CS252 - Midterm Exam Study Guide

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Lecture #01 – General Introduction

Features of Good Programming Languages Reasons for Different Programming Language Design Choices Programming Languages 1. Flexibility 4. Safety (e.g. security and can errors be 1. Simplicity 1. Different domains (e.g. web, 2. Type safety caught at compile time) 2. Readability security, bioinformatics) 3. Performance 5. Machine independence 3. Learnability 6. Efficiency 2. Legacy code and libraries 4. Build Time

Conflict: Type Systems

3. Personal preference

- Advantage: Prevents bad programs.
- Disadvantage: Reduces programmer flexibility.

Blub Paradox: Why do I need advanced programming language techniques (e.g. monads, closures, type inference, etc.)? My language does not have it, and it works just fine.

5. Concurrency

Current Programming Language Issues

- · Multi-code "explosion"
- Big Data
- Mobile Devices

Advantages of Web and Scripting Languages

- Examples: Perl, Python, Ruby, PHP, JavaScript
- · Highly flexible

Goals almost always conflict

- Dynamic typing
- · Easy to get started
- Minimal typing (i.e. type systems)

Major Programming Language Research Contributions

- Garbage collection
- · Sound type systems
- Concurrency tools
- Closures

Programs that Manipulate Other Programs

- Compilers & interpreters
- JavaScript rewriting
- IDFs

Formal Semantics

- Used to share information unambiguously
- Can formally prove a language supports a given property
- Crisply define how a language works

Types of Formal Semantics

- Operational
 - o Big Step "natural"
 - o Small Step "structural"
- Axiomatic
- Denotational

Instrumentation

- Program Analyzers

Haskell

- Purely functional Define "what stuff is"
- No side effects
- Referential transparency A function with the same input parameters will always have the same result.
 - o An expression can be replaced with its value and nothing will change.
- Supports type inference.

Duck Typing – Suitability of an object for some function is determined not by its type but by presence of certain methods and properties.

- o More flexible but less safe.
- Supported by Haskell
- o Common in scripting languages (e.g. Python, Ruby)

Side Effects in Haskell

- · Generally not supported.
- Example of Support Side Effects: File IO
- Functions that do have side effects must be separated from other functions.

Lazy Evaluation

- · Results are not calculated until they are needed
- Allows for the representation of infinite data structures

Lecture #02 - Introduction to Haskell

Lists

Key Traits of Haskell

- 1. Purely functional
- 2. Lazy evaluation
- 3. Statically typed
- 4. Type Inference 5. Fully curried functions
- let Keyword required in ghci to set a
 - variable value. Example: > let f x = x + 1

ghci – Interactive Haskell.

- 4

Run Haskell from Command Line Use runhaskell keyword. Example:

> runhaskell <FileName>.hs

Hello World in Haskell

main :: IO () main = do putStrLn "Hello World"

Primitive Classes in Haskell

- 1.Int Bounded Integers
- 3.Float
- 4.Double
- 5.Bool
- 6.Char

- 2. Integer Unbounded

- **Hello World in Haskell** main :: IO ()

main = doputStrLn "Hello World"

Operators

o: Prepend

o ++ Concatenate

• Comma separated in square brackets

o!! Get element a specific index

o tail All elements after head

o head First element in list

- > f 3

o last Last element in the list

- o init All elements except the last o take n Take first n elements from a
- o replicate 1 m Create a list of length I containing only m
- o repeat m Create an in

List Examples

> putStrLn \$ "Hello " ++ "World" "Hello World"

> let s = bra in s !! 2 : s ++ 'c' : last s : 'd' : s "abracadabra"

Ranges

· Can be infinite or bounded

• Use the "..." notation. Examples: > [1..4]

[1, 2, 3, 4]

> [1,2..6] [1, 2, 3, 4, 5, 6]

> [1,3..10] [1, 3, 5, 7, 9]

