Lecture 2 Image Formation

COMP3204 Computer Vision

What is inside an image?

Book
pp
29-38
57-66

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Content

1. How is an image formed?
2. What restrictions are there on image formation?
3. Go to a different space - Fourier …..

Decomposing an image into its bits

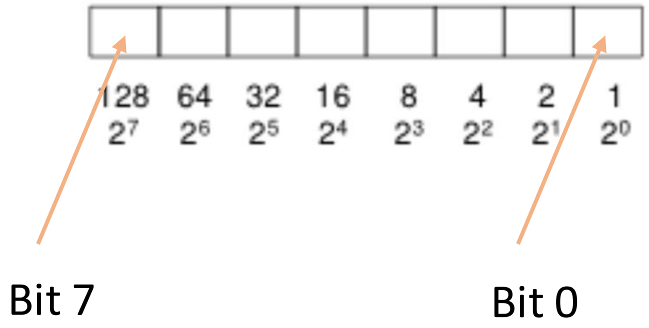
Every pixel is an 8-bit unsigned integer in [0, 255]

(a) original image

$$\begin{bmatrix} & \cdot\cdot\cdot & \\ \vdots & \ddots & \vdots \\ & \cdot\cdot\cdot & \end{bmatrix}$$

N by *N* matrix

For an 8-bit number:

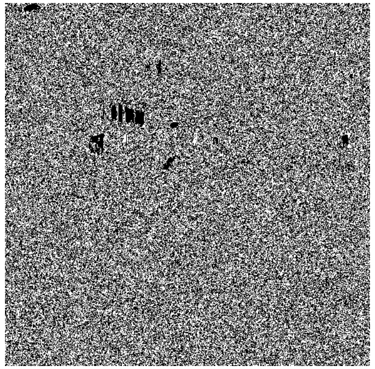


https://en.wikipedia.org/wiki/Bit_plane

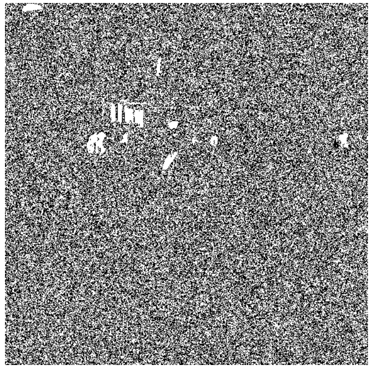
Decomposing an image into its bits

The Most Significant Bit carries the most information whereas bit 0 is noise

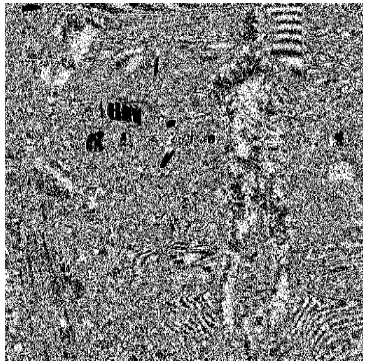
(a) original image



(b) bit 0 (LSB)

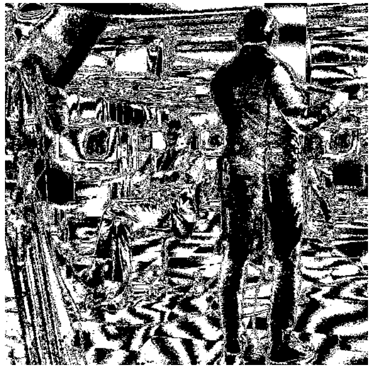


(c) bit 1

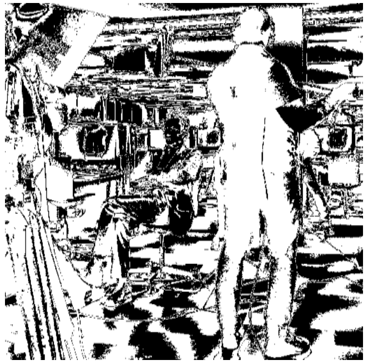


(d) bit 2

(e) bit 3



(f) bit 4



(g) bit 5

(h) bit 6

(i) bit 7 (MSB)

