1. (a) Let $Z = \min(X_1, \ldots, X_n)$

$$1 - P(Z \le z) = P(Z > z)$$

$$= P(\min(X_1, \ldots, X_n) > z)$$

Since $X_1, \ldots, X_n$ are i.i.d and minimum of them is $> z$

$$= P(X_1 > z) . P(X_2 > z) \ldots P(X_n > z)$$

$$= (1 - z)^n$$

$$\therefore 1 - P(Z \le z) = (1-z)^n$$

$$P(Z \le z) = 1 - (1-z)^n$$

$$F_Z(z) = \begin{cases} 0, & z < 0 \\ 1 - (1-z)^n, & 0 \le z \le 1 \\ 1, & z > 1 \end{cases}$$

(b) $Y_n = n \min(X_1, \ldots, X_n)$

$$P(Y_n \le y) = P(n \min(X_1, \ldots, X_n) \le y)$$

$$= P(\min(X_1, \ldots, X_n) \le \frac{y}{n})$$

$$= 1 - (1 - \frac{y}{n})^n$$

$$= 1 - e^{-y} \quad (\because \lim_{n \to \infty} (1 - \frac{y}{n})^n = e^{-y})$$

This is the cdf of Exp(1).

$\therefore Y_n$ converges in distribution to Exp(1).

2. $M_n = \max(X_1, \dots, X_n)$

$P(M_n \le m) = P(\max(X_1, \dots, X_n) \le m)$

$= P(X_1 \le m) \cdot P(X_2 \le m) \dots P(X_n \le m)$ $(\because X_1, \dots, X_n$ i.i.d $)$

$= (1 - e^{-m})^n$ for $m > 0$

$P(M_n - \log n \le m) = P(M_n \le m + \log n)$

$= (1 - e^{-m - \log n})^n$

$$\lim_{n\to\infty} F_{M_n - \log n}(m) = \lim_{n\to\infty} (1 - e^{-m-\log n})^n$$

$$= \lim_{n\to\infty} \left(1 - \frac{e^{-m}}{e^{\log n}}\right)^n$$

$$= \lim_{n\to\infty} \left(1 - \frac{e^{-m}}{n}\right)^n$$

$$= e^{-e^{-m}}$$

$$\therefore F_{M_n - \log n} = \begin{cases} 0, & m < 0 \\ e^{-e^{-m}}, & m > 0 \end{cases}$$