Infinite List Example

> let even = [2,4..]> take 5 even

[2, 4, 6, 8, 10]

```
List Comprehension
                                                                        A Simple Function
• Based off set notation.
                                                              > let inc x = x + 1
                                                              > inc 3
• Supports filtering as shown in second example
                                                                                                                   Pattern Matching
• If multiple variables (e.g. a, b, c) are specified, iterates through
                                                                                                    • Used to handle different input data
 them like nested for loops.
                                                              > inc 4.5
                                                                                                    • Guard uses the pipe (|) operator
• Uses the pipe (|) operator. Examples:
                                                                                                    • Example:
> [ 2*x | x <- [1..5]]
                                                              > inc (-5) -- Negative
                                                                                                    inc :: Int -> Int
[2, 4, 6, 8, 10]
                                                                         Type Signature
                                                                                                      | x < 0 = error "invalid x"
> [(a, b, c) | a <- [1..10], b <-[1..10],
                                                              • Uses symbols ":: " and "->"
                                                                                                    inc x = x + 1
                  c \leftarrow [1..10], a^2 + b^2 = c^2]
                                                              • Example:
                                                              inc :: Int -> Int
 [(3, 4, 5), (4, 3, 5), (6, 8, 10), (8, 6, 10)]
                                                              inc x = x + 1
```

```
Recursion

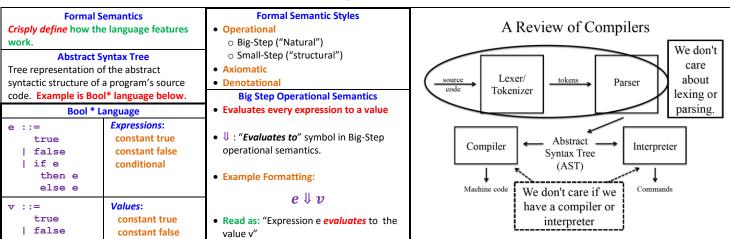
• Base Case – Says when recursion should stop.

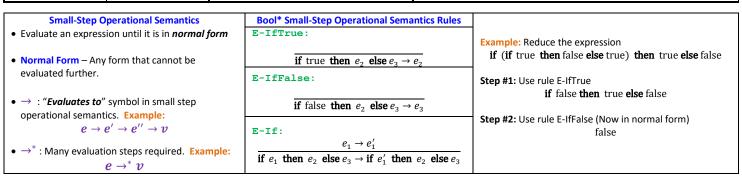
• Recursive Step – Calls the function with a smaller version of the problem

Example:
addNum :: [Int] -> Int
addNum [] = 0
addNum (x:xs) = x + addNum xs

| Cab #01 – Max Number
| Cab #01 – Max
```

Lecture #03 – Operational Semantics





| | Extended Bool * Language | Literate Haskell | Case Statement in Haskell |
|--|---|---|---|
| Bool* Extension: Numbers • 0 : The Number "0" | e ::= true false if e then e else e | • File Extension: ".lhs" | Keywords: case, of, otherwiseOperator: -> |
| • succ 0: Represents "1" • succ succ 0: Represents "2" | 0 succ e | • Code lines begin with ">" | Example: |
| • pred n: Gets the predecessor of "n" | pred e v ::= true false | All other lines are comments. | <pre>case x of val1 -> "Value 1"</pre> |
| | IntV IntV ::= 0 succ IntV | "Essentially swaps code with comments." | val2 -> "Value 2" otherwise -> "Everything else." |

Lab #02 Review

```
Bool Expression Type
> data BoolExp = BTrue
        | BFalse
>
        | Bif BoolExp BoolExp
        | B0
>
        | Bsucc BoolExp
>
>
        | Bpred BoolExp
    deriving Show
```

```
BoolVal Type
> data BoolVal = BVTrue
>
                | BVFalse
                | BVNum BVInt
>
>
    deriving Show
> data BVInt = BV0
               | BVSucc BVInt
    deriving Show
```

Type Constructors: BoolExp, BoolVal, BVInt

Non-nullary Value Constructors: Blf, Bsucc, Bpred, BVSucc, BVNum

Note: Even constants like BO, BTrue, BFalse, BVTrue, and BVFalse are nullary value constructors (since they take no arguments)

Lecture #04 – Higher Order Functions

Lambda

- Analogous to anonymous classes in Java.
- Based off Lambda calculus
- Example:

```
> (\x -> x + 1) 1
>(\x y -> x + y) 2 3
5
```

Function Composition

- Uses the period (.)
- f(g(x)) can be rewritten (f . g) x

Point-Free Style

• Pass function arguments no arguments.

```
> let inc = (+1) - No args
> inc 3
```

```
Example: Lambda with Function
Composition
> let f = (\x -> x - 5)
            . (\y -> y * 2)
9
> let f = (\x y \rightarrow x - y)
          (\z -> z * (-1))
> f 3 4
-7
```

Iterative vs. Recursive

- Iterative tends to be more efficient than recursive.
- Compiler can optimize tail recursive function.

Tail Recursive Function - The recursive call is the last step performed before returning a value.

Not Tail Recursive

```
public int factorial(int n) {
  if (n==1) return 1;
  else {
    return n * factorial(n-1);
```

Last step is the multiplication so not tail recursive.

