA based were used to optimize weight and were combined with the *MV* model to evaluate portfolio risk.
2. GASV & PONSGASV: GA and PONSGA based were used to optimize weight and were combined with the *SV* model to evaluate portfolio risk.
3. GAMVS & PONSGAMVS: GA and PONSGA based were used to optimize weight and were combined with the *MVS* model to evaluate portfolio risk.
4. GAMAD & PONSGAMAD: GA and PONSGA based were used to optimize weight and were combined with the *MAD* model to evaluate portfolio risk.
5. GALPM & PONSGALPM: GA and PONSGA based were used to optimize weight and were combined with the *LPM* model to evaluate portfolio risk.

4.2. **Parameter settings.** The parameters used in the PONSGA runs are shown in Table 3. The population size was 30. The maximum generation was set as 50. The Pareto rate was set to 0.5 intentions in order to remain on the first Pareto front, while the evolutionary mechanisms chose individuals from higher fronts. The simulation was terminated if no improvement was further achieved in the value of the objective function. The crossover operator was a single point crossover, and the crossover rate was set to 0.6. The mutation rate was set to 0.05. The elitist operator was used in this model because it outperformed the non-elitists [15]. The best single individual of each parent population was reserved as an elitist member in the next population. The parameters used in the experiments were based on Deb's suggestions [1]. Minor changes in these parameters did not seem to have a major effect on performance in our preliminary tries. The GA parameter settings were the same as PONSGA. In addition, the investor's preferences for mean, variance and skewness were assumed to be equivalent. In other words, $(\lambda_1 = \lambda_2 = 1)$ and $(\lambda_1 = \lambda_2 = \lambda_3 = 1)$ for the objective functions of GA.

Table 3. Parameter settings of PONSGA

parameters	**Value of setting**
Encoding method	Real number encoded
Population size	30
Number of generations	50
Pareto rate	0.5
Selection operator	Tournament
Crossover operator	Single point
Crossover rate	0.6
Mutation rate	0.05
Reserved elitist	The best single individual of each population

4.3. **Performance analysis.** The experimental performance analyses consisted of several parts: applying the coefficient of variation (CV) index, Sharpe index, Sortino index and portfolio performance index (PPI) to evaluate the portfolio performance; revealing the suitability of asset allocations in the portfolio; and finally, analyzing the significance test. The experimental results were the average values over the last 30 weeks of 570 observations, by the rolling window process.

4.3.1. *CV index.* CV is a better statistical measure than the pure volatility measure for determining the ratio of volatility (risk) to return. CV is calculated as follows.

$$CV = \frac{\sigma}{|\mu|} \tag{17}$$

where σ is standard deviation of portfolio risk; the μ represented by $\bar{r_p}$ is the mean of the portfolio return. CV is particularly helpful when comparing the dispersion of sample data in a data series around the mean. The highest CV were for the GASV model (2.6131) shown in Table 4, followed by GAMAD (2.5943), GAMV (2.5671), GALPM (2.4592) and GAMVS (2.4202). The lowest CV or ranking of five CV were PONSGAMAD (Rank 1, 1.7294), PONSGAMVS (Rank 2, 1.8165), PONSGALPM (Rank 3, 1.8871), PONSGASV (Rank 4, 1.9403) and PONSGAMV (Rank 5, 1.9403); these models obtained lower risk under higher expected returns than the GA models did.

Table 4. Parameter settings of PONSGA

Models	CV	Rank	*Sharpe*	Rank	SI	Rank	PPI	Rank	MRank
GAMV	2.5671	8	-0.1269	10	0.3946	9	0.2906	7	8.50
GASV	2.6131	10	0.4783	7	0.5291	6	0.2682	9	8.00
GAMVS	2.4202	6	0.4396	8	0.3939	10	0.2725	8	8.00
GAMAD	2.5943	9	0.4278	9	0.4620	8	0.1979	10	9.00
GALPM	2.4592	7	0.5986	6	0.4968	7	0.3024	6	6.50
average	2.5308		0.3635		0.4553		0.2663		
PONSGAMV	1.9403	5	1.6050	3	0.6668	3	0.5205	2	3.25
PONSGASV	1.9329	4	1.3866	5	0.6676	2	0.5696	1	3.00
PONSGAMVS	1.8165	2	1.3958	4	0.5965	5	0.3850	5	4.00
PONSGAMAD	1.7294	1	1.6698	2	0.6322	4	0.4804	4	2.75
PONSGALPM	1.8871	3	1.7949	1	0.6982	1	0.4935	3	2.00
average	1.8612		1.5704		0.6522		0.4898		

Note: MRank is average ranking of different performance measurements of portfolio(CV, $Sharpe$, $Sortino$ and PPI).

