Homework #1-1

1) Augmented Matrix

$$\begin{array}{c} E_1 \\ E_2 \end{array}\left[\begin{array}{cc|c} 1 & 5 & 7 \\ -2 & -7 & -5 \end{array}\right]$$

$E_2' = 2E_1 + E_2$

$$\begin{aligned} \cancel{2x_1} + 10x_2 &= 14 \\ -\cancel{2x_1} - 7x_2 &= -5 \\ 3x_2 &= 9 \end{aligned}$$

New Matrix

$$\left[\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 3 & 9 \end{array}\right]$$

Scale E_2

$E_2' = \frac{1}{3}E_2$

New Matrix

$$\begin{bmatrix} 1 & 5 & 7 \\ 0 & 1 & 3 \end{bmatrix}$$

$E_1' = E_1 - 5 \cdot E_2$

$$\begin{aligned} x_1 + \cancel{5x_2} &= 7 \\ -\cancel{5x_2} &= -15 \\ \hline x_1 &= -8 \end{aligned}$$

Final Matrix

$$\begin{bmatrix} 1 & 0 & -8 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\boxed{\begin{aligned} x_1 &= -8 \\ x_2 &= 3 \end{aligned}}$$

3)

$$\begin{aligned} x_1 + 5x_2 &= 7 \\ x_1 - 2x_2 &= -2 \end{aligned}$$

Augmented matrix

$$\begin{array}{c} E_1 \\ E_2 \end{array}\begin{bmatrix} 1 & 5 & 7 \\ 1 & -2 & -2 \end{bmatrix}$$

$E_2' = -E_1 + E_2$

New Matrix

$$\begin{bmatrix} 1 & 5 & 7 \\ 0 & -7 & -9 \end{bmatrix}$$

$E_2' = -\frac{1}{7}E_1$

New Matrix

$$\begin{bmatrix} 1 & 5 & 7 \\ 0 & 1 & \frac{9}{7} \end{bmatrix}$$

$E_1' = -5E_2 + E_1$

New Matrix

$$\begin{bmatrix} 1 & 0 & \frac{4}{7} \\ 0 & 1 & \frac{9}{7} \end{bmatrix}$$

Intersection point

$$\boxed{\left(\frac{4}{7}, \frac{9}{7}\right)}$$

5) Initial Augmented Matrix

$$\begin{bmatrix} 1 & -4 & 5 & 0 & 7 \\ 0 & 1 & -3 & 0 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -5 \end{bmatrix}$$

a) Operation #1: Update the second row of matrix (i.e. E_2) by adding 3 times the third row (i.e. E_3). Hence, the new second row (i.e. E_2') is:

$$E_2' = E_2 + 3 \cdot E_3$$

b) Operation #2: Update the first row of the matrix (i.e. E_1) by adding -5 times the third row (i.e. E_3). Hence the new second row (i.e. E_1') is:

$$E_1' = E_1 - 5 \cdot E_3$$

Homework #1-1

7) The Solution set is empty (i.e. the system is inconsistent) since the third row of the matrix implies that $0=1$.

9) $\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 2 & 4 \end{array}\right]$

$E_4' = \frac{1}{2} E_4$

New Augmented Matrix

$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$

$E_3' = E_3 + 3E_4$

New Augmented Matrix

$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$

$E_2' = 3E_3 + E_2$

New Augmented Matrix

$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$

$E_1' = E_1 + E_2$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$

$\boxed{\begin{array}{l} x_1 = 4 \\ x_2 = 8 \\ x_3 = 5 \\ x_4 = 2 \end{array}}$

11) $\left[\begin{array}{ccc|c} 0 & 1 & 4 & -5 \\ 1 & 3 & 5 & -2 \\ 3 & 7 & 7 & 6 \end{array}\right]$

Interchange E_1 & E_2

$\left[\begin{array}{ccc|c} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 3 & 7 & 7 & 6 \end{array}\right]$

$E_3' = E_3 - 3E_1$

New Augmented Matrix

$\left[\begin{array}{ccc|c} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 0 & -2 & -8 & 0 \end{array}\right]$

$E_3 = E_3 + 2E_2$

New Augmented Matrix

$\left[\begin{array}{ccc|c} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 0 & 0 & 0 & -10 \end{array}\right]$

E_3 is inconsistent

The Solution set is empty.

13) $\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{array}\right]$

Interchange E_2 and E_3

$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 2 & 2 & 9 & 7 \end{array}\right]$

$E_3' = E_3 - 2E_1$

New Augmented Matrix

$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 2 & 15 & -9 \end{array}\right]$

$E_3' = E_3 - 2E_2$

New Augmented Matrix

$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 5 & -5 \end{array}\right]$

$E_3' = \frac{1}{5} E_3$

New Augmented Matrix

$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -1 \end{array}\right]$

$E_1' = E_1 + 3E_3$, $E_2' = E_2 - 5E_3$

New Augmented Matrix

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right]$

$\boxed{\begin{array}{l} x_1 = 5 \\ x_2 = 3 \\ x_3 = -1 \end{array}}$

15)

$$\left[\begin{array}{ccccc} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 3 & 0 & 0 & 7 & -5 \end{array}\right]$$

$E_4' = E_4 - 3E_1$

New Augmented Matrix

$$\left[\begin{array}{ccccc} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 0 & 0 & -9 & 7 & -11 \end{array}\right]$$

$E_3' = E_3 + 2E_2$

New Augmented Matrix

$$\left[\begin{array}{ccccc} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 0 & 0 & -9 & 7 & 11 \end{array}\right]$$

System is consistent as E_3 and E_4 will not cancel each other out.