… and here, bit 4 is the lighting

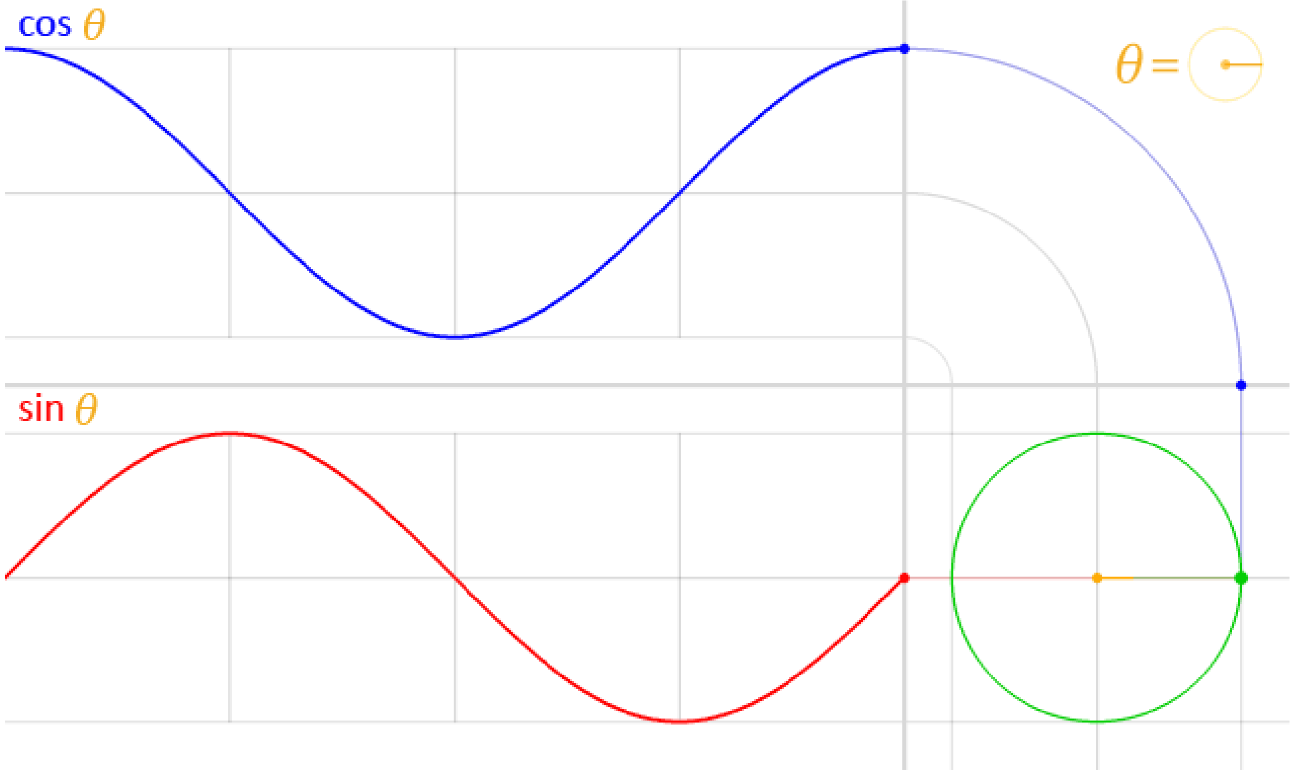
Effects of differing image resolution

(a) 64×64 **(b)** 128×128 **(c)** 256×256

Low resolution lose information but *N* by *N* points implies much storage

How do we choose an appropriate value for *N*?

Recall: What are waves?



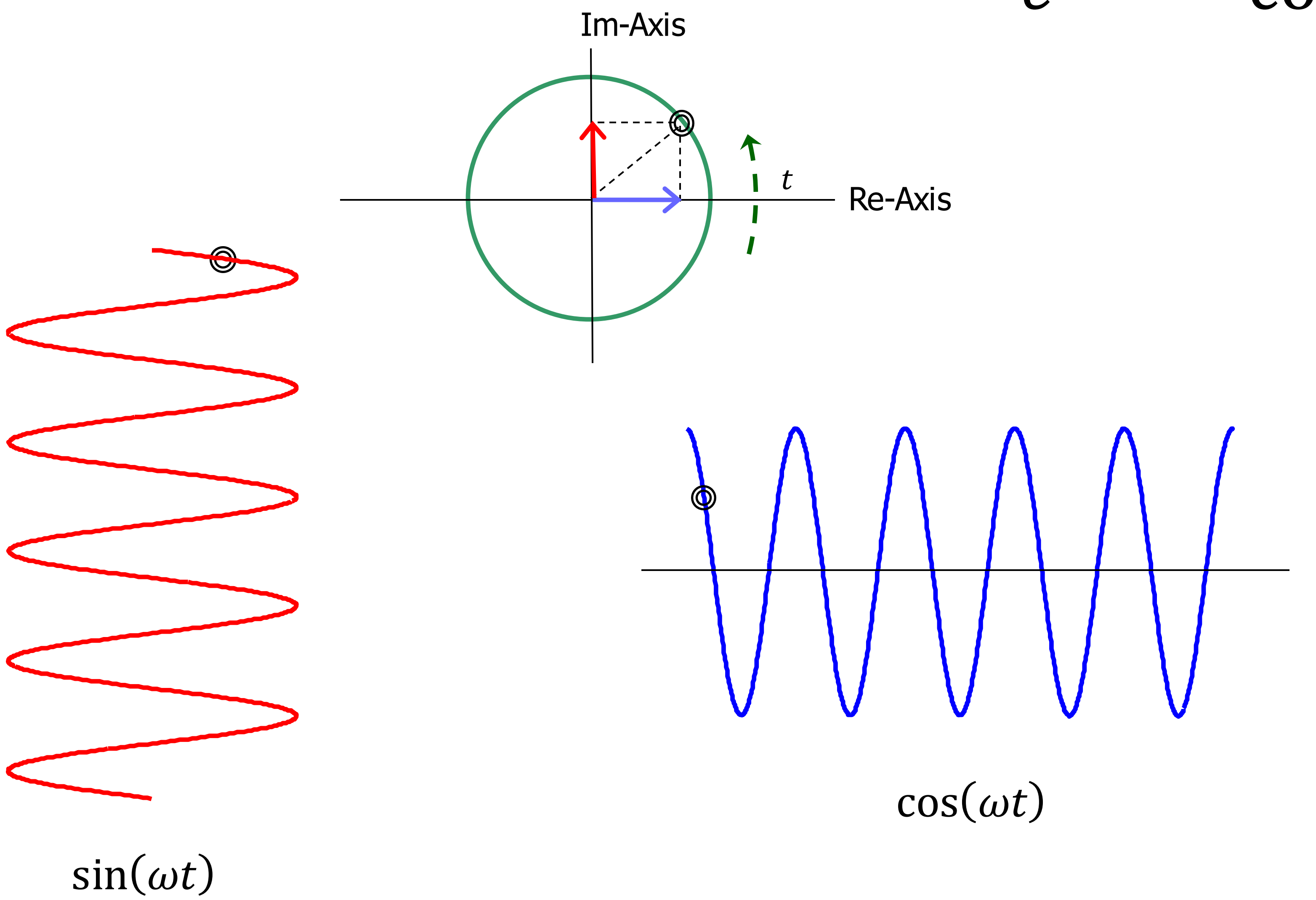
https://en.wikipedia.org/wiki/Sine_and_cosine

2D waves are along x and y axes simultaneously

Recall: Complex Exponential Function?

