- $\mathbf{v} \cdot \mathbf{v} = ||\mathbf{v}||^2$ TRUE by definition.
- For any scalar c, $\mathbf{u} \cdot (c\mathbf{v}) = c(\mathbf{u} \cdot \mathbf{v})$. TRUE
- If the distance from $\mathbf{u}$ to $\mathbf{v}$ equals the distance from $\mathbf{u}$ to $-\mathbf{v}$, then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal. TRUE
- For a square matrix A, vectors in Col A are orthogonal to vectors in Nul A. FALSE Counterexample $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$
- If vectors $\mathbf{v}_1, \ldots, \mathbf{v_p}$ span a subspace W and if $\mathbf{x}$ is orthogonal to each $\mathbf{v_j}$ for $j = 1, \ldots, p$ then x is in $W^\perp$. TRUE since any vector in W can be written as linear combination of basis vectors and dot product splits up nicely over sums and constants.

- $\mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{u} = 0$ TRUE since dot product is commutative.
- For any scalar c, $||c\mathbf{v}|| = c||v||$. FALSE need absolute value of c.
- If $\mathbf{x}$ is orthogonal to every vector in a subspace W, then $\mathbf{x}$ is in $W^{\perp}$. TRUE by definition of $W^{\perp}$
- If $||\mathbf{u}||^2 + ||\mathbf{v}||^2 = ||\mathbf{u} + \mathbf{v}||^2$, then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal. TRUE By Pythagorean Theorem.
- For an $m \times n$ matrix A, vectors in the null space of A are orthogonal to vectors in the row space of A. TRUE by Thm 3

- Not every linear independent set in $\mathbb{R}^n$ is an orthogonal set. TRUE It is only orthogonal if every dot product between two elements is 0.
- If $\mathbf{y}$ is a linear combination of nonzero vectors from an orthogonal set, then the weights in the linear combination can be computed without row operations on a matrix. TRUE $c_j = \frac{\mathbf{y} \cdot \mathbf{u}_j}{\mathbf{u}_j \cdot \mathbf{u}_j}$.
- If the vectors in an orthogonal set of nonzero vectors are normalized, then some of the new vectors may not be orthogonal. FALSE Normalizing just changes the magnitude of the vectors, it doesn't affect orthogonality.
- A matrix with orthonormal columns is and orthogonal matrix. FALSE. It must be a square matrix.
- If L is a line through $\mathbf{0}$ and if $\widehat{\mathbf{y}}$ is the orthogonal projection of y onto L, then $||\widehat{\mathbf{y}}$ gives the distance from $\mathbf{y}$ to L. FALSE The distance is $||\mathbf{y} - \widehat{\mathbf{y}}||$

- Not every orthogonal set in $\mathbb{R}^n$ is linearly independent. FALSE Orthogonal implies linear independence.
- If a set $S = \{\mathbf{u}_1, \ldots, \mathbf{u}_n\}$ has the property that $\mathbf{u}_i \cdot \mathbf{u}_j = 0$ whenever $i \neq j$, then S is an orthonormal set. FALSE Might not be normal (magnitude may not be 1).
- If the columns of an $m \times n$ matrix A are orthonormal, then the linear mapping $\mathbf{x} \mapsto A\mathbf{x}$ preserves length. TRUE Thm 7.
- The orthogonal projection of $\mathbf{y}$ onto $\mathbf{v}$ is the same as the orthogonal projection of $\mathbf{y}$ onto $c\mathbf{v}$ whenever $c \neq 0$. TRUE
- An orthogonal matrix is invertible. TRUE The columns are linear independent since orthogonal. Thus invertible by invertible matrix theorem.

