Homework 1 (due on next Wednesday, 04/18/2018)

Please review the following topics in the textbook; then answer the homework problems listed below.

- Section 2.1 including Theorem 1-5 on the necessary/sufficient optimality conditions.
- Section 3.1 focusing on the Wolfe conditions.
- Section 3.5: the line search algorithm (Algorithm 3.5 and 3.6) for the Wolfe conditions.
- Section 3.2: the global convergence of the linear search methods (Theorem 3.2).
- Section 3.3: the steepest descent method and its convergence rate (Theorem 3.3 and 3.4).

Problem 1. Exercise 2.1 in the textbook.

Problem 2. Exercise 2.9 in the textbook.

Problem 3. Exercise 3.6 in the textbook.

Problem 4. Consider the steepest descent method with exact line searches applied to the convex quadratic function $f(x) = \frac{1}{2}x^T Qx - b^T x$, where Q is symmetric and positive definite. Show that the search direction at step $k+1$ is always orthogonal to the search direction at step k, i.e., $p_k^T p_{k+1} = 0$ for all k.

Problem 5. Program Algorithm 3.5 and Algorithm 3.6 in the textbook. For the selection of trial step length α_i in Algorithm 3.5 and α_j in Algorithm 3.6, you can use bisection.

Problem 6. Consider the problem of minimizing

$$f(x_1, x_2) \quad = \quad (cx_1 - 2)^4 + x_2^2(cx_1 - 2)^2 + (x_2 + 1)^2,$$

where c is a nonzero parameter.

1. Compute $\nabla f(x)$, $\nabla^2 f(x)$, and find the optimal solution.
2. Program the steepest descent method (using your program from Problem 5 to find the step length).
3. Use your program to numerically solve this problem under two cases: i) $c = 1$, ii) $c = 10$. Compare the convergence in these two cases.
4. Using Theorem 3.4 to explain why as c increases the convergence of the steepest descent method deteriorates.