

Richard Price, Bayes' theorem, and God

Bayes' theorem is 250 years old this year. But did the Rev. Thomas Bayes actually devise it? **Martyn Hooper** presents the case for the extraordinary Richard Price, friend of US presidents, mentor, pamphleteer, economist, and above all preacher. And did Price develop Bayes' theorem in order to prove the existence of God?

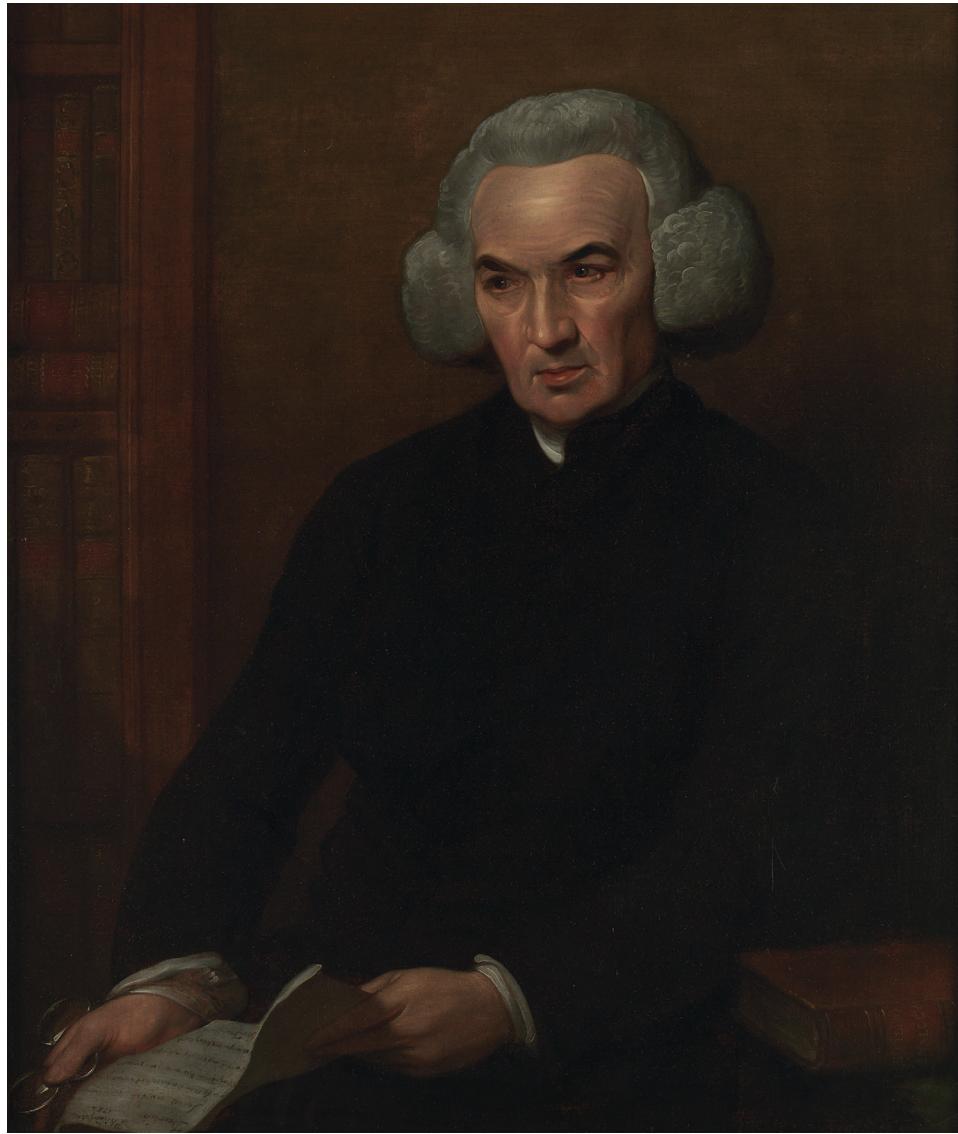
It was 250 years ago that Richard Price (1723–1791), a dissenting minister from Wales who lived and worked in London, wrote to John Canton FRS enclosing "An Essay towards Solving

a Problem in the Doctrine of Chances" by the late Rev. Thomas Bayes¹. The letter was written on November 10th, 1763, and the accompanying essay, which was read to the Royal Society on

December 23rd of the same year, contained ten propositions and three rules together with an appendix that applied these rules to particular problems. The essay by Bayes is rightly regarded as laying the foundation for probability theory based on the theorem that makes its first appearance in the paper.

