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CHAPTER 5

NONLINEAR CONTROL DESIGN FOR MOBILE ROBOTS

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Abstract

Much research in the field of mobile robots has been carried out in mechanical design, posture detection methods, vision, path planning, control design, etc. Higher levels of task specifications rely on the tracking and regulation capabilities of the feedback control design. This chapter provides an overview of nonlinear control design for mobile robots. Structural properties, such as controllability, feedback linearizability and feedback stabilizability of the system under consideration, are discussed. Control strategies designed to stabilize the motion of mobile robots about either a time-indexed trajectory, a geometric path, or a fixed point in the configuration-space of the robot are presented and compared via theoretical discussions and simulations.

1 Introduction

Wheeled Mobile Robots (WMR) have received much consideration in the last decade. WMR are used in the industry as a means of transport, inspection and operation. They are also useful in environments with clusters of obstacles and in hostile environments. Much research has been carried out in mechanical design, posture detection methods, vision, path planning, control design, etc. Higher levels of task specifications rely on the tracking and regulation capabilities of the feedback control design. The design of control laws has been developed to stabilize the motion of mobile robots about either a time-indexed trajectory, a geometric path, or a fixed point in the configuration-space of the robot. The first two problems have received solutions involving relatively classical nonlinear control techniques [14], [24] and [21]. The problem of stabilization about a fixed point in the configuration-space proved to be a more difficult problem. This is due to the non-existence of stabilizing smooth pure state-space feedback laws for the nonlinear models representing nonholonomic WMR. As shown in [24], this negative result follows from a theorem due to Brockett [2]. Hence, for nonholonomic mobile robots, the problem of stabilization about a given configuration cannot be solved as a special case of path following or tracking. Nevertheless, recent solutions to the tracking problem have suggested an alternative control technique, little studied until then, which consists in using *time-varying* feedback laws in order to achieve smooth point-stabilization. This possibility was first pointed out in [22]. There exists a second alternative which also does not violate Brockett's theorem. It consists of using discontinuous or piecewise smooth controllers. Work on the control design of discontinuous controllers for mechanical systems was initiated in [4] and [3] . In [5], the kinematic model of a mobile robot with two inputs and three states was considered. The control law in [5] is a piecewise continuous pure state-feedback law capable, in the ideal case, of making the mobile robot exponentially converge to the desired posture.

The purpose of this chapter is to provide an overview of several controllers, recently proposed, and compare them via simulations and theoretical discussions. The paper is organized as follows: In Section 2 structural properties, such as controllability, feedback linearizability and feedback stabilizability of the system under consideration, are discussed. Subsequent sections deal with the problem of designing feedback control laws for three classes of problems:

- Tracking : Section 3
- Path following : Section 4
- Point stabilization : Section 5

The following kinematic model for a wheeled mobile robot, assumed to be of the unicycle type, is considered throughout this chapter

$$\begin{aligned} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= \omega \end{aligned} \tag{1.1}$$

where the triplet $z = (x, y, \theta)$ describes the position and the orientation of the robot with respect to a fixed frame. The velocity inputs are the translational (or advancement) velocity v and the angular velocity ω. Although this is a simplified model of the vehicle's motion (motor dynamics, elastic deformations of ties and other mechanical effects are neglected) it is sufficient to capture the nonholonomy property which characterizes most WMR and is the core of the difficulties involved in the control problems discussed hereafter.

2 Model Properties

2.1 *Model Transformations*

Control design may in some cases be facilitated by a preliminary change of state coordinates which transforms the model equations of the robot into a simpler "canonical" form.

For instance, the following change of coordinates [22]:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & -\cos\theta & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} \tag{2.1}$$

together with the following change of inputs:

$$\begin{aligned} u_1 &= \omega \\ u_2 &= v - \omega x_3. \end{aligned} \tag{2.2}$$

transforms the system (1.1) into:

$$\begin{aligned} \dot{x}_1 &= u_1 \\ \dot{x}_2 &= u_2 \\ \dot{x}_3 &= x_2 u_1 \end{aligned} \tag{2.3}$$

This system belong to the more general class of so-called *chained* systems [15], [16] characterized by equations in the form:

$$\begin{aligned} \dot{x}_1 &= u_1 \\ \dot{x}_2 &= u_2 \\ \dot{x}_3 &= x_2 u_1 \\ &\vdots \\ \dot{x}_n &= x_{n-1} u_1 \end{aligned} \tag{2.4}$$

Chained systems are of particular interest in the field of mobile robotics because the modeling equations of several nonholonomic systems (unicycle-type and car-like vehicles pulling trailers, for example) can locally be transformed into this form [15].

An alternative *local* change of coordinates is:

$$\begin{aligned} x_1 &= x \\ x_2 &= \tan\theta \\ x_3 &= y \end{aligned} \tag{2.5}$$

associated with the following change of control inputs:

$$u_1 = \cos\theta v, \qquad u_2 = \frac{1}{\cos^2\theta}\omega \tag{2.6}$$

This yields the same system (2.3).

However, the transformation presents in this case a singularity when $\cos\theta = 0$. It is thus only valid in domains where $\theta \in (-\frac{\pi}{2} + k\pi, +\frac{\pi}{2} + k\pi)$, $(k \in \mathbb{Z})$.

Depending on the transformation which is considered, a control law derived for the model (2.3) will yield different transient behaviors of the trajectories for $x(t)$, $y(t)$, and $\theta(t)$.

2.2 Controllability

A system is said to be controllable when it can be steered from any state configuration to any other configuration in finite-time by using finite inputs, [26]. As opposed to linear systems, controllability of nonlinear systems does not imply the existence of stabilizing smooth static state feedbacks.

For driftless nonlinear systems in the form

$$\dot{z} = \sum_{i=1}^{m} g_i(z)u_i \quad z \in I\!R^n, \quad u \in I\!R^m$$

a sufficient condition for controllability is that the dimension of the involutive closure of the distribution generated by the vector fields g_i be equal to n, $\forall z$ (accessibility rank condition), i.e.

$$\dim\{\mathrm{inv}\Delta\} = n; \qquad \Delta \triangleq \mathrm{span}\{g_i\} \tag{2.7}$$

For the system (1.1), the input vector fields are

$$g_1 = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}, g_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

and we have

$$rank\{g_1, g_2, [g_1, g_2]\} = rank \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & -\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = 3 \tag{2.8}$$

Therefore, $dim\{inv\Delta\} = 3$ and the system is controllable.

Since the controllability property is not affected by diffeomorphic changes of coordinates, the model (2.3) is also controllable.

This mathematical verification of the vehicle's controllability just confirms the common experience and knowledge that unicycle-type mobile robots can be steered to any position and orientation on a flat ground.

2.3 Tangent Linearization

For many nonlinear systems, linear approximations can be used as a basis for a first simple control design. Linearization may also provide indications concerning the

controllability and feedback stabilizability of the nonlinear system. More precisely, if the tangent linearized system is controllable, then the original nonlinear system is controllable and feedback stabilizable, at least locally. But the converse is not true. In our case, the tangent linearization of system (1.1) about the equilibrium point $(z = 0, u = 0)$, gives

$$\dot{z} = G(0)\begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} v \\ \omega \end{pmatrix} \tag{2.9}$$

This linear system is not controllable since the rank of the associated controllability matrix

$$\mathcal{C} = G(0) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{2.10}$$

is only two.

Information about the nonlinear model is thus lost when using the linearized model.

2.4 Full-State Feedback Linearization

A nonlinear system $\dot{z} = f(z, u)$ is said to be full-state linearizable via static feedback if there exists a change of coordinates

$$\xi = T(z) \tag{2.11}$$

and a feedback law ν of the form

$$\nu = a(z)\begin{pmatrix} v \\ \omega \end{pmatrix} + b(z) \tag{2.12}$$

which transforms the original system into a linear system. Necessary and sufficient conditions for which this linearization is possible are given in [18], [13]. For driftless systems such as (1.1), the result is the following:

A system $\dot{z} = \sum_{i=1}^{m} g_i(z)u_i$, is locally full-state linearizable about $z = 0$ if and only if:

$$rank[g_1(0), g_2(0), \ldots, g_m(0)] = n \tag{2.13}$$

This condition is clearly not satisfied when $m < n$, i.e. when the system's dimension exceeds the number of inputs. Since this condition is equivalent to the controllability of the tangent linearized model, see Equation (2.10), full-state linearization of unicycle-type mobile robots is not possible.

2.5 Smooth State Feedback Stabilization

The problem of smooth state feedback stabilization can be formulated as follows: find a feedback $u = k(z)$, where $k(z)$ is a smooth function of z, such that the closed-loop system,

$$\dot{z} = G(z)k(z) = f(z) \tag{2.14}$$

is asymptotically stable (i.e. such that the solutions $z(t)$ asymptotically converge to zero for any initial $z(0)$ in the neighborhood of zero). As mentioned earlier, a nonlinear system can be controllable without being feedback stabilizable, in the sense of the above definition. In [2], Brockett gives necessary conditions for smooth-feedback stabilizability. In the particular case of driftless systems, a corollary to Brockett's conditions is the following:

Corollary 2.1 (Brockett 83) *Consider a driftless system of the form*

$$\dot{z} = \sum_{i=1}^{m} g_i(z)u_i \quad z \in I\!R^n \ , \ u \in I\!R^m, \ m \leq n.$$

where the g_i are smooth vector fields. If the vectors $g_i(0)$ are linearly independent, i.e.

$$rank[g_1(0), g_2(0), \cdots, g_m(0)] = m \tag{2.15}$$

then a solution to the above stabilization problem exists if and only if $m = n$.

Note that the condition $m = n$ is again equivalent to the controllability of the linearized system. Since it is not satisfied in the present case ($n = 3$, $m = 2$), stabilizing smooth feedback laws $u = k(z)$ do not exist for the considered mobile robot.

This negative result has recently motivated the exploration of new control structures other than smooth pure state feedback laws. Such structures have the characteristic of being time-varying ($u = k(z,t)$, with $k(z,t)$ a smooth function) or piecewise continuous ($u = k(z)$, with $k(z)$ piecewise continuous function). These control structures will be presented in the last section of this chapter when returning to the stabilization problem. Before this, the simpler problems of tracking and path following will be addressed.

