Year 8 Mathematics

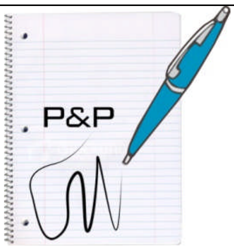
Learning Strategies

Mathematics is often the most challenging subject for students. Much of the trouble comes from the fact that mathematics is about logical thinking, not memorizing rules or remembering formulas. It requires a different style of thinking than other subjects. The students who seem to be “naturally” good at math just happen to adopt the correct strategies of thinking that math requires – often they don’t even realise it. We have isolated several key learning strategies used by successful maths students and have made icons to represent them. These icons are distributed throughout the book in order to remind students to adopt these necessary learning strategies:

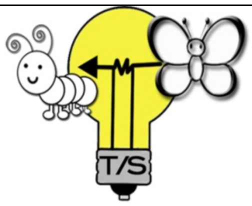
Talk Aloud Many students sit and try to do a problem in complete silence inside their heads. They think that solutions just pop into the heads of ‘smart’ people. You absolutely must learn to talk aloud and listen to yourself, literally to talk yourself through a problem. Successful students do this without realising. It helps to structure your thoughts while helping your tutor understand the way you think.

BackChecking This means that you will be doing every step of the question twice, as you work your way through the question to ensure no silly mistakes. For example with this question: $3 \times 2 - 5 \times 7$ you would do “3 times 2 is 5 ... let me check – no 3×2 is 6 ... minus 5 times 7 is minus 35 ... let me check ... minus 5×7 is minus 35. Initially, this may seem time-consuming, but once it is automatic, a great deal of time and marks will be saved.

Avoid Cosmetic Surgery Do not write over old answers since this often results in repeated mistakes or actually erasing the correct answer. When you make mistakes just put one line through the mistake rather than scribbling it out. This helps reduce silly mistakes and makes your work look cleaner and easier to backcheck.



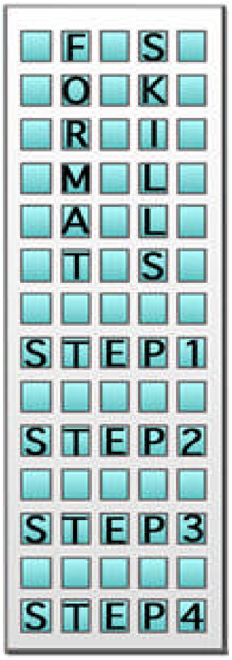
Pen to Paper It is always wise to write things down as you work your way through a problem, in order to keep track of good ideas and to see concepts on paper instead of in your head. This makes it easier to work out the next step in the problem. Harder maths problems cannot be solved in your head alone – put your ideas on paper as soon as you have them – always!



Transfer Skills This strategy is more advanced. It is the skill of making up a simpler question and then transferring those ideas to a more complex question with which you are having difficulty.

For example if you can’t remember how to do long addition because you can’t recall exactly how to carry the one: $\begin{array}{r} 5889 \\ +4587 \\ \hline \end{array}$ then you may want to try adding numbers which you do know how to calculate that also involve carrying the one: $\begin{array}{r} 5 \\ +9 \\ \hline \end{array}$

This skill is particularly useful when you can’t remember a basic arithmetic or algebraic rule, most of the time you should be able to work it out by creating a simpler version of the question.

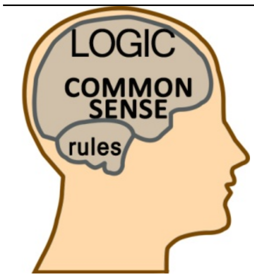


Format Skills These are the skills that keep a question together as an organized whole in terms of your working out on paper. An example of this is using the "=" sign correctly to keep a question lined up properly. In numerical calculations format skills help you to align the numbers correctly.

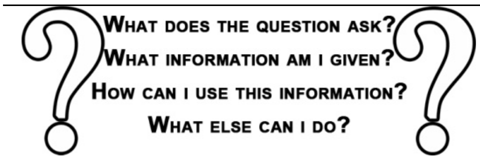
This skill is important because the correct working out will help you avoid careless mistakes. When your work is jumbled up all over the page it is hard for you to make sense of what belongs with what. Your "silly" mistakes would increase. Format skills also make it a lot easier for you to check over your work and to notice/correct any mistakes.

Every topic in math has a way of being written with correct formatting. You will be surprised how much smoother mathematics will be once you learn this skill. Whenever you are unsure you should always ask your tutor or teacher.

Its Ok To Be Wrong Mathematics is in many ways more of a skill than just knowledge. The main skill is problem solving and the only way this can be learned is by thinking hard and making mistakes on the way. As you gain confidence you will naturally worry less about making the mistakes and more about learning from them. Risk trying to solve problems that you are unsure of, this will improve your skill more than anything else. It's ok to be wrong – it is NOT ok to not try.



Avoid Rule Dependency Rules are secondary tools; common sense and logic are primary tools for problem solving and mathematics in general. Ultimately you must understand <u>Why</u> rules work the way they do. Without this you are likely to struggle with tricky problem solving and worded questions. Always rely on your logic and common sense first and on rules second, always ask <u>Why?</u>



Self Questioning This is what strong problem solvers do naturally when they get stuck on a problem or don't know what to do. Ask yourself these questions. They will help to jolt your thinking process; consider just one question at a time and <u>Talk Aloud</u> while putting <u>Pen To Paper</u>.

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Number

Useful formulae and hints

A factor is a number that divides into a given number equally. For example, the factors of 12 are 1, 2, 3, 4, 6 and 12

A multiple is a number that a given number divides into evenly. For example, the multiples of 4 are 4, 8, 12, 16, 20...

$$a + (-b) = a - b$$

$$5 + (-3) = 5 - 3 = 2$$

$$a - (+b) = a - b$$

$$5 - (+3) = 5 - 3 = 2$$

$$a - (-b) = a + b$$

$$5 - (-3) = 5 + 3 = 8$$

$$(-a) \times (-b) = a \times b$$

$$(-5) \times (-3) = 15$$

$$a \times (-b) = (-a) \times b = -(a \times b)$$

$$3 \times (-5) = (-3) \times 5 = -(3 \times 5) = -15$$

$(-a) \div (-b) = a \div b$

$(-15) \div (-3) = 5$

$a \div (-b) = (-a) \div b = -(a \div b)$

$15 \div (-3) = (-15) \div 3 = -(15 \div 3) = -5$

Order of operation is:

Brackets

Indices

Multiplication

Division

Addition

Subtraction

To add mixed numerals, add the whole numbers then add the fractions. Simplify if necessary.

Examples:

$$2\frac{1}{2} + 3\frac{1}{4} = 5 + \frac{1}{2} + \frac{3}{4} = 5\frac{3}{4}$$

$$2\frac{2}{3} + 3\frac{1}{2} = 5 + \frac{2}{3} + \frac{1}{2} = 5 + \frac{7}{6} = 6\frac{1}{6}$$

When multiplying mixed numbers, convert to improper fractions, multiply as usual, simplify if necessary

Example

$$2\frac{1}{2} \times 3\frac{1}{3} = \frac{5}{2} \times \frac{10}{3} = \frac{50}{6} = 8\frac{2}{6} = 8\frac{1}{3}$$

When dividing mixed numbers, convert to improper fractions, invert second fraction and multiply the two fractions as above, simplify

Example

$$5\frac{1}{4} \div 1\frac{1}{2} = \frac{17}{3} \div \frac{3}{2} = \frac{21}{4} \times \frac{2}{3} = \frac{42}{12} = \frac{7}{2} = 3\frac{1}{2}$$

To find a fraction of a mixed numeral, convert the mixed numeral to an improper fraction, multiply as above, simplify

Example

$$\frac{2}{5}\ of\ 6\frac{2}{3} = \frac{2}{5} \times \frac{20}{3} = \frac{40}{15} = \frac{8}{3} = 2\frac{2}{3}$$

To change a fraction to a percentage, multiply the fraction by 100

To change a percentage to a fraction, divide the number by 100

To change a fraction to a decimal, divide the numerator by the denominator

To change a decimal to a fraction, divide the number (without the decimal point) by 10, 100, 1000, etc where the number of zeroes is equal to the number of figures after the decimal point. Simplify the fraction if necessary

Examples: $0.2 = \frac{2}{10} = \frac{1}{5}$

$$0.11 = \frac{11}{100}$$

$$0.255 = \frac{255}{1000} = \frac{51}{200}$$

To change a fraction to a recurring decimal, divide as above and determine where repeating pattern commences. Use correct dot notation to denote recurring section

Examples

$$\frac{2}{9} = 0.22222\ \ldots. = 0.\dot{2}$$

$$\frac{1}{11} = 0.090909\ \ldots = 0.\dot{0}\dot{9}$$

$$\frac{3}{7} = 0.428571428571\ \ldots. = 0.\dot{4}\dot{2}\dot{8}\dot{5}\dot{7}\dot{1}$$

$$\frac{11}{15} = 0.73333\ \ldots = 0.7\dot{3}$$

There are rules that can be used to determine if numbers are divisible by 2, 3, 5, and 9. The finding of these rules is an exercise contained in the questions

The probability of an event occurring $= \frac{Number\ of\ favorable\ outcomes}{Total\ possible\ outcomes}$

Example: probability of drawing a black two from a deck of cards $= \frac{2}{52} = \frac{1}{26}$

The complement of an event is the probability of an event not occurring, and mathematically, probability of event not occurring $= 1 -$probability of event occurring

Example: probability of NOT drawing a black 2 from a deck of cards = $1 -$probability of drawing black two $= 1 - \frac{1}{26} = \frac{25}{26}$

A unit fraction shows one part out the total number of parts. For example, ½ means one part out of two

To add or subtract decimals, line up the two numbers according to their decimal points, then add or subtract as normal, carrying the decimal point down to the same place in the answer

Exercise 1

Number Groups & Families

1) Evaluate the following

a) $(2 \times 3)^2$

b) $2^2 \times 3^2$

c) $(3 \times 2)^3$

d) $3^3 \times 2^3$

e) $(5 \times 5)^2$

f) $5^2 \times 5^2$

g) Use your results above to rewrite the following

$(a \times b)^2 = \quad \times$

2) Evaluate the following

a) $\sqrt{36}$

b) $\sqrt{9 \times 4}$

c) $\sqrt{9} \times \sqrt{4}$

d) $\sqrt{400}$

e) $\sqrt{16 \times 25}$

f) $\sqrt{16} \times \sqrt{25}$

g) Use your results above to rewrite the following

$\sqrt{a \times b} = \sqrt{\quad} \times \sqrt{\quad}$

h) Use your results to simplify

$\sqrt{900} = \sqrt{\quad} \times \sqrt{\quad} =$

3) Evaluate the following

a) 31×9

b) $(31 \times 10) - 31$

c) 17×9

d) $(17 \times 10) - 17$

e) 47×9

f) $(47 \times 10) - 47$

g) $a \times 9 = (\quad) - (\quad)$

4) Evaluate the following

a) $2 \times 15 \times 5$

b) 15×10

c) $2 \times 31 \times 5$

d) 31×10

e) $2 \times 83 \times 5$

f) 83×10

g) $2 \times a \times 5 =$

5) Determine the highest common factor of:

a) $6 \text{ and } 14$

b) $24 \text{ and } 33$

c) $18 \text{ and } 81$

d) $42 \text{ and } 77$

e) $17 \text{ and } 19$

f) $12, 36, \text{ and } 48$

g) $12, 36, \text{ and } 54$

6) Determine the lowest common multiple of the following

a) $2 \text{ and } 3$

b) $4 \text{ and } 5$

c) $4 \text{ and } 6$

d) $11 \text{ and } 13$

e) $8 \text{ and } 12$

f) $2, 3, \text{ and } 9$

g) $6, 9, \text{ and } 108$

7) Complete the following sequence

0, 1, 1, 2, 3, 5, 8, 13, ____, ____, ____, ____

a) What is this famous sequence called?