It was Price, not Bayes, who communicated the essay to the Royal Society; Thomas Bayes had died two years before. And it was Price who submitted a second paper a year later² that demonstrated the second rule in the original essay. I have long been intrigued by just how much of Bayes' essay was the work of Price and now, 250 years after it was first read to the Royal Society, is a good time to try to estimate this. But first a few words about Richard Price, Doctor of Divinity and Fellow of the Royal Society.

I know of nobody acquainted with the works of Price who can understand why this quiet, unassuming Welshman is not more widely known. His contribution to the eighteenth-century Enlightenment was truly great. His close personal friends included Benjamin Franklin, John Adams, Joseph Priestley and Thomas Jefferson as well as Thomas Bayes. (Franklin was one of the ten Fellows who sponsored Price's membership of the Royal Society.) He was a prolific writer on morals and ethics, political theory, economics, mathematics and statistics; but what is truly astonishing is not the volume of written work that he produced but the quality and richness of his writings, which helped formulate the modern world and remain relevant today. His defence of the American and French Revolutions made him a household name to the extent that when Yale University awarded honorary law degrees in 1781 one went to George Washington and the only other one to Price³. He advised the British Prime Minister Pitt on reducing the national debt, was very active in attempts to introduce the first workable system of universal old age pensions (which was passed by the House of Commons and rejected twice by the House of Lords) and he set the insurance industry on a



Richard Price; portrait by Benjamin West (1728–1820); courtesy National Museum of Wales

All this – and Bayes’ theorem too?

On December 5th, 1765, ten Fellows of the Royal Society signed the following citation: “The Revd Mr Richard Price of Newington Green, who hath communicated several curious papers to this R Society, printed in the Philosophical Transactions, being desirous of becoming a member of it, is recommended by us, upon our personal knowledge, as likely to become a very usefull member, from his great skill in Mathematicks and Philosophy.”

Richard Price was admitted as a member of the Royal Society a week later. His admittance was based on many contributions made by Price to the important theological and moral questions of the day. And his election would have been in no small part due to his having written two papers that were concerned with the doctrine of chances.

Richard Price was a preacher, a radical, a pamphleteer, and, above all, an influence in all kinds of areas: not least in statistics, in economics, in rights for women, and in the founding of the United States. Benjamin Franklin, Thomas Jefferson, John Adams and Thomas Paine all visited him at his house in Newington Green or at the church where he ministered, both of which are still standing today. (The house forms part of the oldest brick terrace in London.) So did Prime Minister William Pitt.

Though he never visited America, he was feted there. In 1776 his pamphlet *Observations on the Nature of Civil Liberty, the Principles of Government, and the Justice and Policy of the War with America* argued for independence and against the war and sold 60 000 copies within days of publication. It made Price one of the best-known men in England, and is said to have played no small part in determining the Founding Fathers to declare independence. Hence his honorary degree from Yale – and in 1778 a Congressional invitation, declined, to assist in running the finances of the new nation.

His statistical work, and his US connections, were not limited to his contribution to Bayes’ theorem. In 1769 in a letter to his close friend Benjamin Franklin he wrote on life expectancy and the increasing population of London. This and another on calculating the values of contingent reversions helped reform the inadequate calculations on which many insurance and benefit societies had recently been formed. His 1780 essay on the population of England directly influenced Thomas Malthus, who formulated the idea that population increases geometrically, outstripping food production which only increases arithmetically.