3 Tracking

Definition 3.1 (Tracking) *Consider a virtual reference unicycle-type vehicle the equations of which are*

$$\begin{aligned} \dot{x}_r &= v_r \cos\theta_r \\ \dot{y}_r &= v_r \sin\theta_r \\ \dot{\theta}_r &= \omega_r \end{aligned} \tag{3.1}$$

The subscript "r" stands for reference, and v_r and ω_r are assumed to be bounded and have bounded derivatives. The tracking problem, under the following assumption,

$$\mathbf{A_1} : v_r(t) \not\to 0 \quad or \quad \omega_r(t) \not\to 0 \quad when \ t \to +\infty \tag{3.2}$$

is to find a feedback control law $\begin{pmatrix} v \\ \omega \end{pmatrix} = k(z, z_r, v_r, \omega_r)$, *such that*

$$\lim_{t\to\infty}(z(t) - z_r(t)) = 0 \tag{3.3}$$

Note that assumption $\mathbf{A_1}$, implies that the reference vehicle is not at rest all the time. Hence, stabilization to a fixed posture is not included in the above tracking problem definition.

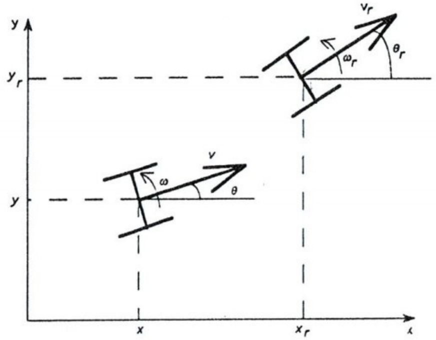

Figure 3.1: Illustration of the tracking problem.

The tracking problem so defined involves error equations which describe the time evolution of the difference $z - z_r$. In [14] and [21], the following change of coordinates is used

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{pmatrix} \tag{3.4}$$

e_1 and e_2 are now the coordinates of the position error vector expressed in the basis of a frame linked to the mobile robot.

The associated tracking error equations are then obtained by differentiating the equality (3.4). Introducing the following change of inputs:

$$\begin{aligned} u_1 &= -v + v_r \cos e_3 \\ u_2 &= \omega_r - \omega \end{aligned}$$

this gives after simple calculations:

$$\dot{e} = \begin{pmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} e + \begin{pmatrix} 0 \\ \sin e_3 \\ 0 \end{pmatrix} v_r + \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \tag{3.5}$$

3.1 Linear Feedback Design

Linearization of system (3.5) about the equilibrium point $(e = 0, u = 0)$ yields the following linear time-varying system

$$\dot{e} = \begin{pmatrix} 0 & \omega_r(t) & 0 \\ -\omega_r(t) & 0 & v_r(t) \\ 0 & 0 & 0 \end{pmatrix} e + \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \tag{3.6}$$

If v_r and ω_r are constant, one falls upon a time-invariant linear system the controllability matrix of which is

$$\mathcal{C} = [B, AB, A^2B] = \begin{bmatrix} 1 & 0 & 0 & 0 & -\omega_r^2 & v_r\omega_r \\ 0 & 0 & -\omega_r & v_r & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.7}$$

In this case, it is simple to verify that the linearized model is controllable provided that either v_r or ω_r is different from zero. When the reference vehicle is at rest $(v_r = \omega_r = 0)$, controllability of the linearized model is lost. Due to this fact, attention should be paid to the manner in which the linear control law is designed. For instance, a linear state feedback obtained by imposing closed-loop poles independent of v_r and ω_r may become ill-conditioned when $|v_r|$ and $|\omega_r|$ tend to become small.

In order to be more specific, let us consider the following linear feedback law:

$$\begin{aligned} u_1 &= -k_1 e_1 \\ u_2 &= -k_2 \mathrm{sgn}(v_r) e_2 - k_3 e_3 \end{aligned} \tag{3.8}$$

calculated so as to set the system's closed-loop poles equal to the roots of the following characteristic polynomial equation:

$$(s + 2\xi a)(s^2 + 2\xi as + a^2) = 0$$

where ξ and a are positive real numbers. The corresponding control gains are

$$\begin{aligned} k_1 &= 2\xi a \\ k_2 &= \frac{a^2 - \omega_r^2}{|v_r|} \\ k_3 &= 2\xi a \end{aligned}$$

With a fixed pole placement strategy (a and ξ are constant), the control gain k_2 increases without bound when v_r tends to zero.

Regularization of the controller is possible by letting the closed-loop poles depend on the values of v_r and ω_r. This procedure will be called *velocity scaling*. Choose, for example, $a = (\omega_r^2 + bv_r^2)^{\frac{1}{2}}$ with $b > 0$. The control gains then become

$$\begin{aligned} k_1 &= 2\xi(\omega_r^2 + bv_r^2)^{\frac{1}{2}} \\ k_2 &= b|v_r| \\ k_3 &= 2\xi(\omega_r^2 + bv_r^2)^{\frac{1}{2}} \end{aligned} \tag{3.9}$$

and the resulting control is now defined for all values of v_r and ω_r. The particular values $v_r = \omega_r = 0$ for which the system is not controllable simply yield zero control action, which makes sense intuitively.

3.2 Nonlinear Feedback Design

It is possible to design nonlinear feedbacks for the nonlinear model (3.5) so as to enlarge the domain where asymptotic stability is granted and cover the case when v_r and ω_r are time varying. Such controllers have been proposed in [14], [21]. One of them, given in [21], is:

$$\begin{aligned} u_1 &= -k_1(v_r, \omega_r) e_1 \\ u_2 &= -k_4 v_r \frac{\sin e_3}{e_3} e_2 - k_3(v_r, \omega_r) e_3 \end{aligned} \tag{3.10}$$

where k_4 is a positive constant and $k_1(.)$ and $k_3(.)$ are continuous functions, strictly positive on $\mathbb{R} \times \mathbb{R} - (0,0)$.

Note the resemblance between this control and the linear control (3.8)-(3.9) derived in the previous section. It can be utilized in the choice of $k_1(\cdot)$ and $k_3(\cdot)$ to have the two controls behave in the same way near the origin $e = 0$.

Proposition 3.1 *Under Assumption* $\mathbf{A_1}$, *the control (3.10) globally asymptotically stabilizes the origin* $e = 0$.

Proof: Consider the Lyapunov function:

$$V(e) = \frac{k_4}{2}(e_1^2 + e_2^2) + \frac{e_3^2}{2}$$

which is non-increasing along any system's solution since

$$\begin{aligned} \dot{V} &= k_4 e_1(u_1 + \omega e_2) + k_4 e_2(v_r \sin e_3 - \omega e_1) + e_3 u_2 \\ &= -k_1 k_4 e_1^2 - k_3 e_3^2 \end{aligned}$$

Along a system's solution, $||e(t)||$, and thus $||\dot{e}(t)||$, are bounded. Since $v_r(t)$ and $\omega_r(t)$, and their time-derivatives, are bounded (by assumption), $k_1(v_r(t), \omega_r(t))$ and $k_3(v_r(t), \omega_r(t))$ are uniformly continuous. As a consequence, $\dot{V}(t)$ is also uniformly continuous. Moreover, $V(t)$ does not increase and thus converges to some limit value, denoted as V_{lim}. From Barbalat's Lemma, $\dot{V}(t)$ tends to zero. This in turn implies, omitting the time index from now on, that $k_1(v_r,\omega_r)e_1$ and $k_3(v_r,\omega_r)e_3$ tend to zero. Using the properties of $k_1(\cdot)$ and $k_3(\cdot)$, one deduces that $(v_r^2+\omega_r^2)e_1^2$ and $(v_r^2+\omega_r^2)e_3^2$ tend to zero. In fact, e_1 and e_3 unconditionally converge to zero if $k_1(\cdot)$ and $k_3(\cdot)$ are chosen strictly positive on $\mathbb{R}\times\mathbb{R}$.

From the third system's equation, and using some of the above results:

$$\dot{e}_3 = -k_4 v_r \frac{\sin(e_3)}{e_3} e_2 + o(t) \quad \text{with} \lim_{t\to+\infty} o(t) = 0$$

Hence, using the fact that $v_r e_3$ tends to zero:

$$\frac{d}{dt}(v_r^2 e_3) = -k_4 v_r^3 \frac{\sin(e_3)}{e_3} e_2 + o(t)$$

$v_r^3\frac{\sin(e_3)}{e_3}e_2$ is uniformly continuous (since its time derivative is bounded). From Barbalat's Lemma (slightly generalized), $\frac{d}{dt}(v_r^2 e_3)$ tends to zero. Therefore, $v_r^3\frac{\sin(e_3)}{e_3}e_2$ also tends to zero. Now, since $v_r e_3$ tends to zero, $v_r^2((\frac{\sin(e_3)}{e_3})^2+e_3^2)e_2^2$ tends to zero. Since $((\frac{\sin(e_3)}{e_3})^2+e_3^2)$ is strictly larger than some positive number, $v_r e_2$ tends to zero.

From the first system's equation, and using the convergence of u_1 and u_2 to zero:

$$\dot{e}_1 = \omega_r e_2 + o(t)$$

Hence, using the fact that $\omega_r e_1$ tends to zero:

$$\frac{d}{dt}(\omega_r^2 e_1) = \omega_r^3 e_2 + o(t)$$

$\omega_r^3 e_2$ is uniformly continuous (its time-derivative is bounded). Thus, from Barbalat's Lemma, $\frac{d}{dt}(\omega_r^2 e_1)$ tends to zero. Therefore, $\omega_r^3 e_2$, and thus $\omega_r e_2$, tend to zero.

Finally, it has been shown that $(v_r^2+\omega_r^2)e_i^2$ $(i=1,2,3)$ tend to zero. Therefore $(v_r^2+\omega_r^2)V(e)$ tends to zero, with $V(e)$ converging to V_{lim}. Since $(v_r^2+\omega_r^2)$ does not tend to zero (assumption $\mathbf{A}_1$), V_{lim} is necessarily equal to zero.

(end of proof)

◇

Remark 3.1 *Comparing the expressions of the linear controller (3.8)(3.9) and the nonlinear controller (3.10) suggests using the following functions:*

$$k_1(v_r,\omega_r) = k_3(v_r,\omega_r) = 2\xi(\omega_r^2 + b v_r^2)^{\frac{1}{2}}, \text{ with } k_4 = b.$$

4 Path Following

Path following has been studied by several authors [17], [20], [6], [25], [29]. We present here the approach developed in [25].

The mobile robot and the path to be followed, denoted as $\mathcal{P}$, are represented in Figure 4.1.