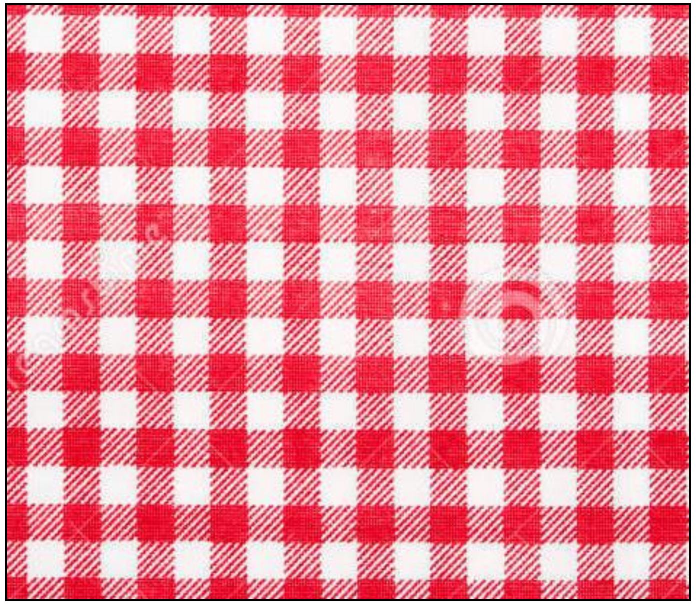
$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

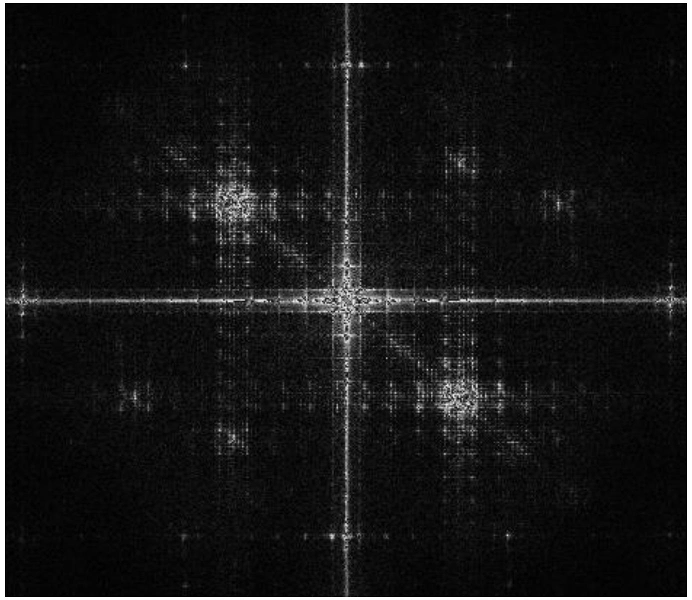
j: the complex number $j = \sqrt{-1}$

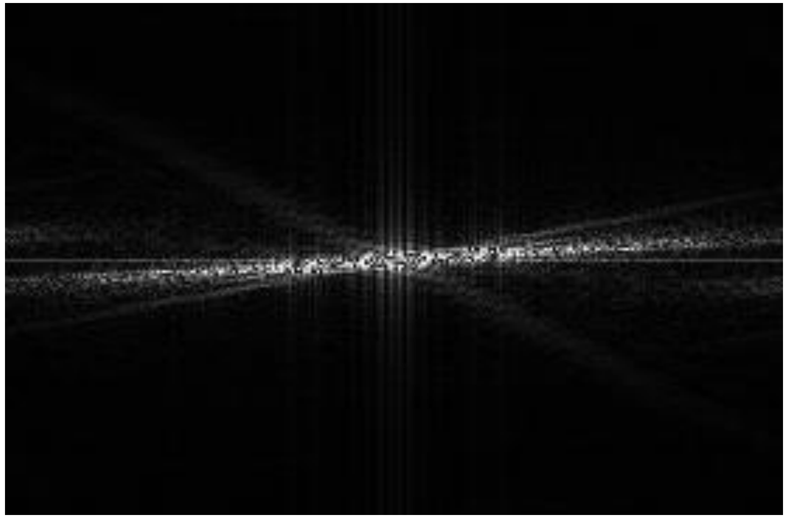


Recall: Frequency

- N.b. colour immaterial (just for visuals)







The relationship between the frequency ξ and the period T is: $\xi = 1/T$

Joseph Fourier

- Any periodic function is the result of adding up sine and cosine waves of different frequencies
- “Fourier’s treatise is one of the very few scientific books that can never be rendered antiquated by the progress of science” James Clerk Maxwell 1878

Step up Fourier...

A time−variant signal

j: the complex number $j = \sqrt{-1}$

Fourier transform of signal p

at angular frequency ω

$$Fp(\omega) = \int_{-\infty}^{\infty} p(t)\, e^{-j\omega t}\, dt$$

$$e^{-j\omega t} = \cos(\omega t) - j \sin(\omega t)$$

ω: angular frequency;