- Is $\mathbf{z}$ is orthogonal to $\mathbf{u}_1$ and $\mathbf{u}_2$ and if $W = Span\{\mathbf{u}_1, \mathbf{u}_2\}$ then $\mathbf{z}$ must be in $W^{\perp}$. TRUE z will be orthogonal to any linear combination of to $\mathbf{u}_1$ and $\mathbf{u}_2$.
- For each $\mathbf{y}$ and each subspace W, the vector $\mathbf{y} - \text{proj}_W \mathbf{y}$ is orthogonal to W. TRUE
- The orthogonal projection $\widehat{\mathbf{y}}$ of $\mathbf{y}$ onto a subspace W can sometimes depend on the orthogonal basis for W used to compute $\widehat{\mathbf{y}}$. FALSE It is always independent of basis.
- If $\mathbf{y}$ is in a subspace W, then the orthogonal projection of $\mathbf{y}$ onto W is $\mathbf{y}$ itself. TRUE
- If the columns of an $n \times p$ matrix U are orthonormal, then $UU^T\mathbf{y}$ is the orthogonal projection of $\mathbf{y}$ onto the column space of U. TRUE

- If W is a subspace of $\mathbb{R}^n$ and if $\mathbf{v}$ is in both W and $W^\perp$, then $\mathbf{v}$ must be the zero vector. TRUE
- In the Orthogonal Decomposition Theorem, each term in formula (2) for $\widehat{\mathbf{y}}$ is itself an orthogonal projection of $\mathbf{y}$ onto a subspace of W. TRUE
- If $\mathbf{y} = \mathbf{z}_1 + \mathbf{z}_2$ where $\mathbf{z}_1$ is in a subspace W and $\mathbf{z}_2$ is in $W^\perp$, then $\mathbf{z}_1$ must be the orthogonal Projection of $\mathbf{y}$ onto W. TRUE
- The best approximation to $\mathbf{y}$ by elements of a subspace W is given by the vector $\mathbf{y} - \mathrm{proj}_W \mathbf{y}$. FALSE The best approximation is $\mathrm{proj}_W \mathbf{y}$.
- If an $n \times p$ matrix U had orthonormal columns, then $UU^T\mathbf{x} = \mathbf{x}$ for all $\mathbf{x}$ in $\mathbb{R}^n$. FALSE This only holds if U is square.

- If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an orthogonal basis for W, then multiplying $\mathbf{v}_3$ by a scalar c gives a new orthogonal basis $\{\mathbf{v}_1, \mathbf{v}_2, c\mathbf{v}_3\}$ FALSE We don't want $c = 0$
- The Gram-Schmidt process produces from a linearly independent set $\{\mathbf{x}_1, \ldots, \mathbf{x}_p\}$ and orthogonal set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ with the property that for each k, the vectors $\mathbf{v}_1, \ldots \mathbf{v}_k$ span the same subspace as the spanned by $\mathbf{x}_1, \ldots \mathbf{x}_k$. TRUE
- If $A = QR$, where Q has orthonormal columns, then $R = Q^T A$. TRUE

- The general least squares problem is to find an $\mathbf{x}$ that makes $A\mathbf{x}$ as close as possible to $\mathbf{b}$. TRUE
- A least-squares solution of $A\mathbf{x} = \mathbf{b}$ is a vector $\widehat{\mathbf{x}}$ that satisfies $A\widehat{\mathbf{x}} = \widehat{\mathbf{b}}$ where $\widehat{\mathbf{b}}$ is the orthogonal projection of $\mathbf{b}$ onto $ColA$. TRUE Remember the projection gives us the best approximation.
- A least-squares solution of $A\mathbf{x} = \mathbf{b}$ is a vector $\widehat{\mathbf{x}}$ such that $||\mathbf{b} - A\mathbf{x} \leq ||\mathbf{b} - A\widehat{\mathbf{x}}||$ for all x in $\mathbb{R}^n$. FALSE the inequality is facing the wrong way.
- Any solution of $A^TA\mathbf{x} = A^T\mathbf{b}$ is a least squares solution of $A\mathbf{x} = \mathbf{b}$. TRUE, this is how we can find the least squares solution.
- If the columns of A are linearly independent, the the equation $A\mathbf{x} = \mathbf{b}$ has exactly one least squares solution. TRUE Then A^TA is invertible so we can solve $A^TA\mathbf{x} = A^T\mathbf{b}$ for $\mathbf{x}$ by taking the inverse.