Study guide for Final – Math 129A Linear Algebra (110 pts).

1. Describe the solutions of a system of linear equations in parametric vector form. Also, give a geometric description of the solution set.

Sections 1.1 and 1.2: Elementary row operations, echelon and reduced echelon forms, pivot positions, the row reduction algorithm (Gaussian elimination and Gauss-Jordan elimination), parametric descriptions of solutions sets (basic and free variables). Theorem 2 (existence and uniqueness theorem).
Section 1.5: Homogeneous and nonhomogeneous linear systems. Writing a solution set (of a consistent system) in parametric vector form.

2. Determine if a given set of vectors is linearly dependent.

Section 1.7: Linearly independent and linearly dependent sets of vectors. Linear dependence relation. Linear independence of matrix columns. Theorem 7 (characterization of linearly dependent sets), Theorem 8 and Theorem 9.

3. *Section 1.8:* Introduction to linear transformations: matrix transformations and linear transformation.

4. *Section 2.1:* Matrix multiplication. Row-column rule for computing AB.

5. Find the inverse of a matrix if it exists.

Section 2.2: The inverse of a 2×2 matrix (theorem 4) and the algorithm for finding A^{-1}. Also, theorems 5, 6 and 7 are useful to know.

6. Compute the determinant of a matrix.

Section 3.1: Cofactor expansion across any row and down any column. Know how to find the determinant of a triangular matrix.
Section 3.2: Properties of determinants (theorem 3 and 4). Know how to find the determinant by row reducing matrix to echelon form. Also, know how to combine the method of row reduction and cofactor expansion to compute the determinant.

7. *Section 4.1:* Vector spaces and subspaces. Know how to determine if a given set is a subspace/ vector space. A subspace spanned by a set (theorem 1).

8. Find a basis for the space spanned by a given set of vectors.

Section 4.3: Linearly independent sets; bases. The spanning set theorem. Bases for Nul A and Col A.

9. *Section 4.4:* Coordinate systems. The unique representation theorem. Know how to find the coordinate vector of **x** relative to basis $\mathcal{B}$: $[\mathbf{x}]_{\mathcal{B}} = P_{\mathcal{B}}^{-1}\mathbf{x}$ and vice versa: $\mathbf{x} = P_{\mathcal{B}}[\mathbf{x}]_{\mathcal{B}}$.

10. Assume that the matrix A is row equivalent to B. Find bases for Col A, Row A, and nul A. List rank A and dim Nul A.

Section 4.3: Linearly independent sets; bases. The spanning set theorem. Bases for Nul A and Col A.
Section 4.5: The dimension of a vector space. The dimensions of Nul A and Col A.
*Section 4.*6: The Row space and the rank theorem.

11. Find the characteristic polynomial and the eigenvalues of a given matrix.

Section 5.2: The characteristic equation, characteristic polynomial, algebraic multiplicity of an eigenvalue, similarity.

12. Diagonalize a given matrix if possible.

Section 5.3: The diagonalization theorem, how to diagonalize a matrix (four step algorithm – see example 3 on page 285), when a matrix is diagonalizable (theorems 6 and 7).

13. *Section 6.1:* The inner product, properties of the inner product (theorem 1), the length of a vector, distance in $\mathbb{R}^n$, orthogonal vectors, orthogonal complements.

14. - 15. *Section 6.2:* An orthogonal set, an orthogonal basis, theorem 5 (how to express a vector as a linear combination of the vectors in an orthogonal basis), an orthogonal projection (how to find the orthogonal projection of **y** onto **u** and then write **y** as the sum of two orthogonal vectors, one in Span{**u**} and one orthogonal to **u**). Orthonormal sets (how to normalize vectors to produce an orthonormal set).