

### Theorem: The Invertible Matrix Theorem

Let  $A$  be a square  $n \times n$  matrix. Then the following statements are equivalent. That is, for a given  $A$ , the statements are either all true or all false.

- a.  $A$  is an invertible matrix.
- b.  $A$  is row equivalent to the  $n \times n$  identity matrix
- c.  $A$  has  $n$  pivot positions.
- d. The equation  $A\mathbf{x} = \mathbf{0}$  has only the trivial solution.
- e. The columns of  $A$  form a linearly independent set.
- f. The linear transformation  $\mathbf{x} \mapsto A\mathbf{x}$  is one-to-one
- g. The equation  $A\mathbf{x} = \mathbf{b}$  has at least one solution for each  $\mathbf{b}$  in  $\mathbb{R}^n$ .
- h. The columns of  $A$  span  $\mathbb{R}^n$ .
- i. The linear transformation  $\mathbf{x} \mapsto A\mathbf{x}$  maps  $\mathbb{R}^n$  onto  $\mathbb{R}^n$ .
- j. There is an  $n \times n$  matrix  $C$  such that  $CA = I$ .
- k. There is an  $n \times n$  matrix  $D$  such that  $AD = I$ .
- l.  $A^T$  is an invertible matrix.