

Chapter 4-5 - Dimension of a Vector Space

1) Given the subspace:

$$\left\{ \begin{bmatrix} s-2t \\ s+t \\ 3t \end{bmatrix} : s, t \in \mathbb{R} \right\}$$

a) Find a basis

$$H = s \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$$

$$B = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} \right\}$$

b) State the dimension

$$\dim H = 2$$

3) Given the subspace:

$$\left\{ \begin{bmatrix} 2c \\ a-b \\ b-3c \\ a+2b \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$$

a) Find a basis:

$$B = \left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -3 \\ 0 \end{bmatrix} \right\}$$

b) State the dimension:

$$\dim H = 3$$

5) Given the subspace:

$$\left\{ \begin{bmatrix} a-4b-2c \\ 2a+5b-4c \\ -a+2c \\ -3a+7b+6c \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$$

a) Find a basis:

$$B = \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ -3 \end{bmatrix}, \begin{bmatrix} -4 \\ 5 \\ 0 \\ 7 \end{bmatrix} \right\}$$

c term is a scalar multiple of a

b) State the dimension:

$$\dim H = 2$$

7) Given the subspace:

$$\{(a, b, c) : a+3b+c=0, b-2c=0, 2b-c=0\}$$

a) Find a basis

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & 1 & -2 \\ 0 & 2 & -1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -3 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 3 \end{bmatrix}$$

$$R_3^1 = R_3 - 2R_2$$

3 pivots - no free variables

$$B = \{ \vec{0} \}$$

b) Find the dimension
 $\dim H = 0$

* a) Find the dimension of the subspace whose first and third entries are equal in \mathbb{R}^3

$$H = c \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + d \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, c, d \in \mathbb{R}$$

$$\dim H = 2$$

11) Find the dimension of the subspace spanned by:

$$\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -7 \\ -3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 9 & -7 \\ 0 & 1 & 4 & -3 \\ 2 & -2 & -2 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 3 & 9 & -7 \\ 0 & 1 & 4 & -3 \\ 0 & -5 & -20 & 15 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 3 & 9 & -7 \\ 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\dim H = 2$$

13) Determine the dimensions of $\text{Nul} A$ and $\text{Col} A$.

$$A = \begin{bmatrix} 1 & -6 & 9 & 0 & -2 \\ 0 & 1 & 2 & -4 & 5 \\ 0 & 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\dim(\text{Col} A) = 3 \quad (\text{three pivots})$$

$$\dim(\text{Nul} A) = 2 \quad (\text{two free variables})$$

15) Determine the dimensions of $\text{Nul} A$ and $\text{Col} A$

$$A = \begin{bmatrix} 1 & 0 & 9 & 5 \\ 0 & 0 & 1 & -4 \end{bmatrix}$$

$$\dim(\text{Col} A) = 2 \quad (\text{two pivots})$$

$$\dim(\text{Nul} A) = 2 \quad (\text{two free variables})$$

17) Determine the dimensions of $\text{Nul} A$ and $\text{Col} A$

$$A = \begin{bmatrix} 1 & -4 & 0 \\ 0 & 4 & 7 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\dim(\text{Col} A) = 3$$

$$\dim(\text{Nul} A) = 0$$

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19)

a) True - By definition in the text.

b) False - The plane must pass through the origin.

c) False - P_4 has five dimensions.

d) False - Set S must have n elements.

e) True - Even if the set is not a basis by Theorem 9 any larger set T cannot be a basis linearly independent.

20)

a) False - \mathbb{R}^2 is not a subspace of \mathbb{R}^3 .

b) False - The number of free variables is the dimension of $N(A)$.

c) False - Being spanned by a finite set does not automatically make it infinite dimensional as a finite basis may otherwise exist.

d) False - S must also have n elements.

e) True

21) Show that the polynomials are a basis of P_4 .

$$1, 2t, -2 + 4t^2, -12t + 8t^3$$

By isomorphic coordinate mappings

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -12 \\ 0 \\ 8 \end{bmatrix} \right\}$$

Set is linearly independent with dimension 4.

$$\dim P_3 = 4$$

By theorem 4-12, the set is a basis since it is linearly independent with the same dimension.