EE5904/ME5404 Part II

Project 1
SVM for Classification of Spam Email Messages

REPORT DUE ON 23 APRIL 2021

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Outline

Project description

Recap

Task 1 : Train

Task 2 : Test

Task 3 : Evaluate

Important Notes

Project Description

Project Goal

- Implement a SVM to classify spam or not a spam for the **Spam Email Data Set**
- Spam Email Data Set
  - **4601 samples** of email metadata taken from **UC Irvine Machine Learning Repository**
  - **57 features** per sample
  - **Label : +1 (spam), -1 (non-spam)**
  - http://archive.ics.uci.edu/ml/datasets/spambase

0.00000	0.01043	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.01043	0.01043	0.02105	0.00000	0.00000	0.00000
0.00000	0.03166	0.06332	0.00000	0.02105	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00196	0.00000	0.00000	0.02601	0.12811
0.98827							

48 continuous real [0,100] attributes of type word_freq_WORD
= percentage of words in the e-mail that match WORD,
i.e. 100 * (number of times the WORD appears in the e-mail) /
total number of words in e-mail. A "word" in this case is any
string of alphanumeric characters bounded by non-alphanumeric
characters or end-of-string.

6 continuous real [0,100] attributes of type char_freq_CHAR
= percentage of characters in the e-mail that match CHAR,
i.e. 100 * (number of CHAR occurrences) / total characters in e-mail

Project Description

Project Goal

- Implement a SVM to classify spam or not a spam for the **Spam Email Data Set**
- Spam Email Data Set
  - **4601 samples** of email metadata taken from **UC Irvine Machine Learning Repository**
  - **57 features** per sample
  - **Label : +1 (spam), -1 (non-spam)**
  - http://archive.ics.uci.edu/ml/datasets/spambase

- Dataset divided into **3 subsets**
  - Training set
  - Test set
  - Evaluation set (Not provided)

Name	Value
eval_data	*57x600 double*
eval_label	*600x1 double*
test_data	*57x1536 double*
test_label	*1536x1 double*
train_data	*57x2000 double*
train_label	*2000x1 double*

Project Description

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Train ➡ Construction: For a given training set $S = \{(\mathbf{x}_1, d_1), \ldots, (\mathbf{x}_N, d_N)\}$, find optimal hyperplane $(\mathbf{w}_\circ, b_\circ)$ such that, for all $i \in \{1, 2, \ldots, N\}$,

$$\xrightarrow{\mathbf{x}_i} \boxed{\mathbf{w}_\circ^T \mathbf{x}_i + b_\circ} \xrightarrow{g(\mathbf{x}_i)} \boxed{\mathrm{sgn}[g(\mathbf{x}_i)]} \xrightarrow{y_i = d_i}$$

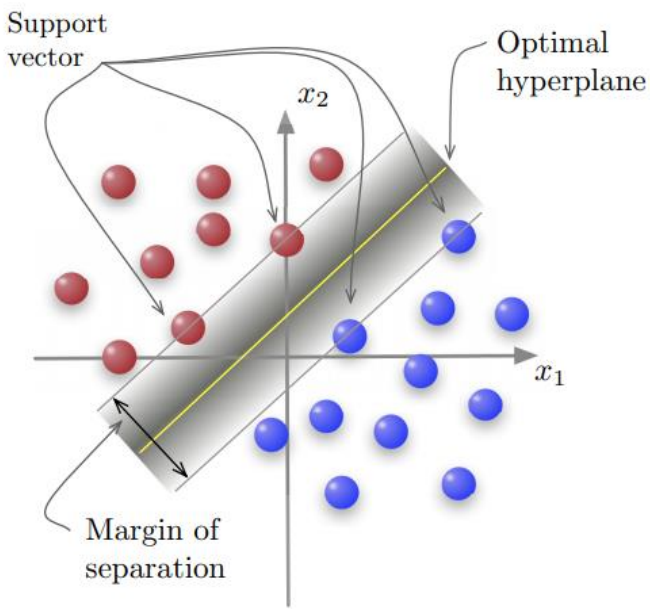
Test ➡ Testing: For a given test set $\bar{S} = \{(\bar{\mathbf{x}}_1, \bar{d}_1), \ldots, (\bar{\mathbf{x}}_{\bar{N}}, \bar{d}_{\bar{N}})\}$, compute output $\bar{y}_i$ of SVM (with $\mathbf{w}_\circ$ and $b_\circ$) for all $i \in \{1, 2, \ldots, \bar{N}\}$, and compare it against the known $\bar{d}_i$ to evaluate performance of SVM

$$\xrightarrow{\bar{\mathbf{x}}_i} \boxed{\mathbf{w}_\circ^T \bar{\mathbf{x}}_i + b_\circ} \xrightarrow{g(\bar{\mathbf{x}}_i)} \boxed{\mathrm{sgn}[g(\bar{\mathbf{x}}_i)]} \xrightarrow{\bar{y}_i}$$

