Year 10 Mathematics

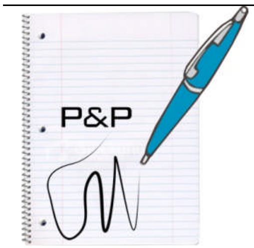
Learning Strategies

Mathematics is often the most challenging subject for students. Much of the trouble comes from the fact that mathematics is about logical thinking, not memorizing rules or remembering formulas. It requires a different style of thinking than other subjects. The students who seem to be "naturally" good at math just happen to adopt the correct strategies of thinking that math requires – often they don't even realise it. We have isolated several key learning strategies used by successful maths students and have made icons to represent them. These icons are distributed throughout the book in order to remind students to adopt these necessary learning strategies:

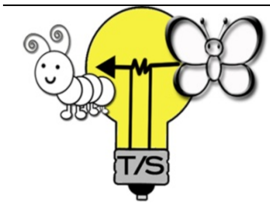
Talk Aloud Many students sit and try to do a problem in complete silence inside their heads. They think that solutions just pop into the heads of 'smart' people. You absolutely must learn to talk aloud and listen to yourself, literally to talk yourself through a problem. Successful students do this without realising. It helps to structure your thoughts while helping your tutor understand the way you think.

BackChecking This means that you will be doing every step of the question twice, as you work your way through the question to ensure no silly mistakes. For example with this question: $3 \times 2 - 5 \times 7$ you would do "3 times 2 is 5 ... let me check – no 3×2 is 6 ... minus 5 times 7 is minus 35 ... let me check ... minus 5×7 is minus 35. Initially, this may seem time-consuming, but once it is automatic, a great deal of time and marks will be saved.

Avoid Cosmetic Surgery Do not write over old answers since this often results in repeated mistakes or actually erasing the correct answer. When you make mistakes just put one line through the mistake rather than scribbling it out. This helps reduce silly mistakes and makes your work look cleaner and easier to backcheck.



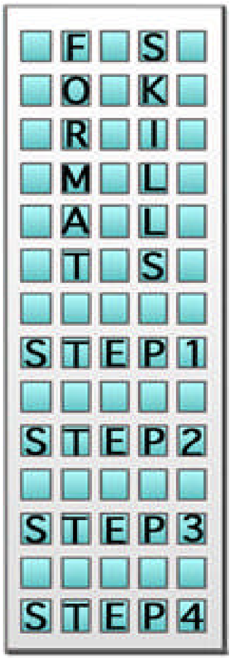
Pen to Paper It is always wise to write things down as you work your way through a problem, in order to keep track of good ideas and to see concepts on paper instead of in your head. This makes it easier to work out the next step in the problem. Harder maths problems cannot be solved in your head alone – put your ideas on paper as soon as you have them – always!



Transfer Skills This strategy is more advanced. It is the skill of making up a simpler question and then transferring those ideas to a more complex question with which you are having difficulty.

For example if you can't remember how to do long addition because you can't recall exactly how to carry the one: $\begin{array}{r} 5889 \\ +\,4587 \\ \hline \end{array}$ then you may want to try adding numbers which you do know how to calculate that also involve carrying the one: $\begin{array}{r} 5 \\ +\,9 \\ \hline \end{array}$

This skill is particularly useful when you can't remember a basic arithmetic or algebraic rule, most of the time you should be able to work it out by creating a simpler version of the question.

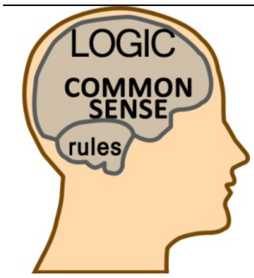


Format Skills These are the skills that keep a question together as an organized whole in terms of your working out on paper. An example of this is using the "=" sign correctly to keep a question lined up properly. In numerical calculations format skills help you to align the numbers correctly.

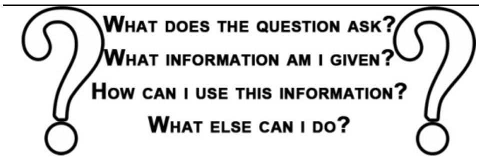
This skill is important because the correct working out will help you avoid careless mistakes. When your work is jumbled up all over the page it is hard for you to make sense of what belongs with what. Your "silly" mistakes would increase. Format skills also make it a lot easier for you to check over your work and to notice/correct any mistakes.

Every topic in math has a way of being written with correct formatting. You will be surprised how much smoother mathematics will be once you learn this skill. Whenever you are unsure you should always ask your tutor or teacher.

Its Ok To Be Wrong Mathematics is in many ways more of a skill than just knowledge. The main skill is problem solving and the only way this can be learned is by thinking hard and making mistakes on the way. As you gain confidence you will naturally worry less about making the mistakes and more about learning from them. Risk trying to solve problems that you are unsure of, this will improve your skill more than anything else. It's ok to be wrong – it is NOT ok to not try.



Avoid Rule Dependency Rules are secondary tools; common sense and logic are primary tools for problem solving and mathematics in general. Ultimately you must understand <u>Why</u> rules work the way they do. Without this you are likely to struggle with tricky problem solving and worded questions. Always rely on your logic and common sense first and on rules second, always ask <u>Why?</u>



Self Questioning This is what strong problem solvers do naturally when they get stuck on a problem or don't know what to do. Ask yourself these questions. They will help to jolt your thinking process; consider just one question at a time and <u>Talk Aloud</u> while putting <u>Pen To Paper</u>.

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Number

Useful formulae and hints

Just as whole number indices are used to denote numbers that are multiples of the base (e.g. $2^3 = 2 \times 2 \times 2$), fractional indices are used to denote numbers that are smaller than the base, but can be combined to equal it

Fractional indices represent the reciprocal of whole number indices.

Just as $2^3 = 8, 8^{\frac{1}{3}} = 2$

Indices that are not unit fractions have two parts; the numerator is applied to the base as usual, but the denominator is applied reciprocally

Example $8^{\frac{2}{3}} = (8^2)^{\frac{1}{3}} = 64^{\frac{1}{3}} = 4$

The order does not matter: $8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = 2^2 = 4$

Negative indices can be converted to positive indices by inverting the number

Example: $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

$$\frac{1}{2^{-3}} = 2^3 = 8$$

A surd is the exact representation of an irrational number e.g. $\sqrt{3}$

Surds can be simplified by using the law $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$

Example: $\sqrt{20} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$ (the multiplication sign is dropped)

To rationalise the denominator of an expression that contains surds, multiply the top and bottom of the fraction by the complement of the expression

Example: the complement of $2 + \sqrt{3}$ is $2 - \sqrt{3}$

By doing this the surd is removed from the denominator

Example: $\frac{4}{2+\sqrt{3}} = \frac{4}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{8-4\sqrt{3}}{4-2\sqrt{3}+2\sqrt{3}-\left(\sqrt{3}\right)^2} = \frac{8-4\sqrt{3}}{4-3} = 8 - 4\sqrt{3}$

There are two methods of calculating interest on loan repayments or investments etc

Simple interest is used when interest is applied to the principal only at a constant rate. The formula is:

$I = PRT$

Where I is the interest, P is the principal, R is the interest rate expressed as a decimal, and T is the length of the loan

The compound interest formula is used when the interest earned attracts interest also. In its simplest form, the formula is:

$A = P(1 + R)^n$, where A is the amount paid including principal and interest, and n is the number of years of the loan

The above assumes a yearly interest rate; however this is not always the case. If interest is to be paid at intervals not equal to a year, the formula has to be modified. The rate (R) is divided into the number of periods in a year, and n becomes the number of these periods over the course of the loan

For example, if the interest is calculated monthly for 5 years, then R becomes $\frac{R}{12}$, since there are 12 months in a year. n becomes 60, since there are 60 months in 5 years

Example:

Amount paid on a loan of $1000 over 4 years, interest calculated yearly at 24%

$A = 1000(1 + 0.24)^4$

If the interest was to be calculated every month the formula would become:

$$A = 1000\left(1 + \frac{0.24}{12}\right)^{48}$$

F the interest was to be calculated every quarter (3months), the formula would become:

$$A = 1000\left(1 + \frac{0.24}{4}\right)^{16}$$

To change a recurring decimal to a fraction, use the following as an example:

Let $x = 0.\dot{3}$

Then $10x = 3.\dot{3} = 3 + x$

So $10x = 3 + x$

$9x = 3$

$$x = \frac{3}{9}$$

There are two parts to be considered when converting between units (e.g. 30 kilometres per hour into metres per minute); apply the following strategy:

Separate the two parts into the units before the "per" (km to m) and those after it (hours to minutes)

For the first set of units, if the conversion is from a large to a smaller unit, multiply by the conversion factor

Example: metres is smaller than km, so multiply by 1000

30 km per hour = 30000 m per hour

(If the conversion is from a small to a larger unit, divide)

For the part after the "per", the rule is reversed; that is if converting from a large to a smaller unit, divide by the conversion factor (and vice versa)

Example: minutes are smaller than hours, so divide by 60

30000 m per hour = 30000 ÷60 = 500 m per minute

When asked to compare a set of rates in different units, convert all of them to the same set of units

Distance = Speed x Time: $d = s \times t$, (100 km per hour for 2 hours = 200 km)

Speed = Distance ÷ Time: $s = \frac{d}{t}$, (200 km in 2 hours is an average speed of 100 km per hour)

Time = Distance ÷ Speed: $t = \frac{d}{s}$, (200 km at an average speed of 100 km per hour takes 2 hours)

(Note: speed is an average speed for the journey; it does indicate the speed at any particular point)

