- Every elementary row operation is reversible. TRUE You can reverse multiplying by a constant by multiplying by its inverse. If you add row one to row two and replace row two, then you can subtract row one from row two to get it back.
- A 5×6 matrix has six rows. FALSE It has 5 rows and six columns.
- The solution set of a linear system involving variables $x_1, ..., x_n$ is a list of numbers $(s_1, ...s_n)$ that makes each equation in the system a true statement when the values $s_1, ..., s_n$ are substituted for $x_1, ..., x_n$, respectively. FALSE This describes one element of the solution set, not the entire set.
- Two fundamental questions about a linear system involve existence and uniqueness. TRUE

- Elementary row operations on an augmented matrix never change the solution set of the associated linear system. TRUE See the document also linked to the website.
- Two matrices are row equivalent if they have the same number of rows. FALSE They are row equivalent if you can get from one to the other using elementary row operations. Having the same number of rows is a necessary condition but is not sufficient to say that they are row equivalent.
- An inconsistent system has more than one solution. FALSE An inconsistent system has no solutions.
- Two linear systems are equivalent if they have the same solution set. TRUE If they have the same solution set, they both reduce to the same matrix in reduced row echelon form, since row operations are reversible, we can then reverse one set of these to get from one matrix to the other by row operations, thus they are row equivalent.

- In some cases a matrix may be row reduced to more than one matrix in reduced row echelon form, using different sequences of row operations. FALSE
- The row reduction algorithm applies only to augmented matrices for a linear system. FALSE
- A basic variable in a linear system is a variable that corresponds to a pivot column in the coefficient matrix. TRUE
- Finding a parametric description of the solution set of a linear system is the same as solving the system. TRUE
- If one row in an echelon form of an augmented matrix is $[0\,0\,0\,5\,0]$, then the associated linear system is inconsistent. FALSE

Section 1.2 22

- The echelon form of a matrix is unique FALSE
- The pivot positions in a matrix depend on whether row interchanges are used n the row reduction process. FALSE
- Reducing a matrix to echelon form is called the forward phase of the row reduction process. TRUE
- Whenever a system has free variables, the solution set contains many solutions. FALSE
- A general solution os a system is an explicit description os all solutions of the system. TRUE

Section 1.3 23

- Another notation for the vector $\begin{bmatrix} -4 \\ 3 \end{bmatrix}$ is $\begin{bmatrix} -4 & 3 \end{bmatrix}$. FALSE
- The points in the place corresponding to $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} -5 \\ 2 \end{bmatrix}$ lie on a line through the origin. FALSE
- An example of a linear combination of vectors v_1 and v_2 is the vector $1/2v_1$ TRUE
- The solution set of the linear system whose augmented matrix is $[a_1\ a_2\ a_3\ b]$ is the same as the solution set of the equation $x_1a_1 + x_2a_2 + x_3a_3 = b$ TRUE
- The set Span $\{u, v\}$ is always visualized as a plane through the origin. FALSE

- Any list of five real numbers is a vector in $\mathbb{R}^5$. TRUE
- The vector u results when a vector $u - v$ is added to the vector v. TRUE
- The weights $c_1, \ldots, c_p$ is a linear combination $c_1 v_1 + \cdots + c_p v_p$ cannot all be zero. FALSE
- When u and v are nonzero vectors, Span $\{u, v\}$ contains the line through u and the origin. TRUE
- Asking whether the linear system corresponding to an augmented matrix $[a_1\ a_2\ a_3\ b]$ has a solution amounts to asking whether b is in Span $\{a_1, a_2, a_3\}$. TRUE

Section 1.4 23

- The equation $Ax = b$ is referred to as the vector equation. FALSE
- The vector b is a linear combination of the columns of a matrix A if and only if the equation $Ax = b$ has at least one solution. TRUE
- The equation $Ax = b$ is consistent if the augmented matrix $[A\, b]$ has a pivot position in every row. FALSE
- The first entry in the product Ax is a sum of products. TRUE
- If the columns of an $m \times n$ matrix span $\mathbb{R}^m$, then the equation $Ax = b$ is consistent for each b in $\mathbb{R}^m$ TRUE
- If A is an $m \times n$ matrix and if the equation $Ax = b$ is inconsistent for some b in $\mathbb{R}^m$, then A cannot have a pivot position in every row. TRUE

Section 1.4 24

- Every matrix equation $Ax = b$ corresponds to a vector equation with the same solution set. TRUE
- Any linear combination of vectors can always be written in the form Ax for a suitable matrix A and vector x. TRUE
- The solution set of the linear system whose augmented matrix is $[a_1\, a_2\, a_3\, b]$ is the same as the solution set of $Ax = b$, if $A = [a_1\, a_2\, a_3]$ TRUE
- If the equation $Ax = b$ is inconsistent, then b is not in the set spanned by the columns of A. TRUE
- If the augmented matrix $[A\, b]$ has a pivot position in every row, then the equation $Ax = b$ is inconsistent. FALSE
- If A is an $m \times n$ matrix whose columns do not span $\mathbb{R}^m$, then the equation $Ax = b$ is inconsistent for some b in $\mathbb{R}^m$. TRUE

Section 1.5 23

- A homogeneous equation is always consistent. TRUE - The trivial solution is always a solution.
- The equation $A\mathbf{x} = \mathbf{0}$ gives an explicit descriptions of its solution set. FALSE - The equation gives an implicit description of the solution set.
- The homogeneous equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution if and only if the equation has at least one free variable. FALSE - The trivial solution is always a solution to the equation $A\mathbf{x} = \mathbf{0}$.
- The equation $\mathbf{x} = \mathbf{p} + t\mathbf{v}$ describes a line through $\mathbf{v}$ parallel to $\mathbf{p}$. False. The line goes through $\mathbf{p}$ and is parallel to $\mathbf{v}$.
- The solution set of $A\mathbf{x} = \mathbf{b}$ is the set of all vectors of the form $\mathbf{w} = \mathbf{p} + \mathbf{v}_h$ where $\mathbf{v}_h$ is any solution of the equation $A\mathbf{x} = \mathbf{0}$ FALSE This is only true when there exists some vector $\mathbf{p}$ such that $A\mathbf{p} = \mathbf{b}$.

