**CS252 – Midterm Exam Study Guide**

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**Lecture #01 – General Introduction**

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| **Reasons for Different**  **Programming Languages**   1. **Different domains** (e.g. web, security, bioinformatics) 2. **Legacy code and libraries** 3. **Personal preference** | **Programming Language Design Choices**   1. **Flexibility** 2. **Type safety** 3. **Performance** 4. **Build Time** 5. **Concurrency** | **Features of Good Programming Languages** | |
| 1. **Simplicity** 2. **Readability** 3. **Learnability** | 1. **Safety** (e.g. security and can errors be caught at compile time) 2. **Machine independence** 3. **Efficiency** |
| **Goals almost always conflict** | |

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| **Conflict: Type Systems**   * **Advantage:** Prevents bad programs. * **Disadvantage:** Reduces programmer flexibility. | **Blub Paradox:** Why do I need advanced programming language techniques (e.g. monads, closures, type inference, etc.)? My language does not have it, and it works just fine. | **Current Programming Language Issues**   * **Multi-code “explosion”** * **Big Data** * **Mobile Devices** | **Advantages of Web and Scripting Languages**   * **Examples:** Perl, Python, Ruby, PHP, JavaScript * **Highly flexible** * **Dynamic typing** * **Easy to get started** * **Minimal typing** (i.e. type systems) |

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| **Major Programming Language Research Contributions**   * **Garbage collection** * ***Sound* type systems** * **Concurrency tools** * **Closures** | **Programs that Manipulate Other Programs**   * **Compilers & interpreters** * **JavaScript rewriting** * **Instrumentation** * **Program Analyzers** * **IDEs** | **Formal Semantics**   * Used to **share information *unambiguously*** * **Can formally prove a language supports a given property** * ***Crisply define* *how a language works*** | **Types of Formal Semantics**   * **Operational**   + Big Step “***natural***”   + Small Step “***structural***” * **Axiomatic** * **Denotational** |

**Haskell**

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| * **Purely functional** – Define “*what stuff is*” * **No side effects** * **Referential transparency** – **A function with the same input parameters will always have the same result**.   + **An expression can be replaced with its value and nothing will change.** * **Supports type inference.** | **Duck Typing** – Suitability of an object for some function is determined not by its type but by presence of certain methods and properties.   * + **More flexible** but **less safe**.   + **Supported by Haskell**   + **Common in scripting languages** (e.g. Python, Ruby) | **Side Effects in Haskell**   * Generally not supported. * **Example of Support Side Effects**: File IO * Functions that do have side effects must be separated from other functions. |
| **Lazy Evaluation**   * **Results are not calculated until they are needed** * **Allows for the representation of infinite data structures** |

**Lecture #02 – Introduction to Haskell**

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| **Key Traits of Haskell**   1. **Purely functional** 2. **Lazy evaluation** 3. **Statically typed** 4. **Type Inference** 5. **Fully curried functions** | **ghci** – Interactive Haskell.  **let** – Keyword required in ghci to set a variable value. **Example**:  **> let f x = x + 1**  **> f 3**  **4** | **Run Haskell from Command Line**  Use **runhaskell** keyword.  **Example**:  **> runhaskell <*FileName*>.hs** | **Hello World in Haskell**  **main :: IO ()**  **main = do**  **putStrLn “Hello World”** |

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| **Primitive Classes in Haskell**   1. **Int** – **Bounded** Integers 2. **Integer** – **Unbounded** 3. **Float** 4. **Double** 5. **Bool** 6. **Char** | **Lists** | | **Ranges** |
| * **Base 0** * Comma separated in square brackets * **Operators**   + **:**  Prepend   + **++** Concatenate   + **!!** Get element a specific index   + **head** First element in list   + **tail** All elements after head | * + **last** Last element in the list   + **init** All elements in the list except the last one   + **take n** Take first n elements from a list   + **replicate l m** Create a list of length l containing only m   + **repeat m** Create an infinite list containing only m | * Can be infinite or bounded * Use the “**..**” notation. **Examples**:   **> [1..4]**  **[1, 2, 3, 4]**  **> [1,2..6]**  **[1, 2, 3, 4, 5, 6]**  **> [1,3..10]**  **[1, 3, 5, 7, 9]**  **> [5, 4..1]**  **[5, 4, 3, 2, 1]** |
| **Hello World in Haskell**  **main :: IO ()**  **main = do**  **putStrLn “Hello World”** | **List Examples**  **> putStrLn $ “Hello “ ++ “World”**  **“Hello World”**  **> let s = bra in s !! 2 : s ++ ‘c’ : last s : ‘d’ : s**  **“abracadabra”** | | **Infinite List Example**  **> let even = [2,4..]**  **> take 5 even**  **[2, 4, 6, 8, 10]** |

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| **List Comprehension**   * **Based off set notation.** * **Supports filtering** as shown in second example * If **multiple variables** (e.g. a, b, c) are specified, **iterates through them like nested for loops**. * Uses the **pipe** (**|**) operator. **Examples:**   **> [ 2\*x | x <- [1..5]]**  **[2, 4, 6, 8, 10]** | **A Simple Function**  **> let inc x = x + 1**  **> inc 3**  **4**  **> inc 4.5**  **5.5**  **> inc (-5) -- Negative**  **-4** | **Pattern Matching**   * Used to handle different input data * Guard uses the pipe (**|**) operator * **Example**:   **inc :: Int -> Int**  **inc x**  **| x < 0 = error “invalid x”**  **inc x = x + 1** |
| **> [(a, b, c) | a <- [1..10], b <-[1..10],**  **c <- [1..10], a^2 + b ^2 == c^2]**  **[(3, 4, 5), (4, 3, 5), (6, 8, 10), (8, 6, 10)]** | **Type Signature**   * Uses symbols “**::**” and “**->**” * **Example**:   **inc :: Int -> Int**  **inc x = x + 1** |

