Chapter 3-1 - Introduction to Determinants

1) Calculate the determinant using cofactor expansion along the first row and second colum

$$A=\begin{bmatrix}3&0&4\\2&3&2\\0&5&-1\end{bmatrix}$$

$\det A = 3C_{11} + 0C_{12} + 4C_{13}$

$\det A = 3(-1)^2(3(-1)-10) + 4(-1)^4(10-0)$

$\det A = 3(-13) + 4\cdot 10$

$\boxed{\det A = 1}$

$\det A = 0C_{12} + 3C_{22} + 5C_{32}$

$\det A = 3(-1)^4(3(-1)-4\cdot 0) + 5(-1)^5(6-8)$

$\det A = 3\cdot(-3)\cdot 1 \quad + 5(-1)(-2)$

$\boxed{\det A = 1}$

3) Calculate the determinant using cofactor expansion along the first row and second column

$C_{11} = (-1)^2((1)(-1) - 2(3)) = -1-6 = -7$

$C_{12} = (-1)^3(3(-1) - 1\cdot 2) = (-1)\cdot(-5) = 5$

$C_{13} = (-1)^4(3\cdot 3 - 1\cdot 1) = \quad 8$

$\det A = (2)\cdot C_{11} + (-2)C_{12} + (3)C_{13}$

$\det A = 2\cdot(-7) + -2\cdot 5 + 3\cdot 8$

$\boxed{\det A = -14 - 10 + 24 = 0}$

$C_{22} = (-1)^4\cdot(2(-1) - 1\cdot 3) = (1)(-5) = -5$

$C_{23} = (-1)^5(2\cdot 2 - 3\cdot 3) = (-1)(-5) = 5$

$\det A = (-2)(5) + (1)\cdot -5 + 3\cdot(5)$

$\boxed{\det A = 0}$

5) Calculate the determinant using cofactor expansion along the first row

$$A=\begin{bmatrix}2&3&-3\\4&0&3\\6&1&5\end{bmatrix}$$

$C_{11} = (-1)^2(5\cdot 0 - 3\cdot 1) = -3$

$C_{12} = (-1)^3(4\cdot 5 - 3\cdot 6) = -2$

$C_{13} = (-1)^4(4\cdot 1 - 0\cdot 6) = 4$

$\det A = 2\cdot C_{11} + 3\cdot C_{12} + (-3)C_{13}$

$\det A = 2\cdot(-3) + (3)(-2) + (-3)\cdot 4$

$\boxed{\det A = -24}$

7) Calculate the determinant using cofactor expansion across the first row.

$$A=\begin{bmatrix}4&3&0\\6&5&2\\9&7&3\end{bmatrix}$$

$C_{11} = (-1)^2(5\cdot 3 - 2\cdot 7) = 1$

$C_{12} = (-1)^3(6\cdot 3 - 2\cdot 9) = 0$

C_{13} - Not needed

$\det A = 4\cdot C_{11} + 3\cdot C_{12} + 0\cdot C_{13}$

$\det A = 4\cdot 1 + 0 + 0$

$\boxed{\det A = 4}$

9) Compute the determinant and select the row or column with the simplest computation

$$A=\begin{bmatrix}4&0&0&5\\1&7&2&-5\\3&0&0&0\\8&3&1&7\end{bmatrix}$$

Use third row

$\det A = 3\cdot C_{31}$

$C_{31} = (-1)^4\cdot \det A_{31} = \det A_{31}$

$$A_{31}=\begin{bmatrix}0&0&5\\7&2&-5\\3&1&7\end{bmatrix}$$

use the first row

$\det(A_{31}) = 5\cdot C_{13}$

$C_{13} = (-1)^4 \det\begin{bmatrix}7&2\\3&1\end{bmatrix}$

$C_{13} = 7-6 = 1$

$\det(A_{31}) = 5$

$\det A = 3\cdot 5 = 15$

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11) Calculate the determinant by cofactor expansion

$$A=\begin{bmatrix}3&5&-6&4\\0&-2&3&-3\\0&0&1&5\\0&0&0&3\end{bmatrix}$$

Use fourth row

$\det A = 3\cdot C_{44}$

$C_{44}=(-1)^8\cdot \det A_{44}$

$$A_{44}=\begin{bmatrix}3&5&-6\\0&-2&3\\0&0&1\end{bmatrix}$$

Use third row

$\det A_{44} = 1\cdot C_{33}$

$\det A_{44} = 1\cdot(-1)^6(3\cdot(-2)-0\cdot 5)$

$\det A_{44} = -6$

$\det A = (3)(1)(-6) = \boxed{-18}$

Also can use theorem 3-2

$$A=\begin{bmatrix}3&5&-6&4\\0&-2&3&-3\\0&0&1&5\\0&0&0&3\end{bmatrix}$$

$\det A = (3)(-2)(1)(3)$

$\det A = -18$

13) Calculate the determinant by cofactor expansion

$$A=\begin{bmatrix}4&0&7&3&-5\\0&0&2&0&0\\7&3&-6&4&-8\\5&0&5&2&-3\\0&0&9&-1&2\end{bmatrix}$$