Exercise 1

Surds & Indices

1) Simplify the following, leaving your answers in index form

a) $(3^3)^4$

b) $(2^5)^2$

c) $(5^3)^5$

d) $(10^4)^2$

2) Evaluate the following

a) $(2^4)^{1/2}$

b) $\left(3^{1/3}\right)^3$

c) $(2^9)^{1/3}$

d) $(10^6)^{1/3}$

e) $\left(6^{1/4}\right)^4$

3) Simplify and evaluate the following

a) $8^{1/3}$

b) $4^{1/2}$

c) $16^{1/4}$

d) $125^{1/3}$

4) Simplify and evaluate the following

a) $\left(\frac{1}{8}\right)^{-2}$

b) $\left(\frac{1}{8}\right)^{\frac{1}{3}}$

c) $\left(\frac{1}{8}\right)^{-\frac{1}{3}}$

d) $8^{-\frac{1}{3}}$

5) Convert the following to index form

a) $\sqrt{2}$

b) $\sqrt{x}$

c) $\sqrt[3]{8}$

d) $\sqrt[3]{5^2}$

e) $\sqrt[5]{x^3}$

f) $\sqrt[3]{6^{-2}}$

g) $\sqrt[-3]{6^2}$

6) Convert the following to surd form

a) $(6)^{\frac{1}{2}}$

b) $(5)^{\frac{1}{3}}$

c) $(7)^{\frac{2}{3}}$

d) $(3)^{-\frac{2}{5}}$

e) $(x)^{\frac{a}{b}}$

7) Simplify the following

a) $\sqrt{4}+\sqrt{9}$

b) $\sqrt{4}\times\sqrt{9}$

c) $\sqrt{4\times 9}$

d) $\frac{\sqrt{64}}{\sqrt{4}}$

e) $\sqrt{\frac{64}{4}}$

8) Expand and simplify

a) $(3-\sqrt{5})(3+\sqrt{5})$

b) $(\sqrt{2}+3)(\sqrt{2}-3)$

c) $(1-\sqrt{7})(\sqrt{7}+1)$

d) $(a+\sqrt{b})(a-\sqrt{b})$

9) Rationalise the denominator

a) $\frac{4}{\sqrt{5}-3}$

b) $\frac{1}{2+\sqrt{3}}$

c) $\frac{1+\sqrt{5}}{1-\sqrt{5}}$

d) $\frac{2-\sqrt{7}}{2+\sqrt{7}}$

10) Show the following in simplest form

a) $\sqrt{20}$

b) $\sqrt{8}$

c) $\sqrt{45}$

d) $\sqrt{8}\sqrt{8}$

e) $\sqrt{75}$

11) Simplify the following

a) $\frac{\sqrt{12}}{\sqrt{75}}$

b) $\frac{\sqrt{50}}{3\sqrt{8}}$

c) $\frac{3\sqrt{24}}{2\sqrt{2}}$

d) $\sqrt{27}-2\sqrt{3}$

Exercise 2

Consumer Arithmetic

1) Calculate the total amount payable over the course of a loan under the following conditions

a) Principal of $20,000 at 12% per annum compound interest calculated yearly for 2 years

b) Principal of $10,000 at 10% per annum compound interest calculated yearly for 4 years

c) Principal of $30,000 at 5% per annum compound interest calculated yearly for 3 years

d) Principal of $8,000 at 15% per annum compound interest calculated yearly for 5 years

2) Calculate the total amount payable over the course of a loan under the following conditions

a) Principal of $20,000 at 15% per annum compound interest calculated yearly for 10 years

b) Principal of $20,000 at 10% per annum compound interest calculated quarterly for 3 years

c) Principal of $20,000 at 12% per annum compound interest calculated monthly for 2 years

d) Principal of $20,000 at 18% per annum compound interest calculated monthly for 4 years

3) Calculate the total **interest** payable over the course of a loan under the following conditions

a) Principal of $10,000 at 24% per annum compound interest calculated monthly for 2 years

b) Principal of $15,000 at 9% per annum compound interest calculated monthly for 2 years

c) a Principal of $20,000 at 5.25% per annum compound interest calculated weekly for 2 years

4) Alan takes a loan to purchase a car, and agrees to pay simple interest on the loan at the rate of 10% per annum. If the loan is for $40,000 how much should he repay per month to have the loan fully repaid in 5 years?

5) If the above loan was calculated using compound interest, and Alan made no repayments, calculate how much would he owe on the loan after 1, 2 and 3 years.

6) A man invests $25,000 at a rate of 5% compound interest paid annually. How much would his investment be worth after 1, 2, and 3 years?

7) Calculate the book value of the following assets (use compound interest formulae)

a) A motor vehicle that originally cost $50,000 that depreciates at a rate of 10% of book value per year for 5 years

b) A computer system that originally cost $20,000 that depreciates at a rate of 5% of book value per year for 10 years

c) A truck that originally cost $80,000 that depreciates at a rate of 15% of book value per year for 4 year

8) Which investment is worth more?

a) An initial deposit of $100,000 that has simple interest paid at a rate of 10% per annum

b) An initial deposit of $100,000 that has compound interest paid at a rate of 8% per annum

9) Alan takes a loan to purchase a car, and agrees to pay simple interest on the loan at the rate of 5% per annum. If the loan is for $40,000 how much should he repay per month to have the loan fully repaid in 10 years?

10) If the above loan was calculated using compound interest, and Alan made no repayments, calculate how much would he owe on the loan after 1, 2 and 3 years.

Exercise 3

Recurring Decimals & Rates

1) Convert the following recurring decimals to fractions

a) $0.\dot{1}$

b) $0.\dot{4}$

c) $0.\dot{5}$

d) $0.\dot{7}$

e) $0.\dot{8}$

f) $0.\dot{9}$

2) Convert the following recurring decimals to fractions

a) $0.\dot{3}\dot{4}$

b) $0.\dot{2}\dot{6}$

c) $0.\dot{1}\dot{5}$

d) $0.\dot{6}\dot{1}$

e) $0.\dot{0}\dot{5}$

f) $0.\dot{5}\dot{1}$

3) Convert the following recurring decimals to fractions

a) $0.4\dot{2}$

b) $0.1\dot{5}$

c) $0.6\dot{3}$

d) $0.3\dot{5}$

e) $0.7\dot{2}$

f) $0.0\dot{2}$

4) Convert the following to metres per second

a) 60 m per minute

b) 1 km per second

c) 10 km per hour

d) 3 km per minute

e) 0.2 km per hour

f) 3600 km per day

5) Convert the following to km per hour

a) 3 m per second

b) 10 m per minute

c) 240 km per day

d) 260 m per hour

e) 10 km per second

6) Convert the following simple interest rates to a monthly interest rate

a) 12% per annum

b) 6% per quarter

c) 8% per half year

d) 50% per decade

e) 0.5% per day

7) Tony wants to buy some meat and checks the prices at three supermarkets

- Scoles Supermarket has meat at $18 per kg
- Bullworths price is $2 per 100 grams
- PGA Supermarket is selling the meat for $4 per 250 grams

Which supermarket is cheapest?

8) Put the following speeds in order from slowest to fastest

50 m per second
1000 km per minute
500000 cm per minute
500 km per hour
17500 m per hour
10000 km per day

9) Put the following hire charges in order from highest to lowest

$ 864 per day

$36 per hour

1 cent per second

60 cents per minute

$25920 for the month of September

10) The speed of light in a vacuum is 3×10^8 metres per second

a) How far does light travel in 10 seconds?

b) How long does light take to travel 90000 metres?

c) There are approximately 31.5 million seconds in a year. How far does light travel in one year?

d) The closest star to Earth is approximately 4.3 light years away. How long would it take a rocket travelling at 40,000 km per hour to reach it?

In all the above, use your knowledge of index laws

Year 10

Mathematics

Chance & Data

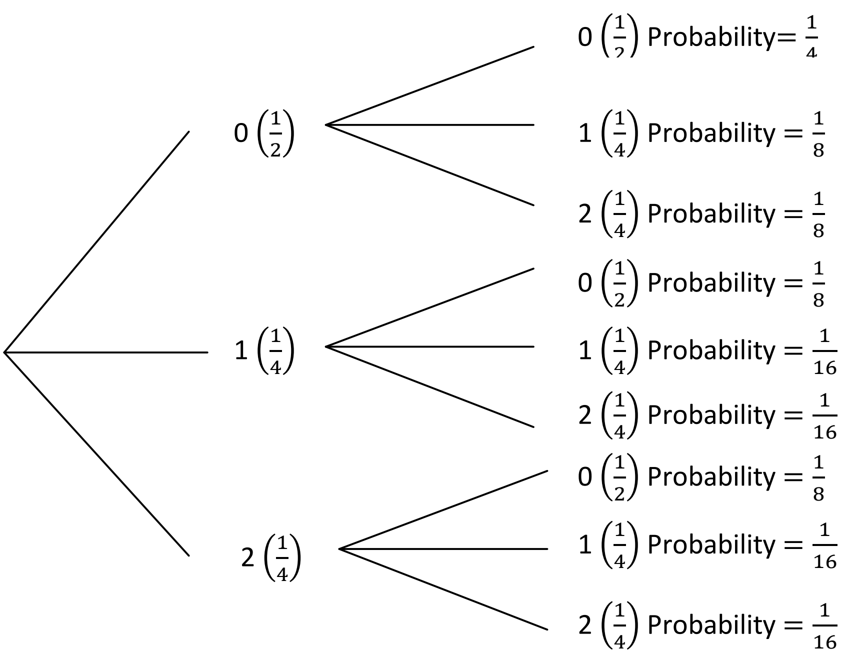
Useful formulae and hints

A tree diagram is used to show and calculate the probability of a certain sequence of events happening, for which the probability of each sequence happening is known. It is a clearer method of listing a sample space for multiple events, and calculating probabilities

Example:

A spinner has a probability of landing on the number 0 of $\frac{1}{2}$, and a probability of landing on a 1 or a 2 of $\frac{1}{4}$. If it was spun twice, the tree diagram would appear as:

First spin Second spin



Every combination of the two spins can be traced through the diagram; for example a 0 then a 1, a 2 then a 0 etc

The individual probabilities for each event are listed, and the probability of any combined events are found by multiplying the individual probabilities

For example, the probability of spinning a 1 on the first spin and a 0 on the second is $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

Ensure that before calculating, the probabilities for each event add to 1: $\frac{1}{2} + \frac{1}{4} + \frac{1}{4}$, and the probability for all combined events also adds to 1: $\frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16}$

Be sure to adjust sample spaces depending on whether a sample is replaced before the second event (e.g. if a card is taken from a pack and not replaced, the sample space is now 51 for the second draw)

Two way tables show information of surveys etc where there are two (or more) groups and two (or more categories). Probabilities can be calculated from these tables

	Green	Blue	
Men	100	900	1000
Women	200	600	800
	300	1500	1800

The above table shows that there were 500 men and 1000 women (total of 1500 people), 100 men and 200 women voted for green; and 400 men and 800 women voted for blue.

