BATCH:

- TRAINING AND TEST DATA GENERATED BY SAME DISTRIBUTION
- IF MODEL CLASS NOT TOO COMPLEX AND ENOUGH EXAMPLES MODEL THAT DOES BEST ON TRAINING DATA IS NOT TOO MUCH WORSE ON TEST DATA

ON-LINE:

- ALL IS IN FLUX
- NO STATISTICAL ASSUMPTIONS
- STILL CAN BOUND "REGRET" :=

TOTAL LOSS OF ON-LINE − TOTAL LOSS OF BEST OFF-LINE CHOSEN IN HIND SIGHT

- BOUNDS HOLD FOR ARBITRARY SEQUENCES OF EXAMPLES

On-Line Learning

	experts						
	E_1	E_2	E_3	E_n	prediction	*true label*	loss
day 1	1	1	0	0	0	1	1
day 2	1	0	1	0	1	0	1
day 3	0	1	1	1	1	1	0
day t	$x_{t,1}$	$x_{t,2}$	$x_{t,3}$	$x_{t,n}$	$\widehat{y}_t$	y_t	$\lvert y_t - \widehat{y}_t \rvert$

Protocol of the Master Algorithm

For $t = 1$ To T Do

Receive $\boldsymbol{x}_t \in \{0,1\}^n$

Predict $\widehat{y}_t \in \{0,1\}$

Get label $y_t \in \{0,1\}$

Incur loss $|y_t - \widehat{y}_t| \in \{0,1\}$

CASE 1: THERE IS A CONSISTENT EXPERT

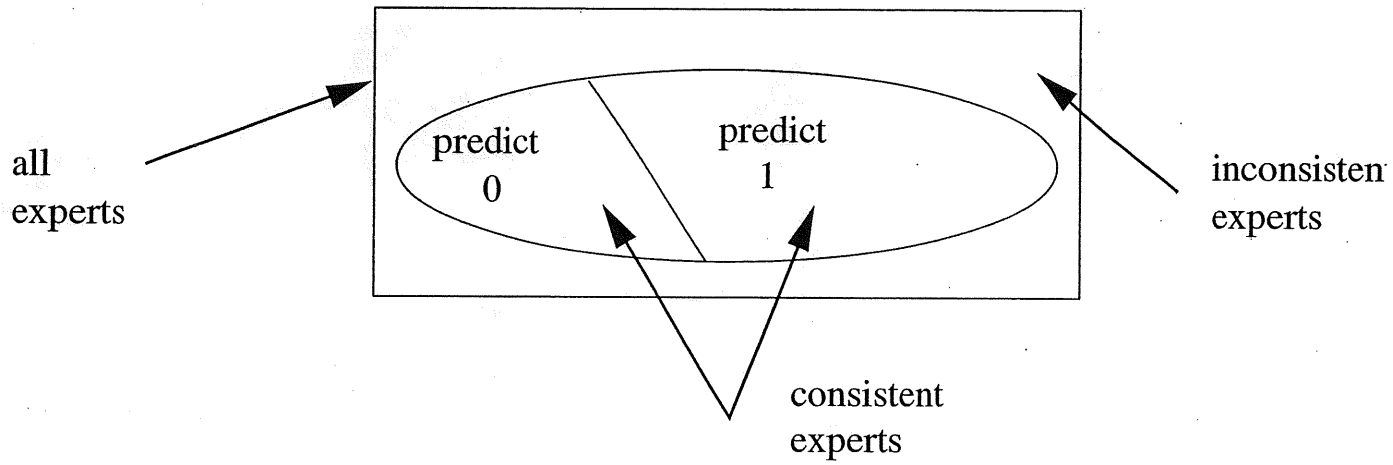
GIVEN SEQUENCE $(\bar{x}_t, y_t)$ s.t

$x_{t,i} = y_t$ for all t

LOSS OF OFF-LINE COMPARATOR IS ZERO

NOISE-FREE CASE

Halving Algorithm [BF]



- Predicts with majority
- If mistake then number of consistent experts is halved

