CS 252:

Advanced Programming Language Principles

The Simply Typed Lambda Calculus

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REVIEW: Arith Language

```
e ::= true
    | false
    | 0
    | succ e
    | pred e
    | iszero e
    | if e then e
else e
```

```
v ::= true
    | false
    | i
```

Types for Arith

```
T ::= Bool
    | Int
```

Typing rules for Arith (in-class)

Adding lambda functions

```
e ::= ...
    | λx.e
    | e e
    | x
```

```
v ::= ...
    | λx.e
```

Small-step evaluation rules for the Lambda Calculus (in-class)

So what is the type of a function?

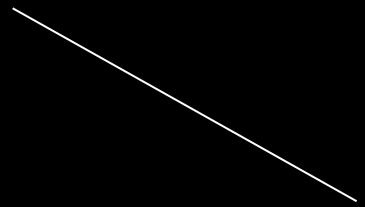
```
T ::= Bool
    | Nat
    | T -> T
```

Options to determine function type

1. type inference
2. require type annotations

Typing Annotations

$\lambda x{:}\mathbf{T}.e$



Type of the argument

Arith w/ Lambdas

```
e ::= true | false | 0
    | succ e | pred e
    | iszero e
    | if e then e else e
    | λx:T.e
    | e e
    | x
```

What is the type of `x`?

- How do we determine a variable's type in a function?
- We use a *typing environment:*
  - maps variables to types
  - we use the Greek letter gamma: Γ

New typing relation

We need to write our typing rules *given our new typing environment.*

Our revised typing relation is:

$$\Gamma \vdash e : T$$

This is known as the "turnstile"

Typing rules for the Simply Typed Lambda Calculus (in-class)

How powerful is this language?

Implement a typechecker for the Simply Typed Lambda Calculus

Download `stlc.hs`.

Complete the `typecheck` method following the rules outlined in class.