4.3.2. *Sharpe index.* The *Sharpe* index is another commonly used reward-volatility measure which calculates the expected return per unit of risk (standard deviation) in a portfolio. It is defined as follows:

$$Sharpe = \frac{R_M(W) - r_f}{\sigma_P} \tag{18}$$

Where r_f is the risk free rate and $\sigma_P = \sqrt{V_M(W)}$ is the standard deviation of the portfolio in different models. In this study, the one-year interest rate of NTD from the Bank of Taiwan was adopted instead of r_f.

The *Sharpe* index is used to recognize how well the return of the portfolio compensates the investors for their risk taking. A higher *Sharpe* index indicates more return under the same amount of risk. Investors often prefer portfolios with a higher *Sharpe* index. Table 4 shows that the highest *Sharpe* index was found in the PONSGALPM (1.7949), followed by the PONSGAMAD (1.6698), PONSGAMV (1.6050), PONSGAMVS (1.3958) and PONSGASV (1.3866), with the GA models having the lower *Sharpe* index, in which the ranking of six to ten were GALPM, GASV, GAMVS, GAMAD and GAMV.

4.3.3. *Sortino index.* The *Sortino* index created by Rom in 1991[24] was used to evaluate the risk-adjusted return of the portfolio. It is the same as the *Sharpe* index but penalizes only those returns lower than the user-required rate of return. It is represented below:

$$SI = \frac{R_M(W) - r_f}{LPM_2} \tag{19}$$

$$LPM_2 = \frac{1}{T}\sum_{i=1}^{T}[max(0, \tau - r_{p,t})]^2 \tag{20}$$

The *SI* ratio is the per unit of downside risk over the actual rate of return in excess of the investor-specific rate of return. The ranking of five models in *SI* index were: PONSGALPM (0.6982), PONSGASV (0.6676), PONSGAMV (0.6668), PONSGAMAD (0.6322) and PONSGAMVS (0.5965), as shown in Table 4. They are higher than the GA models GASV (0.5291), GALPM (0.4968), GAMAD (0.4620), GAMV (0.3946) and GAMVS (0.3939).

4.3.4. *Portfolio performance index.* The portfolio performance index proposed by Stutzer in 2000 [25] is used to measure the investors receive risk with the loss to achieve a target return. The index is calculated as follows:

$$PPI = -ln\frac{1}{T}\sum_{i=1}^{T}e^{\theta r_p}$$

where $\theta < 0$. θ is set to negative of the mean excess return divided by its variance. *PPI* evaluates the ratio at which the probability of underperformance decays to 0. The portfolio with the fastest decay ratio is preferred when the investor signifies the aversion risk of benchmark underperformance. *PPI* captures the investor preference for positive skewness and has a distribution-free to measure the portfolio performance. The five ranking of *PPI* index were: PONSGASV (0.5696), PONSGAMV (0.5205), PONSGALPM (0.4935), PONSGAMAD (0.4804) and PONSGAMVS (0.3850). The lower *PPI* index ranking six to ten were: GALPM (0.3024), GAMV (0.2906), GAMVS (0.2725), GASV (0.2682) and GAMAD (0.1979).

In Table 4, the average *CV* of PONSGA model (1.8612) is lower than that of GA model (2.5308). It represents that the PONSGA models had lower risk and higher expected return than the GA models did. The average of *Sharpe*, *Sortino* and *PPI* of PONSGA (1.5704, 0.6522 and 0.4898) were higher than GA (0.3635, 0.4553 and 0.2663). The average rank (MRank) between different indexes of each model is also shown in Table 4. The better performance models were: PONSGALPM (2.00), PONSGAMAD (2.75), PONSGASV (3.00), PONSGAMV (3.25) and PONSGAMVS (4.00). The worse performance models were: GALPM (6.5), GASV (8.00), GAMVS (8.00), GAMV (8.50) and GAMAD (9.00). Either NSGA model or the GA model which integrated the *LPM* method (PONSGALPM and GALPM) achieved better performance. The all *CV*, *Sharpe*, *SI* and *PPI* ratios suggest that the GA forecasts are not as accurate as the PONSGA performance measurements.

