COMPARATIVE ANALYSIS OF TWO-DIMENSIONAL STEADY STATE HEAT CONDUCTION IN A SLAB USING METHOD OF LINES AND ANSYS

B. Tech Dissertation

Submitted in partial fulfillment of the requirements
for the award of the degree of

BACHELOR OF TECHNOLOGY IN CHEMICAL ENGINEERING

Submitted by

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DECLARATION CERTIFICATE

This is to certify that the thesis entitled **"COMPARATIVE ANALYSIS OF TWO-DIMENSIONAL STEADY STATE HEAT CONDUCTION IN A SLAB USING METHOD OF LINES AND ANSYS" SUBMITTED BY BHARGAV SAHUKARU (N161259), L.N.LAVANYA BHOGADI (N160910), BHARGAV BATTULA (N160153), SRI DEEPIKA CHALLA (N160597), MOHAN KUMAR SATIVADA (N160208), N.DIVYA (N161198)** to the Department of Chemical Engineering, Rajiv Gandhi University of Knowledge Technologies, Nuzvid for the submission of major project report in IV year BTech in Chemical Engineering is a bonafide work carried out under our supervision and guidance during the academic year 2021-2022. This report is, in our opinion, is worthy of consideration for the credits of Major project for IV-year B-Tech in Chemical Engineering in accordance with the regulations of the university.

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DISSERTATION APPROVAL CERTIFICATE

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External Examiner

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NOMENCLATURE

S. No	SYMBOL	Name	Units
1.	k	Thermal conductivity	W/m.°C
2.	T	Temperature of the slab	°C
3.	h	Heat Transfer Coefficient	W/ m^2.°C
4.	U	Eigenvectors of solution matrix	-
5.	V	Eigenvectors of Transpose matrix	-
6.	n	No of lines	-

ABSTRACT

Heat transfer is an important problem in many disciplines including science and engineering. The present work focuses on review of analytical and semi-analytical approaches to solve two-dimensional steady-state heat conduction problem. Two-dimensional steady state conduction is governed by a second order partial differential equation. A solution must satisfy the differential equation and four boundary conditions. Analytical solutions are available only for a very few simple boundary conditions and these are not amenable for complex boundaries. The method of lines is a technique for solving partial differential equations (PDEs) in which all but one dimension is discretized. In this study an analytical approach to a two-dimensional slab with steady state heat conduction under different boundary conditions is considered. The complete description of heat flow through slab by solving obtained partial differential equations using method of lines method has been presented in this study. The temperature distribution profiles in the slab by using method of lines were plotted and compared with the profiles of analytical solution by using MATLAB. The steady state analysis of temperature distribution of heat conduction in a slab with specified Dirichlet boundary conditions was developed by using Method of lines. From the profiles it is observed that as the number of lines is increasing, the error between analytical and method of lines (semi analytical) solutions has been decreased, and the profiles of analytical and MOL converged which indicates the preference of higher number of lines in order to obtain accurate values (with minimal errors).

Key words: Analytical approach, Method of lines, partial differential equations, heat conduction, steady state.

CHAPTER-1

INTRODUCTION & METHODOLOGY

Our physical world is most generally described in scientific and engineering terms with respect to the analysis of unsteady, heat conduction, one dimensional,2-dimensional and 3-dimensional space and time which are abbreviated as space time. PDEs provide a mathematical description of physical space time, and they are therefore among the most widely used forms of mathematics. PDEs play an important role in the mathematical modelling of engineering systems, particularly chemical engineering systems. PDE models, which are derived from mass, energy, and momentum balances, usually describe space and time variations of the state variables that can occur for various reasons e.g., non-ideal mixing conditions in batch reactors, heat and mass transfer in packed columns etc. And hence solving PDEs with ease and efficiency is of great interest in recent times.

A heat transfer chemical engineering problem i.e., Analysis of two-dimensional steady state heat conduction in a slab under different boundary conditions was taken and solved using MOL. The MOL embodies a hybrid procedure for solving PDEs in one or multiple dimensions in a semi-analytical manner. The MOL is a semi analytical or semi-discrete numerical method to calculate accurate numerical solutions of PDEs.

Analytical solution:

Analytical solutions are mathematical functions that define the dependent variable as a function of the independent variables. They are exact and generally difficult to derive mathematically for all but the simplest PDE problems. For PDEs as the non-homogeneity in boundary conditions increases the complexity in evaluation of analytical solution increases. But in real life problems and in chemical engineering systems mostly non-homogeneous boundary conditions exist as homogeneous boundary conditions are complex to maintain. As a consequence, alternative Numerical Difference (effective) methods for the solution of PDEs, such as MOL, are of broad interest in science and engineering.

Method of lines (Semi-Analytical Solution):

MOL discretizes the space derivative while leaving the time derivative continuous. Using MOL, the adjoint system of linear, first order ordinary differential equations are solved analytically with the eigenvalue method. The outcome of the computational procedure provides a discrete sequence of piecewise temperatures-time variations at each line, which is expressed in terms of linear combinations of exponential functions of time containing the eigenvalues and eigenvectors.

The MOL embodies a hybrid procedure for solving partial differential equations in one- or multiple dimensions in a semi-analytical manner. The MOL is a semi analytical or semi-discrete numerical method to calculate accurate numerical solutions of PDEs. The main motivation inherent to MOL is that an adjoint system of ordinary differential equations is easier to solve than the original partial differential equation. In general, the required algebraic approximation of the spatial derivatives in the PDEs can be based on finite differences, finite elements, collocations or Fourier series expansions. The MOL replaces the spatial derivatives in the PDE with algebraic approximations (u_t is subscript notation for $\frac{\partial u}{\partial t}$). Once this is done, the spatial derivatives are no longer stated explicitly in terms of the spatial independent variables. Thus, in effect only the initial value variable time remains. In other words, with only one remaining independent variable, the system of Ordinary Differential Equations (ODEs) approximates to the original PDE. Then formulating the approximating system of ODEs is done. Once this is done, we can apply any integration algorithm for initial value ODEs to compute an approximate numerical solution to the PDE.

$$u_x = \frac{u_i - u_{i-1}}{\Delta x}$$

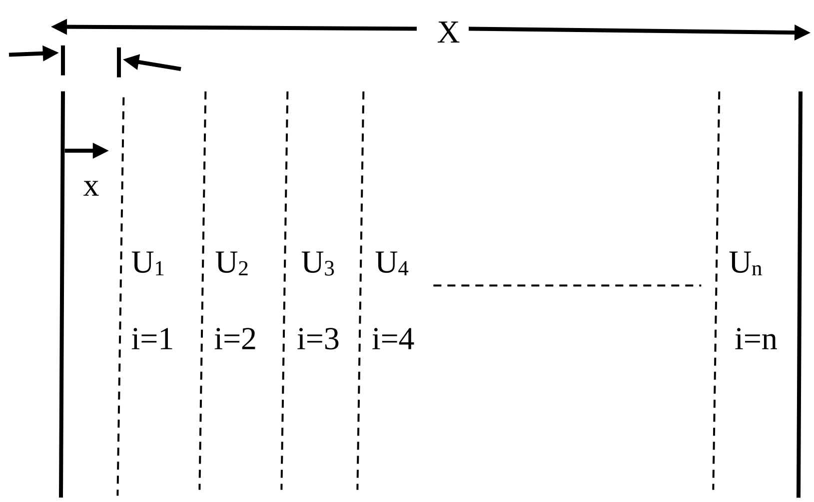

Fig.1.1. Schematic Diagram of MOL analysis

The MOL is the use of existing, and generally well established, numerical methods for ODEs. As to convert the spatial derivative 'u_x' with an algebraic approximation the finite difference (FD) is used.

Where 'i' is an index designating a position along a grid in x and Δx is the spacing in x along the grid, assumed constant for the time being. Thus, for the left end value of x, i=1, and for the right end value of x, i = N, i.e., the grid in x has N points.

The MOL is:

$$\frac{du_i}{dt} = \frac{u_i - u_{i-1}}{\Delta x} \qquad 1 \le i \le N.$$

For further calculation of initial conditions and boundary conditions are needed. As errors also count apart, truncation error, from a truncated Taylor series is also added.

$$u_x = \frac{u_i - u_{i-1}}{\Delta x} + 0\ (\Delta x)$$

Where O(Δx) denotes order Δx i.e., truncation error. The numerical integration of the MODEs of equations. If the derivative $\frac{du_i}{dt}$ is approximated by a first order FD:

$$\frac{du_i}{dt} \approx \frac{u_i^{n+1} - u_i^n}{\Delta t} + 0\ (\Delta t)$$

Where n is an index for the variable t (t moves forward in steps denoted or indexed by n).

Hence, $$\frac{u_i^{n+1} - u_i^n}{\Delta t} = \frac{u_i^n - u_{i-1}^n}{\Delta x}$$

Solve for u_i^{n+1} explicitly can solve for the solution at the advanced point in t, n+1, from the solution at the base point n. This procedure is generally termed the forward Euler method which is the most basic form of ODE integration. The collection of analytical/numerical temperature-time solutions provided by MOL and the eigenvalue method exhibits excellent quality at all times. MOL as a numerical tool is suitable for many other types of PDEs, e.g., second order hyperbolic, elliptic and mixed hyperbolic-parabolic. Due to wide applications of MOL to various real-life problems there are numerous solvers and libraries in Mathematics like MATLAB, Maple C, Fortran.

ANSYS:

The field of chemical engineering is in constant change, so are available calculation tools and software packages. In fast everyday life, it is a considerable challenge for a chemical engineer to know which tool can serve best for solving a certain problem. Different packages can be applied to solve typical problems in mass and energy balance, fluid mechanics, heat and mass transfer, unit operations, reactor engineering, and process and equipment design and control.

ANSYS is an American company based in Canonsburg, Pennsylvania. It develops and markets multi physics engineering simulation software for product design, testing and operation. Ansys is general-purpose, finite element modeling package for numerically solving a wide variety of mechanical problems. Ansys product solutions include 3D design, structural analysis, fluid dynamics, electronics simulation, optics and light simulation, materials intelligence etcAnsys develops and markets engineering simulation software for use across the product life cycle. Ansys mechanical finite element analysis software is used to simulate computer models of structures, electronics, or machine components for analyzing the strength, toughness, elasticity, temperature distribution, electromagnetism, fluid flow and other attributes.

Ansys simulation solutions enable materials and chemical process companies to dramatically improve overall equipment effectiveness, capacity and raw material utilization, resulting in more efficient operations and reduced costs. From equipment and processes to chemical and petrochemical refining to glass, polymer and metals manufacturing, Ansys simulation solutions accelerate for tomorrow's advanced material systems. Engineering simulation enables optimization of even the most complex and resource-intensive processes, minimizing their environmental impact of waste water discharges.

Applications:

1. Ansys fluid mixing simulation tools are used to model the mixing process and blending of one or more fluid like materials.

2. Ansys multi physics simulation enables to efficiently scale up gas-liquid process equipment to maximize yield and improve efficiency.

3. Also enables the design and optimization of technologies that reduce emissions, energy use and waste.

4. Ansys enables heat exchanger design optimization for energy efficiency, material selection, and life prediction under harsh operating conditions.

CHAPTER-2

LITERATURE REVIEW

Paper-1:

Title: Heat conduction simulation of 2D moving Heat source problems using a moving mesh method

Authors: Zicheng Hu and Zhihui Liu

Published Year: 2020

Numerical investigation of two-dimensional heat conduction problems of material subjected to multiple moving Gaussian point heat sources. All heat sources are imposed on the inside of material and assumed to move along some specified straight lines or curves with time-dependent velocities. A simple but efficient moving mesh method, which continuously adjusts the two-dimensional mesh dimension by dimension upon the one-dimensional moving mesh partial differential equation with an appropriate monitor function of the temperature field, has been developed. The physical model problem is then solved on this adaptive moving mesh. Numerical experiments are presented to exhibit the capability of the proposed moving mesh algorithm to efficiently and accurately simulate the moving heat source problems.

