Investigation of Gamma-ray Coincidence and Angular Correlation

Zayd Pandit (PNDZAY001), PHY3004W
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***Abstract*—We investigate the various properties of coincident gamma rays from ^{60}Co and ^{22}Na. We measure the angular correlation for coincident 0.511 MeV gamma-rays from our ^{22}Na source. We also determine the absolute efficiency of our Rexon 50mm x 50mm NaI(Tl) scintillator detector and the activity of ^{60}Co using our knowledge of the coincident gamma-rays of our two sources.**

I. Introduction

THE positron emitted during β^+ decay of ^{22}Na annihilates with electrons in the sample, which then produces a pair produces two $511keV$ photons, travelling in opposite directions[1]. ^{60}Co decays via β^- emission, during which a $1332.5keV$ and a $1173.2keV$ gamma-ray get emitted[1][2], in any direction.

II. Method and Experimental Setup

The modules needed for the "NIM" crate are:

- 2 x Ortec 556 HV power supplies
- 2 x Ortec 113 Preamplifiers(PA)
- 2 x 590A Amplifier (AMP)/ timing single channel analyser (TSCA) combo units
- 2 x Ortec 427A delay amplifiers (DA)
- 2 x Rexon 50mm x 50mm NaI(Tl) scintillator detectors
- 1 x Ortec 416A gate and delay
- 1 x Ortec 418A Unniversal coincidence (UCO) unit
- 1 x Ortec 974 quad counter/timer

We set up two detector channels, red (detector 1) and blue (detector 2). For each of these two channels we use a power supply, a PA, an AMP & TSCA, and a delay amplifier.

The detectors are setup opposite each other as shown in Figure 1

The pulses are directed into the UCS30 ADC/multichannel analyser where they are sorted and converted into a pulse height spectrum.

The spectra are measured for ^{60}Co, ^{22}Na and ^{137}Cs and these are used for the calibration. The linear fitting of the energy calibration are shown in Appendix A.

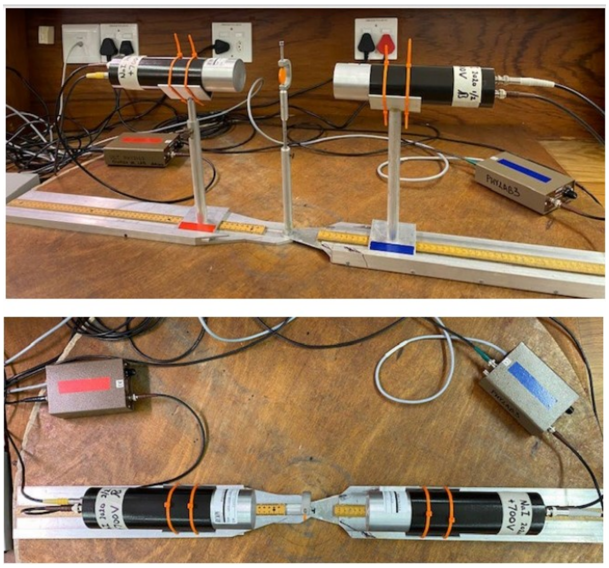

Fig. 1. The detectors setup on opposite sides of the radioactive source

	Detector 1 (red)	Detector 2 (blue)
HV	+600 V	+600 V
PA capacitance	$1000\mu F$	$1000\mu F$
AMP course gain	X 20	X 20
AMP fine gain	0.95	0.80
TSCA lower level	0.10	0.10
TSCA window	10.00	10.00

TABLE I
TABLE OF MODULE SETTINGS

A. Part 1: Coincident Spectra of 60Co and 22Na

We will "gate" the acquisition system. This means that the UCS30 unit will record an event only if the unit receives a 5V signal from the TSCA pulse on the gate line. The TSCA produces a small, narrow in time ($500ns$), 5V pulse, which we then send to the GDG to be stretched (to over $3\mu s$). The stretched signal can be used as a gate on the UCS30 unit, but only if timed correctly. We adjust the timing by delaying the pulses by $2.5\mu s$ such that they overlap with the gate pulse.