His fame was such that he was frequently caricatured and satirised. Gilray’s political cartoon, overleaf, was one of many.

Mary Wollstonecraft, the pioneer of feminism, though an Anglican, attended his chapel, was inspired and was mentored by him, and wrote *A Vindication of the Rights of Men* to defend him when his praise of the French Revolution was attacked by Edmund Burke. Her *Vindication of the Rights of Woman*, perhaps the founding document of the feminist movement, followed two years later.

Price died in 1791. His funeral sermon was preached by Joseph Priestley, the discoverer of oxygen; he was buried in Bunhill Fields, the North London burial ground for Nonconformists. Thomas Bayes is also buried there. A few yards separate their tombs.

sound footing while advising the Society for Equitable Assurances (forerunner of today’s Equitable Life Assurance Society) over many years. There is yet more: the enormity of his contribution to some of the great questions of the day has to be set against the fact that he believed any activity not associated with his preaching to be secondary. So just who was Richard Price and what exactly was his contribution to Bayes’ theorem?

Richard Price was born on February 23rd, 1723, into a Nonconformist family in the village of Llangeinor, just to the north of Bridgend, south Wales. He received his education in various dissenting academies in Wales, but when his father died, when Richard was 16, he made the journey to London where his Uncle Samuel, a popular dissenting preacher, took the young Price under his wing.

It is important to understand the background of how Price came to submit the “Essay towards Solving a Problem in the Doctrine of Chances”. Bayes and Price went back a long way. Both came from a family of Nonconformists and therefore were denied a traditional university education. (Students at Oxford and Cambridge had to confirm their acceptance of the 39 articles of the Anglican Church; Catholics and Nonconformists were therefore excluded.) Price was enrolled at the Tenter Alley dissenter academy at Moorfields in London where he studied under John Eames. Eames was a friend of Isaac Newton and a capable mathematician. A former pupil at the Academy was Thomas Bayes and, even though there may have been as much as 20 years in age difference between the two students (there is some doubt as to the exact year that Bayes was born), it is conceivable that the two met at the academy. Perhaps Bayes would

visit as an old boy – though it is more likely they met through friendship with John Eames whom both of them knew. As well as studying Classics and moral philosophy, Richard was taught applied mathematics including hydrostatics, mechanics, astronomy and optics by Eames himself, and there is every reason to believe that the educational standards at some dissenting academies was every bit as good as, or even better than, those at established universities⁴. If their friendship began through John Eames it certainly grew in later years, cemented, no doubt, by the shared religious beliefs of the two men. When Bayes dies in April 1761 he left Price £100 in his will and asked that his unfinished papers be given to him – although curiously Bayes could “only suppose” that Price was a “preacher at Newington Green”, which is evidence that they had drifted apart over the years. Bayes’ will instructs his family to “examine the papers which he had written on different subjects, and which his own modesty would never suffer him to make public”. One of these papers was an unfinished attempt to solve a problem on the doctrine of chances which Price, with his love of mathematics, recognised as important in relation to probability theory. And that is how Price became involved in submitting Bayes’ essay to John Canton while indicating that its content might be of interest to the Royal Society, which it certainly was.

The essay begins with the problem being set out:

Given the number of times in which an unknown event has happened and failed: Required the chance that the probability of it happening in a single trial lies somewhere between any two degrees of probability that can be named.

This leads us to ask how much Price contributed to solving the problem in the essay. Ideally we would have sight of Bayes’ original unfinished work for comparison with Price’s submission, but we do not. Nor do we have Bayes’ original introduction to the problem. Lacking the ability to compare the unfinished essay with the finished product leaves us having to look for other sources of information that can indicate the extent to which Price contributed to the essay. And this is where I confess that I believe that Price’s contribution to the essay was more than to just act as the messenger to the Royal Society. And I base my belief on three indicators that point towards Price contributing significantly to the content of the essay.

