Year 8 Mathematics Solutions

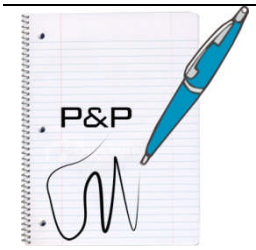
Learning Strategies

Mathematics is often the most challenging subject for students. Much of the trouble comes from the fact that mathematics is about logical thinking, not memorizing rules or remembering formulas. It requires a different style of thinking than other subjects. The students who seem to be "naturally" good at math just happen to adopt the correct strategies of thinking that math requires – often they don't even realise it. We have isolated several key learning strategies used by successful maths students and have made icons to represent them. These icons are distributed throughout the book in order to remind students to adopt these necessary learning strategies:

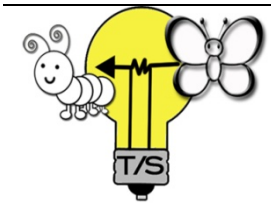
Talk Aloud Many students sit and try to do a problem in complete silence inside their heads. They think that solutions just pop into the heads of 'smart' people. You absolutely must learn to talk aloud and listen to yourself, literally to talk yourself through a problem. Successful students do this without realising. It helps to structure your thoughts while helping your tutor understand the way you think.

BackChecking This means that you will be doing every step of the question twice, as you work your way through the question to ensure no silly mistakes. For example with this question: $3 \times 2 - 5 \times 7$ you would do "3 times 2 is 5 ... let me check – no 3×2 is 6 ... minus 5 times 7 is minus 35 ... let me check ... minus 5×7 is minus 35. Initially, this may seem time-consuming, but once it is automatic, a great deal of time and marks will be saved.

Avoid Cosmetic Surgery Do not write over old answers since this often results in repeated mistakes or actually erasing the correct answer. When you make mistakes just put one line through the mistake rather than scribbling it out. This helps reduce silly mistakes and makes your work look cleaner and easier to backcheck.



Pen to Paper It is always wise to write things down as you work your way through a problem, in order to keep track of good ideas and to see concepts on paper instead of in your head. This makes it easier to work out the next step in the problem. Harder maths problems cannot be solved in your head alone – put your ideas on paper as soon as you have them – always!



Transfer Skills This strategy is more advanced. It is the skill of making up a simpler question and then transferring those ideas to a more complex question with which you are having difficulty.

For example if you can't remember how to do long addition because you can't recall exactly how to carry the one: $\begin{array}{r} 5889 \\ +\,4587 \\ \hline \end{array}$ then you may want to try adding numbers which you do know how to calculate that also involve carrying the one: $\begin{array}{r} 5 \\ +\,9 \\ \hline \end{array}$

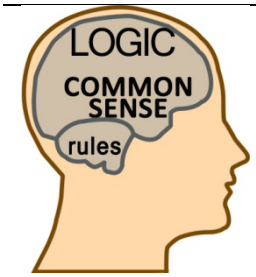
This skill is particularly useful when you can't remember a basic arithmetic or algebraic rule, most of the time you should be able to work it out by creating a simpler version of the question.

Format Skills These are the skills that keep a question together as an organized whole in terms of your working out on paper. An example of this is using the "=" sign correctly to keep a question lined up properly. In numerical calculations format skills help you to align the numbers correctly.

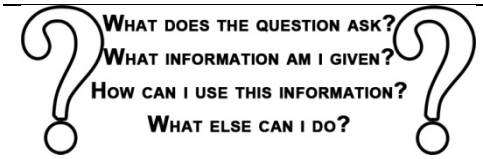
This skill is important because the correct working out will help you avoid careless mistakes. When your work is jumbled up all over the page it is hard for you to make sense of what belongs with what. Your "silly" mistakes would increase. Format skills also make it a lot easier for you to check over your work and to notice/correct any mistakes.

Every topic in math has a way of being written with correct formatting. You will be surprised how much smoother mathematics will be once you learn this skill. Whenever you are unsure you should always ask your tutor or teacher.

Its Ok To Be Wrong Mathematics is in many ways more of a skill than just knowledge. The main skill is problem solving and the only way this can be learned is by thinking hard and making mistakes on the way. As you gain confidence you will naturally worry less about making the mistakes and more about learning from them. Risk trying to solve problems that you are unsure of, this will improve your skill more than anything else. It's ok to be wrong – it is NOT ok to not try.



Avoid Rule Dependency Rules are secondary tools; common sense and logic are primary tools for problem solving and mathematics in general. Ultimately you must understand Why rules work the way they do. Without this you are likely to struggle with tricky problem solving and worded questions. Always rely on your logic and common sense first and on rules second, always ask Why?



Self Questioning This is what strong problem solvers do naturally when they get stuck on a problem or don't know what to do. Ask yourself these questions. They will help to jolt your thinking process; consider just one question at a time and Talk Aloud while putting Pen To Paper.

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Year 8 Mathematics

Number

Exercise 1

Number Groups & Families

1) Evaluate the following

a) $(2 \times 3)^2$

$= 6^2 = 36$

b) $2^2 \times 3^2$

$= 4 \times 9 = 36$

c) $(3 \times 2)^3$

$= 6^3 = 216$

d) $3^3 \times 2^3$

$= 27 \times 8 = 216$

e) $(5 \times 5)^2$

$= 25^2 = 625$

f) $5^2 \times 5^2$

$25 \times 25 = 625$

g) Use your results above to rewrite the following

$(a \times b)^2 =$ $\times$

$a^2 \times b^2$

2) Evaluate the following

a) $\sqrt{36}$

6

b) $\sqrt{9 \times 4}$

$= \sqrt{36} = 6$

c) $\sqrt{9} \times \sqrt{4}$

$= 3 \times 2 = 6$

d) $\sqrt{400}$

20

e) $\sqrt{16 \times 25}$

$= \sqrt{400} = 20$

f) $\sqrt{16} \times \sqrt{25}$

$4 \times 5 = 20$

g) Use your results above to rewrite the following

$\sqrt{a \times b} = \sqrt{\ } \times \sqrt{\ }$

$\sqrt{a} \times \sqrt{b}$

h) Use your results to simplify

$\sqrt{900} = \sqrt{9} \times \sqrt{100} = 3 \times 10 = 30$

3) Evaluate the following

a) 31×9

279

b) $(31 \times 10) - 31$

279

c) 17×9

153

d) $(17 \times 10) - 17$

153

e) 47×9

423

f) $(47 \times 10) - 47$

423

g) $a \times 9 = (a \times 10) - (a)$

4) Evaluate the following

a) $2 \times 15 \times 5$

150

b) 15×10

150

c) $2 \times 31 \times 5$

310

d) 31×10

310

e) $2 \times 83 \times 5$

830

f) 83×10

830

g) $2 \times a \times 5 =$

$10 \times a$

5) Determine the highest common factor of:

a) $6\ and\ 14$

Factors of 6 are

1, 2, 3, 6

Factors of 14 are

1, 2, 7, 14

HCF is 2

b) $24\ and\ 33$

Factors of 24 are

1, 2, 3, 4, 6, 8, 12, 24

Factors of 33 are

1, 3, 11, 33

HCF is 3

c) $18\ and\ 81$

Factors of 18 are

1, 2, 3, 6, 9, 18

Factors of 81 are

1, 3, 9, 27, 81

HCF is 9

d) $42\ and\ 77$

Factors of 42 are

1, 2, 3, 6, 7, 14, 21, 42

Factors of 77 are

1, 7, 11, 77

HCF is 7

e) $17\ and\ 19$

Factors of 17 are

1, 17

Factors of 19 are

1, 19

HCF is 1

f) $12, 36, and\ 48$

Factors of 12 are

1, 2, 3, 4, 6, 12

Factors of 36 are

1, 2, 3, 4, 6, 9, 12, 18, 36

Factors of 48 are

1, 2, 3, 4, 6, 8, 12, 16, 24, 48

HCF is 12

g) $12, 36, and\ 54$

Factors of 12 are

1, 2, 3, 4, 6, 12

Factors of 36 are

1, 2, 3, 4, 6, 9, 12, 18, 36

Factors of 54 are

1, 2, 3, 6, 9, 18, 27, 54

HCF is 6

6) Determine the lowest common multiple of the following

a) $2\ and\ 3$

Multiples of 2 are

2, 4, 6, 8, 10, 12, ...

Multiples of 3 are

3, 6, 9, 12, 15, ...

LCM is 6

b) $4\ and\ 5$

Multiples of 4 are

4, 8, 12, 16, 20, ...

Multiples of 5 are

5, 10, 15, 20, 25, ...

LCM is 20

c) $4\ and\ 6$

Multiples of 4 are

4, 8, 12, 16, 20, ...

Multiples of 6 are

6, 12, 18, 24, 30 ...

LCM is 12

d) $11\ and\ 13$

Multiples of 11 are

11, 22, 33, 44, 55, 66, 77, 99, 110, 121, 132, 143, 154, ...

Multiples of 13 are

13, 26, 39, 52, 65, 78, 91, 104, 117, 130, 143, 156, ...

LCM is 143

e) $8\ and\ 12$

Multiples of 8 are

8, 16, 24, 32, ...

Multiples of 12 are

12, 24, 36, 48, 60, ...

LCM is 24

f) $2, 3, and\ 9$

Multiples of 2 are

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, ...

Multiples of 3 are

3, 6, 9, 12, 15, 18, 21, 24, 27, 30, ...

Multiples of 9 are

9, 18, 27, 36, 45

LCM is 16

g) $6, 9, and\ 108$

Multiples of 6 are

6, 12, 18, 24, 30 ...108

Multiples of 9 are

9, 18, 27, 36, 45, 54, ..., 108

Multiples of 108 are

108, 216, 324, 432, ...

LCM is 108

7) Complete the following sequence

0, 1, 1, 2, 3, 5, 8, 13, ___, ___, ___, ___

21, 34, 55, 89

a) What is this famous sequence called?

Fibonacci sequence

b) How is each term in the sequence generated?

It is the sum of the two previous terms

c) Starting with the second and third terms, divide each term by the term before it, and list your answers in a sequence

1, 2, 1.5, 1.667, 1.6, 1.625, 1.615, 1.619, 1.617, 1.618

d) Describe the sequence

Each term in the sequence is alternatively larger, then smaller than the previous term. The difference between the terms is decreasing, and the sequence is tending toward a final value which is approximately 1.618

Exercise 2

Directed Numbers

1) Evaluate the following

a) $105 + (-15)$

90

b) $226 + (-12)$

214

c) $174 + (-32)$

142

d) $275 - (-45)$

320

e) $410 - (-19)$

429

f) $399 - (-33)$

432

2) Evaluate the following

a) $-505 + (-25)$

-530

b) $-386 + (-43)$

-429

c) $-401 + (-1)$

-402

d) $-137 - (23)$

-160

e) $-294 - (16)$

-310

f) $-412 - (78)$

-490

g) $-299 - (-15)$

-314

h) $-505 - (-35)$

-540

i) $-999 - (-1)$

-1000

3) Evaluate the following

a) $32 \times (-5)$

-160

b) $14 \times (-3)$

-42

c) $5 \times (-8)$

-40

d) -9×11

-99

e) -6×9

-54

f) -15×15

-225

g) $-9 \times (-7)$

63

h) $-11 \times (-8)$

88

i) $-13 \times (-5)$

65

4) Evaluate the following

a) $-81 \div 9$

-9

b) $-64 \div 4$

-16

c) $-100 \div 10$

-10

d) $-54 \div -9$

6

e) $-66 \div -11$

6

f) $-36 \div -12$

3

g) $72 \div (-8)$

-9

h) $144 \div (-12)$

-12

i) $99 \div (-11)$

-9

5) Use order of operations to simplify and evaluate the following

a) $17 \times (-5) + (-4)$

$= -85 + (-4) = -89$

b) $-12 \div (-4) - 8$

$3 - 8 = -5$

c) $80 + (-15) \div (-5)$

$80 + 3 = 83$

d) $3 \times (-12) \times (2) \div (-6) - (8)$

$-36 \times 2 \div (-6) - 8$

$= -72 \div (-6) - 8$

$= 12 - 8 = 4$

e) $1 + (4) \times (-2)^2 - 6$

$1 + 4 \times 4 - 6$

$1 + 16 - 6$

11

f) $-2 \times (-10) + 2^4 - 4$

$-2 \times (-10) + 16 - 4$

$= 20 + 16 - 4$

$= 32$

g) $6 - 6 \times (6 + 6)$

6) To convert from degrees Fahrenheit to degrees Celsius, the following formula is applied

$℃ = (℉ - 32) \times 5 \div 9$

Use the formula to convert the following Fahrenheit temperatures to Celsius

a) 86

$℃ = (86 - 32) \times 5 \div 9$

$= 54 \times 5 \div 9$

$= 30°$

b) 131

$℃ = (131 - 32) \times 5 \div 9$

$= 99 \times 5 \div 9$

$= 55°$

c) 68

$℃ = (68 - 32) \times 5 \div 9$

$= 36 \times 5 \div 9$

$= 20°$

d) 212

$℃ = (212 - 32) \times 5 \div 9$

$= 180 \times 5 \div 9$

$= 100°$

e) 32

$℃ = (32 - 32) \times 5 \div 9$

$= 0 \times 5 \div 9$

$= 0°$

f) –22

$℃ = (22 - 32) \times 5 \div 9$

$= -10 \times 5 \div 9$

$= -5.56°$

7) A medicine has to be administered to a patient according to his weight in kg and the following formula

$$(-0.4) \times weight \times (-10) + weight \div 10$$

What dosage should be given to patients who weigh:

a) 40kg

Dosage $= (-0.4) \times 40 \times (-10) + 40 \div 10$

$= -16 \times (-10) + 4$

$= 164$ units

b) 60kg

Dosage $= (-0.4) \times 60 \times (-10) + 60 \div 10$

$= -24 \times (-10) + 6$

$= 246$ units

c) 80kg

Dosage $= (-0.4) \times 80 \times (-10) + 80 \div 10$

$= -32 \times (-10) + 8$

$= 328$ units

d) 100kg

Dosage $= (-0.4) \times 100 \times (-10) + 100 \div 10$

$= -40 \times (-10) + 10$

$= 410$ units

e) 200kg

Dosage $= (-0.4) \times 200 \times (-10) + 200 \div 10$

$= -80 \times (-10) + 20$

$= 820$ units

Exercise 3

Fractions & Mixed Numerals

1) Add the following

a) $2\frac{1}{3}+3\frac{1}{3}$

$5\frac{2}{3}$

b) $1\frac{1}{5}+4\frac{2}{5}$

$5\frac{3}{5}$

c) $\frac{1}{2}+3\frac{1}{4}$

$=\frac{2}{4}+3\frac{1}{4}$

$=3\frac{3}{4}$

d) $4\frac{1}{6}+3\frac{5}{6}$

$=7\frac{6}{6}$

$=8$

e) $2\frac{2}{3}+1\frac{2}{3}$

$=3\frac{4}{3}$

$=4\frac{1}{3}$

f) $5\frac{3}{4}+1\frac{3}{4}$

$=6\frac{6}{4}$

$=7\frac{1}{2}$

2) Add the following

a) $3\frac{1}{2}+2\frac{1}{3}$

$=3\frac{3}{6}+2\frac{2}{6}$

$5\frac{5}{6}$

b) $1\frac{2}{3}+1\frac{1}{4}$

$=1\frac{8}{12}+1\frac{3}{12}$

$2\frac{11}{12}$

c) $2\frac{1}{2}+3\frac{2}{3}$

$=2\frac{3}{6}+3\frac{4}{6}$

$=5\frac{7}{6}$

$6\frac{1}{6}$

d) $4\frac{3}{5}+1\frac{3}{4}$

$=4\frac{12}{20}+1\frac{15}{20}$

$=5\frac{27}{20}$

$=6\frac{7}{20}$

e) $2\frac{3}{4}+2\frac{2}{5}$

$=2\frac{15}{20}+2\frac{8}{20}$

$=4\frac{23}{20}$

$=5\frac{3}{20}$

f) $1\frac{5}{6}+3\frac{3}{7}$

$=1\frac{35}{42}+3\frac{18}{42}$

$=4\frac{53}{42}$

$=5\frac{11}{42}$

3) Multiply the following

a) $1\frac{1}{2}\times 2\frac{1}{3}$

$2\frac{1}{6}$

b) $3\frac{1}{3}\times 2\frac{2}{3}$

$6\frac{2}{9}$

c) $3\frac{3}{4}\times 1\frac{1}{2}$

$3\frac{3}{8}$

d) $4\frac{1}{4}\times 2\frac{3}{5}$

$8\frac{3}{20}$

e) $1\frac{1}{3}\times 2\frac{3}{4}$

$2\frac{3}{12}$

$2\frac{1}{4}$

f) $2\frac{2}{5}\times 2\frac{1}{5}$

$4\frac{2}{25}$

4) Divide the following

a) $1\frac{1}{2}\div 2\frac{1}{3}$

$=\frac{3}{2}\div\frac{7}{2}$

$=\frac{3}{2}\times\frac{2}{7}$

$=\frac{6}{14}$

$=\frac{3}{7}$

b) $3\frac{1}{3}\div 2\frac{2}{3}$

$=\frac{10}{3}\div\frac{8}{3}$

$=\frac{10}{3}\times\frac{3}{8}$

$= \frac{30}{24}$

$= 1\frac{1}{4}$

c) $3\frac{3}{4} \div 1\frac{1}{2}$

$= \frac{15}{4} \div \frac{3}{2}$

$= \frac{15}{4} \times \frac{2}{3}$

$= \frac{30}{12}$

$2\frac{1}{2}$

d) $4\frac{1}{4} \div 2\frac{3}{5}$

$= \frac{17}{4} \div \frac{13}{5}$

$= \frac{17}{4} \times \frac{5}{13}$

$= \frac{85}{52}$

$1\frac{33}{52}$

e) $1\frac{1}{3} \div 2\frac{3}{4}$

$= \frac{4}{3} \div \frac{11}{4}$

$= \frac{4}{3} \times \frac{4}{11}$

$= \frac{16}{33}$

f) $2\frac{2}{5} \div 2\frac{1}{5}$

$= \frac{12}{5} \div \frac{11}{5}$

$\frac{12}{5} \times \frac{5}{11}$

$\frac{60}{55}$

$= 1\frac{1}{11}$

5) Calculate the following

a) $5 - \frac{2}{3}$

$= \frac{15}{3} - \frac{2}{3}$

$= \frac{13}{3}$

$4\frac{1}{3}$

b) $2 - \frac{3}{4}$

$= \frac{8}{4} - \frac{3}{4}$

$= \frac{5}{4}$

$= 1\frac{1}{4}$

c) $4-\frac{7}{3}$

$=\frac{12}{3}-\frac{7}{3}$

$=\frac{5}{3}$

$1\frac{2}{3}$

d) $2-\frac{3}{2}$

$=\frac{4}{2}-\frac{3}{2}$

$=\frac{1}{2}$

e) $3-\frac{8}{5}$

$=\frac{15}{5}-\frac{8}{5}$

$=\frac{7}{5}$

$1\frac{2}{5}$

6) Calculate the following

a) $\frac{3}{4}\ of\ 2\frac{1}{2}$

$=\frac{3}{4}\times\frac{5}{2}$

$\frac{15}{8}$

$1\frac{7}{8}$

b) $\frac{1}{3}\ of\ 3\frac{6}{7}$

$=\frac{1}{3}\times\frac{27}{7}$

$=\frac{27}{21}$

$=1\frac{6}{27}$

$=1\frac{2}{9}$

c) $\frac{1}{2}\ of\ 2\frac{2}{5}$

$=\frac{1}{2}\times\frac{12}{5}$

$=\frac{12}{10}$

$=1\frac{2}{10}$

$1\frac{1}{5}$

d) $\frac{2}{5}\ of 3\frac{5}{8}$

$=\frac{2}{5}\times\frac{29}{8}$

$=\frac{58}{40}$

$=1\frac{18}{40}$

$=1\frac{9}{20}$

e) $\frac{5}{2}\ of\ \frac{8}{15}$

$= \frac{5}{2} \times \frac{8}{15}$

$= \frac{40}{30}$

$= 1\frac{10}{30}$

$1\frac{1}{3}$

7) Eric ran two and a half laps of the running track, while Peter ran two fifths of Eric's distance

a) How many laps did they run combined?