Use second row

$\det A = a_{23}\cdot C_{23} = 2\cdot C_{23}$

$C_{23} = (-1)^5\cdot\det[A_{23}] = (-1)\det B$

$$B=A_{23}=\begin{bmatrix}4&0&3&-5\\7&3&4&-8\\5&0&2&-3\\0&0&-1&2\end{bmatrix}$$

Use second column

$\det B = b_{22}\cdot C_{B_{22}} = 3\cdot C_{B22}$

$C_{B2} = (-1)^{2+2}\cdot \det B_{22}$

$$D=B_{22}=\begin{bmatrix}4&3&-5\\5&2&-3\\0&-1&2\end{bmatrix}$$

$\det[D] = 0\cdot C_{D31} + (-1)C_{D32} + 2\cdot C_{D32}$

$C_{D32} = (-1)^5\cdot(4(-3)-(-5)(5))$

$C_{D32} = -1(-12+25) = -13$

$C_{D33} = (-1)^6(8-15) = -7$

$\det D = (-1)(-13)+(2)(-7)$

$\boxed{\det D = 13-14=-1}$

$C_{B22}=-1$, $\det B = -3$

$\det A = 2(-1)(-3) = \boxed{6}$

15) Add the downward diagonal products and subtract the upward diagonal products to find a 3x3 determinant

$$\begin{bmatrix}1&0&4\\2&3&2\\0&5&-2\end{bmatrix}$$

$$\begin{array}{ccccc} & & 0 & 10 & 0\\ 1&0&4&1&0\\2&3&2&2&3\\0&5&-2&0&5\\ & & -6 & 0 & 40\end{array}$$

$\Delta = (-6)+0+40-0-10-0$

$\Delta = 40-16$

$\boxed{\Delta = 24}$

17) Add the downward diagonal products and subtract the upward diagonal products to find a 3x3 determinant

$$\begin{bmatrix}2&-3&3\\3&2&2\\1&3&-1\end{bmatrix}$$

$$\begin{array}{ccccc} & & 6 & 12 & 9\\ 2&-3&3&2&-3\\3&2&2&3&2\\1&3&-1&1&3\\ & & -4 & -6 & 27\end{array}$$

$\det = -4-6+27-6-12-9$

$\boxed{\det = -10}$

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19) State the row operation and describe how it affects the determinant.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$

Interchange rows 2 and 1.

$\det(\text{Initial}) = ad - bc$

$\det(\text{swap}) = bc - ad$

Negated the determinant.

21) State the row operation and describe the affect on the determinant.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & \\ ck & d \cdot k \end{bmatrix}$$

$\boxed{\text{Scale row 2 by K}}$

$\det(A) = ad - bc$

$\det(A') = Kad - kbc$

$= k(ad - bc)$

$\boxed{\text{Scaled determinant by K}}$

23) State the row operation and describe how it affects the determinant.

$$A = \begin{bmatrix} a & b & c \\ 3 & 2 & 1 \\ 4 & 5 & 6 \end{bmatrix} \quad A' = \begin{bmatrix} 3 & 2 & 1 \\ a & b & c \\ 4 & 5 & 6 \end{bmatrix}$$

Interchange rows 1 and 2

$\det A = a(12-5) - b(18-4) + c(15-8)$

$\det A = 7a - 14b + 7c$

$\det A' = -1(7a - 14b + 7c)$

$\boxed{\text{Negated the determinant}}$

25) Compute the determinant of the given elementary matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & K & 1 \end{bmatrix}$$

$\det A = 1 \cdot (1 \cdot 1 - K \cdot 0)$

$\det A = 1\ (1 - 0)$

$\boxed{\det A = 1}$

27) Compute the determinant of the given elementary matrix.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ K & 0 & 1 \end{bmatrix}$$

$\det A = (-1)^{3+3}(1 - 0)$

$\det A = 1 \cdot 1$

$\boxed{\det A = 1}$

29) Compute the determinant of elementary matrix.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By theorem 3-2

$\det A = 1 \cdot K \cdot 1 = \boxed{K}$

33) Verify that $\det(EA) = (\det E)(\det A)$ where E is an elementary matrix and A is $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$E = \begin{bmatrix} 1 & K \\ 0 & 1 \end{bmatrix}$$

$\det E = 1$

$(\det E)(\det A) = \det A = ad - bc$

$$EA = \begin{bmatrix} a + cK & b + dK \\ c & d \end{bmatrix}$$

$\det EA = ad + \cancel{cdK} - (bc + \cancel{cdK})$

$\boxed{\det EA = ad - bc}$

35) Calculate the determinants and verify $\det(EA) = (\det E)(\det A)$

$E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\det E = 0 - 1 = -1$

$(\det E)(\det A) = -1(ad - bc)$

$\boxed{(\det E)(\det A) = bc - ad}$

$EA = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$

$\boxed{\det EA = cb - ad}$

37) Let $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$. Write $5A$. Is $\det(5A) = 5\det A$?

$5A = \begin{bmatrix} 15 & 5 \\ 20 & 10 \end{bmatrix}$

$\det(5A) = 150 - 100 = 50$

$\det(A) = 6 - 4 = 2$

$5\det(A) = 10$

No - They are not equal.