

A Fully-Automated Solver for Multiple Square Jigsaw Puzzles Using Hierarchical Clustering

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Thesis Goals

A Fully-Automated
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Primary Goal: Develop a solver that can assemble multiple jigsaw puzzles simultaneously, with performance that exceeds the state of the art.

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Primary Goal: Develop a solver that can assemble multiple jigsaw puzzles simultaneously, with performance that exceeds the state of the art.

Additional Goals:

- ▶ Define the first metrics that quantify the quality of outputs from a multi-puzzle solver
- ▶ Design visualizations for viewing the errors (if any) in a solver output



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- ▶ First jigsaw puzzle introduced in the 1760s. Modern jigsaw puzzles introduced in the 1930s.
- ▶ First computational jigsaw puzzle solver introduced in 1964
- ▶ Solving a jigsaw puzzle is NP-complete [1, 2]



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- ▶ First computational jigsaw puzzle solver introduced in 1964
- ▶ Solving a jigsaw puzzle is NP-complete [1, 2]
- ▶ **Example Applications:** DNA fragment reassembly, shredded document reconstruction, speech descrambling, and image editing



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- ▶ First jigsaw puzzle introduced in the 1760s. Modern jigsaw puzzles introduced in the 1930s.
- ▶ First computational jigsaw puzzle solver introduced in 1964
- ▶ Solving a jigsaw puzzle is NP-complete [1, 2]
- ▶ **Example Applications:** DNA fragment reassembly, shredded document reconstruction, speech descrambling, and image editing
 - ▶ In most cases, the original, “**ground-truth**” image is unknown.



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Jig Swap Puzzles – Variant of the traditional jigsaw puzzle

- ▶ All pieces are equal-sized squares
- ▶ Substantially more difficult



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Ground-Truth Image



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Ground-Truth Image



Randomized Jig Swap Puzzle



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There are three primary jig swap puzzle types as formalized by [3]. In all cases, the “ground-truth” input is **unknown**.

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There are three primary jig swap puzzle types as formalized by [3]. In all cases, the “ground-truth” input is **unknown**.

- ▶ **Type 1:** Puzzle dimensions and piece rotation are known.
Only piece location is unknown.



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There are three primary jig swap puzzle types as formalized by [3]. In all cases, the “ground-truth” input is **unknown**.

- ▶ **Type 1:** Puzzle dimensions and piece rotation are known. Only piece location is unknown.
- ▶ **Type 2:** All piece locations and rotations unknown. Puzzle dimensions may be known.



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There are three primary jig swap puzzle types as formalized by [3]. In all cases, the “ground-truth” input is **unknown**.

- ▶ **Type 1:** Puzzle dimensions and piece rotation are known. Only piece location is unknown.
- ▶ **Type 2:** All piece locations and rotations unknown. Puzzle dimensions may be known.
- ▶ **Mixed-Bag:** Pieces come from multiple puzzles.



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There are three primary jig swap puzzle types as formalized by [3]. In all cases, the “ground-truth” input is **unknown**.

- ▶ **Type 1:** Puzzle dimensions and piece rotation are known. Only piece location is unknown.
- ▶ **Type 2:** All piece locations and rotations unknown. Puzzle dimensions may be known.
- ▶ **Mixed-Bag:** Pieces come from multiple puzzles.

Mixed-Bag puzzles are the focus of this thesis.



Previous Work

Paikin & Tal

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Paikin & Tal [4] – Current State of the Art

- ▶ Greedy, **kernel growing** solver
- ▶ Supports Type 1, Type 2, and Mixed-Bag puzzles
- ▶ Immune to missing pieces



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Paikin & Tal [4] – Current State of the Art

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- ▶ Immune to missing pieces

Limitations:

- ▶ **Poor Seed Selection:** All decisions are made at runtime using as few as 13 pieces
- ▶ **Externally Supplied Information:** The solver must be told the number of input puzzles

The Mixed-Bag Solver





Mixed-Bag Solver

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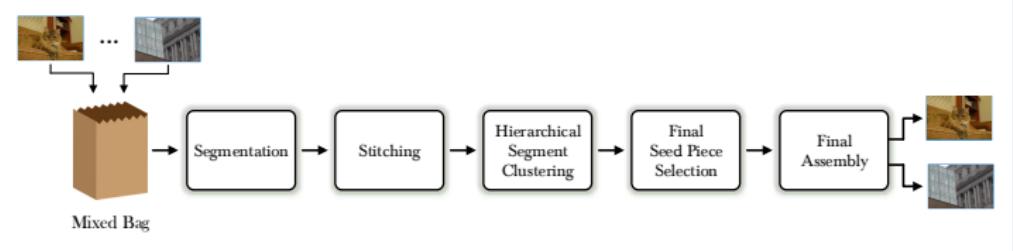
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Paikin & Tal's Algorithm

- ▶ Begin each puzzle with a single piece
- ▶ Place all pieces around the expanding kernel

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Paikin & Tal's Algorithm

- ▶ Begin each puzzle with a single piece
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Alternate Jigsaw Puzzle Solving Strategy

- ▶ Correctly assemble small puzzle regions (i.e., segments)
- ▶ Iteratively merge smaller regions to form large ones



Paikin & Tal's Algorithm

- ▶ Begin each puzzle with a single piece
- ▶ Place all pieces around the expanding kernel

Alternate Jigsaw Puzzle Solving Strategy

- ▶ Correctly assemble small puzzle regions (i.e., segments)
- ▶ Iteratively merge smaller regions to form large ones
- ▶ **Advantages of this Approach:**
 - ▶ Reduces the size of the problem
 - ▶ Provides structure to the unordered set of puzzle pieces.



Paikin & Tal's Algorithm

- ▶ Begin each puzzle with a single piece
- ▶ Place all pieces around the expanding kernel

Alternate Jigsaw Puzzle Solving Strategy

- ▶ Correctly assemble small puzzle regions (i.e., segments)
- ▶ Iteratively merge smaller regions to form large ones
- ▶ **Advantages of this Approach:**
 - ▶ Reduces the size of the problem
 - ▶ Provides structure to the unordered set of puzzle pieces.

