Section 5.1 21

- If $A\mathbf{x} = \lambda\mathbf{x}$ for some vector $\mathbf{x}$, then λ is an eigenvalue of A. FALSE This is true as long as the vector is not the zero vector.
- A matrix A is not invertible if and only if 0 is an eigenvalue of A. TRUE
- A number c is an eigenvalue of A if and only if the equation $(A - cI)\mathbf{x} = \mathbf{0}$ has a nontrivial solution. TRUE This is a rearrangement of the equation $A\mathbf{x} = \lambda\mathbf{x}$.
- Finding an eigenvector of A may be difficult, but checking whether a given vector is in fact an eigenvector is easy. TRUE Just see if $A\mathbf{x}$ is a scalar multiple of $\mathbf{x}$.
- To find the eigenvalues of A, reduce A to echelon form. FALSE Row reducing changes the eigenvectors and eigenvalues.

- If $A\mathbf{x} = \lambda\mathbf{x}$ for some scalar λ, then $\mathbf{x}$ is an eigenvector of A. FALSE The vector must be nonzero.'
- If $\mathbf{v_1}$ and $\mathbf{v_2}$ are linearly independent eigenvectors, then they correspond to different eigenvalues. FALSE The converse if true, however.
- A steady-state vector for a stochastic matrix is actually an eigenvector. TRUE A steady state vector has the property that $Ax\mathbf{x} = \mathbf{x}$. In this case λ is 1.
- The eigenvalues of a matrix are on its main diagonal. FALSE This is only true for triangular matrices.
- An eigenspace of A is a null space of a certain matrix. TRUE The eigenspace is the nullspace of $A - \lambda I$.

Section 5.2 21

- The determinant of A is the product of the diagonal entries in *A*. FALSE in general. True is A is triangular.
- An elementary row operation on *A* does not change the determinant. FALSE interchanging rows and multiply a row by a constant changes the determinant.
- (det A)*(det B)=det(AB) TRUE Yay!
- If $\lambda + 5$ is a factor of the characteristic polynomial of A, then 5 is an eigenvalue of *A*. FALSE -5 is an eigenvalue. (The zeros are the eigenvalues.

- If A is 3×3, with columns $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ then det A equals the volume of the parallelepiped determined $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$. FALSE it's the absolute value of the determinant. We can prove this by thinking of the columns as changing under a linear transformation from the unit cube. When we apply a transformation, the volumes gets multiplied by determinant.
- det A^T=(-1)det A FALSE detA^T=det A.
- The multiplicity of a root r of a characteristic equation of A is called the algebraic multiplicity of r as an eigenvalue of A. TRUE That's the definition.
- A row replacement operation on A does not change the eigenvalues. FALSE.

- A is diagonalizable if $A = PDP^{-1}$ for some matrix D and some invertible matrix P. FALSE D must be a diagonal matrix.
- If $\mathbb{R}^n$ has a basis of eigenvectors of A, then A is diagonalizable. TRUE In this case we can construct a P which will be invertible. And a D.
- A is diagonalizable if and only if A has n eigenvalues, counting multiplicity. FALSE It always has n eigenvalues, counting multiplicity.
- If A is diagonalizable, then A is invertible. FALSE It's invertible if it doesn't have zero an eigenvector but this doesn't affect diagonalizabilty.

- A is diagonalizable if A has n eigenvectors. The eigenvectors must be linear independent.
- If A is diagonalizable, then A had n distinct eigenvalues. FALSE It could have repeated eigenvalues as long as the basis of each eigenspace is equal to the multiplicity of that eigenvalue. The converse is true however.
- If $AP = PD$, with D diagonal then the nonzero columns of P must be the eigenvectors of A. TRUE. Each column of PD is a column of P times A and is equal to the corresponding entry in D times the vector P. This satisfies the eigenvector definition as long as the column is nonzero.
- If A is invertible, then A is diagonalizable. FALSE these are not directly related.