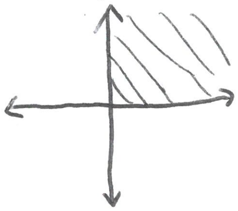
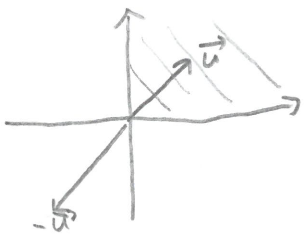
Chapter 2-8 - Subspaces in $\mathbb{R}^n$

1) Why is the set not a subspace of $\mathbb{R}^2$?



If $c<0$, then any vector is not included.

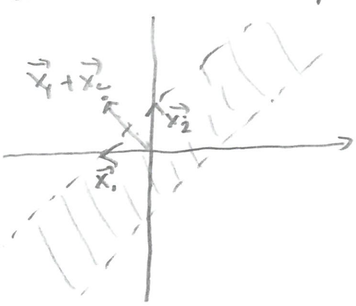
Example:



$-\vec{u}$ not in the set

3) Why is the set not a subspace of $\mathbb{R}^n$

Not all linear combinations are in the set. Example



$\vec{x}_1+\vec{x}_2$ not in the set.

5) Let:

$\vec{v}_1 = \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -4 \\ -5 \\ 8 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 8 \\ 2 \\ -9 \end{bmatrix}$.

Determine if $\vec{w}$ in subspace generated by $\vec{v}_1$ and $\vec{v}_2$

$$\begin{bmatrix} 2 & -4 & 8 \\ 3 & -5 & 2 \\ -5 & 8 & -9 \end{bmatrix}$$

$R_1' = \frac{1}{2}R_1$

$\Downarrow$

$$\begin{bmatrix} 1 & -2 & 4 \\ 3 & -5 & 2 \\ -5 & 8 & -9 \end{bmatrix}$$

$R_2 = R_2 - 3R_1$

$R_3 = R_3 + 5R_1$

$\Downarrow$

$$\begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -10 \\ 0 & -2 & 11 \end{bmatrix}$$

$\vec{w}$ not in span$\{\vec{v}_1, \vec{v}_2\}$

Since not consistent.

7) Let:

$\vec{v}_1 = \begin{bmatrix} 2 \\ -8 \\ 6 \end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix} -3 \\ 8 \\ -7 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} -4 \\ 6 \\ -7 \end{bmatrix}$, $\vec{p} = \begin{bmatrix} 6 \\ -10 \\ 11 \end{bmatrix}$, and $A = [\vec{v}_1 \vec{v}_2 \vec{v}_3]$

a) How many vectors are in $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$

3

b) How many vectors are in Col A?

Infinite

Is $\vec{p}$ in Col A?

$$\begin{bmatrix} 2 & -3 & -4 & 6 \\ -8 & 8 & 6 & -10 \\ 6 & -7 & -7 & 11 \end{bmatrix}$$

$R_2' = R_2 + 4R_1$

$R_3' = R_3 - 3R_1$

$\Downarrow$

$$\begin{bmatrix} 2 & -3 & -4 & 6 \\ 0 & -4 & -10 & 14 \\ 0 & 2 & 5 & -7 \end{bmatrix}$$

$R_3 = R_3 - \frac{1}{2}R_2$

$\Downarrow$

$$\begin{bmatrix} 2 & -3 & -4 & 6 \\ 0 & -4 & -10 & 14 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Yes since system is consistent

9) Is $\vec{p}$ in Nul A where:

$A = \begin{bmatrix} 2 & -3 & -4 \\ -8 & 8 & 6 \\ 6 & -7 & -7 \end{bmatrix}$, $\vec{p} = \begin{bmatrix} 6 \\ -10 \\ 11 \end{bmatrix}$

$$A\vec{p} = \begin{bmatrix} 2\cdot 6 + (-3)(-10) + (-4)\cdot(11) \\ (-8)(6) + (8)(-10) + (6)(11) \\ (6)(6) + (-7)(-10) + (-7)(11) \end{bmatrix} = \begin{bmatrix} 12 + 30 - 44 \\ -48 - 80 + 66 \\ 36 + 70 - 77 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ -62 \\ 29 \end{bmatrix}$$

