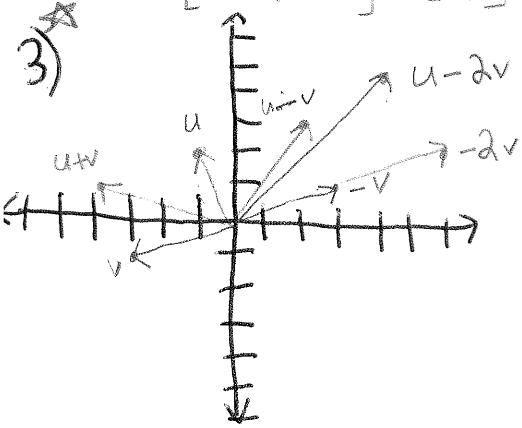
Homework 1-3

1) $u = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ $v = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$

a) $u + v = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$

b) $u - 2v = \begin{bmatrix} -1 - (2 \cdot -3) \\ 2 - (2 \cdot -1) \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

3)



$u - v = \begin{bmatrix} -1 - (-3) \\ 2 - (-1) \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$u - 2v = \begin{bmatrix} -1 - 2(-3) \\ 2 - 2(-1) \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

5) $x_1 \begin{bmatrix} 6 \\ -1 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \\ -5 \end{bmatrix}$

System of equations

$6x_1 - 3x_2 = 1$

$-x_1 + 4x_2 = -7$

$5x_1 = -5$

7)

a) It is a combination of u & v

$u - 2v$

b) It is a combination of u & v

$2u - 2v$

c) It is a combination of u & v

$2u - 3.5v$

d) It is a combination of u & v

$3u - 4v$

9)

$x_2 + 5x_3 = 0$

$4x_1 + 6x_2 - x_3 = 0$

$-x_1 + 3x_2 - 8x_3 = 0$

$x_1 \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -1 \\ -8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Homework #1-3

11) A

$a_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$ $a_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ $a_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}$ $b = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$

Is b a linear combination of a_1, a_2, and a_3?

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{bmatrix}$$

$R_2' = R_2 + 2R_1$

$$\Downarrow$$

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 2 & 8 & 6 \end{bmatrix}$$

$R_3' = R_3 - 2R_2$

$$\Downarrow$$

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

System is consistent so b is a linear combination

13)

$A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix}$ $b = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$

Is b a linear combination of A?

$$\begin{bmatrix} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ -2 & 8 & -4 & -3 \end{bmatrix}$$

$R_3' = R_3 + 2R_1$

$$\Downarrow$$

$$\begin{bmatrix} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$ ← contradiction

15)

$V_1 = \begin{bmatrix} 7 \\ 1 \\ -6 \end{bmatrix}$, $V_2 = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix}$

List five vectors in Span $\{v_1, v_2\}$ and give the vector weights

a) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0v_1 + 0v_2$

b) $\begin{bmatrix} 7 \\ 1 \\ -6 \end{bmatrix} = 1v_1 + 0v_2$

c) $\begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} = 0v_1 + 1v_2$

d) $\begin{bmatrix} -7 \\ -1 \\ 6 \end{bmatrix} = -1v_1 + 0v_2$

e) $\begin{bmatrix} 5 \\ -3 \\ 0 \end{bmatrix} = 0v_1 - 1V_2$

17) $a_1 = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$ $a_2 = \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix}$ $b = \begin{bmatrix} 4 \\ 1 \\ h \end{bmatrix}$

For what values of h is b in the plane spanned by a_1 & a_2?

$$\begin{bmatrix} 1 & -2 & 4 \\ 4 & -3 & 1 \\ -2 & 7 & h \end{bmatrix}$$

$R_2' = R_2 - 4R_1$

$$\Downarrow$$

$$\begin{bmatrix} 1 & -2 & 4 \\ 0 & 5 & -15 \\ -2 & 7 & h \end{bmatrix}$$

$R_3' = R_3 + 2R_1$

$$\Downarrow$$

$$\begin{bmatrix} 1 & -2 & 4 \\ 0 & 5 & -15 \\ 0 & 3 & h+8 \end{bmatrix}$$

$R_2' = \frac{1}{5} R_2$

$$\Downarrow$$

$$\begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 3 & h+8 \end{bmatrix}$$

