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5 pts

1. 2 pts. Determine if the given set is a subspace of $\mathbb{P}_n$ for an appropriate value of n. Justify your answer.

   All polynomials of the form $\mathbf{p}(t) = at^2$, where a is in $\mathbb{R}$.

Yes it is a subspace. if $a=0$, then $p(t)=0$ which means the set contains the zero vector.

It is also closed under vector addition where

$p_1(t)+p_2(t) = a_1t^2+a_2t^2$

$p_1(t)+p_2(t) = (a_1+a_2)t^2$

if we call $a_1+a_2=a$

then:

$p_1(t)+p_2(t) = at^2 \in$ Set of all at^2

Also closed under scalar multiplication

$c(p_1(t)) = c(a_1t^2)$

$c(p_1(t)) = (ca_1)t^2$

if $ca_1=a$, then

$cp_1(t) = at^2 \in$ Set of all at^2

$p(t) = \text{span}\{t^2\}$ which by the theorem in 4.1 makes it a subspace of V which in this case is P_n (i.e. P_2)

2. 3 pts. Let W be the set of all vectors of the form $\begin{bmatrix} 5b+2c \\ b \\ c \end{bmatrix}$, where b and c are arbitrary.

   Find vectors $\mathbf{u}$ and $\mathbf{v}$ such that $W = \text{Span}\{\mathbf{u},\mathbf{v}\}$. Why does this show that W is a subspace of $\mathbb{R}^3$?

$\begin{bmatrix} 5b+2c \\ b \\ c \end{bmatrix} = \begin{bmatrix} 5b \\ b \\ 0 \end{bmatrix} + \begin{bmatrix} 2c \\ 0 \\ c \end{bmatrix} = b\begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ - this is a form of $\text{span}\left\{\begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}\right\}$

Hence: $\vec{u} = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ where $\vec{u},\vec{v} \in \mathbb{R}^3$

By the theorem in chapter 4 (I believe 4-1 but not positive on the theorem number), a span in V is a subspace of V. Hence, $\text{span}\left\{\begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}\right\}$ is a subspace of $\mathbb{R}^3$, as V in this case is $\mathbb{R}^3$.