ω = 2 π ξ, where frequency ξ is $1/t$

Inverse Fourier…

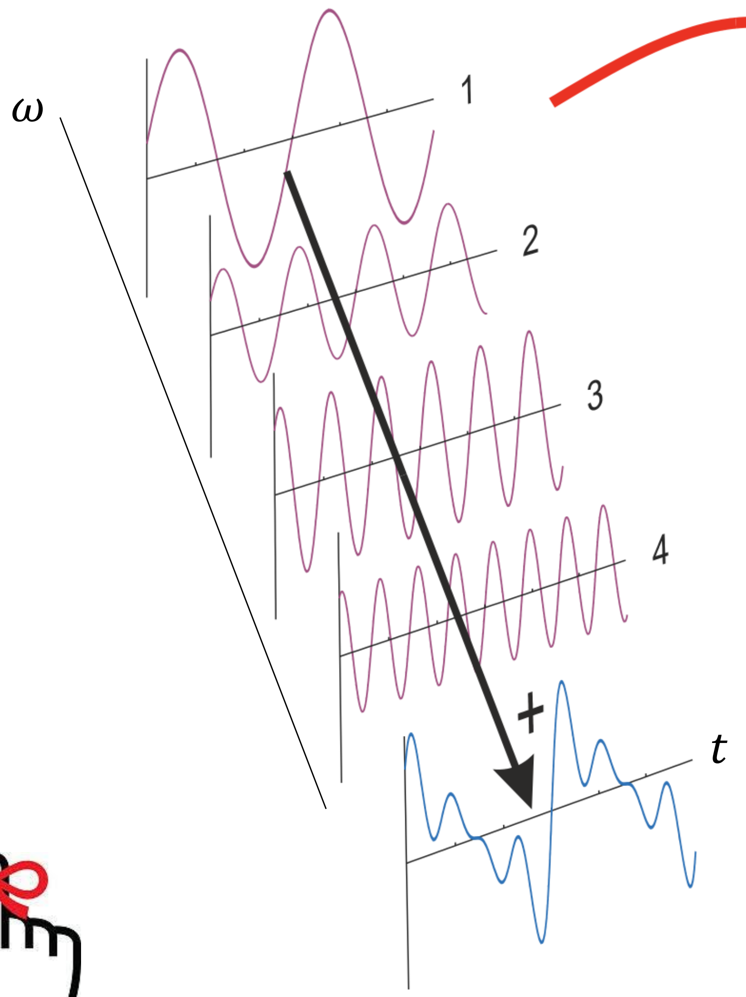
Fourier coefficients

Original signal in time domain

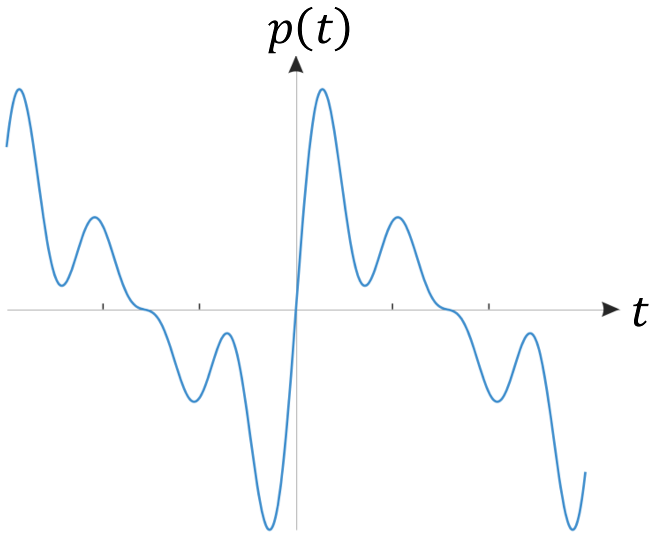
$$p(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Fp(\omega)\, e^{j\omega t}\, d\omega$$

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

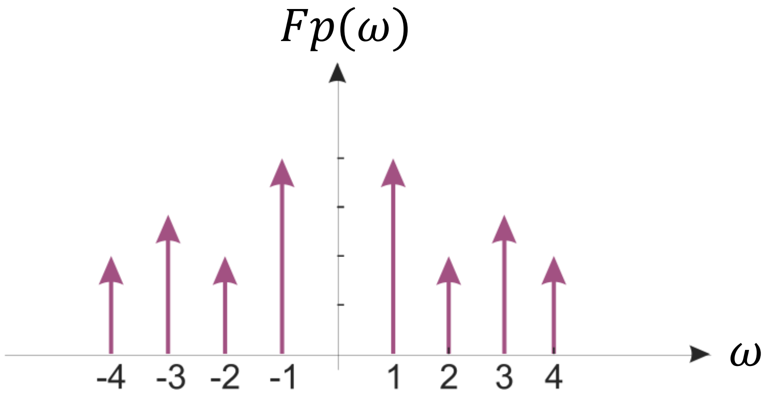
What does the Fourier transform do?



Addition in the frequency domain



Addition in the time domain



Separation in the frequency domain

Other Forms of Fourier …

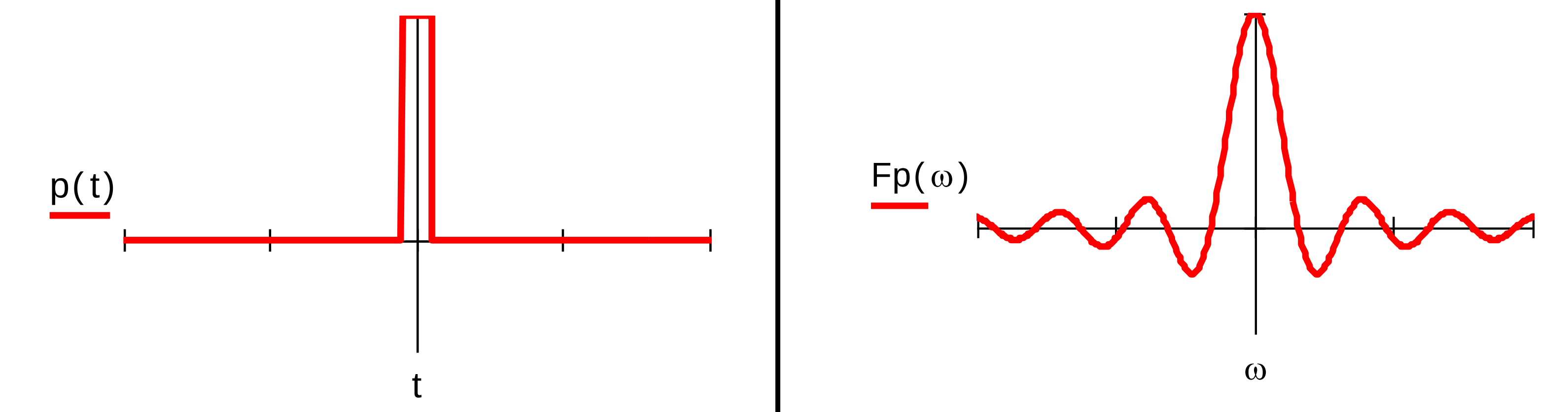
$$Fp(\xi) = \int_{-\infty}^{\infty} p(t)\, e^{-j2\pi\xi t}\, dt$$

