# Math22

Ethan
etran@college.harvard.edu
Andy
andy_li@college.harvard.edu

Fall 2021

# Contents



# 1 Overview

Networks are a common graph structure in mathematics with vertices connected by edges, usually with the purpose of visualizing relationships between objects. The use of network graphs have been extended into other fields, and this project will focus on the use of networks in sociology. Here, networks have nodes that represent agents, which are connected by links (edges) that signify some social interaction or tie between the two agents. Further detail comes in the form of node degrees (number of connections) and strength (weight of connections) to represent networks (Porter, 1082).

A large component of networks lies in communities, which are densely connected subgroups of nodes that in the context of sociology represent extensive human interaction. These communities are discovered through community detection algorithms, which have important real world uses such connecting people with common interests over social media Even with the wonderous applications of community networks in sociology, they are still built on the fundamentals of graphs which require linear algebra for construction. Thus begs the guiding question: How does linear algebra assist in community detection and community network graphing?

# 2 Graph Theory

## 2.1 What is Graph Theory

## 2.2 Adjacency Matrices

We will define adjacency matrices, which are algebraic representations of graphs in Graph Theory (Biggs 7).

**Definition 1:** The adjacency matrix of a network is the $nxn$ matrix $A$

whose entries $a_{ij}$ are given by

$$a_{ij} = \begin{cases} 1 & \text{if } v_i \text{ and } v_j \text{ are adjacent} \\ 0 & \text{otherwise} \end{cases}$$

From these matrices, graphs can be easily constructed

# 3 Community Detection

## 3.1 What is Community Detection?

Community Detection is the process by which researchers can identify groups within a network in which individuals' membership in particular groups are yet to be ascertained. Community Detection is performed by researchers to map out the relationships between isolated nodes, members, or objects within the community. By gathering and ordering network members by the strength of their relationships to each other one can discover communities—otherwise known as groups, clusters, cohesive subgroups, or modules. This information can yield valuable insights. For example, community detection can reveal the key figures and leaders within a group, around which other members gather. The results of community detection are often represented through graphs of nodes and edges, and are generated through the application of algorithms to networks as represented by adjacency matrices.

## 3.2 Lin Alg

# 4 Real World Application

In this simple example, we have interviewed five people about who they eat with most frequently. They will rank their peers from 1-4, One the five by five matrix, zeroes or ones on the diagonal depending on if a person eats with themselves or not Strength of connection given by ranking that people say (how often they eat with someone)

# References

"Communities in Networks" by Mason A. Porter, Jukka-Pekka Onnela, and Peter J. Mucha