Peter ran $\frac{2}{5} \times 2\frac{1}{2} = \frac{2}{5} \times \frac{5}{2} = 1$ lap

Total distance $= 2\frac{1}{2} + 1 = 3\frac{1}{2}$ laps

b) If Eric ran exactly 1km, how far did Peter run?

Peter ran $\frac{2}{5}x1 = 0.4$ km= 400 m

c) What is the length of one lap?

Peter ran 1 lap which is 400 metres

d) What was the total distance run by the two of them?

Together they ran 1 km + 400 m = 1.4 km

8) Brian and John are doing a joint project. In thirty minutes Brian wrote out one and a half pages of work, while John did three quarters of what Brian did

a) How many pages had John finished?

John completed $\frac{3}{4} \times 1\frac{1}{2} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8} = 1\frac{1}{8}$ pages of work

b) How many pages had they finished in total?

$$1\frac{1}{2}+1\frac{1}{8}=1\frac{4}{8}+1\frac{1}{8}=2\frac{5}{8}$$

c) If the project needed 5 pages, what fraction of it have they finished?

$$2\frac{5}{8}\div 5=\frac{21}{5}\div\frac{5}{1}=\frac{21}{5}\times\frac{1}{5}=\frac{21}{25}=84\%$$

Exercise 4

Decimals, Fractions, & Percentages

1) Convert the following fractions to decimals

a) $\frac{3}{4}$

0.75

b) $\frac{3}{5}$

0.6

c) $\frac{7}{20}$

0.35

d) $\frac{1}{8}$

0.125

e) $\frac{9}{40}$

0.225

f) $\frac{11}{50}$

0.22

g) $\frac{9}{1000}$

0.009

h) $\frac{3}{500}$

0.006

2) Convert the following decimals to fractions

a) 0.24

$= \frac{24}{100} = \frac{6}{25}$

b) 0.4

$= \frac{4}{10} = \frac{2}{5}$

c) 0.375

$= \frac{375}{1000}$

$= \frac{3}{8}$

d) 0.45

$= \frac{45}{100}$

$= \frac{9}{20}$

e) 0.08

$= \frac{8}{100}$

$= \frac{2}{25}$

f) 0.002

$= \frac{2}{1000}$

$= \frac{1}{500}$

g) 0.8

$= \frac{8}{10}$

$= \frac{4}{5}$

h) 0.14

$= \frac{14}{100}$

$= \frac{7}{50}$

3) Convert the following fractions to percentages

a) $\frac{3}{8}$

37.5%

b) $\frac{9}{50}$

18%

c) $\frac{1}{12}$

8.5%

d) $\frac{1}{20}$

5%

e) $\frac{4}{25}$

16%

f) $\frac{3}{40}$

7.5%

g) $\frac{3}{1000}$

0.3%

h) $\frac{9}{60}$

15%

4) Convert the following percentages to fractions

a) 32%

$= \frac{32}{100}$

$= \frac{8}{25}$

b) 17.5%

$= \frac{175}{1000}$

$= \frac{7}{40}$

c) 2%

$$= \frac{2}{100}$$

$$= \frac{1}{50}$$

d) 0.85%

$$= \frac{85}{10000}$$

$$= \frac{17}{2000}$$

e) 80%

$$= \frac{80}{100}$$

$$= \frac{4}{5}$$

f) 72%

$$= \frac{72}{100}$$

$$= \frac{18}{25}$$

g) 12.5%

$$= \frac{125}{1000}$$

$$= \frac{1}{8}$$

h) 8%

$$= \frac{8}{100}$$

$$= \frac{2}{25}$$

5) Convert the following decimals to percentages

a) 0.11

11%

b) 0.4

40%

c) 0.001

0.1%

d) 0.35

35%

e) 0.125

12.5%

f) 0.375

37.5%

g) 0.101

10.1%

h) 0.0005

0.05%

6) Convert the following percentages to decimals

a) 42.5%

0.425

b) 18%

0.18

c) 0.03%

0.0003

d) 37.5%

0.375

e) 100%

1

f) 12%

0.12

g) 7.5%

0.075

h) 1%

0.01

7) Convert the following fractions to decimals, using the correct notation

a) $\frac{33}{100}$

0.33

b) $\frac{1}{6}$

$0.1\dot{6}$

c) $\frac{1}{9}$

$0.\dot{1}$

d) $\frac{2}{33}$

$0.\dot{0}\dot{6}$

e) $\frac{3}{11}$

$0.\dot{2}\dot{7}$

f) $\frac{7}{15}$

$0.4\dot{6}$

g) $\frac{5}{33}$

$0.\dot{1}\dot{5}$

h) $\frac{5}{21}$

$0.\dot{2}\dot{3}\dot{8}\dot{0}\dot{9}\dot{5}$

8) A shop is offering 20% off the marked price of shoes. If the marked price is $85, what is the discounted price?

$$85 \times 20\% = 85 \times \frac{20}{100} = \frac{1700}{100} = \$17$$

Discounted price is $85 - 17 = \$68$

9) The rainfall for March this year in Canberra was 15% more than for March last year. If last year's rainfall was 300mm, how much rainfall did Canberra receive this March?

$$300 \times 15\% = 300 \times \frac{15}{100} = \frac{4500}{100} = 45mm$$

Total rainfall was $300 + 45 = 345mm$

10) Alex went on a diet and lost 12.5% of his weight. If he weighed 116 kg before going on his diet, how much does he weigh now?

$$116 \times 12.5\% = 116 \times \frac{125}{1000} = \frac{14500}{1000} = 14.5kg$$

Alex now weighs $116 - 14.5 = 101.5kg$

11) GST adds 10% on certain purchases as a tax. If an item costs $12.50 before GST, how much does it cost after GST?

$$12.50 \times 10\% = 12.5 \times \frac{10}{100} = \$1.25$$

New cost $= 12.50 + 1.25 = \$13.75$

12) Andrew bought an item for $80 and later sold it for $100

a) How much was his profit?

Profit $100 - 80 = \$20$

b) What was his profit as a percentage of his cost?

$$\frac{20}{80} \times 100 = 25\%$$

c) What was his profit as a percentage of his selling price?

$$\frac{20}{100} \times 100 = 20\%$$

13) Karen bought an item for $60 and later sold it for $80

a) How much was her profit?

Profit $= 80 - 60 = \$20$

b) What was her profit as a percentage of her cost?

$$\frac{20}{60} \times 100 = 33.33\%$$

c) What was her profit as a percentage of her selling price?

$$\frac{20}{80} \times 100 = 25\%$$

14) Bill bought an item for $200 and later sold it for $300

a) How much was his profit?

Profit $= 300 - 200 = \$100$

b) What was his profit as a percentage of his cost?

$$\frac{100}{200} \times 100 = 50\%$$

c) What was his profit as a percentage of his selling price?

$$\frac{100}{300} \times 100 = 33.33\%$$

15) (Challenge question) Use your answers from questions 12-14 to investigate the relationship between profit as a percentage of cost price and profit as a percentage of selling price. (Hint: Change the percentage profits on cost and selling to fractions)

From question 12, profit on cost was $\frac{1}{4}$, profit on selling was $\frac{1}{5}$

From question 13, profit on cost was $\frac{1}{3}$, profit on selling was $\frac{1}{4}$

From question 14, profit on cost was $\frac{1}{2}$, profit on selling was $\frac{1}{3}$

The value of the denominator of the profit on selling is one more than the denominator of the profit on cost

In general, if the profit on cost was $\frac{1}{a}$, the profit on selling is $\frac{1}{a+1}$

Exercise 5

Mental Division Strategies

1) Which of the following numbers are exactly divisible by 2?

a) 2

Yes

b) 8

Yes

c) 13

No

d) 1

No

e) 240

Yes

f) 0

No

2) Which of the following numbers are exactly divisible by 3?

a) 3

Yes

b) 15

Yes

c) 23

No

d) 1

No

e) 312

Yes

f) 0

No

3) Which of the following numbers are exactly divisible by 5?

a) 5

Yes

b) 40

Yes

c) 47

No

d) 1

No

e) 95

Yes

f) 0

No

4) Which of the following numbers are exactly divisible by 9?

a) 9

Yes

b) 72

Yes

c) 116

No

d) 1

No

e) 225

Yes

f) 0

No

5) From your answers to questions 1 to 4, state some general rules for determining if a number is divisible by 2, 3, 5, or 9

To be exactly divisible by 2, the number must have a 0 or an even number as its last digit

To be exactly divisible by 3, the digits of the number must add up to a number that is a multiple of 3

To be exactly divisible by 5, the number must end in a 5 or a 0

To be exactly divisible by 9, the digits of the number must add up to a number that is a multiple of 9

6) Perform the following divisions without the use of a calculator

a) $30 \div 10$

3

b) $70 \div 10$

7

c) $130 \div 10$

13

d) $980 \div 10$

98

e) $40 \div 20$

2

f) $80 \div 20$

4

g) $260 \div 20$

13

h) $980 \div 20$

49

7) Perform the following divisions without the use of a calculator

a) $60 \div 12$

5

b) $144 \div 12$

12

c) $228 \div 12$

19

d) $624 \div 12$

52

e) $972 \div 12$

81

8) Perform the following divisions without the use of a calculator

a) $125 \div 25$

5

b) $450 \div 25$

18

c) $775 \div 25$

31

d) $925 \div 25$

37

e) $975 \div 25$

39

9) Express the following numbers as a multiple of 18 plus a remainder.
Example:$200 = 11 \times 18 + 2$

a) 246

$13 \times 18 + 12$

b) 312

$17 \times 18 + 6$

c) 444

$24 \times 18 + 12$

d) 568

$31 \times 18 + 10$

e) 926

$51 \times 18 + 8$

10) Use your answers from question 9 to solve the following: Example: $200 = 18 \times 11 + 2 = 11\frac{2}{18} = 11\frac{1}{9}$

a) $246 \div 18$

$$13\frac{12}{18} = 13\frac{2}{3}$$

b) $312 \div 18$

$$17\frac{6}{18} = 17\frac{1}{3}$$

c) $444 \div 18$

$$24\frac{12}{18} = 24\frac{2}{3}$$

d) $568 \div 18$

$$31\frac{10}{18} = 31\frac{5}{9}$$

e) $926 \div 18$

$$51\frac{8}{18} = 51\frac{4}{9}$$

11) Calculate the following (use the method from questions 9 and 10)

a) $136 \div 44$

$$3\frac{4}{44} = 3\frac{1}{11}$$

b) $222 \div 44$

$$5\frac{2}{44} = 5\frac{1}{22}$$

c) $612 \div 44$

$$13\frac{40}{44} = 13\frac{10}{11}$$

d) $742 \div 44$

$$16\frac{38}{44} = 16\frac{19}{22}$$

e) $906 \div 44$

$$20\frac{26}{44} = 20\frac{13}{22}$$

Exercise 6

Chance

1) List the sample space for the event "rolling a six sided die"

1, 2, 3, 4, 5, 6

2) A normal six sided die is thrown. What is the probability of rolling

a) The number two

$\frac{1}{6}$

b) The number 6

$\frac{1}{6}$

c) The number 3

$\mathbf{\frac{1}{6}}$

3)

a) What is the probability of rolling any particular number on a six sided die?

$\frac{1}{6}$

b) What is the sum of all possible rolls of a six side die?

$6 \times \frac{1}{6} = 1$

c) What is the probability that upon rolling a normal six side die, one of the numbers on the die is rolled?

1 (certain to roll one of the numbers)

4) A six side die is rolled. What is the probability of

a) Rolling an even number

$$\frac{3}{6} = \frac{1}{2}$$

b) Rolling an odd number

$$\frac{3}{6} = \frac{1}{2}$$

c) Not rolling an even number

$$\frac{1}{2}$$

d) Rolling a number that is a multiple of 3

$$\frac{2}{6} = \frac{1}{3}$$

e) Rolling a number that is not a multiple of 3

$$\frac{2}{3}$$

5) Using your answers above, if the probability of a certain event happening is $\frac{11}{23}$, what is the probability of the event NOT happening

$$1 - \frac{11}{23} = \frac{12}{23}$$

6)

a) What is the probability of drawing the ace of hearts from a standard deck of 52 cards?

$$\frac{1}{52}$$

b) What is the probability of drawing any card other than the ace of hearts?

$$1 - \frac{1}{52} = \frac{51}{52}$$

c) What is the probability of drawing a heart from a standard deck of 52 cards?

$$\frac{13}{52} = \frac{1}{4}$$

d) What is the probability of NOT drawing a heart?

$$1 - \frac{1}{4} = \frac{3}{4}$$

7) A man is told by his doctor that out of 250 people with his condition studied, 99 have died within 3 months. Should the man be optimistic about his chance of survival? Explain

$$\frac{99}{250} < \frac{1}{2}$$

So the man statistically has a better than 50% chance of surviving more than 3 months

8) A bag contains 100 yellow discs. What is the probability that a disc chosen at random will NOT be yellow?

Probability of getting yellow disc $= \frac{100}{100} = 1$

Probability of not getting a yellow disc is $1 - 1 = 0$

It is impossible to pull out anything other than a yellow disc

9) A bag contains 24 discs: 9 green, 6 yellow, 5 blue and 4 red discs. If a disc is chosen at random, what is the probability that it is NOT blue?

Probability disc is blue $= \frac{5}{24}$

Probability disc is not blue $= 1 - \frac{5}{24} = \frac{19}{24}$

10) A standard six sided die is rolled. What is the probability that the number rolled is NOT the number 9?

Probability that number is $9 = 0$ (There is no number 9 on the die)

Probability not the number nine $= 1 - 0 = 1$

Year 8 Mathematics

Algebra

Exercise 1

Equivalent Expressions

1) The expression $x + x + x + x$ is equivalent to

a) $4y$

b) $4x$

c) x^4

d) None of the above

b: $4x$

2) The expression $3y - y$ is equivalent to

a) $2y$

b) $3y^2$

c) $4y$

d) None of the above

a:$2y$

3) The expression $r \times r \times r$ is equivalent to

a) 3r

b) r^3

c) $r^2 + r$

d) None of the above

b:r^3

4) The expression $x \div y$ is equivalent to

a) xy

b) $\frac{x}{y}$

c) $x + y$

d) None of the above

b: $\frac{x}{y}$

5) The expression $a \times b$ is equivalent to

a) $a + b$

b) $b + a$

c) Either of the above

d) Neither of the above

d: Neither of them (it is expressed as ab)

6) The expression $3g$ is equivalent to

a) $g + g + g$

b) $2g + g$

c) $g + 2g$

d) None of the above

e) All of the above

a: $g + g + g$

7) The expression x^4 is equivalent to

a) $x^2 + x^2$

b) $x \times x \times x \times x$

c) $2x \times 2x$

d) None of the above

e) All of the above

b: $\boldsymbol{x \times x \times x \times x}$

8) What must the value of x and y be so that $x \times x \times y$ is equivalent to x^2y?

The equality is valid for any values of $x \; and \; y$

9) Five boys each have the same number of lollies in their pockets. If x represents the number of lollies in the pocket of one boy, which of the following expressions could be used to calculate the total number of lollies?

a) x^5

b) $5y$

c) $5x$

d) $\frac{x}{5}$

We need to know how many 5 times the number of lollies is, so expression required is $5x$

10) Ten bins hold a total of x apples. If each bin holds the same number of apples, what is the expression that could be used to calculate the number of apples in each bin?

a) x^{10}

b) $\frac{x}{10}$

c) 10x

d) $10x - 10$

We need to know how many one tenth of the total is, so the expression required is $\frac{x}{10}$

Exercise 2

Rules, Patterns & Tables of Values

1) The temperature in a room is 32 degrees. An air conditioner is turned on and cools the room down at the rate of 2 degrees per minute for 7 minutes

a) Construct a table showing the temperature of the room for the seven minutes

Minutes	Temperature
0	32
1	30
2	28
3	26
4	24
5	22
6	20
7	18

b) What temperature will the room be after 3 minutes?

26 degrees

c) How many minutes will it take the room to cool down to 22 degrees?

5 minutes

d) Describe a rule in words that relates the temperature of the room to the number of minutes that the air conditioner has been turned on

The temperature is equal to 32 subtract 2 times the number of minutes

e) Write the above rule algebraically

Let y be the temperature and x be the number of minutes

$y = 32 - 2x$

2) Tom has $20 in his bank account. Each week he deposits $5 of his pocket money into the account

a) Construct a table that shows how much money is in Tom's account for the next 6 weeks

Weeks	$
0	20
1	25
2	30
3	35
4	40
5	45
6	50

b) How much money will be in Tom's account after 4 weeks?

$40

c) After how many weeks will Tom have $80 in his account?

12 weeks

d) Write a rule in words that describes how to calculate the amount of money in Tom's account after a certain number of weeks

The amount of money equals 20 dollars plus 5 dollars times the number of weeks of deposits

e) Write the above rule algebraically

Let p be the amount of money in the account and q be the number of weeks the account has been open

$p = 20 + 5q$

3) A tap is dripping water into a bucket. The amount of water in the bucket is equal to 500 mL plus 10 mL per minute that the tap has been dripping

a) Write the rule algebraically

Let c be the amount of water in the bucket, and b be the number of minutes the tap has been dripping

$c = 500 + 10b$

b) Construct a table that shows the amount of water in the bucket from 0 to 10 minutes

Minutes	mL
0	500
1	510
2	520
3	530
4	540
5	550
6	560
7	570
8	580
9	590
10	600

c) Use the rule to calculate the amount of water in the bucket after one hour

When $b = 60, c = 500 + (60 \times 10) = 1100$ mL

4) A mechanic charges a $20 call out fee plus $25 per hour

a) Write this rule algebraically

Let d be the charge and s the number of hours

$d = 20 + 25s$

b) How much does the mechanic charge for 5 hours work?

When $s = 5, d = 20 + (25 \times 5) = \$145$

c) Construct a table that shows the above information

Hours	Charge
0	20
1	45
2	70
3	95
4	120
5	145
6	170
7	195

5) If x represents the number of bricks in a wall, and y represents the number of hours a man has been building the wall for

a) Translate the algebraic expression $x = 100 + 50y$ into a word expression

The number of bricks in the wall equals one hundred plus 50 bricks per hour of work

b) How many bricks per hour does the man use?

50

c) How many bricks were already in the wall when the man started working?

100

d) How many bricks will be in the wall after 4 hours?

When $y = 4, x = 100 + (50 \times 4) = 300$ bricks

6) If t represents the number of pellets of fish food in a tank, and w represents the number of fish in the tank

a) Translate the expression $t = 200 - 10w$ to a word expression

The number of pellets in the tank equals 200 subtract 10 times the number of fish.

b) Construct a table that shows the number of pellets left for each quantity of fish in the tank

Number of fish	Number of pellets
0	200
1	190
2	180
3	170
4	160
5	150
6	140
7	130

c) How many pellets were in the tank before there were any fish?

200

d) How many pellets does each fish eat?

10

e) How many fish will be in the tank when the food runs out?

When $t = 0, 200 - 10w = 0, w = 20$

The food will run out when there are 20 fish in the tank

7) A taxi charges 2 dollars when a passenger first gets in, and \$1.50 per kilometre for the journey.

a) If t is the total charge for a journey, and y represents the number of kilometres travelled, write an algebraic expression for the total cost of a journey

$t = 2 + 1.5y$

b) Construct a table that shows the total cost for journeys from 1km to 10km

km	Cost
1	3.50
2	5
3	6.50
4	8
5	9.50
6	11
7	12.5
8	14
9	15.5
10	17

c) How much would a journey of 20km cost?

When $y = 20, t = 2 + (1.50 \times 20) = \$32$

d) A man has $23, how far can he travel?

When $t = 23,\ 2 + (1.50 \times y) = 23$

$y = 14km$

8) A car salesman gets paid a certain amount per day plus another amount per sale (commission). The table below shows how much he earns in a day, for each number of sales

Number of cars sold	1	2	3	4	5	6	7	8
Total pay	150	200	250	300	350	400	450	500

a) How much does he get paid even if he doesn't manage to sell a car?

If you continue the pattern backwards, when number of cars sold equals zero, total pay equals $100

b) How much commission does he get per car sold?

From the table, an increase of 1 car sold gets the salesman an extra $50

c) Write a word expression that describes his total pay in terms of number of cars sold

Total pay equals one hundred dollars plus fifty dollars for each car sold

d) Write the word expression in algebraic form

Let y represent the total pay, and x represent the number of cars sold

Then $y = 100 + 50x$

e) How much would the salesman earn if he were lucky enough to sell 20 cars in one day?