We recorded counts of ^{60}Co and ^{22}Na with 3 different setups to compare, we call these setups Scan 1, Scan 2 and Scan 3.
For Scan 1, the TSCA settings on the blue detector were set to be as wide as possible.
For Scan 2, the TSCA window is set tightly around the 0.511 MeV photopeak for the case of ^{22}Na and the 1.333 MeV

photopeak for the case of ^{60}Co. If the incoming photon has an energy allowed by the gating, the signal is recorded.
For Scan 3, the UCS30 is gated on the blue detector with the window is set tightly around the 0.511 MeV photopeak for ^{22}Na and 1.333 MeV photopeak for ^{60}Co. At the moment the blue detector receives a photon of an energy which is allowed by the gating, we record the signal from the red detector.

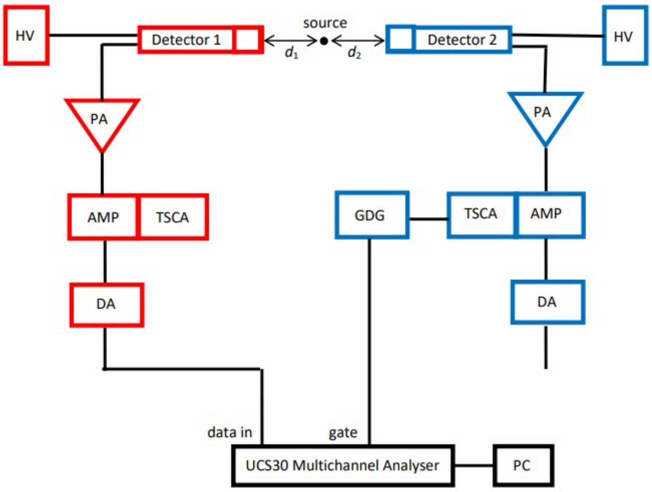

Fig. 2. The diagram depicting the setup for Scan 3

B. Part 2: Measurement of Angular Correlation

We will measure the angular correlation function, $P(\theta)$. This is the relative probability of two gamma's being emitted at a relative angle between the detectors The two detectors are placed opposite each other from the source - this is the 0°

We add the universal coincidence unit (UCO) to the outputs of the TSCA's. If the UCO receives two input pulses which overlap in time, it outputs a simple logic pulse of 5V, 500ns. We then add a scaler unit to count the logic pulses from each detector and the logic pulses from the UCO.

We then record all the pulses from the two TSCA's (we call N_1 and N_2) and the number of coincidences counted by the scaler (we'll call N_{co}).

We introduce the ^{22}Na source and vary the relative angle between the two detectors, recording N_1, N_2 and N_{co} at each angle

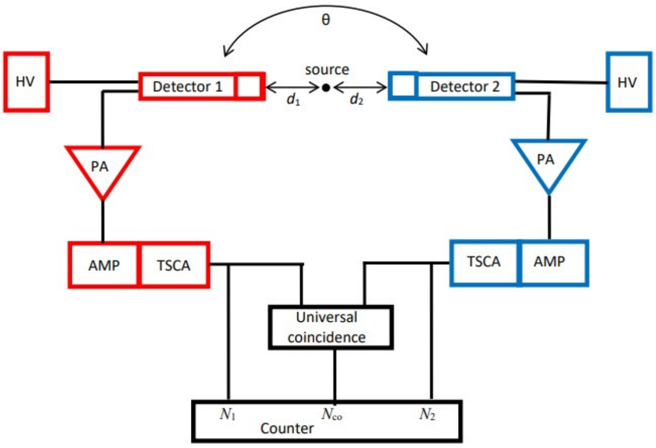

Fig. 3. A diagram depicting the experimental setup for Part B, for measuring the angular correlation

Scans	d_1	d_2	TSCA windows
1	15.0	15.0	Both wide open
2	15.0	15.0	Both tight on 0.511 MeV photopeaks
3	15.0	30.0	Both wide open

TABLE II
DETAILS OF THE DIFFERENT SCANS FOR USE AND COMPARISON IN PART B

C. Part 3: Absolute Efficiency of the Detector

We will determine $\epsilon_1(0.5)$, the absolute efficiency of detector 1 for detecting a 0.511 MeV photon moving axially through the detector. We will then determine its efficiency for detecting a 0.511 MeV photon or the Compton scatter events inside the detector as a result of the 0.511 MeV photon.