Evaluate ➡ Application: Given a SVM with hyperplane $(\mathbf{w}_\circ, b_\circ)$, classify a data point $\mathbf{x}_{\text{new}}$ that is not in $\Sigma = S \cup \bar{S}$:

$$\xrightarrow{\mathbf{x}_{\text{new}}} \boxed{\mathbf{w}_\circ^T \mathbf{x}_{\text{new}} + b_\circ} \xrightarrow{g(\mathbf{x}_{\text{new}})} \boxed{\mathrm{sgn}[g(\mathbf{x}_{\text{new}})]} \xrightarrow{y_{\text{new}}}$$

Recap – Hard Margin



Discriminant function

$$g(\mathbf{x}) = \mathbf{w}_\circ^T \mathbf{x} + b_\circ$$

Support vector: $\mathbf{x}_i$ that satisfies

$$g(\mathbf{x}_i) = \pm 1$$

Primal problem

Given data set : $S = \{(\mathbf{x}_i, d_i)\}$, $i = 1, 2, \ldots, N$

Find : $\mathbf{w}$ and b

Minimizing : $f(\mathbf{w}) = \frac{1}{2}\mathbf{w}^T\mathbf{w}$

Subject to : $d_i\left(\mathbf{w}^T\mathbf{x}_i + b\right) \geq 1$

Known parameters: $\mathbf{x}_i$, d_i

Unknown variables: $\mathbf{w}$, b

Solve → Optimum hyperplane

Recap – Hard Margin

Finding optimal hyperplane (primal problem) Slide 73

Given data set : $S = \{(\mathbf{x}_i, d_i)\}$

Find : $\mathbf{w}$ and b

Minimizing : $f(\mathbf{w}) = \frac{1}{2}\mathbf{w}^T\mathbf{w}$

Subject to : $d_i\left(\mathbf{w}^T\mathbf{x}_i + b\right) \geq 1$

Alternative formulation using method of **Lagrange multipliers**

Finding optimal hyperplane (dual problem) Slide 79

Given : $S = \{(\mathbf{x}_i, d_i)\}$

Find : Lagrange multipliers $\{\alpha_i\}$

Linear kernel

Maximizing : $Q(\boldsymbol{\alpha}) = \sum_{i=1}^{N}\alpha_i - \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\,\alpha_j\,d_i\,d_j\,\mathbf{x}_i^T\mathbf{x}_j$

Subject to : (1) $\sum_{i=1}^{N}\alpha_i\,d_i = 0$

(2) $\alpha_i \geq 0$

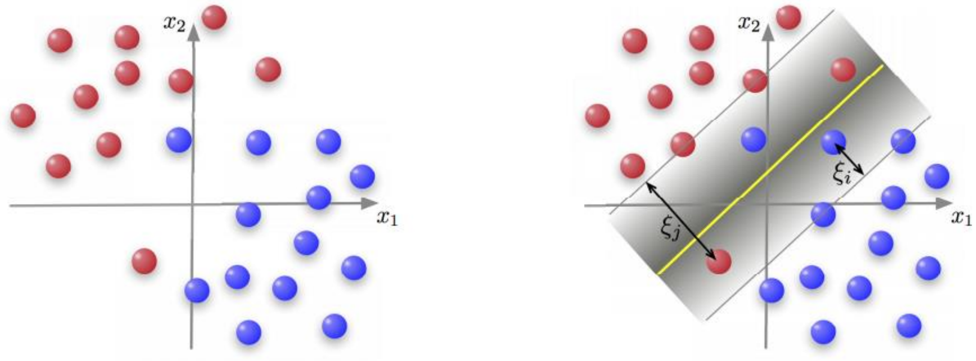
Karush-Kuhn-Tucker conditions

For data point x_i that is a support vector $\alpha_{\circ,i} \neq 0$

Recap – Soft Margin

Dealing with non-separable patterns:

1. Find optimal hyperplane to minimize classification error Slide 94



New function to be minimized

$$f(\mathbf{w}, \boldsymbol{\xi}) = \frac{1}{2}\mathbf{w}^T\mathbf{w} + C\sum_{i=1}^{N}\xi_i$$