When $x = 20, y = 100 + (50 \times 20) = \$1100$

Exercise 3

Factorization

1) Simplify the following expressions by removing the common factor

a) $2x + 4$

$2(x + 2)$

b) $6 + 3y$

$3(2 + y)$

c) $5x - 10$

$5(x - 2)$

d) $6 - 8t$

$2(3 - 4t)$

e) $4x + 6y + 2$

$2(2x + 3y + 1)$

2) Remove the common factor from the following expressions

a) $ax + xy$

$x(a + y)$

b) $6b - 3a$

$3(b - a)$

c) $2x + 3xy$

$x(2 + 3y)$

d) $4pq + 2q$

$2q(2p + 1)$

e) $6x - y$

Cannot be simplified; no common factor

3) Remove the common factor from the following expressions

a) $3xy + 6y + 9$

$3(xy + 2y + 3)$

b) $2tac + 6ta + 4a$

$2a(tc + 3t + 2)$

c) $5pq - 10qr + 15q$

$5q(p - 2r + 3)$

d) $ab + 6b - bc$

$b(a + 6 - c)$

e) $abc + bc - a$

Has no common factor

4) Factorize the following expressions

a) $3x^2 + 6x$

$x(3x + 6)$

b) $2y^2 + 4y$

$2y(y + 2)$

c) $5a - 10a^2$

$5a(1 - 2a)$

d) $7x - 5x^2$

$x(7 - 5x)$

e) $xy^2 + x^2y$

$xy(y + x)$

5) Factorize the following expressions

a) $x^2 - 2xy + xy^2$

$x(x - 2y + y^2)$

b) $abc + a^2b^2c^2 + b^2c$

$bc(a + a^2bc + b)$

c) $rx^2 - rx + r^2x^2$

$rx(x - 1 + rx)$

d) $a - ab - b$

No common factor

e) $3y + 6y^2 - 9y^3$

$3y(1 + 2y - 3y^2)$

6) The following expressions have been factorized, what were the original equations?

a) $.2(x - 3)$

$2x - 6$

b) $5(1 - 2y)$

$5 - 10y$

c) $2(2x - y)$

$4x - 2y$

d) $a(x - 2)$

$ax - 2z$

e) $2r(r + 3)$

$2r^2 + 6r$

7) The following expressions have been factorized, what were the original expressions

a) $ab(a - b + 2)$

$a^2b - ab^2 + 2ab$

b) $x^2(3 + 2y)$

$3x^2 + 2x^2y$

c) $abc(a + b + c)$

$a^2bc + ab^2c + abc^2$

d) $\frac{x}{2}(2x - 5)$

$x^2 - \frac{5x}{2}$

e) $\sqrt{x}(2-\sqrt{x})$

$2\sqrt{x}-x$

8) The following expressions have been factorized incorrectly. Complete the factorization

a) $x(2x-4y)$

$2x(x-2y)$

b) $y(2xy-3x)$

$xy(2y-3)$

c) $5(2x^2+4x)$

$5x(2x+4)$

d) $2p(3-6p)$

$6p(1-2p)$

e) $x^2(3y-6y^2)$

$3x^2y(1-2y)$

9) A rectangle has side lengths of 4cm and $(x+1)$cm

a) What is the area of the rectangle in terms of its measurements?

$4\times(x+1)$

b) Remove the brackets and express the area in terms of x

$4x+4$

c) Assume $x=3$cm. Calculate the area of the rectangle by:

- Calculating the length and using the formula area = length x width

  Length $=4cm$

  Area $=4\times4=16cm^2$

- Substituting the value into the equation obtained in part a

  $4(x+1)=4\times4=16cm^2$

- Substituting the value into the equation obtained in part b

  If $x=3,\ 4x+4=16cm^2$

- What do you notice about your three answers?

  They are the same showing that the different ways of expressing the area are equivalent

10) The base of a triangle has a length of $(x-4)$cm, and a vertical height of 3cm

a) What is the area of the triangle in terms of its measurements?

$$\frac{1}{2} \times (x-4) \times 3$$

b) Remove the brackets and express the area in terms of x

$$\frac{3x}{2} - 6$$

c) Assume $x = 8$cm. Calculate the area of the triangle by:

- Calculating the base and using the formula area = ½ base x height

  Base $= 4cm$

  Area $= \frac{1}{2} \times 4 \times 3 = 6cm^2$

- Substituting the value into the equation obtained in part a

  $$\frac{1}{2} \times (8-4) \times 3$$

  $$= 6cm^2$$

- Substituting the value into the equation obtained in part b

  $$\frac{3 \times 8}{2} - 6 = 6cm^2$$

- What do you notice about your three answers?

  They are the same showing that the different ways of expressing the area are equivalent

Exercise 4

Solving Linear Equations & Inequalities

1) Solve the following equations

a) $x + 2 = 7$

$x = 7 - 2 = 5$

b) $y - 4 = 8$

$y = 8 + 4 = 12$

c) $r + 1 = -3$

$r = -3 - 1 = -4$

d) $x + 4 = 1$

$x = 1 - 4 = -3$

e) $-y + 2 = 6$

$-y = 6 - 2 = 4$

$y = -4$

2) Solve the following equations

a) $2(x - 2) = 6$

$2x - 4 = 6$

$2x = 10$

$x = 5$

b) $3(y + 4) = 6$

$3y + 12 = 6$

$3y = -6$

$y = -2$

c) $5(2 - x) = 10$

$10 - 5x = 10$

$-5x = 0$

$x = 0$

d) $\frac{1}{2}(2x + 1) = 5$

$x + \frac{1}{2} = 5$

$x = \frac{9}{2}$

e) $3\left(\frac{1}{2}x - 1\right) = 12$

$\frac{3x}{2} - 3 = 12$

$\frac{3x}{2} = 15$

$x = \frac{2}{3} \times 15 = 10$

3) Solve the following equations

a) $\frac{x-2}{3} = 4$

$x - 2 = 12$

$x = 14$

b) $\frac{x+1}{2} = 5$

$x + 1 = 10$

$x = 9$

c) $\frac{3-y}{4} = 1$

$3 - y = 4$

$-y = 4 - 3$

$-y = 1$

$y = -1$

d) $\frac{x-4}{2} = \frac{1}{2}$

$x - 4 = 1$

$x = 5$

e) $3x = \frac{x}{x+1}$

$3x(x + 1) = x$

$3x^2 + 3x = x$

$3x^2 + 2x = 0$

$x(3x + 2) = 0$

$x = 0, or\ 3x + 2 = 0$

$x = 0\ or\ x = -\frac{2}{3}$

4) Solve the following equations

a) $2x - 4 = x + 2$

$2x - x = 2 + 4$

$x = 6$

b) $3r + 1 = r - 5$

$3r - r = -5 - 1$

$2r = -6$

$r = -3$

c) $\frac{1}{2}x + 1 = \frac{3}{2}x - 3$

$\frac{1}{2}x - \frac{3}{2}x = -3 - 1$

$-x = -4$

$x = 4$

d) $4y + \frac{1}{2} = 2y + 2$

$4y - 2y = 2 - \frac{1}{2}$

$2y = \frac{3}{2}$

$y = \frac{3}{4}$

e) $4 - 3t = t - 2$

$-3t - t = -2 - 4$

$-4t = -6$

$t = \frac{3}{2}$

5) If you double the number of lollies in a bag and add 6 to the result you get 18. How many lollies in the bag?

Let x represent the number of lollies in the bag

$2x + 6 = 18$

$2x = 12$

$x = 6$

There are 6 lollies in the bag

6) If you triple Peter's age and subtract 12, you get 48. How old is Peter?

Let w represent Peter's age

$3w - 12 = 48$

$3w = 60$

$w = 20$

Peter is 20 years old

7) A car park is currently holding half its maximum capacity plus 6. If its maximum capacity is 100, how many cars are currently in the car park?

Let r represent the number of cars in the car park

$r = \frac{1}{2} \times 100 + 6$

$r = 56$

There are 56 cars currently in the car park

8) I ran a lap of a running track. My friend's time was equal to half my time minus 20 seconds. If my friend's time was 40 seconds, how long did it take me to run the lap?

Let z represent my time

$$40 = \frac{1}{2}z - 20$$

$$\frac{1}{2}z = 40 + 20$$

$$\frac{1}{2}z = 60$$

$$z = 120$$

My time was 2 minutes

9) Solve the following inequalities

a) $2y \leq 6$

$y \leq 3$

b) $3x > 9$

$x > 3$

c) $\frac{1}{2}x \geq 3$

$x \geq 6$

d) $2t - 4 < 6$

$2t < 10$

$t < 5$

e) $x + 2 \geq -4$

$x \geq -6$

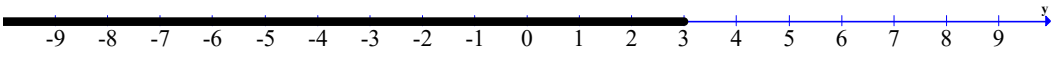
f) $2p - 3 \leq -5$

$2p \leq -2$

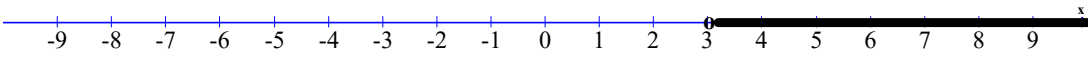
$p \leq -1$

10) Show the solutions to question 9 on a number line

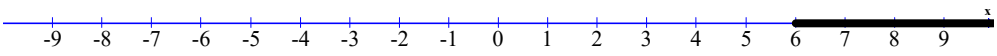
a)



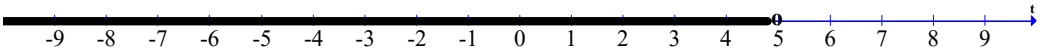
b)



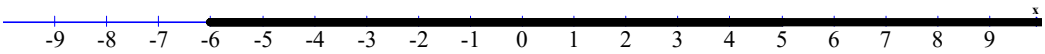
c)



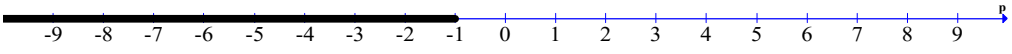
d)



e)



f)



11) The density of an object is calculated by dividing its mass by its volume; $\rho = \frac{M}{V}$. What is the density of an object of mass 10kg and volume of 5m^3?

Density $= \frac{5}{10} = 0.5$ kg per m^3

12) The two smaller sides of a right-angled triangle measure 6 cm and 8 cm respectively. Express the length of the hypotenuse in a formula and calculate its length

Let length of hypotenuse be represented by x

Than $x^2 = 6^2 + 8^2$

So $x = \sqrt{6^2 + 8^2}$

$x = \sqrt{36 + 64} = \sqrt{100} = 10cm$

13) Degrees Celsius is converted from degrees Fahrenheit by the formula

$$C = \frac{5}{9}(F - 32)$$

a) Convert 68 degrees F to C

$$C = \frac{5}{9}(68 - 32) = \frac{5}{9}(36) = 20°C$$

b) Convert 77 degrees F to C

$$C = \frac{5}{9}(77 - 32) = \frac{5}{9}(45) = 25°C$$

c) Convert 32 degrees F to C

$$C = \frac{5}{9}(32 - 32) = \frac{5}{9}(0) = 0°C$$

d) Convert 100 degrees F (the boiling point of water) to C

$$C = \frac{5}{9}(100 - 32) = \frac{5}{9}(68) = 37.78°C$$

e) Convert 30 degrees **C to F**

$$C = \frac{5}{9}(F - 32)$$

$$30 = \frac{5}{9}(F - 32)$$

$$\frac{9}{5} \times 30 = F - 32$$

$$F = 54 + 32 = 86°$$

Exercise 5

Graphing Linear Equations

1) For each of the following equations, construct a table of values for values of x from $x = -3$ to $x = 3$

a) $y = x + 2$

x	y
-3	-1
-2	0
-1	1
0	2
1	3
2	4
3	5

b) $y = 2x + 1$

x	y
-3	-5
-2	-3
-1	-1
0	1
1	3
2	5
3	7

c) $y = -x + 4$

x	y
-3	7
-2	6
-1	5
0	4
1	3
2	2
3	1

d) $y = -2x - 2$

x	y
-3	4
-2	2
-1	0
0	-2
1	-4
2	-6
3	-8

e) $y = 4x + 1$

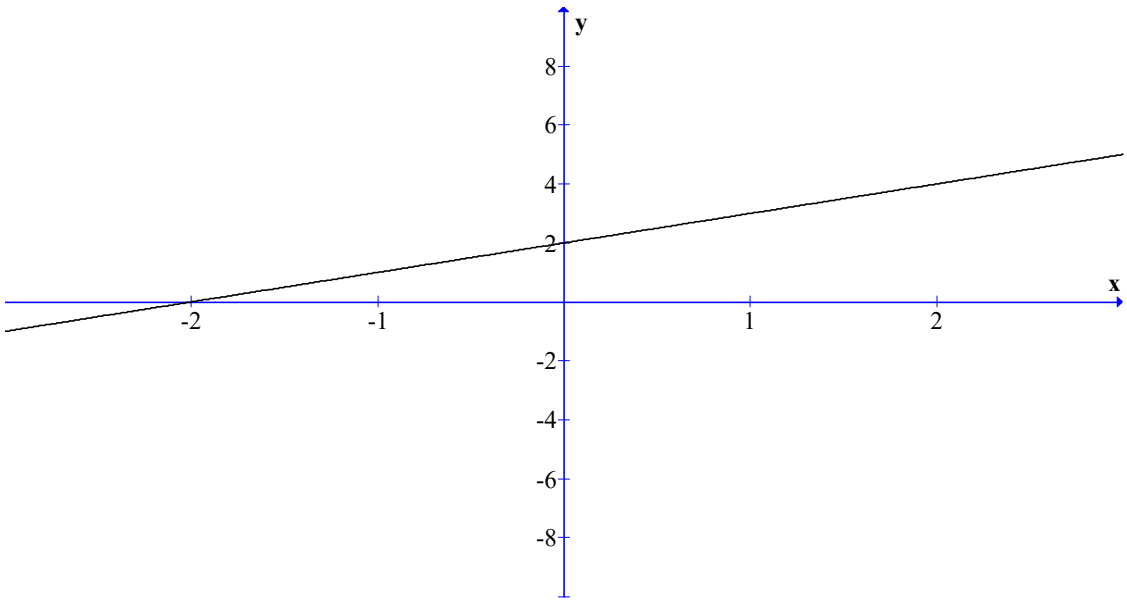
x	y
-3	-11
-2	-7
-1	-3
0	1
1	5
2	9
3	13

f) $y = 2$

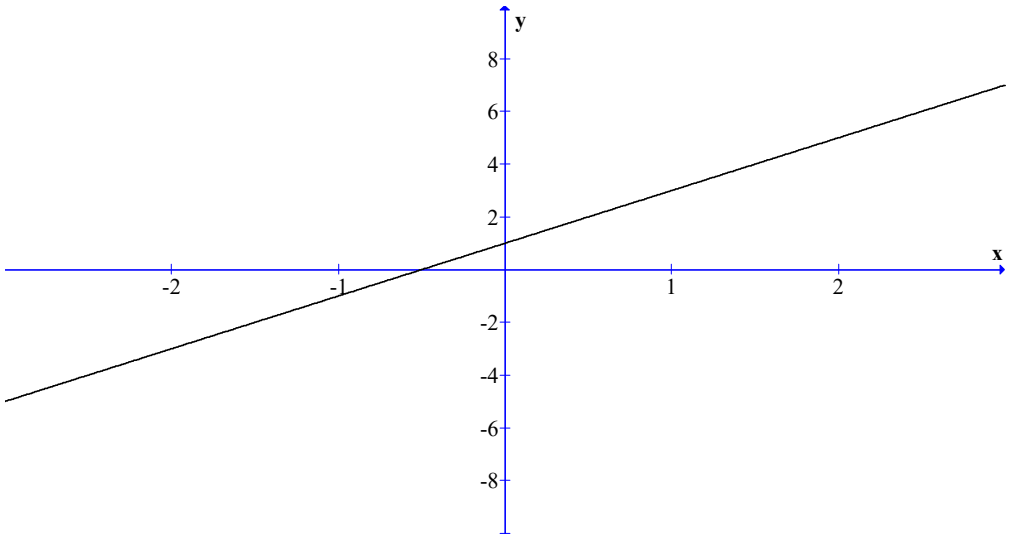
x	y
-3	2
-2	2
-1	2
0	2
1	2
2	2
3	2

2) On the same grid, draw a graph for each part of question 1, and label your lines

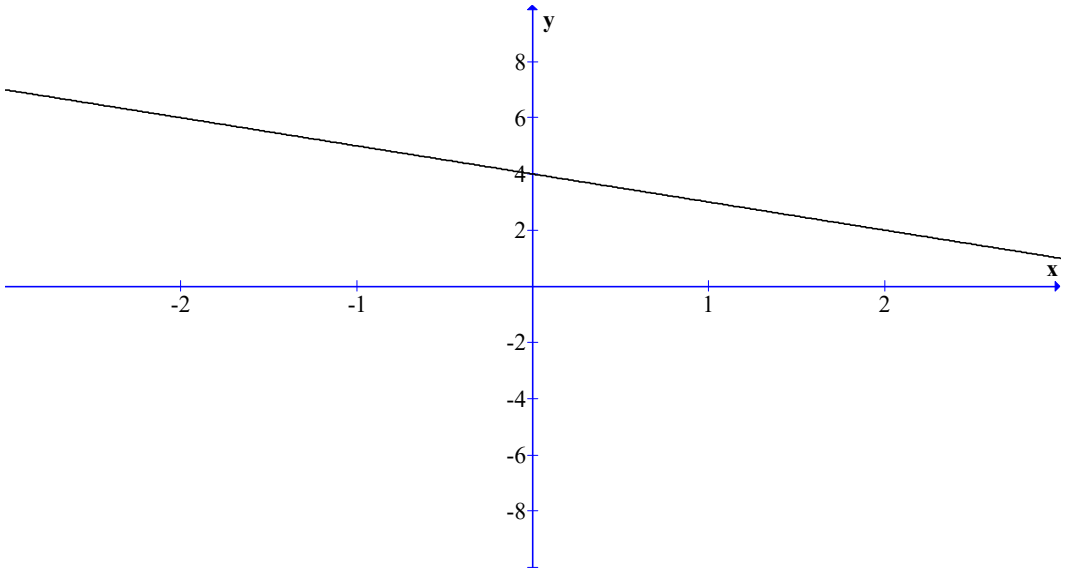
a)



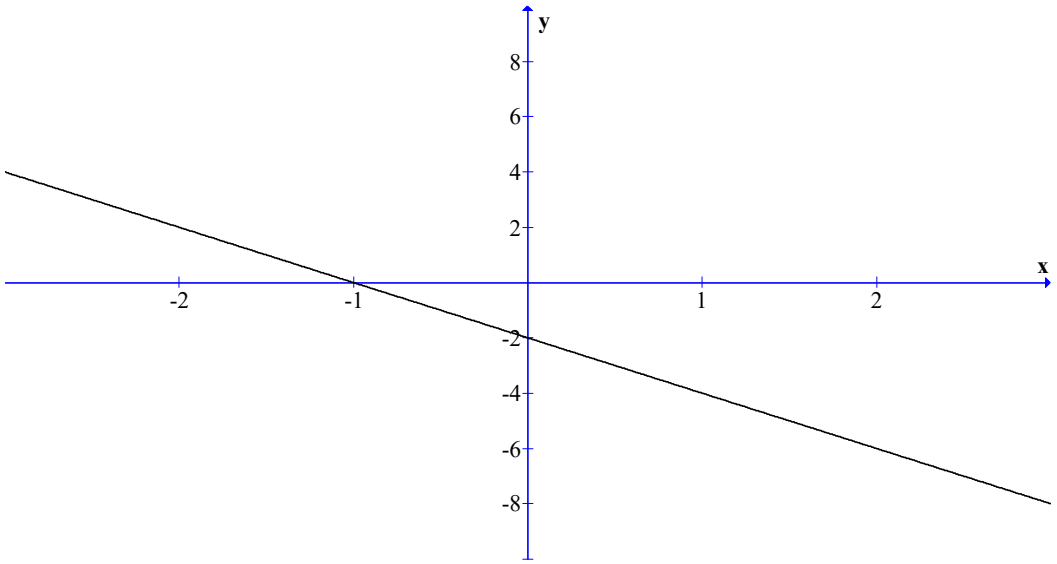
b)