To calculate probabilities only use the sample space that is stated. For example, the probability that a person chosen at random will have voted green is $\frac{300}{1800} = \frac{1}{3}$, but the probability that a woman chosen at random will have voted green is $\frac{600}{800} = \frac{3}{4}$, since the sample space is the women only

A box and whisker plot is a method of representing data when there are a large set of scores and only summary data is required. They use 5 key points which can be calculated from the data set

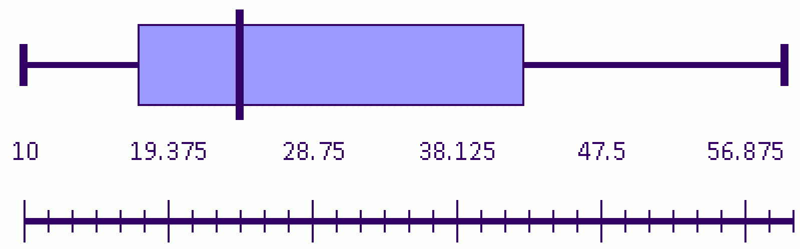
- The minimum
- The maximum
- The median
- The lower quartile
- The upper quartile

You should already be familiar with calculating the median. The median point cuts the data set in half; the numbers less than the median, and those greater than the median. Each of these sets will have their own median and these are the upper and lower quartiles

Example:

10, 15, 20, 22, 24, 35, 40, 45, 60 has a minimum of 10, a maximum of 60 and a median of 24. The lower data set is now 10, 15, 20 and 22 which has a median of 17.5 (the lower quartile), and the upper data set is now 35, 40, 45 and 60 which has a median of 42.5 (the upper quartile)

The five points are plotted as follows



Numbers : 10,15,20,22,24,35,40,45,60

Quartile 1 = 17.5
Median = 24
Quartile 3 = 42.5

25% of the data is contained in the box, with half of this above and below the median line. 25% of the data is contained in the whiskers at either end of the graph

Exercise 1

Probability

1) Construct a tree diagram that details the possible outcomes of tossing a coin three times. From your tree diagram determine the probability of:

a) Three heads

b) Two heads and a tail

c) A head on the third throw given that the first two throws were tails

d) If the first three throws were tails, what is the probability that a fourth throw is a head?

2) A pizza shop has the following choices of ingredients:

- Ham
- Onion
- Tomato
- Pineapple

If you are allowed three toppings only, construct a list of all possible pizzas you could make. From this table:

a) How many possible pizzas can be made?

b) What is the probability that one of these pizzas contains ham?

c) What is the probability that one of these pizzas contains ham and onion?

d) What is the probability that one of these pizzas does not contain Tomato?

3) The table below shows the distribution of cars at a car yard. For example, there are 200, 6 cylinder black cars.

	BLACK	WHITE	
6 CYLINDER	200	100	300
8 CYLINDER	150	350	500
	350	450	800

From this table:

a) What is the probability that a car chosen at random will be white?

b) What is the probability that a car chosen at random will be a 6 cylinder?

c) What is the probability that a car chosen at random will be an 8 cylinder black car?

d) What is the probability that a black car chosen at random will be a 6 cylinder?

e) What is the probability that a white car chosen at random will be an 8 cylinder?

4) There are 4 white and 2 green shirts in a draw. Construct a tree diagram that shows the possible combinations of 2 shirts that can be taken at random, and show the probabilities.

5) A box contains 2 black, 2 white and 1 pink balls. A ball is chosen at random from the box, replaced, and a second ball is then chosen at random.

a) Construct a probability tree diagram, and use it to answer the following:

b) What is the probability that both balls are black?

c) What is the probability that one ball is white and the other is pink?

d) What is the probability that both balls are pink?

6) Repeat question 5, but assume the first ball is not replaced

7) A survey was taken to investigate possible links between eating excessive fast food and diabetes. The results are shown in the table below

	Diabetic	Non-diabetic	
Fast food more than 2 times a week	500	100	600
Fast food less than 3 times a week	200	400	600
	700	500	

a) How many people were studied?

b) What is the probability that a person chosen at random from the group had diabetes?

c) What is the probability that a person chosen at random from the group had fast food more than twice a week?

d) What is the probability that a person chosen at random from the group who had fast food more than twice a week also had diabetes?

e) What is the probability that a person chosen at random from the group who did not have diabetes ate fast food more than twice a week?

8) One thousand students were surveyed regarding how many hours of homework they did each week

- Of the 400 girls surveyed, 100 said they did homework for more than 10 hours per week
- 400 boys did homework for less than 10 hours per week

Construct a probability table from the above information, and use it to determine the probability that a person who did homework for less than 10 hours a week was a girl

Exercise 2

Data Representation & Analysis

1) The top ten test batting averages in history are:

99.94, 60.97, 60.83, 60.73, 59.23, 58.67, 58.61, 58.45, 57.78, 57.02

a) Construct a box and whisker plot of the data

b) Predict if the mean will be higher or lower than the median and justify your answer

c) Calculate the mean and standard deviation

Is the range, inter-quartile range or standard deviation a more realistic measure of the spread of the data in this case? Justify your answer

2) The following data shows the score (in points) of the winning AFL football teams over three weeks

61, 63, 72, 75, 80, 84, 84, 86, 90, 96, 97, 102, 105, 105, 106, 107, 108, 110, 111, 115, 120, 122, 125, 130

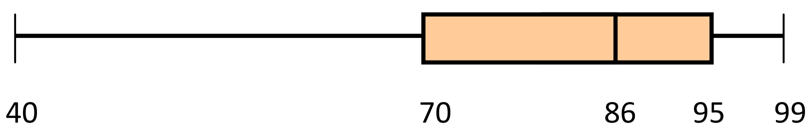
a) Draw a box-and-whisker plot marking the 5 relevant points

b) What is the inter-quartile range?

c) What is the median score?

d) Comment on the spread of the data

3) The box and whisker plot below shows the distribution of students' maths test scores



a) .What was the lowest score in the test?

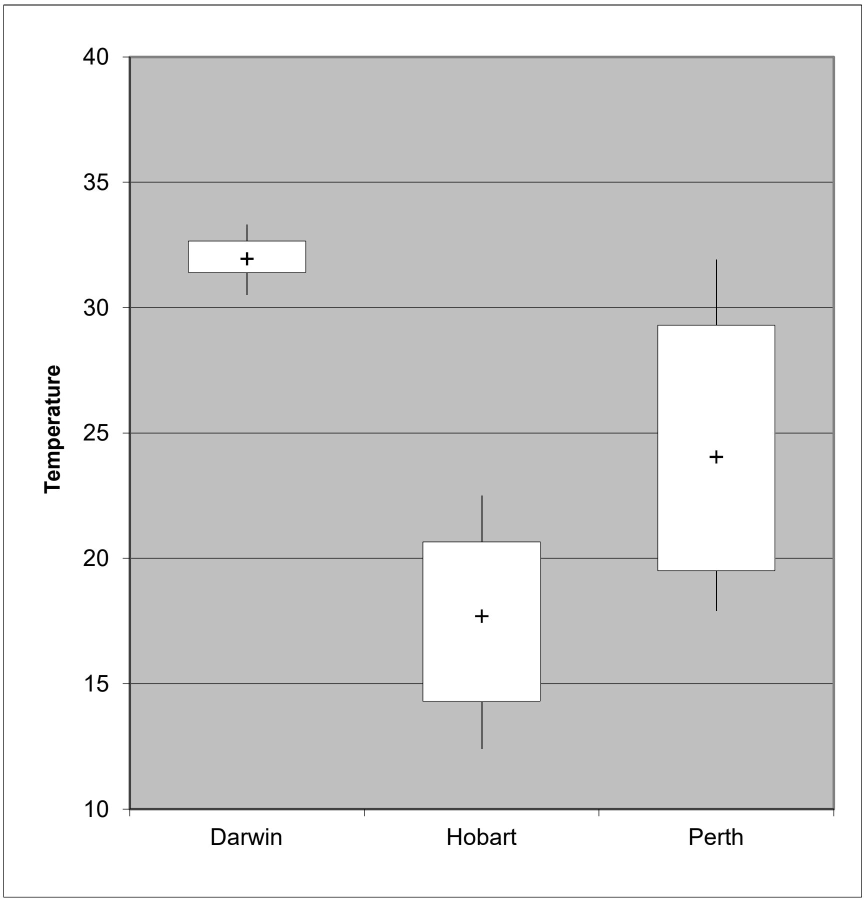
b) What percentage of the class scored above 70%?

