TracIn

Single Step:

$$\ell(w_{t+1}, z') = \ell(w_t, z') + \nabla\ell(w_t, z') \cdot (w_{t+1} - w_t) + O(||w_{t+1} - w_t||^2)$$
$$\ell(w_t, z') - \ell(w_{t+1}, z') \approx -\nabla\ell(w_t, z') \cdot (w_{t+1} - w_t)$$
$$\ell(w_t, z') - \ell(w_{t+1}, z') \approx \eta_t \nabla\ell(w_t, z') \cdot \nabla\ell(w_t, z) \because w_{t+1} - w_t = -\eta_t \nabla\ell(w_t, z)$$

All Steps:

$$\text{TracIn}(z, z') = \sum_t \eta_t \nabla\ell(w_t, z') \cdot \nabla(w_t, z)$$

BoostIn

$$\text{BoostIn}(z, z') = \sum_t I[P(z')_t = P(z)_t] \left(-\frac{\partial L(y', f_t(z'))}{\partial f_t(z')} \cdot (\alpha^t_{P(z')_t} - \alpha^t_{P(z')_t}[\setminus z]) \right)$$
$$= \boxed{\sum_t I[P(z')_t = P(z)_t] \left(\eta_t \frac{\partial L(y', f_t(z'))}{\partial f_t(z')} \cdot \frac{\partial \alpha^t_{P(z')_t}}{\partial z} \right)},$$
$$\frac{\partial \alpha^t_{P(z')_t}}{\partial z} = \frac{g^t_z + h^t_z \alpha^t_{P(z')_t}}{\beta^t_l}$$

LeafInfluence (Single Point)

$$\text{LeafInf}(z, z') = \boxed{-\left(\frac{\partial L(y', f_T(z'))}{\partial f_T(z')} \right) \left(\sum_t I[P(z')_t = P(z)_t] \cdot \frac{\partial \alpha^t_{P(z')_t}}{\partial z} \right)}$$
$$\frac{\partial \alpha^t_{P(z')_t}}{\partial z} = \frac{(g^t_z + h^t_z \alpha^t_{P(z')_t}) + (h^t_z + k^t_z \alpha^t_{P(z')_t}) J^{t-1}_z}{\beta^t_l}$$
$$J^t_z = J^{t-1}_z + \frac{\partial \alpha^t_{P(z)_t}}{\partial z}$$

Notation

z, z' = Train and test examples, respectively.

$P(z)_t$ = Leaf that z ends at for tree t.

α_l^t = Leaf value for leaf l at tree t.

β_l^t = Sum of 2nd derivatives for leaf l in tree t.

g_z^t, h_z^t, k_z^t = 1st, 2nd, and 3rd derivatives for example z in tree t, respectively.

J_z^t = Prediction derivative accumulator; cum. sum for example z at tree t.