First, we have to go back to Bayes’ will. Remember that Bayes had instructed his family to examine his papers on various subjects “which his own modesty would never suffer him to make public”. That is a strange thing to say. Why would Bayes’ “modesty” prevent him making public

much of his work, including the essay on the doctrine of chances? Could it be that, like many of his other papers, the essay on the doctrine of chances was not published in Bayes' lifetime because it went only part way to answering the problem set? I believe that Bayes was not able to publish the essay because it simply was not finished; otherwise surely he would have done so. It seems that Bayes had already revisited his unfinished work to try to complete it, as Price's introduction to the essay, in the form of the letter to Canton, tells the reader that "Bayes had some doubts as to the validity of an earlier version of his demonstration"⁵ and he feared "that the postulate on which he had argued might not perhaps be looked upon by all as reasonable; and therefore he chose to lay down in another form the proposition in which he thought the solution of the problem is contained"². This is interesting. Bayes had obviously been grappling with the problem but was still some way off answering the question posed in a way that would save his "modesty".

There might be another reason why Bayes had not been able to complete the paper to his, and others', satisfaction. The paper was given to Price in 1761 yet had probably been written 5 or 6 years earlier when Bayes began a long battle with illness. It may have been that he was



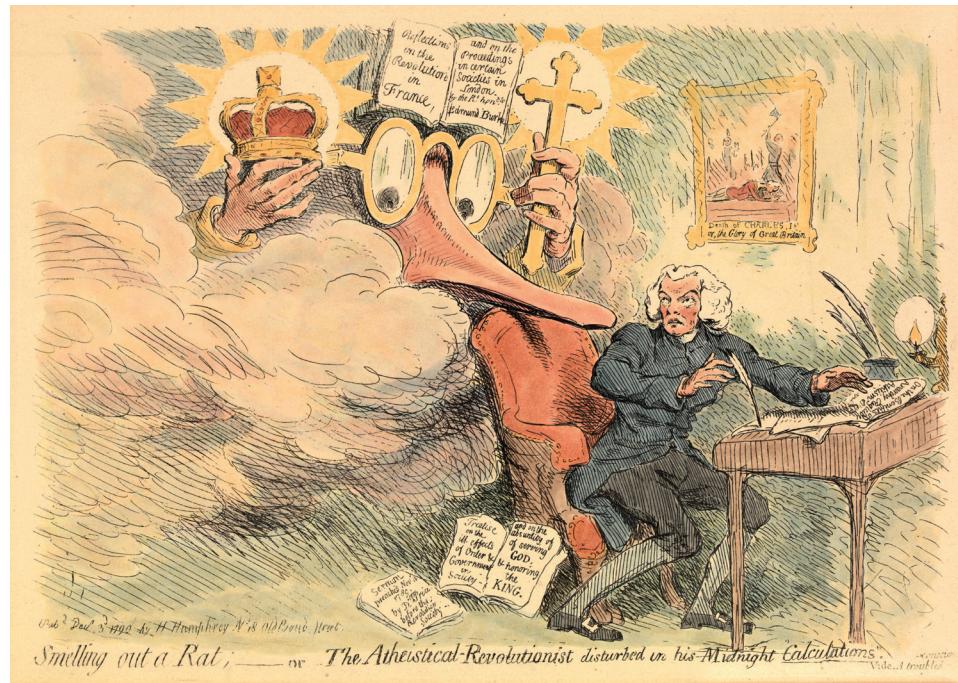
Richard Price's birthplace, Tynton, in Llangeinor, South Wales

simply too unwell to divert all his energy towards finishing the work. What is clear is that the work was unfinished and that nobody other than Price worked on the piece following Bayes' death. We