b) How is each term in the sequence generated?

c) Starting with the second and third terms, divide each term by the term before it, and list your answers in a sequence

d) Describe the sequence

Exercise 2

Directed Numbers

1) Evaluate the following

a) $105 + (-15)$

b) $226 + (-12)$

c) $174 + (-32)$

d) $275 - (-45)$

e) $410 - (-19)$

f) $399 - (-33)$

2) Evaluate the following

a) $-505 + (-25)$

b) $-386 + (-43)$

c) $-401 + (-1)$

d) $-137 - (23)$

e) $-294 - (16)$

f) $-412 - (78)$

g) $-299 - (-15)$

h) $-505 - (-35)$

i) $-999 - (-1)$

3) Evaluate the following

a) $32 \times (-5)$

b) $14 \times (-3)$

c) $5 \times (-8)$

d) -9×11

e) -6×9

f) -15×15

g) $-9 \times (-7)$

h) $-11 \times (-8)$

i) $-13 \times (-5)$

4) Evaluate the following

a) $-81 \div 9$

b) $-64 \div 4$

c) $-100 \div 10$

d) $-54 \div -9$

e) $-66 \div -11$

f) $-36 \div -12$

g) $72 \div (-8)$

h) $144 \div (-12)$

i) $99 \div (-11)$

5) Use order of operations to simplify and evaluate the following

a) $17 \times (-5) + (-4)$

b) $-12 \div (-4) - 8$

c) $80 + (-15) \div (-5)$

d) $3 \times (-12) \times (2) \div (-6) - (8)$

e) $1 + (4) \times (-2)^2 - 6$

f) $-2 \times (-10) + 2^4 - 4$

g) $6 - 6 \times (6 + 6)$

6) To convert from degrees Fahrenheit to degrees Celsius, the following formula is applied

$$℃ = (°\mathrm{F} - 32) \times 5 \div 9$$

Use the formula to convert the following Fahrenheit temperatures to Celsius

a) 86

b) 131

c) 68

d) 212

e) 32

f) --22

7) A medicine has to be administered to a patient according to his weight in kg and the following formula

$$(-0.4) \times weight \times (-10) + weight \div 10$$

What dosage should be given to patients who weigh:

a) 40kg

b) 60kg

c) 80kg

d) 100kg

e) 200kg

Exercise 3

Fractions & Mixed Numerals

1) Add the following

a) $2\frac{1}{3}+3\frac{1}{3}$

b) $1\frac{1}{5}+4\frac{2}{5}$

c) $\frac{1}{2}+3\frac{1}{4}$

d) $4\frac{1}{6}+3\frac{5}{6}$

e) $2\frac{2}{3}+1\frac{2}{3}$

f) $5\frac{3}{4}+1\frac{3}{4}$

2) Add the following

a) $3\frac{1}{2}+2\frac{1}{3}$

b) $1\frac{2}{3}+1\frac{1}{4}$

c) $2\frac{1}{2}+3\frac{2}{3}$

d) $4\frac{3}{5}+1\frac{3}{4}$

e) $2\frac{3}{4}+2\frac{2}{5}$

f) $1\frac{5}{6}+3\frac{3}{7}$

3) Multiply the following

a) $1\frac{1}{2}\times 2\frac{1}{3}$

b) $3\frac{1}{3}\times 2\frac{2}{3}$

c) $3\frac{3}{4}\times 1\frac{1}{2}$

d) $4\frac{1}{4}\times 2\frac{3}{5}$

1

e) $1\frac{1}{3}\times 2\frac{3}{4}$

f) $2\frac{2}{5}\times 2\frac{1}{5}$

4) Divide the following

a) $1\frac{1}{2}\div 2\frac{1}{3}$

b) $3\frac{1}{3}\div 2\frac{2}{3}$

c) $3\frac{3}{4}\div 1\frac{1}{2}$

d) $4\frac{1}{4}\div 2\frac{3}{5}$

1

e) $1\frac{1}{3}\div 2\frac{3}{4}$

f) $2\frac{2}{5}\div 2\frac{1}{5}$

5) Calculate the following

a) $5-\frac{2}{3}$

b) $2-\frac{3}{4}$

c) $4-\frac{7}{3}$

d) $2-\frac{3}{2}$

e) $3-\frac{8}{5}$

6) Calculate the following

a) $\frac{3}{4}\ of\ 2\frac{1}{2}$

b) $\frac{1}{3}\ of\ 3\frac{6}{7}$

c) $\frac{1}{2}\ of\ 2\frac{2}{5}$

d) $\frac{2}{5}\ of 3\frac{5}{8}$

e) $\frac{5}{2}\ of\ \frac{8}{15}$

7) Eric ran two and a half laps of the running track, while Peter ran two fifths of Eric's distance

a) How many laps did they run combined?

b) If Eric ran exactly 1km, how far did Peter run?

c) What is the length of one lap?

d) What was the total distance run by the two of them?

8) Brian and John are doing a joint project. In thirty minutes Brian wrote out one and a half pages of work, while John did three quarters of what Brian did

a) How many pages had John finished?

b) How many pages had they finished in total?

c) If the project needed 5 pages, what fraction of it have they finished?

Exercise 4

Decimals, Fractions, & Percentages

1) Convert the following fractions to decimals

a) $\frac{3}{4}$

b) $\frac{3}{5}$

c) $\frac{7}{20}$

d) $\frac{1}{8}$

e) $\frac{9}{40}$

f) $\frac{11}{50}$

g) $\frac{9}{1000}$

h) $\frac{3}{500}$

2) Convert the following decimals to fractions

a) 0.24

b) 0.4

c) 0.375

d) 0.45

e) 0.08

f) 0.002

g) 0.8

h) 0.14

3) Convert the following fractions to percentages

a) $\frac{3}{8}$

b) $\frac{9}{50}$

c) $\frac{1}{12}$

d) $\frac{1}{20}$

e) $\frac{4}{25}$

f) $\frac{3}{40}$

g) $\frac{3}{1000}$

h) $\frac{9}{60}$

4) Convert the following percentages to fractions

a) 32%

b) 17.5%

c) 2%

d) 0.85%

e) 80%

f) 72%

g) 12.5%

h) 8%

5) Convert the following decimals to percentages

a) 0.11

b) 0.4

c) 0.001

d) 0.35

e) 0.125

f) 0.375

g) 0.101

h) 0.0005

6) Convert the following percentages to decimals

a) 42.5%

b) 18%

c) 0.03%

d) 37.5%

e) 100%

f) 12%

g) 7.5%

h) 1%

7) Convert the following fractions to decimals, using the correct notation

a) $\frac{33}{100}$

b) $\frac{1}{6}$

c) $\frac{1}{9}$

d) $\frac{2}{33}$

e) $\frac{3}{11}$

f) $\frac{7}{15}$

g) $\frac{5}{33}$

h) $\frac{5}{21}$

8) A shop is offering 20% off the marked price of shoes. If the marked price is $85, what is the discounted price?

9) The rainfall for March this year in Canberra was 15% more than for March last year. If last year's rainfall was 300mm, how much rainfall did Canberra receive this March?

10) Alex went on a diet and lost 12.5% of his weight. If he weighed 116 kg before going on his diet, how much does he weigh now?

11) GST adds 10% on certain purchases as a tax. If an item costs $12.50 before GST, how much does it cost after GST?

12) Andrew bought an item for $80 and later sold it for $100

a) How much was his profit?

b) What was his profit as a percentage of his cost?

c) What was his profit as a percentage of his selling price?

13) Karen bought an item for $60 and later sold it for $80

a) How much was her profit?

b) What was her profit as a percentage of his cost?

c) What was her profit as a percentage of her selling price?

14) Bill bought an item for $200 and later sold it for $300

a) How much was his profit?

b) What was his profit as a percentage of his cost?

c) What was his profit as a percentage of his selling price?

15) (Challenge question) Use your answers from questions 12-14 to investigate the relationship between profit as a percentage of cost price and profit as a percentage of selling price. (Hint: Change the percentage profits on cost and selling to fractions)

Exercise 5

Mental Division Strategies

1) Which of the following numbers are exactly divisible by 2?

a) 2

b) 8

c) 13

d) 1

e) 240

f) 0

2) Which of the following numbers are exactly divisible by 3?

a) 3

b) 15

c) 23

d) 1

e) 312

f) 0

3) Which of the following numbers are exactly divisible by 5?

a) 5

b) 40

c) 47

d) 1

e) 95

f) 0

4) Which of the following numbers are exactly divisible by 9?

a) 9

b) 72

c) 116

d) 1

e) 225

f) 0

5) From your answers to questions 1 to 4, state some general rules for determining if a number is divisible by 2, 3, 5, or 9

6) Perform the following divisions without the use of a calculator

a) $30 \div 10$

b) $70 \div 10$

c) $130 \div 10$

d) $980 \div 10$

e) $40 \div 20$

f) $80 \div 20$

g) $260 \div 20$

h) $980 \div 20$

7) Perform the following divisions without the use of a calculator

a) $60 \div 12$

b) $144 \div 12$

c) $228 \div 12$

d) $624 \div 12$

e) $972 \div 12$

8) Perform the following divisions without the use of a calculator

a) $125 \div 25$

b) $450 \div 25$

c) $775 \div 25$

d) $925 \div 25$

e) $975 \div 25$

9) Express the following numbers as a multiple of 18 plus a remainder. Example:$200 = 11 \times 18 + 2$

a) 246

b) 312

c) 444

d) 568

e) 926

10) Use your answers from question 9 to solve the following: Example: $200 = 18 \times 11 + 2 = 11\frac{2}{18} = 11\frac{1}{9}$

a) $246 \div 18$

b) $312 \div 18$

c) $444 \div 18$

d) $568 \div 18$

e) $926 \div 18$

11) Calculate the following (use the method from questions 9 and 10)

a) $136 \div 44$

b) $222 \div 44$

c) $612 \div 44$

d) $742 \div 44$

e) $906 \div 44$

Exercise 6

Chance

1) List the sample space for the event “rolling a six sided die”

2) A normal six sided die is thrown. What is the probability of rolling

a) The number two

b) The number 6

c) The number 3

3)

a) What is the probability of rolling any particular number on a six sided die?

b) What is the sum of all possible rolls of a six side die?

c) What is the probability that upon rolling a normal six side die, one of the numbers on the die is rolled?

4) A six side die is rolled. What is the probability of

a) Rolling an even number

b) Rolling an odd number

c) Not rolling an even number

d) Rolling a number that is a multiple of 3

e) Rolling a number that is not a multiple of 3

5) Using your answers above, if the probability of a certain event happening is $\frac{11}{23}$, what is the probability of the event NOT happening

6)

a) What is the probability of drawing the ace of hearts from a standard deck of 52 cards?

b) What is the probability of drawing any card other than the ace of hearts?

c) What is the probability of drawing a heart from a standard deck of 52 cards?

d) What is the probability of NOT drawing a heart?

7) A man is told by his doctor that out of 250 people with his condition studied, 99 have died within 3 months. Should the man be optimistic about his chance of survival? Explain

8) A bag contains 100 yellow discs. What is the probability that a disc chosen at random will NOT be yellow?

9) A bag contains 24 discs: 9 green, 6 yellow, 5 blue and 4 red discs. If a disc is chosen at random, what is the probability that it is NOT blue?

10) A standard six sided die is rolled. What is the probability that the number rolled is NOT the number 9?