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| **Recursion**   * **Base Case** – Says when recursion should stop. * **Recursive Step** – Calls the function with a ***smaller version*** of the problem   **Example:**  **addNum :: [Int] -> Int**  **addNum [] = 0**  **addNum (x:xs) = x + addNum xs** | **Lab #01 – Max** **Number**  **> maxNum :: [Int] -> Int**  **> maxNum [] = error "Invalid Input"**  **> maxNum [x] = x**  **> maxNum (x:xs) = if x > maxXs then x else maxXs**  **> where maxXs = maxNum xs** | **Reasons for a Large Number of Programming Languages**   * **Different domains** * **Different design choices** |

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| **Recursion**   * **:t** or **:type** – Gets the type of a variable or function.   **Example:**  **> :type ‘A’**  **‘A’ :: Char**  **> :t “Hello”**  **“Hello” :: [Char]** | **Haskell’s Base Typeclasses**   * **Ord** – Can be ordered * **Eq** – Can perform equality check * **Show** – Can convert to String * **Read** – Can convert from String * **Enum** – Sequentially Ordered * **Bounded** – Has upper and lower bound. |  |

**Lecture #03 – Operational Semantics**

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| **Formal Semantics**  ***Crisply define*** **how the language features work**. | | **Formal Semantic Styles**   * **Operational** – **Specify how expressions should be evaluated.**   + Big-Step (“Natural”)   + Small-Step (“structural”) * **Axiomatic** * **Denotational** |  |
| **Abstract Syntax Tree**  Tree representation of the abstract syntactic structure of a program’s source code. **Example is Bool\* language below**. | |
| **Big Step Operational Semantics**   * **Evaluates every expression to a value** * : “***Evaluates to***” symbol in Big-Step operational semantics. * **Example Formatting:** * **Read as:** “Expression e ***evaluates*** to the value v” |
| **Bool \* Language** | |
| **e ::=**  **true**  **| false**  **| if e**  **then e**  **else e** | ***Expressions*:**  **constant true**  **constant false**  **conditional** |
| **v ::=**  **true**  **| false** | ***Values*:**  **constant true**  **constant false** |

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| **Small-Step Operational Semantics**   * Evaluate an expression until it is in ***normal form*** * **Normal Form** – Any form that cannot be evaluated further. * : “***Evaluates to***” symbol in small step operational semantics. **Example:** * : Many evaluation steps required. **Example:** | **Bool\* Small-Step Operational Semantics Rules** | **Example:** Reduce the expression  **Step #1:** Use rule “E-IfTrue” with “E-If”  **Step #2:** Use rule “E-IfFalse” (Now in normal form) |
| **E-IfTrue:** |
| **E-IfFalse:** |
| **E-If:** |

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| **Bool\* Extension: Numbers**   * **0** : The Number “0” * **succ 0** : Represents “1” * **succ succ 0** : Represents “2” * **pred n** : Gets the predecessor of “*n*” | **Extended Bool \* Language** | **Literate Haskell**   * **File Extension:** “.lhs” * **Code lines begin with “>”** * **All other lines are comments**. * “Essentially swaps code with comments.” | **Case Statement in Haskell**   * **Keywords:** **case**, **of**, **otherwise** * **Operator:** **->**   **Example**:  **case x of**  **val1 -> “Value 1”**  **val2 -> “Value 2”**  **otherwise -> “Everything else.”** |
| **e ::=**  **true**  **| false**  **| if e then e else e**  **| 0**  **| succ e**  **| pred e** |
| **v ::= true | false**  **| IntV**  **IntV ::= 0 | succ IntV** |

**Lab #02 Review**

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| **Bool Expression Type**  **> data BoolExp = BTrue**  **> | BFalse**  **> | Bif BoolExp BoolExp BoolExp**  **> | B0**  **> | Bsucc BoolExp**  **> | Bpred BoolExp**  **> deriving Show** | **BoolVal Type**  **> data BoolVal = BVTrue**  **> | BVFalse**  **> | BVNum BVInt**  **> deriving Show**  **> data BVInt = BV0**  **> | BVSucc BVInt**  **> deriving Show** | **Type Constructors:** BoolExp, BoolVal, BVInt  ***Non-nullary* Value Constructors:** BIf, Bsucc, Bpred, BVSucc, BVNum  **Note:** Even constants like B0, BTrue, BFalse, BVTrue, and BVFalse are nullary value constructors (since they take no arguments) |

**Lecture #04 – Higher Order Functions**

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| **Lambda**   * Analogous to anonymous classes in Java. * **Based off Lambda calculus** * **Example**:   **> (**\**x** -> **x + 1) 1**  **2**  **>(**\**x y** -> **x + y) 2 3**  **5** | **Function Composition**   * Uses the **period** (**.**) * **f(g(x))** can be rewritten **(f** . **g) x** | **Point-Free Style**   * Pass function arguments no arguments. Example:   **> let inc = (+1) – No args**  **> inc 3**  **4** | **Example: Lambda with Function Composition**  **> let f = (\x -> x – 5)**  **. (\y -> y \* 2)**  **> f 7**  **9**  **> let f = (\x y -> x – y)**  **. (\z -> z \* (-1))**  **> f 3 4**  **-7** |

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| **Iterative vs. Recursive**   * **Iterative tends to be more efficient than recursive.** * **Compiler can optimize tail recursive function.** | **Not Tail Recursive**  **public int factorial(int n) {**  **if (n==1) return 1;**  **else {**  **return n \* factorial(n-1);**  **}**  **}**  Last step is the multiplication so not tail recursive. | **Tail Recursive Factorial**  **public int factorialAcc(int n, int acc)**  **{**  **if (n==1) return acc;**  **else {**  **return factorialAcc(n-1, n\*acc);**  **}**  **}**  **Tail recursive code often uses the accumulator pattern like above.** |
| **Tail Recursive Function** – The recursive call is the last step performed before returning a value. |

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| **Tail Recursion in Haskell**  **fact' :: Int -> Int -> Int**  **fact' 0 acc = acc**  **fact' n acc = fact' (n - 1) (n \* acc)** |  |  |