Paper-2:

Title: Semi-analytical method of lines for solving elliptic partial differential equations.

Authors: Venkat Subramanian

Published Year: 2003

In this paper, they presented a method for solving Laplace's equation using a semi analytical method of lines. This method consists of using a central difference approximation for the second-order derivative in one of the spatial directions followed by solving analytically the resulting system of second-order differential equations by an analytical method.

Paper-3:

Title: The method of lines for the numerical solution of a mathematical model for capillary formation: The role of endothelial cells in the capillary.

Authors: Serdal Pamuk, Arzu Erdem

Published Year: 2007

In this paper they presented the method of lines to obtain the numerical solution of a mathematical model for capillary formation in tumor angiogenesis. This method is an approach to the numerical solution of partial differential equations that involve a time variable t and space variables x, y, . . . As the number of lines is increased, the accuracy of the method of lines representation of the original system increases. The method provides very accurate numerical solutions for linear and nonlinear problems in comparison with other existing methods. They also provided computer programs written in MATLAB and figures that show the endothelial cell movement.

Paper-4:

Title: The Method of Lines Solution of the Regularized Long-Wave Equation Using Runge-Kutta Time Discretization Method

Authors: H. O. Bakodah and M. A. Banaja

Published Year: 2013

A method of lines approach to the numerical solution of nonlinear wave equations typified by the regularized long wave (RLW) is presented. The method developed uses a finite differences discretization to the space. Solution of the resulting system was obtained by applying fourth Runge-Kutta time discretization method. The RLW equation was introduced to describe the behavior of the undular bore by Peregrine. The RLW equation describes a lot of important physical phenomena, such as shallow water waves and ionic waves.

Paper-5:

Title: A new modification of the method of lines for first order hyperbolic PDEs

Authors: Fatmah M. Alabdali et.al

Published Year: 2014

MOL for the solution of first order partial differential equations. The modification they have done was using new three-point difference approximation which leads to stable scheme and good accuracy. And the proposed modified MOL can be used to solve hyperbolic, parabolic and elliptical partial differential equations. The stability and accuracy of method is analyzed by using matrix analysis. Modified MOL method is applied to a number of test problems, on uniform grids, to compare the accuracy and computational efficiency with the standard method.The new modification they have done was using a new three-point difference approximation and good accuracy. The MOL can be used in to solve Hyperbolic, Parabolic and Elliptical partial differential equations.

Paper -6:

Title:Two dimensional heat conduction in a composite slab with temperature dependent conductivity

Authors: Sami A.AI-Sanea

Published Year: 1994

A numerical model, based on the finite-volume method, is developed and applied for the determination of the two-dimensional, steady-state temperature variation in a composite slab with temperature-dependent thermal conductivity. Results of application to four cases are compared in which the slab can be: homogeneous with constant conductivity, composite with temperature-independent conductivity, composite with linear temperature-dependent conductivity, and (4) composite with nonlinear temperature-dependent conductivity. Analytical solutions are utilized for the first three cases and show excellent agreement with the numerical results. A parametric study is carried out to investigate the effects of grid size, conductivity ratio of materials in the composite and various forms of conductivity-temperature relations. The temperature distribution is found to remain unaltered to change in conductivity for a conductivity ratio greater than about 100. The superiority of using the harmonic man of the nodal face conductivity is demonstrated.

Paper -7:

Title: An analytical solution for Transient Heat Conduction in a Composite slab with Time-Dependent Heat Transfer Coefficient

Authors: Ryoichi Chiba

Published Year: 2018

An analytical solution is derived for one-dimensional transient heat conduction in a composite slab consisting of *n* layers, whose heat transfer coefficient on an external boundary is an arbitrary function of time. The composite slab, which has thermal contact resistance at n-1 interfaces, as well as arbitrary initial temperature distribution and internal heat generation, convectively exchanges heat at the external boundaries with two different time varying surroundings. To obtain the analytical solution, the shifting function method is first used, which yields new partial differential equations under conventional types of external boundary conditions.The solution for the derived differential equations is then obtained by means of an orthogonal expansion technique. Numerical calculations are performed for two composite slabs. The numerical results demonstrated the effects of temporal variations in the HTC on the transient temperature field, in particular, on the temperatures at two external boundaries.

Paper -8:

Title: Research on unsteady state heat conduction in slab and cylinder

Authors: Manoj kumar sheladiya, Vinay Bhatt, Pratik kikani

Published Year: 2013

This paper mainly concerned with the analytic solution of unsteady state one dimensional heat conduction problem. In this report as a base, improved lumped parameter model is taken to find the variation in temperature field in a slab and cylinder type geometry. Polynomial approximation method is used to solve the unsteady state conduction equation for both the geometry. Along with this analysis on other models like heat generation in both slab and cylinder and models with boundary heat flux is performed.

Paper -9:

Title: Analysis of Steel Slab for Unsteady State Heat Conduction by Hermite Approximation

Authors: Vinodh singh yadav, Ashish Patidar, Devender Singh

Published Year: 2018

This paper presents the ideas on improvement of lumped-parameter model for unsteady state (transient) heat conduction in a slab with temperature-dependent thermal conductivity. The transient temperature depends on various model parameter, they are Biot number, heat source and time. Polynomial Approximation Method (PAM) has been possible to derive a unified relation for the transient thermal behavior of solid (slab and tube) with both internal heat generation and boundary heat flux. In all the cases, a closed form solution is obtained between temperature, Biot number, heat source parameter and time.

Paper -10:

Title:Heat Conduction in One-Dimensional using MOL

Authors: Mehdi Dehghan

Published Year: 2008

Developed the solution of one-dimensional wave equation which is hyperbolic equation subject to an integral conservation condition using Method of lines. He proposed two forms of MOL for solving the described problem. These forms involve using second order central difference approximation and fourth order difference approximation for discretizing the spacial variable. Developed new algorithms to solve stiff ODEs and tested those on several problems.

CHAPTER – 3

Problem Statement & Boundary Conditions

Consider a very long square bar of 0.1*0.1m cross section. Assuming two-dimensional steady state heat conduction with no heat generation is taking place in the slab with thermal conductivity K= 1.5 W/m℃ and h= 45 W/m^2 ℃, to determine the temperature profile in the plate with different boundary conditions by using analytical and method of lines.

Case-I: Dirichlet Boundary Conditions:

The bottom edge is maintained at 100ºC while the other three edges are maintained at 0ºC.

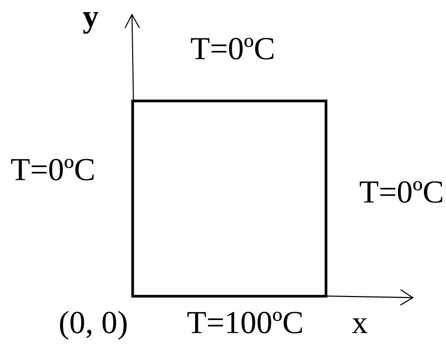

Fig.3.1. 2-D Slab under Dirichlet Boundary Conditions

Dirichlet boundary conditions:

1. At x=0; T=0;

2. At x=a; T=0;

3. At y=b; T=0;

4. At y=0; T=100^0 C.

Case-II: Neumann Boundary Conditions:

The top edge is insulated and the right edge is maintained at 100ºC while the other two edges are maintained at 0ºC.

Neumann Boundary Conditions:

1. At x=0; T=100ºC;

2. At x=a; T=0^{o}C;

3. At y=0; T=0^{o}C;

4. At y=b; $\frac{dT}{dy} = 0°\text{C}$**;**

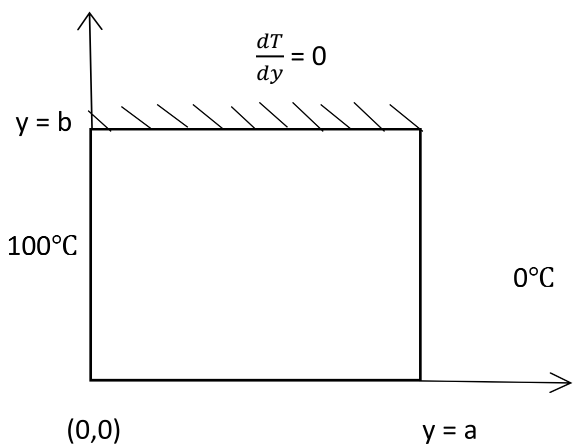

Fig.3.2. 2-D Slab under Neumann Boundary Conditions

CHAPTER-4

ANALYTICAL & SEMI-ANALYTICAL SOLUTIONS FOR BOTH THE CASES

4.1 Analytical Solution for Dirichlet Boundary Conditions:

Assumptions:

1. There is no internal heat generation inside the slab.
2. The slab is under steady state condition i.e., the properties doesn't vary with the time.
3. Two – dimensional heat conduction takes place inside the slab, temperature travels through x & y direction only. Heat flow in z-direction is zero. ($\frac{\partial^2 T}{\partial z^2}$ = 0)
4. The slab material has constant thermal conductivity(k).
5. Temperature is varying independently i.e., when position on X axis is fixed temperature is varying with respect to Y axis is fixed temperature is varying with respect to X axis.

   X(x) = Independent solution of temperature when y is kept constant.

   Y(y) = Independent solution of temperature when x is kept constant.

Final Governing equation after taking all assumptions,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \text{ ------(1)}$$

The complete solution can be assumed as the product of individual solutions (from assumption 5),

$$T\ (x, y) = X(x).\ Y(y) \text{ -------(2)}$$

$$\frac{1}{X}\ \frac{\partial^2 X}{\partial x^2} = -\frac{1}{Y}\ \frac{\partial^2 Y}{\partial y^2} = -\lambda^2$$

From variable separable method:

$$\frac{\partial^2 X}{\partial x^2} + \lambda^2\ X(x) = 0 \text{ -------(a)}$$

$$\frac{\partial^2 Y}{\partial y^2} - \lambda^2\ Y(y) = 0 \text{ -------(b)}$$

Above mentioned equations (a) and (b) are having single independent variable they can be written as:

$$\frac{d^2X}{dx^2} + \lambda^2 X(x) = 0$$

$$\frac{d^2Y}{dy^2} - \lambda^2 Y(y) = 0$$

The equations we get,

D = $\pm$ iλ, Imaginary roots, the solution is in the form of,

X(x) = $C_1\ Cos\lambda x + C_2 \sin \lambda x$

D = ± λ, Real roots (or) D= $\pm$ α + $\sqrt{\beta}$ irrational roots (where λ= $\sqrt{\beta}$, α $= 0$)

The solution is in the form of Y(y) = $C_3 cosh\lambda y + C_4 sinh\lambda y$ ----------(4)

Substitute equation (3) and (4) in (2), we get,

T (x, y) = $[C_1\ Cos\lambda x + C_2 \sin \lambda x]\ [C_3 cosh\lambda y + C_4 sinh\lambda y]$ ----------(5)

Equation (5) is the general solution for the steady state two-dimensional heat conduction with any kind of boundary conditions.

By applying Dirichlet boundary conditions:

At x=0; T=0;

At x=a; T=0;

At y=b; T=0;

At y=0; T=100°C.

From 1st Boundary Condition: At x=0; T=0;

From the equation (5), we get, $C_1 = 0$.

