1. **Найти предел последовательности:**

a.

$$\lim_{x\to 6}\frac{x^2-36}{x^2-x-30}=\left(\frac{0}{0}\right)=\frac{\cancel{(x-6)}(x+6)}{\cancel{(x-6)}(x+5)}=\frac{12}{11}$$

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b.

$$\lim_{x\to 7}\frac{x^2-49}{x^2-13x+42}=\left(\frac{0}{0}\right)=\frac{\cancel{(x-7)}(x+7)}{\cancel{(x-7)}(x-6)}=\frac{14}{1}$$

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c. *

$$\lim_{x\to 7}\frac{\sqrt{x+2}-\sqrt[3]{x+20}}{\sqrt[4]{x+9}-2}=\left(\frac{0}{0}\right)$$

$$=\frac{\sqrt{x+2}-\sqrt[3]{x+20}}{\sqrt[4]{x+9}-2}\cdot\frac{\sqrt{x+2}^2+\sqrt[3]{x+20}\cdot\sqrt{x+2}+\sqrt[3]{x+20}^2}{\sqrt{x+2}^2+\sqrt[3]{x+20}\cdot\sqrt{x+2}+\sqrt[3]{x+20}^2}\cdot$$

$$\cdot\frac{\left(\sqrt[4]{x+9}^3+2\sqrt[4]{x+9}^2+4\sqrt[4]{(x+9)}\quad+8\right)}{\left(\sqrt[4]{x+9}^3+2\sqrt[4]{x+9}^2+4\sqrt[4]{(x+9)}\quad+8\right)}=$$

$$=\frac{\sqrt{x+2}^3-(x+20)}{(x+9-16)}\cdot\frac{\left(\sqrt[4]{x+9}^3+2\sqrt[4]{x+9}^2+4\sqrt[4]{(x+9)}\quad+8\right)}{\sqrt{x+2}^2+\sqrt[3]{x+20}\cdot\sqrt{x+2}+\sqrt[3]{x+20}^2}$$

$$\cdot\frac{\sqrt{x+2}^3+(x+20)}{\sqrt{x+2}^3+(x+20)}$$

$$=\frac{(x+2)^3-(x+20)^2}{(x-7)}\cdot$$

$$\cdot\frac{\left(\sqrt[4]{x+9}^3+2\sqrt[4]{x+9}^2+4\sqrt[4]{(x+9)}\quad+8\right)}{(\sqrt{x+2}^2+\sqrt[3]{x+20}\cdot\sqrt{x+2}+\sqrt[3]{x+20}^2)\cdot(\sqrt{x+2}^3+(x+20))}$$

$$=\frac{(x+2)^3-(x+20)^2}{(x-7)}\cdot b=\frac{x^3+5x^2-28x-392}{x-7}\cdot b$$

$$=\frac{\cancel{(x-7)}(x^2+12x+56)}{\cancel{x-7}}\cdot b$$

$$:=\frac{(x^2+12x+56)\left(\sqrt[4]{x+9}^3+2\sqrt[4]{x+9}^2+4\sqrt[4]{(x+9)}\quad+8\right)}{(\sqrt{x+2}^2+\sqrt[3]{x+20}\cdot\sqrt{x+2}+\sqrt[3]{x+20}^2)\cdot(\sqrt{x+2}^3+(x+20))}=\frac{112}{27}$$

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d.

$$\lim_{x\to 0}\frac{3x\cdot tg4x}{1-cos4x}=\left(\frac{0}{0}\right)=\frac{3x\cdot 4x}{2\cdot sin^2 2x}=\frac{12x^2}{8x^2}=\frac{3}{2}$$

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e. **

$$\lim_{x\to 0}\frac{\sqrt{2}x^2\cdot sin4x}{(1-cos2x)^{3/2}}=\left(\frac{0}{0}\right)=\frac{\sqrt{2}x^2\cdot 4x}{(1-(1-2sin^2x))^{3/2}}=\frac{4\sqrt{2}x^3}{(2x^2)^{3/2}}=\frac{4\sqrt{2}x^3}{2\sqrt{2}x^3}=\pm 2$$

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f.

$$\lim_{x\to\infty}\left(\frac{4x}{4x+3}\right)^{\frac{5x^2}{7x-1}} = (1^{+\infty}) = \lim_{x\to\infty}\left(\frac{4x+3-3}{4x+3}\right)^{\frac{5x^2}{7x-1}}$$

$$= \lim_{x\to\infty}\left(1+\frac{(-3)}{4x+3}\right)^{\frac{4x+3}{(-3)}\cdot\frac{(-3)}{4x+3}\cdot\frac{5x^2}{7x-1}} = e^{\frac{(-3)}{4x+3}\cdot\frac{5x^2}{7x-1}} = e^{-\frac{15}{28}}$$

---

g. *

$$\lim_{x\to\infty}\frac{ln(x^2-x+1)}{ln(x^{10}+x+1)} = \left(\frac{\infty}{\infty}\right) = \frac{2ln|x| + ln(1-\frac{x+1}{x^2})}{10ln|x| + ln(1+\frac{x+1}{x^2})} = \frac{2+\frac{ln(1-\frac{x+1}{x^2})}{ln|x|}}{10+\frac{ln(1+\frac{x+1}{x^2})}{ln|x|}} = \frac{1}{5}$$

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h. *

$$\lim_{x\to+0}\frac{(5^x-1)}{x} = \left(\frac{\infty}{\infty}\right) = \frac{e^{ln(5^x)}-1}{x} = \frac{e^{x\cdot ln(5)}-1}{\frac{x\cdot ln(5)}{ln(5)}} = \frac{1}{\frac{1}{ln5}} = ln5$$

На языке Python предложить алгоритм вычисляющий численно предел последовательности:

$$\lim_{x\to+\infty}\frac{n}{\sqrt[n]{n!}} = \left(\frac{\infty}{\infty}\right) = \frac{n}{\sqrt[n]{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n}} = \frac{e}{\sqrt[n]{\sqrt{2\pi n}}} \to n! \text{ по формуле Стерлинга}$$

В [3]:

```
import math
def func_limit(eps_exponent):
    eps = 10**(-eps_exponent)
    f = 0
    n = 1
    delta = 1
    while delta > eps:
        func = math.e/((math.sqrt(2*math.pi*n))**(1/n))
        delta = func - f
        f = func
        n += 1
    print('limit ', func)
    print(f'число вычислений:{n}, при точности:{eps}')
```

executed in 14ms, finished 16:27:17 2021-02-26

В [7]:

```
% % time
func_limit(10)
```

executed in 311ms, finished 16:28:23 2021-02-26

```
limit  2.718235356975357
число вычислений:433352, при точности:1e-10
Wall time: 294 ms
```