Tail Recursive Factorial

```
public int factorialAcc(int n, int acc)
 if (n==1) return acc;
 else (
   return factorialAcc(n-1, n*acc);
```

Tail recursive code often uses the accumulator pattern like above.

Tail Recursion in Haskell

```
fact' :: Int -> Int -> Int
fact' 0 acc = acc
fact' n acc = fact' (n - 1) (n * acc)
```

Higher Order Functions

Functions in Functional Programming

- Functional languages treat programs as mathematical functions.
- Mathematical Definition of a Function: A function f is a rule that associates to each x from some set X of values a unique y from a set of Y values.

$$(x \in X \land y \in Y) \rightarrow y = f(x)$$

- **f** Name of the function
- X Independent variable
- y Dependent variable
- X Domain
- *Y* Range

Qualities of Functional Programming

- Functions clearly distinguish: Incoming values (parameters)
 - Outgoing Values (results)
- No (re)assignment
- No loops
- · Return values depend only on input parameters
- Functions are first class values; this means they can:
 - o Passed as arguments to a function
 - o Be returned from a function
 - o Construct new functions dynamically

Higher Order Function

Any function that takes a function as a parameter or returns a function as a result.

Function Currying

Transform a function with multiple arguments into multiple functions that each take exactly one argument.

Named after Haskell Brooks Curry.

Currying Example

addNums :: Num a => a -> a -> a

addNums is a function that takes in a number and returns a function that takes in another number.

map

- Built in Haskell higher order function
- Applies a function to all elements of a list.

filter

- Built in Haskell higher order function
- Removes all elements from a list that do not satisfy (i.e. make true) some predicate.

filter :: (a -> Bool) -> [a] -> [a]

> filter (>2) [1, 2, 3, 4]
[3, 4]

foldl

- Built in higher order function
- Does not support infinite lists.
- · Should only be used for special cases.

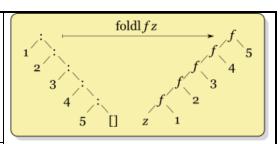
Example:

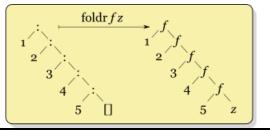
> foldl (
$$x y -> x - y$$
) 0 [1, 2, 3, 4] -10 -- (((0-1) - 2) - 3) - 4

foldr

- Built in higher order function
- Supports infinite lists.
- "Usually the right fold to use"

Example:





Thunk – A delayed computation

Due to lazy evaluation, foldl and foldr build thunks rather than calculate the results as they go.

foldl'

- Data.list.foldl' evaluates its results eagerly (i.e. does not use thunks)
- Good for large, but finite lists.

Lecture #05 – Small-Step Operational Semantics

WHILE Language

 Unlike the Bool* language, WHILE supports mutable references.

| e ::= a | Variable/addresses |
|------------------|--------------------|
| l v | Values |
| a:=e | Assignment |
| e;e | Sequence |
| e op e | Binary Operations |
| if e then e | Conditional |
| else e | |
| while (e) e | While Loops |
| v ::= i | Integers |
| b | Boolean |
| op ::= + - * | / |

Small Step Semantics with State

 Since the WHILE language supports mutable references, the grammar must be updated to support it.

While Relation:

$$e, \sigma \rightarrow e', \sigma'$$

• σ – Store. Maps references to values.

Example Operations:

- $\sigma(a)$ Retrieves the value at address "a"
- σ[a := v] Identical to the original store with the exception that it now stores the value v at address "a"

Evaluation Order Rules

- Tend to be repetitive and clutter the semantics.
- Context based rules tend to represent the same information as evaluation order rules but more concisely.

Reduction Rule

Rewrites the expression. Example:

E-IfFalse:

if false then e2 else e3 \rightarrow e3

Context Rule

Specify the order for evaluating expressions. Example:

E-If:

 $\frac{e_1 \rightarrow e_1'}{\text{if } e_1 \text{ then } e_2 \text{ else } e_3 \rightarrow \text{if } e_1' \text{ then } e_2 \text{ else } e_3}$

Reducible Expression (Redex) – Any expression that can be transformed (reduced) in one step.

| >= | > | <= | <

Example: Redex

if true then (if true then false else false) else true

This reduces to "if true then false else false"

Example: Not a Redex

if (if true then false else false) then true else true

Not a redex as expression "if true then false else false" must be evaluated first.

Evaluation Contexts

- Alternative to evaluation order rules.
- Marker (•) / hole indicate the next place for evaluation (i.e. where we will do the work).

Example:

C[r]

= if (if true then false else false) then true else true

r = **if** true **then** false **else** false

C = **if** • **then** true else true

C[r] is the original expression.