Year 8 Mathematics

Algebra

Useful formulae and hints

To add variables, the variable notation must be the same. Just as you cannot (meaningfully) add two dogs to three oranges, you cannot add say $2x$ to $3y$

$2x + 3y$ is as simplified as the algebraic expression can be

Just as two dogs plus 3 dogs equals 5 dogs, so $2x + 3x = 5x$

The variable must be exactly the same, since it is unique

Note $2x + 3x^2$ is also invalid, since x and x^2 are different variables

When multiplying variables, the same rules as multiplying numbers are followed

Just as $2 \times 2 \times 2 = 2^3 = 8$, $y \times y \times y = y^3$: this cannot be simplified further as the value of y is unknown

$p \times r = pr$; this cannot be simplified as the values of p and r are unknown

NOTE: do not confuse the notation $\boldsymbol{pr}$ with a two digit number such as 23; $\boldsymbol{pr = p \times r}$, the $\times$ sign is removed to avoid unnecessary writing and confusion with variables

Example of converting a word rule to an algebraic expression

The temperature of an oven is 100 degrees and goes up 5 degrees every minute

Let r be the temperature of the oven, and q the number of minutes (these are the two things that change, that is they are the variables)

Simply substitute the variables for the appropriate words, and mathematical symbols for other key words

The temperature of an oven (r)

Is (=)

100 (100)

And goes up (+)

5 degrees every minute (5 x q)

$r = 100 + 5q$

To find a word rule from an algebraic sentence, reverse the above

Example: y is the number of cars in a car park, and w is the number of minutes since 9 o'clock

If $y = 20 + 2w$

Then

Y (the number of cars)

= (is equal to)

20 (20)

+2w (plus 2 times the number of minutes after 9 o'clock)

To factorize an equation, remove the common factor(s), and divide each term by that factor to leave the remainders in the brackets

Examples

$2x + 4y = 2(x + y)$ (2 is the common factor)

$2xy + 3y = y(2x + 3)$ (y is the common factor)

$2x^2 + 3xy = x(2x + 3y)$ (x is the common factor)

$2x^2y + 4xy^2 = 2xy(x + 2y)$ (2xy is the common factor)

To check if factorized completely, expression in brackets should have no common factors

To solve

$x + 3 = 5$

Subtract 3 from both sides; $x = 5 - 3 = 2$

To solve

$2(x + 3) = 8$

Remove brackets by multiplying through by common factor

$2x + 6 = 8$

Subtract 6 from both sides

$2x = 8 - 6 = 2$

Divide both sides by 2

$$x = \frac{2}{2} = 1$$

To solve

$$\frac{x + 4}{2} = 6$$

Multiply both sides by 2

$x + 4 = 6 \times 2 = 12$

Subtract 4 from both sides

$x = 12 - 4 = 8$

To solve

$5x + 3 = 2x + 15$

Subtract $2x$ from both sides of the equation

$5x + 3 - 2x = 15$

Collect like terms

$3x + 3 = 15$

Add 3 to both sides

$3x = 15 - 3 = 12$

Divide both sides by 3

$$x = \frac{12}{3} = 4$$

In all cases, substitute the solution back into the equation and test for equality

Example

$5x + 3 = 2x + 15$

If $x = 4$, does $5 \times 4 + 3 = 2 \times 4 + 15$?

(Using BIMDAS) $20 + 3 = 23 = 8 + 15$

Therefore solution is correct

To solve inequalities, follow the procedure for solving equalities above, preserving the inequality sign

EXCEPT when dividing or multiplying by a negative number in which case the inequality sign is reversed

Examples

$x + 3 < 5$

$x < 5 - 3$

$x < 2$

$3 - 2x < 11$

$-2x < 8$

$x > -4$

When graphing linear equations:

Put independent variable on the horizontal axis, and the dependent variable on the vertical axis

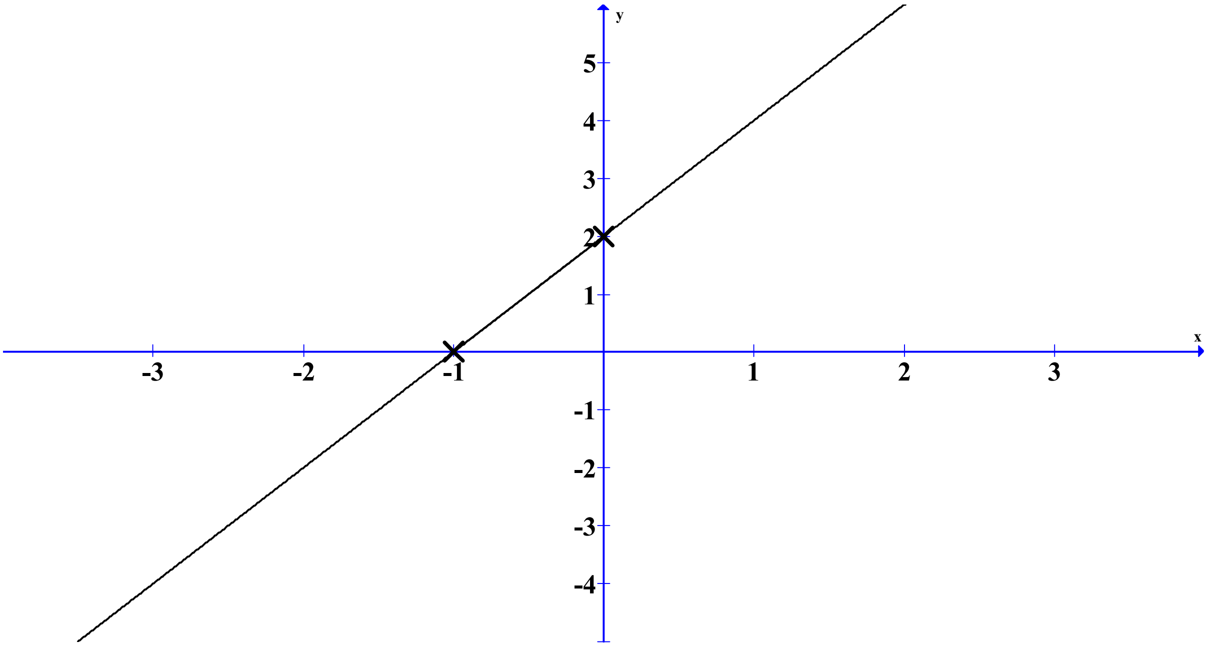
In the expression $y = 2x + 2, y$ is the dependent variable

To graph a line, only two points are needed.

The easiest points to find are the ones where the line crosses the axes, that is where $x = 0$ and $y = 0$

For $y = 2x + 2$, when $x = 0, y = 1$ and when $y = 0, x = -1$

The graph will be



Exercise 1

Equivalent Expressions

1) The expression $x + x + x + x$ is equivalent to

a) $4y$

b) $4x$

c) x^4

d) None of the above

2) The expression $3y - y$ is equivalent to

a) $2y$

b) $3y^2$

c) $4y$

d) None of the above

3) The expression $r \times r \times r$ is equivalent to

a) 3r

b) r^3

c) $r^2 + r$

d) None of the above

4) The expression $x \div y$ is equivalent to

a) xy

b) $\frac{x}{y}$

c) $x + y$

d) None of the above

5) The expression axb is equivalent to

a) $a+b$

b) $b+a$

c) Either of the above

d) Neither of the above

6) The expression $3g$ is equivalent to

a) $g+g+g$

b) $2g+g$

c) $g+2g$

d) None of the above

e) All of the above

7) The expression x^4 is equivalent to

a) x^2+x^2

b) $x \times x \times x \times x$

c) $2x \times 2x$

d) None of the above

e) All of the above

8) What must the value of x and y be so that $x \times x \times y$ is equivalent to x^2y?

9) Five boys each have the same number of lollies in their pockets. If x represents the number of lollies in the pocket of one boy, which of the following expressions could be used to calculate the total number of lollies?

a) x^5

b) $5y$

c) $5x$

d) $\frac{x}{5}$

10) Ten bins hold a total of x apples. If each bin holds the same number of apples, what is the expression that could be used to calculate the number of apples in each bin?

a) x^{10}

b) $\frac{x}{10}$

c) 10x

d) $10x - 10$

Exercise 2

Rules, Patterns & Tables of Values

1) The temperature in a room is 32 degrees. An air conditioner is turned on and cools the room down at the rate of 2 degrees per minute for 7 minutes

a) Construct a table showing the temperature of the room for the seven minutes

b) What temperature will the room be after 3 minutes?

c) How many minutes will it take the room to cool down to 22 degrees?

d) Describe a rule in words that relates the temperature of the room to the number of minutes that the air conditioner has been turned on

e) Write the above rule algebraically

2) Tom has \$20 in his bank account. Each week he deposits \$5 of his pocket money into the account

a) Construct a table that shows how much money is in Tom's account for the next 6 weeks

b) How much money will be in Tom's account after 4 weeks?

c) After how many weeks will Tom have \$80 in his account?

d) Write a rule in words that describes how to calculate the amount of money in Tom's account after a certain number of weeks

e) Write the above rule algebraically

3) A tap is dripping water into a bucket. The amount of water in the bucket is equal to 500 mL plus 10 mL per minute that the tap has been dripping

a) Write the rule algebraically

b) Construct a table that shows the amount of water in the bucket from 0 to 10 minutes

c) Use the rule to calculate the amount of water in the bucket after one hour

4) A mechanic charges a $20 call out fee plus $25 per hour

a) Write this rule algebraically

b) How much does the mechanic charge for 5 hours work?

c) Construct a table that shows the above information

5) If x represents the number of bricks in a wall, and y represents the number of hours a man has been building the wall for

a) Translate the algebraic expression $x = 100 + 50y$ into a word expression

b) How many bricks per hour does the man use?

c) How many bricks were already in the wall when the man started working?

d) How many bricks will be in the wall after 4 hours?

6) If t represents the number of pellets of fish food in a tank, and w represents the number of fish in the tank

a) Translate the expression $t = 200 - 10w$ to a word expression

b) Construct a table that shows the number of pellets left for each quantity of fish in the tank

c) How many pellets were in the tank before there were any fish?

d) How many pellets does each fish eat?

e) How many fish will be in the tank when the food runs out?

7) A taxi charges 2 dollars when a passenger first gets in, and $1.50 per kilometre for the journey.

a) If t is the total charge for a journey, and y represents the number of kilometres travelled, write an algebraic expression for the total cost of a journey

b) Construct a table that shows the total cost for journeys from 1km to 10km

c) How much would a journey of 20km cost?

d) A man has $23, how far can he travel?

8) A car salesman gets paid a certain amount per day plus another amount per sale (commission). The table below shows how much he earns in a day, for each number of sales

Number of cars sold	1	2	3	4	5	6	7	8
Total pay	150	200	250	300	350	400	450	500

a) How much does he get paid even if he doesn't manage to sell a car?

b) How much commission does he get per car sold?

c) Write a word expression that describes his total pay in terms of number of cars sold

d) Write the word expression in algebraic form

e) How much would the salesman earn if he were lucky enough to sell 20 cars in one day?