We position the detectors opposite each other, with detector 1 (red) at 8.0 cm away from the ^{22}Na source and detector 2 (blue) at 24.0 cm away from the source.

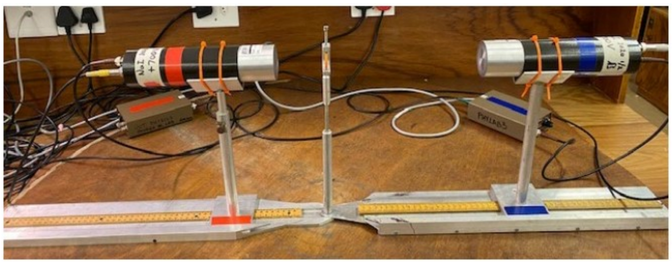

Fig. 4. The experimental setup of the detectors for Part C, with detector 1 (red) at 8.0 cm away from the ^{22}Na source and detector 2 (blue) 24.0 cm away.

D. Part 4: The Activity of Our 60Co Source

The count rate from Detector 1 is:

$$R_1 = D\epsilon_1\Big(\frac{\Omega_1}{4\pi}\Big)$$

Similarly, the count rate from Detector 2 is:

$$R_2 = D\epsilon_2\Big(\frac{\Omega_2}{4\pi}\Big)$$

We know that during the β^- decay of ^{60}Co, a single 1.333 MeV gamma ray is emitted for every time a 1.173 MeV gamma is emitted. If detector 1 detects a gamma ray then the probability that detector 2 will detect the coincident gamma ray will be determined by the fractional solid angle of the detector at its set distance multiplied by the efficiency of the detector:

$$\epsilon_2\left(\frac{\Omega_2}{4\pi}\right)$$

Thus if the count rate measured by detector 1 is R_1 then the coincident rate will be:

$$R_{co} = R_1\epsilon_1\left(\frac{\Omega_2}{4\pi}\right)$$

Substituting for $\epsilon_2\Omega_2$ we find $R_c = \frac{R_1 R_2}{D}$. We therefore get the activity of our source to be:

$$D = \frac{R_1 R_2}{R_{co}} \tag{1}$$

E. Part E: Investigation of the counts as a function of distance from the source

The fractional solid angle subtended from the source to the face of the detector decreases by the square of the distance between the source and the detector (d).

$$\Omega = \frac{A}{r^2} \tag{2}$$

where A is the surface area of the sphere with radius r. We thus expect an inverse square relationship between the detected counts and d.

III. Results and Discussion

A. Part 1: Coincident Spectra of 60Co and 22Na

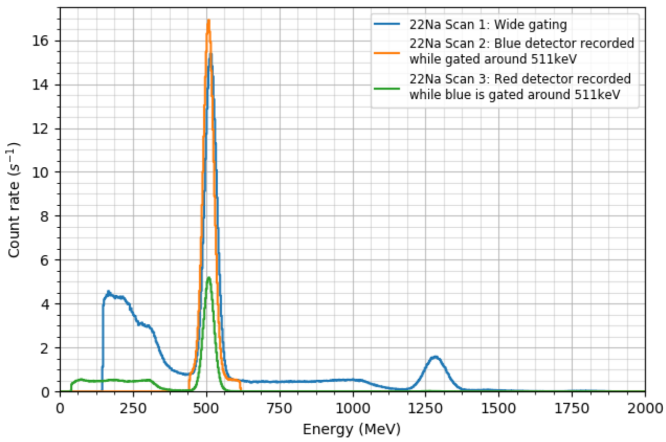

Fig. 5. The coincident spectra of 22Na. For the three different scans

Since Scan 1 has a wide gating on the TSCA, its spectra is basically the complete spectrum of ^{22}Na.
For Scan 2, we set the TSCA window tight around the 0.511 MeV photopeak. Thus all the recorded signals were only those of inputs around 0.511 MeV - as confirmed by the data. The photopeak is slightly taller in Scan 2 than Scan 1. This is unexpected, as the count rate for the 0.511 MeV photons should be constant, no matter the exclusion of the rest of the energies. This could be due to slight error in the experimental setup, such as the movement of the source or detectors between Scan 1 and Scan 2.
For Scan 3 we recorded the input in the red detector when the blue detector received input corresponding to a 0.511 MeV photon. We thus expect the red detector to record, not only 0.511 MeV photons, but Compton events due to 0.511 MeV photons, and other background events which happen to enter the red detector at the time that the blue detector receives the 0.511 MeV photon. This matches the results we observe in Scan 3 perfectly.