From 2nd Boundary Conditions: At x=a; T=0;

From equation (5),

$$\text{Sin}\lambda a = 0;$$

$$\Rightarrow \lambda = \pm \frac{n\pi}{a}$$

Substitute C_1 and λ values in equation (5), the equation becomes,

Let us take, $C_{2.}C_{3=}K_1$; $C_2.C_4 = K_2$

$$T(x,y) = \sin(\frac{n\pi}{a}x)\ [K_1 cosh\frac{n\pi}{a}y + K_2\ sinh\frac{n\pi}{a}y \quad \text{--------(6)}$$

From 3rd boundary condition, @y=b; T=0;

After applying this 3rd boundary condition to equation (6), we get

$$K_1 = -K_2 \frac{sinh(\frac{n\pi}{a})}{\cosh(\frac{n\pi}{a}b)}$$

Substitute K_1 in equation 6, then the equation becomes,

$$T(x,y) = \frac{K_2 \sin(\frac{n\pi}{a})x}{\cosh(\frac{n\pi}{a})b}\ [-\cosh(\frac{n\pi}{a})y\ \sinh(\frac{n\pi}{a})b + \sinh(\frac{n\pi}{a})y\ \cosh(\frac{n\pi}{a})b\ ]$$

From the formula , ∴ sinhA coshB – coshA sinhB = sinh (A-B) the above equation can be written as,

$$T(x,y) = C_n \sin\left(\frac{n\pi}{a}x\right)\left[\sinh\left(\frac{n\pi}{a}\right)(y-b)\right]$$

Where,

$$C_n = \frac{K_2}{\cosh(\frac{n\pi}{a}b)}$$

And the complete solution is,

$$T(x,y) = \sum_{n=1}^{\propto}(C_n sin\frac{n\pi x}{a}\sinh(\frac{n\pi}{a})(y-b)) \text{ -------------(7)}$$

From fourth boundary condition at y=0; T=100℃;

Apply this boundary condition to equation (7), we get

$$100 = \sum_{n=0}^{\propto} -C_n sin\frac{n\pi}{a}x \ [\sinh\left(\frac{n\pi}{a}\right)(b)]$$

$$100 = \sum_{n=0}^{\propto} b_n sin\frac{n\pi}{a}x \ , \quad \text{where} \quad b_n = -C_n[\sinh\left(\frac{n\pi}{a}\right)(b)$$

$$F(x) = \sum_{n=0}^{\propto} b_n sin\frac{n\pi}{a}x$$

Applying Fourier half series expansion,

$$F(x) = \sum_{n=0}^{\propto} b_n sin\frac{n\pi}{L}x$$

$$b_n = \frac{2}{L}\int_0^L f(x) sin\frac{n\pi}{L}x \, dx$$

Here in the case,

$$b_n = \frac{2}{a}\int_0^a 100 sin\frac{n\pi}{a}x \, dx$$

$$C_n = \frac{-200}{\text{asinh}\left(\frac{n\pi}{h}\right)b}\int_0^a sin\frac{n\pi}{a}x \, dx$$

Over all solution for the temperature distribution in taken slab is,

$$T(x,y) = \sum_{n=1}^{\propto}(C_n sin\frac{n\pi x}{a}\sinh(\frac{n\pi}{a})(b-y)) \text{ ---------(8)}$$

Where,

$$C_n = \frac{200n\pi}{\sinh\left(\frac{n\pi}{a}\right)b}\ (1-\cos(n\pi))$$

4.2 Analytical Solution for Neumann Boundary Conditions case:

Assumptions:

1. There is no internal heat generation inside the slab.
2. The slab is under steady state condition i.e., the properties doesn't vary with the time.
3. Two – dimensional heat conduction takes place inside the slab, temperature travels through x & y direction only. Heat flow in z-direction is zero. ($\frac{\partial^2 T}{\partial z^2}$ = 0)
4. The slab material has constant thermal conductivity(k).
5. Temperature is varying independently i.e., when position on X axis is fixed temperature is varying with respect to Y axis is fixed temperature is varying with respect to X axis.

   X(x) = Independent solution of temperature when y is kept constant.

   Y(y) = Independent solution of temperature when x is kept constant.

Final Governing equation after taking all assumptions,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \text{ ------(1)}$$

The complete solution can be assumed as the product of individual solutions (from assumption 5),

$$T(x, y) = X(x).\ Y(y) \text{ -------(2)}$$

$$-\frac{1}{X}\ \frac{\partial^2 X}{\partial x^2} = \frac{1}{Y}\ \frac{\partial^2 Y}{\partial y^2} = -\lambda^2$$

From variable separable method:

$$\frac{\partial^2 X}{\partial x^2} - \lambda^2\ X(x) = 0 \text{ -------(a)}$$

$$\frac{\partial^2 Y}{\partial y^2} + \lambda^2\ Y(y) = 0 \text{ -------(b)}$$

Above mentioned equations (a) and (b) are having single independent variable they can be written as:

$$\frac{d^2 X}{dx^2} - \lambda^2 X(x) = 0$$

$$\frac{d^2Y}{dy^2} + \lambda^2 Y(y) = 0$$

The equations we get,

D = $\pm$ iλ, Imaginary roots, the solution is in the form of,

X(x) = $C_1\ Cosh\lambda x + C_2 \sinh \lambda x$ ---------(3)

D = ± λ, Real roots (or) D= $\pm$ α + $\sqrt{\beta}$ irrational roots (where λ= $\sqrt{\beta}$, $\alpha = 0$)

The solution is in the form of Y(y) $= C_3 cos\lambda y + C_4 sin\lambda y$ ----------(4)

Substitute equation (3) and (4) in (2), we get,

T (x, y) $= [C_1\ Cosh\lambda x + C_2 \sin h\lambda x]\ [C_3 cos\lambda y + C_4 sin\lambda y]$ ----------(5)

Equation (5) is the general solution for the steady state two-dimensional heat conduction with any kind of boundary conditions.

By applying Neumann boundary conditions:

At x=0; T=100℃;

At x=a; T=0℃;

At y=b; T=0℃;

At y=0; $\frac{dT}{dy}$=0℃.

From 1st Boundary Condition: At y=0; T=0;

From the equation (5), we get, $C_3 = 0$.

Substitute C_3 value in equation 5, the equation becomes,

T (x, y) $= [C_1\ Cosh\lambda x + C_2 \sin h\lambda x]\ [\ C_4 sin\lambda y]$ ----------(6)

Differentiate the equation (6) with respect to y, then we get

$$\frac{dT}{dy} = [C_1 cosh\lambda x\ +\ C_2 sinh\lambda x][C_4 \lambda cos\lambda y]$$

From 2[nd] Boundary Conditions: At y=b; $\frac{dT}{dy}$=0;

Apply this boundary condition to above mentioned differentiated equation, then we get,

$$\cos\lambda b = 0;$$

$$\Rightarrow \lambda = (\frac{2n-1}{2b})\pi$$

Substitute λ values in the equation (6), then we get,

$$T(x,y)= [C_1 Cosh(\frac{2n-1}{2b})\pi x+C_2 sinh(\frac{2n-1}{2b})\pi x][\ C_4 sin(\frac{2n-1}{2b})\pi y] \text{ ----------(7)}$$

From 3[rd] boundary condition, @x=a; T=0°C;

Let us take $C_1C_4 = K_1$; $C_2C_4 = K_2$,

$$T(x,y)= [K_1 Cosh(\frac{2n-1}{2b})\pi x+K_2 sinh(\frac{2n-1}{2b})\pi x][\ sin(\frac{2n-1}{2b})\pi y]$$

After applying this 3[rd] boundary condition to equation (7), we get

$$K_1 = -K_2 \frac{sinh(\frac{2n-1}{2b})\pi a}{\cosh(\frac{2n-1}{2b})\pi a}$$

Substitute K_1 in above equation , then the equation becomes,

$$T(x,y) = K_2 \frac{sinh(\frac{2n-1}{2b})\pi a}{\cosh(\frac{2n-1}{2b})\pi a} [-\cosh(\frac{2n-1}{2b})\pi x \sinh(\frac{2n-1}{2b})\pi a + \sinh(\frac{2n-1}{2b})\pi x \cosh(\frac{2n-1}{2b})\pi a]$$

$$T(x,y) = K_2 \frac{sinh(\frac{2n-1}{2b})\pi a}{\cosh(\frac{2n-1}{2b})\pi a} [\sinh(\frac{2n-1}{2b})\ \pi(x-a)]$$

Let,

$$C_n = \frac{K_2}{\cosh(\frac{2n-1}{2b})\pi a}$$

And the complete solution is,

$$T(x,y) = \sum_{n=1}^{\propto}(C_n \sin(\frac{2n-1}{2b})\ \pi y[\sinh(\frac{2n-1}{2b})\pi(x-a)) \text{ -------------(8)}$$

From fourth boundary condition at x=0; T=100℃;

Apply this boundary condition to equation (8), we get

$$100 = \sum_{n=1}^{\propto} -(C_n \sin(\frac{2n-1}{2b})\, \pi y[\sinh(\frac{2n-1}{2b})\pi a)$$

$$100 = \sum_{n=0}^{\propto} b_n sin\frac{2n-1}{2b}\pi y \quad , \quad \text{where} \quad b_n = -C_n[\sin\left(\frac{(2n-1)\pi}{2b}\right)(y)$$

$$F(y) = \sum_{n=0}^{\propto} b_n sin\frac{(2n-1)\pi}{2b}y$$

Applying Fourier half series expansion,

$$F(x) = \sum_{n=0}^{\propto} b_n sin\frac{n\pi}{L}x$$

$$b_n = \frac{2}{L}\int_0^L f(x) sin\frac{n\pi}{L}x\, dx$$

Here in the case,

$$b_n = \frac{2}{b}\int_0^a 100 sin\frac{(2n-1)\pi}{2b}y dy$$

$$C_n = \frac{-200}{\mathrm{bsinh}\left(\frac{2n-1}{2b}\right)\pi a}\int_0^b sin\frac{(2n-1)\pi}{2b}y\, dy$$

Over all solution for the temperature distribution in taken slab is,

$$T(x,y) = \sum_{n=1}^{\propto}(C_n sin\frac{(2n-1)}{2b}\pi y[\sinh(\frac{2n-1}{2b})\pi(a-x)) \text{ ---------(8)}$$

Where,

$$C_n = \frac{200n\pi}{\mathrm{bsinh}\left(\frac{2n-1}{2b}\right)\pi a}\int_b^0 sin\frac{(2n-1)\pi}{2b}y\, dy$$

4.3 semi-analytical solution for the slab under Dirichlet boundary conditions:

Steps for applying MOL to a problem:

1. Discretize the given differential equation in one coordinate direction.
2. Transformation of partial differential equation to ordinary differential equation.
3. Apply all the taken boundary conditions to the solution of obtained differential equation.
4. Solving the equations.