**Higher Order Functions**

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| **Functions in Functional Programming**   * **Functional languages treat programs as mathematical functions**. * **Mathematical Definition of a Function**: A function is a rule that associates to each from some set of values a unique from a set of values. * – Name of the function * – Independent variable * – Dependent variable * – Domain * – Range | **Qualities of Functional Programming**   * **Functions clearly distinguish**:   + Incoming values (**parameters**)   + Outgoing Values (**results**) * **No (re)assignment** * **No loops** * **Return values depend only on input parameters** * ***Functions are first class values***; this means they can:   + **Passed as arguments to a function**   + **Be returned from a function**   + **Construct new functions dynamically** | **Higher Order Function**  Any function that **takes a function as a parameter *or* returns a function as a result**. |  |
| **Function Currying**  **Transform a function with multiple arguments into multiple functions that each take exactly one argument**.  Named after Haskell Brooks Curry.  **Currying Example**  **addNums :: Num a => a -> a -> a**  **addNums** is a **function that takes in a number and returns a function that takes in another number**. |

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| **map**   * Built in Haskell higher order function * **Applies a function to all elements of a list.**   **map :: (a -> b) -> [a] -> [b]**  **> map (+1) [1, 2, 3]**  **[2, 3, 4]** | **foldl**   * Built in higher order function * **Does not support infinite lists**. * **Should only be used for special cases**.   **foldl :: (b -> a -> b) -> b -> a -> b**  **Example:**  **> foldl (\x y -> x - y) 0 [1, 2, 3, 4]**  **-10 -- (((0-1) - 2) - 3) - 4** |  |
| **filter**   * Built in Haskell higher order function * **Removes all elements from a list that do not satisfy (i.e. make true) some predicate**.   **filter :: (a -> Bool) -> [a] -> [a]**  **> filter (>2) [1, 2, 3, 4]**  **[3, 4]** | **foldr**   * Built in higher order function * **Supports infinite lists**. * “***Usually the right fold to use***”   **foldr :: (b -> a -> a) -> a -> b -> a**  **Example:**  **> foldr (\x y -> x + y) 0 [1, 2, 3, 4]**  **-2 -- 1 – (2 – (3 – (4 – 0)))** |

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| **Thunk** – A delayed computation  Due to lazy evaluation, **foldl and foldr build thunks rather than calculate the results as they go**. | **foldl'**   * **Data.list.foldl’** evaluates its results eagerly (i.e. does not use **thunks**) * **Good for large, but finite lists.** |  |

**Lecture #05 – Small-Step Operational Semantics**

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| **WHILE Language**   * Unlike the Bool\* language, **WHILE supports mutable references**. | | **Small Step Semantics with State**   * Since the WHILE language supports mutable references, the grammar must be updated to support it.   **While Relation:**   * – Store. **Maps *references* to values.**   **Example Operations:**   * – Retrieves the value at address “” * – Identical to the original store with the exception that it now stores the value at address “” | **Evaluation Order Rules**   * **Tend to be repetitive and clutter the semantics.** * **Context based rules tend to represent the same information as evaluation order rules but more concisely.** |
| **e ::= a**  **| v**  **| a:=e**  **| e;e**  **| e op e**  **| if e then e**  **else e**  **| while (e) e** | Variable/addresses  Values  Assignment  Sequence  Binary Operations  Conditional  While Loops | **Reduction Rule**  Rewrites the expression. Example:  **E-IfFalse:**  **if false then e2 else e3** → **e3** |
| **Context Rule**  **Specify the order for evaluating expressions**. Example:  **E-If:** |
| **v ::= i**  **| b** | Integers  Boolean |
| **op ::= + | - | \* | /**  **| >= | > | <= | <** | |

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| **Reducible Expression** (**Redex**) – Any expression that can be transformed (reduced) in one step. | **Example: Redex**  This reduces to “” | **Example: Not a Redex**  Not a redex as expression “” must be evaluated first. |

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| **Evaluation Contexts**   * **Alternative to evaluation order rules**. * **Marker** (**●**) / **hole** indicate the **next place for evaluation** (i.e. where we will do the work).   **Example**:  is the original expression. | **Rewriting Evaluation Order Rules**  **Context based rules only apply to reducible expressions** (redexs). **Example**:  **EC-IfFalse**: | **Data.Map**   * **Library:** Data.Map * **Immutable** * **Example Methods**:   + **Map.empty** – Creates and returns an empty map   + **Map.insert k v m** – Inserts a value “**v**” at key “**k**” into map “**m**”. **Returns a new, updated map.**   + **Map.lookup k m** – Returns the value at key “**k**” in map “**m**”. **Wrapped in a maybe**. |
| **Context Syntax**  **C ::=**  **| if C then e else e**  **| C op e**  **| v op C**  **| ...** |

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| **Precondition** – Text above the line in a rule. | **Context Rule for Binary Op:** | **How to Read a Small Step Semantic Rule**: “Given <*Precondition*>, then <*LeftSideArrow*> evaluates to <*RightSideArrow*>.” |

**Lecture #06 – LaTeX**

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| **TeX**   * Created by Donald Knuth * **Domain specific language for typesetting documents**. * Precisely controls the interface of content. * Type of **Literate Programming** – **Logic is in natural language and code is interspersed**. “***Mark code instead of marking comments.***” | **LaTeX**   * Developed by Leslie Lamport. Derives from TeX. * Type of **Domain Specific Language** (DSL) – A **computer language that is specialized for a particular application domain**. * Enforces **separation of concerns** – Design principle for **separating a computer program into different sections, such that each section addresses a separate concern**.   + **Example:** LaTeX separates formatting from content. * **Literate Programming** | **Specify Document Type**  **\documentclass{article}**  **Specify Title Block Content**  **\title{Hello World!}**  **Start Document**  **\begin{document}**  **Generate Title from Title Information**  **\title{Hello World!}**  **Close the Document**  **\end{document}** | **Cross-Reference**  **\ref{*<referenceName>*}**  **Reference a Bibliography Citation**  **\cite{*<citationName>*}**  **Create a Reference**  **\label{*<referenceName>*}**  **Create a Bibliography**  **\bibliography{*<bibFileName>*}**  **Create a List**  **\begin{itemize}**  **\item Text for #1**  **\item Text for #2**  **\end{itemize}** |