4.3.5. *Efficient Portfolio.* In this experiment, 17 assets were selected in a portfolio, as shown in Table 1. The experiments were classified as two parts: one considering two dimensions, namely, expected return and different risk model (*MV*, *SV*, *MAD* and *LPM*) under GA and PONSGA approaches, and the other considering three dimensions, namely, expected return, risk and skewness of *MVS* model under the GA and PONSGA approaches. Figure 3 to Figure 7 show the PONSGA model had lower risk than the GA model did in *MV*, *SV*, *MVS*, *MAD* and *LPM* models, even though there were no significance expected return differences between the GA and PONSGA model. Moreover, a portfolio scatter to the upper left of the graph represents the combination offering the best possible expected return for given risk level. The PONSGA model provides more efficient portfolio selection than the GA does.

The set of portfolio optimal solution on three dimensions (return, risk and skewness) was plotted and is shown in Figure 8. The trade-off point of GAMVS is marked $\bullet$ in black and PONSGA is marked $\star$ in black, which represented a risky portfolio with the highest return and skewness for given risk level. It showed that the PONSGAMVS model performed better than the GAMVS did, with a lower risk and similar expected return and skewness. In these experiments, no matter whether there were two objectives or three objectives, portfolio optimization using PONSGA model outperformed the GA model.

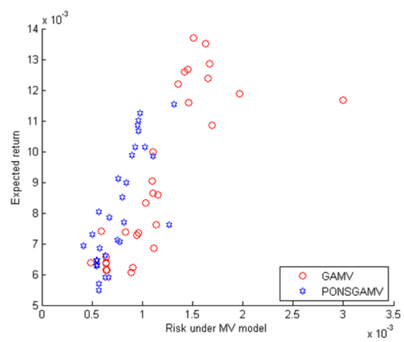

FIGURE 3. MV model

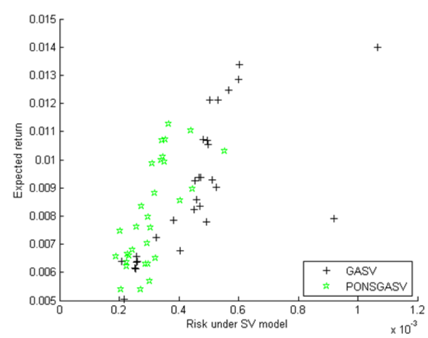

FIGURE 4. SV model

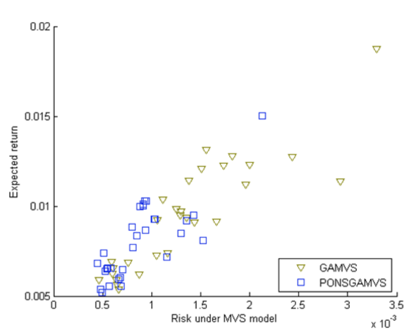

FIGURE 5. MVS model

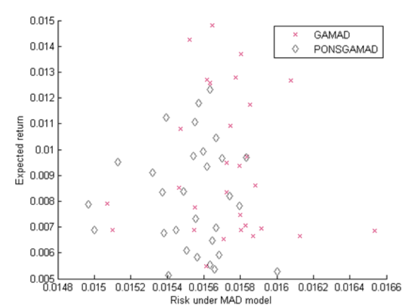

FIGURE 6. MAD model

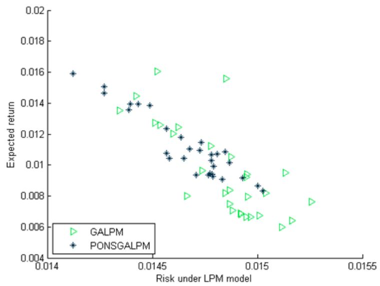

FIGURE 7. LPM model

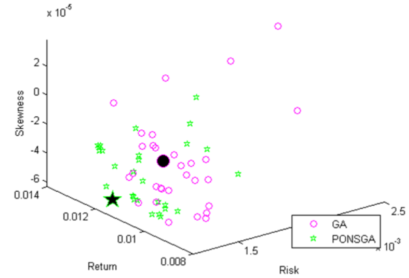

FIGURE 8. Return, risk and skewness

4.3.6. *Significance testing.* Significance testing is used to determine the probability that there is a difference in the observed samples. The significance level (α) and p-value are the criteria used for rejecting the null hypothesis. The null hypotheses were rejected in favor of alternative hypotheses, when the t-statistic (t-stat) is lower than its corresponding critical value at level (0.1, 0.05 and 0.01) or the p-value is less than the chosen level. In this study, the significance test was applied to explain whether the PONSGA model significantly outperformed the GA models under CV, $Sharpe$, SI and PPI.

