MATH 129A (8) **SPRING 2017** **QUIZ 4**

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5 pts

4 pts. Let $A = \begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$. Show that the equation $A\mathbf{x} = \mathbf{b}$ does not have a solution for all possible $\mathbf{b}$, and describe the set of all $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ does have a solution.

Augmented Matrix

$$\begin{bmatrix} 2 & -1 & b_1 \\ -6 & 3 & b_2 \end{bmatrix}$$

$R_2' = R_2 + 3R_1$

$\Downarrow$

$$\begin{bmatrix} 2 & -1 & b_1 \\ 0 & 0 & 3b_1+b_2 \end{bmatrix}$$

It is possible to have a contradiction if $3b_1+b_2 \neq 0$ so no solution for all $\vec{b} \in \mathbb{R}^2$

Set of b for which $Ax=b$ has a solution when

$3b_1+b_2=0$

$\boxed{b_2=-3b_1}$

Verification of work

$2x_1 - x_2 = b_1$

$2 \quad -1 =$
$1 = b_1$

$$\begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

1 pts. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$. Does $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ span $\mathbb{R}^4$? Why or why not?

No it does not span all of $\mathbb{R}^4$ since to span all of $\mathbb{R}^4$, there must be a pivot in every row of the matrix $[\vec{v_1}\ \vec{v_2}\ \vec{v_3}]$. With three columns (i.e. vectors) and four rows, it is not possible to have a pivot in every row since max number of pivots is 3, which is less than the number of rows (4). Essentially end up with reduced echelon matrix which has only 3 pivots. (See squares $\square$)

$$\begin{bmatrix} \boxed{1} & 0 & 0 \\ 0 & \boxed{1} & 0 \\ 0 & 0 & \boxed{1} \\ 0 & 0 & 0 \end{bmatrix} \leftarrow \text{3 pivots}$$