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| **Create Section with Label**  **\section{Section #1}**  **\label{sec:one}**  **Create Subsection with Label**  **\subsection{<S*ubs*ectionName>}**  **\label{sec:<*refName*>}**  **Use of Tilde (~)**  Creates an undividable space so the text **“Section~\ref{sec:one}”** will appear on one line | **BibTeX**   * **References are tedious to reformat and renumber.** * Reference details shorted in a “\***.bib**” file.   **Create a Bibliography**  **\bibliography{*biblio*}**  BibTeX filename for the example would be “**biblio.bib**"  **Define Bibliography Style**  **\bibliographystyle{plainurl}** | **BibTeX Article Reference Example**  **@article{citationName,**  **author = {Donald Knuth},**  **title = {Literate Programming},**  **journal = {},**  **year = {1984},**  **volume = {27},**  **number = {2},**  **pages = {97—111},**  **}** |

**Lecture #07 – Types and Typeclasses**

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| **Maybe Type**   * **Example of an algebraic data type** * Enables behavior similar to **null** in Java * Can be used to provide context. * **Used when:**   + **A function may not return a value**   + **A caller may not pass an argument** * **Definition:**   **data Maybe a = Nothing**  **| Just a** | **Maybe “Divide” Example**  **divide :: Int -> Int -> Maybe Int**  **divide \_ 0 = Nothing**  **divide x y = Just $ x `div` y**  **> divide 5 2**  **2**  **> divide 4 0**  **Nothing**  **DO NOT FORGET THE Just IN CORRECT SOLUTION** | **Maybe Map Example**  **import Data.Map**  **m = Map.empty**  **m’ = Map.insert “a” 42 m**  **case (Map.lookup “a”) of**  **Nothing -> error “Element not in map”**  **Just x -> putStrLn $ show x**  **Since element may not be in the map, you need to use a maybe** |

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| **Algebraic Data Type**   * A **composite data type** (i.e. **a type made from other types**). * **Created via the Keyword:** **data** * **Examples:**   + **Either**   + **Maybe**   + **Tree** | **Example Algebraic Data Type**  **data Tree k = EmptyTree**  **| Node (Tree k) (Tree k) val**  **deriving (Show)**  **k** – **Type parameter**. **Specifies a type not a value**.  **Node**: **Value Constructor** **that creates values of type** “**Tree k**” | * **Tree and Tree Int have no types since they themselves form a concrete type**. * **Node** does have a type:   **> :t Node**  **Node :: (Tree k) -> (Tree k) -> k -> (Tree k)**  **Explanation:** **To make a complete Node object, you pass it two objects of type “Tree k” and another object of type “k” and that returns a “Tree k” object.** |
| **Partially Applying a Value Constructor**   * Value constructors can be partially applied similar to functions. **Example**:   **> let leaf = Node EmptyTree EmptyTree**  **> Node (leaf 3) (leaf 7) 5**  This creates a three node tree with value 5 at the root and values 3 and 7 at the leaves. | **Type of the “+” Operator**  **> :t (+)**  **(+) :: (Num a) => a -> a -> a**  **Explanation:** The plus sign takes two numbers of type “**a**” and returns an object of type “**a**”. |
| **Type of a Number**  **> :t 3**  **3 :: (Num a) => a**  **Explanation:** Since “3” has no explicit type, it can for now be any type that satisfies the “Num” type class. |

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| **Kinds** | | **Typeclasses** | |
| * “***The type of types***”. * **Concrete types have a kind of** “***\****” * **Keyword** **:k**, **:kind** * **Example**:   **> :k Tree**  **Tree :: \* -> \***  **Explanation:** A Tree requires one type parameter (e.g. **Int**) to be made a concrete type. | **String Kind**  **> :kind String**  **String** **:: \***  **Map Kind**  **> :k Map**  **Map** **:: \* -> \* -> \***  **Maybe Kind**  **> :k Maybe**  **Map** **:: \* -> \***  **Map String Kind**  **> :kind (Map String)**  **(Map** **String) :: \* -> \***  **Explanation:** Map String is has one of the two type parameters filled so it has one less asterisk. | * **Similar to interfaces in Java**.   + Like a contract.   + **Implementation details can be included in typeclass definition**. * No relation to classes in object-oriented programming.   + **Example**: Do not have any data associated with them. * **Simplify polymorphism**.   **Example**: **Eq** Typeclass  **class Eq a where**  **(==) :: a -> a -> Bool**  **(/=) :: a -> a -> Bool**  **x == y = not (x /= y)**  **x /= y = not (x == y)**  The last two lines in the type class definition allow the developer to program either (==) or (/=) but not necessarily both. | **Example:** Make **Maybe** an Instance of **Eq**  **instance (Eq a) => Eq (Maybe a) of**  **(==) Nothing Nothing = true**  **(==) (Just x) (Just y) = x == y**  **(==) \_ \_ = false**  Need to ensure type “a” supports “Eq” so add that as a **class constraint**. |
| **Class Constraint**   * **Operator**: => * Ensures that a type parameter satisfies some typeclass requirement. |
| **Kind of Typeclasses**  **> :k Eq**  **Eq :: \* -> Constraint**  **> :k Num**  **Num :: \* -> Constraint**  **Note: Typeclasses are a class constaint (not a type) so their kind is different**. |

**Lecture #08 – Functors**

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| **Functor Type Class Definition**  **class Functor f where**  **fmap :: (a -> b) -> f a -> f b**  This is very similar to the definition of the higher order function “map”  **map :: (a -> b) -> [a] -> [b]** | **Functor** – **Something that can be mapped over**.   * Handles things “inside a box”   **Example**: List (**[]**) as an instance of **Functor**  **instance Functor [] where**  **fmap = map**  **Explanation:** map is a specialized version of fmap for lists. | **Examples: map and fmap on Lists**  **> map (+1) [1, 2, 3]**  **[2, 3, 4]**  **> fmap (+1) [1, 2, 3]**  **[2, 3, 4]**  **> fmap (+1) []**  **[]** | **Examples: fmap on Maybes**  **> fmap (+1) (Just 3)**  **Just 4**  **> fmap (+1) Nothing**  **Nothing** |