M is the "orthogonal projection" of the robot's point P on $\mathcal{P}$. This point exists and is uniquely defined if $\mathcal{P}$ satisfies some conditions and the distance between the robot and $\mathcal{P}$ is not "too large" (see [25]).

$(P;\vec{\eta_T},\vec{\eta_N})$ is a Frenet frame moving along the path.

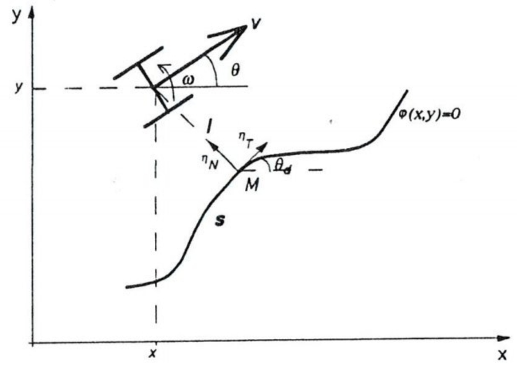

Figure 4.1: Illustration of the path following problem.

l is the signed distance between M and P, i.e. $\vec{MP} = l\vec{\eta_N}$.

s is the signed curvilinear distance along the path, from some initial point to the point M.

$\theta_d(s)$ is the angle between the x-axis and the tangent to the path at the point M.

$c(s)$ is the path's curvature at the point M, assumed to be uniformly bounded and differentiable.

$\tilde{\theta} = \theta - \theta_d$ is the orientation error.

The variables s, l, and $\tilde{\theta}$ constitute a new set of state coordinates for the mobile robot. Note that they coincide with x, y, and θ in the particular case where the path $\mathcal{P}$ coincides with the x-axis.

Using this parametrization, it is rather simple to verify that the following kinematic equations hold:

$$\begin{aligned} \dot{s} &= v\cos\tilde{\theta}/(1-c(s)l) \\ \dot{l} &= v\sin\tilde{\theta} \\ \dot{\tilde{\theta}} &= \omega - v\cos(\tilde{\theta})c(s)/(1-c(s)l) \end{aligned} \tag{4.1}$$

The path following problem can then be postulated as follows:

Definition 4.1 (Path following) *Given a path $\mathcal{P}$ in the xy-plane and the mobile robot translational velocity $v(t)$, assumed to be bounded together with its time-derivative $\dot{v}(t)$, the path following problem consists of finding a (smooth) feedback control law $\omega = k(s, l, \tilde{\theta}, v(t))$ such that:*

$$i) \lim_{t\to+\infty} l(t) = 0$$

$$ii) \lim_{t\to+\infty} \tilde{\theta}(t) = 0$$

Note that the path following problem, as stated above, is less stringent than the tracking problem, in the sense that only the stabilization of the coordinates l and $\tilde{\theta}$ is required. On the other hand, this objective must be achieved by using only one control variable, namely the angular velocity ω.

Introducing the auxiliary control variable:

$$u = \omega - v\cos(\tilde{\theta})c(s)/(1-c(s)l) \tag{4.2}$$

the equations (4.1) are rewritten as :

$$\begin{aligned} \dot{s} &= v\cos\tilde{\theta}/(1-c(s)l) \\ \dot{l} &= v\sin\tilde{\theta} \\ \dot{\tilde{\theta}} &= u \end{aligned} \tag{4.3}$$

4.1 Linear Feedback Design

In the neighborhood of $(l = 0, \tilde{\theta} = 0)$, tangent linearization of the two last system's equations gives:

$$\begin{aligned} \dot{l} &= v\,\tilde{\theta} \\ \dot{\tilde{\theta}} &= u \end{aligned} \tag{4.4}$$

When v is constant and different from zero, this linear system is clearly controllable, and thus asymptotically stabilizable by using linear state feedbacks. In fact, it is rather simple to verify that stabilizing linear feedbacks are in the form:

$$u = -k_2 v l - k_3|v|\tilde{\theta} \quad \text{with } k_2 > 0 \text{ and } k_3 > 0 \tag{4.5}$$

The closed-loop equation for the output l is then:

$$\ddot{l} + k_3|v|\dot{l} + k_2 v^2 l = 0 \tag{4.6}$$

It may also be written as:

$$l'' + k_3 l' + k_2 l = 0 \tag{4.7}$$

with $l' = \frac{\partial l}{\partial \gamma}$ and $\gamma = \int_0^t |v| d\tau$. In the first approximation, γ represents the distance gone by the point M along the path.

The transformation of equation (4.6) into (4.7) is related to the velocity scaling procedure already evoked in Section 3, and the second order linear equation (4.7) suggests selecting the control gains k_2 and k_3 according to:

$$\begin{aligned} k_2 &= a^2 \\ k_3 &= 2\xi a \end{aligned} \tag{4.8}$$

where a must be selected so as to specify the transient "rise distance" (the equivalent of the rise time in the case of time equations), and ξ is the damping coefficient (critical damping is obtained by setting $\xi = 1/\sqrt{2}$).

4.2 Nonlinear Feedback Design

Instead of the linear control (4.5) previously derived, consider the nonlinear control:

$$u = -k_2 v l \frac{\sin\tilde{\theta}}{\tilde{\theta}} - k(v)\tilde{\theta} \tag{4.9}$$

where k_2 is a positive constant, and $k(v)$ is a continuous function strictly positive when $v \neq 0$.

In order to have the two controls behave similarly near $(l = 0, \tilde{\theta} = 0)$ one may choose, for example, $k(v) = k_3|v|$ with k_2 and k_3 given by (4.8).

Proposition 4.1 *Under the assumption:*

$$\mathbf{A_2}: \quad \lim_{t\to\infty} v(t) \neq 0 \tag{4.10}$$

the control (4.9) asymptotically stabilizes $(l = 0, \tilde{\theta} = 0)$, provided that the vehicle's initial configuration is such that:

$$l(0)^2 + \frac{1}{k_2}\tilde{\theta}(0)^2 < \frac{1}{\limsup(c(s)^2)} \tag{4.11}$$

The last condition is needed in the proof to ensure that $(1-c(s)l)$ remains positive (larger than some positive number) and avoid singularities due to the parameterization.

Proof: Consider the Lyapunov function:

$$V = k_2 \frac{l^2}{2} + \frac{\tilde{\theta}^2}{2}$$

Taking the time-derivative of this function along a solution to the closed-loop system:

$$\begin{aligned} \dot{V} &= k_2 l \dot{l} + \tilde{\theta}\dot{\tilde{\theta}} \\ &= k_2 l \sin\tilde{\theta} v + \tilde{\theta} u \\ &= -k(v)\tilde{\theta}^2 \quad (\leq 0) \end{aligned}$$

By invoking arguments similar to the ones used in the proof of Proposition 3.1 (boundedness of l and $\tilde{\theta}$, convergence of V to a limit value V_{lim}, and convergence of $\dot{V}$ to zero), one obtains that $k(v)\tilde{\theta}$ and $v\tilde{\theta}$ tend to zero.

Then, in view of the control expression and the last system's equation:

$$\dot{\tilde{\theta}} = -k_2 v l \frac{\sin\tilde{\theta}}{\tilde{\theta}} + o(t) \quad \text{with} \lim_{t\to+\infty} o(t) = 0$$

Therefore, using the convergence of $v\tilde{\theta}$ to zero and the boundedness of $\dot{v}$:

$$\frac{d}{dt}(v^2\tilde{\theta}) = -k_2 v^3 l \frac{\sin\tilde{\theta}}{\tilde{\theta}} + o(t)$$

Since $v^3 l \frac{\sin\tilde{\theta}}{\tilde{\theta}}$ is uniformly continuous (its time derivative is bounded), $\frac{d}{dt}(v^2\tilde{\theta})$ tends to zero by application of Barbalat's Lemma. Thus $vl\frac{\sin\tilde{\theta}}{\tilde{\theta}}$ also tends to zero. This in turn implies that $v^2 l^2((\frac{\sin\tilde{\theta}}{\tilde{\theta}})^2 + \tilde{\theta}^2)$ tends to zero. Since $((\frac{\sin\tilde{\theta}}{\tilde{\theta}})^2 + \tilde{\theta}^2)$ is larger than some positive real number, vl tends to zero.

Finally, from the convergence of $v\tilde{\theta}$ and vl to zero, one deduces that vV, and thus vV_{lim}, tend to zero. Now, since v does not tend to zero (assumption A_2), V_{lim} must be equal to zero.

(end of proof)

◇

Note that the vehicle's location along the path is characterized by the value of s (the distance gone along the path) and thus depends on the translational velocity v which is not used as a control in the case of path following. This degree of freedom will be used later to stabilize s about a prescribed value s_d. This complementary

problem clearly brings us back to the problem of stabilization about an arbitrary given posture. It will be treated in the next section.

Two path following examples and simulations are now presented.

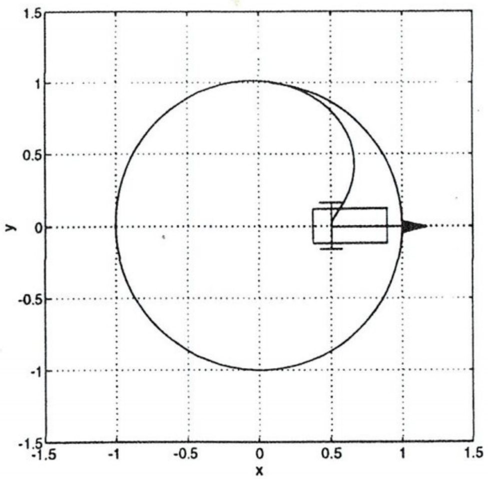

Figure 4.2: Simulated result when following a circle.

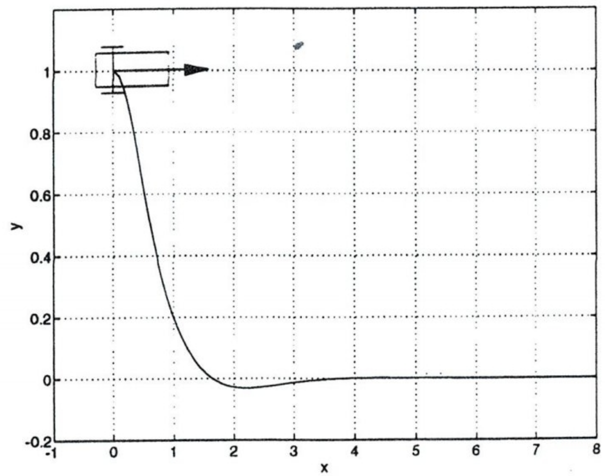

Figure 4.3: Simulated result when following the x-axis.