Exercise 3

Factorization

1) Simplify the following expressions by removing the common factor

a) $2x + 4$

b) $6 + 3y$

c) $5x - 10$

d) $6 - 8t$

e) $4x + 6y + 2$

2) Remove the common factor from the following expressions

a) $ax + xy$

b) $6b - 3a$

c) $2x + 3xy$

d) $4pq + 2q$

e) $6x - y$

3) Remove the common factor from the following expressions

a) $3xy + 6y + 9$

b) $2tac + 6ta + 4a$

c) $5pq - 10qr + 15q$

d) $ab + 6b - bc$

e) $abc + bc - a$

4) Factorize the following expressions

a) $3x^2 + 6x$

b) $2y^2 + 4y$

c) $5a - 10a^2$

d) $7x - 5x^2$

e) $xy^2 + x^2y$

5) Factorize the following expressions

a) $x^2 - 2xy + xy^2$

b) $abc + a^2b^2c^2 + b^2c$

c) $rx^2 - rx + r^2x^2$

d) $a - ab - b$

e) $3y + 6y^2 - 9y^3$

6) The following expressions have been factorized, what were the original equations?

a) $2(x - 3)$

b) $5(1 - 2y)$

c) $2(2x - y)$

d) $a(x - 2)$

e) $2r(r + 3)$

7) The following expressions have been factorized, what were the original expressions

a) $ab(a - b + 2)$

b) $x^2(3 + 2y)$

c) $abc(a + b + c)$

d) $\frac{x}{2}(2x - 5)$

e) $\sqrt{x}(2 - \sqrt{x})$

8) The following expressions have been factorized incorrectly. Complete the factorization

a) $x(2x - 4y)$

b) $y(2xy - 3x)$

c) $5(2x^2 + 4x)$

d) $2p(3 - 6p)$

e) $x^2(3y - 6y^2)$

9) A rectangle has side lengths of 4cm and $(x + 1)$cm

a) What is the area of the rectangle in terms of its measurements?

b) Remove the brackets and express the area in terms of x

c) Assume $x = 3$cm. Calculate the area of the rectangle by:

- Calculating the length and using the formula area = length x width
- Substituting the value into the equation obtained in part a
- Substituting the value into the equation obtained in part b
- What do you notice about your three answers?

10) The base of a triangle has a length of $(x - 4)$cm, and a vertical height of 3cm

a) What is the area of the triangle in terms of its measurements?

b) Remove the brackets and express the area in terms of x

c) Assume $x = 8$cm. Calculate the area of the triangle by:

- Calculating the base and using the formula area = ½ base x height

- Substituting the value into the equation obtained in part a

- Substituting the value into the equation obtained in part b

- What do you notice about your three answers?

Exercise 4

Solving Linear Equations & Inequalities

1) Solve the following equations

a) $x + 2 = 7$

b) $y - 4 = 8$

c) $r + 1 = -3$

d) $x + 4 = 1$

e) $-y + 2 = 6$

2) Solve the following equations

a) $2(x - 2) = 6$

b) $3(y + 4) = 6$

c) $5(2 - x) = 10$

d) $\frac{1}{2}(2x + 1) = 5$

e) $3\left(\frac{1}{2}x - 1\right) = 12$

3) Solve the following equations

a) $\frac{x-2}{3} = 4$

b) $\frac{x+1}{2} = 5$

c) $\frac{3-y}{4} = 1$

d) $\frac{x-4}{2} = \frac{1}{2}$

e) $3x = \frac{x}{x+1}$

4) Solve the following equations

a) $2x - 4 = x + 2$

b) $3r + 1 = r - 5$

c) $\frac{1}{2}x + 1 = \frac{3}{2}x - 3$

d) $4y + \frac{1}{2} = 2y + 2$

e) $4 - 3t = t - 2$

5) If you double the number of lollies in a bag and add 6 to the result you get 18. How many lollies in the bag?

6) If you triple Peter's age and subtract 12, you get 48. How old is Peter?

7) A car park is currently holding half its maximum capacity plus 6. If its maximum capacity is 100, how many cars are currently in the car park?

8) I ran a lap of a running track. My friend's time was equal to half my time minus 20 seconds. If my friend's time was 40 seconds, how long did it take me to run the lap?

9) Solve the following inequalities

a) $2y \leq 6$

b) $3x > 9$

c) $\frac{1}{2}x \geq 3$

d) $2t - 4 < 6$

e) $x + 2 \geq -4$

f) $2p - 3 \leq -5$

10) Show the solutions to question 9 on a number line

11) The density of an object is calculated by dividing its mass by its volume; $\rho = \frac{M}{V}$. What is the density of an object of mass 10kg and volume of $5m^3$?

12) The two smaller sides of a right-angled triangle measure 6 cm and 8 cm respectively. Express the length of the hypotenuse in a formula and calculate its length

13) Degrees Celsius is converted from degrees Fahrenheit by the formula

$$C = \frac{5}{9}(F - 32)$$

a) Convert 68 degrees F to C

b) Convert 77 degrees F to C

c) Convert 32 degrees F to C

d) Convert 100 degrees F (the boiling point of water) to C

e) Convert 30 degrees **C to F**

Exercise 5

Graphing Linear Equations

1) For each of the following equations, construct a table of values for values of x from $x = -3$ to $x = 3$

a) $y = x + 2$

b) $y = 2x + 1$

c) $y = -x + 4$

d) $y = -2x - 2$

e) $y = 4x + 1$

f) $y = 2$

2) On the same grid, draw a graph for each part of question 1, and label your lines

3) Graph the following equations on the same grid, and comment on their similarities

a) $y = 2x + 4$

b) $y = 2x - 1$

c) $y = 2x$

d) $y = 2x + 2$

e) $y = 2x + 1$

4) From your answers to question 3, which of the following equations would produce a line parallel to them?

- $y = 3x - 2$
- $y = x + 4$
- $y = 2x - 7$
- $y = -2x + 4$
- $y = 2x + 100$
- $y = \frac{1}{2}x$

5) Graph the following pair of equations, and determine their point of intersection

$y = 2x + 1$ and $y = x + 2$

6) Graph the following pair of equations and determine their point of intersection

$y = -x + 3$ and $y = 3x + 7$

7) Graph the following pair of equations and determine their point of intersection

$y = 4$ and $y = x - 1$

8) Graph the following pair of equation and determine their point of intersection

$y = 3x + 1$ and $y = 3x - 1$

9)

a) Write two equations for which their lines intersect at one point

b) Write two equations for which their lines do not intersect

c) Write two equations for which the lines intersect at more than one point

Year 8 Mathematics

Data

Useful formulae and hints

A frequency distribution table shows the number of times a certain value or score appears in a data set. It is useful when there are a large number of repeating scores in a set of data.

A grouped frequency distribution table is used to group large data sets when there are few repeating values. A frequency table or graph of such a set would be large and time consuming to construct.

Values are grouped into classes or intervals that are of equal size. The size of the class can be determined from the range of data and number of scores. There are various formulae for determining this, but at this level it is better to get a feel for the size of the group.

Example:

For a data set with 80 scores in the 0 to 100 range, it would be sensible to group the data into sets of 10, that is all scores from 0 to 9.9 in one group, 10 to 19.9 in another and so on

A dot plot is similar to a frequency distribution graph, where a dot on the graph represents one occurrence of a value. The higher the quantity of dots the more of that value occur in the data set. It is useful for quickly comparing the number of times values occur in a set

A stem and leaf plot is a shorthand way of noting numbers that occur in a set of data. In a similar way to a grouped frequency table, values are divided into groups, and the stem is the common number of that group. The "leaves" are the other digits in each number

Example

1	2 3 3 4
2	1 2 8
3	2 2 4 5 5
4	1 1 1 2
5	7 8 8 9

The plot shows the numbers 12, 13, 13, 14, 21, 22, 28, 32, 32, 34, 35, 35, 41, 41, 41, 42, 57, 58, 58, and 59. The left hand values (stems) are the tens values, and the right hand values (leaves) are the units

A stem and leaf plot is also useful for finding the median of a data set, since the scores are in order and the middle score is easy to find.

An outlier is a score that is radically different to all other scores in the data set.

Example

In the data set 2, 4, 5, 6, 1000, 12, 9, 1, 2, 7, and 11

1000 is the outlier

To find the mean of a frequency distribution table, the sum of the scores must still be found.

First calculate the sum of each score by multiplying the score by the number of times it occurs. Do this for each score and ad the totals to get the sum of the data.

Divided by the total of the frequencies (the total number of scores)

Example

Score	Frequency
2	11
3	2
4	7
5	2

The sum of the scores is $(11 \times 2) + (2 \times 3) + (7 \times 4) + (2 \times 5) = 66$

There are 22 scores; the mean is $\frac{66}{22} = 3$

There are 22 scores so the median is the score that is between the 11^{th} and 12^{th} scores, which are 2 and 3. The median is 2.5

The mode can be easily seen to be 2

A scatter graph shows the relationship between two variables (usually from a real life situation) where the relationship shows a trend, but a traditional line graph cannot be drawn through the points due to their spread. The data is usually discrete (that is only certain values are valid) and each dot represents a relationship between the two variables for certain quantities. For example the age of cars and their value would show a general decrease in value as each car got older, but some cars would be better looked after than others and would therefore hold more value; some models may be worth more than others of a similar age etc.

Exercise 1

Data Representation

1) Construct a frequency distribution table for each of the following sets of data

a) 1, 4, 2, 4, 3, 5, 4, 6, 9, 4, 2, 1, 9, 7, 4

b) 8, 7, 6, 7, 10, 11, 1, 7, 4, 7

c) 15, 20, 17, 15, 20, 18, 15, 17, 20

d) 100, 105, 106, 100, 100, 105, 110, 103, 104, 105, 100

e) 1, 1, 1, 1, 1, 1, 2

2) What is the difficulty in constructing a frequency table for the following data set?

20, 30, 32, 41, 24, 23, 25, 38, 37, 36, 22, 26, 44, 33, 36, 48, 46, 38, 37.5, 41.5, 42.5, 44.5, 22.5, 33.5, 34.5, 40.5, 49.5, 53, 23.5

3) Construct a grouped frequency distribution table using a class interval of 5 for the data from question 2

4) Draw frequency histograms for each of the data sets in questions 1a to 1d

5) Draw dot plots for each of the data sets in questions 1a to 1d

6) Organise each of the following data sets into stem and leaf plots

a) 20, 23, 25, 31, 32, 34, 42, 42, 43, 26, 37, 41, 30, 25, 26, 53, 27, 33, 23, 30, 41

b) 73, 62, 66, 76, 78, 80, 83, 99, 92, 75, 74, 88, 99, 70, 71, 69, 66, 73, 81

c) 12, 10, 22, 24, 35, 46, 47, 32, 31, 43, 22, 21, 45, 56, 43, 32, 37, 49, 40, 21, 20, 30, 27, 26, 32, 21, 50, 60, 22

7) Identify the outlier in each of the following data sets

a) 1, 1, 1, 1, 1, 1, 2, 100

b) 100, 105, 3, 106, 100, 100, 105, 110, 103, 104, 105, 100

c) 73, 62, 66, 76, 78, 80, 83, 99, 92, 75, 74, 1225, 88, 99, 70, 71, 69, 66, 73, 81

d) 20, 30, 32, 41, 24, 23, 25, 38, 37, 36, 22, 26, 44, 33, 36, 48, 46, 38, 37.5, 22345, 41.5, 42.5, 44.5, 22.5, 33.5, 34.5, 40.5, 49.5, 53, 23.5

Exercise 2

Data Analysis

1) Find the mean, mode & median of the following data sets

a) 10, 7, 5, 7, 6, 3, 4, 3, 20, 7, 6, 6, 15, 14, 7

b) 4, 20, 8, 13, 12, 15, 8, 15, 18, 7, 13, 9, 20, 17, 1

c) 4, 19, 20, 16, 11, 16, 1, 10, 15, 5, 18, 17, 19, 14, 4

d) 12, 8, 2, 4, 7, 2, 1, 9, 16, 15, 17, 1, 1, 20, 14

e) 17, 5, 3, 15, 19, 12, 5, 1, 3, 11, 18, 17, 14, 1, 7

2) Find the mean mode and median from the following frequency distribution tables

a)

Value	Frequency
1	6
2	4
3	1
4	0
5	5
6	4
7	7

b)

Value	Frequency
20	2
21	2
22	2
23	3
24	2
25	4
26	3

c)

Value	Frequency
11	1
12	1
13	1
14	1
15	1
16	1
17	1

d)

Value	Frequency
1	6
2	4
3	1
4	0
5	5
6	4
7	7
1000	1

3) Using your answers to parts 2a and 2d, what effect does an outlier have on the value of the mode, mean & median?