The alternate strategy is the basis of the **Mixed-Bag Solver**



Mixed-Bag Solver

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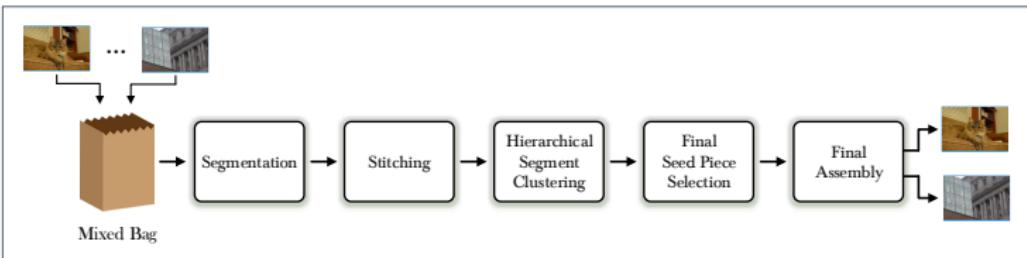
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- ▶ The Mixed-Bag Solver is fully-automated. It makes no assumptions concerning piece orientation, puzzle dimensions, or number of puzzles.
 - ▶ **Input:** A bag of puzzle pieces
 - ▶ **Output:** One or more disjoint, solved puzzles.
- ▶ The Mixed-Bag Solver consists of five distinct stages:





Assembler

Mixed-Bag Solver Component

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- ▶ **Role:** Place the individual pieces in the solved puzzle.
 - ▶ Mixed-Bag Solver is independent of the assembler used, giving the solver significant upgradability and flexibility.



- ▶ **Role:** Place the individual pieces in the solved puzzle.
 - ▶ Mixed-Bag Solver is independent of the assembler used, giving the solver significant upgradability and flexibility.
- ▶ **Assembler Used in this Thesis:** Paikin & Tal
 - ▶ Current state of the art
 - ▶ Allows for more direct comparison of performance
 - ▶ Natively supports Mixed-Bag puzzles
- ▶ **Implementation:** Assembler re-implemented as part of this thesis based off the description in [4]
 - ▶ Written in Python and fully open source [5]



Segmentation

Mixed-Bag Solver Stage #1

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- ▶ **Segment:** Partial puzzle assembly where this is a high degree of confidence pieces are placed correctly.
- ▶ **Role of Segmentation:** Provide structure to the set of puzzle pieces by partitioning them into disjoint segments
 - ▶ **Input:** A bag of puzzle pieces
 - ▶ **Output:** Set of saved segments
- ▶ **Relationship between Puzzle Pieces and Segments:**
 - ▶ Pieces from a single ground-truth input may be separated into multiple segments
 - ▶ A piece can be assigned to at most one segment



Segmentation

Algorithm Overview

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- ▶ Iterative process consisting of one or more rounds
- ▶ In each round, all pieces not yet assigned to a segment are assembled as if all are from the same input image
- ▶ Segments of sufficient size are saved to be used in future Mixed-Bag Solver stages
- ▶ Pieces in a saved segment are not placed in future rounds.
- ▶ Segmentation terminates if all pieces are assigned to a saved segment or when no segment is larger than the minimum allowed size



Segmentation

Composition of a Segment

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- ▶ **Starting a Segment:** Each segment is created iteratively starting with a single seed piece
- ▶ **Definition of Best Buddies:** Any pair of pieces that are more similar to each other than they are to any other piece.
- ▶ **Growing the Segment:** Add to the segment any piece that is a neighbor and best buddy of a segment member
- ▶ **Trimming the Segment**
 - ▶ **Articulation Point:** Any piece whose removal disconnects other pieces from the segment seed.
 - ▶ All articulation pieces pieces are removed from the segment.
 - ▶ After the removal of the articulation points, any pieces no longer connected to the seed are removed.



Segmentation

Example – Input Images

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Image (a) – 805 Pieces [6]

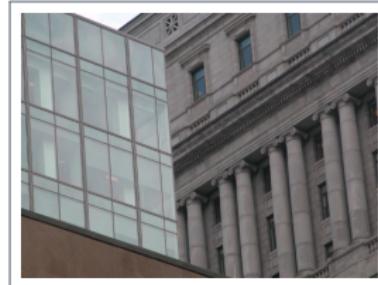


Image (b) – 540 Pieces [7]



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Example – First Segmentation Round Output Image

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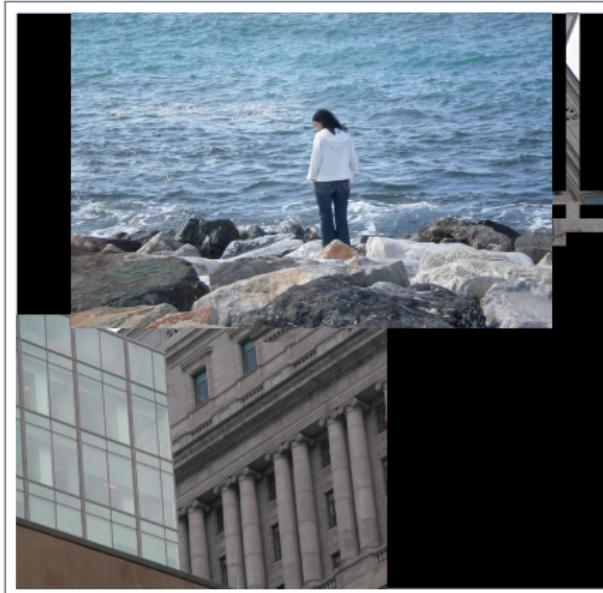
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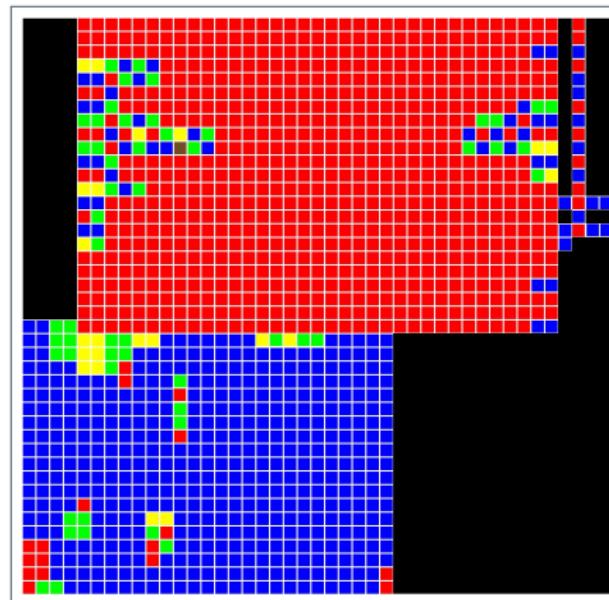
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Mixed-Bag Solver Stage #2

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- ▶ **Role of Stitching:** Quantify the extent that any pair of segments is related.
 - ▶ **Input:** All puzzle pieces and the set of saved segments
 - ▶ **Output:** Segment overlap matrix



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Mixed-Bag Solver Stage #2

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- ▶ **Role of Stitching:** Quantify the extent that any pair of segments is related.
 - ▶ **Input:** All puzzle pieces and the set of saved segments
 - ▶ **Output:** Segment overlap matrix
- ▶ **Theoretical Foundation:** If two segments are from the same ground-truth image, they would eventually **overlap** if one segment were to expand.
 - ▶ Segments should be allowed, but not forced, to expand in all directions.