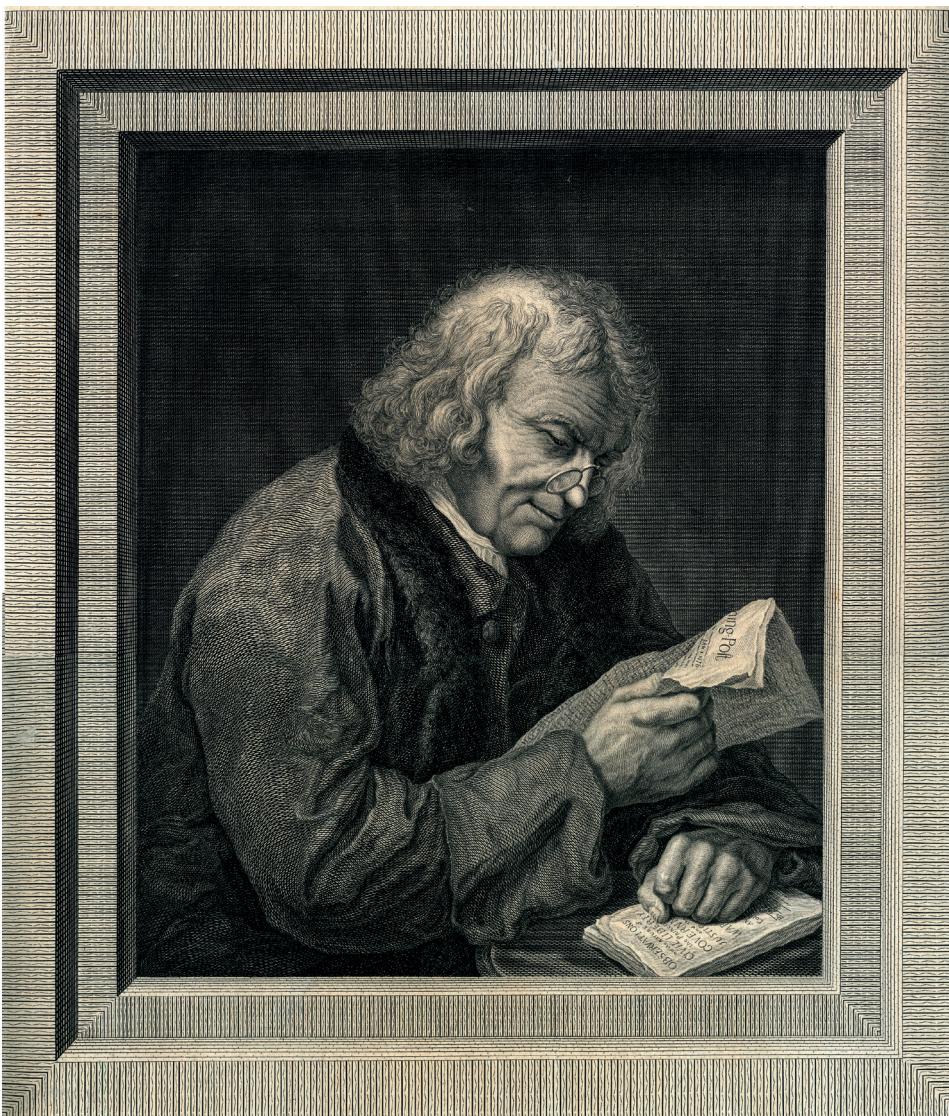
can reasonably conclude, therefore, that Price answered the question set by Bayes.

The second piece of evidence comes from Price's nephew, William Morgan FRS, writing about his uncle in 1815⁶. In his memoir Morgan tells us that his uncle undertook "the task of completing Mr Bayes's solution" (emphasis added). And whilst there is some argument as to the accuracy of Morgan's memoir, there is no reason to doubt this particular recollection. Indeed, Morgan goes on to say that his uncle was dissatisfied with the demonstration in the first paper and that, "notwithstanding the pains he had taken", he went on to write a supplement to the first paper. Price was obviously very well acquainted with the substance of the essay.