$R_3' = R_3 - 3R_2$

$$\Downarrow$$

$$\begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & h+17 \end{bmatrix}$$

If b is in the plane spanned by a_1 and a_2 then

$$h = -17$$

19) ☆

$V_1 = \begin{bmatrix} 8 \\ 2 \\ -6 \end{bmatrix}$, $V_2 = \begin{bmatrix} 12 \\ 3 \\ -9 \end{bmatrix}$

Give a geometric description of span $\{v_1, v_2\}$

$1.5\, V_1 = V_2$

Hence, span $\{v_1 v_2\}$ is the set of all points along the <u>line</u> between the origin and v_1.

Had v_2 not been a scalar multiple of v_1, then the span would have been the <u>plane</u> between the origin, v_1, and v_2.

21) $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $b = \begin{bmatrix} h \\ k \end{bmatrix}$

$$\begin{bmatrix} 2 & 2 & h \\ -1 & 1 & k \end{bmatrix}$$

$R_1' = \frac{1}{2} R_1$

$$\Downarrow$$

$$\begin{bmatrix} 1 & 1 & \frac{h}{2} \\ -1 & 1 & k \end{bmatrix}$$

$R_2' = R_1 + R_2$

$$\Downarrow$$

$$\begin{bmatrix} 1 & 1 & \frac{h}{2} \\ 0 & 2 & k + \frac{h}{2} \end{bmatrix}$$

No value of h or k can make the augmented matrix inconsistent so all combinations of h and k are covered by the span.

Homework 1-3

23)

a) False - $\begin{bmatrix}-4\\3\end{bmatrix}$ is a vector in $\mathbb{R}^2$ while $[-4 \times 3]$ is a 1×2 matrix, not a vector in $\mathbb{R}^2$

b) False - Since the two vectors are not scalar multiples of each other the points corresponding to $\begin{bmatrix}-2\\5\end{bmatrix}$ and $\begin{bmatrix}-5\\2\end{bmatrix}$ are the whole Cartesian plane not a line

c) True - A linear combination of v_1 and v_2 can be written in the form:

$$c_1v_1 + c_2v_2.$$

In this case, $c_1 = \frac{1}{2}$ and $c_2 = 0$

d) True - $[a_1\ a_2\ a_3\ b]$ is a complimentery representation of the linear system:

$$x_1a_1 + x_2a_2 + x_3a_3 = b$$

e) False - The span can take many shapes including a plane, line, or hyperplane.

25)

$$A = \begin{bmatrix}1 & 0 & -4\\0 & 3 & -2\\-2 & 6 & 3\end{bmatrix} = [a_1\ a_2\ a_3],\quad b = \begin{bmatrix}4\\1\\-4\end{bmatrix}$$

$$W = \text{span}\{a_1, a_2, a_3\}$$

a) Is b in the set $\{a_1, a_2, a_3\}$? How many vectors are in $\{a_1, a_2, a_3\}$

No, $\{a_1, a_2, a_3\}$ only contains three vectors namely $\begin{bmatrix}1\\0\\-2\end{bmatrix}$, $\begin{bmatrix}0\\3\\6\end{bmatrix}$ and $\begin{bmatrix}-4\\-2\\3\end{bmatrix}$ none of which are b

b) Is b in W? How many vectors are in W?

$$\begin{bmatrix}1 & 0 & -4 & 4\\0 & 3 & -2 & 1\\-2 & 6 & 3 & -4\end{bmatrix}$$

$R_3' = R_3 + 2R_1$

$$\Downarrow$$

$$\begin{bmatrix}1 & 0 & -4 & 4\\0 & 3 & -2 & 1\\0 & 6 & -5 & 4\end{bmatrix}$$

This matrix is consistent so b is in W.

c) Show a_1 is in W.

$$W = c_1v_1 + c_2v_2 + c_3v_3$$

$$c_1 = 1,\ c_2 = 0,\ c_3 = 0$$

$= v_1$ meaning v_1 is in the span.