Stitching

Stitching Piece Location

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- ▶ **Mini-assembly (MA):** Same as a standard assembly except only a fixed number of pieces are placed .
- ▶ **Stitching Piece (ζ_x):** A piece near the boundary of a segment that is used as the seed of a mini-assembly
- ▶ **Segment Overlap:** Maximum overlap between any mini-assembly for segment, Φ_i and another segment Φ_j .

$$Overlap_{\Phi_i, \Phi_j} = \arg \max_{\zeta_x \in \Phi_i} \frac{|MA_{\zeta_x} \cap \Phi_j|}{\min(|MA_{\zeta_x}|, |\Phi_j|)} \quad (1)$$

- ▶ **Asymmetry:** In most cases:

$$Overlap_{\Phi_i, \Phi_j} \neq Overlap_{\Phi_j, \Phi_i} \quad (2)$$



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Example – Input Image

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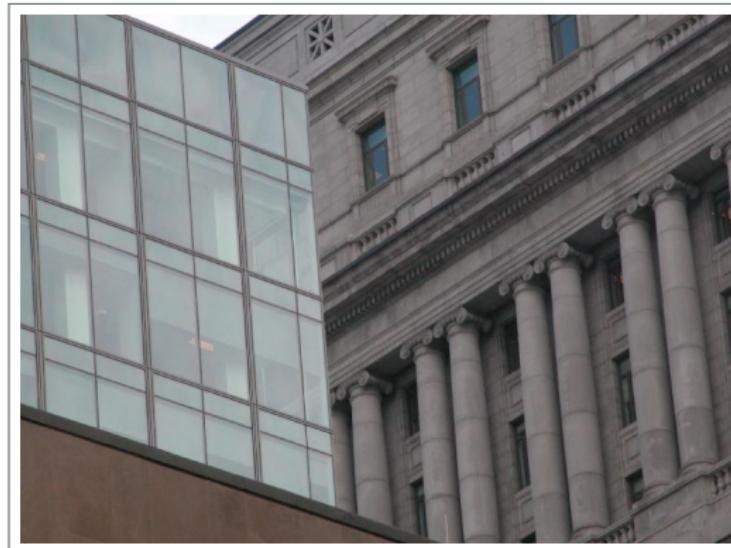
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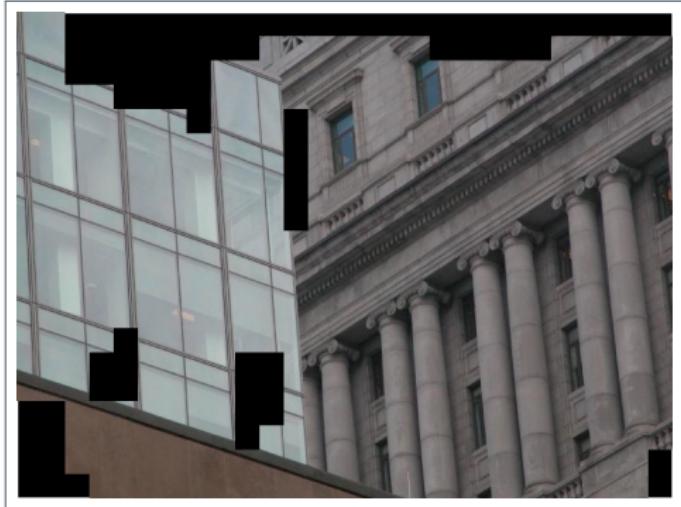
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Segment #1



Segment #2



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Example – Stitching Piece Locations

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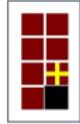
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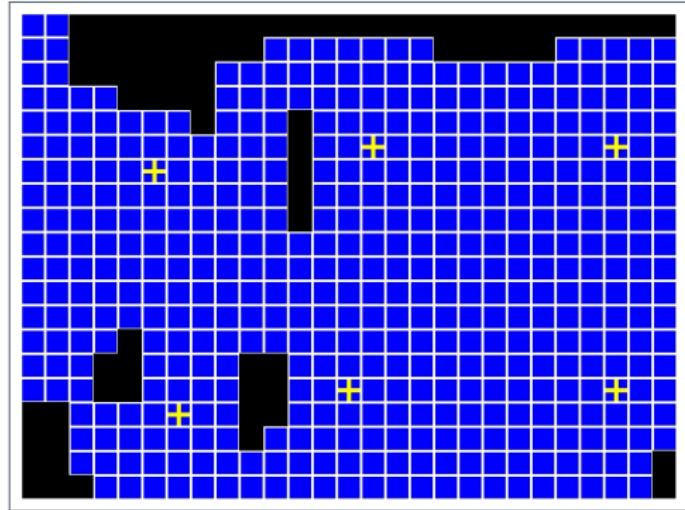
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Stitching Result from Segment #1

Segment Overlap:

$$Overlap_{\Phi_1, \Phi_2} = 0.83 \quad (3)$$



Hierarchical Segment Clustering

Mixed-Bag Solver Stage #3

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- ▶ A single ground-truth image may be comprised of multiple segments.
- ▶ **Role of Hierarchical Clustering:** Merge all segments from the same input image into a single segment cluster.
 - ▶ **Input:** All saved segments and the segment overlap matrix
 - ▶ **Output:** A set of **segment clusters**



Hierarchical Segment Clustering

Calculating the Initial Similarity Matrix

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- ▶ **Segment Overlap Matrix:** A hollow matrix quantifying the relationship between each pair of segments.
- ▶ **Hierarchical Clustering Similarity Matrix:** A diagonal matrix quantifying the similarity between segment pairs.
- ▶ **Quantifying Similarity:** Given n segments, the similarity between segments Φ_i and Φ_j is:

$$\omega_{i,j} = \frac{\text{Overlap}_{\Phi_i, \Phi_j} + \text{Overlap}_{\Phi_j, \Phi_i}}{2} \quad (4)$$



Hierarchical Segment Clustering

Merging Clusters

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- ▶ After two clusters are combined, the similarity between the merged cluster and all other clusters must be recalculated.
- ▶ **Single Link Clustering:** The similarity between any two clusters is equal to the maximum similarity between any two members in the clusters [8]
- ▶ The Mixed-Bag Solver must use single link clustering as two clusters may only have two member segments that are adjacent.