c) What was the median score?

d) What percentage of the class scored between 70 and 86?

e) Comment on the difficulty of the test

4) The following box plots show the distribution of the average monthly temperatures for a year for Hobart, Darwin and Perth. (the cross indicates the median for each data set)



Compare the three sets of data and comment on the similarities and differences in the distributions of average monthly temperatures for the three cities

5) Draw a box and whisker plot of the following data set

5, 9, 9, 12, 13, 22, 25, 30, 100

6) Draw a box and whisker plot for a set of data that has a median of 20, an inter quartile range of 15, and a range of 40

7) The following data is in stem and leaf form. Represent it as a box and whisker plot

Stem	Leaf
2	2 4 7 7 9
3	0 1 1 1 3 3 5 6
4	5 5 5 7 8 9
5	1 1 2 2 5
6	3 3 5 6 7
9	5

Year 10 Mathematics

Algebraic Techniques

Useful formulae and hints

Simultaneous equations are used to calculate values that satisfy two constraints at the same time

There are four main methods of solving them: consider the equations

$y = 2x + 3$ and $y = 3x + 1$

Method 1: guess check and improve

Guess $x = 1$

Substitute into first equation gives $y = 5$

Substitute into second equation gives $y = 4$

Try $x = 4$

Substitute into first equation gives $y = 11$

Substitute into second equation gives $y = 13$

Notice that the values of y are further apart than for x = 1

The required x value is therefore between 1 and 4

Try $x = 2$

Substitute into first equation gives $y = 7$

Substitute into second equation gives $y = 7$

Therefore $x = 2\ and\ y = 7$ is the solution to the equations

Method 2: substitution

If $y = 2x + 3$ and $y = 3x + 1$

Then $2x + 3 = 3x + 1$

Solving for x gives $-x = -2$

$x = 2$

Substituting into either equation gives $y = 7$

Method 3: addition (or subtraction) of equations

If one equation is subtracted from the other they can be solved, providing that one of the pairs of variables has the same coefficient

$y = 2x + 3$

$-(y = 3x + 1)$

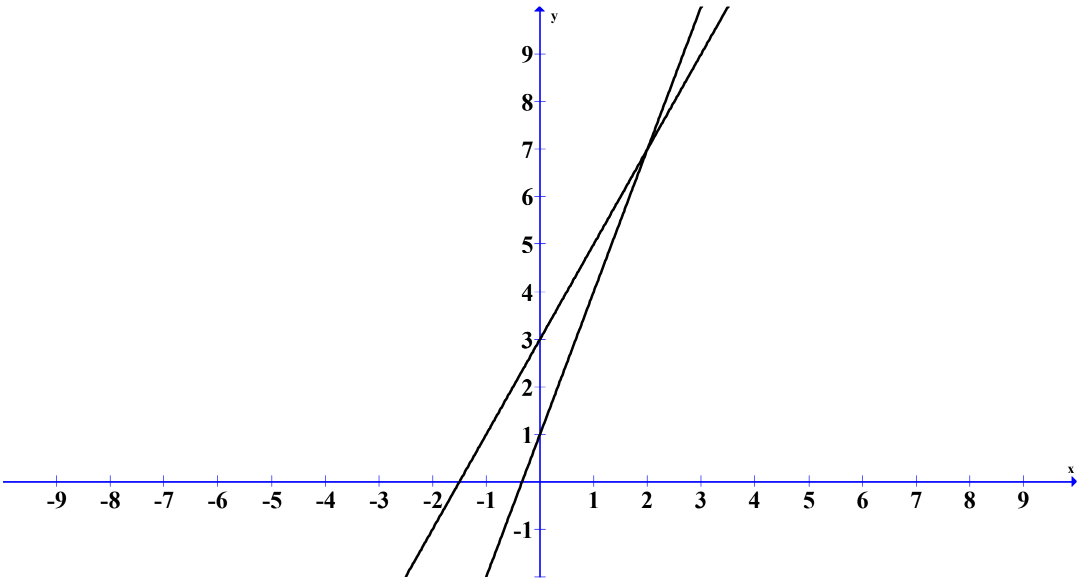
Gives

$0 = (2x - 3x) + (3 - 1)$

$0 = -x + 2$

$x = 2$

Method 4: graph the equations; the solution is the point where the equations meet



The lines cross at the point $x = 2, y = 7$

When problem solving using simultaneous equations, clearly set out what unknown is represented by each variable at the beginning of the question.

Convert the sentences into equations and solve using your preferred method

Example:

Two times my age plus Tom's age is 40, and our combined ages are 25. How old is Tom?

Let x represent my age, and y represent Tom's age

Then $2x + y = 40 \text{ and } x + y = 25$

Subtracting equation 2 from equation 1 gives:

$2x + y - (x + y) = 40 - 25$

$2x + y - x - y = 15$

$x = 15$

Substituting into either equation gives $y = 10$

Therefore I am 15 and Tom is 10

Note the value should be substituted into both equations to check result

To complete the square of a quadratic equation follow this example:

- Put the equation in the form $x^2 + bx + c = 0$;

$x^2 - 4x + 5 = 4$ becomes:

$x^2 - 4x - 1 = 0$

- Halve the coefficient of x and put equation in the form $(x - a)^2 + c$, where a is half the x coefficient

$(x - 2)^2 + c$

- Expand the brackets

$(x - 2)^2 = x^2 - 4x + 4$

- Adjust the constant so that the equation will equal the original equation

The constant in the original equation is (-1), so we need to subtract 5 from the expanded equation

$(x - 2)^2 - 5 = x^2 - 4x - 1$

A quadratic equation is in the from $y = ax^2 + bx + c$ or similar where the largest power of x is 2

A cubic equation is in the form $y = ax^3 + bx^2 + cx + d$ or similar where the largest power of x is 3

An exponential equation is in the form $y = a^x$

A circle has the general equation $(x + a)^2 + (y + b)^2 = c$

There are 4 main methods of solving quadratic equations

Method 1: Factorization

For an equation $x^2 + bx + c = 0$ find two numbers that when multiplied together equal c and when added together equal b

For $x^2 + 8x + 12 = 0$, the factors of c are 1 and 12, 2 and 6, 3 and 4

2 + 6 = 8, therefore these are the numbers to use

Convert the equation to $(x + 2)(x + 6) = 0$, the constants are the factors calculated earlier.

If two numbers multiply together to equal 0, then one of them must be equal to 0

Therefore $x + 2 = 0 \; or \; x + 6 = 0$

So either $x = -2 \; or \; x = -6$ are the solutions to the equation

Method 2: Completing the square

As above $x^2 + 8x + 12 = 0 \; so \; (x + 4)^2 - 4$

$(x + 4)^2 = 4$

Therefore $x + 4 = 2$ or $x + 4 = -2$

$x = -2 \; or \; x = -6$

Method 3: Quadratic equation

When an equation is in the form $ax^2 + bx + c = 0$, the values of x can be found by using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

From the given equation

$$x = \frac{-8 \pm \sqrt{8^2 - 4 \times 1 \times 12}}{2} = \frac{-8 \pm \sqrt{64 - 48}}{2}$$

$$= \frac{-8 + 4}{2} or \frac{-8 - 4}{2}$$

$x = -2 \; or \; x = -6$

Note if the value of $b^2 - 4ac < 0$, the square root is not a real number and the quadratic equation has no solutions. It is often useful to check this value before proceeding

When graphing inequalities consider two things

- Is the dividing line part of the solution? If the inequality contains $\geq or \leq$ then it is and should be shown as a solid line; if not it should be shown as a dashed line
- Shade the correct region by determining which values satisfy the inequality sign. It is often useful to substitute a test point from a region and see if it satisfies the inequality

Exercise 1

Simultaneous Equations

1) Solve the following simultaneous equations by using the guess check and improve method

a) $2x + y = 4 \; and \; x - y = 5$

b) $x + y = 8 \; and \; 2x - y = 10$

c) $2x + y = 3 \; and \; 3x - 2y = 1$

d) $x - 3y = -4 \; and \; x + y = 4$

e) $x + 2y = 5 \; and \; x - 2y = 1$

f) $x + 3y = 7 \; and \; x - y = 3$

g) $2x - y = -1 \; and \; 5x + 7y = 7$

2) For each of the simultaneous equations in question 1, make a table of possible values and use it to check each of your solutions

3) Graph each pair of simultaneous equations from question 1, and use your graphs to check each of your solutions

4) Use an algebraic method (substitution, subtraction or addition of equations) to solve each pair of simultaneous equations from question 1.

5) Solve the following word problems by generating a pair of simultaneous equations and solving them by any of the methods used above. Check your solutions by substituting back into the original equations

a) The sum of two numbers is 8 and the difference is 4. Find the numbers.

b) The cost of two rulers and a pen is \$6.00. The difference of cost between 3 rulers and 2 pens is \$2.00. Find the cost of a ruler and a pen.

c) If I double two numbers and then add them together I get a total of 8. If I multiply the first number by 3, then subtract the second number I get 4. What are the two numbers?

d) The average of two numbers is 9. The difference is 6. Find the numbers

e) There are two angles on a straight line. One angle is 45 more than twice the other. Find the size of each angle.

f) The length of a rectangle is twice its width. The perimeter is 42. Find its dimensions

6) One thousand tickets to a show were sold. Adult tickets cost \$8.50 and children's were \$4.50. \$7300 was raised from the sale of the tickets. How many of each type were sold?