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| **Example**: **Maybe** as an Instance of **Functor**  **instance Functor Maybe where**  **fmap \_ Nothing = Nothing**  **fmap f (Just x) = Just (f x)**  **DO NOT FORGET THE Just IN VALID SOLUTION** | **Either Algebraic Data Type**  **data Either a b = Left a**  **| Right b**  **deriving (Eq,Ord,Read,Show)**   * **Left** – **Error type that is not mappable**. * **Right** – **Expected type** | **Example**: **Either** as an Instance of **Functor**  **instance Functor (Either a) where**  **fmap \_ (Left x) = Left x**  **fmap f (Right y) = Right (f y)**  **> fmap (+1) Leftt 20**  **20 –- No Change**  **> fmap (+1) Right 20**  **21 –- Changed** |

**IO in Haskell**

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| * Haskell avoids side effects but they are inevitable in real programs. * **Monads**   + Related to Functors   + Compartmentalize side effects. * **()**   + Unit type in Haskell | **Type Signature of the main Function in Haskell**  **main :: IO ()**  **Hello World in Haskell**  **main = putStrLn “Hello World”**  **Type Signature of getLine**  **getLine :: IO String** | * **do** – Allows for the chaining of multiple IO/Monad commands together. **Syntactic sugar for bind** “**>>=**” * **<-** Extracts data out of an IO/Monad “Box” * **return** – Places data into an IO/Monad “Box” |  |

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| **do Example**  **main = do**  **line <- getLine**  **if null line -- Checks for empty str**  **then return ()**  **else putStrLn $ reverseWords line**  **reverseWords :: String -> String**  **reverseWords = unwords .**  **map reverse . words** | **return in Haskell**   * **Unrelated to “return” in other languages** * **Better described as “wrap” or “box”**   **Summary:**  **return** – Boxes an IO (**since IO is a monad**)  **<-** Unboxes an IO | **Type of the Unit Type ()**   * Base type   **> :t ()**  **() :: ()** |
| **Type of return**  **> :t (return ())**  **(return ()) :: Monad m => m ()**  **Monad** is a **typeclass**. |

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| **Using IO as a Functor**  **main = do**  **line <- fmap (++“!!!”) getLine**  **putStrLn line**  **Explanation:** This function takes a string input from standard in and appends “!!!” at which point it prints it to the console. | **Definition of** **IO** **as a Functor**  **instance Functor IO where**  **fmap f action = do**  **result *<-* action**  **return (f result)**  **Explanation:** The action object is taken out of the IO box, the function “f” applied to it, and then returned to the IO box. | **id** **Function**   * **Takes one input parameter and returns that input parameter unmodified**. **Examples**:   **> id 3**  **3**  **> id “Hello World”**  **“Hello World”** |

**Functor Laws**

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| **Functor Law #1:** **If we map the id function over a Functor, the Functor that we get back should be the same as the original Functor**.  **Examples:**  **> fmap id (Just 3)**  **Just 3**  **> fmap id Nothing**  **Nothing**  **> fmap id [1, 2, 3]**  **[1, 2, 3]** | **Functor Law #2:** **Composing two functions and then mapping the resulting (composed) function over a Functor should be the same as first mapping one function over the Functor and then mapping the other one**.  **Law #2 Written Formally**  **fmap (f . g) = fmap f . fmap g** | **The Functor laws are NOT enforced. They are good practice that makes the code easier to reason about.** |

**Lecture #09 – Applicative Functors**

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| **Functor** – **Something that can be mapped over**.  **Allow you to map functions over different data types**. **Examples**:   * **Maybe** * **Either** * **IO** * **Lists** * **<\*>**   **Functors return boxed up values**. | **Functor Example**  **> fmap (+1) [1, 2, 3]**  **[2, 3, 4]**  **> let x = fmap (+) [1, 2, 3]**  **Explanation:** In this case **x** is:  **[(1+), (2+), (3+)]** | **Applicative Functor**   * Requires the importing of a special library as shown below:   **import** **Control.Applicative**  Functions in Applicative Typeclass:   * **pure** – Wraps/boxes a value * **<\*>** - **Infix version of** **fmap**. Is itself a Functor. | **Example Uses of pure**  **> pure 7**  **7**  **> pure 7 :: Maybe Int**  **Just 7**  **> pure 7 :: [Int]**  **[7]** |

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| **Type Class Definition of Applicative**  **class (Functor f) => Applicative f where**  **pure :: a -> f a**  **<\*> :: f (a -> b) -> f a -> f b**  **Only difference between** **<\*>** **and** **fmap** **is that the function** **in** **<\*>** **is boxed while it is not in** **fmap** (see the **green** **f**). | **Make Maybe an Instance of Applicative**  **instance Applicative Maybe where**  **pure = Just**  **Nothing <\*> \_ = Nothing**  **(Just f) <\*> x = fmap f x**  **Explanation: pure** simply wraps the value in **Just**. No need to explicitly check if “**x**” is maybe as **fmap** will do that for you. | **Examples of Applicative Maybe**  **> Just (+3) <\*> Just 4**  **Just 7**  **> pure (+3) <\*> Just 4**  **Just 7**  **> pure (+) <\*> Just 3 <\*> Just 4**  **Just 7**  **> (+) <$> Just 3 <\*> Just 4**  **Just 7**  **Explanation:** **x <$>** **is** **fmap** **as an infix operator**. **It is NOT necessarily the same as** **pure x <\*>**. **It should be based off Applicative Functor Law #1.** |

