**MATH129A – Linear Algebra Midterm #1 Study Guide**

**Created By: Zayd Hammoudeh**

**Section 1.1 – Systems of Linear Equations**

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| **Linear Equation** – An equation with **variables** that **can be written in the form**:  **Coefficients:** can be **real or complex** | **Linear System** or **System of Linear Equations** – Collection of **one of more linear equations**. | **Solution:** A **list of numbers** **that makes each equation a true statement** when substituted for variables respectively. | **Solution Set:** Set of **all possible solutions** for a linear system.    **Possible Solution Sets:**   * **No solution** * **One solution** * **Infinite solutions** |

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| **Consistent Linear System** – Has **one or more solutions** | **Coefficient Matrix** – A matrix containing the **coefficients for each variable in each equation** in the linear system. | **Augmented Matrix** – A matrix of a system containing the **coefficient matrix and** **an added column containing the constants** from the **right hand side** of the equation. | **Techniques to Simplify a Linear System**   1. **Replace** one equation with **sum of itself and the multiple of another linear system** (equation) 2. **Interchange** two equations 3. **Multiply all terms** in an equation by a **non-zero constant** |
| **Inconsistent Linear System** – Has no solution |

**Section 1.2 – Row Reduction and Echelon Forms**

**Section 1.3 – Vector Equations**

**Section 1.4 – The Matrix Equation**

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|  |  | **Relationship between Spans and Free Variables**   |  |  | | --- | --- | |  | **General Solution Structure** | | **(Trivial Only)** |  | | **1 Free Variable** | – **Line** through and the origin | | **2 Free Variable** | – **Plane** through , , and the origin | | **Free Variable** | – **Multidimensional sale** through and the origin | |

**Section 1.5 – Solution Sets of Linear Systems**

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| **Homogeneous Equation:**  **Trivial Solution:**   * **Exists for all homogeneous systems**   **Nontrivial Solution:** Any **non-zero vector solution.**   * If it exists, there are infinitely many. * Requires at least one free variable. | **Non-Homogenous System:** where is not the zero vector.   * May be inconsistent. * If its solution exists, it is in the form:   + – Particular solution for the specific non-homogenous system   + – Solution set for the homogenous system |  |

**Section 1.7 – Linear Independence**

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| **Span Review**   * **One Vector** – **At most** a **line**   + **Exception:** vector * **Two Vectors: At most** a **plane**   + **Exception:** Scalar multiples | **Linear Independence**  A set of vectors are **linearly independent** if:  has **only the trivial solution** | **Linear Independence**  A set of vectors are **linearly dependent** if there exists a set of **non-zero weights** such that:  **Note:** This requires **at least one free variable**. |

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| **Linear Dependence Relation:** For vectors , it is defined as:  where are **not all zero.**  **Note:** The values of are not unique. | **Procedure: Checking for Linear Independence**  **Step #1:** Create the coefficient matrix.  **Step #2:** Perform Gaussian elimination to find the echelon matrix.  **Step #3:** Check linear independence   * If **there is a pivot in every row**, the vectors are **linearly independent**. * If **there is a free variable**, the vectors are **linearly dependent**. | **Linear Independence of One Vectors**   * **Zero Vector** () – This is always **linearly dependent**. * **Any Non-Zero Vector** ( where ) then **linearly independent**. |

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| **Linear Independence of Two Vectors** – A set of two vectors is linearly dependent if **at least one** **of the vectors is a scalar multiple** of the other.   * “**At least one**” – Because of the case of the zero vector. |  | **Linear Dependence Summary:** If a set of  **vectors of -dimensions** are linearly independent, then they **span an dimensional shape**. |

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| **Theorem 7:** An indexed set of two or more vectors is **linearly dependent** **if and only if at least one of the vectors in is a linear combination of the others**.  In fact, if is linearly dependent and , then some (where ) is a linear combination of the preceding vectors: . | **Theorem 8:** If a set **contains more vectors than there are entries in each vector**, then the set is **linearly dependent**.  That is any set inis linearly dependent if:  **Proof:** More pivots than columns in an matrix so **at least one free variable**. | **Theorem 9:** If a set  **contains the zero vector**, then the set is **linearly dependent**.  **Proof:** If the zero vector is reordered to be , then |

**Section 1.8 – Introduction to Linear Transformations**

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| **Function** – A **rule** that **assigns to each element** in a set A **exactly one element** from set B. | **Transformation () from to :** A rule that **assigns to each vector** **a vector** | **Domain of :** | **Image of :** For a given , it is the **transformed value** | **Range: Set of all images** .  **Note:** The range may (and often is) be only a subset of the codomain. |
| **Codomain of :** |