Let's consider the slab under Dirichlet boundary conditions:

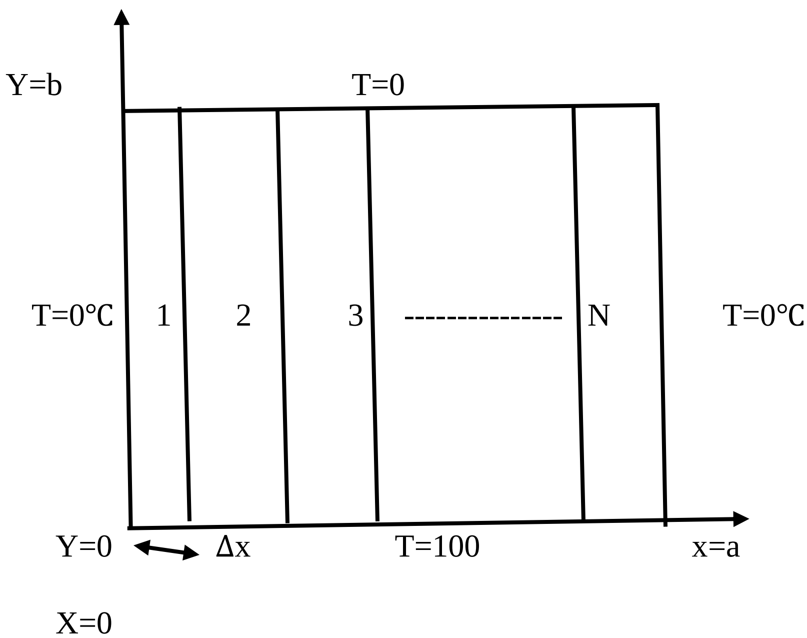

Fig.4.1. 2-D slab used in MOL analysis under Dirichlet boundary conditions

$$\frac{d^2T}{dy^2} = -\frac{T_{i-1} - 2T_i + T_{i+1}}{\Delta x^2} \quad \text{-------(1)}$$

Where, $\Delta x = h = \frac{a-0}{N}$;

$$\frac{d^2T}{dy^2} = AT$$

$$\frac{d^2}{dy^2}\left[\sum_{i=1}^{n} C_i\, U^i\right] = A\left[\sum_{i=1}^{n} C_i\, U^i\right]$$

$\frac{d^2}{dy^2}$ [$\sum_{i=1}^{n} C_i\, U^i$] - [$\sum_{i=1}^{n} C_i A\, U^i$] = 0

$\frac{d^2}{dy^2}$ [$\sum_{i=1}^{n} C_i\, U^i$] - [$\sum_{i=1}^{n} C_i \lambda_i\, U^i$] = 0 (∴ A$U^i = \lambda_i U^i$)

$\sum_{i=1}^{n} \{\frac{d^2}{dy^2} C_i - C_i \lambda_i\}\, U^i = 0$

$\frac{d^2}{dy^2} C_i - C_i \lambda_i = 0$, Let $\frac{d}{dy} = D$,

$D^2 C_i - C_i \lambda_i = 0$

D= $\pm\sqrt{\lambda_i}$

General solution:

$$C_i(y) = K_{i1} e^{\sqrt{\lambda_i} y} + K_{i2} e^{-\sqrt{\lambda_i} y}$$

These K_{i1} *and* K_{i2} can be calculated by applying boundary conditions y=0 and y=b;

$$C_i(y) = \frac{<T_i(y), V^i>}{<U^i, V^i>} \quad \text{(for a non-self-adjoint matrix)}$$

$$C_i(y) = \frac{<T_i(y), U^i>}{<U^i, U^i>} \quad \text{(for a self-adjoint matrix)}$$

And the final overall solution is,

$$T_i(y) = \sum_{i=1}^{n} C_i(y) U^i$$

4.4. semi-analytical solution for the slab under Neumann boundary conditions:

Steps for applying MOL to a problem:

1. Discretize the given differential equation in one coordinate direction.
2. Transformation of partial differential equation to ordinary differential equation.
3. Apply all the taken boundary conditions to the solution of obtained differential equation.
4. Solving the equations.

Let's consider the slab under Neumann boundary conditions:

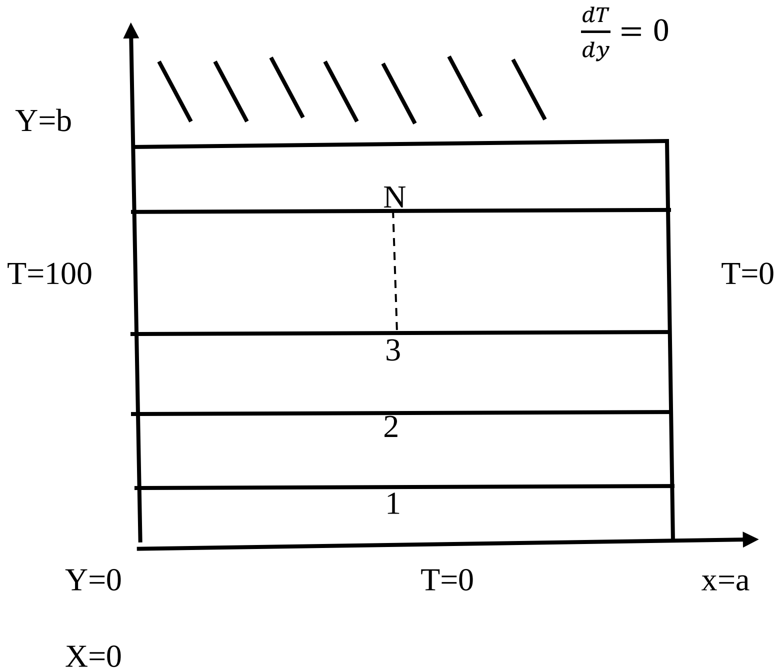

Fig.4.2. 2-D slab used in MOL analysis under Neumann boundary conditions

$\frac{d^2T}{dy^2} = -\frac{T_{j+1}-2T_j+T_{j-1}}{\Delta y^2}$ --------(1)

Where, Δy = h = $\frac{b-0}{N}$;

$\frac{d^2T}{dx^2}$ = AT

$\frac{d^2}{dx^2}$ [$\sum_{j=1}^{n} C_j\, U^j$] = A [$\sum_{j=1}^{n} C_j\, U^j$]

$\frac{d^2}{dx^2}$ [$\sum_{j=1}^{n} C_j\, U^j$] - [$\sum_{j=1}^{n} C_j A\, U^j$] = 0

$\frac{d^2}{dx^2}$ [$\sum_{j=1}^{n} C_j\, U^j$] - [$\sum_{j=1}^{n} C_j \lambda_j\, U^j$] = 0 (∴ A$U^j = \lambda_j U^j$)

$\sum_{j=1}^{n} \{\frac{d^2}{dx^2} C_j - C_j\lambda_j\}\, U^j$ = 0

$\frac{d^2}{dx^2} C_j - C_j\lambda_j$ = 0, Let $\frac{d}{dx} = D,$

$$D^2C_j - C_j\lambda_j = 0$$

D= $\pm\sqrt{\lambda_j}$

General solution:

$$C_j(x) = K_{j1}e^{\sqrt{\lambda_j}x} + K_{j2}e^{-\sqrt{\lambda_j}x}$$

These K_{i1} *and* K_{i2} can be calculated by applying boundary conditions x=0 and x=a;

$$C_j(x) = \frac{<T_j(x),V^j>}{<U^j,V^j>}$$ (for a non-self-adjoint matrix)

$$C_j(x) = \frac{<T_j(x),U^j>}{<U^j,U^j>}$$ (for a self-adjoint matrix)

And the final overall solution is,

$$T_j(x) = \sum_{j=1}^{n} C_j(x)U^j$$

CHAPTER-5

RESULTS & DISCUSSIONS

5.1 Temperature profiles for two-dimensional steady state heat conduction in a slab without internal heat generation under Dirichlet boundary conditions:

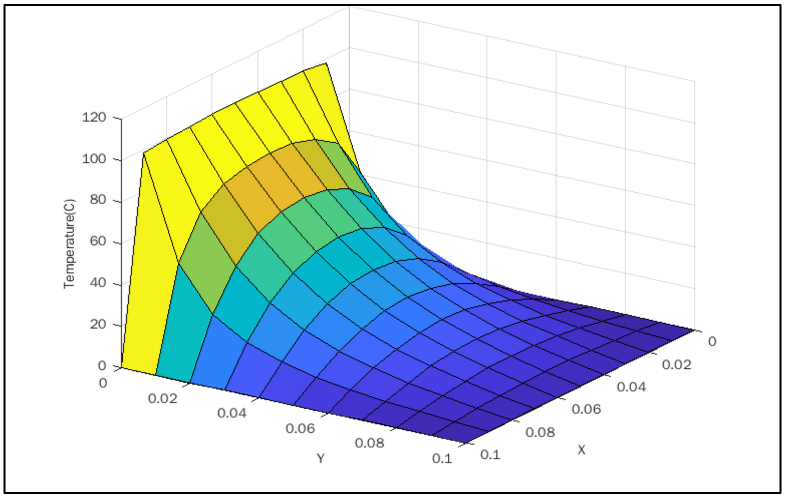

Fig.5.1. Temperature distribution in a given slab by using Analytical method (Case-I) at n=10:

The temperature distribution profile is plotted for analytical method under Dirichlet boundary conditions when 'n' (number of lines/strips) is taken as 10. The specified boundary conditions for the above plot are; At y=0, T=100°C; At x=0, T=0°C; At x=0.1, T=0°C; At y=0.1,T=0°C.

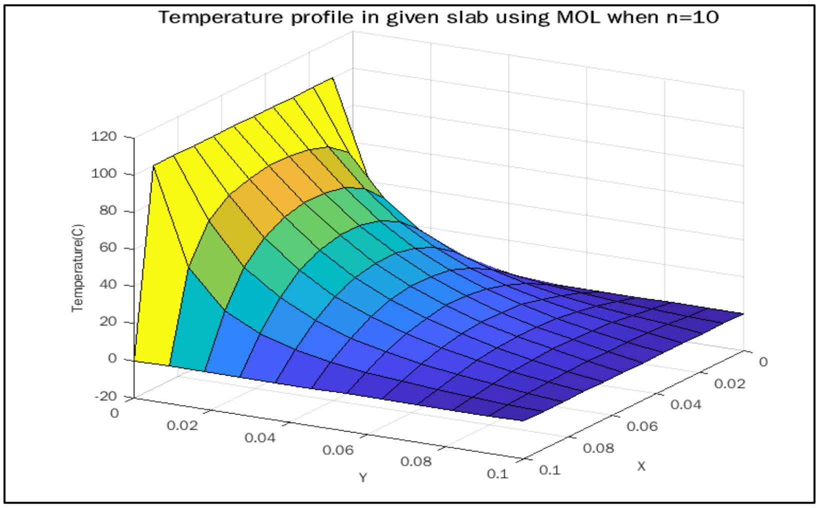

Fig.5.2.Temperature distribution in a given slab by using MOL method (Case-I) at n=10

The temperature distribution profile is plotted for MOL method under Dirichlet boundary conditions when 'n' (number of lines/strips) is taken as 10. The specified boundary conditions for the above plot are ; At y=0,T=100°C; At x=0, T=0°C; At x=0.1,T=0°C; At y=0.1,T=0°C.The x-variables are discretized into 'n' strips with respect to y-axis.

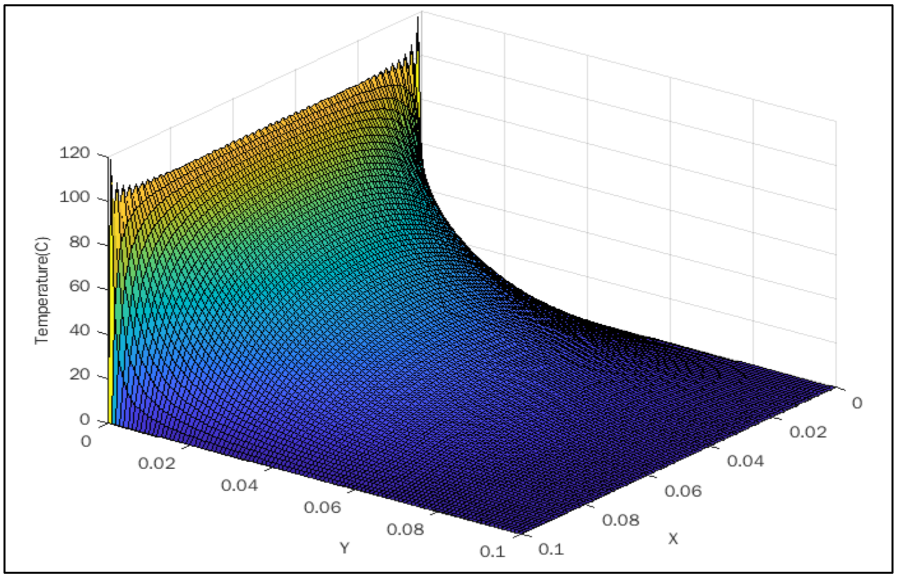

Fig.5.3. Temperature distribution in a given slab by using Analytical method (Case-I) at n=100:

The temperature distribution profile is plotted for analytical method under Dirichlet boundary conditions when 'n' (number of lines/strips) is taken as 100. The specified boundary conditions for the above plot are; At y=0, T=100°C; At x=0, T=0°C; At x=0.1,T=0°C; At y=0.1,T=0°C.