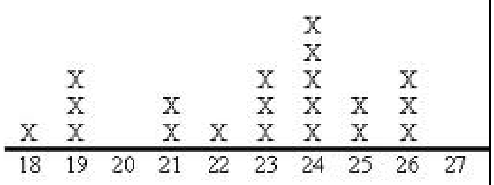
4) Represent the following test scores in a stem and leaf plot, and use it to calculate the mean, mode, median & range of the data

a) 83, 80, 48, 71, 61, 58, 47, 52, 56, 78, 86, 47, 62, 57, 77, 60, 46, 89, 81, 72

b) 48, 88, 50, 49, 54, 56, 57, 47, 48, 84, 62, 82, 69, 79, 51, 48, 89, 49, 65, 75

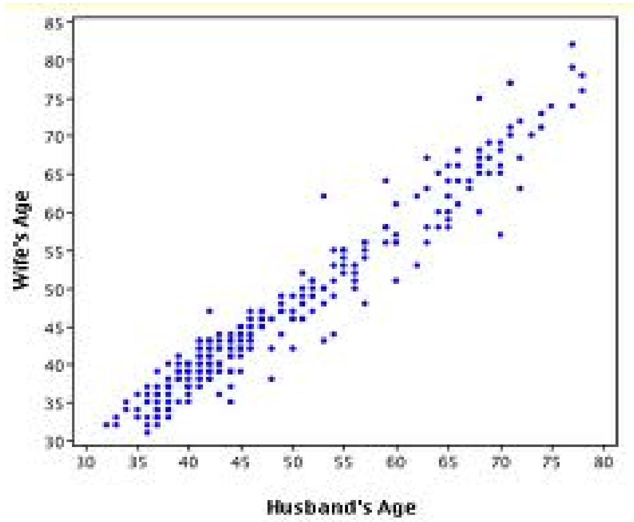
c) 74, 84, 69, 61, 79, 81, 77, 56, 50, 48, 51, 61, 90, 76, 53, 47, 56, 52, 89, 88

5) Calculate the mean, mode, median & range for the following dot plot



6) The mean of a set of data is 25, its mode is 30 (there are 10 scores of 30), and its median is 28. A new score of 200 is added to the set. What effect will this new score have on the mean, mode & median?

7) The following scatter graph shows the relative ages of husbands and wives. Each dot represents a married couple



a) Describe what conclusions can be drawn from the graph in relation to the relative ages of married couples

b) Why are there more data points toward the bottom left of the graph?

c) Would the graph appear the same if the data had been collected 1000 years ago? Explain your answer

Year 8 Mathematics

Measurement

Useful formulae and hints

Pythagoras' Theorem relates the length of the hypotenuse of a right angled triangle to the lengths of the other two sides.

This is represented mathematically by the formula $a^2 + b^2 = c^2$, where c is the length of the hypotenuse, and a and b the lengths of the other two sides

The equation can be expressed in other forms to help find the length of any side when two others (including the hypotenuse) are known

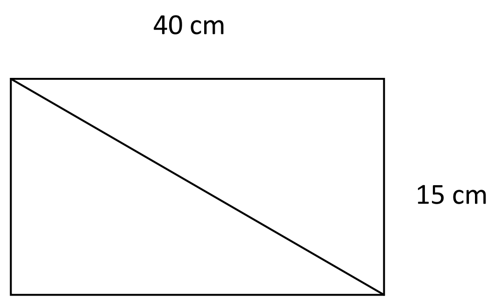
$b^2 + c^2 = a^2$, etc

To solve word problems involving Pythagoras', use or construct a diagram, determine the lines that make up a right angled triangle, and label these sides to show which is the hypotenuse

Put in the known lengths on the appropriate sides and calculate the missing value as per any problem involving Pythagoras' Theorem

Example

A computer screen measures 40cm by 15 cm. What is the length of the diagonal across it?



The diagonal forms a right angled triangle with the two known sides. The diagonal is the hypotenuse, so $d^2 = 40^2 + 15^2$, $d = \sqrt{1825} \cong 42.72\ cm$

The perimeter of a circle equals π times the diameter of the circle

The diameter also equals twice the radius

The area of a circle equals π times the square of the radius of the circle

The volume of a right cylinder equals the area of the base (which is a circle) times the height of the cylinder

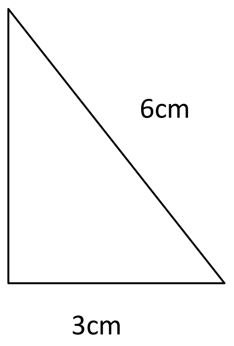
The volume of an irregular prism equals the area of the base times the height of the prism. This is true for all cylinders; normally one calculates the area of the base (since it is a rectangle, triangle, etc), but for irregular prisms the area of the base must be given.

Exercise 1

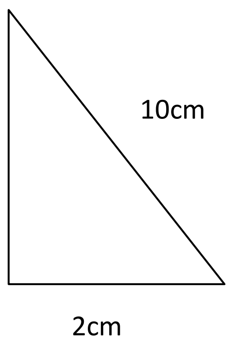
Applications of Pythagoras' Theorem

1) Calculate the length of the unknown side in the following diagrams, leaving your answer in surd form if necessary

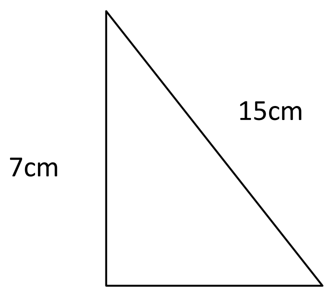
a)



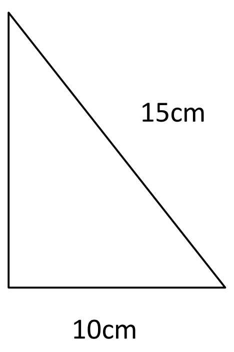
b)



c)



d) A

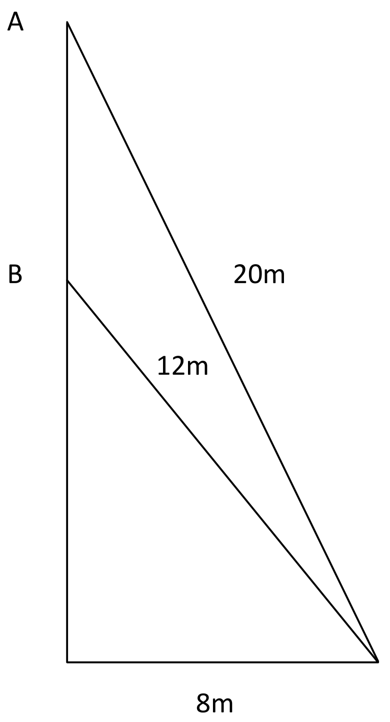


2) The equal sides of an isosceles right-angled triangle measure 8cm. What is the length of the third side?

3) A man stands at the base of a cliff which is 120 metres high. He sees a friend 100 metres away along the beach. What is the shortest distance from his friend to the top of the cliff?

4) A steel cable runs from the top of a building to a point on the street below which is 80 metres away from the bottom of the building. If the building is 40 metres high, how long is the steel cable?

5) What is the distance from point A to point B?



6) A right angled triangle has an area of 20 cm^2. If its height is 4cm, what is the length of its hypotenuse?

7) What is the length of a diagonal of a square of side length 5cm?

8) A man is laying a slab for a shed. The shed is to be 6m wide and 8m long. To check if he has the corners as exactly right angles, what should the slab measure from corner to corner?

9) A box is in the shape of a cube. If the length of each side is 4cm, what is the length of a line drawn from the top left to the bottom right of the box?

10) The path around the outside of a rectangular park is 60m long and 40m wide. How much less will the walk from one corner of the park to another be if a path is built directly across the park from corner to corner?

Exercise 2

Circles: Area & Perimeter

1) Calculate the perimeter of the following circles

- **a)** Radius of 3cm
- **b)** Diameter of 10cm
- **c)** Radius of 2cm
- **d)** Radius of 0.5cm
- **e)** Diameter of π cm

2) Calculate the area of the following circles

- **a)** Diameter of 8cm
- **b)** Radius of 2cm
- **c)** Diameter of 1cm
- **d)** Radius of π cm
- **e)** Diameter of 2π cm

3) A circular plate has an area of 40 cm^2. What is its diameter?

4) Calculate the area of a semi circle of diameter 18cm

5)

- **a)** Calculate the area and perimeter of a circle of radius 2cm (leave your answers in terms of π
- **b)** Calculate the area and perimeter of a circle of radius 4cm
- **c)** If we double the radius of a circle, by what factor does its perimeter change?
- **d)** If we double the radius of a circle, by what factor does its area change?

6)

- **a)** Calculate the area and perimeter of a circle of radius 6cm (leave your answers in terms of π)
- **b)** If we triple the radius of a circle by what factor does its perimeter change?
- **c)** If we triple the radius of a circle, by what factor does its area change?

7) Using your results from questions 5 and 6, if we multiply the radius of a circle by x, by what factor does its perimeter change, and by what factor does its area change?

8) A circle has a radius of 10cm. Every minute its radius decreases by 2cm. Compete the following table that shows the change in perimeter and area each minute (leave answers in terms of π)

Time	Radius	Perimeter	Area	Change in perimeter	Change in area
0	10	20π	100π	_____	_____
1	8	16π	64π	4π	36π
2					
3					
4					
5					

When there is a constant change in radius, does the perimeter or the area of a circle change at a constant rate?

9) A circular track field has an area of approximately 31400 square metres. The world champion bean bag thrower can hurl his bean bag 105 metres. If the bag throw is made from the centre of the field, is the field big enough for him? Explain your answer

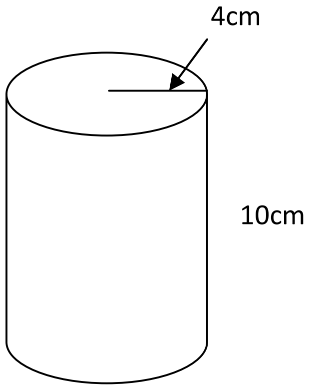
10) A painter wishes to paint a circular floor of diameter 10 metres. If one can of paint covers 20 square metres, how many cans of paint will he need? Explain your answer

Exercise 3

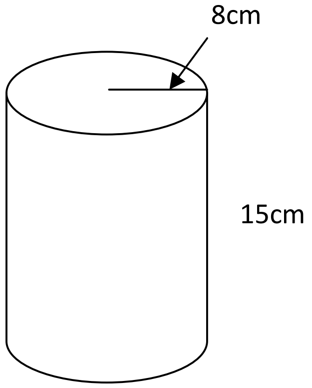
Volume of Cylinders

1) Calculate the volume of the following cylinders

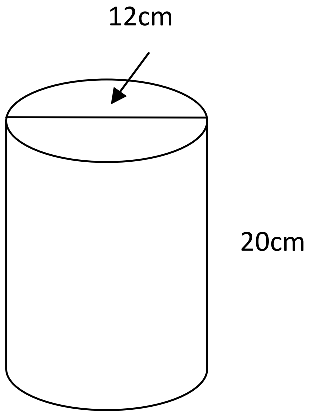
a)



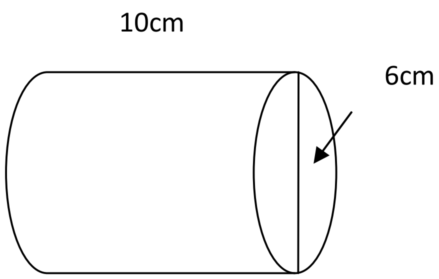
b)



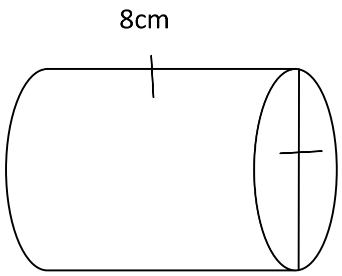
c)



d)



e)



2) Calculate the volume of a cylinder of base radius 25cm and a height of 100cm

3) Calculate the volume of a cylinder of base diameter 40cm and a height of 200cm

4) A cylinder has a height which is three times its base radius

a) Express the formula for the volume of the cylinder in terms of its radius

b) Use the formula from part a to calculate the volume if its radius is 2cm

5) What is the height of a cylinder with a volume of 240 cm^3 and a base radius of 4cm?

