Homework 3 (due on Wednesday 05/16/2018)

Problem 1. Consider the dog-leg path

$$\tilde{p}(\tau) \;=\; \begin{cases} \tau p^U, & 0 \le \tau \le 1 \\ p^U + (\tau - 1)(p^B - p^U), & 1 \le \tau \le 2 \end{cases}$$

where

$$p^U \;=\; -\frac{\|g\|^2}{g^\intercal B g} g, \qquad B \cdot p^B \;=\; -g.$$

Suppose the symmetric matrix B and the vector g satisfy: 1)$g^T Bg > 0$; 2)$(p^U)^T(p^B - p^U) > 0$. Prove that

1. $\|\tilde{p}(\tau)\|$ is an increasing function of τ;
2. $m(\tilde{p}(\tau))$ is a decreasing function of τ,

where $m(p) = g^T p + \frac{1}{2} p^T Bp$.

Problem 2. Code Algorithm 4.1 in the textbook with 1). the Cauchy point method for the subproblem, 2) the dog-leg method based on the results from Problem 1. Test and compare the performance of the methods on the following problem:

$$\min_{x \in R^n} f(x) = \log(1 + x^\intercal Q x),$$

where Q is a symmetric and positive definite matrix.