

**Linear Algebra I (Math 129A)**  
**Study guide for Midterm 2 (75 pts).**

*You are allowed to use The Invertible Matrix Theorem printout.  
It can be downloaded from Canvas (look under Files in the left sidebar)*

**Problem 1.** Find the inverse of a matrix if it exists.

*Section 2.2:* The inverse of a  $2 \times 2$  matrix (theorem 4) and the algorithm for finding  $A^{-1}$ . Also, theorems 5, 6 and 7 are useful to know.

**Problems 2 and 3** will be on the invertible matrix theorem such as “Determine which of the matrices are invertible. Use as few calculations as possible. Justify your answer” and the like.  
*Section 2.2 and 2.3:* The invertible matrix theorem.

**Problem 4.** Compute the determinant of a matrix.

*Section 3.1:* Cofactor expansion across any row and down any column. Know how to find the determinant of a triangular matrix.

*Section 3.2:* Properties of determinants (theorem 3 and 4). Know how to find the determinant by row reducing matrix to echelon form. Also, know how to combine the method of row reduction and cofactor expansion to compute the determinant.

**Problem 5.** *Section 4.1:* Vector spaces and subspaces. Know how to determine if a given set is a subspace/ vector space. A subspace spanned by a set (theorem 1).

**Problem 6.** *Section 4.2:* The null space of a matrix. Theorem 2. The explicit description of  $\text{Nul } A$ . The column space of a matrix. Theorem 3. The contrast between  $\text{Nul } A$  and  $\text{Col } A$ .

**Problem 7.** Find bases for the null space and the column space of a matrix. Also, find the dimensions of  $\text{Nul } A$  and  $\text{Col } A$ .

*Section 4.3:* Linearly independent sets; bases. The spanning set theorem. Bases for  $\text{Nul } A$  and  $\text{Col } A$ .

*Section 4.5:* The dimension of a vector space. The dimensions of  $\text{Nul } A$  and  $\text{Col } A$ .

**Problem 8.** *Section 4.4:* Coordinate systems. The unique representation theorem. Know how to find the coordinate vector of  $\mathbf{x}$  relative to basis  $\mathcal{B}$ :  $[\mathbf{x}]_{\mathcal{B}} = P_{\mathcal{B}}^{-1}\mathbf{x}$  and vice versa:  $\mathbf{x} = P_{\mathcal{B}}[\mathbf{x}]_{\mathcal{B}}$ . The coordinate mapping. Isomorphism. Know how to test the linear independence of a set of polynomials.