7) Mrs. Brown. invested \$30,000; part at 5%, and part at 8%. The total interest on the investment was \$2,100. How much did she invest at each rate?

8) Tyler is catering a banquet for 250 people. Each person will be served either a chicken dish that costs \$5 each or a beef dish that costs \$7 each. Tyler spent \$1500. How many dishes of each type did Tyler serve?

9) Your teacher is giving you a test worth 100 points containing 40 questions. There are two-point and four-point questions on the test. How many of each type of question are on the test?

Exercise 2

Parabolas & Hyperbolae

1) Graph each quadratic equation below, by first making a table of values

a) $y = x^2$

b) $y = 2x^2$

c) $y = 4x^2$

d) $y = 0.5x^2$

e) $y = \frac{1}{2}x^2$

f) $y = -2x^2$

2) From your answers to question 1, comment on the effect of the value of the constant a on the graph of equations of the form $y = ax^2$

3) Graph each quadratic equation below, by first making a table of values

a) $y = x^2 + 1$

b) $y = x^2 + 3$

c) $y = x^2 - 2$

d) $y = x^2 - \frac{1}{2}$

e) $y = 2x^2 - 3$

4) From your answers to question 3, comment on the effect of the value of the constant c on the graph of equations of the form $y = ax^2 + c$

5) Graph the following

a) $y = (x + 1)^2$

b) $y = (x + 2)^2$

c) $y = (x - 2)^2$

d) $y = (x - 1)^2$

6) Comment on the effect of the value of the constant a on the graph of equations of the form $y = (x + a)^2$

7) Graph each parabolic equation below, by first making a table of values

a) $y = \frac{1}{x}$

b) $y = \frac{3}{x}$

c) $y = \frac{5}{x}$

d) $y = \frac{1/2}{x}$

e) $y = \frac{-2}{x}$

8) From your answers to question 5, comment on the effect of the value of the constant k on the graph of equations of the form $y = \frac{k}{x}$

9) Identify which of the following equations produce lines, parabolas or hyperbolae when graphed