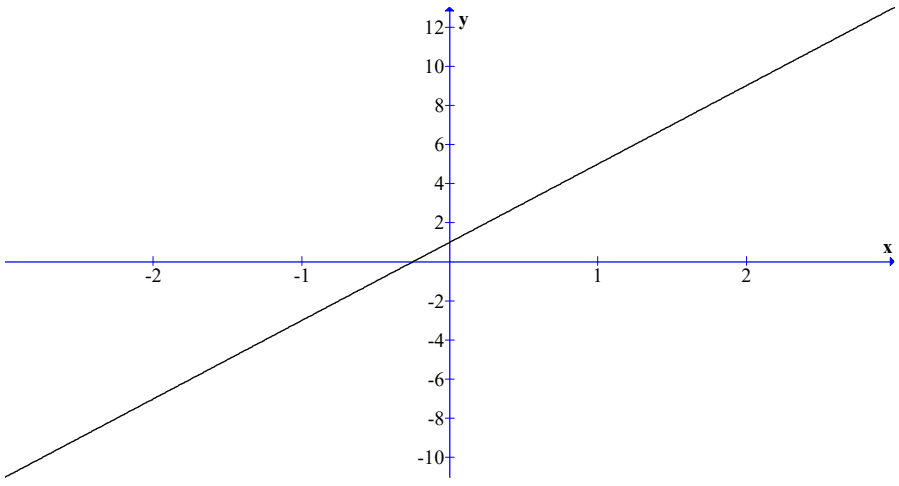
c)



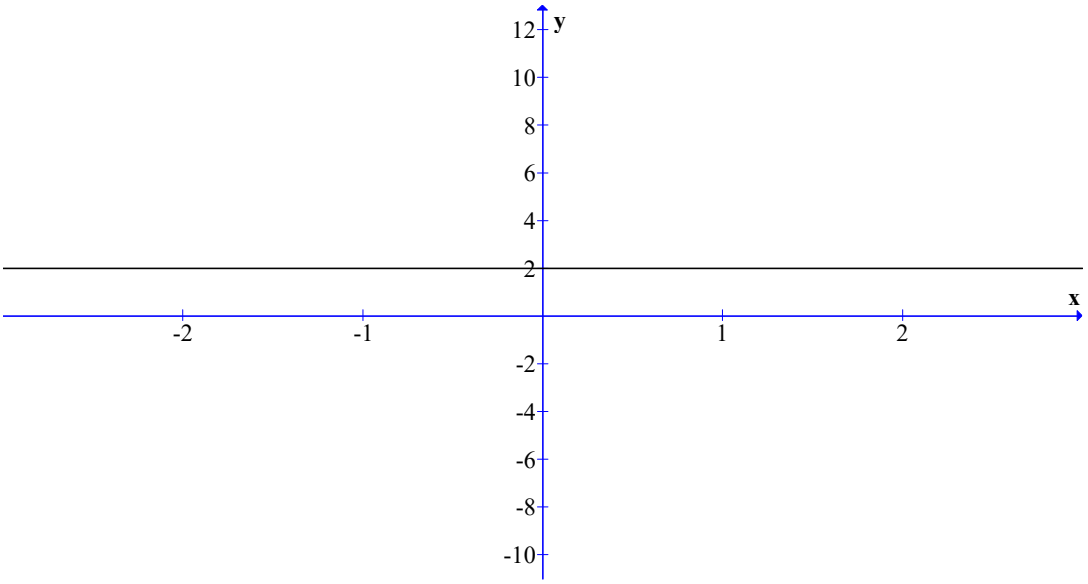
d)



e)

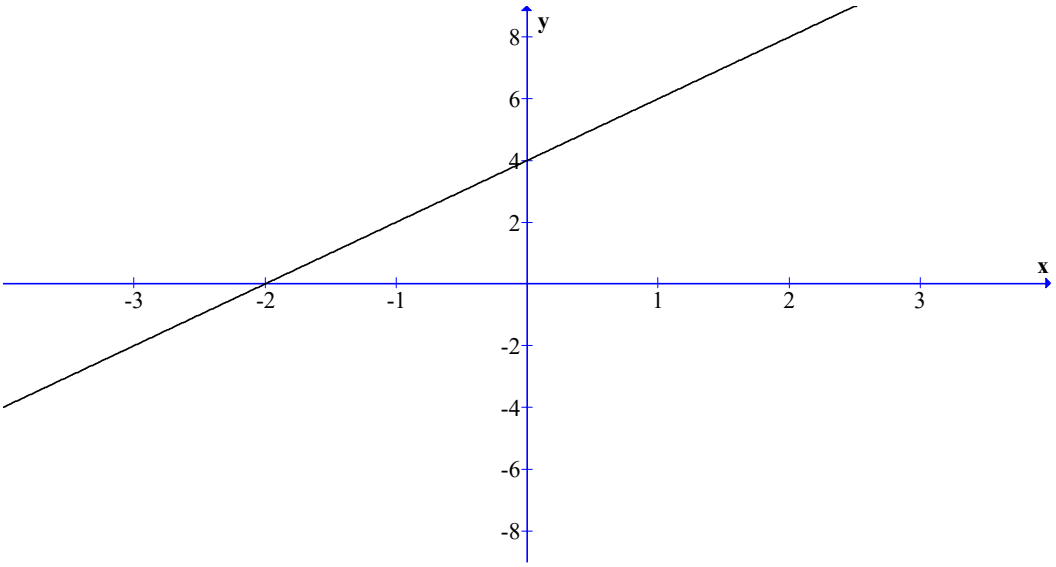


f)

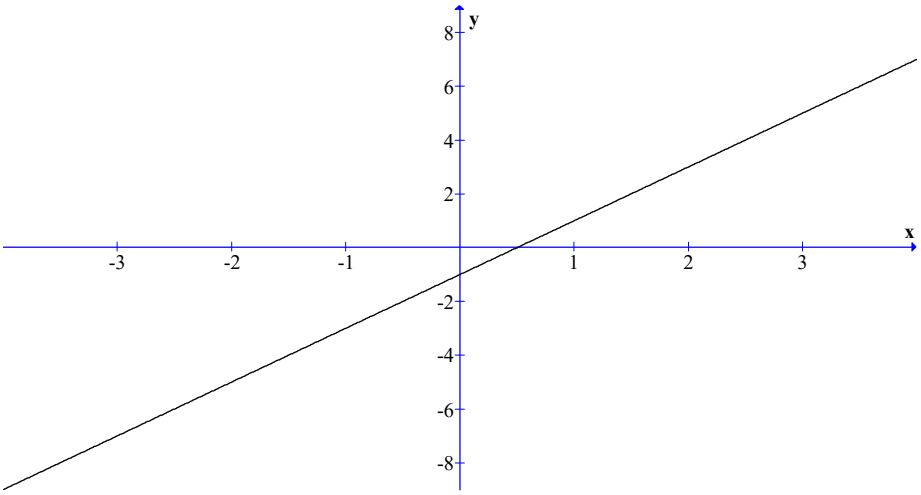


3) Graph the following equations on the same grid, and comment on their similarities

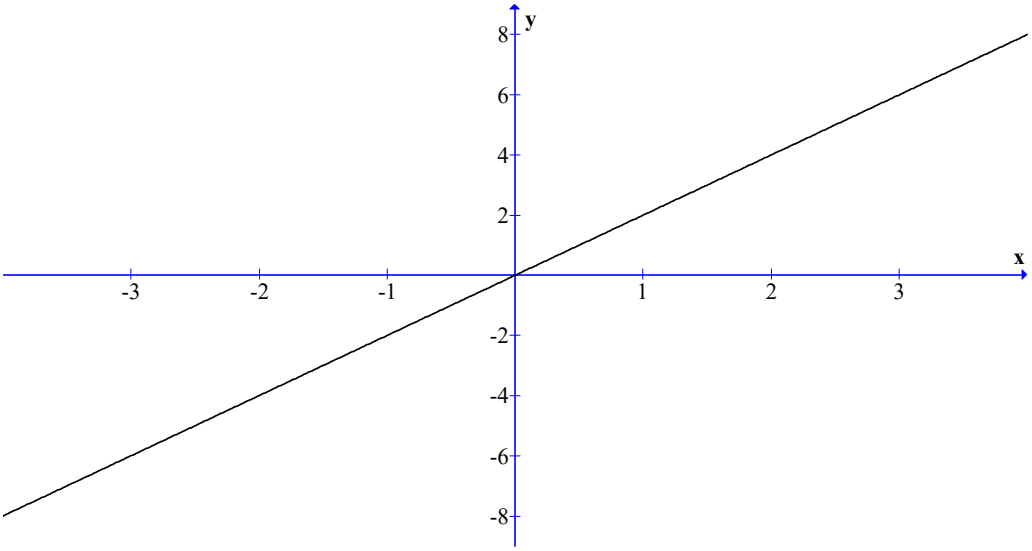
a) $y = 2x + 4$



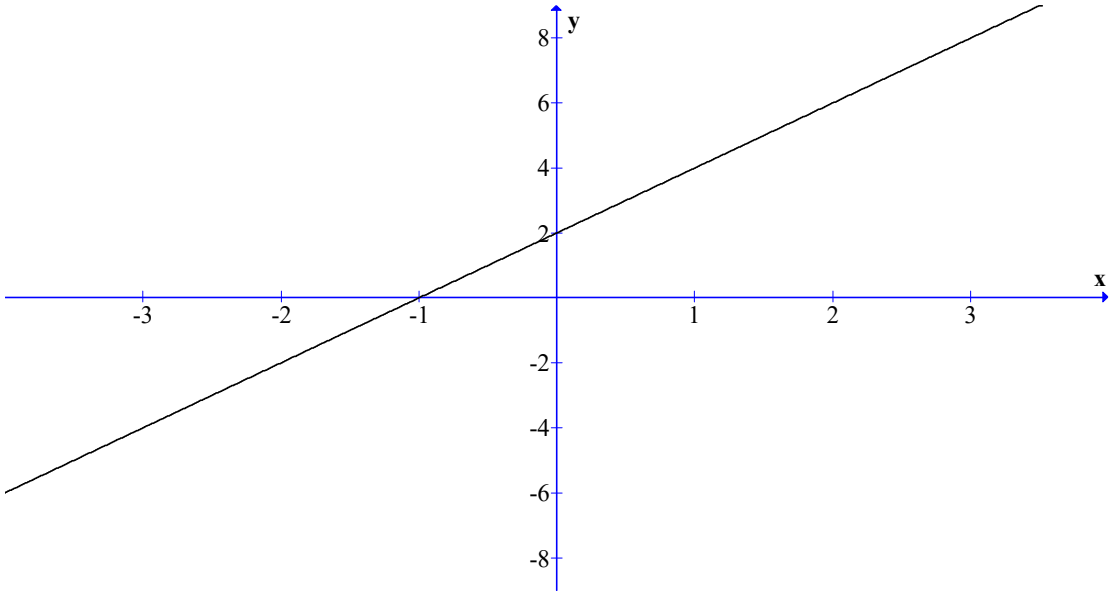
b) $y = 2x - 1$



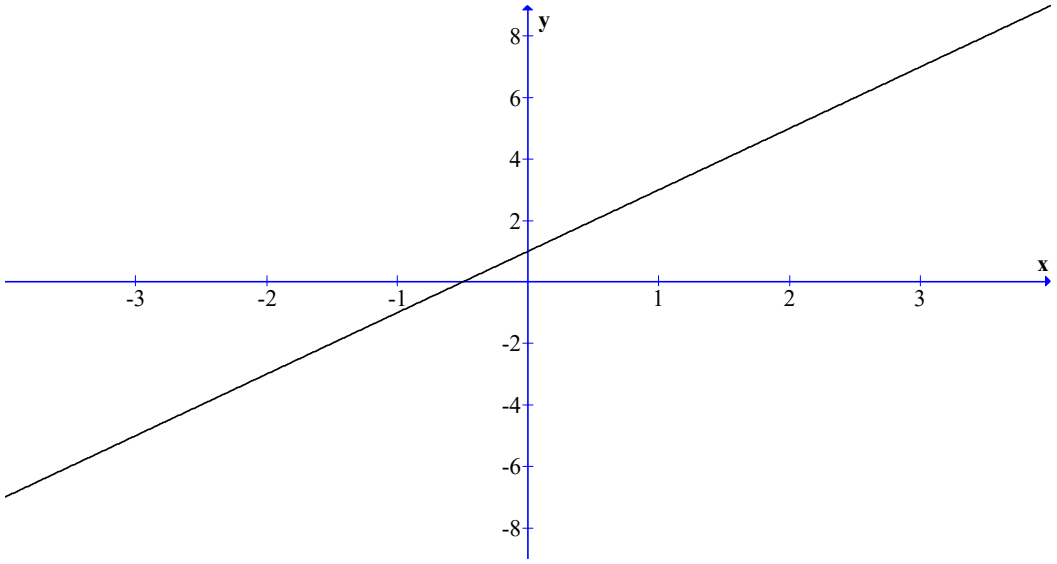
c) $y = 2x$



d) $y = 2x + 2$



e) $y = 2x + 1$



The lines are all parallel

4) From your answers to question 3, which of the following equations would produce a line parallel to them?

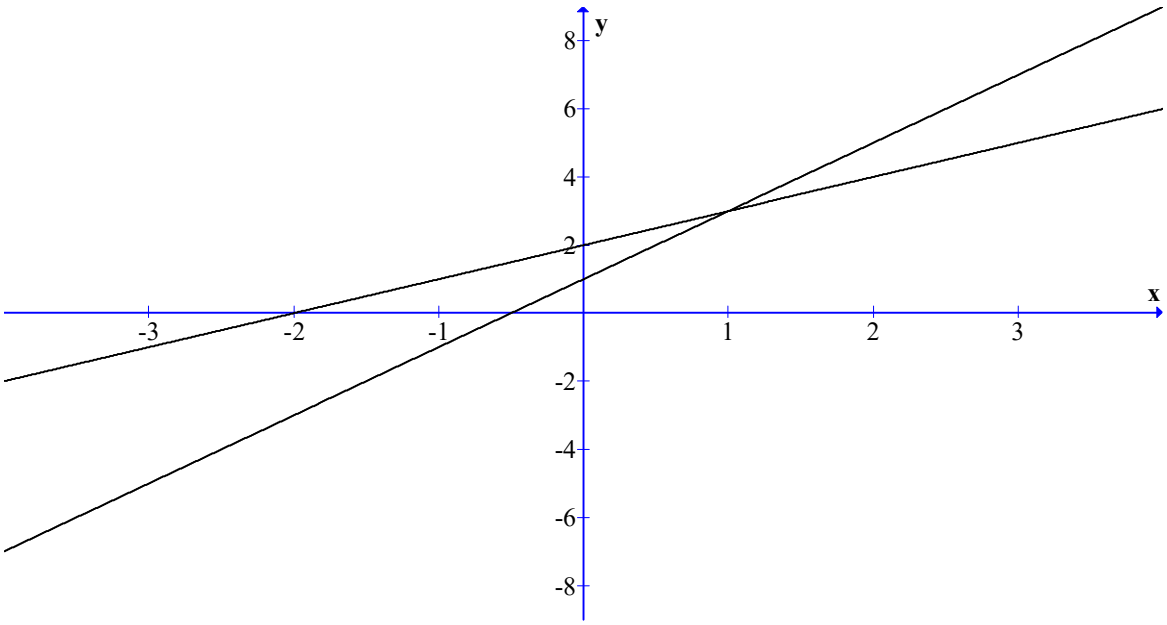
- $y = 3x - 2$
- $y = x + 4$
- $y = 2x - 7$
- $y = -2x + 4$
- $y = 2x + 100$

- $y = \frac{1}{2}x$

$y = 2x - 7$
$y = 2x + 100$

5) Graph the following pair of equations, and determine their point of intersection

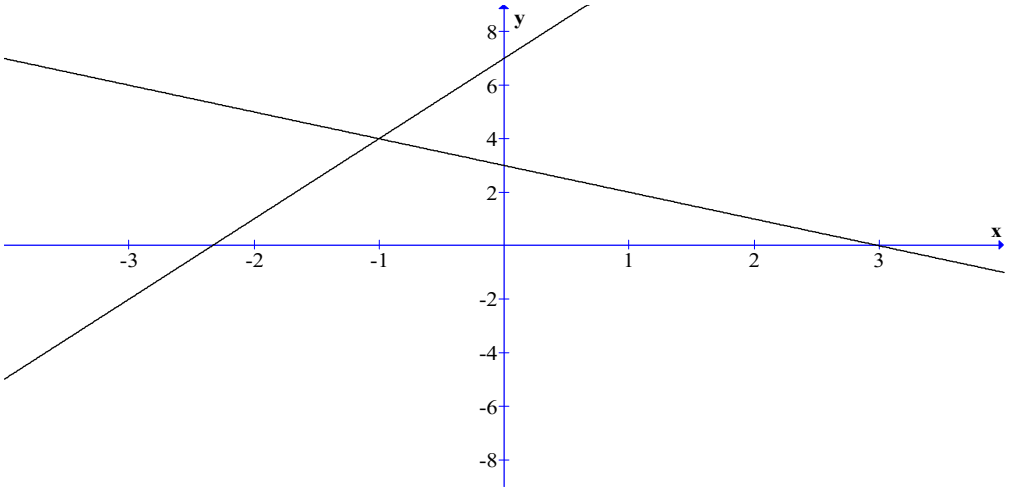
$y = 2x + 1$ and $y = x + 2$



Intersection is point $(1,3)$

6) Graph the following pair of equations and determine their point of intersection

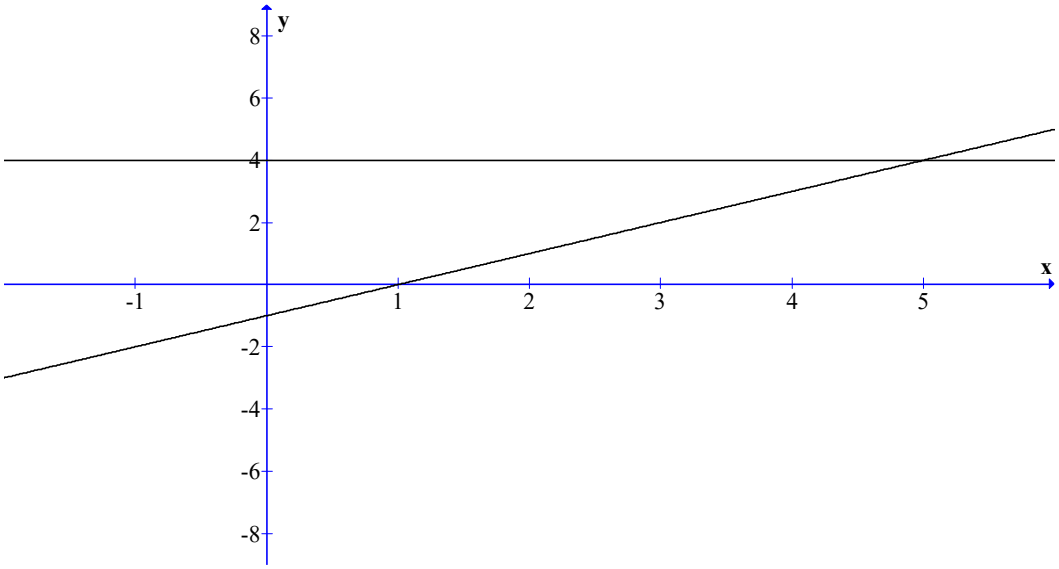
$y = -x + 3$ and $y = 3x + 7$



Intersection is point $(-1,4)$

7) Graph the following pair of equations and determine their point of intersection

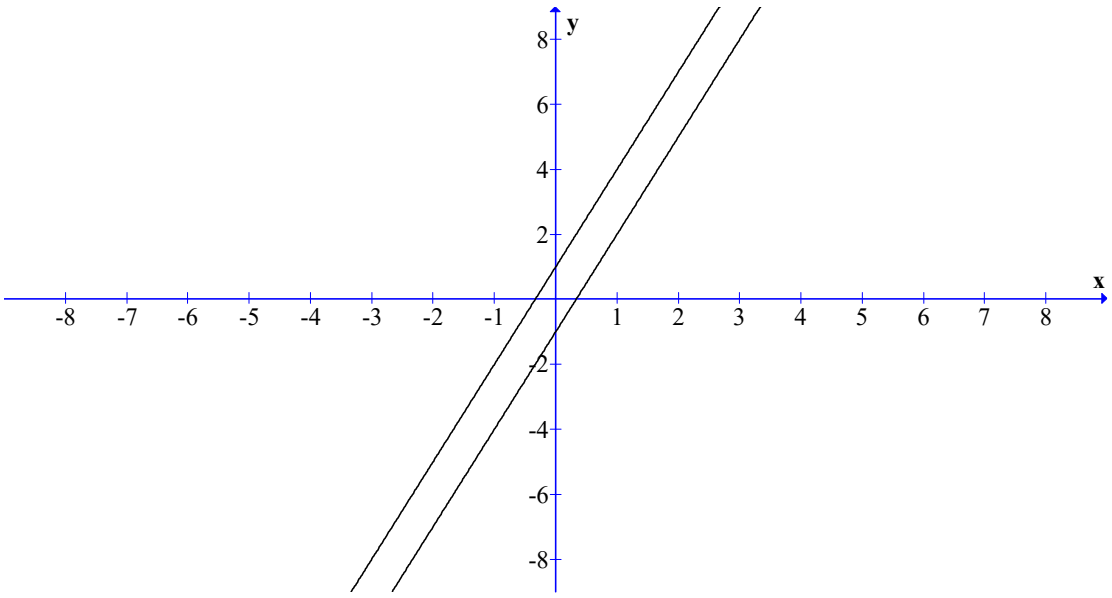
$y = 4$ and $y = x - 1$



Intersection is point $(5,4)$

8) Graph the following pair of equation and determine their point of intersection

$y = 3x + 1$ and $y = 3x - 1$



There is no point of intersection; the lines are parallel

9)

a) Write two equations for which their lines intersect at one point

The equation of any two lines that are not parallel will intersect at one point

b) Write two equations for which their lines do not intersect

Any equations for which the two lines have the same value in front of the x

c) Write two equations for which the lines intersect at more than one point

Two lines never intersect at more than one point

Year 8 Mathematics

Data

Exercise 1

Data Representation

1) Construct a frequency distribution table for each of the following sets of data

a) 1, 4, 2, 4, 3, 5, 4, 6, 9, 4, 2, 1, 9, 7, 4

Number	Frequency
1	2
2	2
3	1
4	5
5	1
6	1
7	1
9	2

b) 8, 7, 6, 7, 10, 11, 1, 7, 4, 7

Number	Frequency
1	1
4	1
6	1
7	4
8	1
10	1
11	1

c) 15, 20, 17, 15, 20, 18, 15, 17, 20

Number	Frequency
15	2
17	2
18	1
20	5

d) 100, 105, 106, 100, 100, 105, 110, 103, 104, 105, 100

Number	Frequency
100	4
103	1
104	1
105	3
106	1
110	1

e) 1, 1, 1, 1, 1, 1, 2

Number	Frequency
1	6
2	1

2) What is the difficulty in constructing a frequency table for the following data set?