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Direct Accuracy

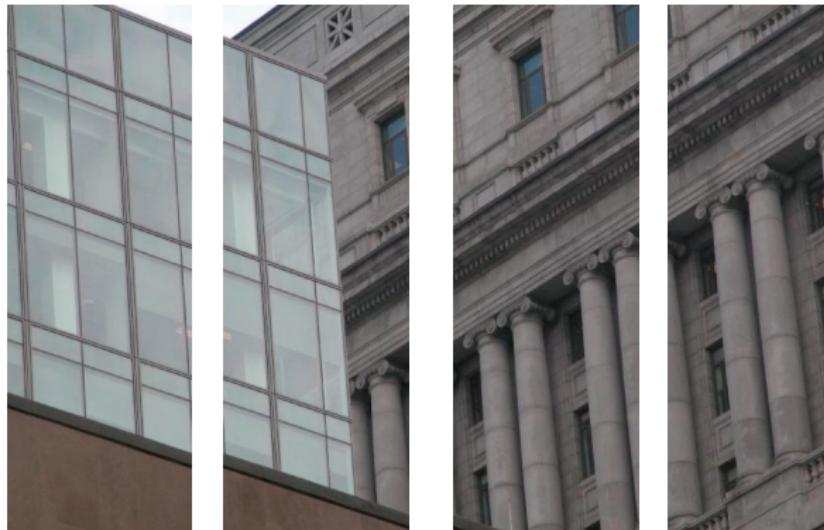
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Segment 1

Segment 2

Segment 3

Segment 4



Hierarchical Segment Clustering

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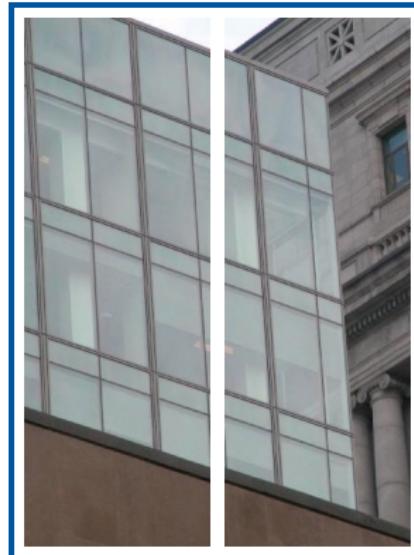
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Segment Cluster 1



Segment 3



Segment 4



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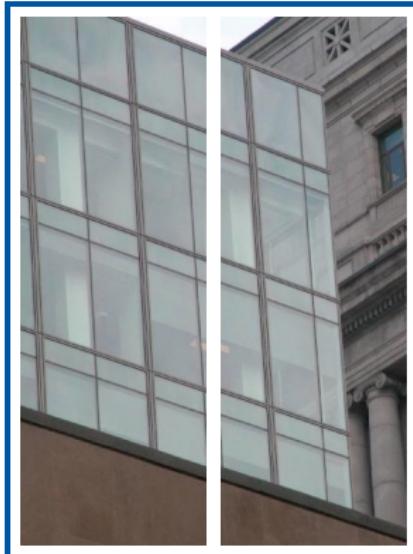
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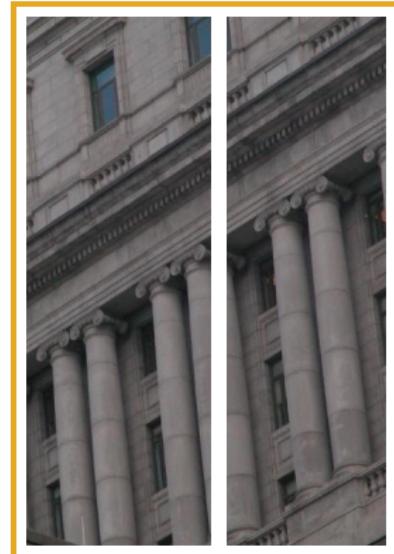
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Segment Cluster 1



Segment Cluster 2



Hierarchical Segment Clustering

Terminating Clustering

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- ▶ The solver continues merging segment clusters until one of two criteria is satisfied:
 - ▶ Only a single segment cluster remains
 - ▶ Maximum similarity between any segment clusters is below a predefined threshold
- ▶ All remaining segment clusters are passed to the next solver stage.

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Final Seed Piece Selection

Mixed-Bag Solver Stage #4

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Mixed-Bag Solver

- ▶ **Role of Final Seed Selection:** Determine the pieces that will be used as the seed for the final output puzzles.
 - ▶ **Input:** Set of segment clusters
 - ▶ **Output:** Final seed pieces
- ▶ A single seed piece is selected from each segment cluster



Final Seed Piece Selection

Mixed-Bag Solver Stage #4

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Mixed-Bag Solver

- ▶ **Role of Final Seed Selection:** Determine the pieces that will be used as the seed for the final output puzzles.
 - ▶ **Input:** Set of segment clusters
 - ▶ **Output:** Final seed pieces
- ▶ A single seed piece is selected from each segment cluster

Paikin & Tal

- ▶ All puzzle seeds are selected greedily at run time, which often leads to poor decisions.



Final Assembly Stage

Mixed-Bag Solver Stage #5

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- ▶ **Role of Final Assembly:** Generate the solved puzzles that are output by the Mixed-Bag Solver.
 - ▶ **Input:** Set of puzzle pieces with the seeds marked
 - ▶ **Output:** Final solved puzzles
- ▶ All pieces are placed around the seeds selected in the previous stage.
- ▶ Assembly proceeds in this stage normally without any custom modifications.

Quantifying Solver Quality





Quantifying Solver Quality

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- ▶ Jigsaw puzzle solvers are not able to always correctly reconstruct the input puzzle(s)
 - ▶ Metrics compare the quality of solver outputs
- ▶ **Two Most Common Quality Metrics:**
 - ▶ Direct Accuracy
 - ▶ Neighbor Accuracy



Quantifying Solver Quality

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- ▶ Jigsaw puzzle solvers are not able to always correctly reconstruct the input puzzle(s)
 - ▶ Metrics compare the quality of solver outputs
- ▶ **Two Most Common Quality Metrics:**
 - ▶ Direct Accuracy
 - ▶ Neighbor Accuracy
- ▶ **Disadvantages of Current Metrics:** Neither account for:
 - ▶ Pieces misplaced in different puzzles
 - ▶ Extra pieces from other puzzles
- ▶ **Goal:** Define new quality metrics for Mixed-Bag puzzles



Quantifying Solver Quality

Standard and Enhanced Direct Accuracy

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- ▶ **Standard Direct Accuracy:** Fraction of pieces, c placed in the same location in both the ground-truth and solved image versus the total number of pieces, n

$$DA = \frac{c}{n} \quad (5)$$



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- ▶ **Standard Direct Accuracy:** Fraction of pieces, c placed in the same location in both the ground-truth and solved image versus the total number of pieces, n

$$DA = \frac{c}{n} \quad (5)$$

- ▶ **Enhanced Direct Accuracy Score (EDAS):** Modified direct accuracy that accounts for missing and extra pieces.

$$EDAS_{P_i} = \arg \max_{S_j \in S} \frac{c_{i,j}}{n_i + \sum_{k \neq i} (m_{k,j})} \quad (6)$$



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- ▶ **Standard Direct Accuracy:** Fraction of pieces, c placed in the same location in both the ground-truth and solved image versus the total number of pieces, n

$$DA = \frac{c}{n} \quad (5)$$

- ▶ **Enhanced Direct Accuracy Score (EDAS):** Modified direct accuracy that accounts for missing and extra pieces.

$$EDAS_{P_i} = \arg \max_{S_j \in S} \frac{c_{i,j}}{n_i + \sum_{k \neq i} (m_{k,j})} \quad (6)$$

- ▶ **Direct Accuracy Range:** 0 to 1
- ▶ **Perfectly Reconstructed Image:** All pieces are placed in their original location ($DA = EDAS = 1$)



Direct Accuracy

Example – Effect of Shifts

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Problem: Direct accuracy is highly vulnerable to shifts, in particular when puzzle dimensions are not fixed



Direct Accuracy

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Ground-Truth Image



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Ground-Truth Image



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Ground-Truth Image



Solver Output

Conclusion: Direct accuracy can be overly punitive.