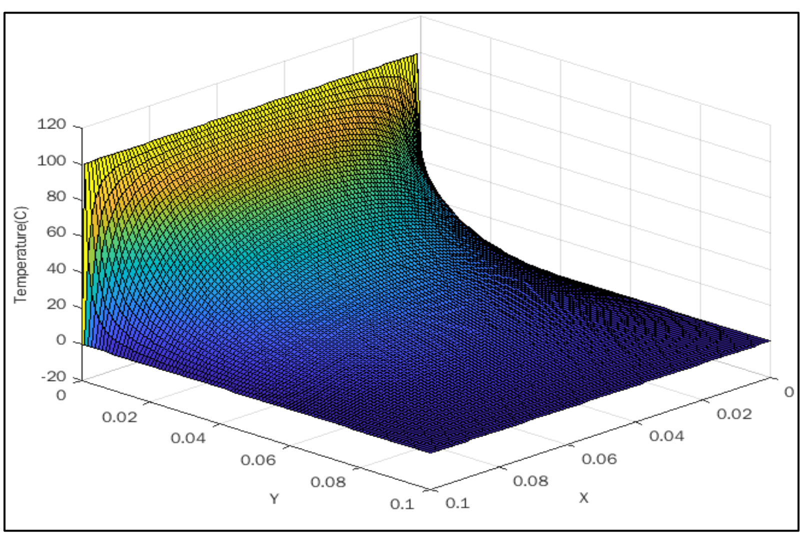

Fig.5.4. Temperature distribution in a given slab by using MOL method (Case-I) at n=100:

The temperature distribution profile is plotted for MOL method under Dirichlet boundary conditions when 'n' (number of lines/strips) is taken as 100. The specified boundary conditions for the above plot are ; At y=0,T=100°C; At x=0, T=0°C; At x=0.1,T=0°C; At y=0.1,T=0°C.The x-variables are discretized into 'n' strips with respect to y-axis.

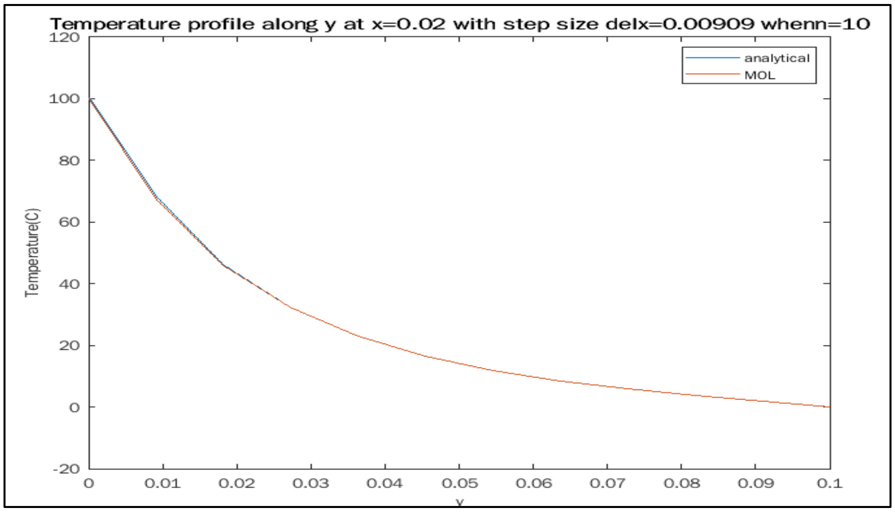

Fig.5.5. Comparison between Analytical and MOL method for case-1 at x=0.02 when n=10:

For the Dirichlet boundary conditions, the above graph is plotted for comparing the solution results obtained from both analytical and MOL (Semi-Analytical) method at x=0.02 for n=10.

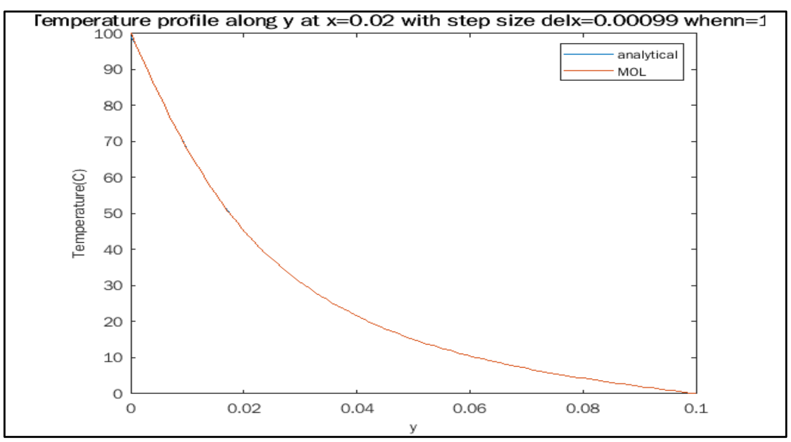

Fig.5.6. Comparison between Analytical and MOL method for case-1 at x=0.02 when n=100:

For the Dirichlet boundary conditions, the above graph is plotted for comparing the solution results obtained from both Analytical and MOL(Semi-Analytical) method at x=0.02 for n=100.

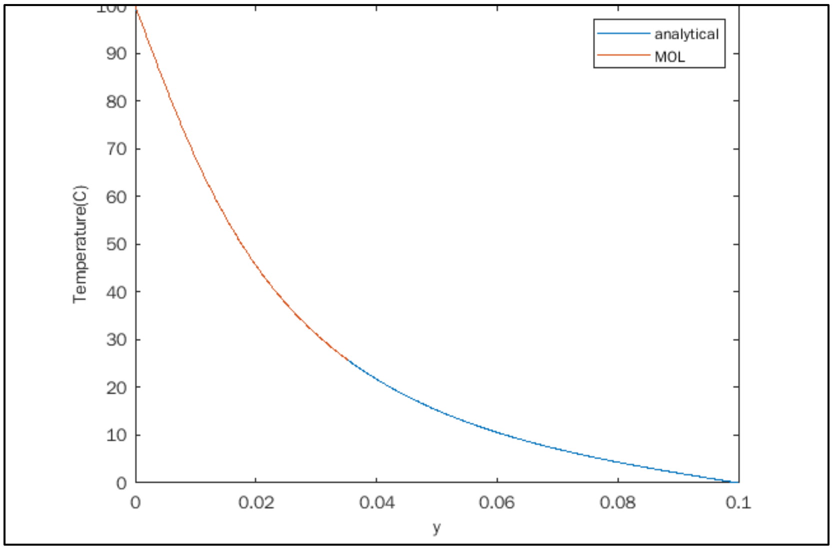

Fig.5.7. Comparison between Analytical and MOL method for case-1 at x=0.02 when n=1000:

For the Dirichlet boundary conditions, the above graph is plotted for comparing the solution results obtained from both Analytical and MOL(Semi-Analytical) method at x=0.02 for n=1000.

S. No	Analytical	MOL using n=10	MOL using n=100	MOL using n=1000	Error when n=10	Error when n=100	Error when n=1000
1	99.6584	100	100	100	0.3416	0.3416	0.3416
2	68.2262	67.3594	67.9942	68.1882	0.8668	0.232	0.038
3	45.6341	45.2717	45.5426	45.6195	0.3624	0.0915	0.0146
4	31.2202	31.1236	31.1986	31.2169	0.0966	0.0216	0.0033
5	21.7757	21.7559	21.7719	21.7751	0.0198	0.0038	0.0006
6	15.2754	15.2822	15.2780	15.2759	0.0068	0.0026	0 .0005
7	10.6035	10.609	10.6048	10.6037	0.0055	0.0013	0.0002
8	7.1076	7.1122	7.1090	7.1078	0.0046	0.0014	0.0002

9	4.3659	4.3694	4.3671	4.3661	0.0035	0.0012	0.0002
10	2.0766	2.0792	2.0772	2.0767	0.0026	0.0006	0.0001
11	0	0	0	0	0	0	0

Table.5.1. Tabulated results of analytical and method of lines (MOL) solutions (Case-I) at x=0.02:

5.2. Temperature profiles for two-dimensional steady state heat conduction in a slab without internal heat generation under Neuman boundary conditions:

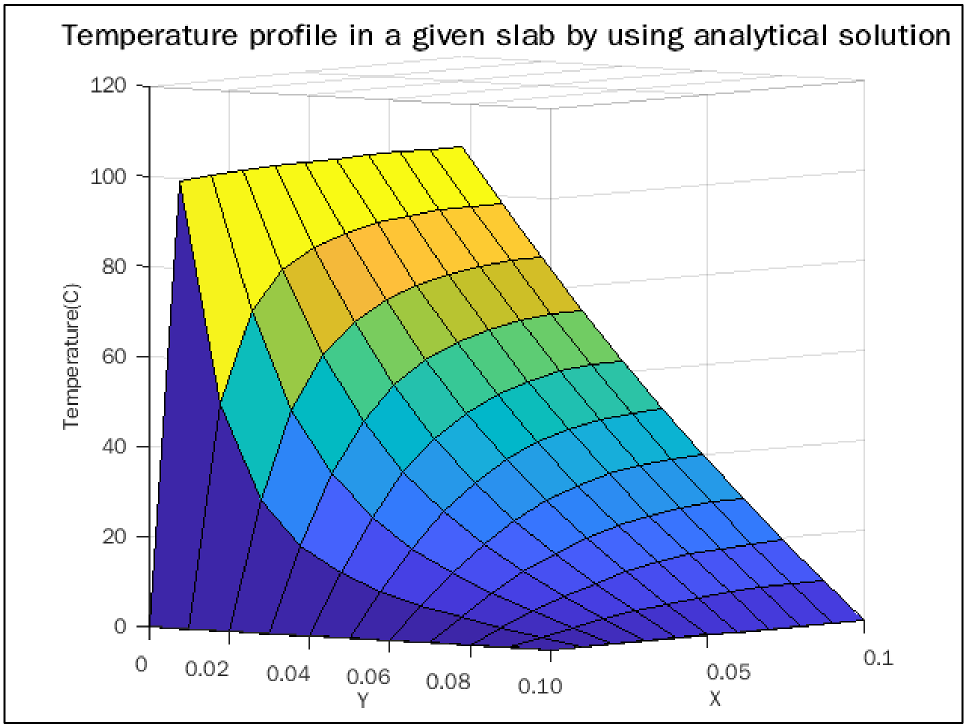

Fig.5.8. Temperature distribution in a given slab by using Analytical method at n=10:

The temperature distribution profile is plotted for analytical method under Neumann Boundary Conditions when 'n' (number of lines/strips) is taken as 10. The specified boundary conditions for the above plot are: At y=0,T=0°C; At x=0,T=100°C; At x=0.1,T=0°C, At y=0.1, The surface is insulated(Derivative of temperature with respect to spatial gradient is zero).