20, 30, 32, 41, 24, 23, 25, 38, 37, 36, 22, 26, 44, 33, 36, 48, 46, 38, 37.5, 41.5, 42.5, 44.5, 22.5, 33.5, 34.5, 40.5, 49.5, 53, 23.5

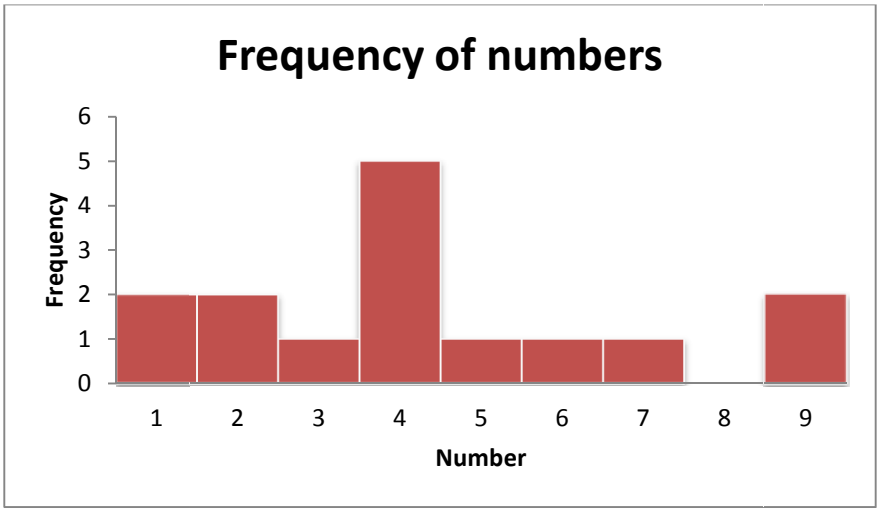
There are many numbers, there is a large range, and most of the numbers have a frequency of 1

3) Construct a grouped frequency distribution table using a class interval of 5 for the data from question 2

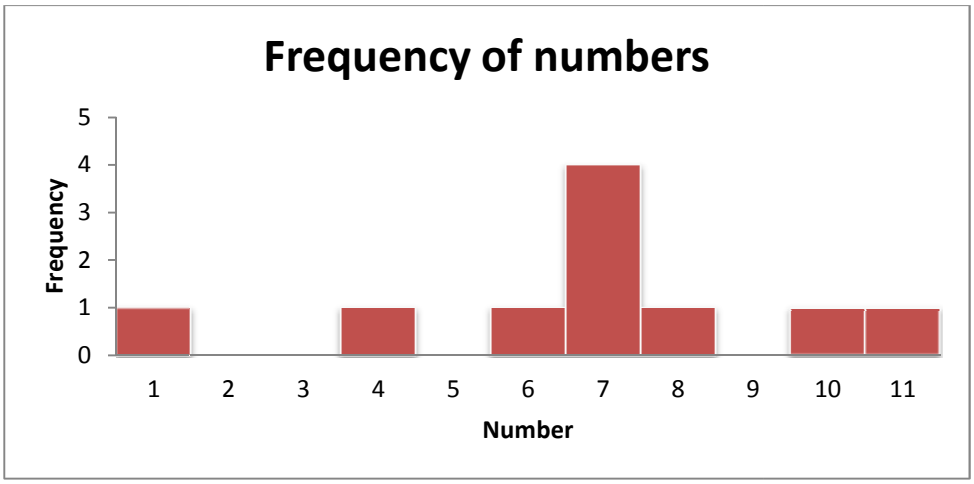
Class	Frequency
20-24.9	2
25-29.9	2
30-34.9	1
35-39.9	5
40-44.9	1
45-49.9	1
50-54.9	1

4) Draw frequency histograms for each of the data sets in questions 1a to 1d

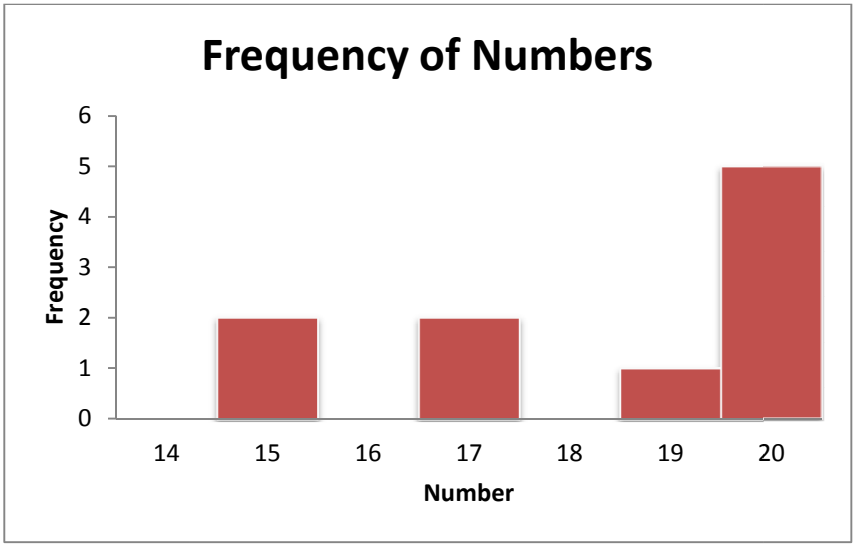
a)



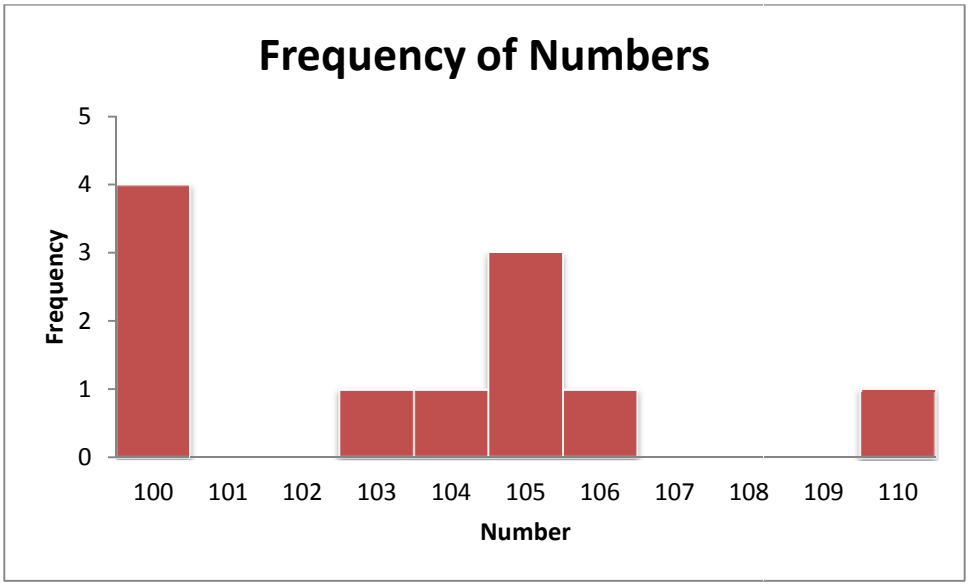
b)



c)

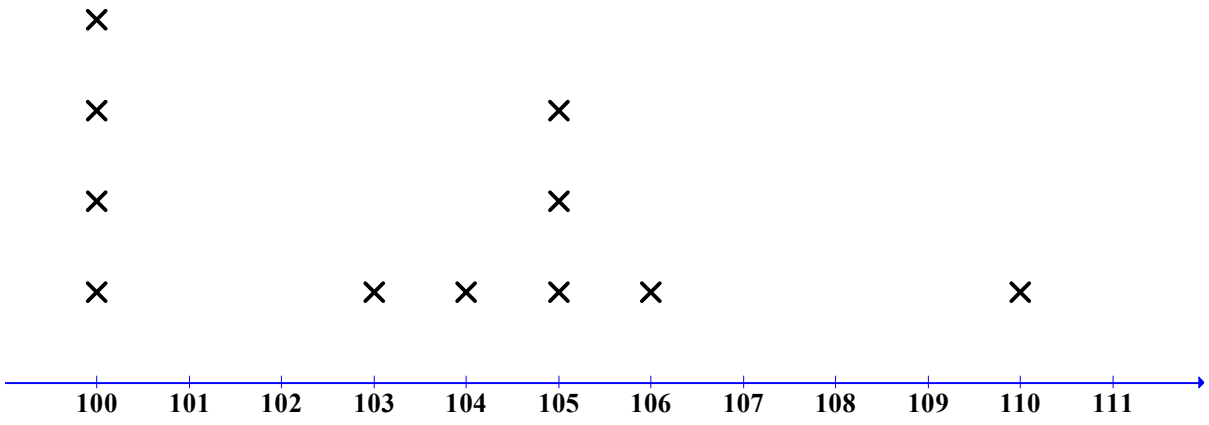


d)

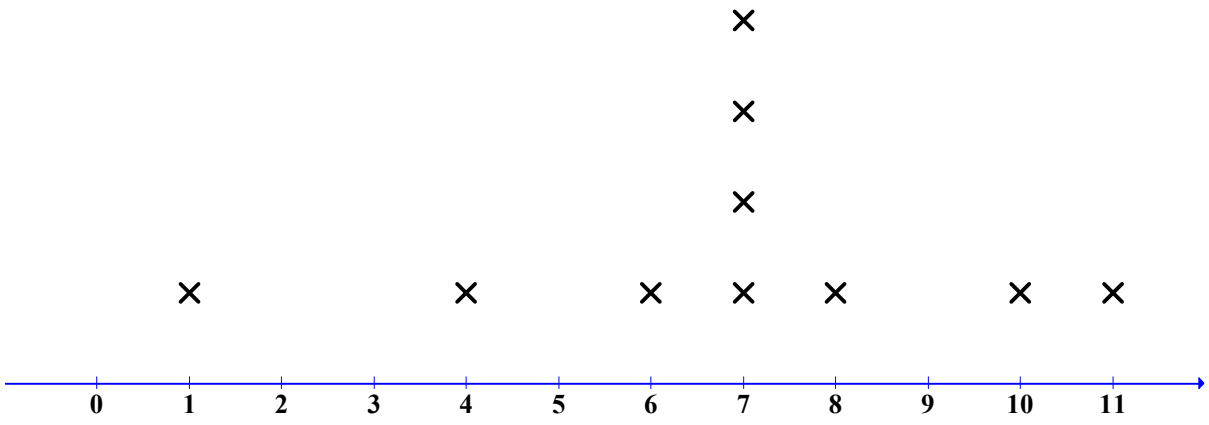


5) Draw dot plots for each of the data sets in questions 1a to 1d

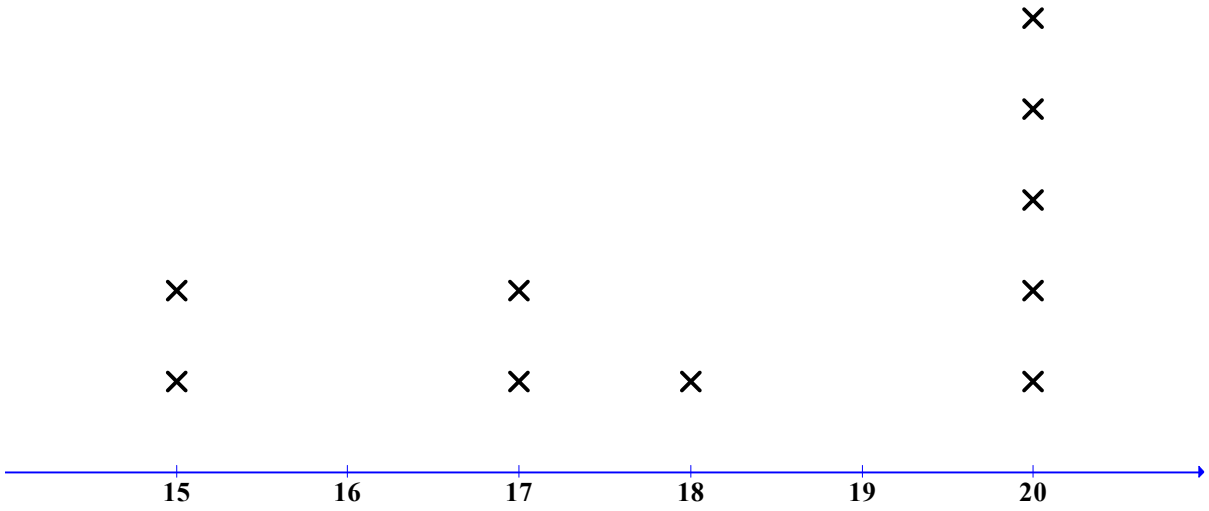
a)



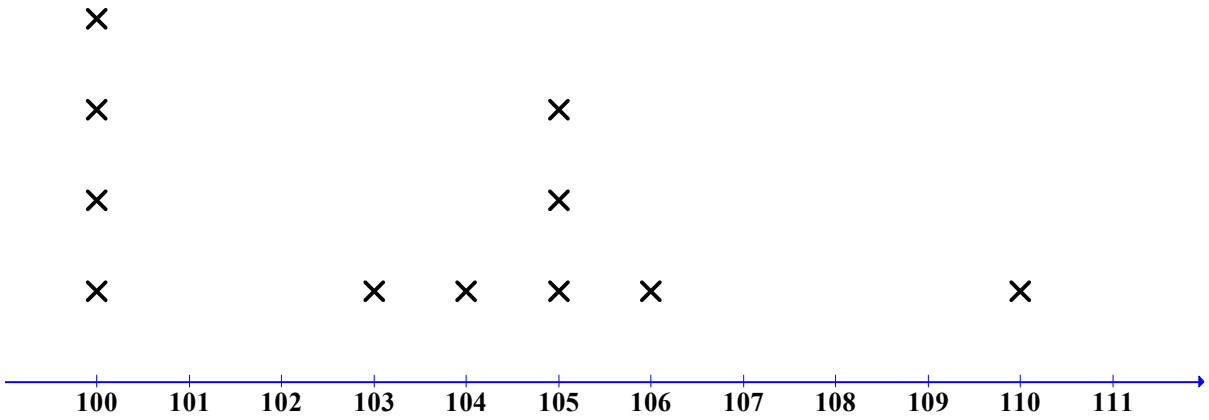
b)



c)



d)



6) Organise each of the following data sets into stem and leaf plots

a) 20, 23, 25, 31, 32, 34, 42, 42, 43, 26, 37, 41, 30, 25, 26, 53, 27, 33, 23, 30, 41

2	0 3 3 5 5 6 6 7
3	0 0 1 2 3 4 7

4	1 1 2 2 3
5	3

b) 73, 62, 66, 76, 78, 80, 83, 99, 92, 75, 74, 88, 99, 70, 71, 69, 66, 73, 81

6	2 6 6 9
7	0 1 3 3 4 5 6 8
8	0 1 3 8
9	2 9 9

c) 12, 10, 22, 24, 35, 46, 47, 32, 31, 43, 22, 21, 45, 56, 43, 32, 37, 49, 40, 21, 20, 30, 27, 26, 32, 21, 50, 60, 22

1	0 2
2	0 1 1 1 2 2 2 4 6 7
3	0 1 2 2 2 5 7
4	0 3 3 6 5 7 9
5	0 6
6	0

7) Identify the outlier in each of the following data sets

a) 1, 1, 1, 1, 1, 1, 2, 100

100

b) 100, 105, 3, 106, 100, 100, 105, 110, 103, 104, 105, 100

3

c) 73, 62, 66, 76, 78, 80, 83, 99, 92, 75, 74, 1225, 88, 99, 70, 71, 69, 66, 73, 81

1225

d) 20, 30, 32, 41, 24, 23, 25, 38, 37, 36, 22, 26, 44, 33, 36, 48, 46, 38, 37.5, 22345, 41.5, 42.5, 44.5, 22.5, 33.5, 34.5, 40.5, 49.5, 53, 23.5

22345

Exercise 2

Data Analysis

1) Find the mean, mode & median of the following data sets

a) 10, 7, 5, 7, 6, 3, 4, 3, 20, 7, 6, 6, 15, 14, 7

$$\text{Mean} = \frac{Sum\ of\ scores}{Number\ of\ scores}$$

$$= \frac{120}{15} = 8$$

$$\text{Mode} = 7$$

To find median put scores in order

3, 3, 4, 5, 6, 6, 6, 7, 7, 7, 7, 10, 14, 15, 20

Median is middle score $= 7$

b) 4, 20, 8, 13, 12, 15, 8, 15, 18, 7, 13, 9, 20, 17, 1

$$\text{Mean} = \frac{Sum\ of\ scores}{Number\ of\ scores}$$

$$= \frac{180}{15} = 12$$

$$\text{Mode} = 8,13,15\ and\ 20$$

To find median put scores in order

1, 4, 7, 8, 8, 9, 12, 13, 13, 15, 15, 17, 18, 20, 20

Median is middle score $= 13$

c) 4, 19, 20, 16, 11, 16, 1, 10, 15, 5, 18, 17, 19, 14, 4

$$\text{Mean} = \frac{Sum\ of\ scores}{Number\ of\ scores}$$

$$= \frac{189}{15} = 12.6$$

$$\text{Mode} = 4\ and\ 19$$

To find median put scores in order

1, 4, 4, 5, 10, 11, 14, 15, 16, 16, 17, 18, 19, 19, 20

Median is middle score $= 15$

d) 12, 8, 2, 4, 7, 2, 1, 9, 16, 15, 17, 1, 1, 20, 14

$$\text{Mean} = \frac{Sum\ of\ scores}{Number\ of\ scores}$$

$$= \frac{129}{15} = 8.6$$

$$\text{Mode} = 1$$

To find median put scores in order

1, 1, 1, 2, 2, 4, 7, 8, 9, 12, 14, 15, 16, 17, 20

Median is middle score $= 8$

e) 17, 5, 3, 15, 19, 12, 5, 1, 3, 11, 18, 17, 14, 1, 7

Mean $= \frac{Sum\ of\ scores}{Number\ of\ scores}$

$= \frac{148}{15} = 9.87$

Mode $= 1, 3, 5, 17$

To find median put scores in order

1, 1, 3, 3, 5, 5, 7, 11, 12, 14, 15, 17, 17, 18, 19

Median is middle score $= 11$

2) Find the mean mode and median from the following frequency distribution tables

a)

Value	Frequency
1	6
2	4
3	1
4	0
5	5
6	4
7	7

Sum of scores $= (6 \times 1) + (4 \times 2) + (1 \times 3) + (5 \times 5) + (4 \times 6) + (7 \times 7) = 115$

Number of scores $= 27$

Mean $= \frac{115}{27} = 4.26$

Mode $= 7$

Median $= 5$

b)

Value	Frequency
20	2
21	2
22	2
23	3
24	2
25	4
26	3

Sum of scores $= (2 \times 20) + (2 \times 21) + (2 \times 22) + (3 \times 23) + (2 \times 24) + (4 \times 25) + (3 \times 26) = 421$

Number of scores $= 18$

Mean $= \frac{421}{18} = 23.39$

Mode $= 25$

There is an even number of scores, so median is between the two middle scores (scores 9 and 10)

The 9th score is 23, the 10th score is 24, so median $= 23.5$

c)

Value	Frequency
11	1
12	1
13	1
14	1
15	1
16	1
17	1

Sum of scores $= 11 + 12 + 13 + 14 + 15 + 16 + 17 = 98$

Number of scores $= 7$

Mean $= \frac{98}{7} = 14$

There is no mode

Median $= 14$

d)

Value	Frequency
1	6
2	4
3	1
4	0
5	5
6	4
7	7
1000	1

Sum of scores $= (6 \times 1) + (4 \times 2) + (1 \times 3) + (5 \times 5) + (4 \times 6) + (7 \times 7) + (1 \times 1000) = 1115$

Number of scores $= 28$

Mean $= \frac{1115}{28} = 39.82$

Mode $= 7$

Median $= 5$

3) Using your answers to parts 2a and 2d, what effect does an outlier have on the value of the mode, mean & median?