Rewriting Evaluation Order Rules

Context based rules only apply to reducible expressions (redexs). Example:

EC-IfFalse:

 $C[if false then e_2 else e_3] \rightarrow C[e_3]$

Context Syntax

C ::= •
 | if C then e else e
 | C op e
 | v op C

Data.Map

- Library: Data.Map
- Immutable
- Example Methods:
 - o Map.empty Creates and returns an empty map
 - Map.insert k v m Inserts a value "v" at key "k" into map "m". Returns a new, updated map.
 - Map.lookup k m Returns the value at key "k" in map "m". Wrapped in a maybe.

Precondition – Text above the line in a rule.

Context Rule for Binary Op:

 $\frac{v_3=v_1 \text{ op } v_2}{C[v_1 \text{ op } v_2] \rightarrow C[v_3]}$

How to Read a Small Step Semantic Rule: "Given <*Precondition>*, then <*LeftSideArrow>* evaluates to <*RightSideArrow>*."

Lecture #06 - LaTeX

TeX

- Created by Donald Knuth
- · Precisely controls the interface of content.
- Type of Literate **Programming - Logic is** in natural language and code is interspersed.

LaTeX

- Developed by Leslie Lamport. Derives from TeX.
- Type of Domain Specific Language (DSL) A computer language that is specialized for a particular application domain.
- Enforces separation of concerns Design principle for separating a computer program into different sections, such that each section addresses a separate
 - o Example: LaTeX separates formatting from content.
- Literate Programming

```
Specify Document Type
\documentclass{article}
```

Specify Title Block Content \title{Hello World!}

> **Start Document** \begin{document}

Generate Title from Title Information \title{Hello World!}

> Close the Document \end{document}

Cross-Reference \ref{<referenceName>}

Reference a Bibliography Citation \cite{<citationName>}

Create a Reference \label{<referenceName>}

Create a Bibliography \bibliography{<bibFileName>}

Create a List

\begin{itemize} \item Text for #1 \item Text for #2 \end{itemize}

Create Section with Label \section{Section #1} \label{sec:one}

Create Subsection with Label \subsection{<SubsectionName>} \label{sec:<refName>}

Use of Tilde (~)

Creates an undividable space so the text "Section~\ref{sec:one}" will appear on one line

RihTeX

- · References are tedious to reformat and
- Reference details shorted in a "*.bib" file.

Create a Bibliography \bibliography{biblio}

BibTeX filename for the example would be "biblio.bib"

Define Bibliography Style \bibliographystyle{plainurl}

BibTeX Article Reference Example

```
@article { citationName ,
   author = {Donald Knuth},
   title = {Literate Programming},
   journal = {},
   year = {1984},
volume = {27},
   number = \{2\},
   pages = {97-111},
```

Maybe Map Example

Lecture #07 – Types and Typeclasses

Maybe Type

- · Example of an algebraic data type
- Enables behavior similar to null in Java
- Used when:
 - o A function may not return a value
- o A caller may not pass an argument
- Definition:

data Maybe a = Nothing | Just a

Algebraic Data Type

 A composite data type (i.e. a type

made from other

Created via the

Keyword: data

• Examples: Either,

Maybe, Tree

types).

Maybe "Divide" Example

```
divide :: Int -> Int -> Maybe Int
divide 0 = Nothing
divide x y = Just $ x `div` y
> divide 5 2
> divide 4 0
Nothing
```

DO NOT FORGET THE Just IN CORRECT SOLUTION

import Data.Map

m = Map.empty

m' = Map.insert "a" 42 m case (Map.lookup "a") of Nothing -> error "Element not in map" Just x -> putStrLn \$ show x

Since element may not be in the map, you need to use a maybe

Example Algebraic Data Type

```
data Tree k = EmptyTree
           | Node (Tree k) (Tree k) val
           deriving (Show)
```

k - Type parameter. Specifies a type not a value.

Node: Value Constructor that creates values of type "Tree k"

• Tree and Tree Int have no types since they themselves form a concrete

• Node does have a type:

```
> :t Node
Node :: (Tree k) \rightarrow (Tree k) \rightarrow k \rightarrow (Tree k)
```

Explanation: To make a complete Node object, you pass it two objects of type "Tree k" and another object of type "k" and that returns a "Tree k" object.

Partially Applying a Value Constructor

- Value constructors can be partially applied similar to functions.
- > let leaf = Node EmptyTree EmptyTree

This creates a three node tree with value 5 at the root and values

Type of the "+" Operator

```
> :t (+)
(+) :: (Num a) => a -> a -> a
```

Explanation: The plus sign takes two numbers of type "a" and returns an object of type "a".