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| **Making** **[]** **an Instance of** **Applicative**  **instance Applicative [] where**  **pure x = [x]**  **fs <\*> xs = [f x | f <- fs, x <- xs]**  **Explanation:** The function is actually a list of functions so list comprehension is needed. | **Example Use of** **Applicative on Lists**  **> (\*) <$> [1, 2, 3] <\*> [1,0,0,1]**  **[1,0,0,1,2,0,0,2,3,0,0,3]**  **> pure 7**  **7 -- No change**  **> pure 7 :: [Int]**  **[7]** | **Definition of** **IO** **as an Instance of** **Applicative**  **instance Applicative IO where**  **pure = return**  **a <\*> b = do**  **f <- a**  **x <- b**  **return (f x)** |

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| **Example of Applicative** **IO**  **import Control.Applicative**  **main = do**  **a <- (++) <$> getLine <\*> getLine**  **putStrLn a** | **liftA2**  A function that simplifies the application of a normal function to two Functors.  **liftA2 :: (Applicative f) => (a -> b -> c) -> f a -> f b -> fc**  **liftA2 f x y = f <$> a <\*> b** |

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| **Example of liftA2**  **> (:) <$> Just 3 <\*> Just [4]**  **Just [3, 4]**  **> liftA2 (:) (Just 3) (Just [4])**  **Just [3, 4]** |  | **Applicative Functor Definition**  **A functor you can apply to other Functors.** |

**Applicative Functor Laws**

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| **Law 1:**  **pure f <\*> x = fmap f x** | **Law 2:**  **pure id <\*> v = v** | **Law 3:**  **pure (.) <\*> u <\*> v <\*> w = u <\*> (v <\*> w)** |
| **Law 4:**  **pure f <\*> pure x = pure (f x)** | **Law 5:**  **u <\*> pure y = pure ($y) <\*> u** | **Similar to Functor Laws, these are not strictly enforced but are good practice to make it easier to reason about the code.** |

**Monoids**

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| **Monoid:** An **associative** **binary function and a value that acts as an identity with respect to that function**.  **Examples**   * x \* 1 Identity of **Multiplication** * lst ++ [] Identity of **Concatenation** * x + 0 Identity of **Addition** | **Definition of Monoid Typeclass**  **class Monoid m where**  **mempty :: m**  **mappend :: m -> m -> m**  **mconcat :: [m] -> m**  **mconcat = foldr mappend mempty** |  |

**Monoid Rules**

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| **Rule #1:**  **mempty `mappend` x = x** | **Rule #2:**  **x `mappend` mempty = x** | **Rule #3:**  **(x `mappend` y) `mappend` z = x `mappend` (y `mappend` z)** |

**Lecture #10 – Monads**

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| **Functor** – **Something that can be mapped over**.  **Definition:**  **instance Functor f where**  **fmap :: (a -> b) -> f a -> f b** | **Problem with Functors:** Do not support chaining of multiple commands. **Example**:  **> fmap (+) (Just 3) (Just 4)**  Returns an error since it cannot resolve **(Just 3+)** and **(Just 4)** | **Applicative Functor:** A **Functor that can be applied to other Functors**.  **class (Functor f) => Applicative f where**  **(<\*>) :: f (a -> b) -> f a -> f b**  **Requires library Control.Applicative** |

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| Even **with Applicative Functors, it is not possible to chain together multiple commands**. **Example:**  **> Just (+3) <\*> Just (+4) <\*> Just (+5)**  **Returns error** | **Monads:** **Can chain through a series of functions**.  **Key Operator**:  **>>=** (**Bind**) | **Example #1:** Using Just  **> (Just 3) >>= (\x -> Just (x + 4)) >>= (\y -> Just (y+5))**  **12**  **Example #2:** Using **return**  **> (return 3) >>= (\x -> return (x + 4)) >>= (\y -> return (y+5))**  **12** |

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| **Comparing <\*> and >>=**  **Functor**:  **(<\*>) :: Applicative f => f (a -> b) -> f a -> f b**  **Monad**:  **(>>=) :: Monad m => m a -> (a -> m b) -> m b**  **Differences:**   1. **Order of the arguments changed.** 2. **The function is boxed in Functor but not Monad** 3. **Monad function returns a boxed result.** | **Example of <$>, <\*>** **and** **>>=**  **> (\x -> x + 1) <$> Just 3**  **Just 4**  **> Just (\x -> x + 1) <\*> Just(3)**  **Just 4**  **> (Just 3) >>= (\x -> Just(x+1))**  **Just 4** | **Example:** Implement **applyMaybe** that applies a function to a **Maybe**  **applyMaybe :: Maybe a -> (a -> b) -> (Maybe b)**  **applyMaybe Nothing \_ = Nothing**  **applyMaybe (Just x) f = Just (f x)** |

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| **Example:** Implement **applyMaybe** that applies a function to a **Maybe**  **applyMaybe :: Maybe a -> (a -> Maybe b)**  **-> (Maybe b)**  **applyMaybe Nothing \_ = Nothing**  **applyMaybe (Just x) f = Just (f x)** | **Chaining** **applyMaybe**  **> (Just 3) `applyMaybe` (\x -> Just (x\*2))**  **`applyMaybe` (\y -> Just (y-1))**  **Just 5**  **> (Just 3) `applyMaybe` (\\_ -> Nothing)**  **`applyMaybe` (\y -> Just (y-1))**  **Nothing** | **Additional Names for Monoids**   * “Programmable Semicolons” * “Applicative Functors you can chain.” |

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| **Monad Typeclass Definition**  **class Monad m where**  **return :: a -> m a**  **(>>=) :: m a -> (a -> m b) -> m b**  **(>>) :: m a -> m b -> m b**  **x >> y = x >>= (\\_ -> y) –-Lamda**  **fail :: String -> m a**  **fail msg = error msg** | **Example a Robot Moving Towards a Goal (Not Failure)** | |
| **–-Location**  **type Robot = (Int, Int)**  **-- Functions**  **up (x,y) = (x, y+1)**  **down (x,y) = (x, y-1)**  **left (x,y) = (x-1, y)**  **right (x,y) = (x+1, y)** | **-- Define Operator and start location**  **x -: f = f x**  **start = (0, 0)**  **> start -: up -: right**  **(1, 1)**  **> start -: up -: left -: left -: right -: down**  **(-1, 0)** |