It increases the mean (if the outlier is higher than the majority of the numbers in the data set, or decreases the mean if it is less)

It has no effect on the mode, since an outlier is only 1 additional number

It has had no effect on the median, although there may be cases where the inclusion of an outlier increases (or decreases) the median slightly

4) Represent the following test scores in a stem and leaf plot, and use it to calculate the mean, mode, median & range of the data

a) 83, 80, 48, 71, 61, 58, 47, 52, 56, 78, 86, 47, 62, 57, 77, 60, 46, 89, 81, 72

4	6, 7, 7, 8
5	2, 6, 7, 8
6	0, 1, 2
7	1, 2, 7, 8
8	0, 1, 3, 6, 9

Sum of scores $= 1311$

Number of scores $= 20$

Mean $= \frac{1311}{20} = 65.55$

Mode $= 47$

Median is between the 10^{th} and 11^{th} scores

$= 61.5$

b) 48, 88, 50, 49, 54, 56, 57, 47, 48, 84, 62, 82, 69, 79, 51, 48, 89, 49, 65, 75

4	7, 8, 8, 8, 9, 9
5	0, 1, 4, 6, 7
6	2, 5, 9
7	5, 9
8	2, 4, 8, 9

Sum of scores $= 1250$

Number of scores $= 20$

Mean $= \frac{1250}{20} = 62.5$

Mode $= 48$

Median is between 10^{th} and 11^{th} score $= 56.5$

c) 74, 84, 69, 61, 79, 81, 77, 56, 50, 48, 51, 61, 90, 76, 53, 47, 56, 52, 89, 88

4	7, 8
5	0, 1, 2, 3, 6, 6
6	1, 1, 9
7	4, 6, 7, 9
8	1, 4, 8, 9
9	0

Sum of scores $= 1342$

Number of scores $= 20$

Mean $= 67.1$

Mode $56\ and\ 61$

Median is between 10^{th} and 11^{th} scores $= \frac{61+69}{2} = 65$

5) Calculate the mean, mode, median & range for the following dot plot

Sum of scores $= (1 \times 18) + (3 \times 19) + (2 \times 21) + (1 \times 22) + (3 \times 23) + (5 \times 24) + (2 \times 25) + (3 \times 26) + (2 \times 28) = 512$

Number of scores $= 22$

Mean $= \frac{512}{22} = 23.27$

Mode $= 24$

Median is between 11^{th} and 12^{th} scores. In this case both of these scores are 24, so median is 24

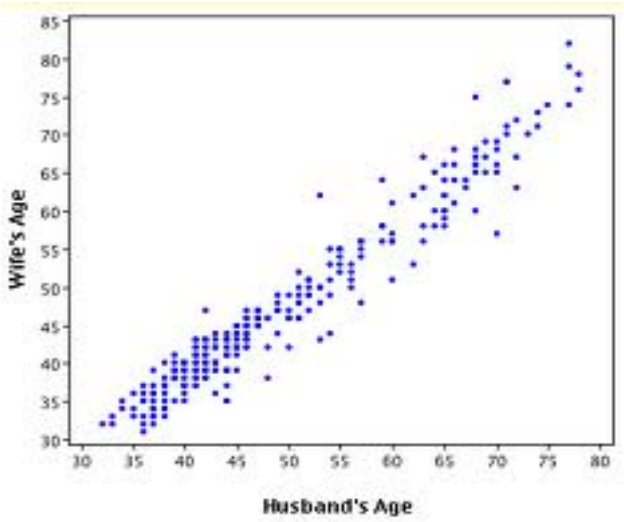
6) The mean of a set of data is 25, its mode is 30 (there are 10 scores of 30), and its median is 28. A new score of 200 is added to the set. What effect will this new score have on the mean, mode & median?

The mean will increase slightly.

The mode will remain unchanged

The median will probably remain unchanged, but could increase slightly

7) The following scatter graph shows the relative ages of husbands and wives. Each dot represents a married couple



a) Describe what conclusions can be drawn from the graph in relation to the relative ages of married couples

The majority of married couples consist of a male who is between 2 and 5 years older than his wife

There are very few people married to anyone much older than themselves

Most married couples are in the 35 to 50 year old range

b) Why are there more data points toward the bottom left of the graph?

As the ages of the couples increase, factors may cause the marriage to dissolve, most probably divorce or

death of at least one of the couple

c) Would the graph appear the same if the data had been collected 1000 years ago? Explain your answer

One thousand years ago people tended to be married earlier, and more often than today outside their age group.

Life expectancy one thousand years ago was a lot less than today, so there would be even fewer points toward the top right of the graph

Year 8 Mathematics

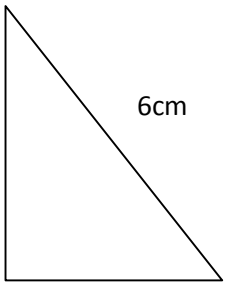
Measurement

Exercise 1

Applications of Pythagoras' Theorem

1) Calculate the length of the unknown side in the following diagrams, leaving your answer in surd form if necessary

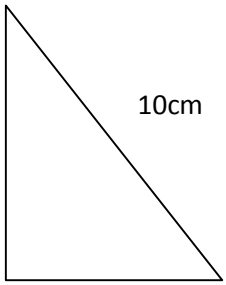
a)



$x^2 + 3^2 = 6^2$

$x = \sqrt{6^2 - 3^2} = \sqrt{27} (= 3\sqrt{3})$

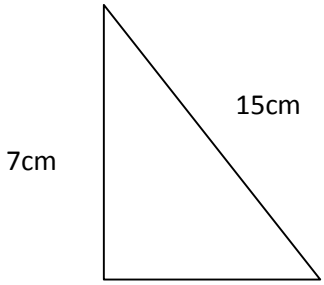
b)



$x^2 + 2^2 = 10^2$

$x = \sqrt{10^2 - 2^2} = \sqrt{96} (= 4\sqrt{6})$

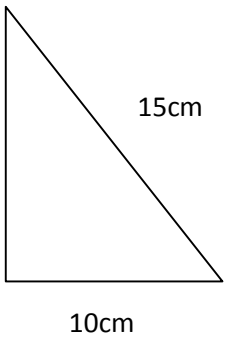
c)



$x^2 + 7^2 = 15^2$

$x = \sqrt{15^2 - 7^2} = \sqrt{176} (= 4\sqrt{11})$

d)



$x^2 + 10^2 = 15^2$

$x = \sqrt{15^2 - 10^2} = \sqrt{125} (= 5\sqrt{5})$

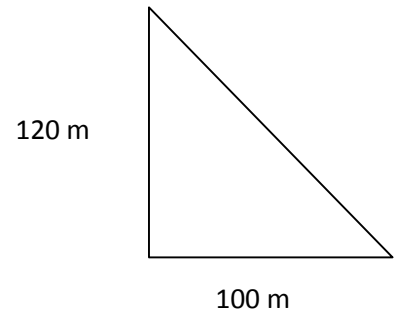
2) The equal sides of an isosceles right-angled triangle measure 8cm. What is the length of the third side?

The third side must be the hypotenuse, since it cannot be equal in length to any other side

So $x^2 = 8^2 + 8^2$

$x = \sqrt{64 + 64} = \sqrt{128} = 8\sqrt{2}$

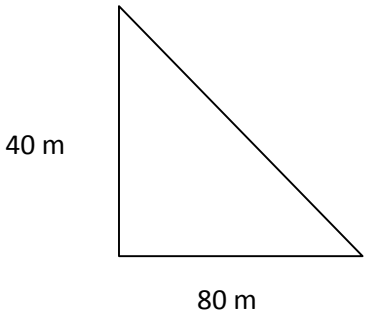
3) A man stands at the base of a cliff which is 120 metres high. He sees a friend 100 metres away along the beach. What is the shortest distance from his friend to the top of the cliff?



$x^2 = 120^2 + 100^2$

$x = \sqrt{14400 + 10000} = \sqrt{24400} \cong 156.2\ m$

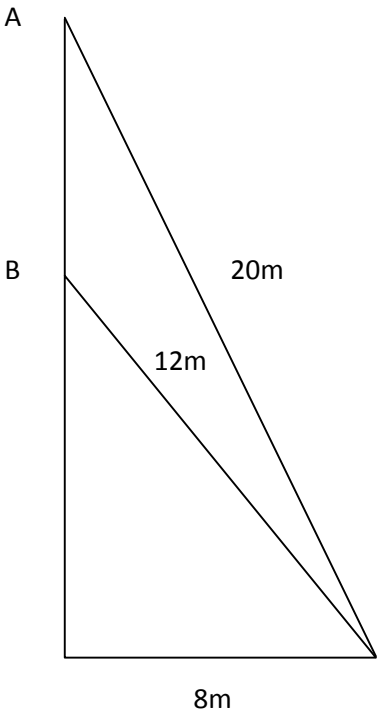
4) A steel cable runs from the top of a building to a point on the street below which is 80 metres away from the bottom of the building. If the building is 40 metres high, how long is the steel cable?



$x^2 = 40^2 + 80^2$

$x = \sqrt{1600 + 6400} = \sqrt{80000} \cong 282.84\ m$

5) What is the distance from point A to point B?



Let distance from ground to point A be x

$x^2 + 8^2 = 20^2$

$x = \sqrt{20^2 - 8^2} = \sqrt{400 - 64} = \sqrt{336} \cong 18.33\ m$

Let distance from ground to point B be y

$y^2 + 8^2 = 12^2$

$y = \sqrt{12^2 - 8^2} = \sqrt{144 - 64} = \sqrt{80} \cong 8.94\ m$

Distance from point B to point A is $18.33 - 8.94 = 9.39\ m$

6) A right angled triangle has an area of 20 cm^2. If its height is 4cm, what is the length of its hypotenuse?

Area of triangle $= \frac{1}{2} \times base \times height$

Area $= 20 = \frac{1}{2} \times base \times 4$

Base $= 10\ cm$

$x^2 = 10^2 + 4^2$

$x = \sqrt{100 + 16} \cong 10.77\ cm$

7) What is the length of a diagonal of a square of side length 5cm?

$x^2 = 5^2 + 5^2$

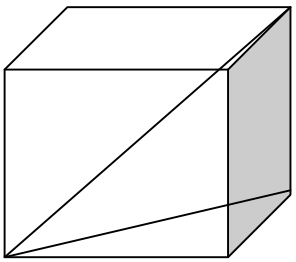
$x = \sqrt{25 + 25} = \sqrt{50} \cong 7.07\ cm$

8) A man is laying a slab for a shed. The shed is to be 6m wide and 8m long. To check if he has the corners as exactly right angles, what should the slab measure from corner to corner?

$x^2 = 6^2 + 8^2$

$x = \sqrt{36 + 64} = \sqrt{100} = 10\ m$

9) A box is in the shape of a cube. If the length of each side is 4cm, what is the length of a line drawn from the top left to the bottom right of the box?



The said line is the hypotenuse of a right angled triangle with its other two sides being a side of the cube (4cm), and the diagonal of the base

Need length of diagonal of the base

The diagonal forms a right angled triangle with two of the sides of the base (4 cm each)

$x^2 = 4^2 + 4^2$

$x = \sqrt{32}$

Let required measurement be y

Then $y^2 = x^2 + 4^2$

From above $x = \sqrt{32}$, so $x^2 = 32$

Therefore $y^2 = 32 + 16 = 48$

$y = \sqrt{48} \cong 6.93\ cm$

10) The path around the outside of a rectangular park is 60m long and 40m wide. How much less will the walk from one corner of the park to another be if a path is built directly across the park from corner to corner?

Let x represent distance from corner to corner

Then $x^2 = 60^2 + 40^2$

$x = \sqrt{3600 + 1600} = \sqrt{5200} \cong 72.11\ m$

Distance from one corner to another going around park is $= 100\ m$

Therefore diagonal path will save $100 - 72.11 = 27.89\ m$

Exercise 2

Circles: Area & Perimeter

1) Calculate the perimeter of the following circles

a) Radius of 3cm

Perimeter $= 2\pi r = 2 \times \pi \times 3 \cong 18.84\ cm$

b) Diameter of 10cm

Perimeter $= \pi d = \pi \times 10 \cong 31.4\ cm$

c) Radius of 2cm

Perimeter $= 2\pi r = 2 \times \pi \times 2 \cong 12.56\ cm$

d) Radius of 0.5cm

Perimeter $= 2\pi r = 2 \times \pi \times 0.5 \cong 3.14\ cm$

e) Diameter of π cm

Perimeter $= \pi d = \pi \times \pi \cong 9.86\ cm$

2) Calculate the area of the following circles

a) Diameter of 8cm

Area $= \pi r^2$

Radius $= 4\ cm$

Area $= \pi \times 4 \times 4 \cong 50.24\ cm^2$

b) Radius of 2cm

Area $= \pi r^2 = \pi \times 2 \times 2 \cong 12.56\ cm^2$

c) Diameter of 1cm

Area $= \pi r^2$

Radius $= 0.5\ cm$

Area $= \pi \times 0.5 \times 0.5 \cong 0.785\ cm^2$

d) Radius of π cm

Area $= \pi r^2 = \pi \times \pi \times \pi \cong 30.96\ cm^2$

e) Diameter of 2π cm

Area $= \pi r^2$

Radius $= \pi\ cm$

Area $= \pi \times \pi \times \pi \cong 30.96\ cm^2$

3) A circular plate has an area of 40 cm^2. What is its diameter?

Area $\pi \times r \times r$

$40 = \pi \times r \times r$

$r^2 = \dfrac{40}{\pi}$

$r = \sqrt{12.74} \cong 3.57\ cm$

Diameter $\cong 7.14\ cm$

4) Calculate the area of a semi circle of diameter 18cm

Area $= \pi \times r \times r$

Radius $= 9\ cm$

Area $= \pi \times 9 \times 9 = 254.34\ cm^2$

Area of semi circle $127.17\ cm^2$

5)

a) Calculate the area and perimeter of a circle of radius 2cm (leave your answers in terms of π

Area $\pi \times 2 \times 2 = 4\pi\ cm^2$

Perimeter $= 2 \times \pi \times 2 = 4\pi\ cm$

b) Calculate the area and perimeter of a circle of radius 4cm

Area $\pi \times 4 \times 4 = 16\pi\ cm^2$

Perimeter $2 \times \pi \times 4 = 8\pi\ cm$

c) If we double the radius of a circle, by what factor does its perimeter change?

The perimeter also doubles

d) If we double the radius of a circle, by what factor does its area change?

The area changes by a factor of 4

6)

a) Calculate the area and perimeter of a circle of radius 6cm (leave your answers in terms of π)

Area $= \pi \times 6 \times 6 = 36\pi\ cm^2$

Perimeter $= 2 \times \pi \times 6 = 12\pi\ cm$

b) If we triple the radius of a circle by what factor does its perimeter change?

The perimeter also triples

c) If we triple the radius of a circle, by what factor does its area change?

The area increases by a factor of 9

7) Using your results from questions 5 and 6, if we multiply the radius of a circle by x, by what factor does its perimeter change, and by what factor does its area change?

Perimeter changes by a factor of x

The area changes by a factor of x^2

8) A circle has a radius of 10cm. Every minute its radius decreases by 2cm. Compete the following table that shows the change in perimeter and area each minute (leave answers in terms of π)

Time	Radius	Perimeter	Area	Change in perimeter	Change in area
0	10	20π	100π	____	____
1	8	16π	64π	4π	36π
2	**6**	**12π**	**36π**	**4π**	**28π**
3	**4**	**8π**	**16π**	**4π**	**20π**
4	**2**	**4π**	**4π**	**4π**	**12π**
5	**0**	**0**	**0**	**4π**	**4π**

When there is a constant change in radius, does the perimeter or the area of a circle change at a constant rate?

