Name (F, L): ——————————————

Student ID #: ——————————————

1. (10 points) The following identity holds for any three events:

$$\mathbb{P}(A \cup B \cup C) = \mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C)$$
$$- \mathbb{P}(A \cap B) - \mathbb{P}(B \cap C) - \mathbb{P}(C \cap A) + \mathbb{P}(A \cap B \cap C). \tag{1}$$

Prove (1) in two ways:

 (a) (5 points) Decompose $A \cup B \cup C$ as a disjoint union. Hint: Draw a large Venn diagram for three sets, showing all possible intersections. The disjoint union should have 7 pieces.

 (b) (5 points) Use the identity $\mathbb{P}(E_1 \cup E_2) = \mathbb{P}(E_1) + \mathbb{P}(E_2) - \mathbb{P}(E_1 \cap E_2)$ for two events. Hint: Write $A \cup B \cup C = A \cup (B \cup C)$ as a union of two events and use the identity as needed.

2. (10 points) In the experiment of throwing a fair dice twice, let

$$A = \{\text{The first number equals } 5\},$$
$$B = \{\text{The sum of two numbers is 6, 7 or 8}\},$$
$$C = \{\text{The second number is even}\}.$$

Use (1) to find $\mathbb{P}(A \cup B \cup C)$. (This means you must compute the values of all terms on the right hand side of (1).)

Hint: Draw a matrix whose entries are the realizations of the sum.