6) What is the base radius of a cylinder with a volume of 3000 cm^3 and a height of 200cm?

7) A tin can is in the shape of a cylinder. It has a height of 11cm and a base radius of 4cm. To the nearest ten millilitres, how many millilitres can it hold?

8) A tin can has a capacity of 500mL, and a height of 15cm. What is the approximate radius of the can?

9) A children's paddle pool is in the shape of a cylinder and holds 5000 litres of water. It has a diameter of 3m. What is its height?

10) A cylinder has a height of 8cm and a base radius of 2cm

a) Calculate its volume

b) Double its height and calculate the new volume

c) Double its radius and calculate the new volume

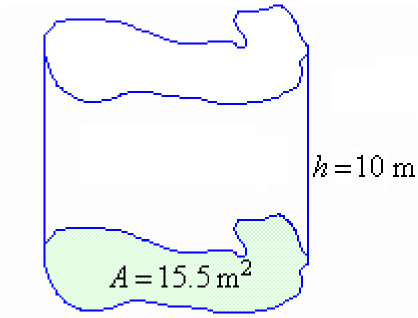
d) What effect does doubling the height of a cylinder have on its volume?

e) What effect does doubling the base radius of a cylinder have on its volume?

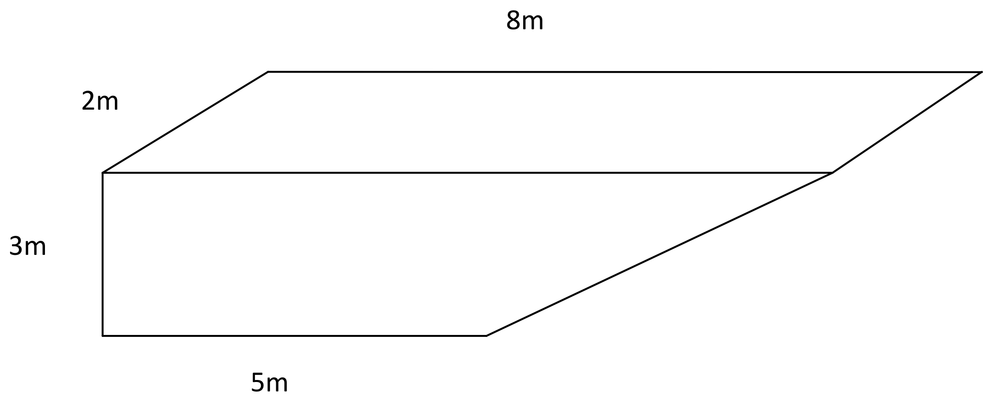
Exercise 4

Irregular Prisms

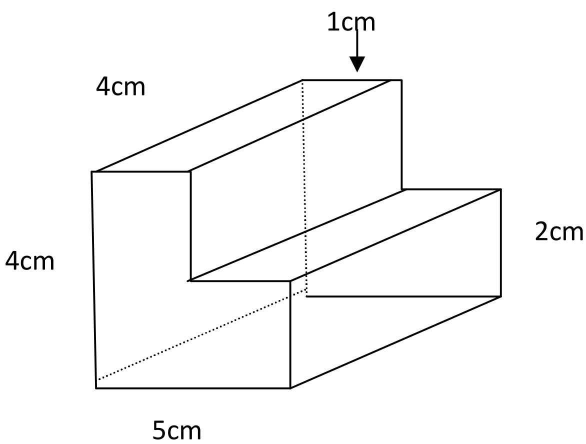
1) Calculate the volume of the following prism



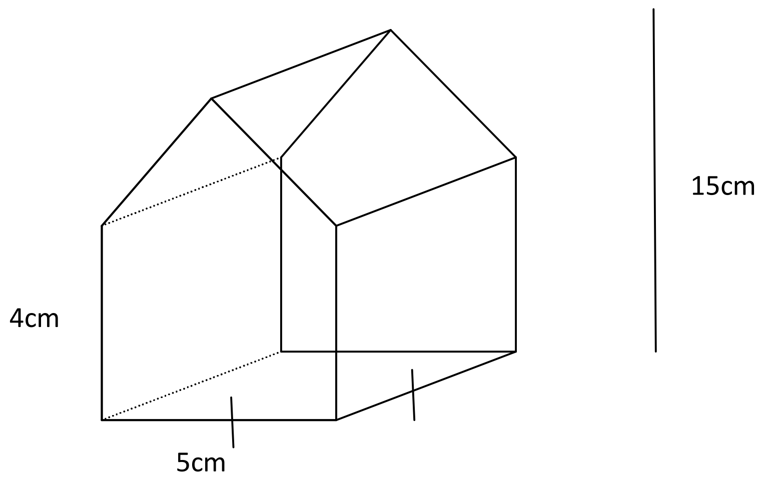
2) Calculate the volume of the following prism



3) Calculate the volume of the following prism



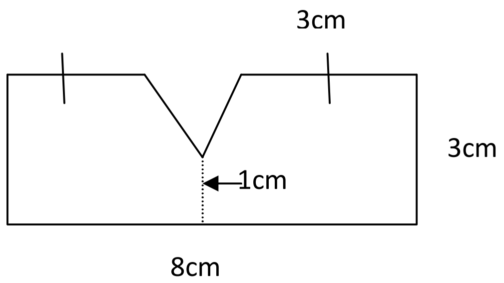
4) Calculate the following volume



5) An irregular prism has a base that has an area of 100 cm^2. If its volume is 1200 cm^3 what is its length?

6) An irregular prism has a volume of 3000 cm^3. If its height is 150 cm, what is the area of its base?

7) The following shape is the base of a prism of length 20cm. What is the volume of the prism?



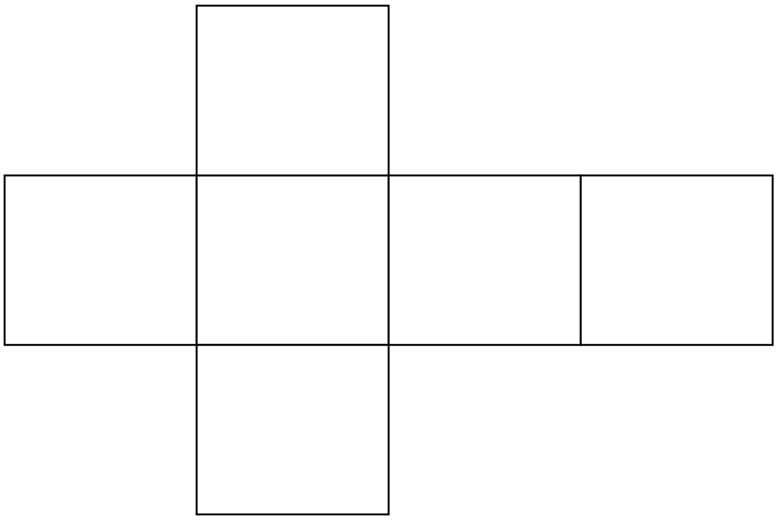
Year 8 Mathematics

Space

Useful formulae and hints

A net is a 2 dimensional representation of a 3 dimensional shape, drawn as if the shape had been unfolded and laid flat

The net of a cube is



Three dimensional shapes have edges, faces and vertices (singular vertex). Each shape has a unique number of these, and there is a special relationship between the three quantities (see exercises)

A Platonic Solid is a 3D shape where each face is the same regular polygon, and the same number of polygons meet at each vertex (corner)

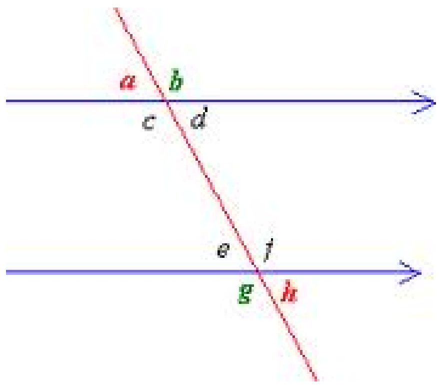
There are five Platonic Solids

- Regular Tetrahedron (Triangular pyramid)
- Regular Hexahedron (Cube)

- Octahedron
- Dodecahedron
- Icosahedron

There is a reason for the existence of exactly five Platonic Solids, which is also explored in the exercises

Transversals are lines that cut across parallel lines, forming pairs of angles that have special relationships



a and h are alternate exterior angles, and are equal

c and f are alternate interior angles, and are equal

c and e are co-interior angles, and add to 180°

e and g are complementary angles, and add to 180°

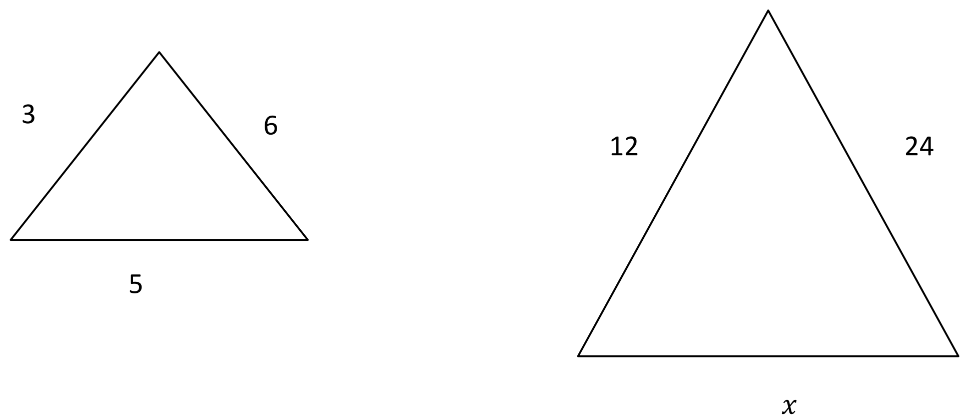
d and h are corresponding angles, and are equal

a and d are vertically opposite angles, and are equal

Supplementary angles are those that add to 90°

Shapes that are scaled versions of each other have exactly the same relationship between each pair of sides

For example; if the following triangles are scaled:



Then the scale factor is 4, since each known side in the smaller triangle has been multiplied by 4 to equal the equivalent side in the larger triangle. Therefore the value of x *is* $5 \times 4 = 20$