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| **Maybe as an Instance of the Monad Typeclass**  **instance Monad Maybe where**  **return = Just**  **(>>=) Nothing \_ = Nothing**  **(>>=) (Just x) f = Just (f x)**  **fail \_ = Nothing** | **Example a Robot Moving Towards a Goal (with Failure)** | |
| **-- Once the goal is reached,**  **-- the robot stops**  **goal := Map.empty**  **-: (Map.insert (0, 2) True)**  **-: (Map.insert (-1, 3) True)**  **-: (Map.insert (-3, -8) True)**  **moveTo :: Pos -> Maybe Pos**  **moveTo p = if Map.member p goal**  **then Nothing**  **else Just p**  **-- Since these are in bind, no need**  **-- to handle Nothing. Bind handles it.**  **up (x,y) = moveTo (x, y+1)**  **down (x,y) = moveTo (x, y-1)**  **left (x,y) = moveTo (x-1, y)**  **right (x,y) = moveTo (x+1, y)** | **start = (0, 0)**  **> return start >>= up >>= left >>= left**  **>>= right >>= down**  **Just (-1, 0)**  **> return start >>= left >>= left >>= up**  **>>= up >>= right >>= up**  **>>= right >>= right >>= down**  **Nothing**  **Explanation:** Reached one of the goals (-1, 3) at the red **up** |

**Integer Division Using Monads**

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| **Integer Division with Bind and No “do”**  **mydiv :: Maybe Int -> Maybe Int -> Maybe Int**  **mydiv x y = x >>= (\numer ->**  **y >>= (\denom ->**  **if denom > 0**  **then Just (div numer denom)**  **else fail “Div by zero”))** | **Integer Division with Bind with “do”**  **mydiv :: Maybe Int -> Maybe Int -> Maybe Int**  **mydiv x y = do**  **numer <- x**  **denom <- y**  **if denom > 0**  **then Just (div numer denom)**  **else fail “Div by 0”** | **Integer Division with Bind with “do” and return**  **mydiv :: Maybe Int -> Maybe Int -> Maybe Int**  **mydiv x y = do**  **numer <- x**  **denom <- y**  **if denom > 0**  **then return $ div numer denom**  **else fail “Div by 0”** |

**List Monad**

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| **Making List an Instance of Monad**  **instance Monad [] where**  **return x = [x]**  **(>>=) xs f = concat(map f xs)**  **fail \_ = []**  **Explnation:** **concat** is needed here as **f** returns elements already in a list. As such, **concat** merges the individual lists (from each call to **f**) into a single list. | **Example Use of** **List as a Monad**  **listOfTuples :: [(Int, Char)]**  **listOfTuples = do**  **n <- [1, 2]**  **ch <- [‘a’, ‘b’]**  **return (n, ch)**  **> listOfTuples**  **[(1,‘a’), (1,‘b’), (2,‘a’), (2, ‘b’)]** | **Combining a Maybe and a List Monad**  **> Just [2,3] >>= (\x -> Just( fmap (+1) x))**  **[3, 4]** |

**Lecture #11 – Parsing Combinators**

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| **Semantics:** Enumerate **what a program means.** **Defined by the interpreter or compiler**.  **Syntax:** Enumerate **how a program Is structured**. **Defined by the lexer and parser**. | **Compilation Flow**  **Step #1:** Tokenizer/lexer generates a set of tokens.  **Step #2:** Parser turns the tokens into an abstract syntax tree.  **Step #3:** Compilers and interpreters convert the AST into machine code or commands respectively. | **Lexer**  **Converts the characters of the program into words of the language**.  **Examples:**   * Lex/Flex (C/C++) * ANTLR & JavaCC (Java) * Parsec (Haskell) |

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| **Categories of Tokens**   * **Reserved Words/Keywords**.   + **Examples**: while, if, then, else * **Literals/Constants**.   + **Examples**: 123, “Hello World!” * **Special symbols**.   + **Examples**: “;”, “=>”, “&&” * **Identifiers**.   + **Examples**: “balance”, “myFunction” | **Parsing**   * **Parser converts tokens to abstract syntax trees**. * **Defined by context free grammars** (CFG) * **Types of Parsers:**   + **Bottom-up**/**Shift-Reduce** Parsers   + **Top-down** parsers | **Context Free Grammars**   * Grammars specify the language. * Specified in Backus-Naur form format. **Example**:   **Expr -> Number**  **| Number + Expr**   * **Terminal** – **Cannot be broken down** further. * **Non**-**terminals** – **Can be broken down** further.   **Example:** “0”, “1”, “2”, … , “9” are terminals but digit, number, and expression are not. | **Example Grammar**  **expr -> expr + expr**  **| expr – expr**  **| ( expr )**  **| number**  **number -> number digit**  **| digit**  **digit -> 0 | 1 | 2 | … | 9** |

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| **Bottom-Up / Shift-Reduce Parser**   * **Shift** tokens onto a stack * **Reduce** the stack to a non-terminal. * **LR** – ***L***eft to right, ***R***ightmost derivation * **LALR** – ***L***ook-***A***head ***LR*** parsers are the most popular type of LR parsers.   + **Examples:** YACC/Bison * **Fading from popularity** | **Top-Down Parser**   * **Non-terminals are expanded to match tokens**. * **LL** – ***L***eft to right, ***L***eftmost derivation * **LL(k) Parser** – Looks ahead up to *k* elements. **Examples**: Java CC, ANTLR   + The higher the *k*, the more difficult language is to parse. ***k* can be arbitrary**.   + ***LL(1)*** - Easy to parse using either LL or recursive descent parsers. **Many computer languages are designed to be LL(1)**. | **Parser Combinator**  **Combine simpler parsers to make a more complex parser**.  **Example**: Parsec | **Useful Parsec Functions**   * **many** – Parses **zero or more** occurrences of the given parser. * **many1** – Parses **1 or more** occurrences of the given parser. * **noneOf** – Anything but the specified value * **spaces** – Whitespace characters * **char** – The specific specified character * **string** – The specific specified string. * **sepBy** – Separate tokens by some token. |