Dual problem (with soft margin) Slide 100

Linear kernel

Find : α_i

Maximize : $Q(\boldsymbol{\alpha}) = \sum_{i=1}^{N}\alpha_i - \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j d_i d_j \mathbf{x}_i^T\mathbf{x}_j$

Subject to : $\sum_{i=1}^{N}\alpha_i d_i = 0$ and $0 \leq \alpha_i \leq C$

Soft Margin

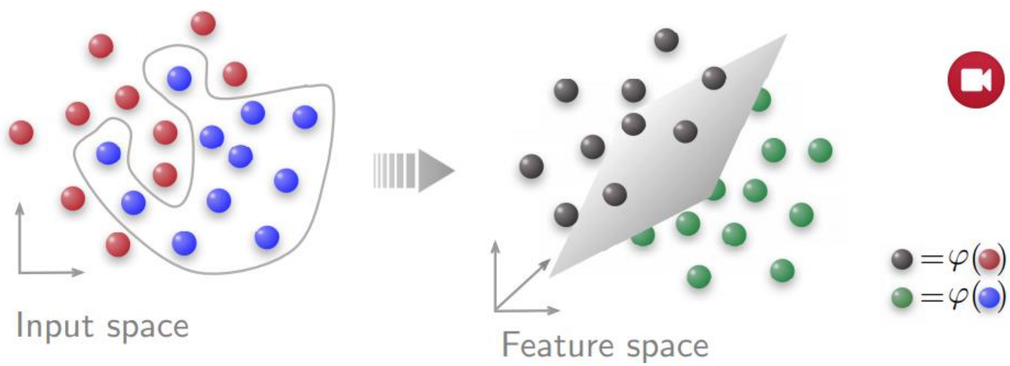
Slide 96

- Value of $C > 0$ reflects cost of violating constraints
  - A large C generally leads to smaller margin but also fewer misclassification of training data
  - A small C generally leads to larger margin but more misclassification of training data
- As a design parameter, value of C is set by user

Recap – Soft Margin

Dealing with non-separable patterns:

2. Transform data into higher dimension space for separation Slide 94



Dual problem with soft margin and transformation Slide 115

Find : α_i

Maximize : $Q(\boldsymbol{\alpha}) = \sum_{i=1}^{N} \alpha_i - \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_i \alpha_j d_i d_j \boldsymbol{\varphi}^T(\mathbf{x}_i)\boldsymbol{\varphi}(\mathbf{x}_j)$

Subject to : $\sum_{i=1}^{N} \alpha_i d_i = 0\,,\ 0 \leq \alpha_i \leq C$

Nonlinear kernel

Soft Margin

Task 1 - Data

Training set – 2000 samples

- Given – ‘train.mat’
  - Features – (57 x 2000)
  - Label (2000 x 1)

- Features of a sample

0.00000	0.01043	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.01043	0.01043	0.02105	0.00000	0.00000	0.00000
0.00000	0.03166	0.06332	0.00000	0.02105	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00196	0.00000	0.00000	0.02601	0.12811
0.98827							

- Label : +1 (spam), -1 (non-spam)

Task 1 – Training set

Import the training set (i.e. train.mat)

- train_data (57 x 2000)
- train_label (2000 x 1)

Preprocess the ‘data’ (Various methods can be used including Sample scaling and Standardization [CHOOSE ONE METHOD])[a, b]

- Scale the data – Rescale the individual sample x such that $||x|| = 1$
- Standardize the data – Transform each feature by removing the mean value of each feature and then dividing by each feature’s standard deviation

Please ensure the ‘label’ is mapped into the set of {-1, +1}

[a] https://scikit-learn.org/stable/modules/preprocessing.html
[b] https://en.wikipedia.org/wiki/Feature_scaling

Task 1 – Kernels

Hard-margin SVM with the linear kernel

$K(x_1, x_2) = x_1^T x_2$

Hard-margin SVM with a polynomial kernel

$K(x_1, x_2) = (x_1^T x_2 + 1)^p$

Soft-margin SVM with a polynomial kernel

$K(x_1, x_2) = (x_1^T x_2 + 1)^p$

Task 1 – Hard and Soft Margins

Hard margin $0 \leq \alpha_i$

- C = + ∞ (In theory)

  $0 \leq \alpha_i \leq C$

- C = Large value (In practice e.g. 10^6)