Table 5 shows that the p-values of the CV index for the left-tailed test under five models are less than the value of the level of significance. The $Sharpe$, SI and PPI index adopted the right-tailed test. The p-value of the $Sharpe$ index under PONSGASV & GASV hypothesis model is significant at α=0.1 (*); the PONSGAMV & GAMV model is significant at α=0.01 (***); the other three models had level of significance at α=0.05 (**). The p-value of the SI index under five hypotheses models are less than the value of α=0.05. The p-value of PPI under PONSGASV & GASV and PONSGAMVS & GAMVS models denote results significant at 1%; the PONSGAMV & GAMV and PONSGALPM & GALPM models are less than the value of 5%; the PONSGAMAD & GAMAD model is significance at 10%.

TABLE 5. Results of the significance test

Indexes	CV		$Sharpe$		SI		PPI	
Models	p-value	t-stat	p-value	t-stat	p-value	t-stat	p-value	t-stat
PONSGAMV & GAMV	0.0004***	-3.73	0.0009***	3.42	0.0013***	3.29	0.0251**	2.04
PONSGASV & GASV	0.0001***	-4.18	0.0713*	1.51	0.0185**	2.19	0.0015***	3.25
PONSGAMVS & GAMVS	0.0000***	-4.87	0.0426**	1.78	0.0057***	2.70	0.0003***	3.91
PONSGAMAD & GAMAD	0.0000***	-5.21	0.0456**	1.75	0.0105**	2.44	0.0820*	1.43
PONSGALPM & GALPM	0.0007***	-3.51	0.0420**	1.79	0.0057***	2.70	0.0185**	2.19

Note: CV, $Sharpe$, SI and PPI denote significance at $p <= 0.1$(*), $p <= 0.05$(**), $p <= 0.01$(***).

In general, the CV ratios of five models denote results significant at 1%. The other models achieve significance above 5% except for the PPI of the PONSGAMAD & GAMAD model, and the $Sharpe$ index of the PONSGASV & GASV model. Therefore the PONSGA model provided the best explanation capability for the asset allocation in a portfolio performance measurement.

5. **Conclusions.** Expected return and risk are two important objectives with regard to portfolio optimization problems. Portfolio optimization applies state-of-the-art technology GA to find an efficient frontier. The greatest restriction of GA to multiple-objective optimization is to mix all of the objective functions into a single composite objective function. It can be difficult to precisely and accurately set each objective function weight. The NSGA-II algorithm improves the GA technique to effectively identify the optimal frontier, which satisfies all of the objectives at a reasonable level without being dominated by any other solution. This article introduced the PONSGA model to apply NSGA-II methodology to maximize both the expected return and the skewness of portfolio, while simultaneously minimizing the different portfolio risk measurements such as mean-variance, semi-variance, mean-variance-skewness, mean absolute deviation and lower partial moment.

The experimental results indicated that PONSGA had a lower coefficient of variation, a higher Sharpe index, Sortino index and PPI index, a relatively higher expected return and lower risk than the other benchmark models did. It also revealed that the efficient portfolio optimization approached more to the upper left of the graph. On the contrary, the portfolios scattering to the lower right of the graph represents that the portfolio had lower returns and higher risk. The proposed PONSGA model provided an efficient frontier with improved two objectives: the higher expected return and lower risk, or three objectives: the maximum expected return and skewness, and minimum risk than the GA benchmark model did. The statistical significance tests also showed that the PONSGA model outperformed the other models and provided the best explanation capability for asset allocation in a portfolio, at significance levels of 10%, 5% and 1%. This shows that PONSGA is an efficient portfolio optimization mechanism to help investors obtain higher returns while simultaneously lowering risk. Future studies should investigate the PONSGA model in different risk measures, such as VaR and conditional VaR (CVaR), in fat tail distributions of return.

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