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| **Example Parsec Code** | | |
| **import Text.ParserCombinators.Parsec**  **num :: GenParser st String**  **num = many1 digit**  **main = do**  **print $ parse num “Hello” “42”** | **import Text.ParserCombinators.Parsec**  **num :: GenParser st Integer**  **num = do**  **str <- many1 digit**  **return $ read str**  **main = do**  **print $ parse num “World” “42”** | * **st** – “State.” Always required for our purposes. * **String/Integer** – Parser return type * **many1** – Select one of more digits. * **digit** – 0, 1, 2, 3, …, 9 (**terminal**) * **num** – Parser entry function * **“Hello”**/**“World”** – Debug string. * **“42”** – String to parse. |

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| **Example with** **try, <|>, and <?>**  **eol = try (string “\n”)**  **<|> string “\n\r”**  **<?> “end of line”**   * **try** – If an incomplete match is found, rewind. * **<|>** – “Or” Operator for matching tokens. * **<?>** – Otherwise with an accompanying error message. |  |  |

**Practice Midterm and Review Notes**

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| **Question #1** | **Question #2** | **Question #3** | **Question #4** | **Question #5** |
| 1. **True** 2. **False** – Lazy evaluation 3. **False** – Lazy evaluation 4. **False** – Statically type 5. **True** | 1. **True** 2. **False** – Applicative functor 3. **True** 4. **True** 5. **True** | 1. **False** – Big step 2. **True** 3. **False** – Use store 4. **True** 5. **False** | 1. **False** – Imperative 2. **True** 3. **False** 4. **True** 5. **True** | 1. **True** 2. **False** – Typeclass 3. **True** 4. **False** 5. **False** – Algebraic data type |

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| **Haskell**   * **Purely Functional** * **Lazy evaluation** * **Fully Curried Language** * **Statically Typed** * **Type Inference** – Via context, Haskell can deduce the type. | **Purely Functional**   * **Referential Transparency** – A **function call** can be replaced with its equivalent value without affecting the program * **No (re)assignment** * **No loop** * **No side effects** | **Functional Languages**   * **Functions are first class objects** meaning they can be passed to a function, returned from it, or created on the fly. * **Higher order function support** | **Operational Semantics**   * **Small Step** – Structural Semantics * **Big Step** – Natural Semantics * “**Get stuck**” – When a function is encountered that does not have an associated rule. |

**CSV Parser Example**

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| **Verbose Approach**  **import Text.ParserCombinator.Parsec**  **import System.Environment**  **csvFile :: GenParser st [[String]]**  **csvFile = do**  **arr <- many line**  **char eof**  **return arr**  **line :: GenParser st [String]**  **line = do**  **result <- many1 cell**  **char ‘\n’**  **return result**  **cells :: GenParser st [String]**  **cells = do**  **firstCell <- cellContents**  **nextCells <- remainingCells**  **return (firstCell:nextCells)**  **cellContent :: GenParser st String**  **cellContent = many $ noneOf “,\n” -- Two characters**  **remainingCells :: GenParser st [String]**  **remainingCells = do**  **(char “,” >> cells)**  **<|> return []**  **main = do**  **args <- getArgs**  **p <- parseFromFile csvFile “example 1” (head args)**  **case p of**  **Left msg -> error msg**  **Right csv -> print csv** | **Concise Approach**  **import Text.ParserCombinator.Parsec**  **import System.Environment**  **csvFile = lines `sepBy` eol**  **line = cells `sepBy` string “,”**  **cells = many (noneOf “\n”)**  **eol = try (string “\n”)**  **<|> string “\n\r”**  **<?> “end of line”**  **main = do**  **args <- getArgs**  **p <- parseFromFile csvFile “example 1” (head args)**  **case p of**  **Left msg -> error msg**  **Right csv -> print csv** |

**Miscellaneous**

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| **Kind of Show and show**  **> :k Show**  **Show :: \* -> Constraint**  **Type and Kind of show**  **> :k show**  **Error (A function not a type)**  **> :t show**  **show :: (Show a) => a -> String** | **Lambda and ADT Combined**  **> (\x -> Just (x+1)) 1**  **Just 2** | **Example**: **applyMaybe** that takes a **(Maybe a)** and applies to it a function that takes a normal **a** and returns a **(Maybe b)**  **applyMaybe :: (Maybe a) -> (a -> Maybe b) -> (Maybe b)**  **applyMaybe Nothing \_ = Nothing**  **applyMaybe (Just x) f = f x**  **Explanation**: Since the function “**f**” already returns a Maybe, you do not need to re-box it. However, since it does not take a Maybe, you need to unbox the first input parameter. |
| **Creating Type Alias**  **type String = [Char]**  Allows for more readable code as developer can use a type name that makes more sense for a given application. |

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| **Applying return to Items**  **> return 7**  **7**  **> return 7 *::* Maybe Int**  **Just 7**  **> return 7 :: [Int]**  **[7] -- Need Int or get an error**  **Conclusion**: **Behavior for return is the same as pure**. Both put the object in the **minimum default context that still yields that value**. | **List comprehension is syntactic sugar for using lists as monads.** |  |

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| **Monads and Lambda**  When trying to chain multiple functions together in a Monad, remember the Monad must return a **boxed value**. Hence, L**ambda often work well as they simplifying boxing**. | **Applicative Typeclass** – Allows you to use normal functions on values that have a context (i.e. are inside a Functor). | **return** – Monad equivalent of “pure” for Applicative Functors.  **Cannot use fmap in the definition of a Monad since fmap returns a boxed value while the function of the Monad returns a boxed value. Hence, if you used fmap with a Monad, you would return a double boxed value.** |
| **Monad**: Given a value of type, **a**, in a context, **m**, apply a function that takes a normal value of type **a** and returns a value in the context **m**.  **(>>=) :: (Monad m) => m a -> (a -> m b) -> m b**  **Monads** **are just applicative functors that support** **bind** (**>>=**).  **Key Difference**: **Applicative functors support normal functions that take and return unboxed values while Monads return boxed values**. |