Type of a Number

```
> :t 3
3 :: (Num \ a) => a
```

Explanation: Since "3" has no explicit type, it can for now be any type that satisfies the "Num" type class.

- > Node (leaf 3) (leaf 7) 5

3 and 7 at the leaves.

```
Typeclasses
                            Kinds
                                                                                                            Example: Make Maybe an Instance of Eq

    Similar to interfaces in Java.

                                                                                                            instance (Eq a) => Eq (Maybe a) of

    Like a contract.

                                                                                                                   (==) Nothing Nothing = true
                                                                   o Implementation details can be included
                                        String Kind
                                                                                                                   (==) (Just x) (Just y) = x == y
                                                                     in typeclass definition.
                             > :kind String
                                                                                                                                                 = false
                             String:: *
                                                                • No relation to classes in object-oriented

 "The type of types".

                                                                                                            Need to ensure type "a" supports "Eq" so add that as
                                                                  programming.
                                        Map Kind
• Concrete types have a kind
                                                                                                            a class constraint.
                             > :k Map
                                                                   o Example: Do not have any data
 of "*"
                             Map :: * -> * -> *
                                                                     associated with them.
• Keyword :k, :kind
                                                                                                            Class Constraint
• Example:
                                       Maybe Kind

    Simplify polymorphism.

                                                                                                            Operator: =>
                             > :k Maybe
> :k Tree
                                                                                                            • Ensures that a type parameter satisfies some type
                             Map:: * -> *
Tree :: * -> *
                                                                Example: Eq Typeclass
                                                                                                              class requirement.
                                     Map String Kind
                                                                class Eq a where
Explanation: A Tree requires
                                                                                                                           Kind of Typeclasses
                             > :kind (Map String)
                                                                      (==) :: a -> a -> Bool
one type parameter to be
                             (Map String) :: * -> *
                                                                      (/=) :: a -> a -> Bool
made a concrete type.
                                                                                                            > :k Eq
                                                                     x == y = not (x /= y)
                                                                                                            Eq :: * -> Constraint
                             Explanation: Map String is has one
                                                                      x \neq y = not (x == y)
                             of the two type parameters filled so
                                                                                                            > :k Num
                             it has one less asterisk.
                                                                The last two lines in the type class definition
                                                                                                            Num :: * -> Constraint
                                                                allow the developer to program either (==) or
                                                                (/=) but not necessarily both.
                                                                                                            Note: Typeclasses are a class constaint (not a type)
                                                                                                            so their kind is different.
```

Lecture #08 – Functors

```
Functor – Something that can be mapped over.
                                                                                          Examples: map and fmap on Lists
        Functor Type Class Definition
                                             • Handles things "inside a box"
                                                                                                                           Examples: fmap on Maybes
                                                                                          > map (+1) [1, 2, 3]
class Functor f where
                                              Example: List ([]) as an Instance of Functor
                                                                                          [2, 3, 4]
  fmap :: (a \rightarrow b) \rightarrow f a \rightarrow f b
                                                                                                                           > fmap (+1) (Just 3)
                                                                                                                           Just 4
                                                                                          > fmap (+1) [1, 2, 3]
                                             instance Functor [] where
This is very similar to the definition of the
                                                                                          [2, 3, 4]
                                                fmap = map
                                                                                                                           > fmap (+1) Nothing
higher order function "map"
                                                                                                                           Nothing
                                                                                          > fmap (+1) []
                                             Explanation: map is a specialized version of
map :: (a -> b) -> [a] -> [b]
                                                                                          []
                                             fmap for lists.
```

```
Example: Either as an Instance of Functor
                                                     Either Algebraic Data Type
Example: Maybe as an Instance of Functor
                                                                                       instance Functor (Either a) where
                                           data Either a b = Left a
                                                                                          fmap _ (Left x) = Left x
fmap f (Right y) = Right (f y)
                                                              | Right b
instance Functor Maybe where
                                                   deriving (Eq,Ord,Read,Show)
   fmap _ Nothing = Nothing
   fmap f (Just x) = Just (f x)
                                                                                       > fmap (+1) Right 20
                                           • Left - Error type that is not mappable.
                                                                                       20 -- No Change
DO NOT FORGET THE Just IN VALID SOLUTION
                                                                                       > fmap (+1) Left 20
                                           • Right - Expected type
                                                                                       21 -- Changed
```