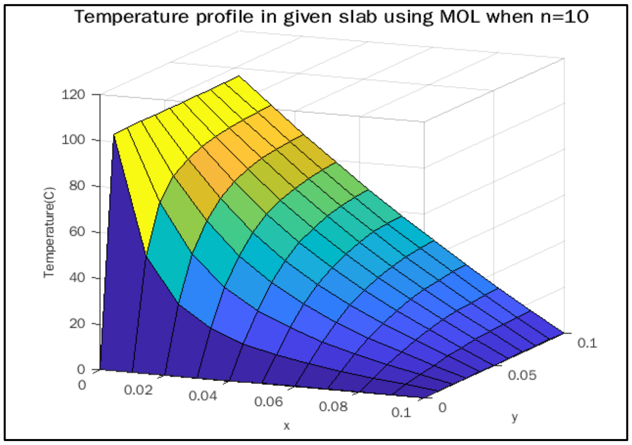

Fig.5.9. Temperature distribution in a given slab by using MOL method (Case-II) at n=10:

The temperature distribution profile is plotted for MOL method under Neumann Boundary Conditions when 'n' (number of lines/strips) is taken as 10. The specified boundary conditions for the above plot are: At y=0,T=0°C; At x=0,T=100°C; At x=0.1,T=0°C, At y=0.1, The surface is insulated(Derivative of temperature with respect to spatial gradient is zero). The y-variables are discretized into 'n' strips with respect to x-axis.

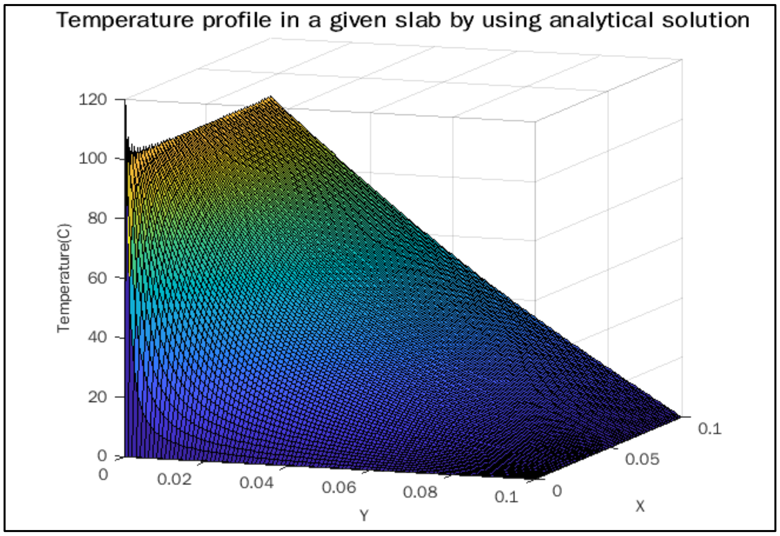

Fig.5.10. Temperature distribution in a given slab by using Analytical method (Case-I) at n=100:

The temperature distribution profile is plotted for analytical method under Neumann Boundary Conditions when 'n' (number of lines/strips) is taken as 100. The specified boundary conditions for the above plot are: At y=0, T=0°C; At x=0,T=100°C; At x=0.1,T=0°C, At y=0.1, The surface is insulated(Derivative of temperature with respect to spatial gradient is zero).

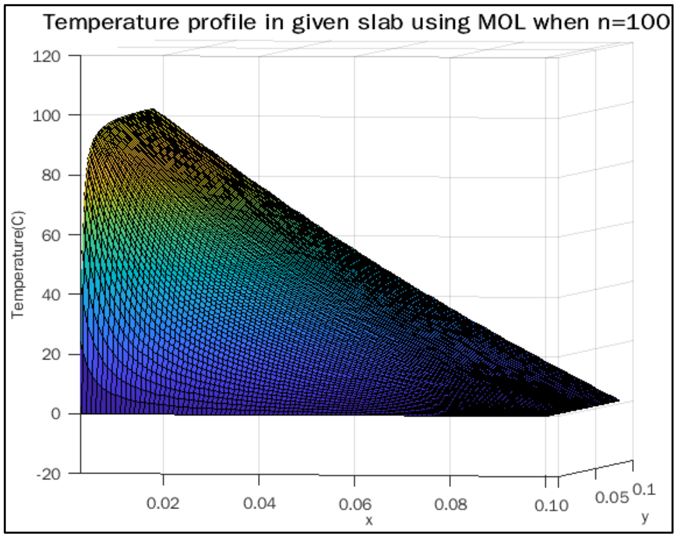

Fig.5.11 Temperature distribution in a given slab by using MOL method (Case-II) at n=100:

The temperature distribution profile is plotted for MOL method under Neumann Boundary Conditions when 'n' (number of lines/strips) is taken as 10. The specified boundary conditions for the above plot are: At y=0,T=0°C; At x=0,T=100°C; At x=0.1,T=0°C, At y=0.1, The surface is insulated(Derivative of temperature with respect to spatial gradient is zero). The y-variables are discretized into 'n' strips with respect to x-axis.

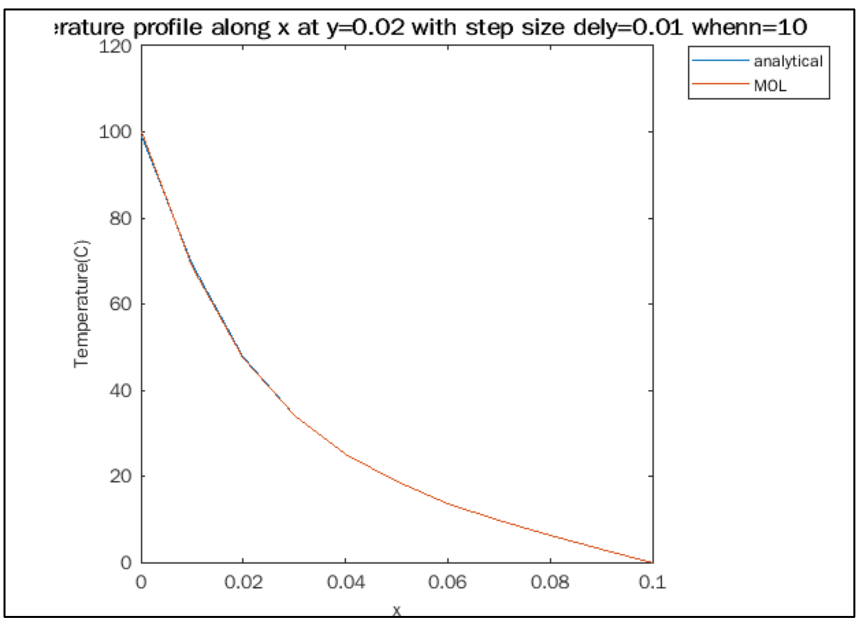

Fig.5.12. Comparison between Analytical and MOL method for case-II at y=0.02 when n=10:

For the Neumann Boundary conditions, the above graph is plotted for comparing the solution results obtained from both Analytical and MOL methods at y=0.02 for 'n'=10.

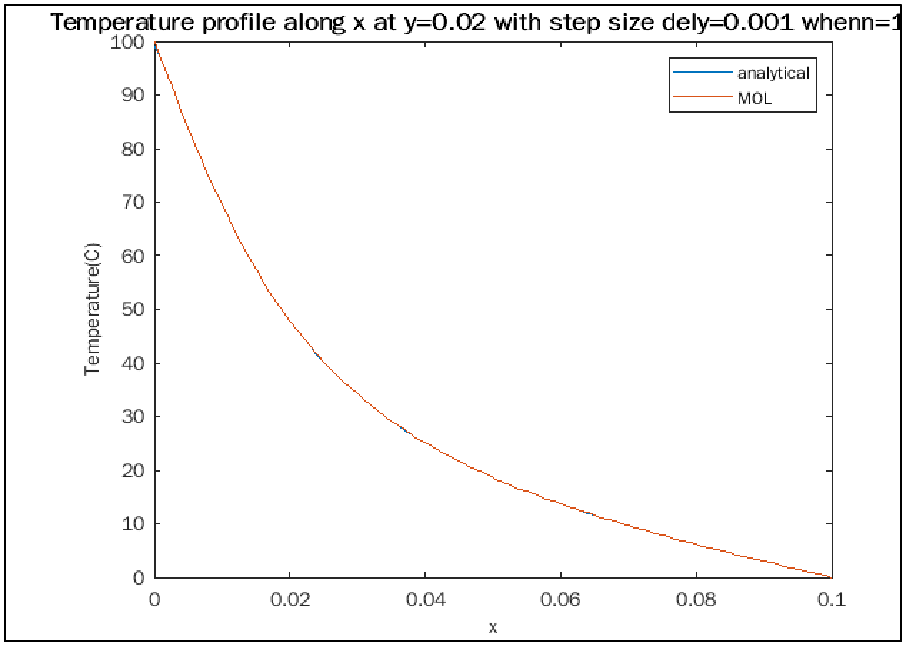

Fig.5.13. Comparison between Analytical and MOL method for case-II at y=0.02 when n=100:

For the Neumann Boundary conditions, the above graph is plotted for comparing the solution results obtained from both Analytical and MOL methods at y=0.02 for 'n'=100.

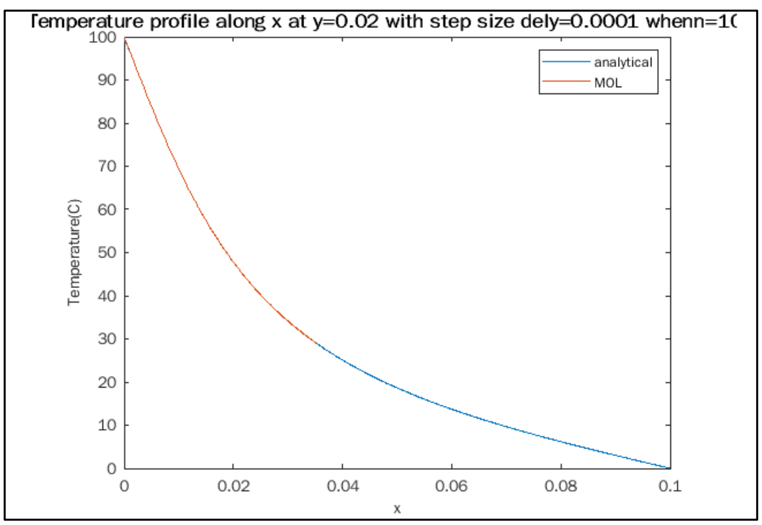

Fig. 5. 14. Comparison between Analytical and MOL method for case-II at y=0.02 when n=1000:

For the Neumann Boundary conditions, the above graph is plotted for comparing the solution results obtained from both Analytical and MOL methods at y=0.02 for 'n'=1000.

S.No	Analytical	MOL using n=10	MOL using n=100	MOL using n=1000	Error when n=10	Error when n=100	Error when n=1000
1	99.6584	100	100	100	0.3416	0.3416	0.3416
2	68.2262	67.3594	67.9942	68.1882	0.8668	0.232	0.038
3	45.6341	45.2717	45.5426	45.6195	0.3624	0.0915	0.0146
4	31.2202	31.1236	31.1986	31.2169	0.0966	0.0216	0.0033
5	21.7757	21.7559	21.7719	21.7751	0.0198	0.0038	0.0006
6	15.2754	15.2822	15.2780	15.2759	0.0068	0.0026	0 .0005
7	10.6035	10.609	10.6048	10.6037	0.0055	0.0013	0.0002
8	7.1076	7.1122	7.1090	7.1078	0.0046	0.0014	0.0002
9	4.3659	4.3694	4.3671	4.3661	0.0035	0.0012	0.0002
10	2.0766	2.0792	2.0772	2.0767	0.0026	0.0006	0.0001
11	0	0	0	0	0	0	0

Table.5.2. Tabulated results of analytical and method of lines (MOL) solutions (Case-II) at y=0.02:

CHAPTER-6

Comparison Studies using Method of Lines and ANSYS

To study the temperature distributions in a 2-D slab under Dirichlet boundary conditions with the same assumptions above taken used ANSYS. Using ANSYS 19.2 version constructed a slab and assigned bounder conditions to that. And that figure given below.