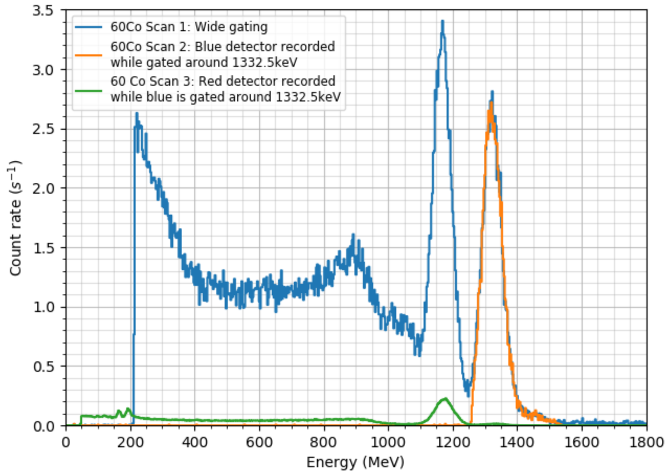

Fig. 6. The coincident spectra of 60Co and 22Na. For the three different scans

Scan 1 has the complete spectrum of ^{60}Co, as expected.
For Scan 2, we set the TSCA window tight around the 1.333 MeV photopeak. Thus we expected that Scan 2 should contain all the 1.333 MeV events - Figure 6 confirms this perfectly.
For Scan 3 we recorded the input in the red detector when the blue detector received input corresponding to a 1.333 MeV photon. We know that during the β^- decay of ^{60}Co, a 1.333 MeV and 1.173 MeV photon are emitted basically instantaneously. These are emitted isotropically, so we expect a small photopeak at 1.173 MeV, none at 1.333 MeV and as well as Compton events and other gamma's entering the red detector at the same time as the blue detector received the 1.333 MeV photon. All of these features are observed clearly in Figure 6.

B. Part 2: Measurement of Angular Correlation

We expected that the angular correlation should roughly fit the Gaussian distribution, due the fact that we expect the photon position to be at $\theta = 0$, but its recorded position is a naturally random variable. This argument holds on the premise of the applicability of the Central Limit Theorem - which says that the sum of independent random variables will tend towards a normal distribution.

Scan 2 has the TSCA windows tight on the 0.511 photopeak for both the red and blue detector channels. This means that we can be certain that the recorded data are only 0.511 MeV photons. Due to conservation of momentum, the pair-produced gamma rays should be going in exactly opposite directions.

Thus we expect that the events recorded should have less non pair-produced gammas and thus the distribution will be narrower than the distribution of Scan 1.

For Scan 3 we moved the second detector to a distance of 30cm away from the source. For an event to be triggered in this setup, we need that both detectors receive a signal simultaneously, if the one detector is further away, the solid angle in which the second photon can be in to be detected is much smaller - thus, we are much more likely to only have events recorded which are due to the pair production (since these have photons move in exactly opposite directions).

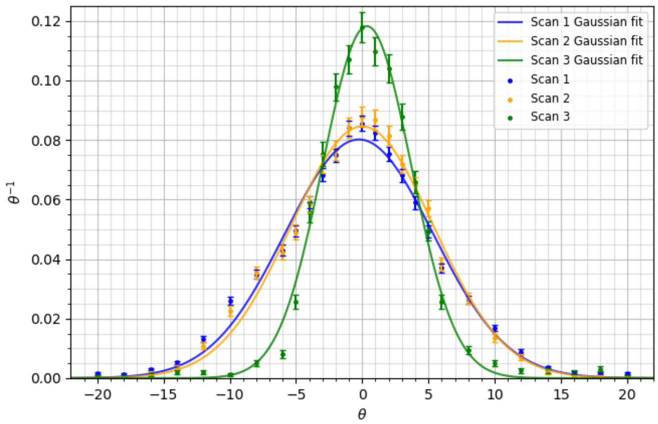

Fig. 7. The probability distribution for angular correlation for Scan 1, Scan 2 and Scan 3. A weighted fit on the data is done into a Gaussian distribution