Example 4.1. [circle following] The path $\mathcal{P}$ is a circle with radius equal to one ($c(s) = 1$, $\forall s$). In the simulation, the translational velocity v is kept constant and equal to one, $k_2 = 4$, and $k(v)$ is chosen equal to $k_3|v|$ with $k_3 = 2\sqrt{2}$. The initial values for l and $\tilde{\theta}$ are $l(0) = -0.5$ and $\tilde{\theta}(0) = 0$. The resulting robot's trajectory in the xy-plane is presented in Figure 4.2. $\bigtriangledown\bigtriangledown\bigtriangledown$

Example 4.2. [straight line following] In this case, $c(s) = 0$, $\forall s$. v, k_2, and $k(v)$ are chosen as in the previous example.The initial values for l and $\tilde{\theta}$ are $l(0) = 1$ and $\tilde{\theta}(0) = 0$. The resulting robot's trajectory in the xy-plane is presented in Figure 4.3. $\bigtriangledown\bigtriangledown\bigtriangledown$

5 Stabilization About a Desired Posture

Definition 5.1 (Point-stabilization) *Given an arbitrary posture z_d, the problem is to find a control law* $\begin{pmatrix} v \\ \omega \end{pmatrix} = k(z - z_d, t)$, *which asymptotically stabilizes $z - z_d$ about zero, whatever the initial robot's posture $z(0)$. Without loss of generality, we may take $z_d = 0$.*

Recall that, from Corollary 2.1, there is no smooth control law $k(z)$ that can solve the point-stabilization problem for the class of systems considered in this chapter.

Three alternatives, the exploration of which is still the object of active research, will be considered here:

- Smooth (differentiable) time-varying nonlinear feedbacks $k(z,t)$.
- Piecewise continuous control laws $k(z)$.
- Time-varying piecewise continuous control laws $k(z,t)$.

For each category, control examples illustrated by simulation results will be given. Corresponding asymptotic convergence rates will also be discussed.

5.1 Smooth Time-Varying Controllers

The possibility of stabilizing a nonholonomic mobile robot (with restricted mobility) about a desired posture by using smooth time-varying feedbacks was first pointed out in ([22]). In this reference, the considered vehicle was also of the unicycle type and the proposed control was, in its simplest form:

$$\begin{aligned} v &= -k_1 x \\ \omega &= -g(t)y - k_3\theta \end{aligned} \tag{5.1}$$

with:

- x and y: the cartesian coordinates, expressed in the basis of a frame linked to the mobile robot, of the point P located on the actuated wheels' axle.
- θ: the orientation of the vehicle's main body.
- k_1 and k_3: positive numbers.
- $g(t)$: a bounded function of class C^1 such that $\frac{\partial g}{\partial t}(t)$ does not tend to zero when t tends to infinity. For example, $g(t) = \sin(t)$.

This control globally asymptotically stabilizes the origin ($x = 0, y = 0, \theta = 0$). However, it appeared in simulation (see [24], for example) that the asymptotic convergence rate, for the state variables, was not better than $1/\sqrt{t}$ for most initial configurations. In other words, instead of the usual exponential stability associated with stable linear systems and characterized by the inequality:

$$||z(t)|| \leq a||z(0)||\exp(-bt) \quad \text{for some positive numbers a, b and c}$$

one can only expect to have with this type of control:

$$||z(t)|| \leq c||z(0)||\sqrt{\frac{1}{t+1}}$$

Nevertheless, it is important to realize that the asymptotic convergence rate is not, by itself, sufficient to properly evaluate the overall control performance. For instance, the time needed to reach a small neighborhood of zero is not directly linked to the asymptotic convergence rate. This time can be reasonably short, while the final convergence phase is slow. In the case of the control (5.1), for example, the time required to reach a small neighborhood to zero can be much reduced by replacing the term $g(t)y$ by another time-varying function $g(y,t)$. An example of such a function is given ([25]). Another possibility will be pointed out further in this section.

The aforementioned time-varying feedback stabilization result has subsequently motivated several studies, mostly within the Automatic Control community. For example, general existence results can be found in ([9]) ([10]), and explicit time-varying feedbacks for a class of nonlinear systems are derived in ([19]). Research work has also been initiated to better understand the convergence properties of such feedbacks ([12]) ([8]).

From these studies (and others), it appears that various methods can be used to derive smooth stabilizing time-varying feedbacks. The one chosen here consists of extending the tracking and path following problems discussed before to the more difficult point-stabilization problem. This two-steps method presents, in our opinion, some advantages. It gives the intuition of where and why time dependence is needed in the control expression. It shows that solutions to the point-stabilization problem are at hand once the tracking and path following problems have been solved. And, most importantly for the practitioner, it shows that the controls calculated for tracking or path following can also be used without modification for point-stabilization.

5.1.1 Extension of the Tracking Problem

Consider the tracking problem described in Section 3, and assume that the virtual reference vehicle moves along a path which passes through the point $(x_d = 0, y_d = 0)$. Assume also that the tangent to the path at this point coincides with the x-axis. The point-stabilization problem may then be treated as a tracking problem (convergence of the tracking errors to zero) with the additional requirement that the reference vehicle should itself be asymptotically stabilized about the desired configuration.

To simplify, we will assume here, although this is not technically necessary, that the reference vehicle moves along the x-axis. Therefore, $y_r(t) = 0$, $\forall t$. We will further assume that $\theta_r(t) = 0$, $\forall t$ $(\Rightarrow \omega_r(t) = 0, \forall t)$.

Consider now the tracking nonlinear control law proposed in Section 3:

$$\begin{aligned} u_1 &= -k_1(v_r, \omega_r)e_1 \\ u_2 &= -k_2 v_r \frac{\sin(e_3)}{e_3} e_2 - k_3(v_r, \omega_r)e_3 \end{aligned} \qquad (5.2)$$

which has been shown to be globally stabilizing when either $v_r(t)$ or $\omega_r(t)$ does not tend to zero.

The idea consists of using v_r (equal to $\dot{x}_r$ in this case) as a new control variable the selection of which must be made so as to ensure both convergence of the tracking errors e_1, e_2, and e_3 to zero and convergence of the reference vehicle's coordinate x_r to zero, when the control (5.2) is used.

A possible choice is the following time-varying control:

$$v_r = -k_4 x_r + g(e, t) \qquad (5.3)$$

where k_4 is a positive number and $g(e, t)$ is a function of class C^{p+1} $(p \geq 1)$, uniformly bounded with respect to t, with partial derivatives up to the order p also uniformly bounded with respect to t, and such that:

C1: $g(0, t) = 0$, $\forall t$

C2: There exists a diverging time-sequence $\{t_i\}_{i \in N}$ and a continuous function $\alpha(\cdot)$ such that

$$||e|| > \epsilon > 0 \Rightarrow \sum_{j=1}^{j=p} \left(\frac{\partial^j g}{\partial t^j}(e, t_i)\right)^2 > \alpha(\epsilon) > 0 \quad , \forall t_i$$

Proposition 5.1 *The control law (5.2), with v_r calculated according to (5.3), globally asymptotically stabilizes the point $(e = 0, x_r = 0)$ and thus solves the point-stabilization problem.*

Proof: The proof is performed by considering an arbitrary solution to the closed-loop system. All functions involved in the proof should thus be seen as time-functions. However, in order to lighten the notations, the time index will be systematically omitted.

Since the Lyapunov function V, used in the proof of Proposition 3.1, is non-increasing the tracking errors e_i $(i = 1, 2, 3)$ are bounded. Therefore $g(e, t)$ (read $g(e(t), t)$) is bounded. Equation (5.3) may then be interpreted as the equation of a stable linear system subjected to the additive bounded perturbation $g(e, t)$. The state x_r associated with this equation remains bounded, and, according to (5.3), v_r is also bounded.

By taking the time-derivative of (5.3), it can be shown in the same way that $\dot{v}_r$ is bounded.

Since v_r and $\dot{v}_r$ are bounded, the Proposition 3.1 applies to the present situation. In particular, if v_r does not tend to zero, then e must tend to zero. By uniform continuity, and by using the condition C_1, $g(e, t)$ tends to zero. Now, in view of (5.3), x_r and v_r must also tend to zero, yielding a contradiction. Therefore, v_r tends to zero.

By taking the time-derivative of (5.3) and using the unconditional convergence of $\dot{e}$ to zero (as proven in the proof of Proposition 3.1):

$$\dot{v}_r = \frac{\partial g}{\partial t}(e, t) + o(t) \qquad \text{with} \quad \lim_{t \to +\infty} o(t) = 0$$

Since $\frac{\partial g}{\partial t}(e, t)$ is uniformly continuous (its time-derivative is bounded), and since v_r tends to zero, $\dot{v}_r$ tends to zero (by application of Barbalat's Lemma). Therefore, $\frac{\partial g}{\partial t}(e, t)$ tends to zero.

By taking the time-derivative of $\frac{\partial g}{\partial t}(e, t)$, using the convergence of $\dot{e}$ to zero, and applying Barbalat's Lemma, one obtains that $\frac{\partial^2 g}{\partial t^2}(e, t)$ tends to zero. Repeating the same procedure as many times as necessary, one obtains that $\frac{\partial^j g}{\partial t^j}(e, t)$ tends to zero, for $1 \leq j \leq p$.

Now, V is non-increasing and converges to some limit value denoted as V_{lim}. If $V_{lim} \neq 0$, there exists a positive real number ϵ such that $||e(t)|| > \epsilon > 0$, $\forall t$. Hence, in view of the condition C_2, $\sum_{j=1}^{j=p}(\frac{\partial^j g}{\partial t^j}(e(t_i), t_i))^2 > \alpha(\epsilon) > 0$. This contradicts the previously established fact that $\sum_{j=1}^{j=p}(\frac{\partial^j g}{\partial t^j}(e(t), t))^2$ tends to zero. The only alternative is $V_{lim} = 0$, which proves that e, and subsequently $g(e, t)$, tend to zero. Finally, one deduces from equation (5.3), that x_r also tends to zero.

(end of proof)
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Note that the time-varying function $g(e,t)$ does not need to depend upon e_1 and e_3 when the functions $k_1(v_r,\omega_r)$ and $k_3(v_r,\omega_r)$ are chosen strictly positive on $I\!R \times I\!R$. The reason is that e_1 and e_3 unconditionally converge to zero in this case, due to the convergence of $\dot{V}$ to zero.