Direct Accuracy

Shiftable Enhanced Direct Accuracy Score (SEDAS)

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- ▶ **Solution:** Allow the reference point for direct accuracy to **shift beyond the upper left corner** of the image
- ▶ **Shiftable Enhanced Direct Accuracy Score (SEDAS):** Select the reference point, l , within radius d_{min} of the upper left corner of the solved puzzle
 - ▶ d_{min} – Manhattan distance between the upper left corner of the solved image and the nearest puzzle piece
- ▶ **Formal Definition of SEDAS:**

$$SEDAS_{P_i} = \arg \max_{l \in L} \left(\arg \max_{S_j \in S} \frac{c_{i,j,l}}{n_i + \sum_{k \neq i} (m_{k,j})} \right) \quad (7)$$

- ▶ **SEDAS Range:** 0 to 1



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Example – Shiftable Reference Point

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Direct Accuracy Reference Point



Direct Accuracy

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Solver Output



SEDAS Reference Points



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- ▶ **Standard Neighbor Accuracy:** Ratio of puzzle piece sides adjacent in both the original and solved images, a , versus the total number of sides, $n \cdot q$

$$NA = \frac{a}{n \cdot q} \quad (8)$$



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- ▶ **Standard Neighbor Accuracy:** Ratio of puzzle piece sides adjacent in both the original and solved images, a , versus the total number of sides, $n \cdot q$

$$NA = \frac{a}{n \cdot q} \quad (8)$$

- ▶ **Enhanced Neighbor Accuracy Score (ENAS):** Modified neighbor accuracy that accounts for missing and extra pieces.

$$ENAS_{P_i} = \arg \max_{S_j \in S} \frac{a_{i,j}}{q(n_i + \sum_{k \neq i} m_{k,j})} \quad (9)$$



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$$NA = \frac{a}{n \cdot q} \quad (8)$$

- ▶ **Enhanced Neighbor Accuracy Score (ENAS):** Modified neighbor accuracy that accounts for missing and extra pieces.

$$ENAS_{P_i} = \arg \max_{S_j \in S} \frac{a_{i,j}}{q(n_i + \sum_{k \neq i} m_{k,j})} \quad (9)$$

- ▶ **Neighbor Accuracy Range:** 0 to 1
- ▶ **Advantage of Neighbor Accuracy:** Less vulnerable to shifts than direct accuracy

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- ▶ Paikin & Tal's algorithm is the current state of the art and was used as the reference for performance comparison



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Input Puzzle Count

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Conclusions

- ▶ Paikin & Tal's algorithm is the current state of the art and was used as the reference for performance comparison
- ▶ **Standard Test Conditions:**
 - ▶ **Puzzle Type:** 2
 - ▶ **Dimensions Fixed:** No
 - ▶ **Piece Width:** 28 pixels
 - ▶ **Benchmark:** Twenty 805 piece images [6]



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- ▶ Paikin & Tal's algorithm is the current state of the art and was used as the reference for performance comparison
- ▶ **Standard Test Conditions:**
 - ▶ **Puzzle Type:** 2
 - ▶ **Dimensions Fixed:** No
 - ▶ **Piece Width:** 28 pixels
 - ▶ **Benchmark:** Twenty 805 piece images [6]
- ▶ **Number of Ground-Truth Inputs:** 1 to 5

# Puzzles	1	2	3	4	5
# Iterations	20	55	25	8	5



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- ▶ Paikin & Tal's algorithm is the current state of the art and was used as the reference for performance comparison
- ▶ **Standard Test Conditions:**
 - ▶ **Puzzle Type:** 2
 - ▶ **Dimensions Fixed:** No
 - ▶ **Piece Width:** 28 pixels
 - ▶ **Benchmark:** Twenty 805 piece images [6]
- ▶ **Number of Ground-Truth Inputs:** 1 to 5

# Puzzles	1	2	3	4	5
# Iterations	20	55	25	8	5

- ▶ **Test Condition Variation:** Only Paikin & Tal's algorithm was provided the number of input puzzles.



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Determining Input Puzzle Count

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- ▶ **Goal:** Measure the Mixed-Bag Solver's accuracy determining the number of input puzzles
 - ▶ **Importance** – The Mixed-Bag Solver must estimate this accurately to provide meaningful outputs.



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- ▶ **Goal:** Measure the Mixed-Bag Solver's accuracy determining the number of input puzzles
 - ▶ **Importance** – The Mixed-Bag Solver must estimate this accurately to provide meaningful outputs.
- ▶ **Single Puzzle Accuracy** – Represents the solver's performance ceiling
- ▶ **Multiple Puzzle Accuracy** – A more general estimate of the solver's performance



Determining Input Puzzle Count

Single Input Puzzle Results

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- ▶ **Summary:** 17 out of the 20 images were correctly identified as a single ground-truth input
- ▶ **Misclassified Images:** 3 out of the 20 images misclassified as if they were two images.
 - ▶ All three images have large areas with little variation (e.g., a blue sky, smooth water)
 - ▶ The solver's poor performance on these puzzles is due to the assembler as noted in [4]



Determining Input Puzzle Count

Single Input Puzzle Results

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- ▶ **Misclassified Images:** 3 out of the 20 images misclassified as if they were two images.
 - ▶ All three images have large areas with little variation (e.g., a blue sky, smooth water)
 - ▶ The solver's poor performance on these puzzles is due to the assembler as noted in [4]
- ▶ **Note:** 85% (17/20) represents the accuracy ceiling when solving multiple puzzles.



Determining Input Puzzle Count

Visual Comparison of a Misclassified Image

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Perfectly Reconstructed
Image (a)



Misclassified Image (b)

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Multiple Input Puzzles

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- ▶ **Goal:** Measure the Mixed-Bag Solver's accuracy determining the input puzzle count for multiple images
- ▶ **Procedure:** Randomly select a specified number of images (between 2 and 5) from the 20 image data set.



