- If $\mathbf{f}$ is a function in the vector space V of all real-valued functions on $\mathbb{R}$ and if $\mathbf{f}(t) = 0$ for some t, then $\mathbf{f}$ is the zero vector in V. FALSE We need $\mathbf{f}(t) = 0$ for all t.
- A vector is an arrow in three-dimensional space. FALSE This is an example of a vector, but there are certainly vectors not of this form.
- A subset H of a vector space V, is a subspace of V if the zero vector is in H FALSE We also need the set to be closed under addition and scalar multiplication.
- A subspace is also a vector space. TRUE This is the definition of subspace, a subset that satisfies the vector space properties.
- Analogue signals are used in the major control systems for the space shuttle, mentioned in the introduction to the chapter. FALSE Digital signals are used...

- A vector is any element of a vector space. TRUE This is the definition. Remember it.
- If $\mathbf{u}$ is a vector in a vector space V, then $(-1)\mathbf{u}$ is the same as the negative of $\mathbf{u}$. TRUE By definition of "negative"
  $\mathbf{u} + -\mathbf{u} = 0$ But
  $0 = (1 + (-1)) * \mathbf{u} = 1 * \mathbf{u} + (-1)\mathbf{u} = \mathbf{u} + (-1)\mathbf{u}$ so $(-1)\mathbf{u}$ must be negative $\mathbf{u}$.
- A vector space is also a subspace. TRUE (Its always a subspace of itself, at the very least.)
- $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$. FALSE The elements in $\mathbb{R}^2$ aren't even in $\mathbb{R}^3$.
- A subset H of a vector space V is a subspace of V if the following conditions are satisfied: (i) the zero vector of V is in H, (ii)$\mathbf{u}$, $\mathbf{v}$ and $\mathbf{u} + \mathbf{v}$ are in H, and (iii) c is a scalar and $c\mathbf{u}$ is in H. FALSE The second and third parts aren't stated correctly.

- The null space of A is the solution set of the equation $A\mathbf{x} = \mathbf{0}$. TRUE
- The null space of an $m \times n$ matrix is in $\mathbb{R}^m$. False. It's $\mathbb{R}^n$
- The column space of A is the range of the mapping $\mathbf{x} \mapsto A\mathbf{x}$. TRUE
- If the equation $A\mathbf{x} = \mathbf{b}$ is consistent, then Col A is $\mathbb{R}^m$. FALSE must be consistent for all b
- The kernel of a linear transformation is a vector space. TRUE To show this we show it is a subspace
- Col A is the set of a vectors that can be written as $A\mathbf{x}$ for some $\mathbf{x}$. TRUE Remember that $A\mathbf{x}$ gives a linear combination of columns of A using $\mathbf{x}$ entries as weights.

- The null space is a vector space. TRUE
- The column space of an $m \times n$ matrix is in $\mathbb{R}^m$ TRUE
- Col A is the set of all solutions of $A\mathbf{x} = \mathbf{b}$. FALSE It is the set of all b that have solutions.
- Nul A is the kernel of the mapping $\mathbf{x} \mapsto A\mathbf{x}$. TRUE
- The range of a linear transformation is a vector space. TRUE It's a subspace(check), thus vector space.
- The set of all solutions of a homogenous linear differential equation is the kernel of a linear transformation. TRUE

- A single vector is itself linearly dependent. FALSE unless it in the zero vector
- If $H = \text{Span}\{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$ then $\{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$ is a basis for H. FALSE They may not be linearly independent.
- The columns of an invertible $n \times n$ matrix form a basis for $\mathbb{R}^n$ TRUE They are linerly independent and span $\mathbb{R}^n$. (why?)
- A basis is a spanning set that is as large as possible. FALSE It is is too large, then it is no longer linearly independent.
- In some cases, the linear dependence relations among the columns of a matrix can be affected by certain elementary row operations on the matrix. FALSE They are not affected.

- A linearly independent set in a subspace H is a basis for H. FALSE It may not span.
- If a finite set S of nonzero vectors spans a vector space V, the some subset is a basis for V. TRUE by Spanning Set Theorem
- A basis is a linearly independent set that is as large as possible. TRUE
- The standard method for producing a spanning set for Nul A, described in this section, sometimes fails to produce a basis. FALSE It NEVER fails!!!
- If B is an echelon form of a matrix A, then the pivot columns of B form a basis for Col A. FALSE Must look at corresponding columns in A.

- The number of pivot columns of a matrix equals the dimension of its column space. TRUE Remember these columns and linearly independent and span the column space.
- A plane in $\mathbb{R}^3$ is a two dimensional subspace of $\mathbb{R}^3$. FALSE unless the plane is through the origin.
- The dimension of the vector space $\mathbb{P}_4$ is 4. FALSE It's 5.
- If $\dim V = n$ and S is a linearly independent set in V, then S is a basis for V. FALSE S must have exactly n elements.
- If a set $\{\mathbf{v}_1 \ldots \mathbf{v}_n\}$ spans a finite dimensional vector space V and if T is a set of more than n vectors in V, then T is linearly dependent. TRUE The number of linearly independent vectors that span a set is unique.

- $\mathbb{R}^2$ is a two dimensional subspace of $\mathbb{R}^3$. FALSE Not a subset, as before.
- The number of variables in the equation $A\mathbf{x} = 0$ equals the dimension of Nul A. FALSE It's the number of free variables.
- A vector space is infinite dimensional is it is spanned by an infinite set. FALSE It must be impossible to span it by a finite set.
- If dim $V = n$ and if S spans V. then S is a basis for V. FALSE S must have exactly n elements or be noted as linearly independent.
- The only three dimensional subspace of $\mathbb{R}^3$ is $\mathbb{R}^3$ itself. TRUE If spanned by three vectors must be all of $\mathbb{R}^3$.

- The row space of A is the same as the column space of A^T. TRUE The rows become the columns of A^T so this makes sense.
- If B is an echelon form of A, and if B has three nonzero rows, then the first three rows of A form a basis of Row A. FALSE The nonzero rows of B form a basis. The first three rows of A may be linear dependent.
- The dimensions of the row space and the column space of A are the same, even if A if A is not square. TRUE by the Rank Theorem. Also since dimension of row space = number of nonzero rows in echelon form = number pivot columns = dimension of column space.

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- The sum of the dimensions of the row space and the null space of A equals the number of rows in A. FALSE Equals number of columns by rank theorem. Also dimension of row space = number pivot columns, dimension of null space = number of non-pivot columns (free variables) so these add to total number of columns.
- On a computer, row operations can change the apparent rank of a matrix. TRUE Due to rounding error.

- If B is any echelon form of A, the the pivot columns of B form a basis for the column space of A. FALSE It's the corresponding columns in A.
- Row operations preserve the linear dependence relations among the rows of A. FALSE For example, Row interchanges mess things up.
- The dimension of null space of A is the number of columns of A that are not pivot columns. TRUE These correspond with the free variables.
- The row space of A^T is the same as the column space of A. TRUE Columns of A go to rows of A^T.
- If A and B are row equivalent, then their row spaces are the same. TRUE. This allows us to find row space of A by finding the row space of its echelon form..