A particular function $g(e,t)$ which satisfies the conditions C_1 and C_2 is:

$$g(e,t) \;=\; ||e||^2 \sin(t)$$

Another possibility, when the functions $k_1(\cdot)$ and $k_3(\cdot)$ are strictly positive, is the bounded function:

$$g(e,t) \;=\; \frac{\exp(k_5 e_2) - 1}{\exp(k_5 e_2) + 1} \sin(t)\;, \quad \text{with} \quad k_5 > 0 \tag{5.4}$$

By choosing a large constant k_5, $g(e,t)$ is approximately equal to $\sin(t)$ as long as $|e_2|$ is not small. Integration of (5.3) then gives:

$$x_r(t) \;\approx\; \frac{k_4}{1+k_4^2} \sin(t) - \frac{1}{1+k_4^2} \cos(t)$$

This relation shows that the reference vehicle maintains a periodic motion the amplitude of which is approximately equal to two (i.e. the amplitude of $\sin(t)$) as long as $|e_2|$ does not become small. This motion produces in turn a quick reduction of $||e||$, as can be checked in simulation. This fast transient is then followed by the final asymptotic convergence phase, which is slow as already pointed out.

Going further in this direction, one may think of the following choice:

$$g(e,t) \;=\; \begin{cases} \sin(t) & \text{when } ||e|| \geq \epsilon > 0 \\ 0 & \text{otherwise} \end{cases}$$

Note that in this case the control law is no longer smooth, and that asymptotic convergence to the desired posture is not granted. However, it is still possible to show that $||e||$ becomes, and stays, smaller than ϵ after a finite time, and that the reference vehicle asymptotically exponentially converges to the desired posture. Thus, by choosing a small value for ϵ, the mobile robot will get very close to the desired posture (at a distance smaller than ϵ) after a finite and reasonably short time, while the control inputs will asymptotically exponentially converge to zero. From a practical point of view, this solution may be quite acceptable. Simulation results are shown in Figures 5.1 and 5.2. The function $g(e,t)$ is given by (5.4).

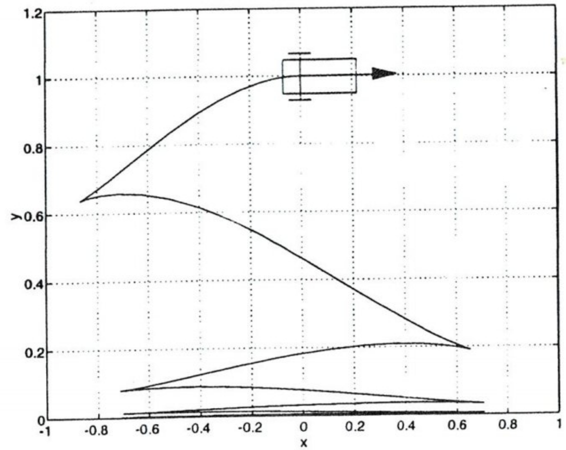

Figure 5.1: The resulting path in the xy-plane with $k_1 = 0.5$, $k_2 = 2$, $k_3 = k_4 = 1$, $k_5 = 10^4$. Initial conditions are: $z(0) = [0, 1, 0]^T$.

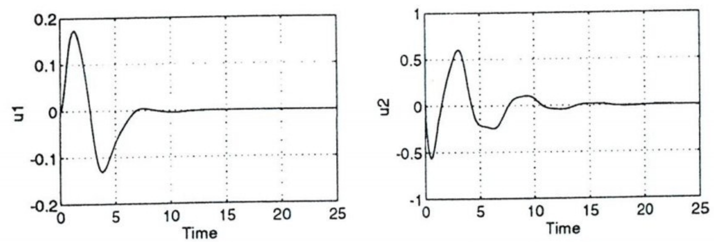

Figure 5.2: Timeplots of theinputs $u_1(t)$ and $u_2(t)$.

5.1.2 Extension of the Path Following Problem

Consider the path following problem described in Section 4, and assume that the chosen path passes through the point ($x_d = 0, y_d = 0$), and that the tangent to the path at this point coincides with the x-axis. The curvilinear coordinate may arbitrarily be set equal to zero at this point ($s_d = 0$).

Stabilization of the mobile robot about the desired posture is then equivalent to having the variables l, $\tilde{\theta}$, and s asymptotically converge to zero.

In order to achieve this objective, one may again consider the nonlinear angular velocity control proposed in Section 4.2 for path following:

$$u = -k(v)\tilde{\theta} - k_2 v l \frac{\sin(\tilde{\theta})}{\tilde{\theta}} \qquad (5.5)$$

It has been established in Section 4.2 (cf. Proposition 4.1) that, if the robot's translational velocity $v(t)$ and its time derivative are bounded, and if $v(t)$ does not asymptotically tend to zero, then l and $\tilde{\theta}$ asymptotically converge to zero provided that some initial conditions are satisfied.

The idea now consists of using the translational velocity v as a second control input the selection of which must be made in order to also have s converge to zero.

A possible choice, among others, is the following time-varying feedback:

$$v = -k_1 \cos(\tilde{\theta}) \frac{\exp(k_3 s) - 1}{\exp(k_3 s) + 1} + g(l, \tilde{\theta}, t) \qquad (5.6)$$

where k_1 and k_3 are positive real numbers, and $g(l, \tilde{\theta}, t)$ is a function having the same properties as the ones described previously for the extension of the tracking problem. In particular, it satisfies the following conditions:

$\mathbf{C_1}$: $g(0, 0, t) = 0$, $\forall t$

$\mathbf{C_2}$: There exists a diverging time-sequence $\{t_i\}_{i \in N}$ and a continuous function $\alpha(\cdot)$ such that

$$(l^2 + \tilde{\theta}^2)^{\frac{1}{2}} > \epsilon > 0 \ \Rightarrow \ \sum_{j=1}^{j=p} (\frac{\partial^j g}{\partial t^j}(l, \tilde{\theta}, t_i))^2 > \ \alpha(\epsilon) > 0 \quad , \forall t_i$$

Proposition 5.2 *The control (5.5)(5.6) asymptotically stabilizes the point ($l = 0, \tilde{\theta} = 0, s = 0$) provided that:*

$$l(0)^2 + \frac{1}{k_2}\tilde{\theta}(0)^2 \ < \ \frac{1}{\limsup\ (c(s)^2)}$$

Proof: The proof is quite similar to the one given for the extension of the tracking problem. It is first established that l, $\tilde{\theta}$, v, and $\dot{v}$ are bounded along any solution to the closed-loop system. Then, using Proposition 4.1, it is shown that v must tend to zero. One deduces from there that $\frac{\partial^j g}{\partial t^j}(l, \tilde{\theta}, t)$ tends to zero, for $1 \leq j \leq p$. This in turn implies, using $\mathbf{C_2}$ and the convergence of the Lyapunov function used in the proof of Proposition 4.1, that l and $\tilde{\theta}$ tend to zero. The convergence of $g(l, \tilde{\theta}, t)$ to zero then follows from $\mathbf{C_1}$. Finally the convergence of s to zero can be worked out from the following equation:

$$\dot{s} = -k_1 \frac{\exp(k_3 s) - 1}{\exp(k_3 s) + 1} + o(t) \qquad \text{with} \lim_{t \to +\infty} o(t) = 0$$

(end of proof)

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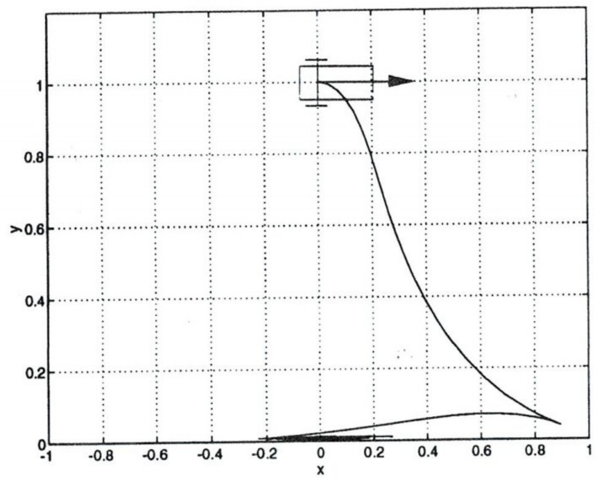

Figure 5.3: The resulting path in the xy-plane with, $k_2 = 9$, $k_1 = 1$ and $k_5 = 50$. Initial conditions are: $z(0) = [0, 1, 0]^T$.

Previous comments and remarks concerning the choice of the time-varying function $g(\cdot, t)$ also hold in this case. In particular, the dependence upon the variable $\tilde{\theta}$ is not necessary when $k(v)$ is chosen strictly positive ($\Rightarrow$ unconditional convergence of $\tilde{\theta}$ to zero). Simulation results are given in Figures 5.3 and 5.4 with

$$g(l, \tilde{\theta}, t) = \frac{\exp(k_5 l) - 1}{\exp(k_5 l) + 1} \sin t \ \ \textit{and} \ k(v) = 3\sqrt{2} \mid v \mid .$$

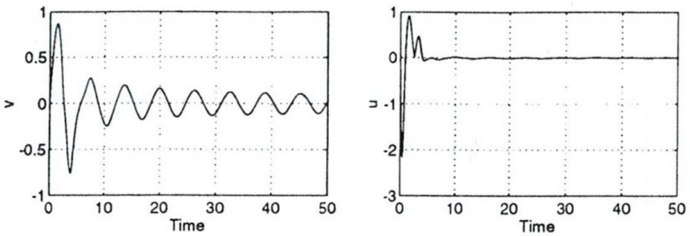

Figure 5.4: Timeplots of the inputs $v(t)$ and $u(t)$.

5.2 *Piecewise Continuous Control*

In this section we present two alternative approaches for the stabilization of the kinematic model of a cart. The first one is based on [5] and consists of dividing the state space into disjoint subspaces. The manifold that divides these complementary subspaces defines non-attractive discontinuous surface. A piecewise continuous feedback law is presented which makes the cart exponentially converge to the origin. This approach will be called the coordinate projection approach. The second approach is based on recent works of [7] on stabilization of nonlinear systems in chained forms. It relies on the concept of mixing piecewise *constant* feedback with piecewise *continuous* feedback laws. This approach will be named hybrid piecewise control since its combines a discrete-time law with a continuous-time one. Although this second approach is more general than the first one, the convergence rate is slower.