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Multiple Input Puzzles

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- ▶ **Goal:** Measure the Mixed-Bag Solver's accuracy determining the input puzzle count for multiple images
- ▶ **Procedure:** Randomly select a specified number of images (between 2 and 5) from the 20 image data set.
- ▶ **Input Puzzle Count Error:** Difference between the actual number of input puzzles and the number determined by the Mixed-Bag Solver.
 - ▶ **Example:** If 3 images were supplied to the solver but it determined there were 4, the error would be 1.



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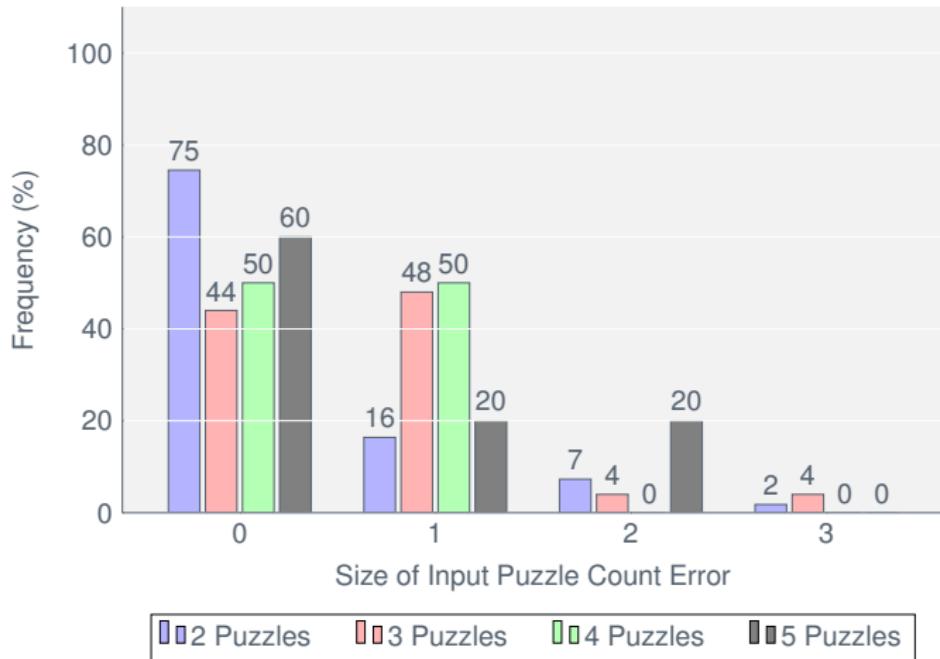
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Mixed-Bag Solver's Input Puzzle Count Error Frequency





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- ▶ **Overall Accuracy: 65%**
- ▶ **Iterations with Error Greater than One: 8%**
- ▶ Accuracy did not significantly degrade as the number of input puzzles increased.



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- ▶ **Overall Accuracy:** 65%
- ▶ **Iterations with Error Greater than One:** 8%
- ▶ Accuracy did not significantly degrade as the number of input puzzles increased.
- ▶ **Over-Rejection of Cluster Mergers:** The Mixed-Bag Solver never underestimated the number of input puzzles.
 - ▶ Performance may be improved by reducing the minimum clustering similarity threshold or minimum segment size



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- ▶ **Goal:** Compare the performance of the Mixed-Bag Solver (**MBS**) and Paikin & Tal's algorithm
- ▶ **Procedure:** Randomly select a specified number of images and input them into both solvers.
- ▶ **Quality Metrics Used:**
 - ▶ Shiftable Enhanced Direct Accuracy Score (SEDAS)
 - ▶ Enhanced Neighbor Accuracy Score (ENAS)
 - ▶ Perfect Reconstruction Percentage



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- ▶ **Goal:** Compare the performance of the Mixed-Bag Solver (**MBS**) and Paikin & Tal's algorithm
- ▶ **Procedure:** Randomly select a specified number of images and input them into both solvers.
- ▶ **Quality Metrics Used:**
 - ▶ Shiftable Enhanced Direct Accuracy Score (SEDAS)
 - ▶ Enhanced Neighbor Accuracy Score (ENAS)
 - ▶ Perfect Reconstruction Percentage
- ▶ **Note:** The results include the Mixed-Bag Solver's performance when it correctly estimated the puzzle count.
 - ▶ This represents the performance ceiling for optimal hierarchical clustering.



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Shiftable Enhanced Direct Accuracy Score (SEDAS)