The perimeter changes at a constant rate of 4π

9) A circular track field has an area of approximately 31400 square metres. The world champion bean bag thrower can hurl his bean bag 105 metres. If the bag throw is made from the centre of the field, is the field big enough for him? Explain your answer

Area $\pi \times r \times r$

$31400 = \pi \times r^2$

$r^2 = \frac{31400}{\pi}$

$r = \sqrt{10000} = 100\,m$

The bean bag would clear the outside of the field if thrown from the centre in any direction

10) A painter wishes to paint a circular floor of diameter 10 metres. If one can of paint covers 20 square metres, how many cans of paint will he need? Explain your answer

Area $\pi \times r \times r$

Radius $= 5\ m$

Area $= \pi \times 25 \cong 78.5\ m^2$

$78.5 \div 20 = 3.925$ cans

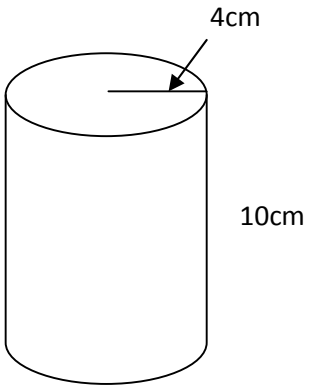
The painter will need to buy 4 cans of paint since part cans cannot be bought

Exercise 3

Volume of Cylinders

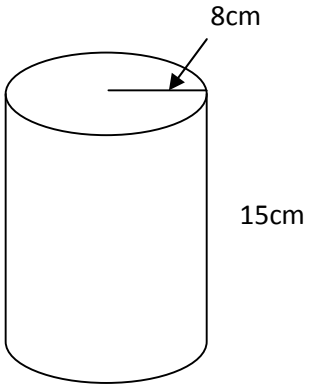
1) Calculate the volume of the following cylinders

a)



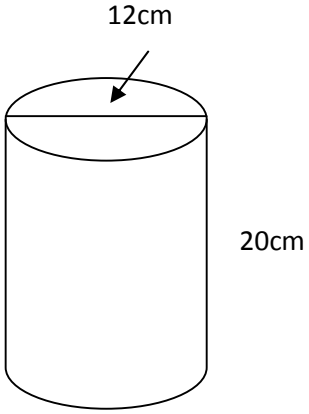
Volume of cylinder $= \pi r^2 h = \pi \times 4 \times 4 \times 10 = 160\pi \cong 502.4\ cm^3$

b)



Volume of cylinder $= \pi r^2 h = \pi \times 8 \times 8 \times 15 = 960\pi \cong 3014.4\ cm^3$

c)

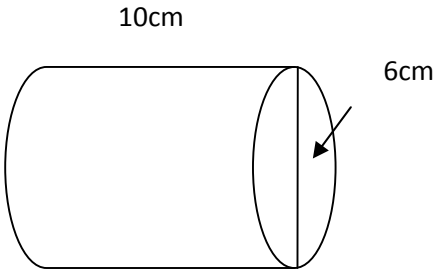


Diameter $= 12\ cm$

Radius $= 6\ cm$

Volume of cylinder $= \pi r^2 h = \pi \times 6 \times 6 \times 20 = 720\pi \cong 2260.8\ cm^3$

d)

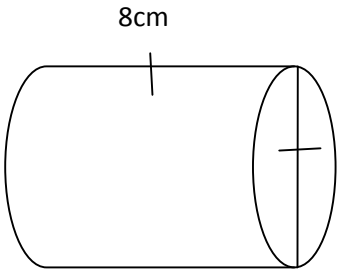


Diameter $= 6\ cm$

Radius $= 3\ cm$

Volume of cylinder $= \pi r^2 h = \pi \times 3 \times 3 \times 10 = 90\pi \cong 282.\ cm^3$

e)



Diameter $= 8\ cm$

Radius $= 4\ cm$

Volume of cylinder $= \pi r^2 h = \pi \times 4 \times 4 \times 8 = 128\pi \cong 401.9\ cm^3$

2) Calculate the volume of a cylinder of base radius 25cm and a height of 100cm

Volume of cylinder $= \pi r^2 h = \pi \times 25 \times 25 \times 100 = 62500\pi \ cm^3$

Alternatively:

Volume of cylinder $= \pi r^2 h = \pi \times 0.25 \times 0.25 \times 1 = 0.0625\pi \ m^3$

3) Calculate the volume of a cylinder of base diameter 40cm and a height of 200cm

Diameter $= 40 \ cm$

Radius $= 20 \ cm$

Volume of cylinder $= \pi r^2 h = \pi \times 20 \times 20 \times 200 = 80000\pi \ cm^3$

Alternatively:

Volume of cylinder $= \pi r^2 h = \pi \times 0.2 \times 0.2 \times 2 = 0.08\pi \ cm^3$

4) A cylinder has a height which is three times its base radius

a) Express the formula for the volume of the cylinder in terms of its radius

Volume of cylinder $= \pi r^2 h$

Here, $h = 3r$

So volume $= \pi \times r^2 \times 3r = \pi \times 3r^3$

b) Use the formula from part a to calculate the volume if its radius is 2cm

When $r = 2$

Volume $= \pi \times 3 \times 2^3 = 24\pi \cong 75.36 \ cm^3$

5) What is the height of a cylinder with a volume of 240 cm^3 and a base radius of 4cm?

Volume of cylinder $= \pi r^2 h = \pi \times 4 \times 4 \times h$

So, $240 = 16 \times \pi \times h$

$h = \frac{240}{16\pi} \cong 4.78\ cm$

6) What is the base radius of a cylinder with a volume of 3000 cm^3 and a height of 200cm?

Volume of cylinder $= \pi r^2 h = \pi \times r^2 \times 200$

So, $3000 = 200 \times \pi \times r^2$

$r^2 = \frac{3000}{200\pi} \cong 47.77$

$r \cong 6.91\ cm$

7) A tin can is in the shape of a cylinder. It has a height of 11cm and a base radius of 4cm. To the nearest ten millilitres, how many millilitres can it hold?

Volume of cylinder $= \pi r^2 h = \pi \times 4 \times 4 \times 11 = 176\pi \cong 552.64\ cm^3$

$1\ mL = 1\ cm^3$

Therefore the can holds approximately 550 mL

8) A tin can has a capacity of 500mL, and a height of 15cm. What is the approximate radius of the can?

Volume of can is therefore $500\ cm^3$

Volume of cylinder $= \pi r^2 h = \pi \times r^2 \times 15$

So, $500 = \pi \times r^2 \times 15$

$r^2 = \frac{500}{15\pi} \cong 10.62$

$r \cong 3.25\ cm$

9) A children's paddle pool is in the shape of a cylinder and holds 5000 litres of water. It has a diameter of 3m. What is its height?

$5000\ litres = 5\ m^3$

Diameter $= 3\ m$

Radius $= 1.5\ m$

Volume of cylinder $= \pi r^2 h = \pi \times 1.5 \times 1.5 \times h = 2.25 \times \pi \times h$

So, $5 = 2.25 \times \pi \times h$

$h = \dfrac{5}{2.25\pi} \cong 0.7\ m$

The pool is approximately 70 cm high

10) A cylinder has a height of 8cm and a base radius of 2cm

a) Calculate its volume (leave in terms of π)

Volume of cylinder $= \pi r^2 h = \pi \times 2 \times 2 \times 8 = 32\pi$

b) Double its height and calculate the new volume

Volume of cylinder $= \pi r^2 h = \pi \times 2 \times 2 \times 16 = 64\pi$

c) Double its radius and calculate the new volume

Volume of cylinder $= \pi r^2 h = \pi \times 4 \times 4 \times 8 = 128\pi$

d) What effect does doubling the height of a cylinder have on its volume?

The volume doubles

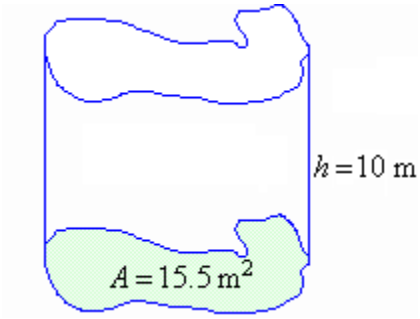
e) What effect does doubling the base radius of a cylinder have on its volume?

The volume increases by a factor of 4 $(= 2^2)$

Exercise 4

Irregular Prisms

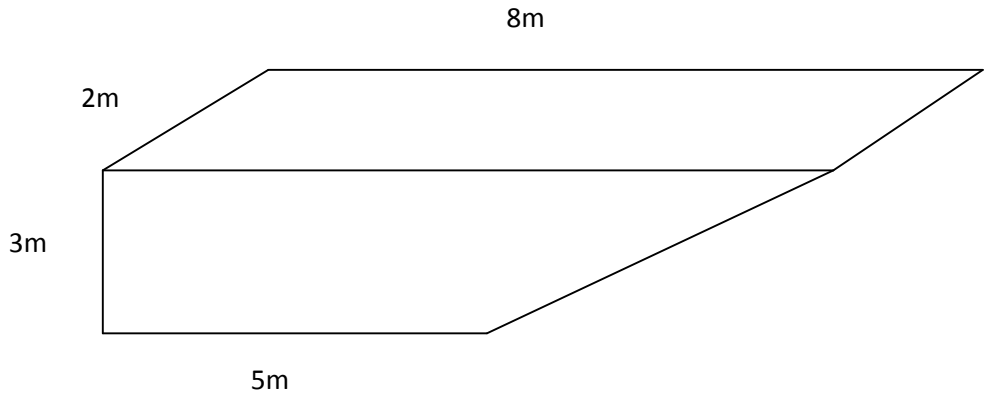
1) Calculate the volume of the following prism



Volume of a prism = Area of base × height

Volume $15.5 \times 10 = 155\ cm^3$

2) Calculate the volume of the following prism

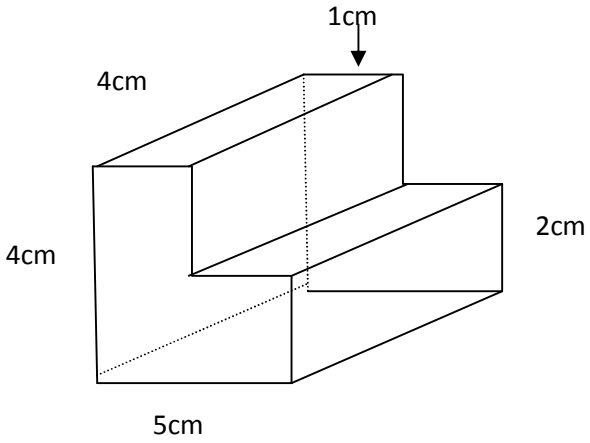


Base is a trapezoid of height 3 m and side lengths of 8 m and 5 m

Area of trapezoid (base) $= \left(\frac{8+5}{2}\right) \times 3 = 19.5\ m^2$

Volume of prism $= 19.5\ \times 2 = 39\ m^3$

3) Calculate the volume of the following prism

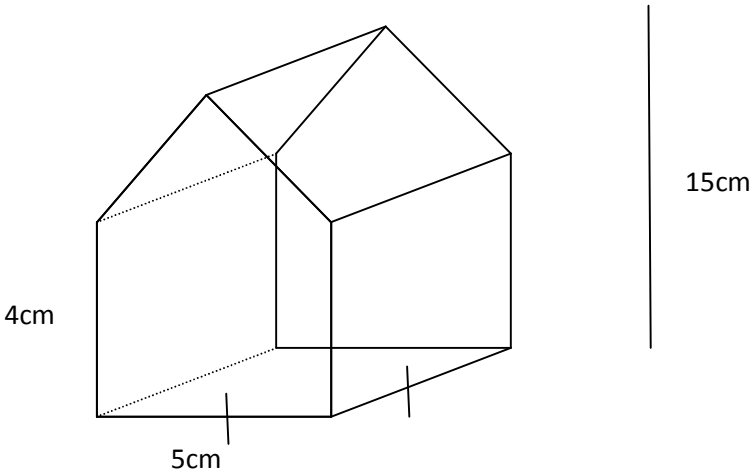


If shape was a full rectangular prism, volume $= 4 \times 4 \times 5 = 80\ cm^3$

Volume of missing piece $= 4 \times 2 \times 4 = 32\ cm^3$

Volume of shape $= 80 - 32 = 48\ cm^3$

4) Calculate the following volume



Volume = Volume of rectangular prism + volume of triangular prism "roof"

Volume of rectangular prism $= 4 \times 5 \times 5 = 100\ cm^3$

Height of triangular prism $= 15 - 4 = 11\ cm$

Volume of triangular prism = Area of triangle $\times 5$

Area of triangular base $= \frac{1}{2} \times 11 \times 5 = 27.5\ cm^2$

Volume of triangular prism $= 27.5 \times 5 = 137.5\ cm^3$

Total volume $= 100 + 137.5 = 237.5\ cm^3$

5) An irregular prism has a base that has an area of 100 cm^2. If its volume is 1200 cm^3 what is its length?

Volume = Area of base × height

$1200 = 100 \times h$

$h = \frac{1200}{100} = 12\ cm$

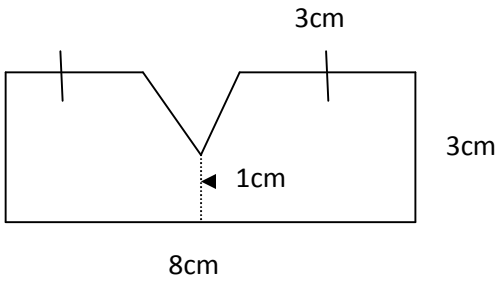
6) An irregular prism has a volume of 3000 cm^3. If its height is 150 cm, what is the area of its base?

Volume = Area of base × height

$3000 =$ Area of base $\times 150$

Area of base $= \frac{3000}{150} = 20\ cm^2$

7) The following shape is the base of a prism of length 20cm. What is the volume of the prism?



Volume = Area of base × height

Area of base = Area of rectangle – area of triangle "cut out"

Area of triangle $= \frac{1}{2} \times$ base$\times$ height

Base $= 8 - 3 - 3 = 2\ cm$

Height $= 3 - 1 = 2\ cm$

Area of triangle $\frac{1}{2} \times 2 \times 2 = 2\ cm^2$

Area of rectangle $8 \times 3 = 24\ cm^2$

Area of base $24 - 2 = 22\ cm^2$

Volume of shape $= 22 \times 20 = 440\ cm^3$

Year 8 Mathematics

Space

Exercise 1

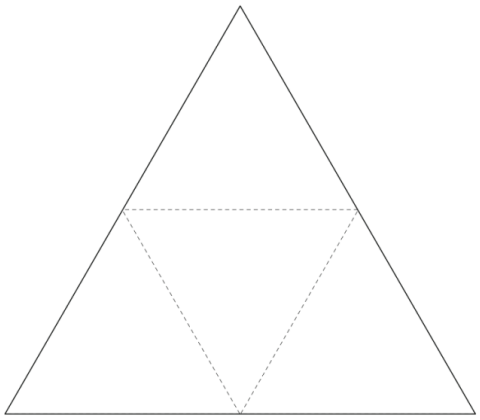
Polyhedra

1) .Complete the following table of convex polyhedra properties

Shape	Faces	Edges	Vertices
Triangular Prism	5	9	6
Rectangular Prism	6	12	8
Triangular Pyramid	4	6	4
Square Pyramid	5	8	5
Hexagonal Prism	8	18	12

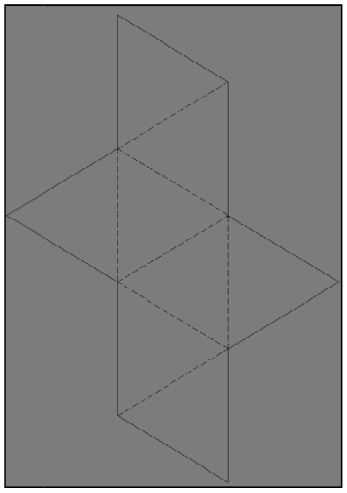
2) Identify the solid formed from the following nets

a)



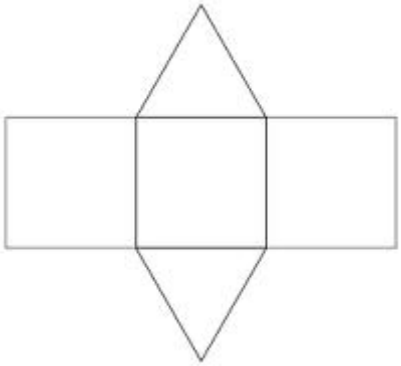
Triangular pyramid

b)



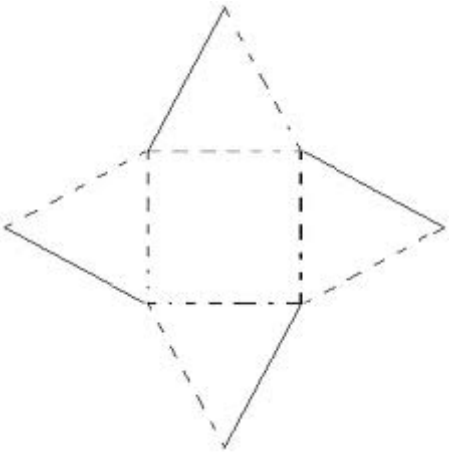
Octahedron

c)



Triangular prism

d)



Square pyramid

3) Complete the following table

Name	Faces	Edges	Vertices	Made up of
Tetrahedron	4	6	4	Equilateral Triangles
Cube	6	12	8	Squares
Octahedron	8	12	6	Equilateral Triangles
Dodecahedron	12	30	20	Regular Pentagon
Icosahedron	20	30	12	Equilateral Triangles

4)

a) What is the size of each internal angle of an equilateral triangle?

60 degrees

b) How many triangles meet at each vertex of a tetrahedron?

3

c) What is the sum of the angles at each vertex of a tetrahedron?

180 degrees

d) How many triangles meet at each vertex of an octahedron?

4

e) What is the sum of the angles at each vertex of a tetrahedron?

240 degrees

f) How many triangles meet at each vertex of an icosahedron?

5

g) What is the sum of the angles at each vertex of an icosahedron?

300 degrees

5)

a) What is the size of each internal angle of a square?

90 degrees

b) How many squares meet at each vertex of a cube

3

c) What is the sum of the angles at each vertex of a cube?

270 degrees

6)

a) What is the size of each internal angle of a pentagon?

108 degrees

b) How many pentagons meet at each vertex of a dodecahedron?

3

c) What is the sum of the angles at each vertex of a dodecahedron?

324 degrees

7) What is the size of each internal angle of a hexagon?

120 degrees

8) Use your answers from questions 4 to 7 to help answer the following

a) Why are platonic solids only made up of regular triangles, squares, and pentagons?

There must be at least three of the shapes meeting at the vertex of a solid. The angle formed must be less than 360 degrees (which is the angle formed by a circle), else a vertex cannot be formed. The only shapes for which 3 or more can meet and form an angle of less than 360 degrees are the above shapes (as per questions 4 to 7)

b) Why is there a limit to the number of each shape that can join at a vertex to make a platonic solid?

The angle formed must be less than 360 degrees

c) Why are there only 5 platonic solids?

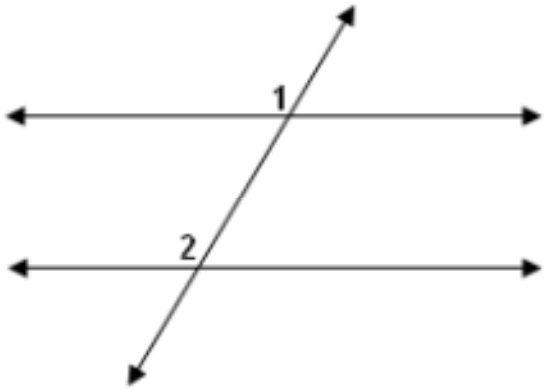
Only 3 regular shapes can form polyhedral. Only a combination of 3, 4 or 5 triangles form an angle at a vertex of less than 360 degrees. Only a combination of 3 squares will form an angle at a vertex of less than 360 degrees. Only a combination of 3 pentagons will form an angle of less than 360 degrees. Hence there are only 5 possible platonic solids

Exercise 2

Angles Relationships

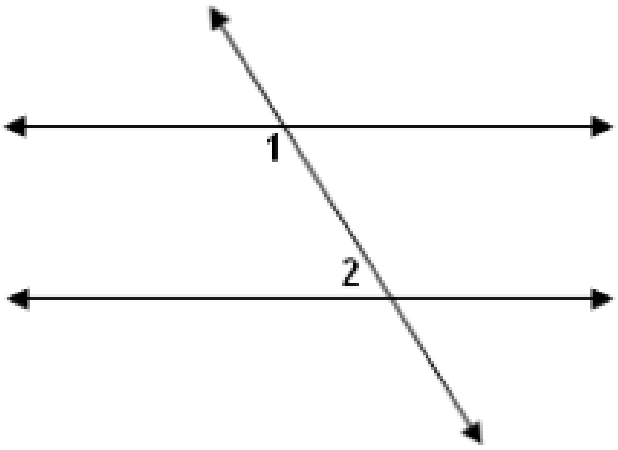
In each of the following, identify the relationship between the numbered angles

1)



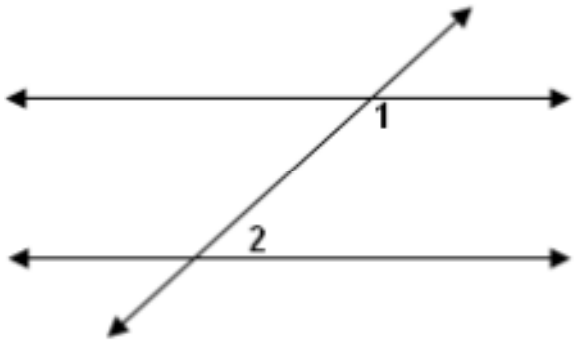
Corresponding

2)



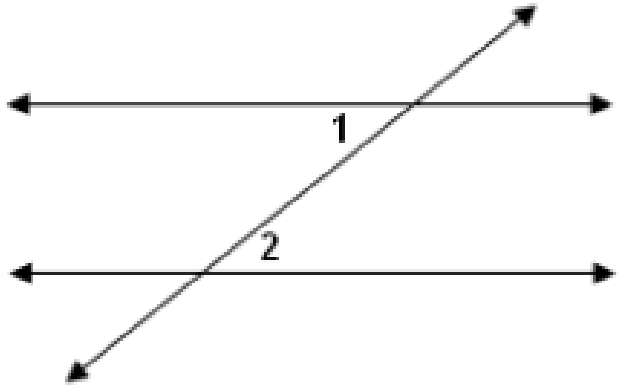
Co-interior

3)