The fitted Gaussian characteristic width's were:
Scan 1: 5.58562
Scan 2: 5.36554
Scan 3: 3.33651

Figure 7 shows the data for all 3 scans fit the Gaussian distribution brilliantly.

The probability angular distribution for Scan 2 as seen in Figure 7 is only slightly narrower than that of scan 1. The difference is not large enough to conclude any significant change in angular correlation. In a further investigation we could redo Scan 1 and Scan 2 and perhaps include another weighted fitting in which the uncertainty of the fit parameters can be accurately calculated.

The probability angular distribution for Scan 3 is far narrower than that of Scan 2 and Scan 1. This shows that the events recorded were much more likely to be pair-produced 0.511 MeV photons than in Scan 1 and 2, where we'd have a lot more events of similar energy and occurring at the same time, but not due to the pair-production.

C. Part 3: Absolute Efficiency of the Detector

N_1	253901	Both TSCA windows wide open
N_2	79100	
N_{co}	12607	
N_1	115940	Tight window on 0.511 MeV photopeaks on both TSCAs
N_2	32444	
N_{co}	7871	
N_1	258432	Tight window on 0.511 MeV photopeaks on TSCA 2 (blue) only
N_2	31578	
N_{co}	12236	

TABLE III
MEASURED COUNTS

Since we know that pair production occurs after the β^+ decay in ^{22}Na, we expect that when a 0.511 MeV photon (γ_1) enters the blue detector, 0.511 MeV photon (γ_2) is entering the red detector. Bringing the red detector closer to the source than the blue detector further guarantees that γ_2 enters the red detector. By comparing the number of events we measure in the second detector out of the number of photons we believe to have entered it, we can determine its absolute efficiency.

In Scan 2, we have a tight window around 0.511 MeV photopeaks on both TSCA's. Here N_2 will be the number of counts in which detector 2 (blue) detects 0.511 MeV photon, and we'll expect that the same number of photons entered detector 2. Since we're gating detector 1, N_{co} will be the number of times 0.511 MeV photons were detected in both. Thus the efficiency of detector 1 (red) at detecting 0.511 MeV photons will be:

$$\epsilon_1(0.5) = \frac{N_{co}}{N_2} = 0.24260 \pm 0.01236 \tag{3}$$

If we use Scan 3, we have that N_2 is once again the number of 0.511 MeV photons entering detector 2. But this time, N_{co} will count all the events in which a 0.511 MeV photon was detected in detector 2 (blue) while another event was detected in the detector 1 (red). Unlike in Scan 2, Scan 3's N_{co} will include events like Compton scatter. This means that using Scan 3 we can determine the efficiency of detector 1 for detecting 0.511 MeV photons and the Compton scatter associated with 0.511 MeV photons. This efficiency will be:

$$\epsilon_1(0.5 + Compton) = \frac{N_{co}}{N_2} = 0.38748 \pm 0.01065 \tag{4}$$

D. Part 4: The Activity of Our ^{60}Co Source

Measurement	Total counts	Count rate (s^{-1})
N_1	231123	192.603
N_2	223368	186.140
N_{co}	4582	3.818

TABLE IV
MEASURED COUNTS AND COUNT RATE FROM DETECTOR 1 (N_1), FROM DETECTOR 2 (N_2) AND COUNTS WHICH OCCURRED SIMULTANEOUSLY (N_{co})

Substituting the above data into (1), gives us our desired result for the activity of our ^{60}Co source.