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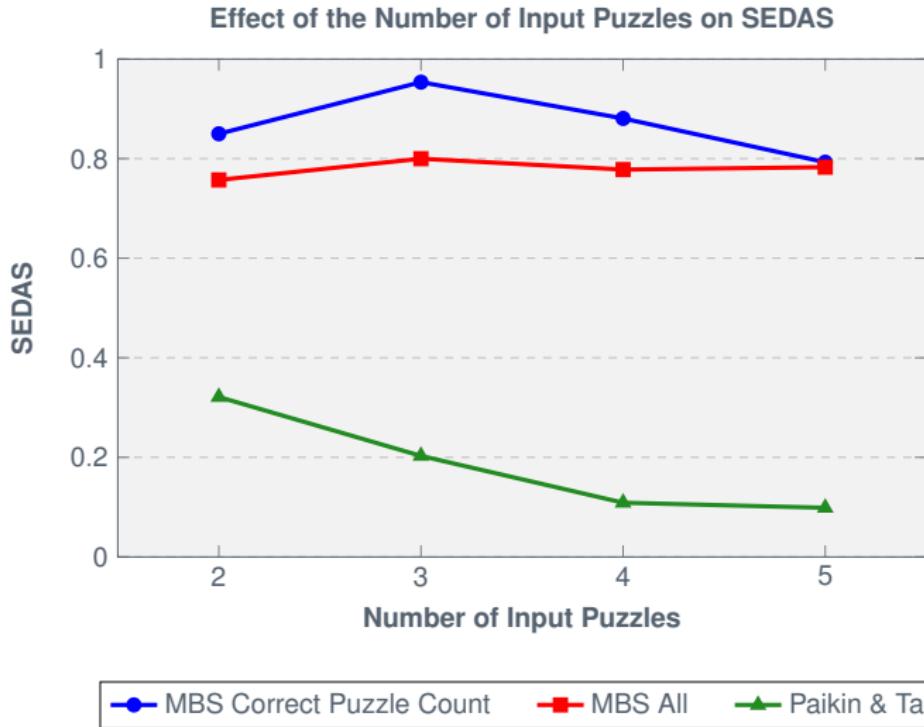
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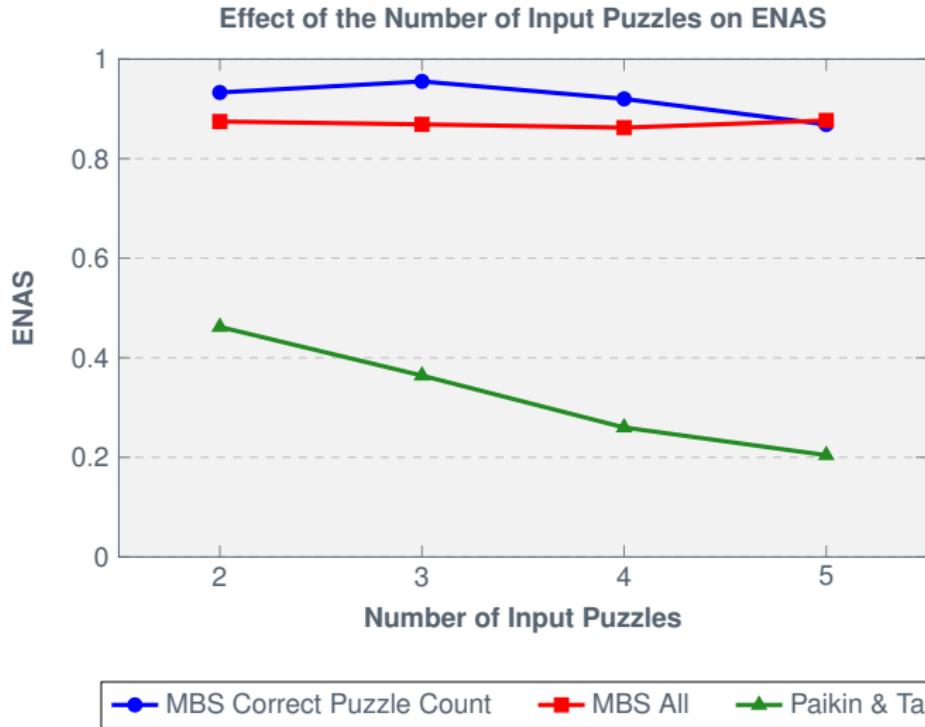
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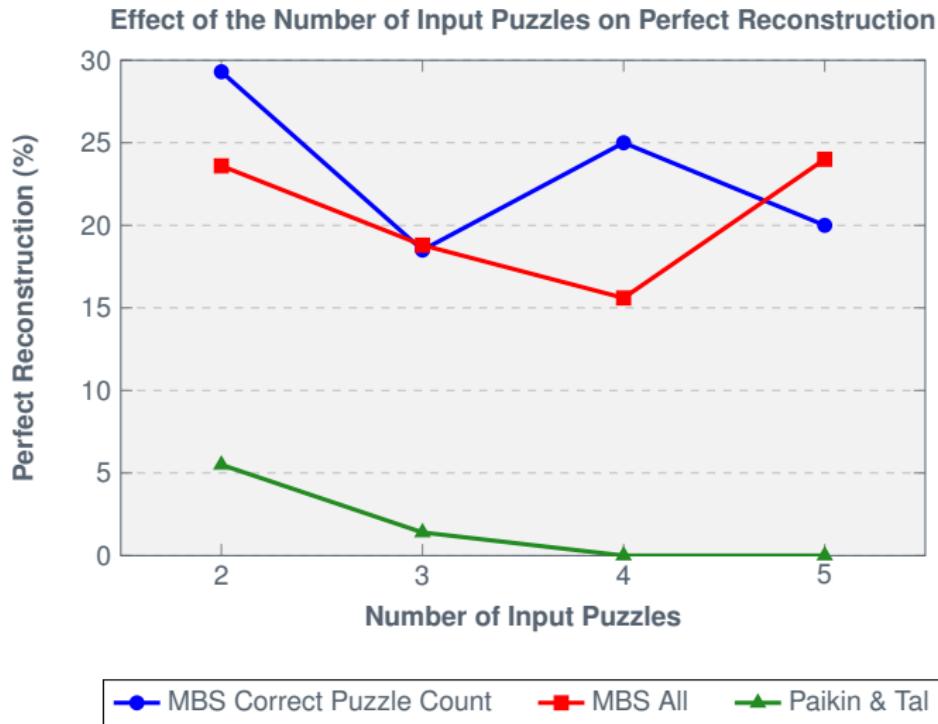
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- ▶ **Summary:** The Mixed-Bag Solver significantly outperformed Paikin & Tal's algorithm across all metrics.
 - ▶ This notwithstanding that only their algorithm was supplied with the number of input puzzles.



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- ▶ **Summary:** The Mixed-Bag Solver significantly outperformed Paikin & Tal's algorithm across all metrics.
 - ▶ This notwithstanding that only their algorithm was supplied with the number of input puzzles.
- ▶ **Puzzle Input Count:** Unlike Paikin & Tal's algorithm, the Mixed-Bag Solver saw no significant decrease in performance with additional input puzzles



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- ▶ **Summary:** The Mixed-Bag Solver significantly outperformed Paikin & Tal's algorithm across all metrics.
 - ▶ This notwithstanding that only their algorithm was supplied with the number of input puzzles.
- ▶ **Puzzle Input Count:** Unlike Paikin & Tal's algorithm, the Mixed-Bag Solver saw no significant decrease in performance with additional input puzzles
- ▶ **Effect of Clustering Errors:** Performance only decreased slightly when incorrectly estimated input puzzle count.
 - ▶ Many of the extra puzzles were relatively insignificant in size

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- ▶ This thesis presented a fully-automated solver for Mixed-Bag puzzles.
- ▶ Mixed-Bag Solver significantly outperforms the current state of the art while receiving no externally supplied information.
- ▶ Introduced the first set of solver quality metrics for Mixed-Bag puzzles.

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- ▶ Improved Assembler
 - ▶ Prioritize placement using multiple best buddies
 - ▶ Address placement performance in regions with low best buddy density

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- ▶ Improved Assembler
 - ▶ Prioritize placement using multiple best buddies
 - ▶ Address placement performance in regions with low best buddy density
- ▶ Dynamic determination of the segment clustering threshold

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- ▶ Improved Assembler
 - ▶ Prioritize placement using multiple best buddies
 - ▶ Address placement performance in regions with low best buddy density
- ▶ Dynamic determination of the segment clustering threshold
- ▶ Expanded stitching piece selection



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There are four primary jig swap puzzle types as formalized by [3]. In all cases, the “ground-truth” input is unknown.

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There are four primary jig swap puzzle types as formalized by [3]. In all cases, the “ground-truth” input is unknown.

- ▶ **Type 1:** Puzzle dimensions and piece rotation are known.
May have “anchor” piece(s) fixed in the correct location(s).

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There are four primary jig swap puzzle types as formalized by [3]. In all cases, the “ground-truth” input is unknown.