Fourier Transform

$$p(t) = \int_{-\infty}^{\infty} Fp(\xi)\, e^{j2\pi\xi t}\, d\xi$$

Inverse Fourier Transform

A rectangular pulse and its Fourier transform



(a) pulse of amplitude $A = 1$ **(b)** Fourier transform

- Pulse $$p(t) = \begin{cases} A & \text{if} \quad -T/2 \le t \le T/2 \\ 0 & \text{otherwise} \end{cases}$$

- Use Fourier $$Fp(\omega) = \int_{-T/2}^{T/2} A e^{-j\omega t} \, dt$$

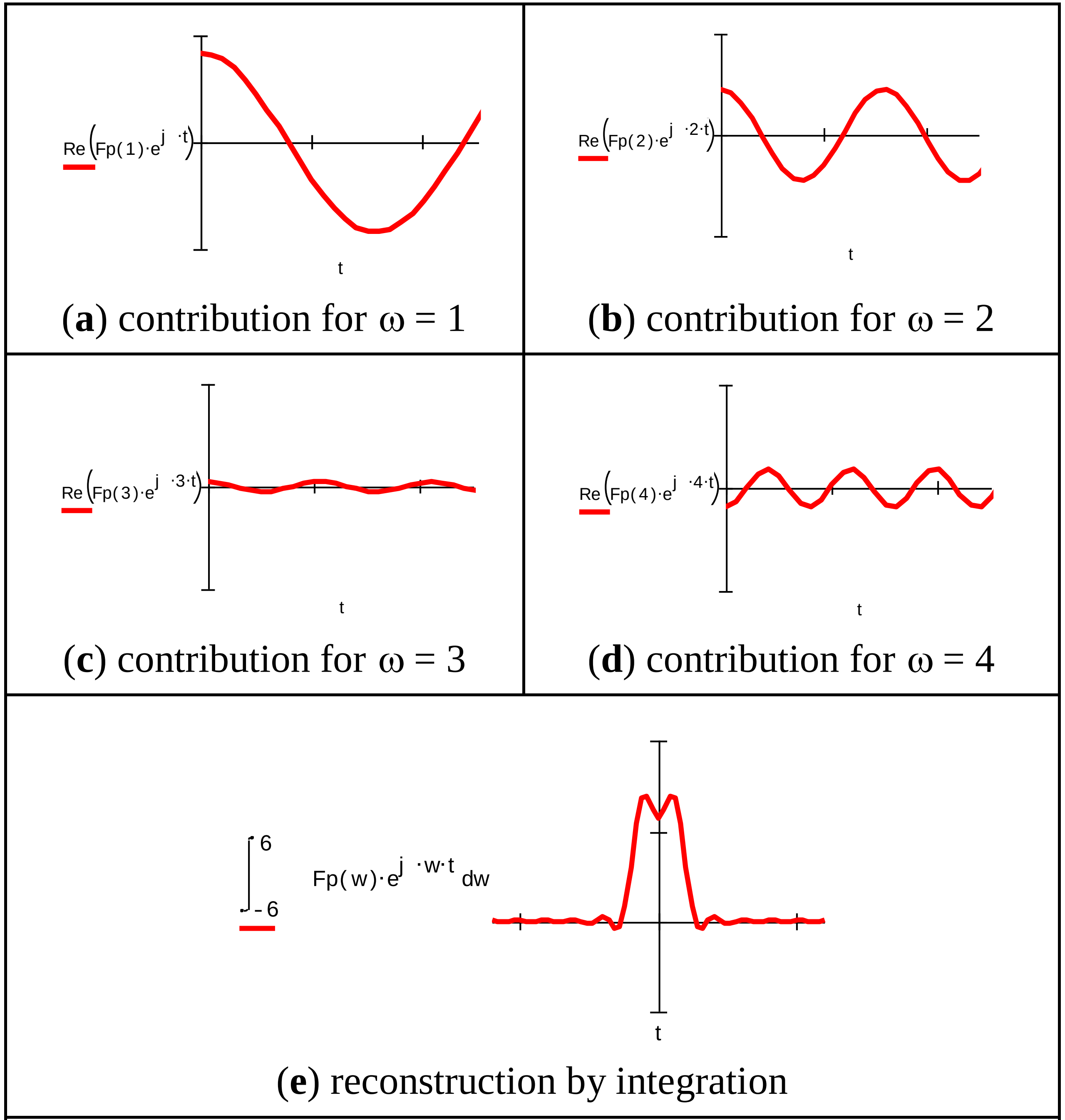
- Evaluate integral $$Fp(\omega) = -\frac{A e^{-j\omega T/2} - A e^{j\omega T/2}}{j\omega}$$

- And get result $$Fp(\omega) = \begin{cases} \frac{2A}{\omega} \sin\left(\frac{\omega T}{2}\right) & \text{if} \quad \omega \ne 0 \\ AT \quad \text{if} \quad \omega = 0 \end{cases}$$

Reconstructing a signal from its Fourier transform

This is the inverse Fourier transform

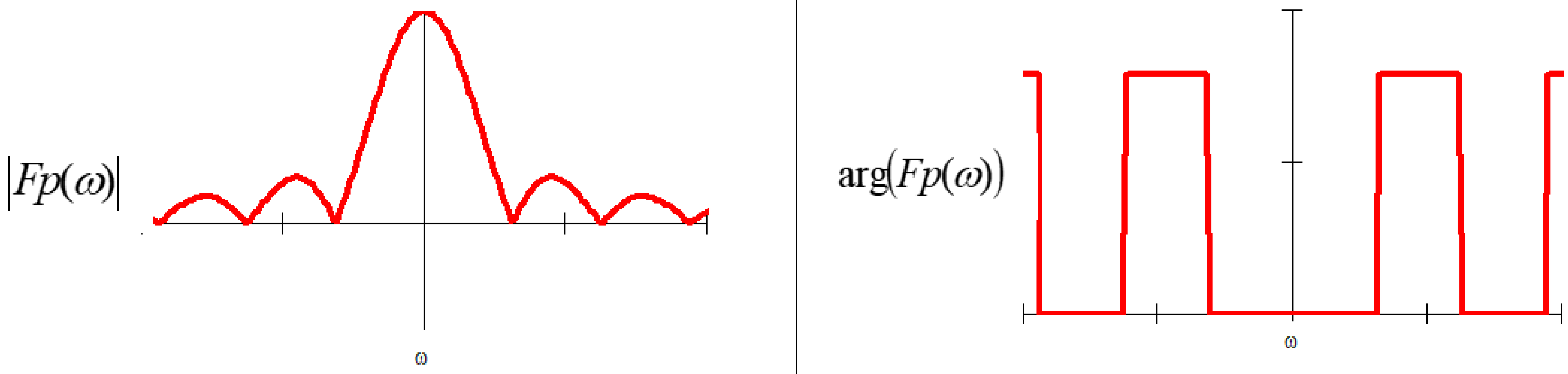
$$p(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Fp(\omega)\ e^{j\omega t}\, d\omega$$