Soft margin $0 \leq \alpha_i \leq C$

- C = 0.1, 0.6, 1.1, 2.1

Task 1 – Calculate α_i

How to calculate α_i

Maximize : $Q(\boldsymbol{\alpha}) = \sum_{i=1}^{N} \alpha_i - \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_i \alpha_j d_i d_j K(\mathbf{x}_i, \mathbf{x}_j)$

Subject to : $\sum_{i=1}^{N} \alpha_i d_i = 0\,,\ 0 \leq \alpha_i \leq C$

Use quadprog function (Quadratic programming)

Description

Solver for quadratic objective functions with linear constraints.

quadprog finds a minimum for a problem specified by

$$\min_x \frac{1}{2} x^T H x + f^T x \text{ such that } \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq, \\ lb \leq x \leq ub. \end{cases}$$

H, A, and Aeq are matrices, and f, b, beq, lb, ub, and x are vectors.

You can pass f, lb, and ub as vectors or matrices; see Matrix Arguments.

`x = quadprog(H,f,A,b,Aeq,beq,lb,ub,x0,options)` solves the preceding problem using the optimization options specified in `options`. Use `optimoptions` to create `options`. If you do not want to give an initial point, set `x0 = []`.

Task 1 – Calculate α_i - quadprog

Maximize : $Q(\boldsymbol{\alpha}) = \sum_{i=1}^{N} \alpha_i - \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_i \alpha_j d_i d_j K(\mathbf{x}_i, \mathbf{x}_j)$

Subject to : $\sum_{i=1}^{N} \alpha_i d_i = 0\,,\ 0 \le \alpha_i \le C$

Description

Solver for quadratic objective functions with linear constraints.

quadprog finds a minimum for a problem specified by

$$\min_x \frac{1}{2} x^T H x + f^T x \text{ such that } \begin{cases} A \cdot x \le b, \\ Aeq \cdot x = beq, \\ lb \le x \le ub. \end{cases}$$

Convert the problem from 'Max' to 'Min'

- Max Q(α) → Min - Q(α)

If f is to be maximized instead, such a maximization problem can be expressed as a minimization problem by the transformation

Slide 62

$$\max_{\mathbf{w}} f(\mathbf{w}) = -\min_{\mathbf{w}} [-f(\mathbf{w})]$$

Task 1 – Calculate α_i - quadprog

Maximize : $Q(\boldsymbol{\alpha}) = \sum_{i=1}^{N} \alpha_i - \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_i \alpha_j d_i d_j K(\mathbf{x}_i, \mathbf{x}_j)$

Subject to : $\sum_{i=1}^{N} \alpha_i d_i = 0$, $0 \le \alpha_i \le C$

Description

Not used

Solver for quadratic objective functions with linear constraints.

quadprog finds a minimum for a problem specified by

$$\min_x \frac{1}{2} x^T H x + f^T x \text{ such that } \begin{cases} A \cdot x \le b, \\ Aeq \cdot x = beq, \\ lb \le x \le ub. \end{cases}$$

$$\min_x \frac{1}{2} x^T H x + f^T x \quad \begin{cases} H_{ij} = d_i d_j K(x_i, x_j) \\ f = (-1, -1, \cdots\cdots, -1)^T \end{cases}$$

$$A \cdot x \le b, \quad \begin{cases} A = [\,] \\ b = [\,] \end{cases}$$

$$Aeq \cdot x = beq, \quad \begin{cases} Aeq = (d_1, d_2, \cdots\cdots, d_N) \\ beq = 0 \end{cases}$$

$$lb \le x \le ub. \quad \begin{cases} lb = (0, 0, \cdots\cdots, 0)^T \\ \text{ub} = (C, C, \cdots\cdots, C)^T \end{cases}$$

```
x = quadprog(H,f,A,b,Aeq,beq,lb,ub,x0,options)
```

Task 1 – Calculate α_i - quadprog

```
x = quadprog(H,f,A,b,Aeq,beq,lb,ub,x0,options)
```

Hard-margin SVM with the linear kernel

$$K(x_1, x_2) = x_1^T x_2$$

For illustration only

$$\min_x \frac{1}{2} x^T H x + f^T x \quad \begin{cases} H(i,j) = d_i d_j x_i^T x_j \\ f = -\text{ones}(2000,1) \end{cases}$$

$$A \cdot x \le b, \quad \begin{cases} A = [\,] \\ b = [\,] \end{cases}$$

$$Aeq \cdot x = beq, \quad \begin{cases} Aeq = train\_label' \\ beq = 0 \end{cases}$$

$$lb \le x \le ub. \quad \begin{cases} lb = zeros(2000,1) \\ ub = \text{ones}(2000,1) * C \end{cases}$$