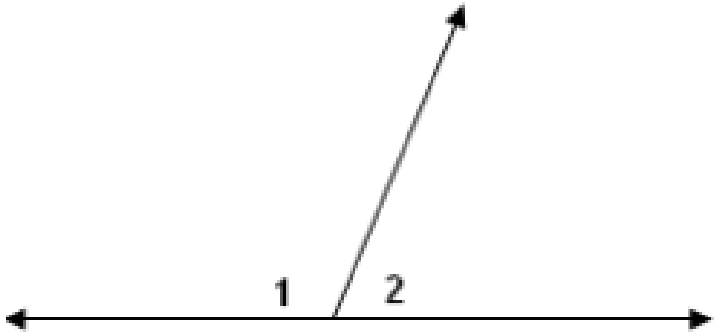
Co-interior

4)



Alternate interior

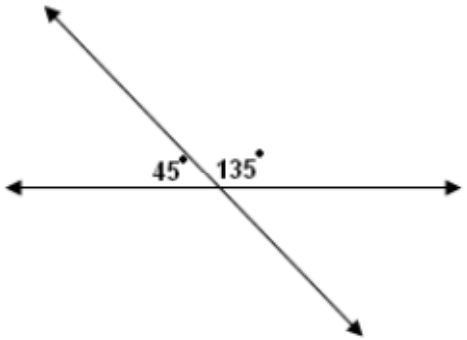
5)



Supplementary

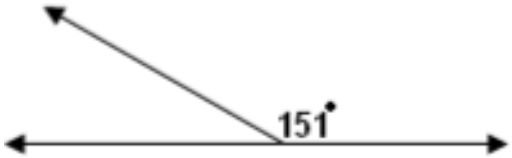
In each of the problems in this section, calculate the size of the missing angles

6)



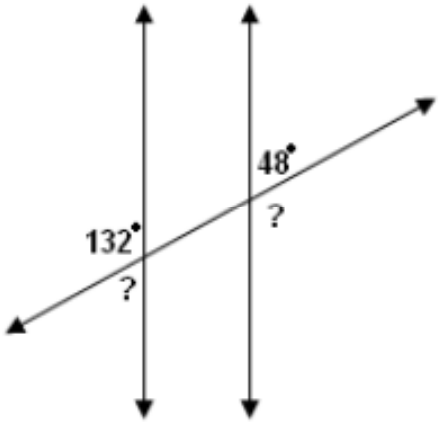
There are two pairs of vertically opposite angles. The missing angles are 135° and 45°. Alternatively they are also supplementary angles giving the same result

7)



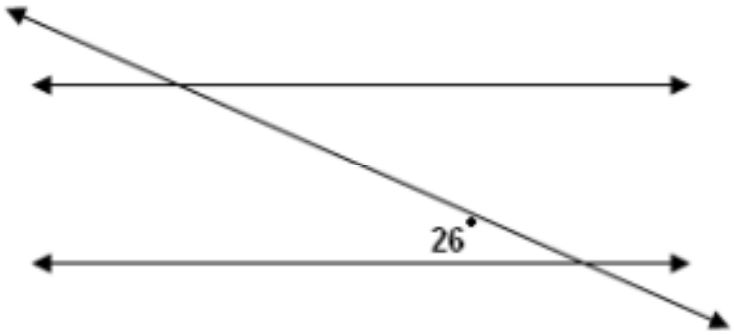
The missing angle is supplementary with the given angle and is therefore 29°

8)



The missing angles are supplementary and are 48° and 132°. Alternatively they are alternate exterior angles, giving the same result

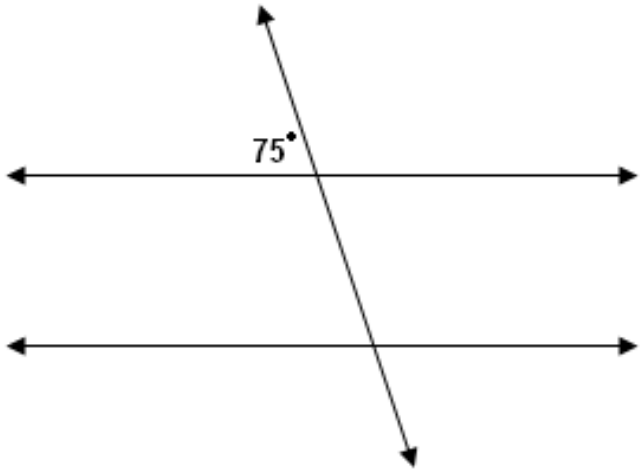
9)



Each intersection has 4 angles. The given angle is 26° as is the angle vertically opposite it. The other pair of angles are both 154°, being supplementary with the given angle

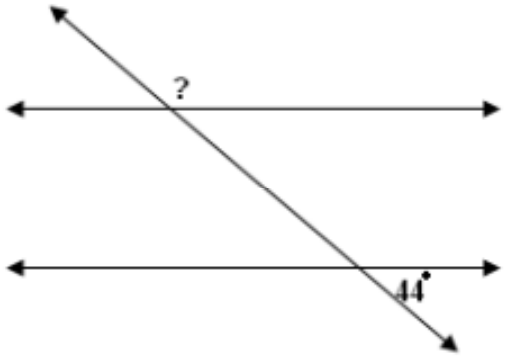
The other set of angles are equivalent to the first set, and can be calculated by associating any of the angles with one from the first set. The two sets form co-interior, alternate interior, and alternate exterior angles, and any of the relationships can be used

10)



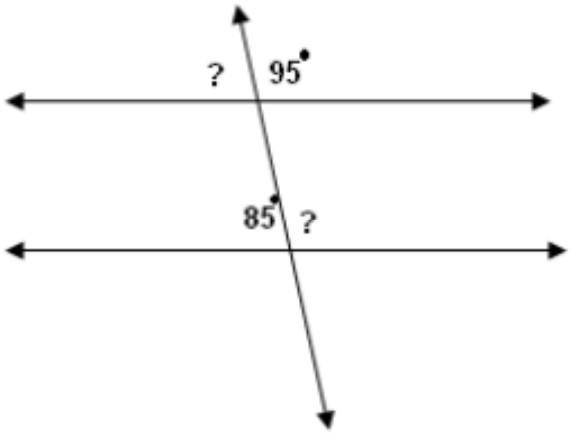
The missing angles clockwise from the known angle are; 105° (supplementary), 75° (vertically opposite), and 105° (supplementary to the previous angle. The second set of angles is equivalent similarly to question 9

11)



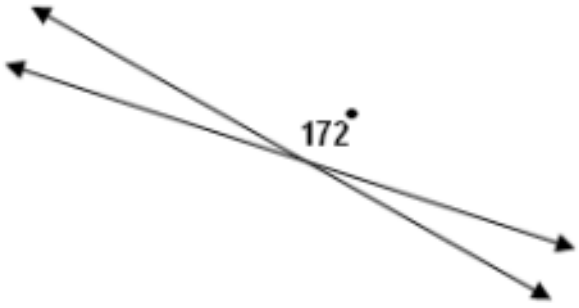
The value of the "?" is 136°. It is corresponding and hence equal to the angle that is supplementary to the given angle

12)



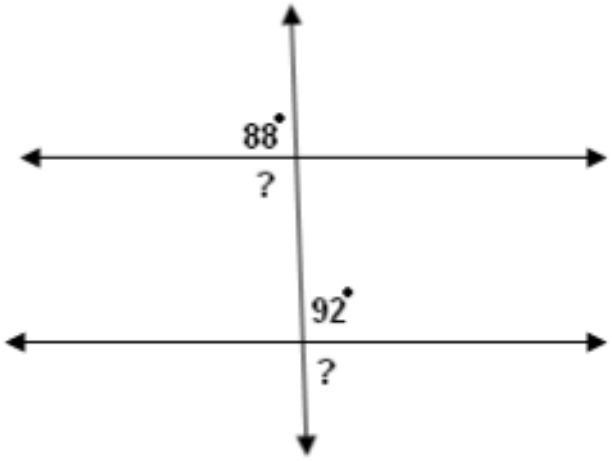
The required angles are 85° and 95° respectively. They are supplementary to the given angles calculating left to right, or alternatively corresponding and hence equal to the angle s above and below them

13)



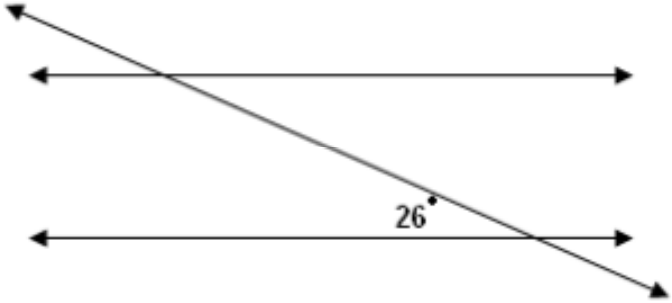
The large unknown angle is 172°, being vertically opposite the given angle, and hence equal. The other two angles are equivalent (being also vertically opposite), and are equal to 8°, forming a supplementary pair with the given angle

14)



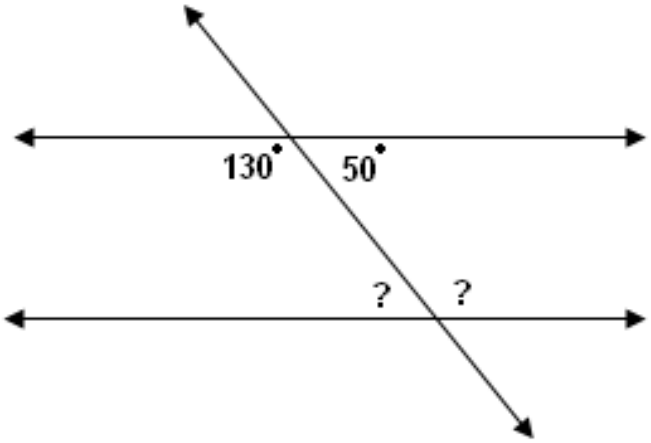
The first angle is 92°, being a corresponding angle with the given angle of 92°. The other angle is alternate exterior to the angle of 88°, and is hence also 88°

15)



The missing angles are 154°, 26°, and 154° due to them being supplementary, vertically opposite and supplementary to the given angle respectively. The other set of angles are equivalent from any of the relationships known. For example there are sets of alternate interior, co-interior and alternate exterior angles present that can be used to calculate the missing values

16)

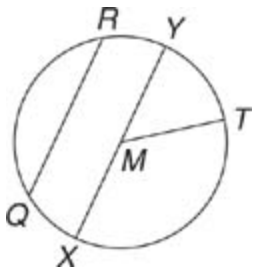


There are two sets of co-interior angles, giving values of 50° and 130°respectively.
There are also two pairs of corresponding angles, giving the same values

Exercise 3

Circles & Scale Factors

1) In the following diagram identify the centre, radius, diameter, secant and chords



Centre is point M

MT is a radius

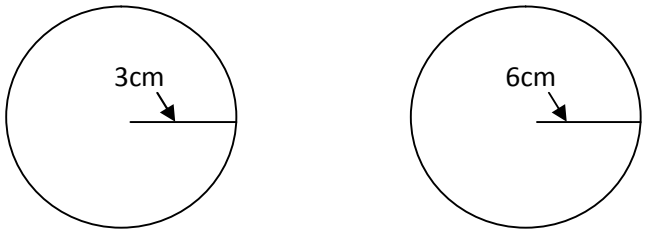
XY is a diameter

RQ is a chord

RQ and XY are also secants

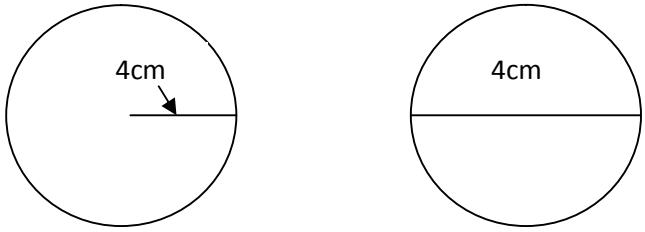
2) Determine the scale factor for the following pairs of circles

a)



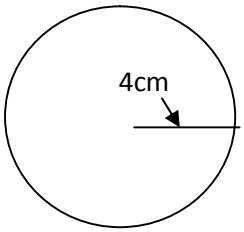
The radius has doubled, scale factor is 2

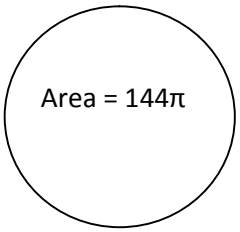
b)



Radius of circle 2 is 2 cm (diameter of 4cm), so scale factor is 0.5

c)





The area of the second circle is 144π.

Area $= \pi r^2$

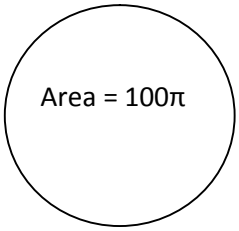
$144\pi = \pi r^2$

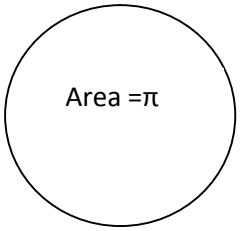
$r^2 = \frac{144\pi}{\pi} = 144$

$r = 12\ cm$

Scale factor is therefore 3

d)





Area of circle 1 is 100π

$100\pi = \pi r^2$

$r^2 = \frac{100\pi}{\pi} = 100$

$r = 10$

Area of circle 2 is π

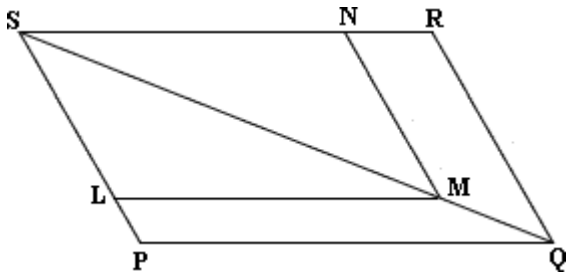
$\pi = \pi r^2$

$$r^2 = \frac{\pi}{\pi} = 1$$

$$r = 1$$

Therefore scale factor is 10

3) If the ratio of SM to QM is 3:1, LM = 9cm, MN = 6cm, and the two quadrilaterals are parallelograms, find the perimeter of PQRS



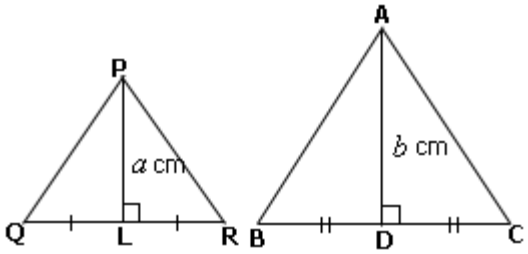
Parallelogram LMNS has been scaled by a factor of $\frac{4}{3}$ to form parallelogram PQRS.

PQ $= \frac{4}{3} \times 9 = 12\ cm$

QR $= \frac{4}{3} \times 6 = 8\ cm$

Perimeter of PQRS $12 + 8 + 12 + 8 = 40\ cm$

4) The perimeter of the two triangles are x and $x + 4$ respectively, find the value of x if $a = 2$ and $b = 3$



The triangles are similar with a scale factor of 3:2

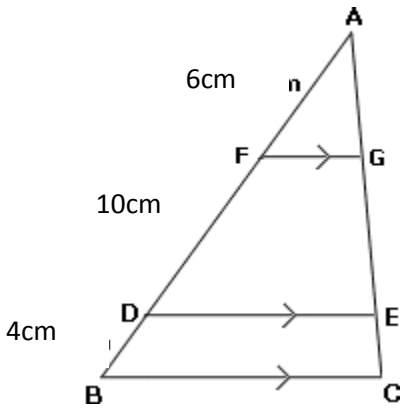
Therefore $\frac{3}{2}x = x + 4$

$\frac{3}{2}x - x = 4$

$\frac{1}{2}x = 4$

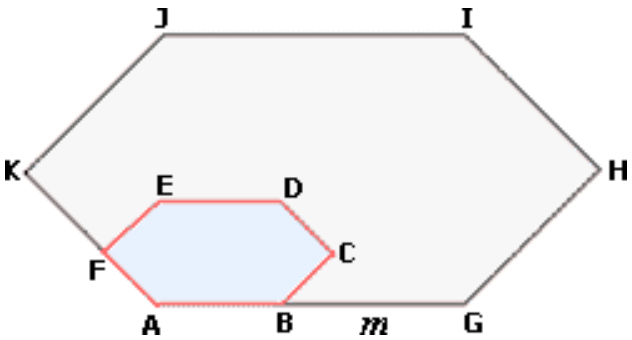
$x = 8\ cm$

5) If the distance from A to E is 12cm, find the length of EC



The triangles are similar; the length of AD is 16 cm. The ratio of DB to AD is 1:4. Therefore EC $= \frac{1}{4} \times 12 = 3\ cm$

6) If AF = 12cm, AK = 30 cm, and AB = 10cm, find the value of m



The small hexagon has been scaled by a factor of 30:12 = 5:2

Therefore AG $= \frac{5}{2} \times 10 = 25\ cm$

So, $m = 25 - 10 = 15\ cm$

7) If a = 12cm, b = 6cm, find the value of x

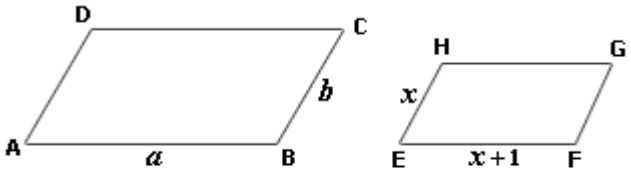


Figure 1

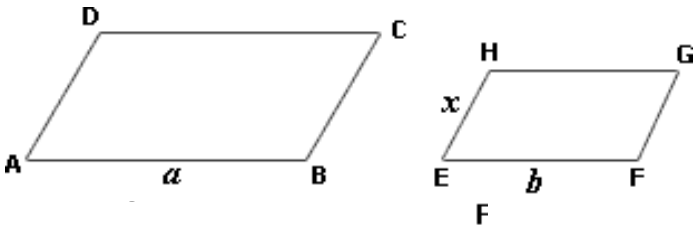
The ratio of a to b is 2:1

Therefore the ratio of $x + 1$ to x is also 2:1

$2x = x + 1$

$x = 1\ cm$

8) If the perimeter of the larger parallelogram is 30cm, a=12, and b = 4, find x



Scale factor is 3:1

Perimeter of EFGH is therefore $30 \times \frac{1}{3} = 10\ cm$

$b + x + b + x = 10$

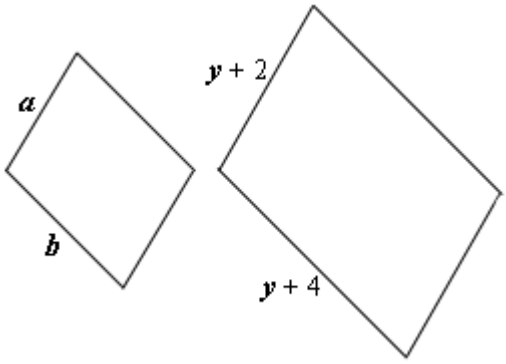
$4 + x + 4 + x = 10$

$2x + 8 = 10$

$2x = 2$

$x = 1\ cm$

9) If a = 5 and b = 6, find the value of y



$b = \frac{6}{5} \times a$

Therefore $y + 4 = \frac{6}{5} \times (y + 2)$

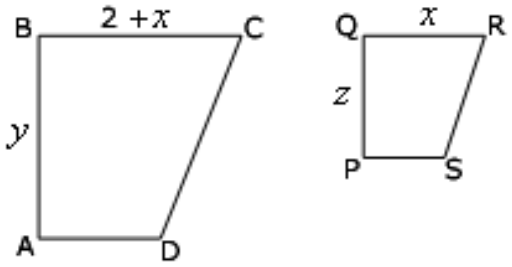
$y + 4 = \frac{6}{5}y + \frac{12}{5}$

$\frac{6}{5}y - y = 4 - \frac{12}{5}$

$\frac{1}{5}y = \frac{8}{5}$

Therefore $y = 8$

10) If $y = 15$ and $z = 5$, find the value of x



The ratio of y to z is 3:1

Therefore $3x = 2 + x$

$2x = 2$

$x = 1\ cm$