A run of the Halving Algorithm

E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	*majority*	*true label*	loss
1	1	0	0	1	1	0	0	1	0	1
x	x	0	1	x	x	1	1	1	1	0
x	x	x	1	x	x	0	0	0	1	1
x	x	x	↑	x	x	x	x			
			consistent							

For any sequence with a consistent expert, HA makes $\leq \log_2 n$ mistakes

GAME AGAINST NATURE (ADVERSARY)
WHICH CHOOSES THE PREDICTION VECTOR $\bar{x}_t$ AND LABEL y_t

IF THERE IS ONE CONSISENT EXPERT
THEN ALG. $\leq \log_2 n$ MISTAKES

Case 2:

What if no expert is consistent?

For any sequence $S = (x_1, y_1), (x_2, y_2), \ldots, (x_T, y_T)$

- $L_A(S)$ is total loss of alg. A and
- $L_i(S)$ is the total loss of expert E_i

RELATIVE LOSS

Want bounds of the form:

$$\forall S: \quad L_A(S) \leq a \min_i L_i(S) + b \log(n)$$

where a, b are constants

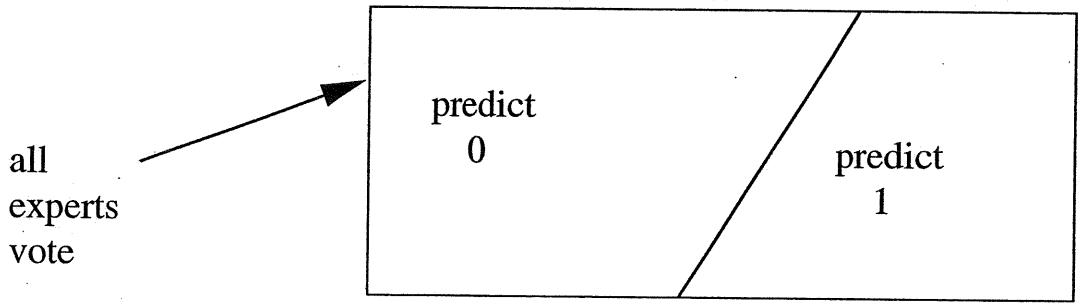
Bounds loss of algorithm
relative to
loss of best expert

$a = 1$

$L_A(S) - \min_i L_i(S)$ CALLED REGRET

Can't wipe out experts!
One weight per expert

Weighted Majority Algorithm [LW]



- Predicts with larger side

- Weights of wrong experts are multiplied by $\beta \in (0, 1]$

- β IS FITNESS FACTOR

- HA : $\beta = 0$

Number of mistakes of the WM algorithm

$M_{t-1,i}$ = # of mistakes of E_i before trial t

$w_{t-1,i} = \beta^{M_{t-1,i}}$ weight of E_i at trial t

$W_{t-1} = \sum_{i=1}^{n} w_{t-1,i}$ total weight at trial t

Minority $\leq \frac{1}{2} W_{t-1}$

Majority $\geq \frac{1}{2} W_{t-1}$

If no mistake then

minority multiplied by β

$W_t \leq 1\, W_{t-1}$

If mistake then
majority multiplied by β

$$W_t \leq \underbrace{\tfrac{1}{2}W_{t-1}}_{\text{minority}} + \beta \underbrace{\tfrac{1}{2}W_{t-1}}_{\text{majority}}$$

$$= \frac{1+\beta}{2} W_{t-1}$$

$$\underset{\substack{total\,final\\ weight}}{W_T} \leq \left(\frac{1+\beta}{2}\right)^M W_0$$

$$W_T = \sum_{j=1}^{n} w_{T,j} = \sum_{j=1}^{n} \beta^{M_j} \geq \beta^{M_i}$$

$$\left(\frac{1+\beta}{2}\right)^M \underbrace{W_0}_{n} \geq \beta^{M_i}$$

$$M \leq \frac{-\ln\beta}{\ln\frac{2}{1+\beta}} M_i + \frac{1}{\ln\frac{2}{1+\beta}} \ln n$$

$$M \underset{\beta = 1/e}{\leq} \underbrace{2.63}_{a} \underbrace{\min_i M_i}_{M^*} + \underbrace{2.63}_{b} \ln n$$

