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5 pts

1. 2 pts. Find the inverse of the following matrix
- $A = \begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix}$
- .

$$\det A = (1 \cdot 7) - (2 \cdot 4)$$

$$\boxed{\det A = 7 - 8 = -1}$$

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} 7 & -2 \\ -4 & 1 \end{bmatrix}$$

$$A^{-1} = (-1) \begin{bmatrix} 7 & -2 \\ -4 & 1 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} -7 & 2 \\ 4 & -1 \end{bmatrix}}$$

Using augmented matrix on the back

Check work

$$\begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} -7 & 2 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} -7+8 & 2-2 \\ -28+14 & 8-2 \end{bmatrix}$$

$$A A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -7 & 2 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} (-7)(1) + (2)(4) & (-7)(2) + (2)(7) \\ (4)(1) + (-1)(4) & (4)(2) + (-1)(7) \end{bmatrix}$$

$$A^{-1} A = \begin{bmatrix} -7+8 & 0 \\ 4-4 & 6-7 \end{bmatrix}$$

$$A^{-1} A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2. 3 pts. If
- A, B
- , and
- C
- are
- $n \times n$
- invertible matrices, does the equation
- $C^{-1}(A+X)B^{-1} = I_n$
- have a solution,
- X
- ? If so, find it.

Find X

$$C^{-1}(A+X)B^{-1} = I_n$$

Multiply both sides by C - property matrix multiplication

$$C C^{-1}(A+X)B^{-1} = C I_n$$

By definition of inverse identity matrix

$$(A+X)B^{-1} = C$$

Multiply by B on both sides

$$(A+X)B^{-1}B = CB$$

By definition of inverse matrix

$$A+X = CB$$

Subtract A from both sides by

$$\boxed{X = CB - A}$$

Has a solution

Verification

$$C^{-1}(A+X)B^{-1} = I_n$$

$$C^{-1}(A+CB-A)B^{-1} = I_n \quad \text{substitute for matrix } X$$

$$C^{-1}(CB)B^{-1} = I_n$$

property matrix subtraction

$$(C^{-1}C)(BB^{-1}) = I_n$$

Associative property of matrix multiplication

$$I_n I_n = I_n$$

Definition of inverse

$$\boxed{I_n = I_n}$$

property of identity matrix.