MATH 129A (8) **SPRING 2017** **QUIZ 5**

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5pts

5 pts. Let $T(\mathbf{x}) = A\mathbf{x}$, find a vector $\mathbf{x}$ whose image under T is $\mathbf{b}$, and determine whether $\mathbf{x}$ is unique.

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & 1 & 6 \\ 3 & -2 & -5 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -1 \\ 7 \\ -3 \end{bmatrix}$$

If $\vec{x}$ exists, then it is a solution to $A\vec{x} = \vec{b}$ so solve the linear system via the augmented matrix.

$$\begin{bmatrix} 1 & 0 & -2 & -1 \\ -2 & 1 & 6 & 7 \\ 3 & -2 & -5 & -3 \end{bmatrix}$$

$R_2' = R_2 + 2R_1$
$R_3' = R_3 - 3R_1$

$\Downarrow$

$$\begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & -2 & 1 & 0 \end{bmatrix}$$

$R_3' = R_3 + 2R_2$

$$\begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 5 & 10 \end{bmatrix}$$

$R_3' = \frac{1}{5} R_3$

$$\begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$R_1' = 2R_3 + R_1$
$R_2' = R_2 - 2R_3$

$\Downarrow$

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$\boxed{\vec{x} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \text{ \underline{unique} since pivot in every column}}$

Verification of Solution

$$A\vec{x} = \begin{bmatrix} 1 & 0 & -2 \\ -2 & 1 & 6 \\ 3 & -2 & -5 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3\cdot 1 + 0\cdot 1 + (-2)\cdot 2 \\ -2\cdot 3 + 1\cdot 1 + 6\cdot 2 \\ 3\cdot 3 + (-2)(1) + (-5)(2) \end{bmatrix}$$

$$A\vec{x} = \begin{bmatrix} 3 - 4 \\ -6 + 1 + 12 \\ 9 - 2 - 10 \end{bmatrix}$$

$$A\vec{x} = \begin{bmatrix} -1 \\ 7 \\ -3 \end{bmatrix} = \vec{b}$$