Chapter 5-2 - The Characteristic Equation

1) Find the characteristic polynomial and eigenvalues of:

$$\begin{bmatrix} 2 & 7 \\ 7 & 2 \end{bmatrix}$$

$\Delta = (2-\lambda)(2-\lambda) - 49$

$\Delta = 4 - 4\lambda + \lambda^2 - 49$

$\boxed{\Delta = \lambda^2 - 4\lambda - 45}$

$\Delta = (\lambda - 9)(\lambda + 5)$

$\boxed{\lambda = 9, \lambda = -5}$

3) Find the characteristic polynomial and eigenvalues of:

$$\begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}$$

$\Delta = (3-\lambda)(-1-\lambda) + 2$

$\Delta = -3 - 2\lambda + \lambda^2 + 2$

$\boxed{\Delta = \lambda^2 - 2\lambda - 1}$

$\lambda = \dfrac{2 \pm \sqrt{4+4}}{2}$

$\boxed{\lambda = 1 \pm \sqrt{2}}$

5) Find the characteristic polynomial and eigenvalues of:

$$\begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$$

$\Delta = (2-\lambda)(4-\lambda) + 1$

$\Delta = 8 - 6\lambda + \lambda^2 + 1$

$\boxed{\Delta = \lambda^2 - 6\lambda + 9}$

$\boxed{\lambda = 3}$

7) Find the characteristic polynomial and eigenvalues of:

$$\begin{bmatrix} 5 & 3 \\ -4 & 4 \end{bmatrix}$$

$\Delta = (5-\lambda)(4-\lambda) + 12$

$\Delta = 20 - 9\lambda + \lambda^2 + 12$

$\boxed{\Delta = \lambda^2 - 9\lambda + 32}$

$\lambda = \dfrac{9 \pm \sqrt{81 - 128}}{2}$

$\lambda = \dfrac{9 \pm i\sqrt{47}}{2}$

No real eigenvalues

9) Find the characteristic polynomial of each 3x3 matrix

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 3 & -1 \\ 0 & 6 & 0 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 2 & 3-\lambda & -1 \\ 0 & 6 & -\lambda \end{vmatrix} = 6(-1)^{3+2}\begin{vmatrix} 1-\lambda & -1 \\ 2 & -1 \end{vmatrix} + (-\lambda)(-1)^{3+3}\begin{vmatrix} 1-\lambda & 0 \\ 2 & 3-\lambda \end{vmatrix}$$

$\Delta = (-6)(\lambda - 1 + 2) + (-\lambda)(3 - 4\lambda + \lambda^2)$

$\Delta = -6\lambda - 6 - 3\lambda + 4\lambda^2 - \lambda^3$

$\boxed{\Delta = -\lambda^3 + 4\lambda^2 - 9\lambda - 6}$

11) Find the characteristic polynomial of the matrix:

$$\begin{bmatrix} 4 & 0 & 0 \\ 5 & 3 & 2 \\ -2 & 0 & 2 \end{bmatrix}$$

$$\begin{vmatrix} 4-\lambda & 0 & 0 \\ 5 & 3-\lambda & 2 \\ -2 & 0 & 2-\lambda \end{vmatrix} = (4-\lambda)(-1)^2(3-\lambda)(2-\lambda)$$

$= (4-\lambda)(6 - 5\lambda + \lambda^2)$

$= 24 - 20\lambda + 4\lambda^2 - 6\lambda + 5\lambda^2 - \lambda^3$

$\boxed{= -\lambda^3 + 9\lambda^2 - 26\lambda + 24}$

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13) Find the characteristic polynomial of the matrix

$$\begin{bmatrix} 6 & -2 & 0 \\ -2 & 9 & 0 \\ 5 & 8 & 3 \end{bmatrix}$$

$$\begin{vmatrix} 6-\lambda & -2 & 0 \\ -2 & 9-\lambda & 0 \\ 5 & 8 & 3-\lambda \end{vmatrix}$$

$= (3-\lambda)(-1)^{3+3}\left((6-\lambda)(9-\lambda)-4\right)$

$= (3-\lambda)(54-15\lambda+\lambda^2-4)$

$= (3-\lambda)(50-15\lambda+\lambda^2)$

$= 150-45\lambda+3\lambda^2-50\lambda+15\lambda^2-\lambda^3$

$= -\lambda^3+18\lambda^2-95\lambda+150$

15) List the eigenvalues, repeated according to their multiples.

$$\begin{bmatrix} 4 & -7 & 0 & 2 \\ 0 & 3 & -4 & 6 \\ 0 & 0 & 3 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4, 3, 3, 1

17) List the eigenvalues repeated according to their multiplicity

$$\begin{bmatrix} 3 & 0 & 0 & 0 & 0 \\ -5 & 1 & 0 & 0 & 0 \\ 3 & 8 & 0 & 0 & 0 \\ 0 & -7 & 2 & 1 & 0 \\ -4 & 1 & 9 & -2 & 3 \end{bmatrix}$$

0, 1, 1, 3, 3

21)

a) False - This is only true if the matrix is triangular.

b) False - Elementary row operations can change eigenvalues.

c) True $\det(AB) = (\det A)(\det B)$

d) False - If $\lambda+5$ is a factor, then $\lambda = -5$ is an eigenvalue.

22)

a) False - It is the absolute value of the determinant.

b) False - $\det A^T = \det A$

c) True - This is the definition of algebraic multiplicity.

d) False - Row replacement can change eigenvalues.