The third reason why I believe that Price's contribution to the essay was substantial is the fact that it took Price two years or more to answer the question set to a standard that would be acceptable to the Royal Society. Now we know that Price considered any work not associated with his calling to be of secondary importance to his ministry, and the fact that he had recently moved his ministry to a new chapel would mean that his calling would have occupied a great amount of his time during this period. It was also around this time that his wife became ill and suffered her first bout of palsy. These events could all explain the length of time it took Price to work on the essay. However, Price believed the essay to be important for what, to him, was the most fundamental reason of all: he believed it could be used to explain the probability of the existence of God. He believed also that it could show how probable it was that miracles



Cartoon of Richard Price and Edmund Burke, by James Gilray (1757–1815). The caption reads "Smelling out a Rat – or the Atheistical-Revolutionist disturbed in his midnight calculations". Price, seated, is the atheistical revolutionist – atheistical because he is a Nonconformist and also because he supported the American and French revolutions challenging the divine right of kings. The painting above his head is of the beheading of Charles I. The creature above him is the arch-conservative and monarchist Edmund Burke, holding the cross of Christianity and the orb of the monarchy. Above his head is a copy of his book *Reflections on the Revolutions in France*, which was written in outraged response to a sermon of Price's welcoming the French Revolution. Many others joined in the pamphlet war between them. Courtesy National Museum of Wales



The correspondence between Richard Price and Benjamin Franklin continued for at least ten years, on topics as diverse as life expectancy, the constitution of American states and the effect of the Aberration of Light on the Time of a Transit of Venus. In 1780 Franklin wrote to Price lamenting the existence of religious tests in the constitution of Massachusetts, but ending "But I shall be out of my Depth, if I wade any deeper in Theology, and I will not trouble you with Politicks, nor with News which are almost as uncertain; but conclude with a heartfelt Wish to embrace you once more, and enjoy your sweet Society in Peace, among our honest, worthy, ingenuous Friends." The portrait above, by Stephen Elmer (d. 1796), believed to be of Franklin, shows him reading the *Morning Post*, held in his right hand, but with his left hand resting on Price's *Observations on the American War*. The portrait of Price on page 36 is sometimes catalogued as showing him with a letter from Benjamin Franklin in his hand.

had taken place in the past. He wrote this plainly in his letter to Canton:

The Purpose I mean is, to shew what reason we have for believing that there are, in the constitution of things fixt laws according to which events happen, and that, therefore, the frame of the world must be the effect of the wisdom and power of an intelligent cause; and thus to confirm the argument taken from final causes for the existence of the Deity¹.

Price would not, therefore, have believed this excursion from his duty to be in any way secondary to his calling as a minister.

Indeed the linkage of the theorem to a proof of God's existence would mean that he would devote as much time as he could to helping to answer the question set by Bayes. Of course, his extra ministry work and his wife's illness may have prevented him from spending as much time as he would want on solving the problem, but even so it still took him two years or more until he was happy with his solution.

Sharon Bertsch McGayre has written an entire book on the theorem³. Her subtitle, *How Bayes' Rule Cracked the Enigma Code, Hunted Down Russian Submarines, and Emerged Triumphant from Two Centuries of Controversy*, does not mention proving the existence of God among its many achievements; but Bayes' (and Price's) theorem, she writes, "transformed probability from a gamblers' measure of frequency into a measure of informed belief"³ and Price's application of it to insurance and macroeconomics should be seen as a cornerstone of modern life. It is time to give this gentle Welshman more credit for his contribution to probability theory, and I am not alone in believing that Price's contribution to formulating Bayes' theorem was a substantial one and that he deserves to be recognised for that contribution. To quote from Sharon McGayre again:

By modern standards, we should refer to the Bayes-Price rule. Price discovered Bayes' work, recognized its importance, corrected it, contributed to the article, and found a use for it. The modern convention of employing Bayes' name alone is unfair but so entrenched that anything else makes little sense³.

And I did not say that – a statistician did.

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Martyn Hooper is chairman of the Richard Price Society, which was formed last year; its aim is to get Price's contribution to the eighteenth century enlightenment and the modern world more widely known through talks and lectures, and to provide a permanent exhibition at Llangeinor, Price's birthplace. See www.richardpricesociety.org.uk for details.