$$D = \frac{R_1 R_2}{R_{co}} = 9390.02 \pm 4912.73\ Bq \tag{5}$$

E. Part E: Investigation of the counts as a function of distance from the source

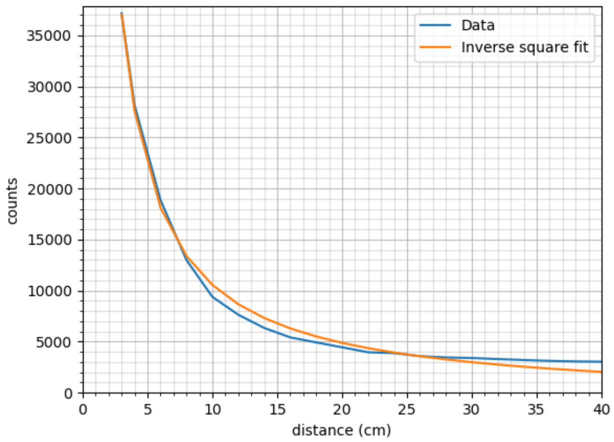

Fig. 8. The number of counts detected as function of the distance between the source and the detector. An inverse square function is fitted with a scaling and vertical shift parameter. The Chi square per degree of freedom is calculated to be 30.68

When an inverse square function is fitted to the data, the fit is fairly good, but seems to have a fundamentally different shape - the data appears to have some decreasing exponential quality. This could be due to attenuation of photons through the air, as the attenuation is exponential as a function of distance. This disparity could also be due to exclusion of the background in this simplistic model.

The extremely high chi-square statistic also suggests that our model does not match our data well.

IV. Conclusion

For Part A we plotted the spectra of ^{60}Co and ^{22}Na 3 scans with different TSCA gating settings. We found that the the data matched exactly the behaviour we expected apart from Scan 2 of ^{22}Na having a higher count rate at 0.511 MeV than Scan 1.

For Part B we found that the angular distribution of coincident gamma rays were normally distributed. We also found that Scan 2, which gated the TSCAs to allow only0.511 MeV photons did not result in a significant narrowing of the angular correlation probability function. We found that moving detector 2 to twice the distance from the source than it was originally significantly decreased the characteristic width of the angular correlation probability distribution - and thus we can say that it recorded a higher proportion of true pair-production events than the other two scans.

For Part C we estimated the absolute efficiency of detector 1 for detecting 0.511 MeV photons axially through the detector to be $\epsilon_1(0.5) = 0.24260 \pm 0.01236$. We also found estimated the absolute efficiency of detector 1 for detecting 0.511 MeV photons and its associated Compton events to be $\epsilon_1(0.5 + Compton) = 0.38748 \pm 0.01065$.

For Part D we calculated the activity of our ^{60}Co source to be $D = 9390.02 \pm 4912.73\ Bq$

In Part E we attempted to model the counts detected as function of distance from the source using an inverse-square model. We reject this hypothesis due to the model not fitting the data sufficiently. We suggest further investigation and a more complete model including the subtraction of background and modelling of the attenuation of photons through the air.

V. Acknowledgements

I'd like to acknowledge the contributions of Joshua Browne and Samson Thornhill for their help with understanding of the investigation

References

[1] Brookhaven National Laboratory NNDC. *Nudat 2.8*. URL: https://www.nndc.bnl.gov/nudat2/.

[2] Steven Sesselmann. *Gamma Spectrum Database*. URL: https://www.gammaspectacular.com/blue/gamma_spectra.

VI. Appendix

A.

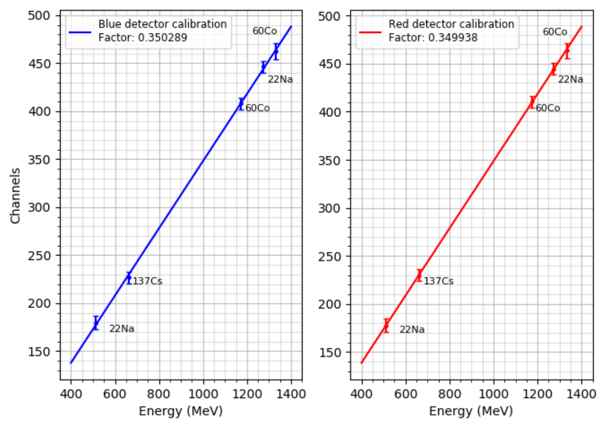

Fig. 9. Weighted linear calibration fit