Hard margin $0 \le \alpha_i$

C = $+\infty$ (In theory)

C = Large value (In practice e.g. 10^6)

$$x0 = [\,] \qquad options = optimset('LargeScale','off','MaxIter',1000)$$

Task 1 – Calculate α_i - quadprog

```
x = quadprog(H,f,A,b,Aeq,beq,lb,ub,x0,options)
```

Hard-margin SVM with a polynomial kernel

$$K(x_1, x_2) = (x_1^T x_2 + 1)^p$$

For illustration only

$$\min_x \frac{1}{2} x^T H x + f^T x \quad \begin{cases} H(i,j) = d_i d_j (x_1^T x_2 + 1)^p \\ f = -\text{ones}(2000,1) \end{cases}$$

$$A \cdot x \le b, \quad \begin{cases} A = [\,] \\ b = [\,] \end{cases}$$

$$Aeq \cdot x = beq, \quad \begin{cases} Aeq = train\_label' \\ beq = 0 \end{cases}$$

$$lb \le x \le ub. \quad \begin{cases} lb = zeros(2000,1) \\ ub = \text{ones}(2000,1) * C \end{cases}$$

Hard margin $0 \le \alpha_i$

C = + ∞ (In theory)

C = Large value (In practice e.g. 10^6)

$$x0 = [\,] \qquad options = optimset('LargeScale','off','MaxIter',1000)$$

Task 1 – Calculate α_i - quadprog

```
x = quadprog(H,f,A,b,Aeq,beq,lb,ub,x0,options)
```

Soft-margin SVM with a polynomial kernel

$$K(x_1, x_2) = (x_1^T x_2 + 1)^p$$

For illustration only

$$\min_x \frac{1}{2} x^T H x + f^T x \quad \begin{cases} H(i,j) = d_i d_j (x_1^T x_2 + 1)^p \\ f = -\text{ones}(2000,1) \end{cases}$$

$$A \cdot x \le b, \quad \begin{cases} A = [\,] \\ b = [\,] \end{cases}$$

$$Aeq \cdot x = beq, \quad \begin{cases} Aeq = train\_label' \\ beq = 0 \end{cases}$$

$$lb \le x \le ub. \quad \begin{cases} lb = zeros(2000,1) \\ ub = \text{ones}(2000,1) * \text{C} \end{cases}$$

Soft margin $0 \le \alpha_i \le C$

C = 0.1, 0.6, 1.1, 2.1

$$x0 = [\,] \qquad options = optimset('LargeScale','off','MaxIter',1000)$$

Task 1 – Select support vectors

Based on KKT conditions

- For a support vector, $\alpha_i \neq 0$ (In theory, $\alpha_i > 0$)

- However, in practice, $\alpha_i > threshold$

- How to decide?
  - Choose an appropriate threshold (e.g. 1e-4) to determine the corresponding α_i to the support vectors

Task 1 – Discriminant function g(x)

Hard Margin SVM with Linear Kernel

Maximizing : $Q(\boldsymbol{\alpha}) = \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j d_i d_j \mathbf{x}_i^T \mathbf{x}_j$

Subject to : (1) $\sum_{i=1}^{N} \alpha_i d_i = 0$

(2) $\alpha_i \geq 0$

Discriminant function

$$g(\mathbf{x}) = \mathbf{w}_\circ^T \mathbf{x} + b_\circ$$

After $\alpha_{\circ,i}$ is obtained, we can calculate $\mathbf{w}_\circ$ and $b_\circ$ as follows: Slide 85

$$\mathbf{w}_\circ = \sum_{i=1}^{N} \alpha_{\circ,i} d_i \mathbf{x}_i \,, \quad b_\circ = \frac{1}{d^{(s)}} - \mathbf{w}_\circ^T \mathbf{x}^{(s)}$$

where $\mathbf{x}^{(s)}$ is a support vector with label $d^{(s)}$

Task 1 – Discriminant function g(x)

Soft Margin SVM with Linear Kernel

Maximize : $Q(\boldsymbol{\alpha}) = \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j d_i d_j \mathbf{x}_i^T \mathbf{x}_j$

Subject to : $\sum_{i=1}^{N} \alpha_i d_i = 0$ and $0 \le \alpha_i \le C$

Discriminant function

$$g(\mathbf{x}) = \mathbf{w}_\circ^T \mathbf{x} + b_\circ$$

After $\alpha_{\circ,i}$ is obtained, we can calculate $\mathbf{w}_\circ$ as follows:

Slide 104,105

$$\mathbf{w}_\circ = \sum_{i=1}^{N} \alpha_{\circ,i} \, d_i \, \mathbf{x}_i$$

After $\mathbf{w}_\circ$ is obtained, we can calculate $b_\circ$ as follows:

1. For each example $\mathbf{x}_i$ with $0 < \alpha_i \le C$,

$$b_{\circ,i} = \frac{1}{d_i} - \mathbf{w}_\circ^T \mathbf{x}_i$$

2. Take $b_\circ$ as the average of all such $b_{\circ,i}$

$$b_\circ = \frac{\sum_{i=1}^{m} b_{\circ,i}}{m}$$

where m is the total number of $\mathbf{x}_i$ with $0 < \alpha_i \le C$.

Task 1 – Discriminant function g(x)

Soft Margin SVM with Nonlinear Kernel

Maximize : $Q(\boldsymbol{\alpha}) = \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j d_i d_j \boldsymbol{\varphi}^T(\mathbf{x}_i) \boldsymbol{\varphi}(\mathbf{x}_j)$

Subject to : $\sum_{i=1}^{N} \alpha_i d_i = 0\,,\ 0 \le \alpha_i \le C$

Discriminant function

$$g(\mathbf{x}) = \sum_{i=1}^{N} \alpha_{\circ,i} d_i K(\mathbf{x}, \mathbf{x}_i) + b_\circ$$

Determine $b_\circ$ in Slide 123

$$g(\mathbf{x}) = \sum_{i=1}^{N} \alpha_{\circ,i} d_i K(\mathbf{x}, \mathbf{x}_i) + b_\circ$$

using the fact that for a support vector $\mathbf{x}^{(s)}$

$$g(\mathbf{x}^{(s)}) = \pm 1 = d^{(s)}$$

Take $b_\circ$ as the average of all such $b_{\circ,i}$

$$b_\circ = \frac{\sum_{i=1}^{m} b_{\circ,i}}{m}$$

where m is the total number of $\mathbf{x}_i$ with $0 < \alpha_i \le C$.

Task 1 – Summary

Given a training set $S = \{(\mathbf{x}_i, d_i)\}$, $i = 1, \ldots, N$

Slide 123

1 Find a suitable kernel

Choose expression then check Mercer's condition

2 Choose a value for C

3 Solve for $\alpha_{\circ,i}$

4 Determine $b_\circ$ in

$$g(\mathbf{x}) = \sum_{i=1}^{N} \alpha_{\circ,i} d_i K(\mathbf{x}, \mathbf{x}_i) + b_\circ$$

using the fact that for a support vector $\mathbf{x}^{(s)}$

$$g(\mathbf{x}^{(s)}) = \pm 1 = d^{(s)}$$

Kernel

Soft Margin

Quadratic programming

Maximize : $Q(\boldsymbol{\alpha}) = \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j d_i d_j K(\mathbf{x}_i, \mathbf{x}_j)$

Subject to : $\sum_{i=1}^{N} \alpha_i d_i = 0\,,\ 0 \le \alpha_i \le C$

Support vector machine: $\mathbf{x} \rightarrow \boxed{\operatorname{sgn}[g(\mathbf{x})]} \xrightarrow{y}$

Task 2 - Data

Test set – 1536 samples

- Given – ‘test.mat’
  - Features – (57 x 1536)
  - Label (1536 x 1)

- Features of a sample

0.00000	0.01043	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.01043	0.01043	0.02105	0.00000	0.00000	0.00000
0.00000	0.03166	0.06332	0.00000	0.02105	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00196	0.00000	0.00000	0.02601	0.12811
0.98827							

- Label : +1 (spam), -1 (non-spam)

Task 2 – Test set

Import the test set (i.e. test.mat)

- test_data (57 x 1536)
- test_label (1536 x 1)

Preprocess the 'data' (Various methods can be used including Sample scaling and Standardization [CHOOSE ONE USE for TRAINING])[a, b]

- Scale the data – Rescale the individual sample x such that $||x|| = 1$

- Standardize the data – Transform each feature in the same manner with the training data. Use the mean and variance of each feature from your training set.

