Chapter 4-5 - Dimension of a Vector Space

1) Given the subspace:

$$\left\{ \begin{bmatrix} s-2t \\ s+t \\ 3t \end{bmatrix} : s,t \in \mathbb{R} \right\}$$

a) Find a basis

$$H = s\begin{bmatrix}1\\1\\0\end{bmatrix} + t\begin{bmatrix}-2\\1\\3\end{bmatrix}$$

$$\boxed{B = \left\{ \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}-2\\1\\3\end{bmatrix} \right\}}$$

b) State the dimension

$$\boxed{\dim H = 2}$$

3) Given the subspace:

$$\left\{ \begin{bmatrix} 2c \\ a-b \\ b-3c \\ a+2b \end{bmatrix} : a,b,c \in \mathbb{R} \right\}$$

a) Find a basis:

$$\boxed{B = \left\{ \begin{bmatrix}0\\1\\0\\1\end{bmatrix}, \begin{bmatrix}0\\-1\\1\\2\end{bmatrix}, \begin{bmatrix}2\\0\\-3\\0\end{bmatrix} \right\}}$$

b) State the dimension:

$$\boxed{\dim H = 3}$$

5) Given the subspace:

$$\left\{ \begin{bmatrix} a-4b-2c \\ 2a+5b-4c \\ -a+2c \\ -3a+7b+6c \end{bmatrix} : a,b,c \in \mathbb{R} \right\}$$

a) Find a basis:

$$\boxed{B = \left\{ \begin{bmatrix}1\\2\\-1\\-3\end{bmatrix}, \begin{bmatrix}-4\\5\\0\\7\end{bmatrix} \right\}}$$

C term is a scalar multiple of a

b) State the dimension:

$$\boxed{\dim H = 2}$$

7) Given the subspace:

$$\{(a,b,c) : a-3b+c=0,\ b-2c=0,\ 2b-c=0\}$$

a) Find a basis

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & 1 & -2 \\ 0 & 2 & -1 \end{bmatrix} \underset{R_3' = R_3 - 2R_1}{\sim} \begin{bmatrix} 1 & -3 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 3 \end{bmatrix}$$

3 pivots - no free variables

$$B = \{\vec{0}\}$$

b) Find the dimension

$\dim H = 0$

9) Find the dimension of the subspace whose first and third entries are equal in $\mathbb{R}^3$.

$$H = c\begin{bmatrix}1\\0\\1\end{bmatrix} + d\begin{bmatrix}0\\1\\0\end{bmatrix},\ c,d \in \mathbb{R}$$

$\dim H = 2$

11) Find the dimension of the subspace spanned by:

$$\begin{bmatrix}1\\0\\2\end{bmatrix} \begin{bmatrix}3\\1\\1\end{bmatrix} \begin{bmatrix}9\\4\\-2\end{bmatrix} \begin{bmatrix}-7\\-3\\1\end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 9 & -7 \\ 0 & 1 & 4 & -3 \\ 2 & 1 & -2 & 1 \end{bmatrix} \underset{R_3' = R_3 - 2R_1}{\sim} \begin{bmatrix} 1 & 3 & 9 & -7 \\ 0 & 1 & 4 & -3 \\ 0 & -5 & -20 & 15 \end{bmatrix} \underset{R_3' = R_3 + 5R_2}{\sim} \begin{bmatrix} 1 & 3 & 9 & -7 \\ 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\dim H = 2$

13) Determine the dimensions of Nul A and Col A.

$$A = \begin{bmatrix} 1 & -6 & 9 & 0 & -2 \\ 0 & 1 & 2 & -4 & 5 \\ 0 & 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\dim(\text{Col}A) = 3$ (three pivots)

$\dim(\text{Nul}A) = 2$ (two free variables)

15) Determine the dimensions of Nul A and Col A

$$A = \begin{bmatrix} 1 & 0 & 9 & 5 \\ 0 & 0 & 1 & -4 \end{bmatrix}$$

$\dim(\text{Col}A) = 2$ (two pivots)

$\dim(\text{Nul}A) = 2$ (two free variables)

17) Determine the dimensions of Nul A and Col A

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 4 & 7 \\ 0 & 0 & 5 \end{bmatrix}$$

$\dim(\text{Col}A) = 3$

$\dim(\text{Nul}A) = 0$

Chapter 4-5 – Dimension of a Vector Space

19)

a) True – By definition in the text.

b) False – The plane must pass through the origin.

c) False – P_4 has five dimensions.

d) False – Set S must have n elements

e) True – Even if the set is not a basis by Theorem 9 any larger set T cannot be a basis/linearly independent.

20)

a) False – $\mathbb{R}^2$ is not a subspace of $\mathbb{R}^3$

b) False – The number of free variables is the dimension of Nul A.

c) False – Being spanned by a finite set does not automatically make it infinite dimensional as a finite basis may otherwise exist.

d) False – S must also have n elements.

e) True

21) Show that the polynomials are a basis of P_4.

$1, 2t, -2+4t^2, -12t+8t^3$

By isomorphic coordinate mappings

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2t \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -12 \\ 0 \\ 8 \end{bmatrix} \right\}$$

Set is linearly independent with dimension 4.

$\dim P_3 = 4$

By theorem 4-12, the set is a basis since it is linearly independent with the same dimension.

MATH 129A (8) SPRING 2017 QUIZ 8

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5 pts

1. 2 pts. Determine if the given set is a subspace of $\mathbb{P}_n$ for an appropriate value of n. Justify your answer.

All polynomials of the form $\mathbf{p}(t) = at^2$, where a is in $\mathbb{R}$.

Yes it is a subspace. if $a=0$, then $p(t)=0$ which means the set contains the zero vector.

It is also closed under vector addition where

$p_1(t)+p_2(t)=a_1t^2+a_2t^2$

$p_1(t)+p_2(t)=(a_1+a_2)t^2$

if we call $a_1+a_2=a$

then:

$p_1(t)+p_2(t)=at^2 \in$ Set of all at^2

Also closed under scalar multiplication

$c(p_1(t))=c(a_1t^2)$

$c(p_1(t))=(ca_1)t^2$

if $ca_1=a$, then

$cp_1(t)=at^2 \in$ Set of all at^2

$p(t)=\text{span}\{t^2\}$ which by the theorem in 4.1 makes it a subspace of V which in this case is P_n (i.e. P_2)

2. 3 pts. Let W be the set of all vectors of the form $\begin{bmatrix} 5b+2c \\ b \\ c \end{bmatrix}$, where b and c are arbitrary.

Find vectors $\mathbf{u}$ and $\mathbf{v}$ such that $W = \text{Span}\{\mathbf{u}, \mathbf{v}\}$. Why does this show that W is a subspace of $\mathbb{R}^3$?

$$\begin{bmatrix} 5b+2c \\ b \\ c \end{bmatrix} = \begin{bmatrix} 5b \\ b \\ 0 \end{bmatrix} + \begin{bmatrix} 2c \\ 0 \\ c \end{bmatrix} = b\begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$ - this is a form of $\text{span}\left\{\begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}\right\}$

Hence: $\vec{u}=\begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$, $\vec{v}=\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ where $\vec{u},\vec{v} \in \mathbb{R}^3$

By the theorem in chapter 4 (I believe 4-1 but not positive on the theorem number), a span in V is a subspace of V. Hence, $\text{span}\left\{\begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}\right\}$ is a subspace of $\mathbb{R}^3$, as V in this case is $\mathbb{R}^3$.