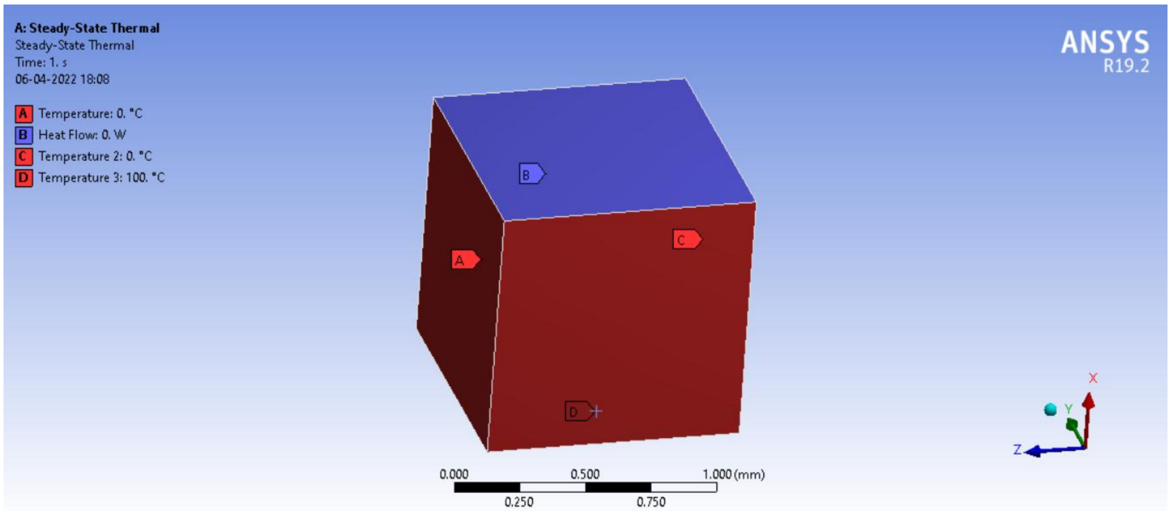

Fig 6.1. Slab under Dirichlet boundary conditions using ANSYS.

Under steady state thermal conditions using stainless steel as material temperature profiles were developed and given below.

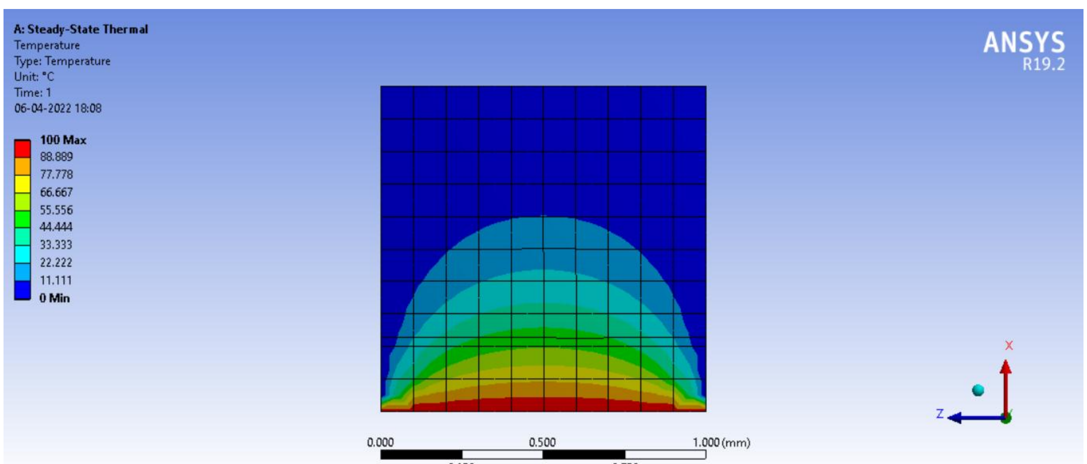

Fig.6.2. Temperature profiles in slab developed with ANSYS

S. No	Analytical Results	MOL using n=10	Ansys Results	Error Between Analytical and ANSYS	Error Between MOL & ANSYS
1	99.6584	100	100	0.3416	0
2	68.2262	67.3594	66.667	1.5592	0.6924
3	45.6341	45.2717	44.444	1.1901	0.8317
4	31.2202	31.1236	33.334	2.1138	2.2104
5	21.7757	21.7559	22.223	0.4473	0.4671
6	10.6035	10.609	11.113	0.5095	0.504
7	0	0	0	0	0

Table 6.1 Comparison results of Ansys & Analytical & MOL

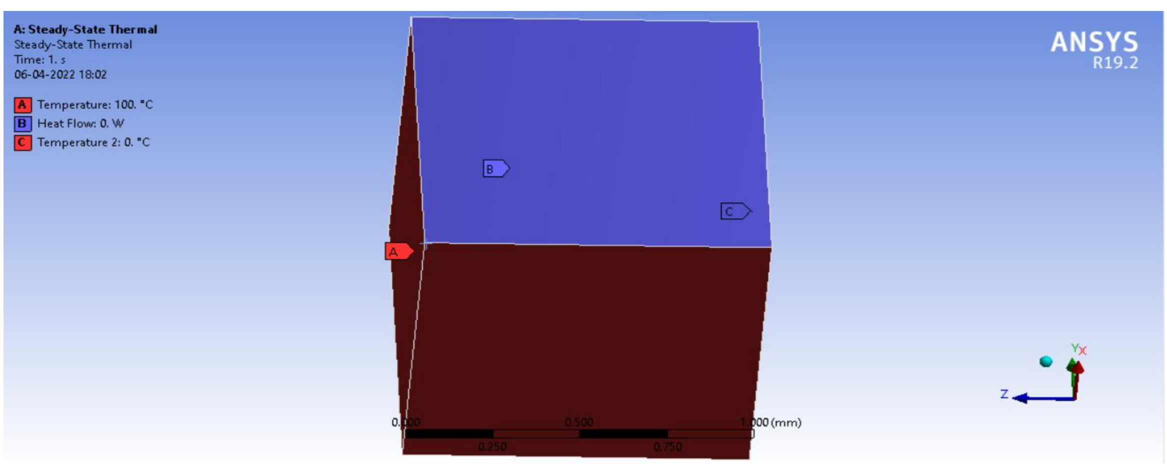

Fig 6.3. Slab under Neumann boundary conditions using ANSYS.

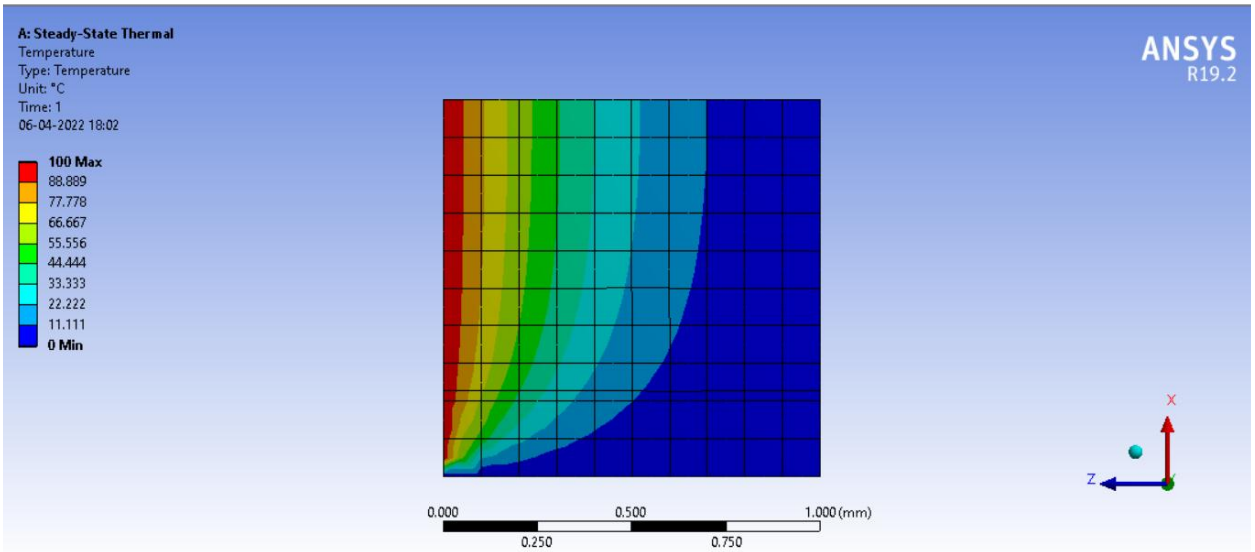

Fig.6.4. Temperature profiles in slab developed with ANSYS

DISCUSSIONS:

The temperature distribution profiles in the two-dimensional steady state heat conduction in a slab are plotted with the profiles of analytical solutions by using MATLAB. The steady state analysis of temperature distribution with specified Dirichlet and Neumann Boundary conditions was developed by using Method of lines.

1. From the graphs 5.5, 5.6, 5.7 it is observed that the temperature profiles obtained from the method of lines are similar to that of the analytical method, showing that it can achieve an approximately accurate temperature profile using the Method of lines.
2. From comparison of graphs and tables for both boundary conditions, it is clear that as the number of lines increasing, the error between analytical and method of lines (semi analytical) solutions has been decreased, and the profiles of analytical and MOL converged which indicates the preference of higher number of lines in order to obtain accurate values (with minimal errors).
3. The temperature distribution profiles are compared obtained from solutions using MOL and Ansys software. The profiles almost converged and the error between the two methods is appreciably negligible.

CONCLUSIONS:

In this study and review on, a two-dimensional slab with steady state heat conduction under different boundary conditions was modeled. The temperature distribution profiles for both analytical and semi analytical methods under Dirichlet and Neumann boundary conditions were plotted. For the semi analytical approach, it was solved by using Method of Lines. The effect of the number of lines on the convergence of solution from method of lines (semi analytical method) to analytical solution was studied under graphical and individual nodal data analysis in a detailed way. It is observed that the two methods converge and give almost similar data with minimal error. As the non-homogeneity in the boundary conditions increases, solving analytically becomes more complex, also time taking, for such non-homogeneous boundary conditions it is assumed that the MOL doesn't much increase the complexity and further study should be needed to prove this.