Reconstructing a Signal from its Transform

Magnitude and phase of Fourier transform of a pulse

$$Fp(\omega) = \int_{-\infty}^{\infty} p(t)e^{-j\omega t}\,dt = \mathrm{Re}\big(Fp(\omega)\big) + j\,\mathrm{Im}\big(Fp(\omega)\big)$$



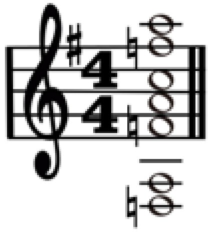
(a) magnitude **(b)** phase

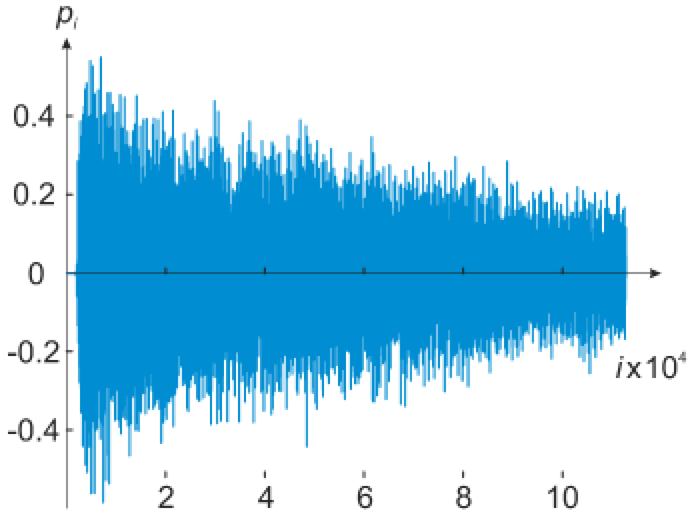
$$|Fp(\omega)| = \sqrt{\mathrm{Re}\big(Fp(\omega)\big)^2 + \mathrm{Im}\big(Fp(\omega)\big)^2}$$

$$\arg\big(Fp(\omega)\big) = \tan^{-1}\left(\frac{\mathrm{Im}\big(Fp(\omega)\big)}{\mathrm{Re}\big(Fp(\omega)\big)}\right)$$

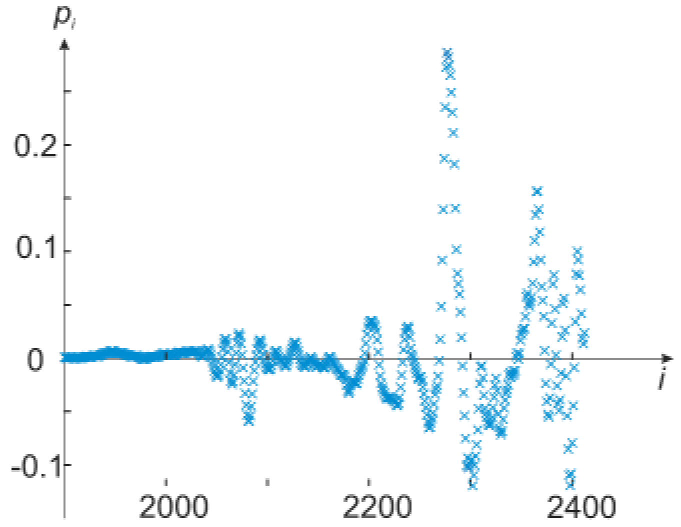
Hard day?