Exercise 1

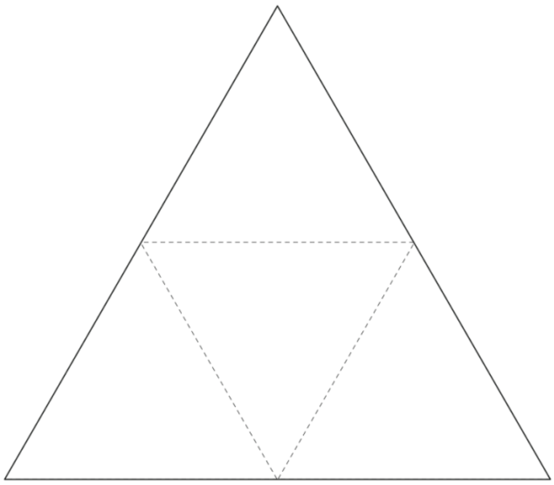
Polyhedra

1) .Complete the following table of convex polyhedra properties

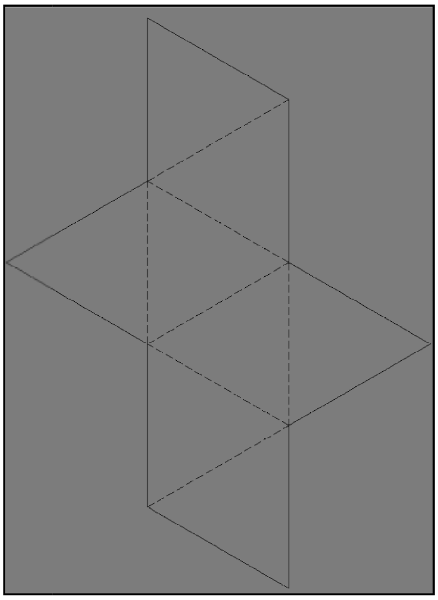
Shape	Faces	Edges	Vertices
Triangular Prism			
Rectangular Prism			
Triangular Pyramid			
Square Pyramid			
Hexagonal Prism			

2) Identify the solid formed from the following nets

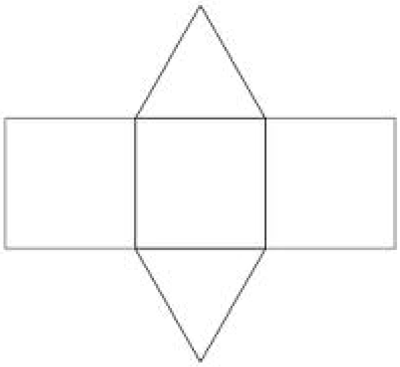
a)



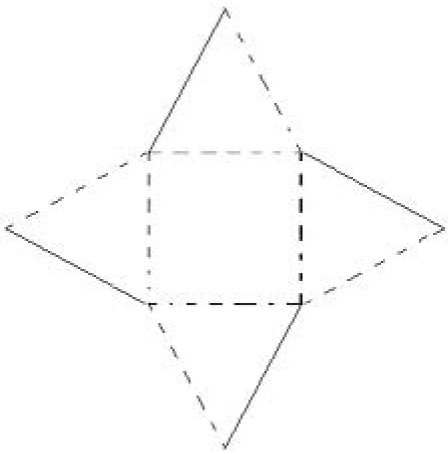
b)



c)



d)



3) Complete the following table

Name	Faces	Edges	Vertices	Made up of
Tetrahedron				
Cube				
Octahedron				
Dodecahedron				
Icosahedron				

4)

a) What is the size of each internal angle of an equilateral triangle?

b) How many triangles meet at each vertex of a tetrahedron?

c) What is the sum of the angles at each vertex of a tetrahedron?

d) How many triangles meet at each vertex of an octahedron>

e) What is the sum of the angles at each vertex of a tetrahedron?

f) How many triangles meet at each vertex of an icosahedron?

g) What is the sum of the angles at each vertex of an icosahedron?

5)

a) What is the size of each internal angle of a square?

b) How many squares meet at each vertex of a cube

c) What is the sum of the angles at each vertex of a cube?

6)

a) What is the size of each internal angle of a pentagon?

b) How many pentagons meet at each vertex of a dodecahedron?

c) What is the sum of the angles at each vertex of a dodecahedron?

7) What is the size of each internal angle of a hexagon?

8) Use your answers from questions 4 to 7 to help answer the following

a) Why are platonic solids only made up of regular triangles, squares, and pentagons?

b) Why is there a limit to the number of each shape that can join at a vertex to make a platonic solid?

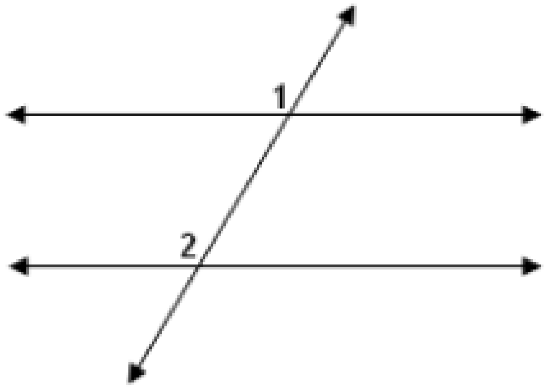
c) Why are there only 5 platonic solids?

Exercise 2

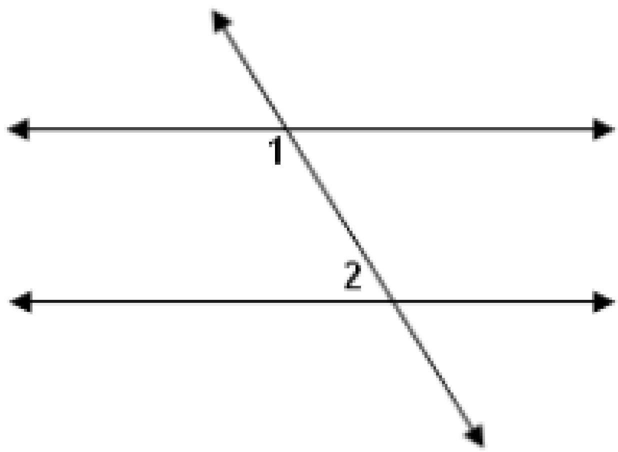
Angle Relationships

In each of the following, identify the relationship between the numbered angles

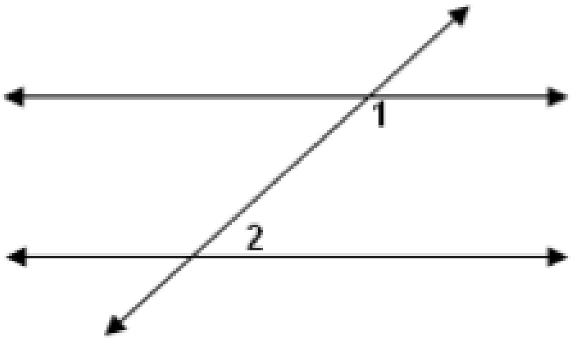
1)



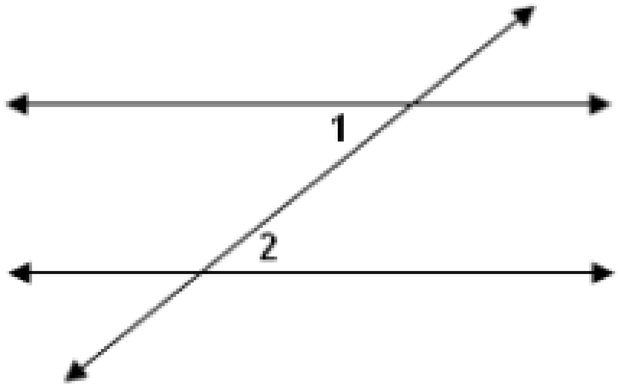
2) A



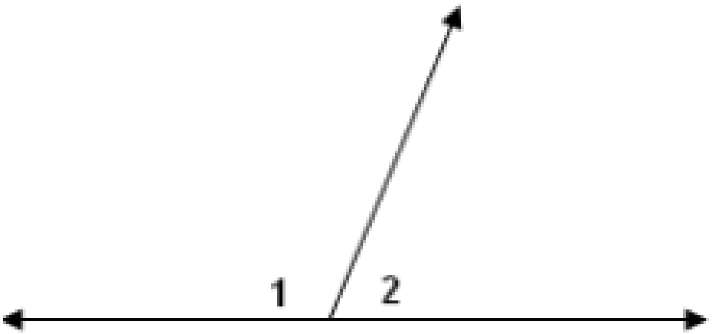
3)



4)

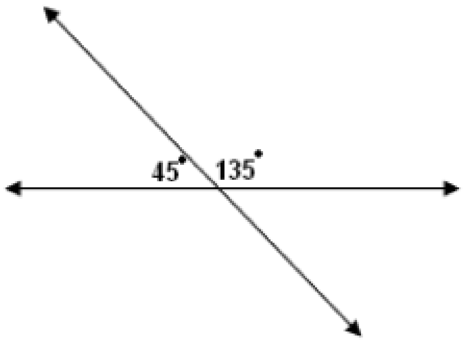


5)

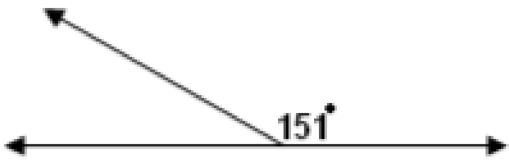


In each of the problems in this section, calculate the size of the missing angles

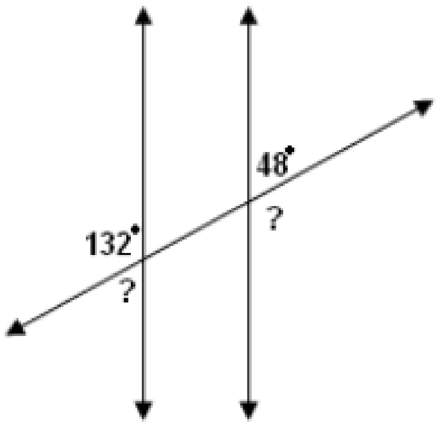
6)



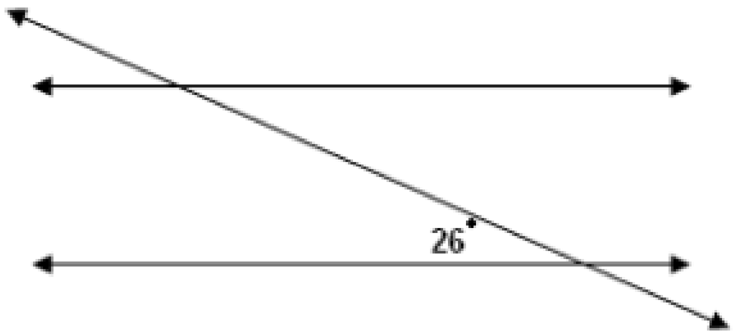
7)



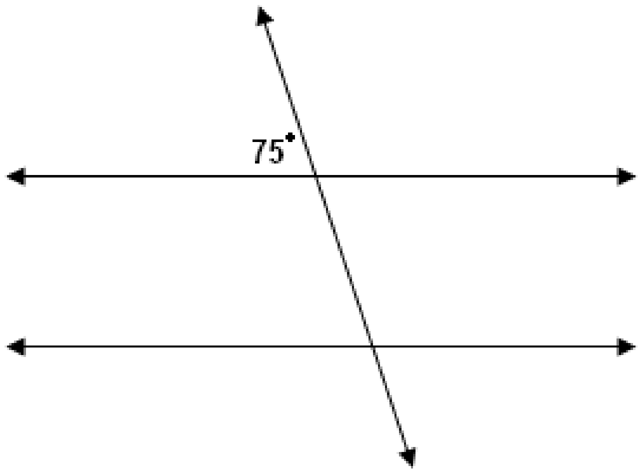
8)



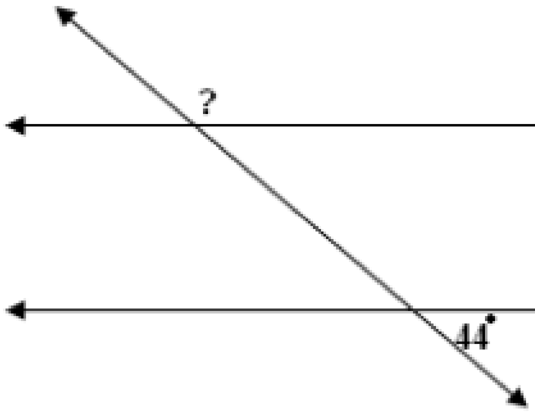
9)