IO in Haskell

| Haskell avoids side effects but they are inevitable in real programs. Monads | Type Signature of the main Function in Haskell main :: IO () | do – Allows for the chaining of multiple IO commands together | |
|---|--|--|--|
| Related to FunctorsCompartmentalize side effects. | Hello World in Haskell main = putStrLn "Hello World" | • <- Extracts data out of an "IO Box" | |
| • () O Unit type in Haskell | Type Signature of getLine getLine :: IO String | • return – Places data into an "IO Box" | |

do Example

```
return in Haskell
```

- Unrelated to "return" in other languages
- Better described as "wrap" or "box"

```
Summary:
return – Boxes an IO
<- Unboxes an IO
```

```
Type of the Unit Type ()

• Base type

> :t ()
() :: ()

Type of return
> :t (return ())
(return ()) :: Monad m => m ()
```

Using IO as a Functor

```
main = do
     line <- fmap (++"!!!") getLine
    putStrLn line</pre>
```

Explanation: This function takes a string input from standard in and appends "!!!" at which point it prints it to the console.

Definition of IO as a Functor

Explanation: The action object is taken out of the IO box, the function "f" applied to it, and then returned to the IO box.

id Function

• Takes one input parameter and returns that input parameter unmodified. Examples:

```
> id 3
```

Monad is a typeclass.

> id "Hello World"
"Hello World"

Functor Laws

Functor Law #1: If we map the id function over a Functor, the Functor that we get back should be the same as the original Functor.

```
Examples:
> fmap id (Just 3)
Just 3
> fmap id Nothing
Nothing
> fmap id [1, 2, 3]
[1, 2, 3]
```

Functor Law #2: Composing two functions and then mapping the resulting (composed) function over a Functor should be the same as first mapping one function over the Functor and then mapping the other one.

The Functor laws are NOT enforced. They are good practice that makes the code easier to reason about.

Lecture #09 – Applicative Functors

Functor – Something that can be mapped over. Allow you to map functions over different data types. Examples:

- Maybe
- Either
- 10
- Lists
- <*>

Functors return boxed up values.

Functor Example

```
> fmap (+1) [1, 2, 3]
[2, 3, 4]
> let x = fmap (+) [1, 2, 3]
```

Explanation: In this case \mathbf{x} is: [(1+), (2+), (3+)]

Applicative Functor

• Requires the importing of a special library as shown below:

import Control.Applicative

Functions in Applicative Typeclass:

- pure Wraps/boxes a value
- <*>- Infix version of fmap. Is itself a Functor.

```
Example Uses of pure
> pure 7
7
> pure 7 :: Maybe Int
Just 7
> pure 7 :: [Int]
```

Type Class Definition of Applicative

```
class (Functor f) => Applicative f where
    pure :: a -> f a
    <*> :: f (a -> b) -> f a -> f b
```

Only difference between <*> and fmap is that the function in <*> is boxed while it is not in fmap (see the green f).

Make Maybe an Instance of Applicative

```
instance Applicative Maybe where
  pure = Just
  Nothing <*> = Nothing
  (Just f) <*> x = fmap f x
```

Explanation: pure simply wraps the value in **Just**. No need to explicitly check if "**x**" is maybe as **fmap** will do that for you.

Examples of Applicative Maybe

```
> Just (+3) <*> Just 4

Just 7
> pure (+3) <*> Just 4

Just 7
> pure (+) <*> Just 3 <*> Just 4

Just 7
> (+) <$> Just 3 <*> Just 4

Just 7

Explanation: x <$> is fmap as an infix operator. It is NOT necessarily the same as pure x <*>
```

```
Making [] an Instance of Applicative
```

```
instance Applicative [] where
  pure x = [x]
  fs <*> xs = [f x | f <- fs, x <- xs]</pre>
```

Explanation: The function is actually a list of functions so list comprehension is needed.

```
Example Use of Applicative on Lists
> (*) <$> [1, 2, 3] <*> [1,0,0,1]
[1,0,0,1,2,0,0,2,3,0,0,3]
> pure 7
7 -- No change
```

> pure 7 :: [Int]

[7]

```
Definition of IO as an Instance of Applicative
instance Applicative IO where
  pure = return
  a <*> b = do
      f <- a
      x <- b
      return (f x)</pre>
```

| Example of Applicative IO | liftA2 |
|---|--|
| import Control.Applicative | A function that simplifies the application of a normal function to two Functors. |
| <pre>main = do a <- (++) <\$> getLine <*> getLine putStrLn a</pre> | liftA2 :: (Applicative f) => (a -> b -> c) -> f a -> f b -> fc liftA2 f x y = f <\$> a <*> b |
| Example of liftA2 | |
| > (:) <\$> Just 3 <*> Just [4] Just [3, 4] > liftA2 (:) (Just 3) (Just [4]) | |