17)

$x_1 - 4x_2 = 1$

☆ $2x_1 - x_2 = -3$

$-x_1 - 3x_2 = 4$

Do they have a common intersection?

$$\left[\begin{array}{cc|c} 1 & -4 & 1 \\ 2 & -1 & -3 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & -4 & 1 \\ 0 & 7 & -5 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & -4 & 1 \\ 0 & 1 & -\frac{5}{7} \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & -\frac{13}{7} \\ 0 & 1 & -\frac{5}{7} \end{array}\right]$$ Intersect at $\left(-\frac{13}{7}, -\frac{5}{7}\right)$

Line #1 and #2 Intersection

$$\left[\begin{array}{ccc} 1 & -4 & 1 \\ -1 & -3 & 4 \end{array}\right]$$

$$\left[\begin{array}{ccc} 1 & -4 & 1 \\ 0 & -7 & 5 \end{array}\right]$$

$$\left[\begin{array}{ccc} 1 & -4 & 1 \\ 0 & 1 & \frac{5}{7} \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & -\frac{13}{7} \\ 0 & 1 & -\frac{5}{7} \end{array}\right]$$

Line #1 and #3 intersection $\left(-\frac{13}{7}, -\frac{5}{7}\right)$

Yes they intesect at point $\left(-\frac{13}{7}, -\frac{5}{7}\right)$

Alternate approach

$$\left[\begin{array}{ccc} 1 & -4 & 1 \\ 2 & -1 & -3 \\ -1 & -3 & 4 \end{array}\right]$$ New Augmented Matrix

$E_2' = E_2 - 2E_1$

$$\left[\begin{array}{ccc} 1 & -4 & 1 \\ 0 & 7 & -5 \\ -1 & -3 & 4 \end{array}\right]$$ New Augmented Matrix

$E_3' = E_3 + E_1$

$$\left[\begin{array}{ccc} 1 & -4 & 1 \\ 0 & 7 & -5 \\ 0 & -7 & 5 \end{array}\right]$$ New Augmented Matrix

$$\left[\begin{array}{ccc} 1 & -4 & 1 \\ 0 & 1 & -\frac{5}{7} \\ 0 & 1 & -\frac{5}{7} \end{array}\right]$$ New Augmented Matrix

$$\left[\begin{array}{ccc} 1 & 0 & -\frac{13}{7} \\ 0 & 1 & -\frac{5}{7} \\ 0 & 1 & -\frac{5}{7} \end{array}\right]$$ Final Augmented Matrix

Homework #1-1

19) ☆

$$\begin{bmatrix} 1 & h & 4 \\ 3 & 6 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & h & 4 \\ 0 & 6-3h & -4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & h & 4 \\ 0 & 1 & \frac{-4}{6-3h} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 4+\frac{4h}{6-3h} \\ & 1 & \frac{-4}{6-3h} \end{bmatrix}$$

$\boxed{h \neq 2}$ otherwise the solution set is undefined

21) ☆

$$\begin{bmatrix} 1 & 3 & -2 \\ -4 & h & 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 12+h & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 0 \end{bmatrix}$$

0 divided by anything is 0.

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \end{bmatrix}$$

consistent for all h since cannot get a contradiction in the bottom row

23)

a) True - Every row operation is reversible.

b) False - A 5x6 matrix has 5 rows and 6 colums.

c) False - A solution is a list of numbers $(s_1, s_2, \ldots, s_n)$ that makes the statement true. The solution set is the set of all possible solutions.

d) True - Two fundamental questions.

1) Is the system consistent meaning does a solution exist?
2) If the solution exists, is it the only one (i.e. unique)?

☆ ☆ For #19 and #21, make sure you plug "h" back in to prove it is inconsistent.

19) Alternate

If the system is consistent

$6-3h \neq 0$

$6 \neq 3h$

$h \neq 2$

21) Alternate

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 12+h & 0 \end{bmatrix}$$

even if $12+h=0$, still consistent

Homework #1-1

27) $x_1 + 3x_2 = f$

$cx_1 + dx_2 = g$

$\Downarrow$

$$\begin{bmatrix} 1 & 3 & f \\ c & d & g \end{bmatrix}$$

$E_2' = E_2 - ce_1$

$$\begin{bmatrix} 1 & 3 & f \\ 0 & d-3c & g-cf \end{bmatrix}$$

By the question, f and g can be any number. Hence, to ensure a consistent solution

$\boxed{3c \neq d}$

Approach

$d - 3c \neq 0$

$d \neq 3c$

2a)

$$\begin{bmatrix} 0 & -2 & 5 \\ 1 & 4 & -7 \\ 3 & -1 & 6 \end{bmatrix}$$ Swap E_1 and E_2

$\Downarrow$

$$\begin{bmatrix} 1 & 4 & -7 \\ 0 & -2 & 5 \\ 3 & -1 & 6 \end{bmatrix}$$ Swap E_1 and E_2 again

$\Downarrow$

$$\begin{bmatrix} 0 & -2 & 5 \\ 1 & 4 & -7 \\ 3 & -1 & 6 \end{bmatrix}$$