Homework #1.2

1)

a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

Yes, it is in reduced echelon form.

b) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

This is in reduced echelon form.

c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

It is in neither reduced nor standard echelon form.

d) $\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

Standard echelon form

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3) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$

$R_3' = R_3 - 6R_1$
$R_2' = R_2 - 4R_1$

$\Downarrow$

$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & -5 & -10 & -15 \end{bmatrix}$

$\Downarrow$

$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -3 \\ 0 & -1 & -2 & -3 \end{bmatrix}$ $R_3' = \frac{1}{5}R_3$
$R_2' = \frac{1}{3}R_2$

$\Downarrow$

$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3' = R_3 - R_2$

$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_2' = -1 \cdot R_2$

$\Downarrow$ $R_1' = R_1 - 2R_2$

$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Pivot Colums: 1 & 2

Pivot Positions Original:

$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$

Pivot Positions RREF:

$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

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5) Describe the echelon forms for a 2x2 matrix.

Case #1

$\begin{bmatrix} \blacksquare & * \\ 0 & \blacksquare \end{bmatrix}$

Case #2

$\begin{bmatrix} \blacksquare & * \\ 0 & 0 \end{bmatrix}$

Case #3

$\begin{bmatrix} 0 & \blacksquare \\ 0 & 0 \end{bmatrix}$

* - Can be zero or number

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7) $\begin{bmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{bmatrix}$

$R_2' = R_2 - 3R_1$

$\Downarrow$

$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 0 & 0 & -5 & -15 \end{bmatrix}$

$\Downarrow$ $R_2' = -\frac{1}{5} \cdot R_2$

$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

$\Downarrow$ $R_1' = R_1' - 4R_2$

$\begin{bmatrix} 1 & 3 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

$$\boxed{\begin{array}{l} x_3 = 3 \\ x_1 = -5 - 3x_2 \\ x_2 \text{ is free} \end{array}}$$

9) $\begin{bmatrix} 0 & 1 & -6 & 5 \\ 1 & -2 & 7 & -6 \end{bmatrix}$

$R_1 \Leftrightarrow R_2$ (interchange)

$\Downarrow$

$\begin{bmatrix} 1 & -2 & 7 & -6 \\ 0 & 1 & -6 & 5 \end{bmatrix}$

$R_1' = R_1 + 2R_2$

$\Downarrow$

$\begin{bmatrix} 1 & 0 & -5 & 4 \\ 0 & 1 & -6 & 5 \end{bmatrix}$

$$\boxed{\begin{array}{l} x_1 = 4 + 5x_3 \\ x_2 = 5 + 6x_3 \\ x_3 \text{ is free} \end{array}}$$

11) $\begin{bmatrix} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{bmatrix}$

$R_2' = R_2 + 3R_1$

$\Downarrow$

$\begin{bmatrix} 3 & -4 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ -6 & 8 & -4 & 0 \end{bmatrix}$

$R_3' = R_3 + 2R_1$

$\Downarrow$

$\begin{bmatrix} 3 & -4 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_1' = \frac{1}{3} R_1$

$\Downarrow$

$\begin{bmatrix} 1 & -\frac{4}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$\boxed{\begin{array}{l} x_1 = \frac{4}{3}x_2 - \frac{2}{3}x_3 \\ x_2 \text{ is free} \\ x_3 \text{ is free} \end{array}}$$

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13)

$$\left[\begin{array}{cccccc} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_1' = R_1 + 3R_2$

$\Downarrow$

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & -1 & -12 & 1 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_1' = R_1 + R_3$

$\Downarrow$

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & 0 & -3 & 5 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\boxed{\begin{array}{l} x_1 = 5 + 3x_5 \\ x_2 = 1 + 4x_5 \\ x_3 \text{ is free} \\ x_4 = 4 - 9x_5 \\ x_5 \text{ is free} \end{array}}$$

15) ☆

a)

$$\left[\begin{array}{cccc} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & \blacksquare & 0 \end{array}\right]$$

Consistent and unique since each variable has a pivot

b)

$$\left[\begin{array}{ccccc} 0 & \blacksquare & * & * & * \\ 0 & 0 & \blacksquare & * & * \\ 0 & 0 & 0 & 0 & \blacksquare \end{array}\right]$$

Inconsistent since last row has a contradiction as $0 \neq \square$

17)

$$\left[\begin{array}{ccc} 2 & 3 & h \\ 4 & 6 & 7 \end{array}\right]$$

$R_2' = R_2 - 2R_1$

$\Downarrow$

$$\left[\begin{array}{ccc} 2 & 3 & h \\ 0 & 0 & 7-2h \end{array}\right]$$

$0 = 7 - 2h$

$2h = 7$

$$\boxed{h = \frac{7}{2}}$$

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19)

$x_1 + h x_2 = 2$

$4x_1 + 8x_2 = K$

$$\begin{bmatrix} 1 & h & 2 \\ 4 & 8 & K \end{bmatrix}$$

$R_2' = R_2 - 4R_1$

$\Downarrow$

$$\begin{bmatrix} 1 & h & 2 \\ 0 & 8-4h & K-8 \end{bmatrix}$$

$R_2' = \frac{1}{8-4h} \cdot R_2$

$\Downarrow$

$$\begin{bmatrix} 1 & h & 2 \\ 0 & 1 & \frac{K-8}{8-4h} \end{bmatrix}$$

$R_1' = R_1 - h R_2$

$\Downarrow$

$$\begin{bmatrix} 1 & 0 & 2 - \frac{h(K-8)}{8-4h} \\ 0 & 1 & \frac{K-8}{8-4h} \end{bmatrix}$$

a) No Solution

$8 - 4h = 0$

$h = 2$

if $K \neq 8$, then no solution if $h = 2$

b) Unique Solution

This is the set of all cases not infinite and not inconsistent.

$h \neq 2$

c) Infinite Solution

$K = 8$, $h = 2$ since last row becomes all zeros

21) Mark each selection as true or false

a) False - The reduced echelon matrix is unique

b) False - The row reduction algorithm applies to all matrices not just augmented ones.

c) True - Basic variables correspond to pivot columns in the coefficient and augmented matrix

d) True - Solving a system amounts to finding a parametric description of the solution set

e) False - This is not enough information to determine if the system is consistent as the row [0 0 0 5 0] is not a contradiction.

23)

Yes it is consistent, since with three rows and three pivot columns, there cannot be a row in the form:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & h \end{bmatrix}$$

where $h \neq 0$ in the augmented matrix

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25)

If a linear system has a pivot position in every row, it means there are no rows in the augmented matrix that can have a contradiction in the form

$[0 \quad 0 \quad 0 \ldots 0 \quad h]$

where $h \neq 0$

Hence, there is no room for a pivot in the augmented column.

27)

If a linear system is consistent, then the solution is unique if and only if each colum in the coefficient matrix is a pivot column.

29) An undetermined system always has more variables than equations. There cannot be more basic variables than there are equations. As such, there must be at least one free variable. Such a variable may be assigned infinitely many values. If a system is consistent, each different value of a free variable will produce a different solution.