Section 2.1 15

- If A and B are 2×2 with columns $\mathbf{a}_1, \mathbf{a}_2$ and $\mathbf{b}_1, \mathbf{b}_2$ then $AB = [\mathbf{a}_1\mathbf{b}_1, \mathbf{a}_2\mathbf{b}_2]$. FALSE Matrix multiplication is "row by column".
- Each column of AB is a linear combination of the columns of A using weights from the corresponding column of B. FALSE Swap A and B then its true
- $AB + AC = A(B + C)$ TRUE Matrix multiplication distributes over addition.
- $A^T + B^T = (A + B)^T$ TRUE See properties of transposition. Also should be able to think through to show this. When we add we add corresponding entries, these will remain corresponding entries after transposition.
- The transpose of a product of matrices equals the product of their tranposes in the same order. FALSE The transpose of a product of matrices equals the product of their tranposes in the reverse order.

Section 2.1 16

- If A and B are 3×3 and $B = [\mathbf{b}_1\ \mathbf{b}_2\ \mathbf{b}_3]$, then $AB = [A\mathbf{b}_1 + A\mathbf{b}_2 + A\mathbf{b}_3]$. FALSE This is right but there should not be $+$'s in the solution. Remember the answer should also be 3×3.
- The second row of AB is the second row of A multiplied on the right by B. TRUE
- $(AB)C = (AC)B$ FALSE Matrix multiplication is not commutative.
- $(AB)^T = A^T * B^T$ FALSE $(AB)^T = B^T * A^T$
- The transpose of a sum of matrices equals the sum of their transposes. TRUE

- In order for a matrix B to be the inverse of A, both equations $AB = I$ and $BA = I$ must be true. TRUE We'll see later that for square matrices AB=I then there is some C such that BC=I. CHALLENGE: Can you find an inverse for any non-square matrix. If so find one, if not explain why. Also in above statement about square matrices, does C=A?
- If A and B are $n \times n$ and invertible, then $A^{-1}B^{-1}$ is the inverse of AB. FALSE $AB^{-1} = B^{-1}A^{-1}$. Remember "shoes and socks."
- If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $ab - cd \neq 0$, then A is invertible. False. A is invertible is $ad - bc \neq 0$
- If A is an invertible $n \times n$ matrix, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for each $\mathbf{b}$ in $\mathbb{R}^n$. TRUE. Since A is invertible we have that $\mathbf{x} = A^{-1}\mathbf{b}$

- Each elementary matrix is invertible. True. Let K be the elementary row operation required to change the elementary matrix back into the identity. If we preform K on the identity, we get the inverse.

Section 2.2 10

- A product of invertible $n \times n$ matrices is invertible, and the inverse of the product of their matrices in the same order. FALSE. It is invertible, but the inverses in the product of the inverses in the reverse order.
- If A is invertible, then the inverse of A^{-1} is A itself. TRUE
- If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $ad = bc$, then A is not invertible. TRUE. A is invertible is $ad - bc \neq 0$ but if $ad = nc$ then $ad - bc = 0$ so A is not invertible.
- If A can be row reduced to the identity matrix, then A must be invertible. TRUE The algorithm presented in this chapter tells us how to find the inverse in this case.
- If A is invertible, then elementary row operations then reduce A to to the identity also reduce A^{-1} to the identity. FALSE They also reduce the identity to A^{-1}

Section 2.3 11

- If the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, then A is row equivalent to the $n \times n$ identity matrix. TRUE From Thm 8
- If the columns of A span $\mathbb{R}^n$, then the columns are linearly independent. TRUE Again from Thm 8. Also is n vector span $\mathbb{R}^n$ they must be linearly independent.
- If A is an $n \times n$ matrix then the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^n$. FALSE we need to know more about A like if it in invertible (or anything else in Thm 8)
- If the equation $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution, then A has fewer than n pivot positions. TRUE This comes from the "all false" part of THM 8. (The statements are either all true or all false.)
- If A^T is not invertible, then A is not invertible. TRUE Also from the all false part of theorem 8.

- If there is an $n \times n$ matrix D such that $AD = I$, then there is also an $n \times n$ matrix C such that $CA = I$. TRUE Thm 8
- If the columns of A are linearly independent, then the columns of A span $\mathbb{R}^n$. TRUE Thm 8
- If the equation $A\mathbf{x} = \mathbf{b}$ has at least solution for each $\mathbf{b}$ in $\mathbb{R}^n$, then the solution is unique for each $\mathbf{b}$. TRUE Thm 8
- If the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ maps $\mathbb{R}^n$ into $\mathbb{R}^n$ then A has n pivot points. FALSE. Since A is $n \times n$ the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ maps $\mathbb{R}^n$ into $\mathbb{R}^n$. This doesn't tell us anything about A.
- If there is a $\mathbf{b}$ in $\mathbb{R}^n$ such that the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent, then the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is not one-to-one. TRUE Thm 8