For all sequences, loss of the master algorithm is comparable to the loss of the best expert

Relative loss bounds [F]

WITH FANCY CHOICE OF β THAT DEPENDS ON n, M^* :

$$M \leq 2M^* + 2\sqrt{M^* \ln(N)} + \log_2 n$$

↑ NECCESSARY FOR DETERMINISTC PREDICTION

STREAMLINE SETUP (NO LABELS)

FOR $t = 1$ TO T DO
  CHOOSE AN EXPERT i
  GET LOSS VECTOR $\bar{L}_t \in [0,1]^N$
  INCUR LOSS $L_{t,i}$

GOAL: ACHIEVE SMALL REGRET

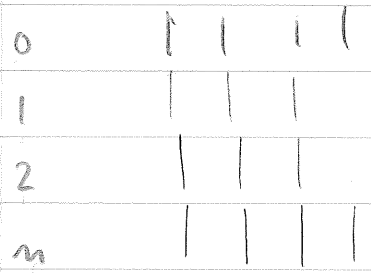
TOTAL LOSS OF ALG - TOTAL LOSS OF BEST

ALG I: DETERMINISTIC FOLLOW THE LEADER

- ALWAYS CHOOSE AN EXPERT OF MINIMAL LOSS

ADVERSARY:
- CHOSEN EXPERT ↑ UNIT OF LOSS
- ALL OTHERS LOSS 0

(T IS # OF TRIALS)



LOSS OF ALG ≈ n LOSS OF BEST

LOSS OF ALG — LOSS OF BEST

T — $\lfloor T/n \rfloor$

ALG III: PERTURB LOSSES OF EXPERTS
PREDICT W. PERTURBED LEADER (LATER!)

ALG IV: HEDGE ALGORITHM
(SIMILAR TO RANDOMIZED WEIGHTED MAJORITY ALGORITHM)

PRABABILISTIC CHOICE OF EXPERT

$\bar{w}_{t-1}$ PROBABILITY VECTOR USED AT TRIAL t

$w_{t-1,i}$ "BELIEVE" AT TRIAL t THAT i IS BEST

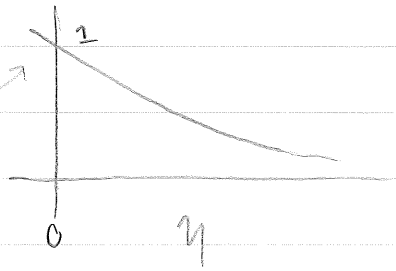
$$w_{0,i} = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)$$

$$w_{t,i} := \frac{w_{t-1,i}\, e^{-\eta\, l_{t,i}}}{Z_t}$$

↑ NORMALIZATION

$\eta > 0$ LEARNING RATE

$e^{-\eta} = \beta$ $\qquad$ $e^{-\infty} = 0$



$$w_{t-1,i} = \frac{e^{-\eta L_{t-1,i}}}{Z_t'}$$

AS $\eta \to \infty$, ALL WEIGHT PLACED ON BEST
& HEDGE BECOMES "FOLLOW THE LEADER"
(TIES BROKEN UNIFORMLY)

$\eta = 0$ WEIGHTS UNCHANGED

$\eta > 0$ GRADUALLY MOVE WEIGHT
TO EXPERTS W. LOW LOSS
"SOFT MIN"

$\eta < 0$ → HIGH LOSS
"SOFTMAX"

NEXT CLASS:

IF η TUNED AS FUNCTION OF
n & $\hat{L}$ THEN

$$\sum_{t=1}^{T} \vec{w}_{t-1} \cdot \vec{L}_t - \underbrace{\min_i L_{T,i}}_{L^*} \leq \sqrt{2\hat{L} \ln n} + \ln n$$

IF $L^* \leq \hat{L}$

LOSS OF ALG − LOSS OF BEST

REGRET