Let’s see the Fourier transform of the Hard Day’s night chord

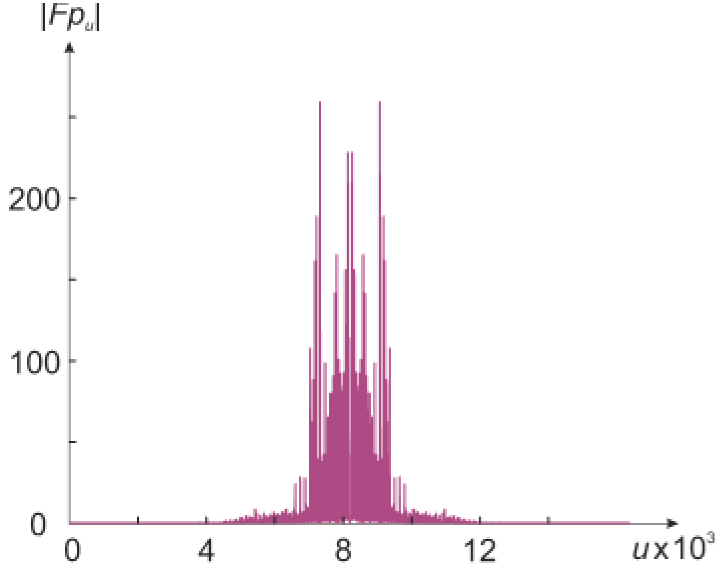




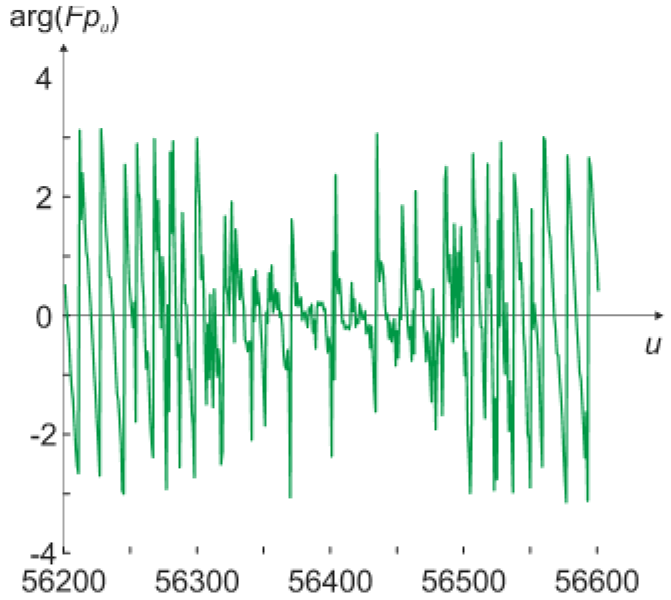
(a) Recorded data



(b) A closer look at the start of (a)

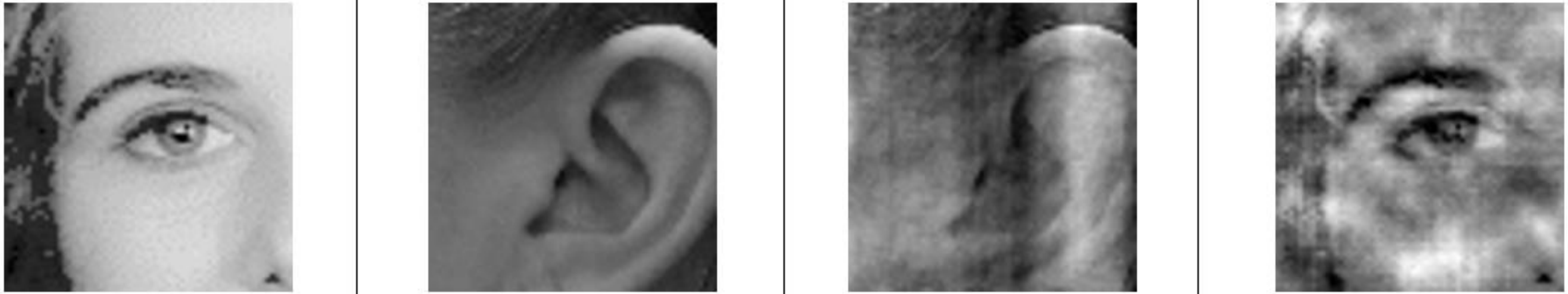


(c) Fourier Spectrum



(d) Phase of central portion

Illustrating the importance of phase

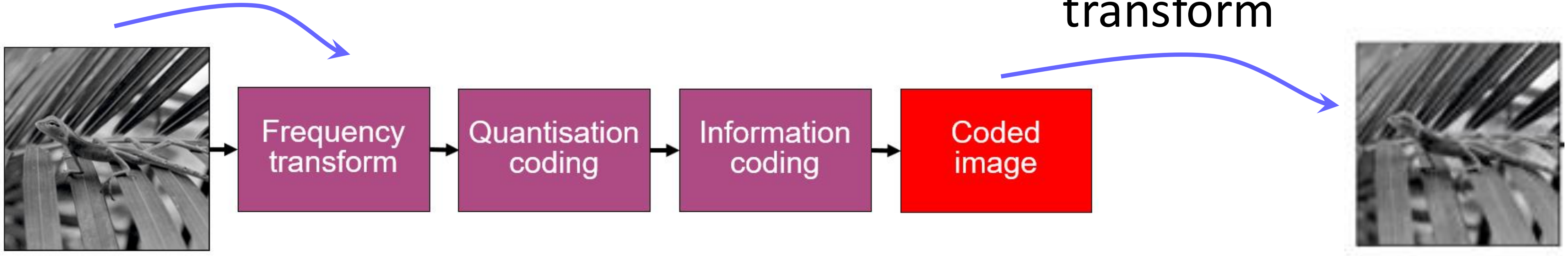


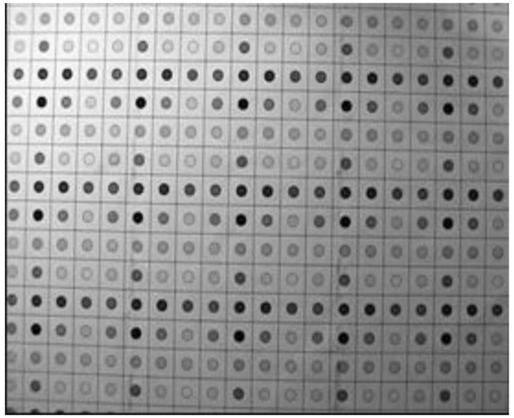
(a) eye image	(b) ear image	(c) reconstruction from magnitude(eye) and phase(ear)	(d) reconstruction from magnitude(ear) and phase(eye)