5.2.1 Coordinate Projection

The idea behind the control design based on a coordinate projection is the following: first we introduce the circle family $\mathcal{P}$ as:

$$\mathcal{P} = \{(x,y) : x^2 + (y-r)^2 = r^2\} \tag{5.7}$$

as the set of circles with radius , $r = r(x,y)$, passing through the origin and centered on the y-axis with $\frac{\partial y}{\partial x} = 0$ at the origin. Associated with these circles we can define θ_d as being the angle of the tangent of $\mathcal{P}$ at (x,y), as:

$$\theta_d(x,y) = \begin{cases} 2\arctan(y/x) & : \ (x,y) \neq (0,0) \\ 0 & : \ (x,y) = (0,0) \end{cases} \tag{5.8}$$

and

$$\begin{aligned} a(x,y) &= r(x,y)\theta_d \qquad a(x,0) = x && (5.9) \\ e(x,y,\theta) &= \theta - \theta_d && (5.10) \end{aligned}$$

where $a(x,y)$ defines the arc length from the origin to (x,y) along a circle which is centered on the y-axis and passes through these two points. $a(x,y)$ may be positive or negative according to the sign of x. When $y = 0$, we define $a(x,0) = 0$ which makes $a(x,y)$ continuous with respect to y since $a(x,\varepsilon) \approx x$ when $\varepsilon \approx 0$. Discontinuities in $a(x,y)$ only take place in the set $d(z) = \{z : x = 0, y \neq 0\}$. The variable e is the orientation error. An illustration of these definitions is shown in Figure 5.5.

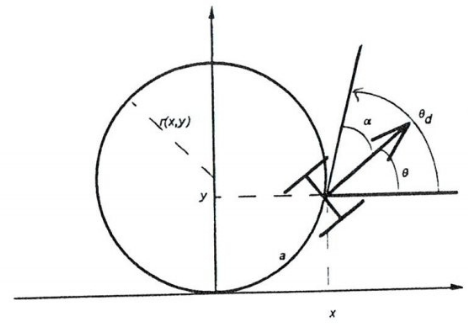

Figure 5.5: Illustration of the variables a, α and θ_d.

The cart can then be stabilized by finding a feedback control law that orients the angle θ according to the tangent of one of the members of the circle family $\mathcal{P}$ and then decreases the arc length of the associated circle. The design of such a control law can be easily understood by writing the open-loop equations in the projected coordinates a and e

$$\begin{aligned} \dot{a} &= b_1(z)v && (5.11) \\ \dot{e} &= b_2(z)v + \omega && (5.12) \end{aligned}$$

where, as before, z is the original system state vector coordinates, i.e $z = (x,y,\theta)^T$ and the functions $b_i(z)$ have the following properties, see [5]:

1. $b_{min}(e) \leq b_1(z) \leq b_{max}(e)$
2. $b_1(e,x,y)$ is continuous in e.
3. $\lim_{e \to 0} b_1(z) = 1$

4. $| b_2(z)a(z) | \leq N$ for some constant $N > 0$.

Here, $b_{min}(e)$ and $b_{max}(e)$ are bounded functions independent of e. These properties are useful for establishing the exponential stability of the closed-loop equations. Taking the following control law with $\gamma > 0$ and $k > 0$,

$$v = -\gamma b_1 a \tag{5.13}$$

$$\omega = -b_2 v - ke = \gamma b_1 b_2 a - ke \tag{5.14}$$

gives, away from the discontinuous surface $d(z)$, the following closed-loop equations:

$$\begin{aligned} \dot{a} &= -\gamma b_1^2 a \\ \dot{e} &= -ke \end{aligned} \tag{5.15}$$

From (5.15) we can see that e exponentially converges to zero and from Property 3 we see that the function $b_1(z)$ tends to a positive constant and therefore the arc length a converges exponentially to zero. Note also that the boundedness of $b_2(z)$ indicated in Property 4 implies the boundedness of the control vector ω. Finally, it can also be proved that the surface $d(z)$ is not attractive and that the convergence rate in the Ψ-space (away from $d(z)$)is not influenced by the crossing of the discontinuous surface $d(z)$.

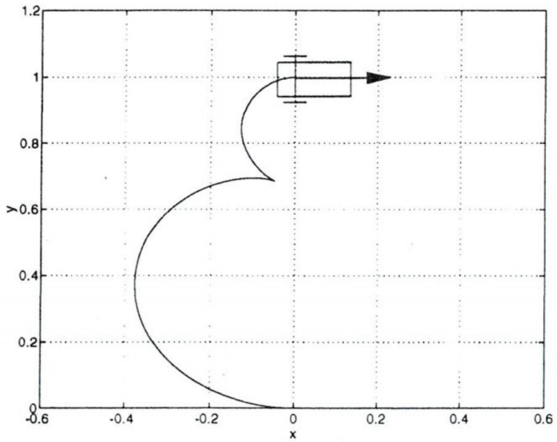

Figure 5.6: The resulting path in the xy-plane with $z(0) = [0, 1, 0]^T$.

It can also be proved that the discontinuous surface can only be crossed twice in finite time. Therefore it is always possible to find an exponentially decaying time function that gives an upperbound on the solution of the projected coordinates a and

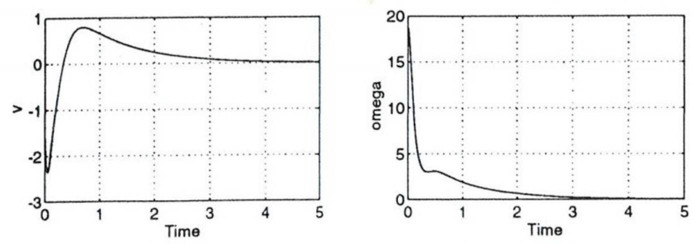

Figure 5.7: Timeplots of the inputs, $v(t)$ and $\omega(t)$.

e. Finally, these properties can be transferred to the original coordinates z. Details on the proofs of these statements can be found in [5] and [27]. Simulation was done for k and γ to 5 and 1, respectively. Figure 5.6 shows the resulting path in the xy-plane for $z(0) = [0, 1, 0]^T$. We see that the cart converges asymptotically to the origin. Figure 5.7 shows the corresponding time history of $v(t)$ and $\omega(t)$.

5.2.2 Hybrid Piecewise Control

Another alternative approach of piecewise control design recently investigated in [7] consists of mixing constant and continuous piecewise control design. The main idea is presented below using the model transformation (2.5) and (2.6), which leads to the following 3-dimensional chained structure:

$$\dot{x}_1 = u_1 \tag{5.16}$$

$$\dot{x}_2 = u_2 \tag{5.17}$$

$$\dot{x}_3 = x_2 u_1 \tag{5.18}$$

The input $u_1(t)$ is piecewise constant for the time interval $I_k = [k\delta, (k+1)\delta]$, $\forall k \in \{0, 1, 2, 3, \ldots\}$, and some $\delta > 0$, namely,

$$u_1(t) = u_1(k\delta) \qquad \forall t \in I_k \tag{5.19}$$

where $u_1(k\delta)$, describes a discrete-time control law for the sub-system $\dot{x}_1 = u_1$. System (5.18) is then written as:

$$\dot{x}_1(t) = u_1(k\delta) \tag{5.20}$$

$$\dot{z}(t) = \begin{pmatrix} 0 & 0 \\ u_1(k\delta) & 0 \end{pmatrix} z(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u_2(t) \tag{5.21}$$

where $z = (x_2, x_3)$. This representation describes two sub-systems where one of them (x_1) has $u_1(k\delta)$ as a piecewise *constant* input and the other (z), has $u_2(t)$ as a piecewise *continuous* input function. For this reason the approach can be understood as a hybrid piecewise control design. Note that sub-system (5.20), describes a piecewise continuous controllable LTI-system as long as $u_1(k\delta)$ does not vanish. Clearly this indicated that somehow the stabilization of the $z(t)$-coordinates should be performed faster than the stabilization of the x_1-coordinate.

The following control law has been proposed in [7], ensuring the uniform asymptotic stabilization of the x-coordinates to the origin:

$$u_1(k\delta) = k_1(k\delta) + sgn(k_1(k\delta))\gamma(\| z(k\delta) \|) \quad (5.22)$$
$$u_2(t) = -|u_1(k\delta)|k_2x_2(t) - u_1(k\delta)k_3x_3(t) \quad (5.23)$$

where:

- $k_1(k\delta)$ is any discrete-time stabilizing feedback law for the subsystem $\dot{x}_1(t) = k_1(k\delta)$, i.e.
$$k_1(k\delta) = \frac{(a-1)}{\delta}x_1(k\delta); \quad 0 < |a| < 1 \quad (5.24)$$
- $\gamma(\| z(k\delta) \|)$ is a positive definite function vanishing only when $\| z(k\delta) \| = 0$, i.e.
$$\gamma(\| z(k\delta) \|) = \frac{\gamma_0(1-a)}{\delta}\,\frac{\exp(c \| z(k\delta) \|^2) - 1}{\exp(c \| z(k\delta) \|^2) + 1} \quad (5.25)$$
where $\gamma_0 > 0$, $c > 0$ and $n > 0$.
- k_2 and k_3 are positive definite constants and the function $\text{sgn}(x)$ is defined as:
$$\text{sgn}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{otherwise} \end{cases}$$

Asymptotic stability results from the analysis of the following closed-loop equation:

$$x_1(k) = \frac{1}{1 - aq^{-1}}s(k)\gamma(\| z(k\delta) \|) \quad (5.26)$$
$$\dot{z}(t) = |u_1(k)|A(s(k))z(t) \quad (5.27)$$

where $s(k) = sgn(k_1(k\delta))$ and $A(s(k))$ is defined as:

$$A(s(k)) = \begin{pmatrix} -k_2 & -s(k)k_3 \\ s(k) & 0 \end{pmatrix} \quad (5.28)$$

Since suitable choices of k_2 and k_3 can render $A(s(k))$ stable with eigenvalues invariant with respect to $s(k)$, the solution of (5.27) in I_k writes as:

$$z(t) = \exp(|u_1(k)|A(s(k))(t - k\delta))z(k\delta) \quad (5.29)$$

hence its norm is bounded as:

$$\| z(t) \| \leq c \| z(k\delta) \| \exp(-|u_1(k)|\lambda_0(t - k\delta)).$$
$$\| z(t) \| \leq c \| z(k\delta) \| \exp(-\gamma(\| z(k\delta) \|)\lambda_0(t - k\delta)) \quad (5.30)$$

where c is a constant and $\lambda_0 = |\lambda_{min}(A)|$ and the last inequality is obtained from the definition of the control law $u_1(k)$. Then for $t = (k+1)\delta$, we get

$$\| z((k+1)\delta) \| \leq c \| z(k\delta) \| \exp(-\lambda_0\delta\gamma(\| z(k\delta) \|)) \quad (5.31)$$

from here we can conclude that $\| z(t) \|$ tends to zero; for details see [7]. Finally Equation (5.26) describes a stable discrete-time system driven by a bounded and asymptotically decaying input $(\gamma(\|z(k\delta)\|))$. Therefore, it is easy to conclude that $x_1(k)$ also tends asymptotically to zero. Convergence rates can be modified by the proper choice of the function $\gamma(\|z(k\delta)\|)$. For instance if $\gamma(\|z(k\delta)\|)$ is not Locally Lipstchitz, then higher convergence rates are reached. An interesting discussion on these convergence rates can be found in works of [8], [11] [12] in connection with time-varying controllers.