Please ensure the 'label' is mapped into the set of {-1, +1}

[a] https://scikit-learn.org/stable/modules/preprocessing.html
[b] https://en.wikipedia.org/wiki/Feature_scaling

Task 2 – Test set

Discriminant function

$$g(\mathbf{x}) = \mathbf{w}_\circ^T \boldsymbol{\varphi}(\mathbf{x}) + b_\circ = \sum_{i=1}^{N} \alpha_{\circ,i} d_i \underbrace{\boldsymbol{\varphi}^T(\mathbf{x}_i)\boldsymbol{\varphi}(\mathbf{x})}_{K(\mathbf{x}_i,\mathbf{x})} + b_\circ$$

To classify a new data point $\mathbf{x}_{\text{new}}$

$$d_{\text{new}} = \text{sgn}\left[g(\mathbf{x}_{\text{new}})\right]$$

<u>For illustrations only</u>

$$g(x_{test}) = \sum_{i=1}^{N} \alpha_{o,i} d_i K(x_i, x_{test}) + b_0$$

If g(x_test) > 0

    x_test_label = +1

Else

    x_test_label = -1

Task 2 – Test set

Discriminant function

$$g(\mathbf{x}) = \mathbf{w}_\circ^T \varphi(\mathbf{x}) + b_\circ = \sum_{i=1}^{N} \alpha_{\circ,i} d_i \underbrace{\varphi^T(\mathbf{x}_i)\varphi(\mathbf{x})}_{K(\mathbf{x}_i,\mathbf{x})} + b_\circ$$

To classify a new data point $\mathbf{x}_{\text{new}}$

$$d_{\text{new}} = \text{sgn}\left[g(\mathbf{x}_{\text{new}})\right]$$

Type of SVM	Training accuracy				Test accuracy			
Hard margin with Linear kernel	?				?			
Hard margin with polynomial kernel	$p=2$	$p=3$	$p=4$	$p=5$	$p=2$	$p=3$	$p=4$	$p=5$
	?	?	?	?	?	?	?	?
Soft margin with polynomial kernel	$C=0.1$	$C=0.6$	$C=1.1$	$C=2.1$	$C=0.1$	$C=0.6$	$C=1.1$	$C=2.1$
$p=1$	?	?	?	?	?	?	?	?
$p=2$	?	?	?	?	?	?	?	?
$p=3$	?	?	?	?	?	?	?	?
$p=4$	?	?	?	?	?	?	?	?
$p=5$	?	?	?	?	?	?	?	?

If g(x_test) > 0

    x_test_label = +1

Else

    x_test_label = -1

Task 3 - Data

Evaluation set – 600 samples

- Not Given – 'eval.mat'
  - eval_data (57 x 600)
  - eval_label (600 x 1)

Task 3 - Evaluation

Design your own SVM

- Hard margin or Soft margin?
- Linear or Polynomial kernel?
- What are the values for p and C?

→ To produce the best performance

To classify the 600 samples in the evaluation set

Not Given – 'eval.mat'

eval_data (57 x 600)

eval_label (600 x 1)

Output : A column vector (600 x 1) named 'eval_predicted'

Task 3 - Evaluation

Hardcode the discriminant function g(x) in the file for evaluation

- If necessary, store the required variables in a separate *.mat file and load at the beginning of the code

Prepare the code so that it could handle the evaluation dataset and able to preprocess the evaluation dataset

- Note: the eval_data is a (57 x 600) matrix

Your code should generate a column vector (600 x 1) named 'eval_predicted'

Task 3 - Evaluation

Please name your m-file for Task 3 as ‘svm_main.m’

Do Not clear any variable in the ‘svm_main’ script

Before submitting your code, please ensure that the code runs without errors by testing it with a dummy data set (Dummy dataset can be created using the training set and test set)

Important Notes

Preprocess your data – Choose one method

- Sample scaling/ Mean normalization/ standardization/ Rescaling …

Use the training set statistics to preprocess the other data sets

Check Mercer Condition for Kernel suitability

Important Notes

Procedure to build SVM

- Preprocess data
- Choose a suitable kernel
  - Linear/ Nonlinear ?
- Choose C
  - Hard margin/ Soft margin
    - Hard margin $0 \leq \alpha_i$
      - C = $+\infty$ (In theory)
      - C = Large value (In practice e.g. 10^6)
- Solve for α_i
  - Quadratic programming
- Support vector selection
  - Choose an appropriate threshold (e.g. 1e-4) to determine the corresponding α_i to the support vectors
- Determine the discriminant function g(x)

Important Notes : Submission

Submit all your codes that you have implemented for the entire project

Make sure your codes run without error

All codes should be executable with the given datasets in the workspace without any additional inputs