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Appendix – I

MATLAB codes for 2-D steady state heat conduction under Dirichlet Boundary conditions

MATLAB Code for analytical method solutions:

```
%program for solving 2D steady state heat equation by analytical method
%over the interval 0<x<a,0<y<b
clear all
clc
a=0.1;b=0.1;
ns= input('Enter the no.of iterations:');
h=a/ns;
x=0:h:a;
y=0:h:b;
[X,Y]=meshgrid(x,y);
sum=0;
T=[];
for n=1:2:101
Cn=(200*(1-cos(n*pi))/(n*pi*sinh((n*pi/a)*b)));
f1=sin((n*pi/a)*x);
f2=sinh((n*pi/a)*(b-y));
f=Cn.*(f1'*f2);
sum=sum+f
end
T(:,:)=sum;
surf(X,Y,sum')
s1=sprintf('Temperature profile in a given slab by using analytical solution');
```

```
title(s1,'fontsize',14);
xlabel('X','fontsize',10);
ylabel('Y','fontsize',10);
zlabel('Temperature(C)','fontsize',10);
```

MATLAB Code for Semi-analytical method (MOL) solutions:

```
%program for solving 2D heat conduction under steady state condition by
%method of lines
%over the interval 0<x<a,0<y<b
clear all
clc
n=input('Enter the number of lines:');
a=0.1;b=0.1;Tr=0;Tl=0;
ns=n+1; %number of iterations
delx=(a)/ns;
d1=2.*ones(1,n);
d2=-1.*ones(1,n-1);
A=diag(d1)+diag(d2,1)+diag(d2,-1);
I=eig(A.*(1/delx^2));
[U,Id]=eig(A.*(1/delx^2));
T0i=100.*ones(n,1);
Tbi=0.*ones(n,1);
K=[];T=[];C=[];
for i=1:1:n
c0(i)=sum(T0i.*U(:,i))/sum(U(:,i).*U(:,i));
cb(i)=sum(Tbi.*U(:,i))/sum(U(:,i).*U(:,i));
end
```

```
for i=1:1:n
P=[1 1 ; exp(b*sqrt(I(i))) exp(-b*sqrt(I(i)))];
L=[c0(i);cb(i)];
k=P\L;
K(:,i)=k;
end
l=1;
dely=delx;
for j=0:dely:b
for i=1:1:n
Cj(i)=(K(1,i)*exp(j*sqrt(I(i))))+(K(2,i)*exp(-j*sqrt(I(i))));
end
C(:,l)=Cj;
T=Cj*U';
Ts(:,l)=T;
l=l+1;
end
Ts;
Tx0=0.*ones(1,l-1);
Txa=0.*ones(1,l-1);
Ts
x=0:delx:a;
y=0:dely:b;
[X,Y]=meshgrid(x,y);
Ts1=[Ts;Txa];
T=[];
T(2:n+2,:)=Ts1
```

```
Z=reshape(T',size(X));
surf(X,Y,Z);
s1=sprintf('Temperature profile in given slab using MOL when n=%d',n);
title(s1,'fontsize',14);
xlabel('X','fontsize',10);
ylabel('Y','fontsize',10);
zlabel('Temperature(C)','fontsize',10);
```

MATLAB Code for comparing Analytical & Semi-Analytical method solutions:

```
 %program for solving 2D heat conduction under steady state condition by
%method of lines
%over the interval 0<x<a,0<y<b
clear all
clc
n=input('Enter the number of lines:');
a=0.1;b=0.1;Tr=0;Tl=0;
ns=n+1; %number of iterations
delx=(a)/ns;
d1=2.*ones(1,n);
d2=-1.*ones(1,n-1);
A=diag(d1)+diag(d2,1)+diag(d2,-1);
I=eig(A.*(1/delx^2));
[U,Id]=eig(A.*(1/delx^2));
T0i=100.*ones(n,1);
Tbi=0.*ones(n,1);
K=[];T=[];C=[];
```

```
for i=1:1:n
c0(i)=sum(T0i.*U(:,i))/sum(U(:,i).*U(:,i));
cb(i)=sum(Tbi.*U(:,i))/sum(U(:,i).*U(:,i));
end
for i=1:1:n
P=[1 1 ; exp(b*sqrt(I(i))) exp(-b*sqrt(I(i)))];
L=[c0(i);cb(i)];
k=P\L;
K(:,i)=k;
end
l=1;
dely=delx;
for j=0:dely:b
for i=1:1:n
Cj(i)=(K(1,i)*exp(j*sqrt(I(i))))+(K(2,i)*exp(-j*sqrt(I(i))));
end
C(:,l)=Cj;
T=Cj*U';
Ts(:,l)=T;
l=l+1;
end
Ts;
Tx0=0.*ones(1,l-1);
Txa=0.*ones(1,l-1);
Ts;
x=0:delx:a;
y=0:dely:b;
```

```
[X,Y]=meshgrid(x,y);
Ts1=[Ts;Txa];
T=[];
T(2:n+2,:)=Ts1;
Z=reshape(T',size(X));
%program for solving 2D steady state heat equation by analytical method
%over the interval 0<x<a,0<y<b
a=0.1;b=0.1;
nsa= n+1;
ha=a/nsa;
x=0:ha:a;
y=0:ha:b;
[X,Y]=meshgrid(x,y);
sum=0;
Ta=[];
for na=1:2:101
Cna=(200*(1-cos(na*pi))/(na*pi*sinh((na*pi/a)*b)));
f1=sin((na*pi/a)*x);
f2=sinh((na*pi/a)*(b-y));
f=Cna.*(f1'*f2);
sum=sum+f;
end
Ta(:,:)=sum;
Ta;
%graph of analytical and method of lines methods
x=0:delx:a;
y=0:dely:b;
```

```
p=input('x value at which you need temperature profile:');

r=round(p/delx);

Tm=T(r+1,:); %temperature data by MOL

Tn=Ta(r+1,:); %Temperature data by analytical

plot(y,Tn,y,Tm,'-')

xlabel('y','fontsize',10)

ylabel('Temperature(C)','fontsize',10)

s1=sprintf('Temperature profile along y at x=%0.3g with step size delx=%0.3g
whenn=%d',p,delx,n);

title(s1,'fontsize',12)

%title('Comparison of Temperature profile between Analytical and MOL')

legend('analytical','MOL')

Tm',Tn'
```

Appendix-II

MATLAB codes for 2-D steady state slab under Neumann boundary Conditions

MATLAB Code for Analytical method solutions:

```
%program for solving 2D steady state heat equation by analytical method
%over the interval 0<x<a,0<y<b
clc
clear all
a=0.1;b=0.1;
ns= input('Enter the no.of iterations:');
h=a/ns;
x=0:h:a;
y=0:h:b;
[X,Y]=meshgrid(x,y);
sum=0;
T=[];
for n=1:1:101
q1=(200)/(b*sinh((((2*n)-1)/(2*b))*(pi*a)));
q2 = @ (y) (sin((((2*n)-1)/(2*b))*(pi*y)));
q3 = integral(q2,0,b);
Cn = q1*q3;
f1=sin((((2*n)-1)*(pi*y))/(2*b));
f2=sinh(((((2*n)-1)*pi)/(2*b))*(a-x));
f=Cn.*(f1'*f2);
sum=sum+f;
end
T(:,:)=sum;
```

```
surf(X,Y,sum')
s1=sprintf('Temperature profile in a given slab by using analytical solution');
title(s1,'fontsize',14);
xlabel('X','fontsize',10);
ylabel('Y','fontsize',10);
zlabel('Temperature(C)','fontsize',10);
```

MATLAB Code for Semi-analytical method (MOL) solutions:

```
%program for solving 2D heat conduction under steady state condition by
%Neumann method of lines
%over the interval 0<x<a,0<y<b
clear all
clc
n=input('Enter the number of lines:');
a=0.1;b=0.1;Tr=0;Tl=0;
ns=n; %number of iterations
dely=(b)/ns;
de=2.*ones(1,n-1);
d1=[de,1];
d2=-1.*ones(1,n-1);
A=diag(d1)+diag(d2,1)+diag(d2,-1);
I=eig(A.*(1/dely^2));
[U,Id]=eig(A.*(1/dely^2));
T0j=100.*ones(n,1);
Taj=0.*ones(n,1);
K=[];T=[];C=[];
for j=1:1:n
```

```
c0(j)=sum(T0j.*U(:,j))/sum(U(:,j).*U(:,j));
ca(j)=sum(Taj.*U(:,j))/sum(U(:,j).*U(:,j));
end
for j=1:1:n
P=[1 1 ; exp(a*sqrt(I(j))) exp(-a*sqrt(I(j)))];
L=[c0(j);ca(j)];
k=P\L;
K(:,j)=k;
end
l=1;
delx=dely;
for i=0:delx:a
for j=1:1:n
Ci(j)=(K(1,j)*exp(i*sqrt(I(j))))+(K(2,j)*exp(-i*sqrt(I(j))));
end
C(:,l)=Ci;
T=Ci*U';
Ts(:,l)=T;
l=l+1;
end
Ts;
Ty0=0.*ones(1,l-1);
Tyb=0.*ones(1,l-1);
Ts;
x=0:delx:a;
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y=0:dely:b;
```

```
[X,Y]=meshgrid(x,y);
Ts1=[Ty0;Ts];
T=[];
T=[Ts1];
Z=reshape(T',size(X));
surf(X,Y,Z);
s1=sprintf('Temperature profile in given slab using MOL when n=%d',n);
title(s1,'fontsize',14);
xlabel('y','fontsize',10);
ylabel('x','fontsize',10);
zlabel('Temperature(C)','fontsize',10);
T'
```

MATLAB Code for Comparing both analytical & Semi-Analytical methods Solutions:

```
%program for solving 2D heat conduction under steady state condition by
%Neumann method of lines
%over the interval 0<x<a,0<y<b
clear all
clc
n=input('Enter the number of lines:');
a=0.1;b=0.1;Tr=0;Tl=0;
ns=n; %number of iterations
dely=(b)/ns;
de=2.*ones(1,n-1);
d1=[de,1];
d2=-1.*ones(1,n-1);
A=diag(d1)+diag(d2,1)+diag(d2,-1);
I=eig(A.*(1/dely^2));
```

```
[U,Id]=eig(A.*(1/dely^2));
T0j=100.*ones(n,1);
Taj=0.*ones(n,1);
K=[];T=[];C=[];
for j=1:1:n
c0(j)=sum(T0j.*U(:,j))/sum(U(:,j).*U(:,j));
ca(j)=sum(Taj.*U(:,j))/sum(U(:,j).*U(:,j));
end
for j=1:1:n
P=[1 1 ; exp(a*sqrt(I(j))) exp(-a*sqrt(I(j)))];
L=[c0(j);ca(j)];
k=P\L;
K(:,j)=k;
end
l=1;
delx=dely;
for i=0:delx:a
for j=1:1:n
Ci(j)=(K(1,j)*exp(i*sqrt(I(j))))+(K(2,j)*exp(-i*sqrt(I(j))));
end
C(:,l)=Ci;
T=Ci*U';
Ts(:,l)=T;
l=l+1;
end
Ts;
Ty0=0.*ones(1,l-1);
```

```
Tyb=0.*ones(1,l-1);
Ts;
x=0:delx:a;
y=0:dely:b;
[X,Y]=meshgrid(x,y);
Ts1=[Ty0;Ts];
T=[];
T=[Ts1];
%program for solving 2D steady state heat equation by analytical method
%over the interval 0<x<a,0<y<b
a=0.1;b=0.1;
nsa=n;
ha=a/nsa;
x=0:ha:a;
y=0:ha:b;
[X,Y]=meshgrid(x,y);
sum=0;
Tan=[];
for na=1:1:101
q1=(200)/(b*sinh((((2*na)-1)/(2*b))*(pi*a)));
q2 = @ (y) (sin((((2*na)-1)/(2*b))*(pi*y)));
q3 = integral(q2,0,b);
Cn = q1*q3;
f1=sin((((2*na)-1)*(pi*y))/(2*b));
f2=sinh(((((2*na)-1)*pi)/(2*b))*(a-x));
f=Cn.*(f1'*f2);
sum=sum+f;
```

```
end

Tan(:,:)=sum;

%graph of analytical and method of lines methods

x=0:delx:a;

y=0:dely:b;

p=input('y value at which you need temperature profile:');

r=round(p/dely);

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Tm=T(r+1,:); %temperature data by MOL

Tn=Tan(r+1,:); %Temperature data by analytical

plot(x,Tn,x,Tm,'-')

xlabel('x','fontsize',10)

ylabel('Temperature(C)','fontsize',10)

s1=sprintf('Temperature profile along x at y=%0.3g with step size dely=%0.3g
whenn=%d',p,dely,n);

title(s1,'fontsize',12)

legend('analytical','MOL')

Tm',Tn’
```