a) $y = \frac{2}{x} + 4$

b) $y = 3x - 6$

c) $y = \frac{1/3}{x}$

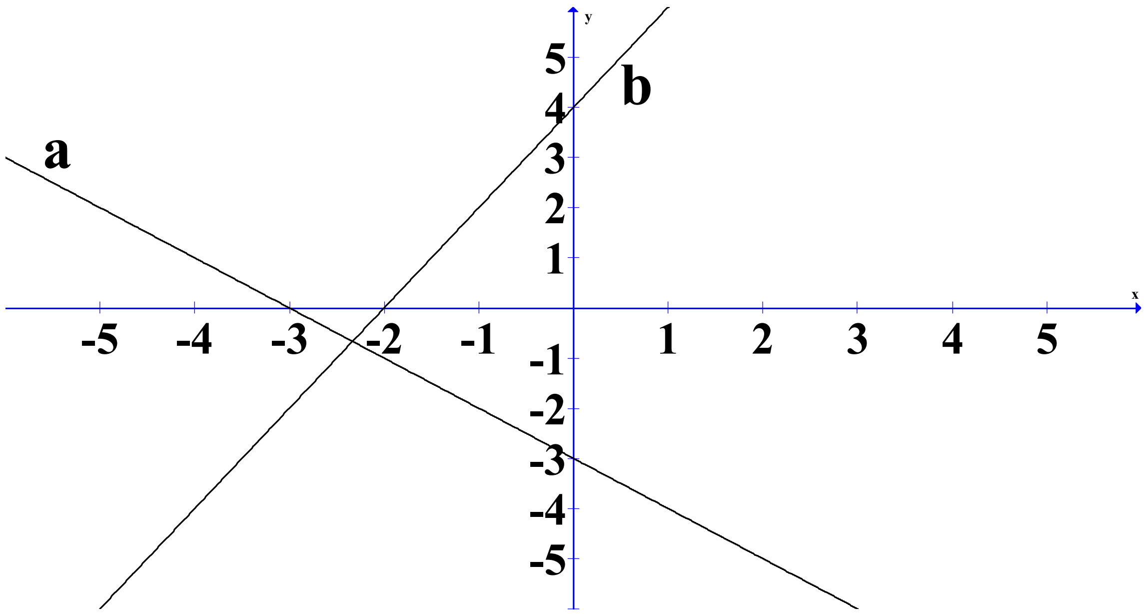
d) $y = \frac{1}{2}x^2 + 1$

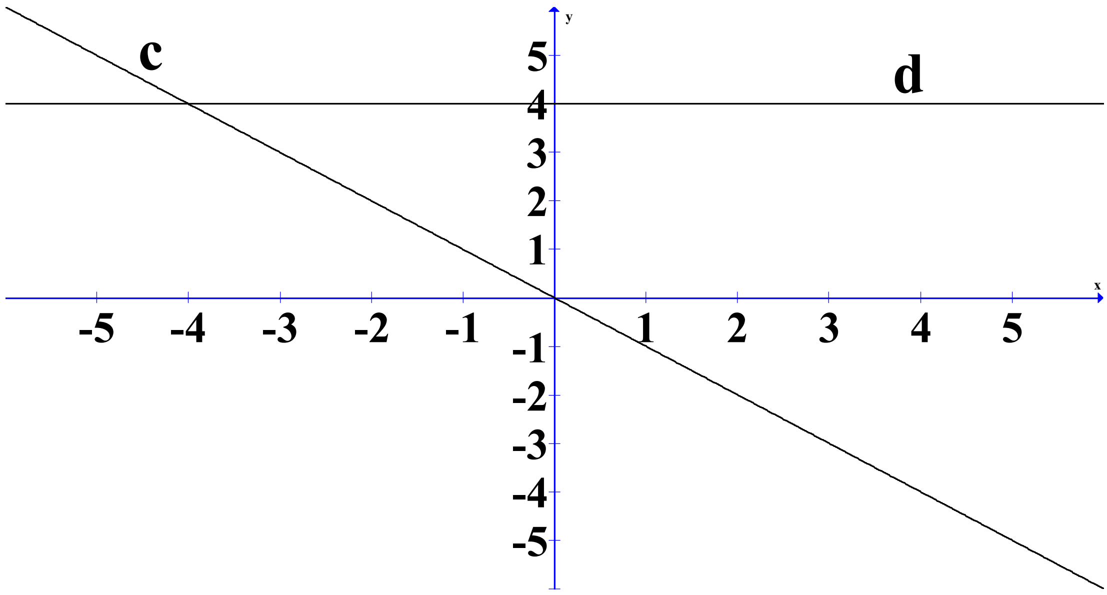
e) $y = \frac{1}{2}x$

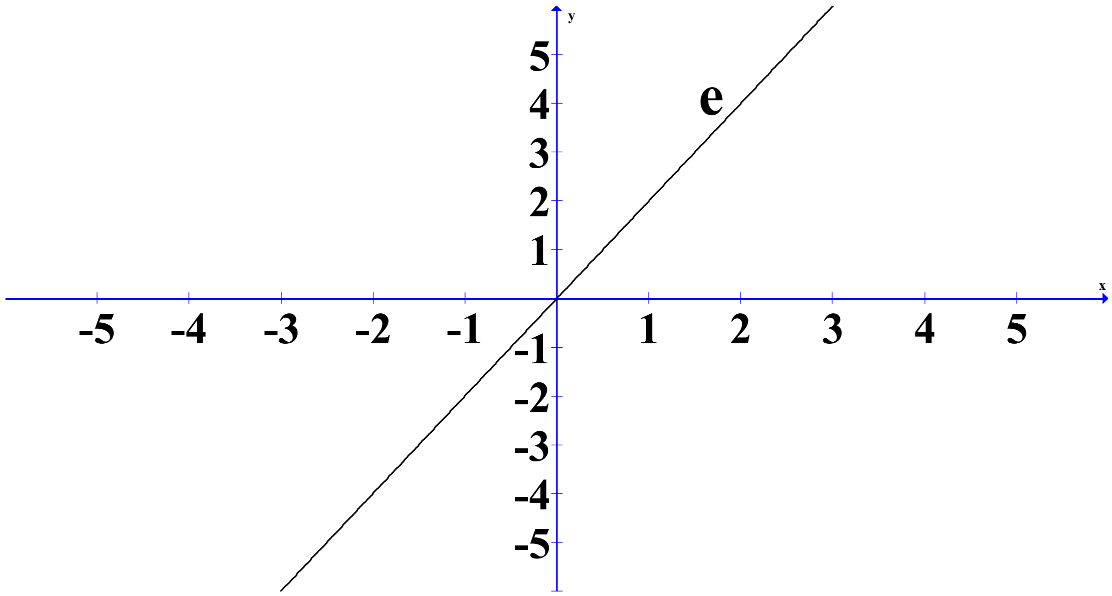
f) $y = -\frac{1}{2x} + 2$

g) $y = -2 - \frac{1}{4}x^2$

10) Match each of the following graphs to its correct equation

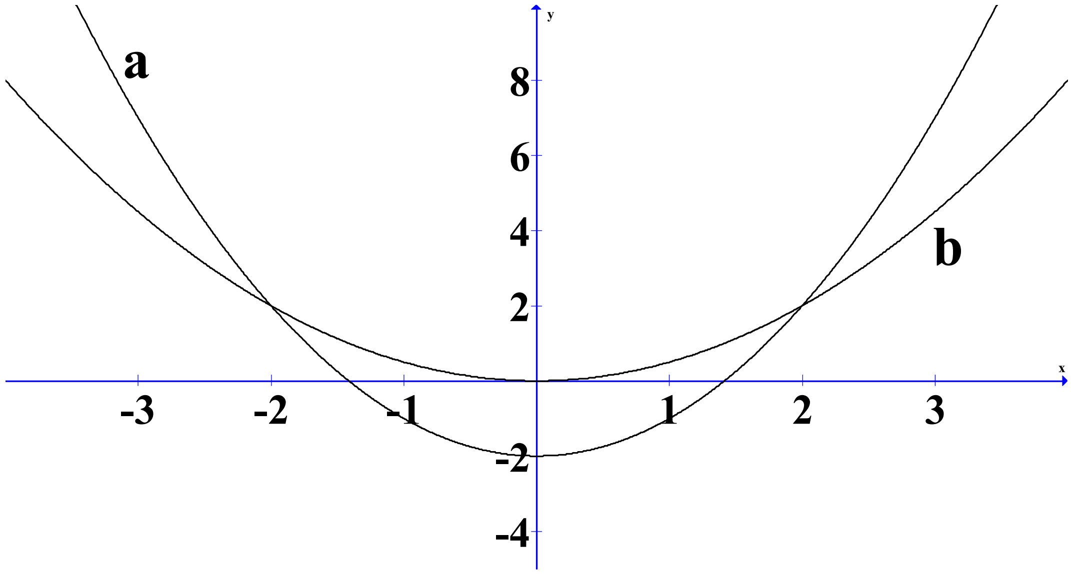


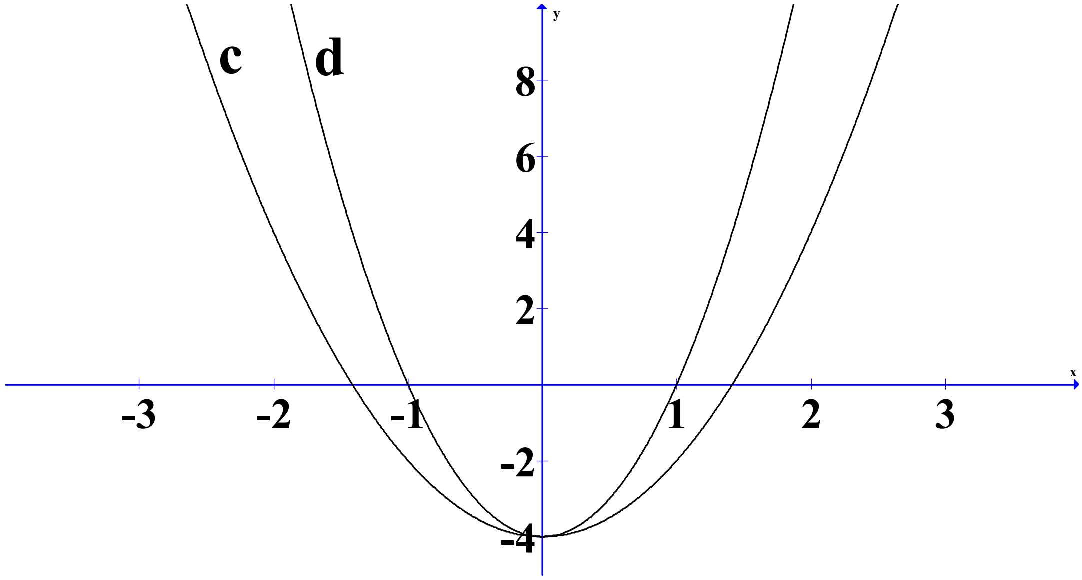


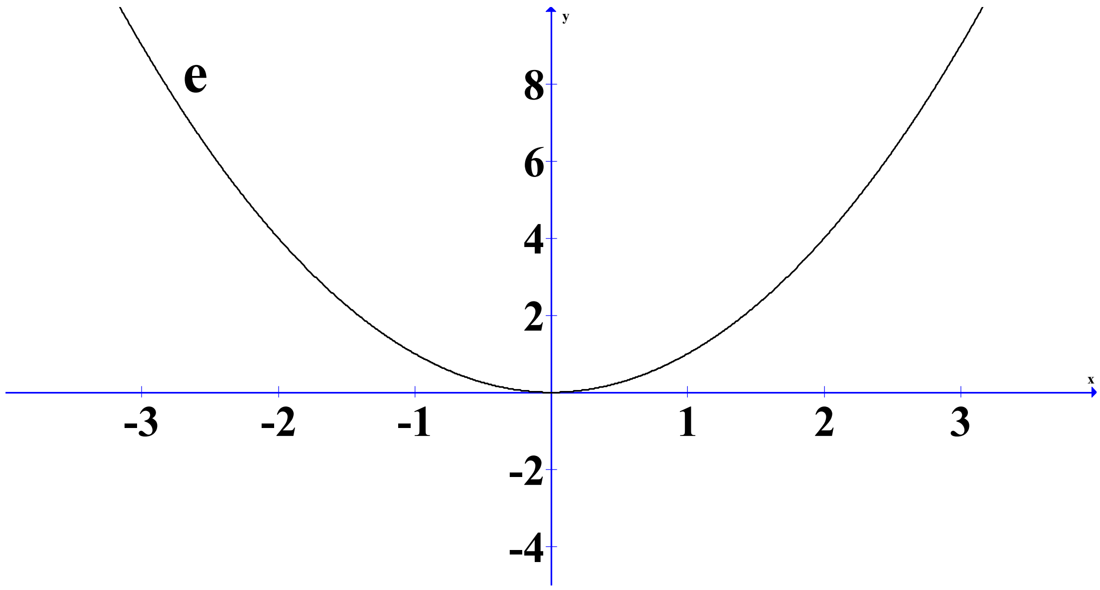


- $y = -x$
- $y = 2x + 4$
- $y = 4$
- $y = -x - 3$
- $y = 2x$

11) Match each of the following graphs to its correct equation

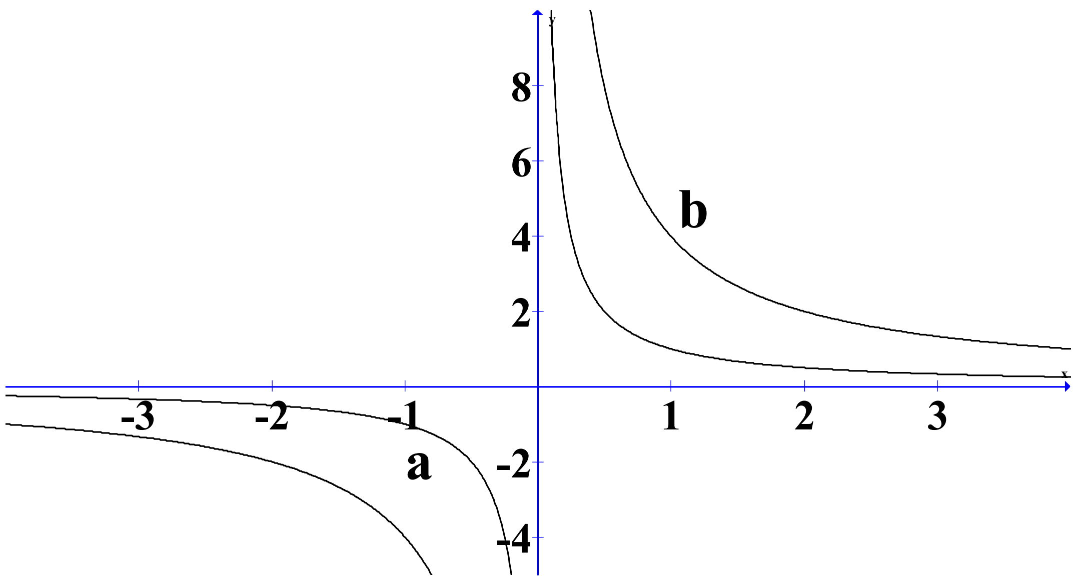


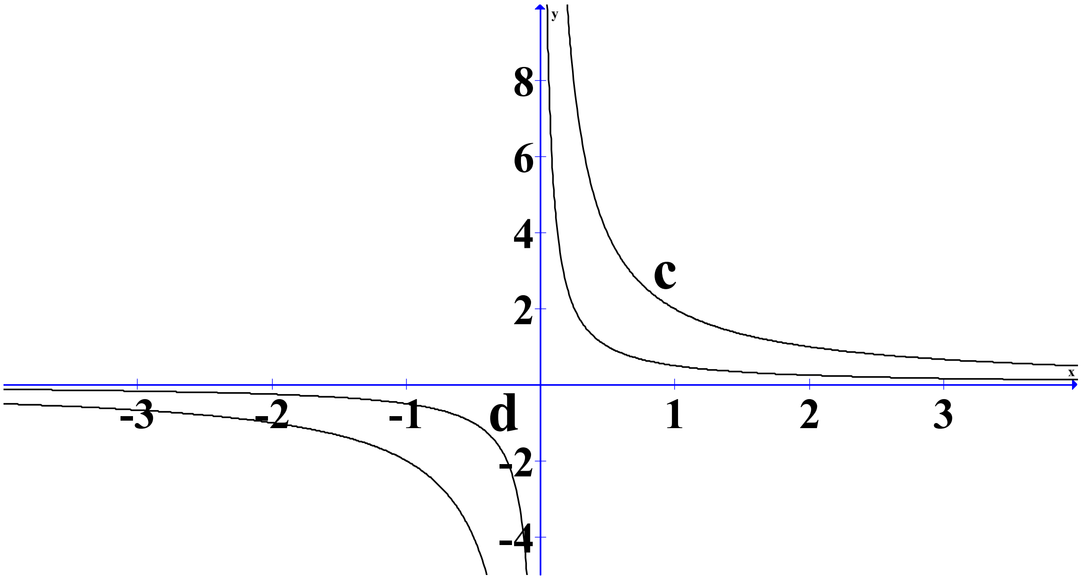




- $y = 4x^2 - 4$
- $y = \frac{1}{2}x^2$
- $y = x^2$
- $y = 2x^2 - 4$
- $y = x^2 - 2$