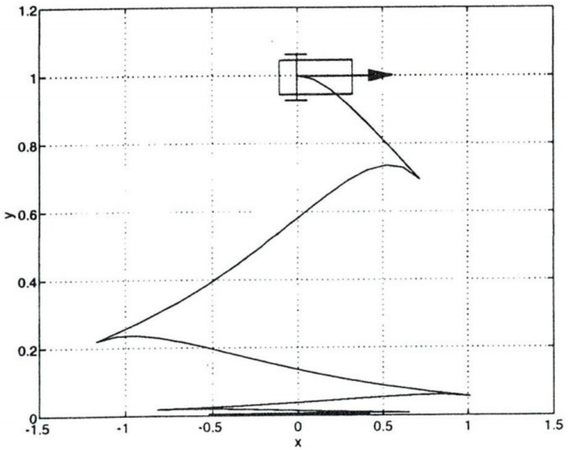

Figure 5.8: The resulting path in the xy-plane when a hybrid piecewise control is used with $\gamma_0 = 1$, $c = 1.8$, $\delta = 1$, $a = 0.8$, $k_2 = 4$ and $k_3 = 2.8$.The initial state is $x_1(0) = 0, z(0) = [1, 0]^T$.

Simulations in Figure 5.8, show that the behavior of the state-trajectories is similar to the behavior that has been obtained with the time-varying controllers but using piecewise continuous changes in the velocity inputs. Details on the proof of the above mentioned stability result and an extension of the previous control design to the case where the inputs are smoothed by adding an integrator in cascade are discussed in

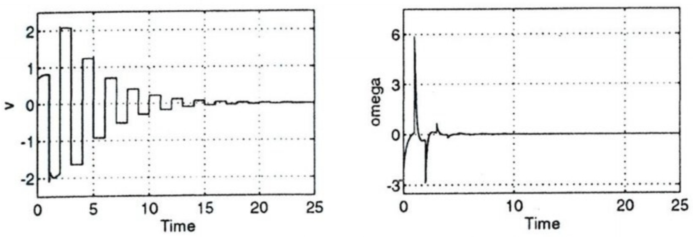

Figure 5.9: Timeplots of the inputs, $v(t)$ and $\omega(t)$.

[7]. Another way to introduce smoothness into the control design is to combine the piecewise continuous and periodic time-varying controllers. The advantage of doing this is that exponential convergence rates can be obtained. This idea is explained below.

5.2.3 Time-Varying Piecewise Continuous Control

In this section, a stabilizing time-varying feedback control law is presented which depends smoothly on the state in continuous time and non-smoothly on the state at discrete instants of time. This approach is developed in [27] and [28] for a n-dimensional, two-input chained form. This system represents locally a car with $n-3$ trailers, [30]. The control law for the three-dimensional case, which represents the cart, is shown in this section. The kinematic model is hence given by

$$\begin{aligned} \dot{x}_1 &= u_1 \\ \dot{x}_2 &= u_2 \\ \dot{x}_3 &= x_2 u_1 \end{aligned} \tag{5.32}$$

By using time-periodic functions in the control law, the inputs are smoothed out and become continuous with respect to time. By letting the feedback be non-smooth with respect to the state at discrete instants of time, exponential convergence is obtained.

The idea behind the control law is to choose the input u_1 such that the subsystem

$$\begin{aligned} \dot{x}_2 &= u_2 \\ \dot{x}_3 &= x_2 u_1 \end{aligned} \tag{5.33}$$

becomes linear and time-varying in the time intervals $[k\delta, (k+1)\delta)$, $k \in \{0, 1, 2, \ldots\}$, where $k\delta, (k+1)\delta, \ldots$ are discrete instants of time, as defined in the previous section. This is obtained by the following structure for u_1:

$$u_1 = k_1(x(k\delta))\, f(t) \tag{5.34}$$

$k\delta$ denotes the last element in the sequence $(0, \delta, 2\delta, \ldots)$ such that $t \geq k\delta$, and $x = [x_1, x_2, x_3]^T$. The function $f(t)$ is smooth and periodic. One possible choice of $f(t)$ is

$$f(t) = (1 - \cos\omega t)/2\ , \quad \omega = \frac{2\pi}{\delta}. \tag{5.35}$$

The other input u_2 will be chosen such that $\|[x_2(t), x_3(t)]^T\|$ converges exponentially to zero.

The sign and the magnitude of the parameter $k_1(x(k\delta))$ is chosen such that u_1 makes $x_1(t)$ converge exponentially to zero as $\|[x_2(t), x_3(t)]^T\|$ converges to zero. Let $k_1(x(k\delta))$ be given by

$$k_1(x(k\delta)) = -\left[x_1(k\delta) + \mathrm{sgn}(x_1(k\delta))\gamma(\|z(k\delta)\|)\right]\beta \tag{5.36}$$

where $z = [x_2, x_3]^T$ and

$$\begin{aligned} \gamma(\|z(k\delta)\|) &= \kappa\|z(k\delta)\|^{\frac{1}{2}} = \kappa(x_2^2(k\delta) + x_3^2(k\delta))^{\frac{1}{4}} & (5.37) \\ \beta &= 1/\int_{k\delta}^{(k+1)\delta} f(\tau)d\tau & (5.38) \end{aligned}$$

where κ is a positive constant and $\mathrm{sgn}(x_1(k\delta))$ is defined in the previous section. We see that $\gamma(\cdot)$ is a function of class $\mathcal{K}$.

To find the control law for u_2 we introduce the following auxiliary variable:

$$x_2^d = -\frac{\lambda_3}{k_1(x(k\delta))} f^2(t) x_3 \tag{5.39}$$

Differentiation gives together with (5.33) and (5.34)

$$\dot{x}_2^d = -\lambda_3(2f\dot{f}\frac{x_3}{k_1(x(k\delta))} + f^3 x_2) \tag{5.40}$$

The input u_2 is chosen to have the following structure:

$$u_2 = -\lambda_2(x_2 - x_2^d) + \dot{x}_2^d \tag{5.41}$$

The controller parameters λ_2 and λ_3 are positive constants.

The control law for System (5.32) is then given from (5.34), (5.36), (5.39), (5.40), and (5.41):

$$\begin{aligned} u_1 &= k_1(x(k\delta))\, f(t), \qquad x = [x_1, x_2, x_3] & (5.42) \\ u_2 &= -(\lambda_2 + \lambda_3 f^3)x_2 - \lambda_3(\lambda_2 f^2 + 2f\dot{f})\frac{1}{k_1(x(k\delta))}x_3 & (5.43) \end{aligned}$$

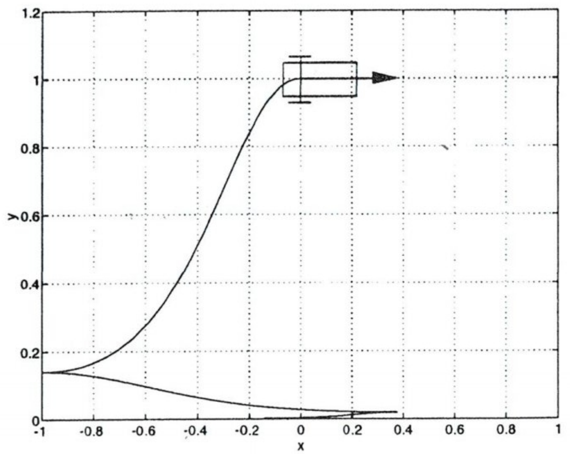

Figure 5.10: The resulting path in the xy-plane with $\delta = 1$, $\beta = 1/\pi$, $\kappa = 3$, $\lambda_2 = \lambda_3 = 1$. Initial conditions are:$x_1(0) = 0$, $x_2(0) = 1$ and $x_3(0) = 0$.

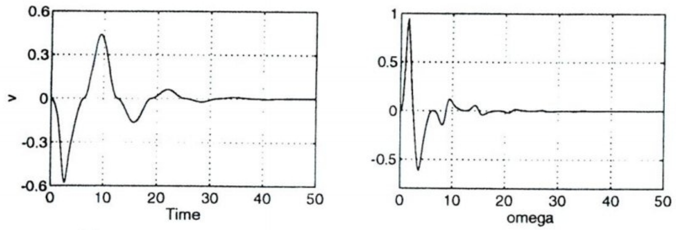

Figure 5.11: Timeplots of the inputs, $v(t)$ and $\omega(t)$.

where $f(t)$ and $k_1(x(k\delta))(x(k\delta))$ are given by (5.35) and (5.36).

To analyze the convergence and the stability of the system we introduce the following variable:

$$\tilde{x}_2 = x_2 - x_2^d$$

The control law (5.42)-(5.43) and (5.33) imply

$$\dot{\tilde{x}}_2 = -\lambda_2 \tilde{x}_2 \tag{5.44}$$

$$\dot{x}_3 = -\lambda_3 f^3(t) x_3 + k_1(x(k\delta)) f(t) \tilde{x}_2 \tag{5.45}$$

As shown in [28], $\|[\tilde{x}_2(t), x_3(t)]^T\|$ converges exponentially to zero. This can then be used to show that, [28], $|k_1(x(k\delta))| \geq c|x_3(t)|^{\frac{1}{2}}$, $\forall t \geq 0$, where c is a positive constant. Since

$$x_2 = x_2^d + \tilde{x}_2 = -\lambda_3 f^2(t) \frac{x_3}{k_1(x(k\delta))} + \tilde{x}_2$$

and $\|[\tilde{x}_2(t), x_3(t)]^T\|$ converges exponentially to zero, $x_2(t)$ converges exponentially to zero as well. From the control law for u_1 (5.42) and from the system equations (5.32), we obtain

$$|x_1((k+1)\delta)| = \gamma(\|z(k\delta)\|) = \kappa(x_2^2(k\delta) + x_3^2(k\delta))^{\frac{1}{4}}, \quad \forall k \in \{0, 1, 2, \ldots\}$$

This can be used to show that $x_1(t)$ converges exponentially to zero as $\|z(t)\| = \|[x_2(t), x_3(t)]^T\|$ converges to zero, [28].

