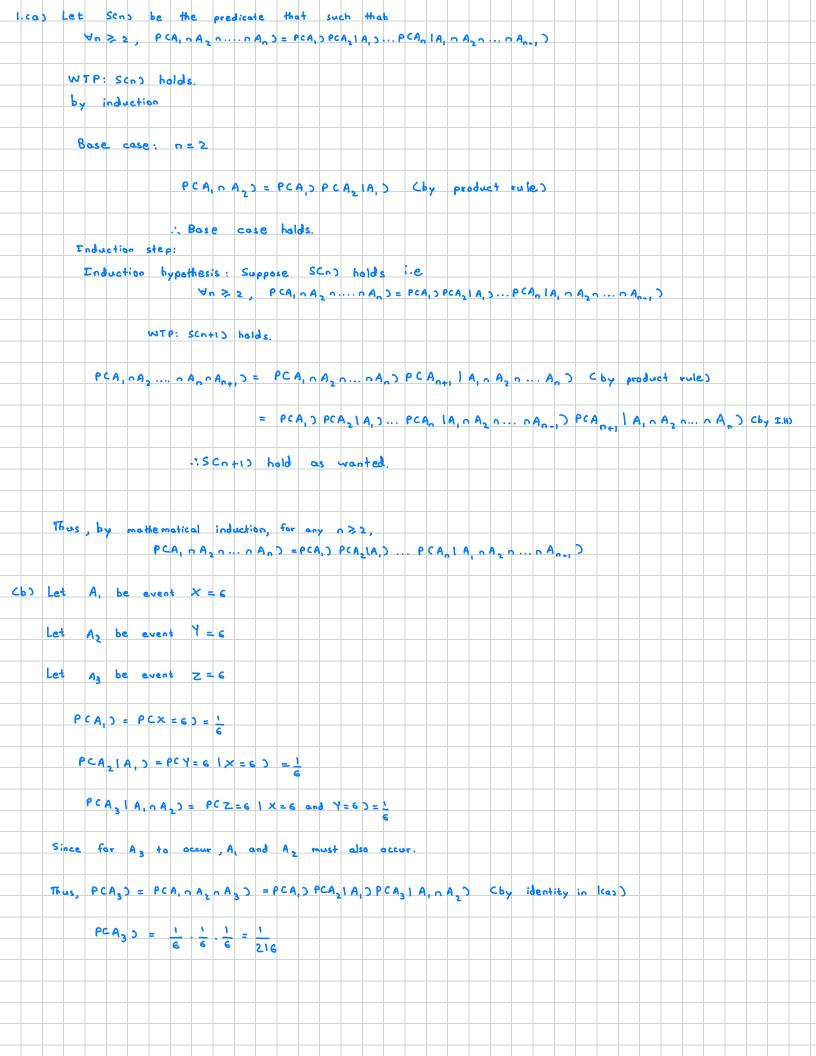
- 1. (10 points)
  - (a) (5 points) Use the identity  $\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B|A)$  and mathematical induction to prove that for any  $n \geq 2$  and arbitrary events  $A_1, ..., A_n$ , the following identity holds:

$$\mathbb{P}(A_1 \cap A_2 \cap \dots \cap A_n) = \mathbb{P}(A_1)\mathbb{P}(A_2|A_1) \cdots \mathbb{P}(A_n|A_1 \cap A_2 \cap \dots \cap A_{n-1}).$$

- (b) (5 points) Consider the following experiment. You first roll a fair 6-sided die and obtain the value X. You then roll a fair X-sided die and obtain the value Y. Finally, you roll a fair Y-sided die and obtain the value Z. Use the result in (a) (with n=3) to find the probability that you obtain Z=6 with the last die. For example, if we first get X=4, then Y is randomly chosen from  $\{1,2,3,4\}$  (each has equal probability  $\frac{1}{4}$ ). Say we get Y=3. Then Z is randomly chosen from  $\{1,2,3\}$ . In this case Z is at most 3.
  - Hint: Formulate  $A_1$ ,  $A_2$  and  $A_3$  suitably. The problem statement provides values of the (conditional) probabilities involved.
- 2. (10 points) Consider  $N \geq 2$  empty boxes lined up from left to right. A coin is fliped where the probability of heads is p, 0 . If heads, a box is selected uniformly at random and a treasure is placed inside it. If tails, no treasure is placed.
  - (a) (4 points) Construct a probability space  $(\Omega, \mathbb{P})$  to model the above experiment. That is, define  $\Omega$  and  $\mathbb{P}$  suitably. Note: There are many possible constructions, but it is possible to choose  $\Omega$  such that  $|\Omega| = N + 1$ .
  - (b) (6 points) Suppose we open all but the rightmost box and find no treasure in the N-1 opened boxes. What is the conditional probability there is a treasure in the remaining rightmost box?

Note: If needed, you may argue using the probability space you construct in (a). The answer is independent of your construction in (a) (see the discussion in Example 3.2 of the lecture notes).



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