Inverse Fourier transform is used for reconstruction

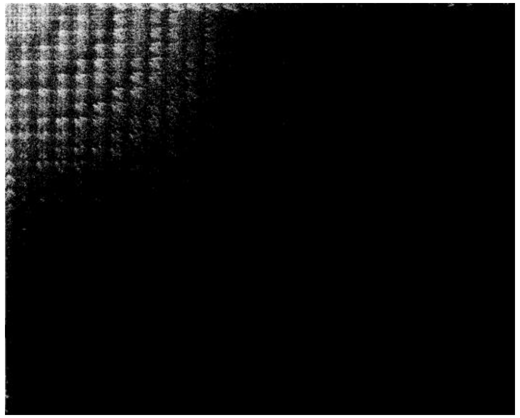
Fourier transform

Inverse Fourier transform

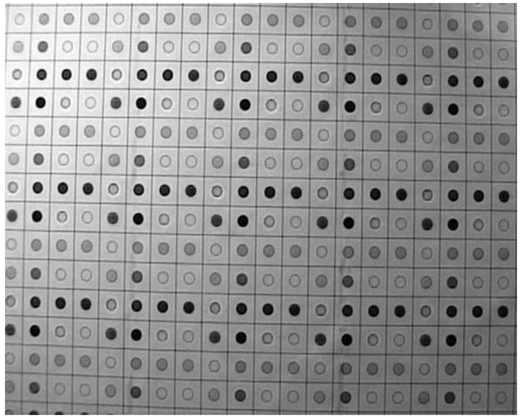




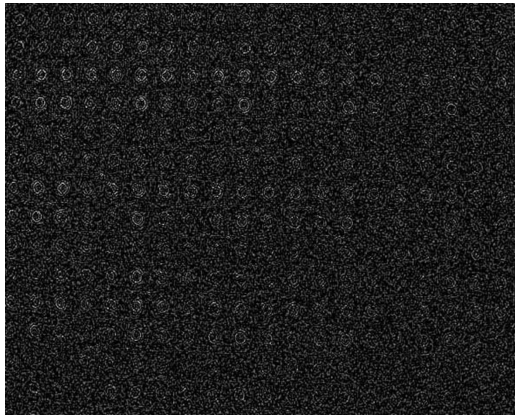
image



5% of transform components



reconstructed image



error

Main points so far

1 – sampling data is not as simple as it appears

2 – sampling affects space and brightness

3 – Fourier allows us to understand frequency

4 – Fourier allows for coding and more

Next, Fourier will allow us to understand
sampling

Other transforms

- Discrete Cosine (Sine) Transform
- Discrete Hartley Transform

Wavelets

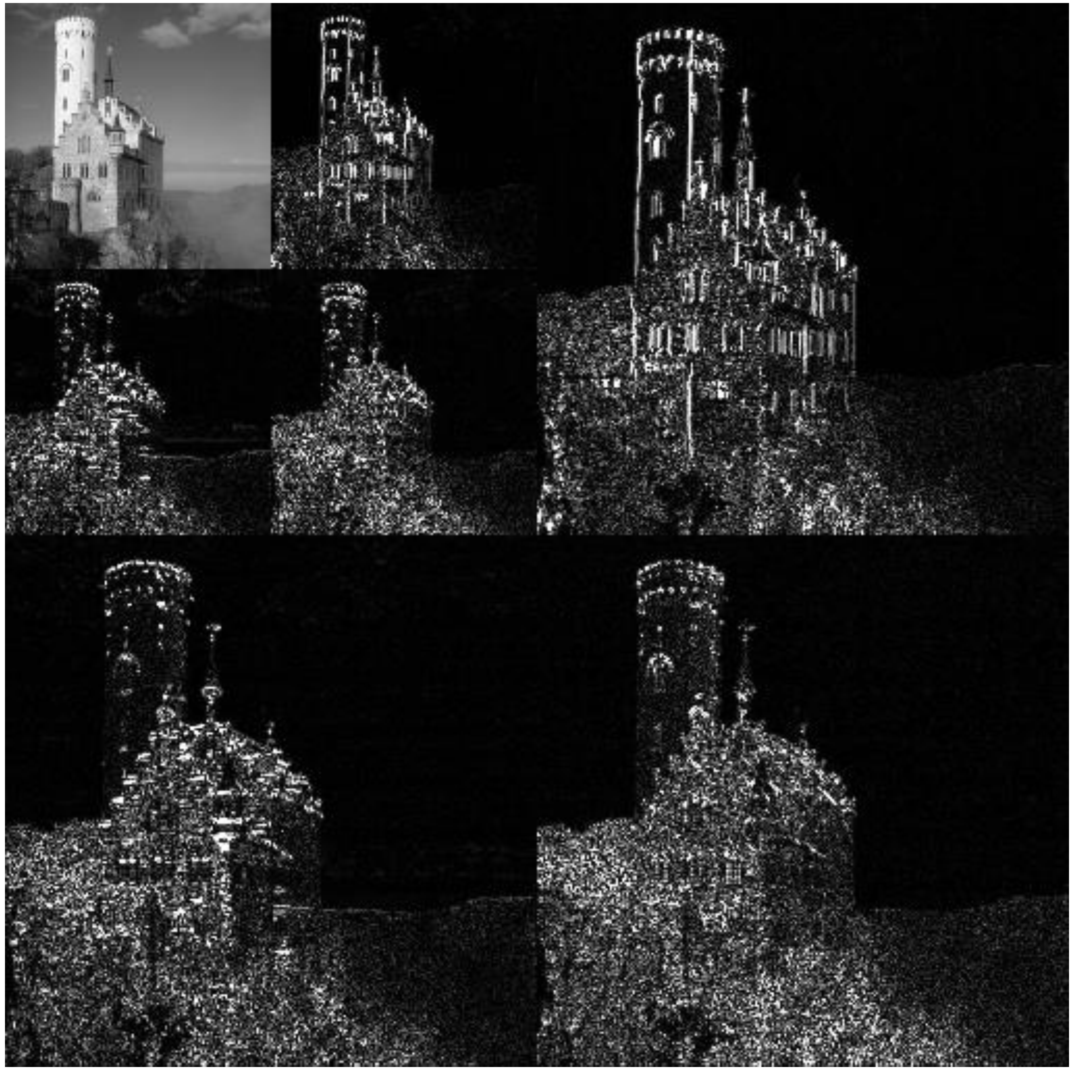
Continuous wavelet
Discrete wavelet
Complex wavelet
Stationary wavelet
Dual wavelet
Haar wavelet
Daubechies wavelet
Morlet wavelet
Gabor wavelet
……

Curvelets

Shearlets

Bandelet
Contourlet
Fresnelet
Chirplet
Noiselet
……

Wavelet transform



An example of the 2D discrete wavelet transform that is used in JPEG2000 [Credit: Wikipedia]