Section 1.5 24

- If $\mathbf{x}$ is a nontrivial solution of $A\mathbf{x} = \mathbf{0}$, then every entry in $\mathbf{x}$ is nonzero. FALSE. At least one entry in $\mathbf{x}$ is nonzero.
- The equation $\mathbf{x} = x_2\mathbf{u} + x_3\mathbf{v}$, with x_2 and x_3 free (and neither $\mathbf{u}$ or $\mathbf{v}$ a multiple of the other), describes a plane through the origin. TRUE
- The equation $A\mathbf{x} = \mathbf{b}$ is homogeneous if the zero vector is a solution. TRUE. If the zero vector is a solution then $\mathbf{b} = A\mathbf{x} = A\mathbf{0} = \mathbf{0}$. So the equation is $A\mathbf{x} = \mathbf{0}$, thus homogenous.
- The effect of adding $\mathbf{p}$ to a vector is to move the vector in the direction parallel to $\mathbf{p}$. TRUE. We can also think of adding $\mathbf{p}$ as sliding the vector along $\mathbf{p}$.
- The solution set of $A\mathbf{x} = \mathbf{b}$ is obtained by translating the solution set of $A\mathbf{x} = \mathbf{0}$. FALSE. This only applies to a consistent system.

- The columns of the matrix A are linearly independent if the equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution. FALSE. The trivial solution is always a solution.
- If S is a linearly dependent set, then each vector is a linear combination of the other vectors in S. FALSE- For example, $[1, 1]$, $[2, 2]$ and $[5, 4]$ are linearly dependent but the last is not a linear combination of the first two.
- The columns of any 4×5 matrix are linearly dependent. TRUE. There are five columns each with four entries, thus by Thm 8 they are linearly dependent.
- If $\mathbf{x}$ and $\mathbf{y}$ are linearly independent, and if $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent, then $\mathbf{z}$ is in $\text{Span}\{\mathbf{x}, \mathbf{y}\}$. TRUE Since $\mathbf{x}$ and $\mathbf{y}$ are linearly independent, and $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent, it must be that $\mathbf{z}$ can be written as a linear combination of the other two, thus in in their span.

Section 1.7 22

- Two vectors are linearly dependent if and only if they lie on a line through the origin. TRUE. If they lie on a line through the origin then the origin, the zero vector, is in their span thus they are linearly dependent.
- If a set contains fewer vectors then there are entries in the vectors, then the set is linearly independent. FALSE For example, $[1, 2, 3]$ and $[2, 4, 6]$ are linearly dependent.
- If $\mathbf{x}$ and $\mathbf{y}$ are linearly independent, and if $\mathbf{z}$ is in the Span$\{\mathbf{x}, \mathbf{y}\}$ then $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent. TRUE If $\mathbf{z}$ is in the Span$\{\mathbf{x}, \mathbf{y}\}$ then $\mathbf{z}$ is a linear combination of the other two, which can be rearranged to show linear dependence.

Section 1.7 22 Continued

- If a set in $\mathbb{R}^n$ is linearly dependent, then the set contains more vectors than there are entries in each vector. False. For example, in $\mathbb{R}^3$ $[1, 2, 3]$ and $[3, 6, 9]$ are linearly dependent.

Section 1.8 21

- A linear transformation is a special type of function. TRUE The properties are (i) $T(\mathbf{u}+\mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ and (ii) $T(c\mathbf{u}) = cT(\mathbf{u})$.
- If A is a 3×5 matrix and T is a transformation defined by $T(\mathbf{x}) = A\mathbf{x}$, then the domain of T is $\mathbb{R}^3$. FALSE The domain is $\mathbb{R}^5$.
- If A is an $m \times n$ matrix, then the range of the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is $\mathbb{R}^m$ FALSE $\mathbb{R}^m$ is the codomain, the range is where we actually land.
- Every linear transformation is a matrix transformation. FALSE. The converse (every matrix transformation is a linear transformation) is true, however. We (probably) will see examples of when the original statement is false later.

Section 1.8 21 Continued

- A transformation T is linear if and only if $T(c_1\mathbf{v}_1 + c_2\mathbf{v}_2) = c_1 T(\mathbf{v}1) + c_2 T(\mathbf{v}_2)$ for all $\mathbf{v}_1$ and $\mathbf{v}_2$ in the domain of T and for all scalars c_1 and c_2. TRUE If we take the definition of linear transformation we can derive these and if these are true then they are true for $c_1, c_2 = 1$ so the first part of the definition is true, and if $\mathbf{v} = 0$, then the second part if true.

Section 1.8 22

- Every matrix transformation is a linear transformation. TRUE To actually show this, we would have to show all matrix transformations satisfy the two criterion of linear transformations.
- The codomain of the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is the set of all linear combinations of the columns of A. FALSE The If A is $m \times n$ codomain is $\mathbb{R}^m$. The original statement in describing the range.
- If $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation and if $\mathbf{c}$ is in $\mathbb{R}^m$, then a uniqueness question is "Is $\mathbf{c}$ is the range of T." FALSE This is an existence question.
- A linear transformation preserves the operations of vector addition and scalar multiplication. TRUE This is part of the definition of a linear transformation.
- The superposition principle is a physical description of a linear transformation. TRUE The book says so. (page 77)