1) Augmented Matrix

E_1 $\begin{bmatrix} 1 & 5 & \vdots & 7 \\ -2 & -7 & \vdots & -5 \end{bmatrix}$ E_2

$E_2' = 2E_1 + E_2$

$$2x_1 + 10x_2 = 14$$
$$-2x_1 - 7x_2 = -5$$

$$3x_2 = 9$$

New Matrix

$\begin{bmatrix} 1 & 5 & \vdots & 7 \\ 0 & 3 & \vdots & 9 \end{bmatrix}$

Scale E_2

$E_2' = \frac{1}{3}E_2$

New Matrix

$\begin{bmatrix} 1 & 5 & 7 \\ 0 & 1 & 3 \end{bmatrix}$

$E_1' = E_1 - 5 \cdot E_2$

$$x_1 + 5x_2 = 7$$
$$\underline{-5x_2 = -15}$$
$$x_1 = -8$$

Final Matrix

$\begin{bmatrix} 1 & 0 & -8 \\ 0 & 1 & 3 \end{bmatrix}$

$\boxed{\begin{array}{l} x_1 = -8 \\ x_2 = 3 \end{array}}$

3)

$$x_1 + 5x_2 = 7$$
$$x_1 - 2x_2 = -2$$

Augmented matrix

E_1 $\begin{bmatrix} 1 & 5 & 7 \\ 1 & -2 & -2 \end{bmatrix}$ E_2

$E_2' = -E_1 + E_2$

New Matrix

$\begin{bmatrix} 1 & 5 & 7 \\ 0 & -7 & -9 \end{bmatrix}$

$E_2' = \frac{-1}{7}E_1$

New Matrix

$\begin{bmatrix} 1 & 5 & 7 \\ 0 & 1 & \frac{9}{7} \end{bmatrix}$

$E_1' = -5E_2 + E_1$

New Matrix

$\begin{bmatrix} 1 & 0 & \frac{4}{7} \\ 0 & 1 & \frac{9}{7} \end{bmatrix}$

Intersection point

$\boxed{\left(\frac{4}{7}, \frac{9}{7} \right)}$

5) Initial Augmented Matrix

$\begin{bmatrix} 1 & -4 & 5 & 0 & 7 \\ 0 & 1 & -3 & 0 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -5 \end{bmatrix}$

a) Operation #1: Update the second row of matrix (i.e. E_2) by adding 3 times the third row (i.e. E_3). Hence, the new second row (i.e. E_2') is:

$$E_2' = E_2 + 3 \cdot E_3$$

b) Operation #2: Update the first row of the matrix (i.e. E_1) by adding -5 times the third row (i.e. E_3). Hence the new second row (i.e. E_1') is:

$$E_1' = E_1 - 5 \cdot E_3$$

①

7) The Solution set is empty (i.e. the system is inconsistent) since the third row of the matrix implies that $0 = 1$,

9)
$$\begin{bmatrix} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 2 & 4 \end{bmatrix}$$

$E_4' = \frac{1}{2} E_4$

New Augmented Matrix

$$\begin{bmatrix} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$E_3' = E_3 + 3E_4$

New Augmented Matrix

$$\begin{bmatrix} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$E_2' = 3E_3 + E_2$

New Augmented Matrix

$$\begin{bmatrix} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$E_1' = E_1 + E_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$x_1 = 4$
$x_2 = 8$
$x_3 = 5$
$x_4 = 2$

11) ☆
$$\begin{bmatrix} 0 & 1 & 4 & -5 \\ 1 & 3 & 5 & -2 \\ 3 & 7 & 7 & 6 \end{bmatrix}$$

Interchange E_1 & E_2

$$\begin{bmatrix} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 3 & 7 & 7 & 6 \end{bmatrix}$$

$E_3' = E_3 - 3E_1$

New Augmented Matrix

$$\begin{bmatrix} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 0 & -2 & -8 & 0 \end{bmatrix}$$

$E_3 = E_3 + 2E_2$

New Augmented Matrix

$$\begin{bmatrix} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 0 & 0 & 0 & -10 \end{bmatrix}$$

E_3 is inconsistent

The Solution set is empty.

13)
$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{bmatrix}$$

Interchange E_2 and E_3

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 2 & 2 & 9 & 7 \end{bmatrix}$$

$E_3' = E_3 - 2E_1$

New Augmented Matrix

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 2 & 15 & -9 \end{bmatrix}$$

$E_3' = E_3 - 2E_2$

New Augmented Matrix

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 5 & -5 \end{bmatrix}$$

$E_3' = \frac{1}{5} E_3$

New Augmented Matrix

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$E_1' = E_1 + 3E_3$, $E_2' = E_2 - 5E_3$

New Augmented Matrix

$$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$x_1 = 5$
$x_2 = 3$
$x_3 = -1$

②

15)

$$\begin{bmatrix} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 2 & 3 & 2 & 1 \\ 3 & 0 & 0 & 7 & -5 \end{bmatrix}$$

$E_4' = E_4 - 3E_1$

New Augmented Matrix

$$\begin{bmatrix} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 2 & 3 & 2 & 1 \\ 0 & 0 & -9 & 7 & -11 \end{bmatrix}$$

$E_3' = E_3 + 2E_2$

New Augmented Matrix

$$\begin{bmatrix} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 0 & 0 & -9 & 7 & 11 \end{bmatrix}$$

System is consistent as E_3 and E_4 will not cancel each other out.