12) Match each of the following graphs to its correct equation





- $y = \frac{1/2}{x}$
- $y = \frac{4}{x}$
- $y = \frac{1}{x}$
- $y = \frac{2}{x}$

13) Graph each of the following and identify the line of symmetry

a) $y = x^2 - 4x + 1$

b) $y = x^2 - 2x + 1$

c) $y = x^2 + 4x + 1$

d) $y = x^2 + 2x + 1$

e) $y = 2x^2 - 4x + 1$

f) $y = 2x^2 - 2x + 1$

g) $y = 2x^2 + 4x + 1$

h) $y = 2x^2 + 2x + 1$

14) From your answers to question 13, or otherwise, comment on the relationship between the equation of the line of symmetry and the equation of the parabola of the form $y = ax^2 + bx +$

15) From your graphs in question 13, determine the co-ordinate of the turning point of the parabola and relate the co-ordinate to your answer to question 14

16) Complete the square of the equations in question 13, and determine how the equations of the form $y = (x + a)^2 + c$ relate to the co-ordinates of the turning pint of the parabola

Exercise 3

Cubics, Exponentials & Circles

1) Graph the following equations

a) $y = x^3$

b) $y = x^3 + 1$

c) $y = x^3 - 1$

d) $y = x^3 + 2$

e) $y = x^3 - 3$

2) Comment on the effect of the value of the constant a on the graph of the form $y = x^3 + a$

3) Graph the following equations

a) $y = 2^x$

b) $y = 3^x$

c) $y = \left(\frac{1}{2}\right)^x$

d) $y = 5^x$

4) Comment on the effect of the value of the constant a on graphs of the form $y = a^x$

5) Graph the following equations

a) $y = 2^{x+1}$

b) $y = 2^{x-1}$

c) $y = 2^{x+2}$

d) $y = 2^{x-3}$

6) Comment on the effect of the value of the constant a on the graph of the form $y = 2^{x+a}$

7) Graph the following

a) $x^2 + y^2 = 1$

b) $x^2 + y^2 = 4$

c) $x^2 + y^2 = 9$

d) $x^2 + y^2 = 16$

8) Comment on the effect of the value of the constant a on the graph of the form $x^2 + y^2 = a$

9) Graph the following

a) $(x - 2)^2 + y^2 = 4$

b) $(x - 1)^2 + y^2 = 4$

c) $(x + 1)^2 + y^2 = 4$

d) $(x + 2)^2 + y^2 = 4$

10) Graph the following

a) $x^2 + (y - 1)^2 = 4$

b) $x^2 + (y - 2)^2 = 4$

c) $x^2 + (y + 1)^2 = 4$

d) $x^2 + (y + 2)^2 = 4$

11) Comment on the effect of the values of the constants a and b on graphs of the form
$(x - a)^2 + (y - b)^2 = c$

Exercise 4

Solving Quadratic Equations

1) Solve the following by factorization

a) $x^2 + 2x - 3 = 0$

b) $x^2 - 6x + 8 = 0$

c) $x^2 + 6x + 5 = 0$

d) $x^2 - x - 6 = 0$

e) $x^2 - 1 = 0$

f) $x^2 - 2x + 10 = 0$

2) Solve the equations from question 1 by completing the square

3) Solve the equations from question 1 by using the quadratic formula

4) Graph the following equations, and relate your graph to the answers to questions 1 to 3

a) $y = x^2 + 2x - 3$

b) $y = x^2 - 6x + 8$

c) $y = x^2 + 6x + 5$

d) $y = x^2 - x - 6$

e) $y = x^2 - 1$

f) $y = x^2 - 2x + 10$

5) Solve the following

a) $2x^2 + 5x - 3 = 0$

b) $2x^2 - 7x - 4 = 0$

c) $4x^2 - 4x + 1 = 0$

d) $6x^2 - x - 2 = 0$

e) $6x^2 + x - 5 = 0$

f) $3x^2 + x + 6 = 0$

6) Solve the following

a) $x^2 + 2x - 6 = 0$

b) $x^2 - 4 = 0$

c) $(x - 2)^2 = 9$

d) $x(x - 4) = 4$

e) $2x^2 = 9$

7) The product of two consecutive negative numbers is 110. What are the numbers?

8) A pool measuring 8 by 12 meters is to have paving of equal width around it. When completed, the total area covered by the pool and the path will be 140 m^2. What will be the width of the path?

9) To fence a rectangular paddock takes 90 metres of fencing. If the area of the paddock is 450 m^2, what are its dimensions?

10) When an integer is added to its inverse, the sum is $\frac{50}{7}$. Use a quadratic expression to calculate the value of the integer.

11) The base of a box has a length 10cm longer than its width. When a 2cm square is cut from each corner, the area of the cardboard left is 128 cm^2. What are the dimensions of the box before the corners are cut out?

Exercise 5

Rearranging Equations

1) The formula relating the voltage (V), current (I) and resistance (R) is

$V = RI$

Rearrange the equation to make R the subject

2) Einstein's famous equation relating the mass of an object to the energy it could release is given by

$E = mc^2$, where c is the speed of light

Rearrange the equation so it is shown in terms of c

3) Pythagoras' theorem states

$a^2 + b^2 = c^2$, where a, b and c are the three side lengths of a triangle

Rearrange the equation to show the relationship in terms of b

4) The force (F) between two objects of mass m_1 and m_2 is given by the equation

$F = \gamma \frac{m_1 m_2}{r^2}$, where r is the distance between the objects and ɣ is a constant.

Express the relationship in terms of r

5) When two resistors are placed in a circuit in parallel, the total resistance R can be calculated from the values of the two resistors by the formula

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$$

Show how the value of one of the resistors can be calculated if the value of the other resistor and the total resistance is known.

6) Find an expression for $y^2 - 2$ if $y = 3x$

7) Write the expression $x^2 - 2x + 3$ in terms of t if $x = t - 1$

8) Substitute u for x^2 to solve $x^4 + 4x^2 - 5 = 0$

9) Use the substitution $x = (r - 1)^2$ to simplify and hence solve the equation $(r^2 - 2r - 3)^2 = 0$
Hint: Complete the square for r

Exercise 6

Graphing Regions

1) Graph the following regions

a) $y < 2$

b) $y \geq 3$

c) $y \leq 0$

d) $y \geq -2$

e) $x > 4$

f) $x < -1$

g) $x > -2$

2) Graph the following regions

a) $y > x$

b) $y \leq x + 1$

c) $y > 2x$

d) $y < x - 1$

e) $y \geq -x - 2$

3) Graph the following regions

a) $x - y < 4$

b) $2x + 1 > 2$

c) $x + y \leq 1$

d) $3x - 2y > 2$

e) $-x - y < 2$

f) $-2x + 2y \geq 3$

4) Find the area of the shape enclosed by the lines $y = 1, y = x$ and $y = -x + 4$

5) Find the area of the shape enclosed by the lines $y = 6$, $y = 2$, $x = 3$ and $x = 7$

6) Find the area of the shape enclosed by the lines $y = 4$, $y = 2$, $y = 2x + 4$ and $y = -x + 7$

7) Determine if the point (1, 1) lies within the regions bounded by

a) The x axis, the y axis, and the line $y = -x + 2$

b) $y \leq x + 1$

c) $2x + 2y > 3$

d) $y > -x - 2$

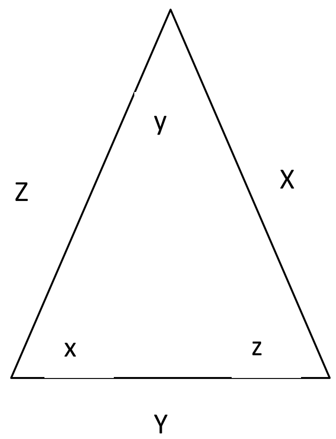
Year 10 Mathematics

Measurement:

Useful formulae and hints

When dealing with non right angled triangles, the sine, cosine and area formulae used with right angled triangles must be modified to account for the angle

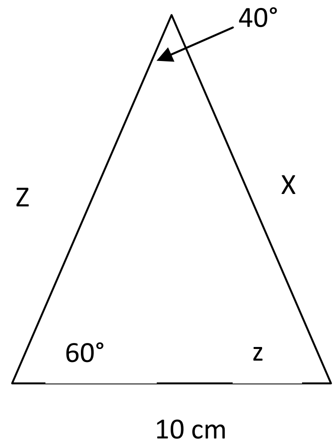
In any triangle



$$\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$$

This formula is used when one side and two angles or two angles and one side is known and the angle is not between the known sides

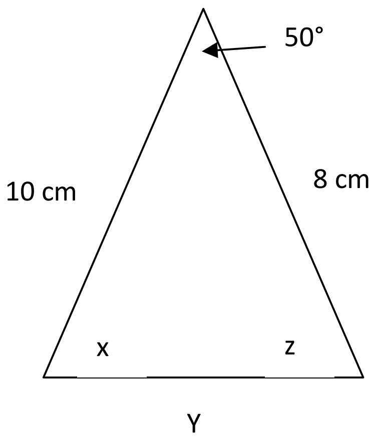
Example



$$\frac{x}{\sin 60°} = \frac{10}{\sin 40°}$$

$$x = \frac{10 \sin 60°}{\sin 40°} = \frac{10 \times 0.866}{0.643} \cong 13.47\ cm$$