Applicative Functor Laws

Just [3, 4]

| Law 1: pure f <*> x = fmap f x | Law 2: pure id <*> v = v | Law 3: pure (.) <*> u <*> v <*> w = u <*> (v <*> w) |
|--|---|--|
| Law 4: pure f <*> pure x = pure (f x) | Law 5: u <*> pure y = pure (\$y) <*> u | |

Monoids

| | ociative binary function and a value that ty with respect to that function. | Definition of Monoid Typeclass | |
|-------------|---|---|--|
| | Examples | class Monoid m where | |
| • x/1 | Identity of Division | mempty :: m | |
| • x * 1 | Identity of Multiplication | mappend :: m -> m -> m | |
| • lst ++ [] | Identity of Concatenation | mconcat :: [m] -> m | |
| • x + 0 | Identity of Addition | <pre>mconcat = foldr mappend mempty</pre> | |
| • x−0 | Identity of Subtraction | | |

Monoid Rules

| Rule #1: | Rule #2: | Rule #3: |
|------------------------|-----------------------------|--|
| mempty `mappend` x = x | $x \rightarrow mappend = x$ | <pre>(x `mappend` y) `mappend` z = x `mappend` (y `mappend` z)</pre> |

Lecture #10 - Monads

Problem with Functors: Do not support chaining of

| Functor – Something that can be mapped over. Definition: | multiple commands. Example: | | Functors. |
|---|------------------------------|--|---|
| instance Functor f where | > fmap (+) (Just 3) (Just 4) | | <pre>class (Functor f) => Applicative f where (<*>) :: f (a -> b) -> f a -> f b</pre> |
| fmap :: (a -> b) -> f a -> f b | and (Just 4) | cannot resolve (Just 3+) Requires library Control. Applicative | |
| Even with Applicative Functors, it is not possible to chain together multiple commands. Example: Monads: Each the chaining together of multiple functions. | | Example #1: Using Just > (Just 3) >>= (\x 12 | -> Just (x + 4)) >>= (\y -> Just (y+5)) |
| > Just (+3) <*> Just (+4) <*> Just (+5) Returns error Key Operator: >>= (Bind) | | <pre>Example #2: Using return > (return 3) >>= (\x 12</pre> | -> return (x + 4)) >>= (\y -> return (y+5)) |

| Comparing <*> and >>= | Example of <\$>, <*> and >>= | |
|---|---|--|
| Functor: (<*>) :: Applicative f => f (a -> b) -> f a -> f b | > (\x -> x + 1) <\$> Just 3 Just 4 | Example: Implement applyMaybe that applies a function to a Maybe |
| Monad: (>>=) :: Monad m => m a -> (a -> m b) -> m b | > Just (\x -> x + 1) <*> Just(3) Just 4 | applyMaybe :: Maybe a -> (a -> b) - > (Maybe b) |
| Differences: | | applyMaybe Nothing _ = Nothing |
| 1. Order of the arguments changed. | | applyMaybe (Just x) $f = Just (f x)$ |
| 2. The function is boxed in Functor but not Monad | > (Just 3) >>= (\x -> Just(x+1)) | |
| 3. Monad function returns a boxed result. | Just 4 | |

Applicative Functor: A Functor that can be applied to other

```
Monad Typeclass Definition
                                                                  Example a Robot Moving Towards a Goal (Not Failure)
                                                                               -- Define Operator and start location
                                                                              x -: f = f x
class Monad m where
                                            --Location
      return :: a -> m a
                                            type Robot = (Int, Int)
                                                                              start = (0, 0)
      (>>=) :: m a -> (a -> m b) -> m b
                                            -- Functions
                                                                              > start -: up -: right
      (>>) :: m a -> m b -> m b
                                            up (x,y) = (x, y+1)
                                                                              (1, 1)
      x \gg y = x \gg (\ -> y) --Lamda
                                            down (x,y) = (x, y-1)
                                            left (x,y) = (x-1, y)
                                                                              > start -: up -: left -: left -: right -: down
      fail :: String -> m a
                                            right (x,y) = (x+1, y)
                                                                              (-1.0)
      fail msg = error msg
```

```
Example a Robot Moving Towards a Goal (with Failure)
                                     -- Once the goal is reached,
                                     -- the robot stops
                                     goal := Map.empty
                                                                                start = (0, 0)
                                             -: (Map.insert (0, 2) True)
Maybe as an Instance of the Monad Typeclass
                                             -: (Map.insert (-1, 3) True)
                                             -: (Map.insert (-3, -8) True)
                                                                                > return start >>= up >>= left >>= left
instance Monad Maybe where
                                                                                               >>= right >>= down
                                     moveTo :: Pos -> Maybe Pos
                                                                                Just (-1, 0)
     return = Just
                                     moveTo p = if Map.member p goal
                                                                                > return start >>= left >>= left >>= up
                                                      then Nothing
     (>>=) Nothing
                      = Nothing
                                                                                               else Just p
     (>>=) (Just x) f = Just (f x)
                                                                                               >>= right >>= right >>= down
                                                                               Nothing
                                     -- Since these are in bind, no need
    fail _
                      = Nothing
                                     -- to handle Nothing. Bind handles it.
                                     up(x,y) = moveTo(x, y+1)
                                                                                Explanation: Reached one of the goals (-1, 3) at the red up
                                     down (x,y) = moveTo (x, y-1)
                                     left (x,y) = moveTo (x-1, y)
                                     right(x,y) = moveTo(x+1, y)
```