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11) Provide integers p and q such that Nul A is a subspace in $\mathbb{R}^p$ and Col A is a subspace of $\mathbb{R}^q$

$$A=\begin{bmatrix} 3 & 2 & 1 & -5 \\ -9 & -4 & 1 & 7 \\ 9 & 2 & -5 & 1 \end{bmatrix}$$

A is 3×4

$p=4, \; q=3$

13) Find a nonzero vector in Nul A and a nonzero vector in Col A.

$$A=\begin{bmatrix} 3 & 2 & 1 & -5 \\ -9 & -4 & 1 & 7 \\ 9 & 2 & -5 & 1 \end{bmatrix}$$

Vector in Col A: $\begin{bmatrix} 3 \\ -9 \\ 9 \end{bmatrix}$

- Could be any column in A.

$$\begin{bmatrix} 3 & 2 & 1 & -5 \\ -9 & -4 & 1 & 7 \\ 9 & 2 & -5 & 1 \end{bmatrix}$$

$R_2' = R_2 + 3R_1$

$R_3' = R_3 - 3R_1$

$\Downarrow$

$$\begin{bmatrix} 3 & 2 & 1 & -5 \\ 0 & 2 & 4 & -8 \\ 0 & -4 & -8 & 16 \end{bmatrix}$$

$R_3' = R_3 + 2R_2$

$\Downarrow$

$$\begin{bmatrix} 3 & 2 & 1 & -5 \\ 0 & 2 & 4 & -8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2' = \frac{1}{2}R_2$

$$\begin{bmatrix} 3 & 2 & 1 & -5 \\ 0 & 1 & 2 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1' = R_1 - 2R_2$

$\Downarrow$

$$\begin{bmatrix} 3 & 0 & -3 & 3 \\ 0 & 1 & 2 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{\vec{x} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \end{bmatrix}}$$

15) Are the vectors a basis for $\mathbb{R}^2$?

$$\begin{bmatrix} 5 \\ -2 \end{bmatrix} \begin{bmatrix} 10 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 10 \\ -2 & -3 \end{bmatrix}$$

$R_1' = \frac{1}{5}R_1$

$$\begin{bmatrix} 1 & 2 \\ -2 & -3 \end{bmatrix}$$

$R_2' = R_2 + 2R_1$

$\Downarrow$

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

Yes since two pivots.

17) Are the vectors a basis of $\mathbb{R}^3$

$$\begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} \begin{bmatrix} 5 \\ -7 \\ 4 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 5 & 6 \\ 1 & -7 & 3 \\ -2 & 4 & 5 \end{bmatrix}$$

Swap R_1 and R_2

$$\begin{bmatrix} 1 & -7 & 3 \\ 0 & 5 & 6 \\ -2 & 4 & 5 \end{bmatrix}$$

$R_3' = R_3 + 2R_1$

$\Downarrow$

$$\begin{bmatrix} 1 & -7 & 3 \\ 0 & 5 & 6 \\ 0 & -10 & 11 \end{bmatrix}$$

$R_3' = R_3 + 2R_2$

$\Downarrow$

$$\begin{bmatrix} 1 & -7 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 23 \end{bmatrix}$$

Yes since vectors are linearly independent.

19) Do the vectors form a basis for $\mathbb{R}^3$?

$$\begin{bmatrix} 3 \\ -8 \\ 1 \end{bmatrix} \begin{bmatrix} 6 \\ 2 \\ -5 \end{bmatrix}$$

No, with two vectors it is never possible to span $\mathbb{R}^3$ as the maximum spannable shape would be a plane.

21) a) False - For this statement to be true, it must cover all $\vec{u}$ and $\vec{v}$.

b) True - Column space of A is all linear combinations of the vectors of A which is the same as the span.

c) False - They form a subspace of $\mathbb{R}^n$ not of $\mathbb{R}^m$.

d) True - Since there are n pivot columns in an invertible $n \times n$ matrix, they form the basis of $\mathbb{R}^n$.

e) True

22)

a) False - Containing the zero vector is not sufficient alone to form a subspace.

b) True - All linear combinations of a set of vectors forms a subspace.

c) True - Null space is a subspace in $\mathbb{R}^n$.

d) False - The column space is the set of all $\vec{b}$ that satisfy

$$A\vec{x} = \vec{b}$$

e) False - The pivot columns of the original matrix must be used for the column space