- ▶ **Type 1:** Puzzle dimensions and piece rotation are known.
May have “anchor” piece(s) fixed in the correct location(s).
- ▶ **Type 2:** All piece locations and rotations unknown. Puzzle dimensions may be known.



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- ▶ **Type 1:** Puzzle dimensions and piece rotation are known.
May have “anchor” piece(s) fixed in the correct location(s).
- ▶ **Type 2:** All piece locations and rotations unknown. Puzzle dimensions may be known.
- ▶ **Type 3:** All piece locations are known. Only rotation is unknown.



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There are four primary jig swap puzzle types as formalized by [3]. In all cases, the “ground-truth” input is unknown.

- ▶ **Type 1:** Puzzle dimensions and piece rotation are known.
May have “anchor” piece(s) fixed in the correct location(s).
- ▶ **Type 2:** All piece locations and rotations unknown. Puzzle dimensions may be known.
- ▶ **Type 3:** All piece locations are known. Only rotation is unknown.
- ▶ **Mixed-Bag:** Pieces come from multiple puzzles



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There are four primary jig swap puzzle types as formalized by [3]. In all cases, the “ground-truth” input is unknown.

- ▶ **Type 1:** Puzzle dimensions and piece rotation are known.
May have “anchor” piece(s) fixed in the correct location(s).
- ▶ **Type 2:** All piece locations and rotations unknown. Puzzle dimensions may be known.
- ▶ **Type 3:** All piece locations are known. Only rotation is unknown.
- ▶ **Mixed-Bag:** Pieces come from multiple puzzles

Mixed-Bag puzzles are the focus of this thesis.



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- ▶ **Cho et al.** [9] – Introduced the first modern jig swap puzzle solver
 - ▶ Graphical model-based Type 1 solver
 - ▶ Puzzle dimensions are known
 - ▶ Used one or more anchor pieces
 - ▶ Defined quality metrics for Type 1 and Type 2 puzzles
 - ▶ Established the standard comparative test conditions

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- ▶ **Cho et al.** [9] – Introduced the first modern jig swap puzzle solver
 - ▶ Graphical model-based Type 1 solver
 - ▶ Puzzle dimensions are known
 - ▶ Used one or more anchor pieces
 - ▶ Defined quality metrics for Type 1 and Type 2 puzzles
 - ▶ Established the standard comparative test conditions
- ▶ **Pomeranz et al.** [10] – Iterative, greedy Type 1 puzzle solver
 - ▶ Eliminated the use of anchor pieces
 - ▶ Created multiple solver benchmarks of various sizes



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- ▶ **Basis of all Modern Jig Swap Solvers:** The more compatible two pieces are, the more likely they are to be adjacent.
- ▶ **Best Buddies:** A pair of puzzle pieces that are more compatible with each other on their respective sides than they are to any other piece [10]
 - ▶ **Note:** Not all puzzle pieces will have a best buddy.

$$\forall p_k \forall s_z, C(p_i, s_x, p_j, s_y) \geq C(p_i, s_x, p_k, s_z)$$

and

(10)

$$\forall p_k \forall s_z, C(p_j, s_y, p_i, s_x) \geq C(p_j, s_y, p_k, s_z)$$

- ▶ **Importance of Best Buddies:** Key adjacency indicator



Quantifying Solver Quality

Best Buddy Density

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- ▶ **Best Buddy Density (BBD):** A metric for quantifying the best buddy profile of an image that is independent of image size.

$$BBD = \frac{b}{n \cdot q} \quad (11)$$

- ▶ A greater BBD means the pieces are more differentiated making the puzzle easier to solve.

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Best Buddy Density

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Visualizing Best Buddy Density

- ▶ Transform each puzzle piece into a square consisting of four isosceles triangles.
- ▶ Color each triangle according to whether the adjacent piece is a best buddy. The scheme used in this thesis:

No Best Buddy	Non-Adjacent Best Buddy	Adjacent Best Buddy	No Piece Present

- ▶ Areas with higher best buddy density will have more green triangles.



Best Buddy Density

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Original Image [11]



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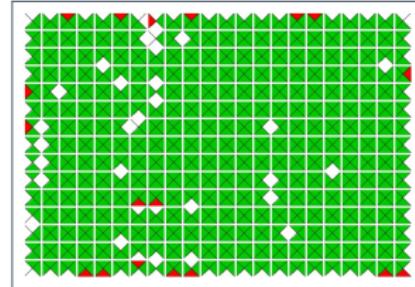
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Original Image [11]



Best Buddy Visualization



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Perfectly Reconstructed
Image (a)



Misclassified Image (b)



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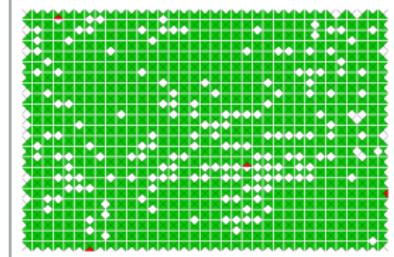
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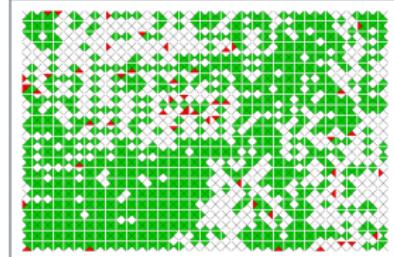
Perfectly Reconstructed
Image (a)



Best Buddy Visualization (a)



Misclassified Image (b)



Best Buddy Visualization (b)



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- ▶ As the number of puzzles increases, the difficulty of simultaneously reconstructing them also increases.



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- ▶ As the number of puzzles increases, the difficulty of simultaneously reconstructing them also increases.
- ▶ **Current State of the Art:** Paikin & Tal [4] solved up to five puzzles simultaneously.



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- ▶ As the number of puzzles increases, the difficulty of simultaneously reconstructing them also increases.
- ▶ **Current State of the Art:** Paikin & Tal [4] solved up to five puzzles simultaneously.
- ▶ **Goal:** Compare the performance of the Mixed-Bag Solver and Paikin & Tal's algorithm on 10 puzzles.



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► Paikin & Tal

- ▶ Seed of nine images came from just three input images
- ▶ SEDAS and EDAS greater than 0.9 for only one image
- ▶ No perfectly reconstructed images



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Experimental Results

Single Input Puzzle

Ten Puzzle Results

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► Paikin & Tal

- ▶ Seed of nine images came from just three input images
- ▶ SEDAS and EDAS greater than 0.9 for only one image
- ▶ No perfectly reconstructed images

► Mixed-Bag Solver

- ▶ SEDAS and EDAS greater than 0.9 for all images
- ▶ Four images perfectly reconstructed
- ▶ Results comparable to Paikin & Tal's algorithm solving each puzzle individually