Integer Division Using Monads

```
Integer Division with Bind with "do"
                                                                                                            Integer Division with Bind with "do" and return
       Integer Division with Bind and No "do"
                                                    mydiv :: Maybe Int -> Maybe Int -> Maybe Int
                                                                                                        mydiv :: Maybe Int -> Maybe Int -> Maybe Int
mydiv :: Maybe Int -> Maybe Int -> Maybe Int
                                                    mydiv x y = do
                                                                                                        mydiv x y = do
mydiv x y = x >>= (\numer ->
                                                                 numer <- x
                                                                                                                     numer <- x
             y >>= (\denom ->
                                                                 denom <- y
                                                                                                                     denom <- y
             if denom > 0
                                                                 if denom > 0
                                                                                                                      if denom > 0
                 then Just (div numer denom)
else fail "Div by zero")
                                                                       then Just (div numer denom)
                                                                                                                         then return $ div numer denom
                                                                       else fail "Div by 0"
                                                                                                                         else fail "Div by 0"
```

List Monad

```
Making List an Instance of Monad
                                                                Example Use of List as a Monad
instance Monad [] where
                                                        listOfTuples :: [(Int, Char)]
        return x = [x]
                                                        listOfTuples = do
         (>>=) xs f = concat(map f xs)
                                                                         n <- [1, 2]
        fail _
                     = []
                                                                         ch <- ['a', 'b']
                                                                         return (n, ch)
Explnation: concat is needed here as f returns elements
                                                        > listOfTuples
already in a list. As such, concat merges the individual lists
                                                        [(1, 'a'), (1, 'b'), (2, 'a'), (2, 'b')]
(from each call to f) into a single list.
```

Miscellaneous

Kind of Show and show > :k Show Show :: * -> Constraint

Type and Kind of show

```
> :k show
Error (A function not a type)
> :t show
show :: (Show a) => a -> String
```

```
Lambda and ADT Combined
> (\x -> Just (x+1)) 1
Just 2
```

Creating Type Alias

```
type String = [Char]
```

Allows for more readable code as developer can use a type name that makes more sense for a given application.

```
Example: applyMaybe that takes a (Maybe a) and applies to it a function
            that takes a normal a and returns a (Maybe b)
```

```
applyMaybe :: (Maybe a) -> (a -> Maybe b) -> (Maybe b)
applyMaybe Nothing _ = Nothing
applyMaybe (Just x) f = f x
```

Explanation: Since the function "f" already returns a Maybe, you do not need to re-box it. However, since it does not take a Maybe, you need to unbox the first input parameter.

Applying return to Items

```
7return
> return 7 :: Maybe Int
Just 7
> return 7 :: [Int]
[7] -- Need Int or get an error
```

Conclusion: Behavior for return is the same as pure. Both put the object in the minimum default context that still yields that value.

List comprehension is syntactic sugar for using lists as monads.

Monads and Lambda

When trying to chain multiple functions together in a Monad, remember the Monad must return a boxed value. Hence, Lambda often work well as they simplifying boxing.

Applicative Typeclass - Allows you to use normal functions on values that have a context (i.e. are inside a Functor).

Monad: Given a value of type, a, in a context, m, apply a function that takes a normal value of type a and returns a value in the context m.

$$(>>=)$$
 :: (Monad m) => m a -> (a -> m b) -> m b

Monads are just applicative functors that support bind (>>=).

Key Difference: Applicative functors support normal functions that take and return unboxed values while Monads return boxed values.

return - Monad equivalent of "pure" for Applicative Functors.

Cannot use fmap in the definition of a Monad since fmap returns a boxed value while the function of the Monad returns a boxed value. Hence, if you used fmap with a Monad, you would return a double boxed value.