Important Notes : Submission

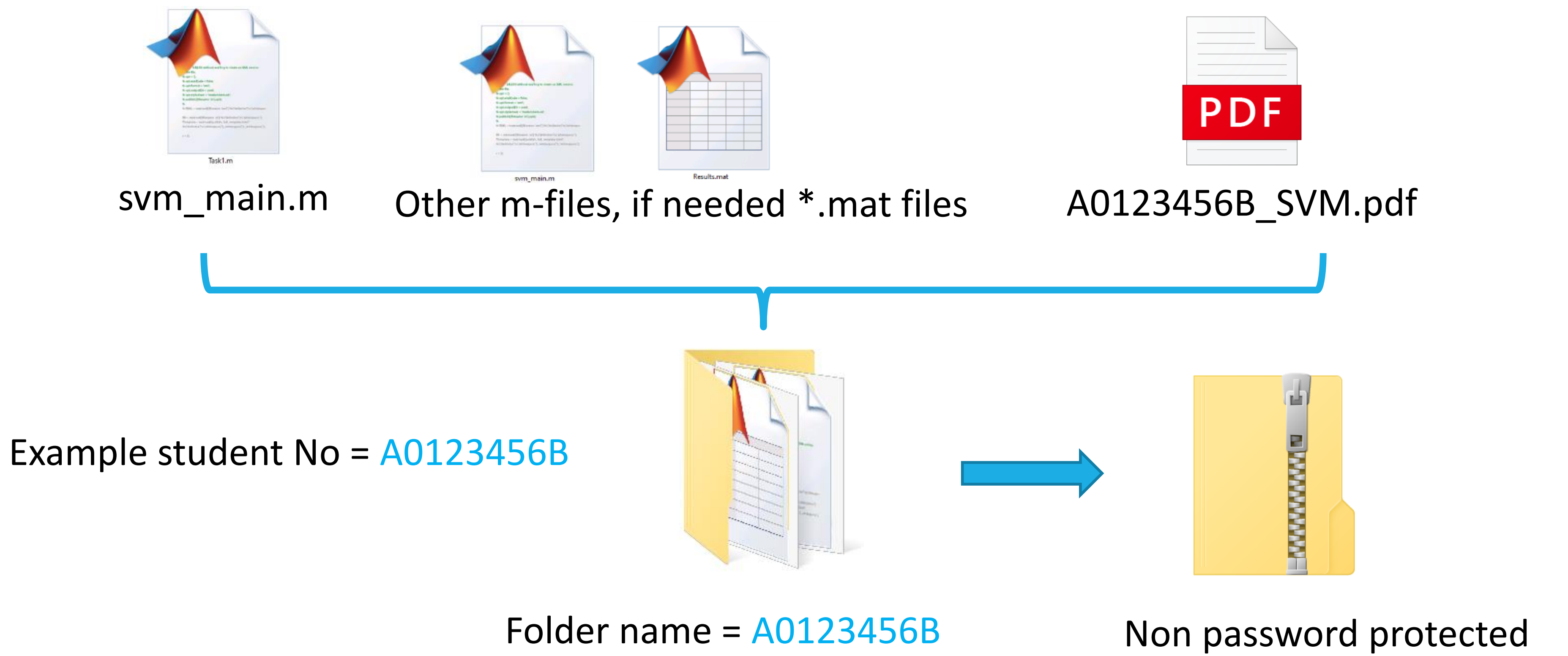
Report

- Details on Implementation
- Completed Table 1
- Discuss the results and their implications
  - Admissibility of the kernels
  - Exitance of optimal hyperplanes
  - Comments on results (with supporting arguments)

Type of SVM	Training accuracy				Test accuracy			
Hard margin with Linear kernel	?				?			
Hard margin with polynomial kernel	$p=2$	$p=3$	$p=4$	$p=5$	$p=2$	$p=3$	$p=4$	$p=5$
	?	?	?	?	?	?	?	?
Soft margin with polynomial kernel	$C=0.1$	$C=0.6$	$C=1.1$	$C=2.1$	$C=0.1$	$C=0.6$	$C=1.1$	$C=2.1$
$p=1$	?	?	?	?	?	?	?	?
$p=2$	?	?	?	?	?	?	?	?
$p=3$	?	?	?	?	?	?	?	?
$p=4$	?	?	?	?	?	?	?	?
$p=5$	?	?	?	?	?	?	?	?

TABLE I: Results of SVM classification.

Important Notes : Submission



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