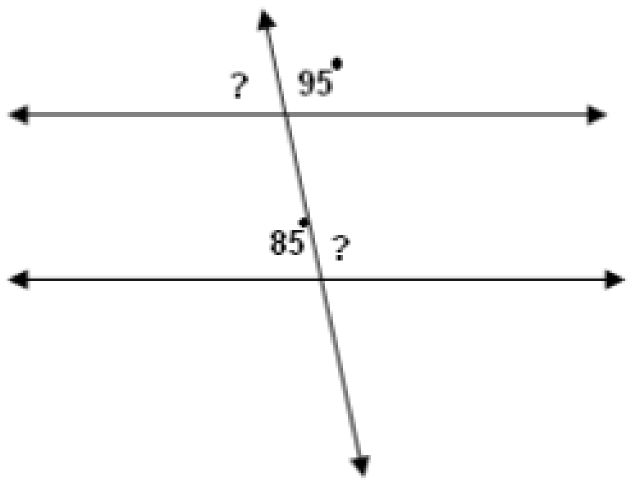
10)



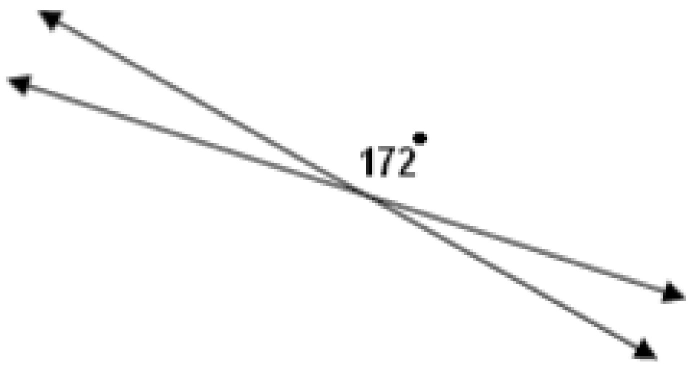
11)



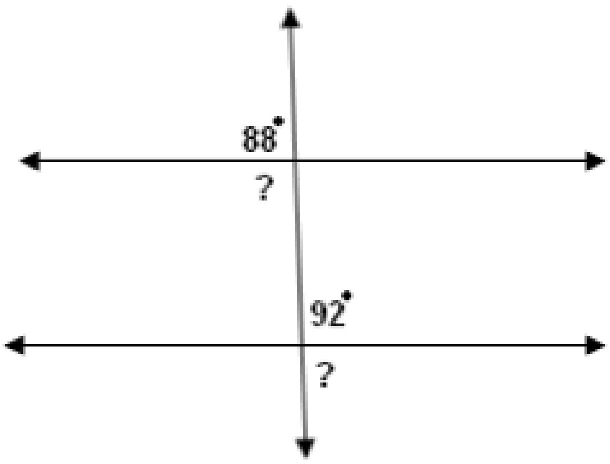
12)



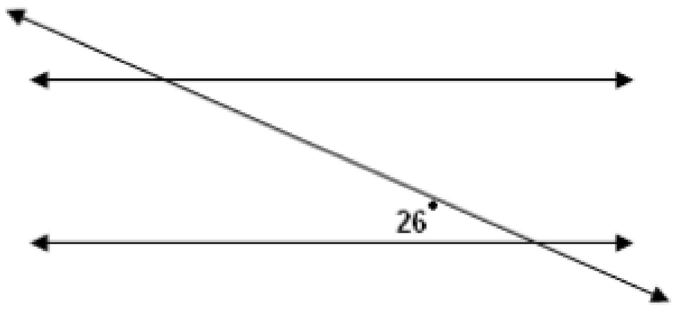
13)



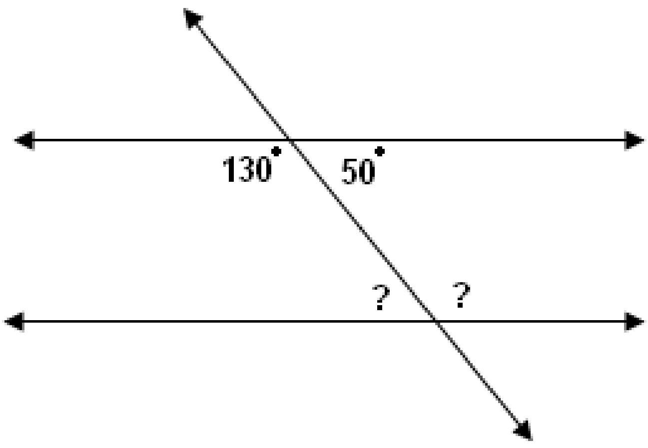
14)



15)



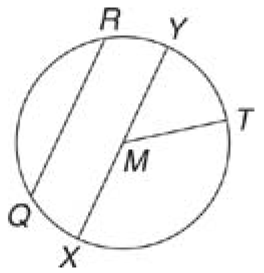
16)



Exercise 3

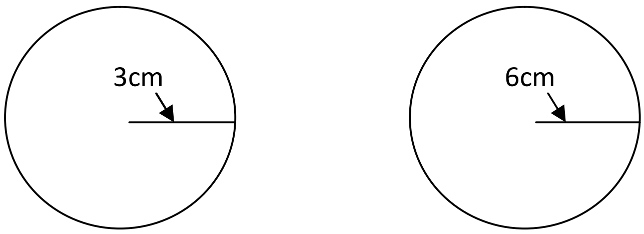
Circles & Scale Factors

1) In the following diagram identify the centre, radius, diameter, secant and chords

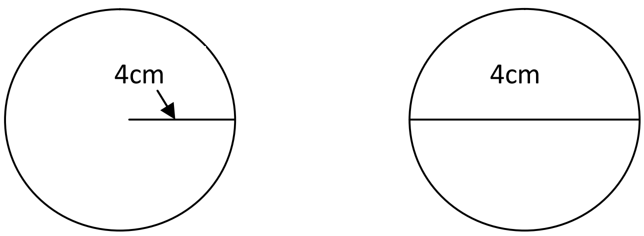


2) Determine the scale factor for the following pairs of circles

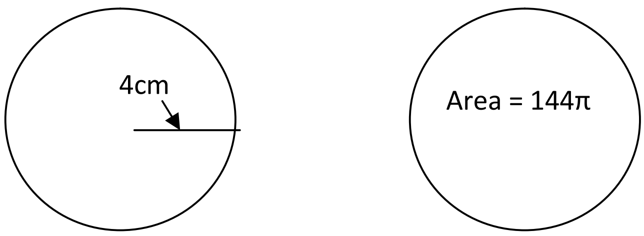
a) A



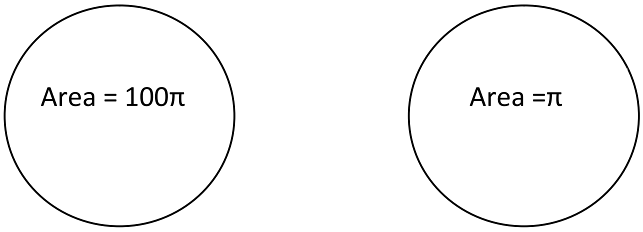
b) A



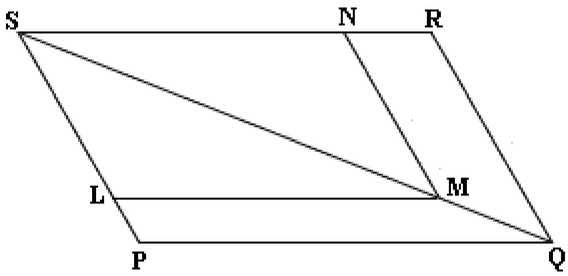
c) A



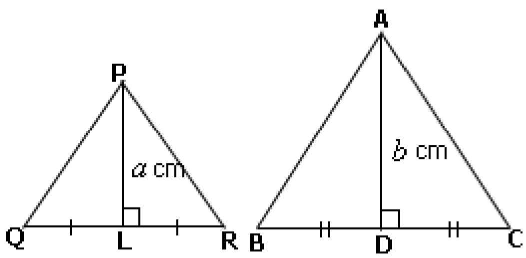
d) A



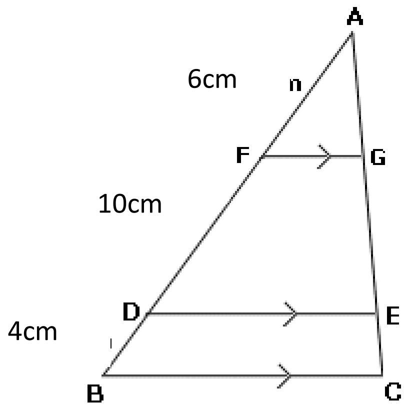
3) If the ratio of SM to QM is 3:1, LM = 9cm, MN = 6cm, and the two quadrilaterals are parallelograms, find the perimeter of PQRS



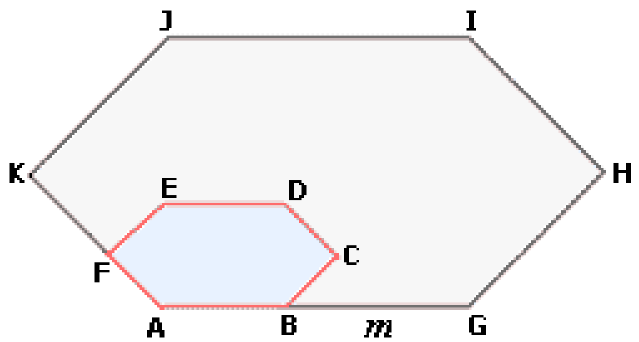
4) The perimeter of the two triangles are x and $x + 4$ respectively, find the value of x if $a = 2$ and $b = 3$



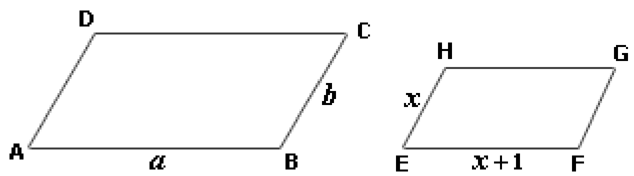
5) If the distance from A to E is 12cm, find the length of EC



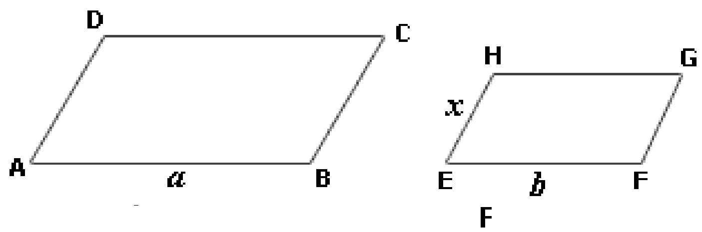
6) If AF = 12cm, AK = 30 cm, and AB = 10cm, find the value of m



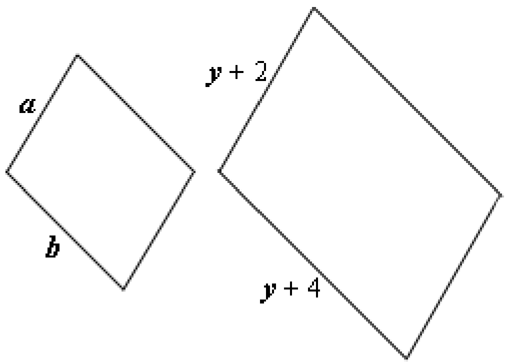
7) If a = 12cm, b = 6cm, find the value of x



8) If the perimeter of the larger parallelogram is 30cm, a=12, and b = 4, find x



9) If a = 5 and b = 6, find the value of y



10) If $y = 15$ and $z = 5$, find the value of x