17) $x_1 - 4x_2 = 1$

★ $2x_1 - x_2 = -3$

$-x_1 - 3x_2 = 4$

Do they have a common intersection?

$$\begin{bmatrix} 1 & -4 & | & 1 \\ 2 & -1 & | & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -4 & | & 1 \\ 0 & 7 & | & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -4 & | & 1 \\ 0 & 1 & | & -\frac{5}{7} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & | & -\frac{13}{7} \\ 0 & 1 & | & -\frac{5}{7} \end{bmatrix}$$ Intersect at $\left(-\frac{13}{7}, -\frac{5}{7}\right)$

Line #1 and #2 Intersection

$$\begin{bmatrix} 1 & -4 & 1 \\ -1 & -3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -4 & 1 \\ 0 & -7 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -4 & 1 \\ 0 & 1 & \frac{5}{7} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & | & -\frac{13}{7} \\ 0 & 1 & | & -\frac{5}{7} \end{bmatrix}$$

Line #1 and #3 intersection $\left(-\frac{13}{7}, \frac{-5}{7}\right)$

Alternate approach

$$\begin{bmatrix} 1 & -4 & 1 \\ 2 & -1 & -3 \\ -1 & -3 & 4 \end{bmatrix}$$ New Augmented Matrix

$E_2' = E_2 - 2E_1$

$$\begin{bmatrix} 1 & -4 & 1 \\ 0 & 7 & -5 \\ -1 & -3 & 4 \end{bmatrix}$$ New Augmented Matrix

$E_3' = E_3 + E_1$

$$\begin{bmatrix} 1 & -4 & 1 \\ 0 & 7 & -5 \\ 0 & -7 & 5 \end{bmatrix}$$ New Augmented Matrix

$$\begin{bmatrix} 1 & -4 & 1 \\ 0 & 1 & \frac{-5}{7} \\ 0 & 1 & \frac{-5}{7} \end{bmatrix}$$ New Augmented Matrix

$$\begin{bmatrix} 1 & 0 & \frac{-13}{7} \\ 0 & 1 & \frac{-5}{7} \\ 0 & 1 & \frac{-5}{7} \end{bmatrix}$$ Final Augmented Matrix

Yes they intersect at point $\left(\frac{-13}{7}, \frac{-5}{7}\right)$

·19) ⭐

$$\begin{bmatrix} 1 & h & 4 \\ 3 & 6 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & h & 4 \\ 0 & 6-3h & -4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & h & 4 \\ 0 & 1 & \frac{-4}{6-3h} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 4+\frac{4h}{6-3h} \\ & 1 & \frac{-4}{6-3h} \end{bmatrix}$$

$\boxed{h \neq 2}$ otherwise
the solution set is
undefined

21) $\begin{bmatrix} 1 & 3 & -2 \\ -4 & h & 8 \end{bmatrix}$ ⭐

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 12+h & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 0 \end{bmatrix} \nwarrow$$

0 divided
by anything
is 0.

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \end{bmatrix}$$

consistent for all h since
Cannot get a contradiction
in the bottom row

⭐ ⭐
For #19 and #21, make
sure you plug "h" back in
to prove it is inconsistent.

23)

a) True - Every row operation
is reversible.

b) False - A 5x6 matrix
has 5 rows and 6 colums.

c) False - A solution is a list of
numbers $(s_1, s_2, ..., s_n)$ that make the
statement true. The solution set
is the set of all possible solutions.

d) True - Two fundamental
questions.
1) Is the system consistent
meaning does a solution exist?
2) If the solution exists, is
it the only one (i.e. unique)?

19) Alternate
If the system is consistent
6-3h ≠ 0
6 ≠ 3h
h ≠ 2

21) Alternate
$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 12+h & 0 \end{bmatrix}$$
even if 12+h=0, still
consistent

④

27) $\begin{aligned} x_1 + 3x_2 &= f \\ cx_1 + dx_2 &= g \end{aligned}$ ✗

$\Downarrow$

$\begin{bmatrix} 1 & 3 & f \\ c & d & g \end{bmatrix}$

$E_2' = E_2 - ce_1$

$\begin{bmatrix} 1 & 3 & f \\ 0 & d-3c & g-cf \end{bmatrix}$

By the question, f and g can be any number. Hence, to ensure a consistent solution

$\boxed{3c \neq d}$

Approach

$d - 3c \neq 0$

$d \neq 3c$

$\begin{bmatrix} 0 & -2 & 5 \\ 1 & 4 & -7 \\ 3 & -1 & 6 \end{bmatrix}$ Swap E_1 and E_2

$\Downarrow$

$\begin{bmatrix} 1 & 4 & -7 \\ 0 & -2 & 5 \\ 3 & -1 & 6 \end{bmatrix}$ Swap E_1 and E_2 again

$\Downarrow$

$\begin{bmatrix} 0 & -2 & 5 \\ 1 & 4 & -7 \\ 3 & -1 & 6 \end{bmatrix}$