In conclusion, if the control law is given by (5.42)-(5.43) where $k_1(x(k\delta))$ is given by (5.36) then the solution $x(t)$ of system (5.32) is globally stable around the origin and exponentially bounded as follows, [28]:

$$\forall x(0) \in I\!R^3 \quad \|x(t)\| \leq h(\|x(0)\|)e^{-\gamma t}, \quad \forall t \geq 0 \tag{5.46}$$

Here, $h(\cdot)$ is of class $\mathcal{K}$ and γ is a positive constant. Figure (5.10) shows the trajectory obtained with this control law.

6 Conclusion

This chapter has presented an overview of nonlinear control design for mobile robots. First structural properties, such as controllability, feedback linearizability and feedback stabilizability of the system under consideration, were discussed. It clearly appears that, in spite of the model controllability, the stabilization of the motion of mobile robots about a fixed point in the configuration-space can not be performed by means of standard and well established control methods. For instance, linear controllers are useless even in the neighbourhood of the equilibrium positions. Also nonlinear control approaches such as feedback linearization fails in this case. It turns out that unusual nonlinear control design are therefore required. These methods are:

smooth time-varying control laws and continuous piecewise feedback structures. Also a combination of these two is possible. We have presented several of these methods and compared them via theoretical discussions and simulations. The main differences between these approaches concerns the convergence rates. Conceptually, the smooth time-varying laws has slower convergence rate than the continuous piecewise controllers, although this difference can be substantially reduced by a proper choice of the controller gains (in a nonlinear fashion).

The presented control laws were designed on the basis of kinematic models; inputs were assumed to be the cart wheels velocities. In some applications, it is suitable to control the mobile robot by using torque as inputs instead of velocities. This can be done by performed a so called dynamical extension where integrators are added in cascade to the kinematic model. Methods such as the hybrid piecewise control [7] and time-varying feedback [22] allow for this possibility. Finally, robustness issues (unmodelled dynamics, measurement noise, load disturbances, etc.) were not addressed in this chapter and certainly require more investigation.

References

[1] D. Aeyels (1985) Stabilisation of a class of nonlinear systems by a smooth feedback control, *System and Control Letters 5*, pp. 289-294.

[2] R. W. Brockett (1983), Asymptotic stability and feedback stabilisation, *in* R. W. Brockett, R. S. Millman & H. J. Sussman, eds, 'Differential Geometric Control Theory', Birkhauser, pp. 181-208.

[3] A. M. Bloch, N. H. McClamroch, M. Reyhanoglu (1990), Controllability and stabilizability properties of a nonholonomic control system, *in* 'Proc. 29th Conference on Decision and Control', IEEE, Honolulu, Hawaii, pp. 1312-1314.

[4] A. M. Bloch & N. H. McClamroch (1989), Control of mechanical systems with classicals nonholonomic constraints, *in* 'Proc. 28th Conference on Decision and Control', IEEE, Tempa, Florida, pp. 201-205.

[5] C. Canudas de Wit & O. J. Sørdalen (1991), Exponential stabilization of mobile robots with nonholonomic constraints, *in* 'Proc. 30th Conference on Decision and Control', IEEE, Brighton, England, pp. 692-697. See also IEEE *Trans. Aut. Contr.*, Vol.37, No.11, 1791-1797.

[6] C. Canudas de Wit & C. Samson (1991), Path following of a 2-dof wheeled mobile robot under kinematic constraints, *in* 'Proc. First European Control Conference', Grenoble, France, pp. 2084-2088.

[7] C.Canudas de Wit, H. Berghuis & H. Nijmeijer (1993), Hybrid stabilization of nonlinear systems in chained form, Øsubmitted to IEEE *Trans. Aut. Contr.*.

[8] R.T. M'Closkey, R.M. Murray (1993), Convergence rates for nonholonomic systems in power form, 1993 American Control Conference, San Francisco.

[9] J. M. Coron (1992), Global asymptotic stabilisazation for controllable systems without drift, *Mathematics of Control, Signals, and Systems* 5, pp. 295-312.

[10] J. M. Coron & J. B. Pomet (1992) A remark on the design of time varying stabilizing feedback laws for controllable systems without drift, *in* 'Proc. Nonlinear Control Systems Design Symposium', IFAC, Bordeaux, France, pp. 413-417.

[11] L. Gurvits (1992), Averaging approach to nonholonomic motion planning, *in* 'Proc. 1992 IEEE International Conference on Robotics and Automation', Nice, France, pp. 2541-2546.

[12] L. Gurvits & Z. X. Li (1992), Smooth time-periodic feedback solutions for nonholonomic motion plannig, *in* Z. X. Li & J. Canny, eds, 'Progress in Nonholonomic Motion Plannig', Kluwer Academic Publisher.

[13] A. Isidori (1989), *Nonlinear Control Systems*, Springer-Verlag.

[14] Y. Kanayama, Y. Kimura, F. Miyazaki & T. Noguchi (1990), A stable tracking control method for an autonomous mobile robot, *in* 'Proc. 1990 IEEE International Conference on Robotics and Automation', Cincinati, Ohio, pp. 384-389.

[15] R.M. Murray & S.S. Sastry, (1991),Nonholonomic motion planning: steering using sinusoids,*in Trans. Aut. Contr.*, Vol.38, No.5, pp. 700-713.

[16] R.M. Murray & S.S. Sastry, (1991),Steering nonholonomic systems in chained form, *in* 'Proc. 30th Conference on Decision and Control', IEEE, Brighton, England, pp. 1121-1126.

[17] W.L. Nelson & I.J. Cox (1988), Local path control for an autonomous vehicle,'Proc. 1988 IEEE Int. Conf. on Robotics and Automation', Philadelphia, pp.1504-1510.

[18] H. Nijmeijer & A. J. Van der Schaft (1990), *Nonlinear Dynamical Control Systems*, Springer-Verlag.

[19] J. B. Pomet (1992), Explicit design of time-varying stabilizing control laws for a class of controllable systems without drift, *System and Control Letters* 18, pp. 147-158.

[20] M. Sampei, T. Tamura, T. Itoh, M. Nakamichi (1991), Path tracking control of trailer-like mobile robot, Proc. IEEE/RSJ Int. Workshop on Intelligent Robots and Systems, IROS'91, Osaka, pp. 193-198.

[21] C. Samson & K. Ait-Abderrahim (1990), Mobile robot control, part 1 : Feedback control of a nonholonomic wheeled cart in cartesian space, Technical report 1288, INRIA Sophia-Antipolis.

[22] C. Samson (1990), Velocity and torque feedback control of a nonholonomic cart, Int. Workshop in Adaptive and Nonlinear Control: Issues in Robotics, Grenoble, France, 1990, Canudas de Wit (Ed). Proc. in Adavanced Robot Control, vol. 162, Springer Verlag.

[23] C. Samson & K. Ait-Abderrahim (1991), Feedback control of a nonholonomic wheeled cart in cartesian space, Proc. 1991 IEEE Int. Conf. on Robotics and Automation, Sacramento, pp. 1136-1141.

[24] C. Samson & K. Ait-Abderrahim (1991), Feedback stabilization of a nonholonomic wheeled mobile robot, Proc. IEEE/RSJ Int. Workshop on Intelligent Robots and Systems, IROS'91, Osaka, pp. 1242-1247.

[25] C. Samson (1992) Path following and time-varying feedback stabilization of a wheeled mobile robot, Proc. Int. Conf. ICARCV'92, Singapour, RO-13.1

[26] E. D. Sontag (1990), *Mathematical control Theory. Deterministic Finite Dimensional Systems*, Springer-Verlag.

[27] O. J. Sørdalen (1993), Feedback Control of Nonholonomic Mobile Robots, Ph.D-thesis, The Norwegian Institute of Technology.

[28] O. J. Sørdalen and O. Egeland (1993), Exponential Stabilization of Chained Nonholonomic Systems *in* European Control Conference, Groningen, The Netherlands, pp. 1438-1443.

[29] O. J. Sørdalen and C. Canudas de Wit (1992), Exponential autonomous vehicle,'Proc. 1992 IEEE Int. Conf. on Robotics and Automation', Nice, pp.2158-2163. Control Law for a Mobile Robot: Extension to the Path Following,

[30] O. J. Sørdalen (1993), Conversion of the Kinematics of a Car with n Trailers into a Chained Form, IEEE Conf. on Robotics and Automation, Atlanta, Georgia.

CHAPTER 6

COORDINATING LOCOMOTION AND MANIPULATION OF A MOBILE MANIPULATOR

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ABSTRACT

A mobile manipulator in this study is a manipulator mounted on a mobile platform. Assuming the end point of the manipulator is guided, *e.g.*, by a human operator to follow an arbitrary trajectory, it is desirable that the mobile platform is able to move as to position the manipulator in certain preferred configurations. Since the motion of the manipulator is unknown *a priori*, the platform has to use the measured joint position information of the manipulator for its own motion planning. This chapter presents a planning and control algorithm for the platform so that the manipulator is always positioned at the preferred configurations measured by its manipulability. Simulation results are presented to illustrate the efficacy of the algorithm. Also the algorithm is implemented and verified on a real mobile manipulator system. The use of the resulting algorithm in a number of applications is also discussed.

1 Introduction

When a human writes across a board, he positions his arm in a comfortable writing configuration by moving his body rather than reaching out his arm. Also when humans transport a large and/or heavy object cooperatively, they tend to prefer certain configurations depending on various factors, *e.g.*, the shape and the weight of the object, the transportation velocity, the number of people involved in the task, and so on. A mobile manipulator consists of a mobile platform and a robot manipulator. When a mobile manipulator performs a manipulation task, it is desirable to bring the manipulator into certain preferred configurations by appropriately planning the