The cosine rule is used when two sides are known as is the angle between them



The cosine rule is $a^2 = b^2 + c^2 - 2bc \times \cos A$, where A is the angle opposite side a

In the above example:

$$y^2 = 10^2 + 8^2 - 2 \times 10 \times 8 \times \cos 50° = 164 - 102.85 = 61.15$$

$$y = \sqrt{61.15} = 7.82\ cm$$

A different version of the cosine rule can be used to find the size of an angle when the lengths of the three sides are known

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

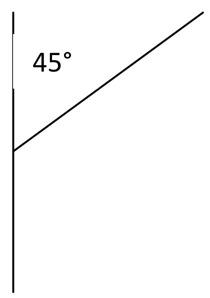
The area of a non right angled triangle can be found by using the formula

Area $= \frac{1}{2} bc \sin A$, where A is the angle between b and c

Above area $= \frac{1}{2} \times 10 \times 8 \times \sin 50° = 40 \times 0.766 = 30.64\ cm^2$

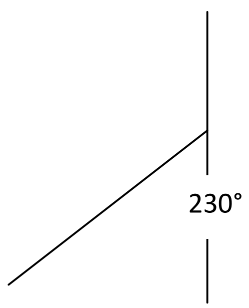
When problem solving: draw a diagram, label each side or angle and use the appropriate formula to find the required value

Bearings are measured clockwise from north

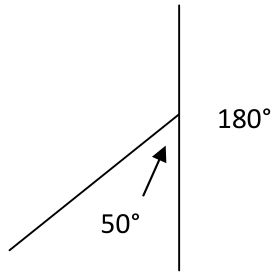


When drawing diagrams with bearings, remember that due east is 90 degrees, due south is 180 degrees, and due west is 270 degrees. Often an angle can be found in a diagram by subtracting these values and considering the angle only in a quadrant

Example: A bearing of 230 degrees can be shown



But also



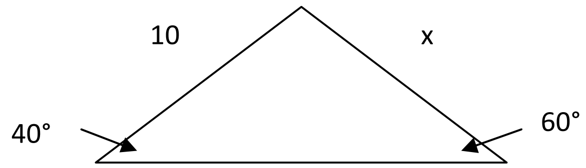
Since due south is 180 degrees. This now produces a right angled triangle with another known angle that can be used in problem solving

Exercise 1

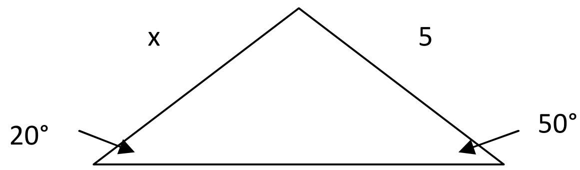
Non Right Angled Triangles

1) .Use the sine rule to calculate the value of x

a)



b)



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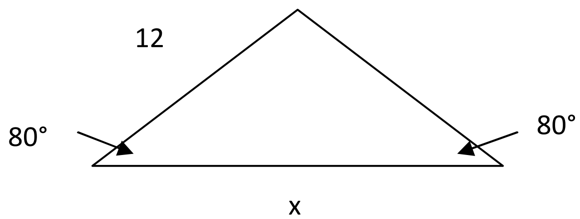
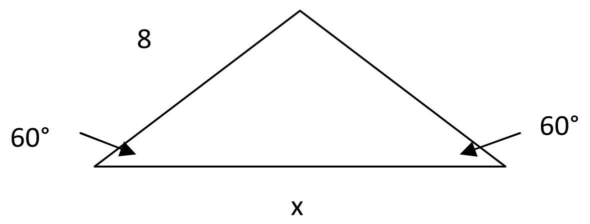
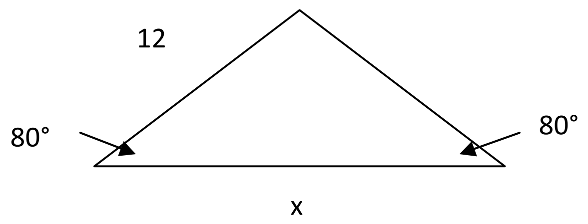


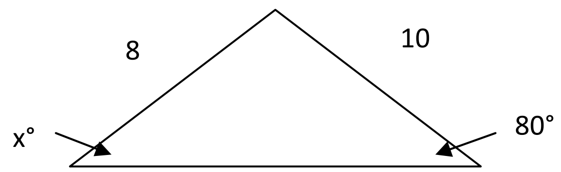
d)



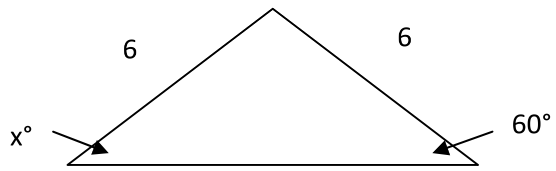
e)



f)

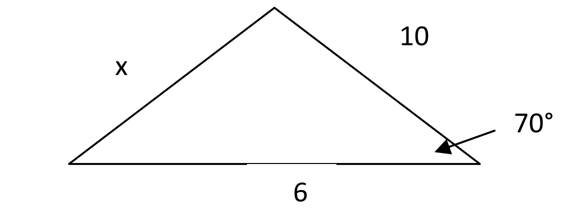


g)



2) Calculate the value of x by using the cosine rule

a)



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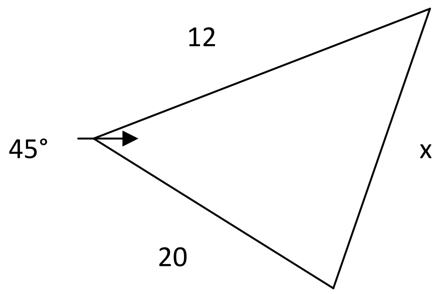

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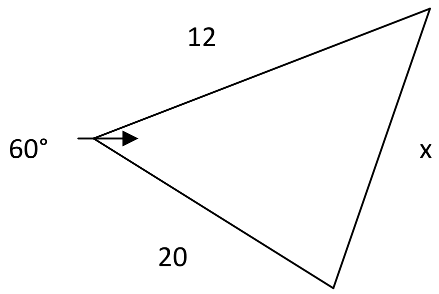

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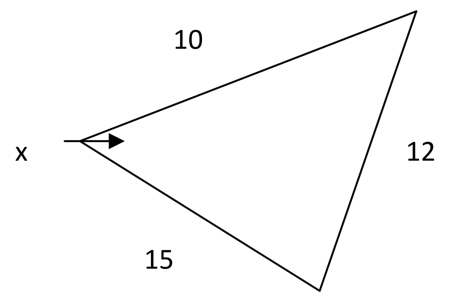

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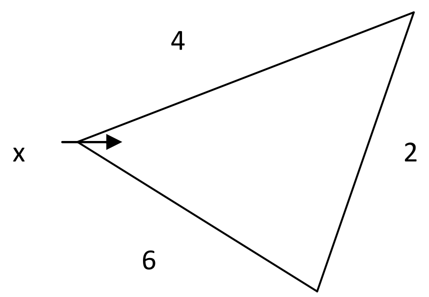


3) Calculate the areas of the triangles from questions 1 and 2

4) A man walks on a bearing of 20 degrees for 10km, then turns and walks on a bearing of 135 degrees until he is due east of his starting position. How far east of his starting position is he?

5) A man walks on a bearing of 40 degrees for 15km, then turns and walks on a bearing of 150 degrees for 10km. How far is he away from his starting position? (Use cosine rule)

6) Two airplanes leave an airport, and the angle between their flight paths is 40º. An hour later, one plane has travelled 300km while the other has travelled 200km. How far apart are the planes at this time?

7) A bicycle race follows a triangular course. The three legs of the race in order are 2.5km, 6km and 6.5km. Find the angle between the starting leg and the finishing leg to the nearest degree.

8) A straight road slopes upward at 15 degrees from the horizontal. The hill forms an angle of 128 degrees at its apex. If the distance up the hill is 5 km, how far is the distance down the hill?

9) A building is of unknown height. At a distance of 100 metres away from the building, an observer notices that the angle of elevation to the top is 41º and that the angle of elevation to a poster on the side of the building is 21º. How far is the poster from the roof of the building

10) An observer is near a river and wants to calculate the distance across the river to a point on the other side. He ties a rope to a point on his side of the river that is 125 metres away from him. The angle formed by him, the point on his side of the river, and the point on the opposite side of the river is 128°, and the distance from point to point is 40 metres. What is the distance from him to the point across the river?

Exercise 2

Bearings

1) A point K is 12km due west of a second point L and 25km due south of a third point M. Calculate the bearing of L from M

2) Point Y is 1km due north of point X. The bearings of point Z from X and Y are 26° and 42° respectively. Calculate the distance from point Y to point Z.

3) A ship steams 4km due north of a point then 3km on a bearing of 040°. Calculate the direct distance between the starting and finishing points.

4) The bearings of a point Z from two points X and Y are 30° and 120°. The distance from X to Z is 220km. What is the distance of X from Y?

5) A man walked along a road for 6km on a bearing of 115°. He then changed course to a bearing of 25° and walked a further 4km. Find the distance and bearing from his starting point

6) Directly east of a lookout station, there is a small forest fire. The bearing of this fire from another station 12.5 km. south of the first is 57°. How far is the fire from the southerly lookout station?

7) Mark and Ron leave a hostel at the same time. Mark walks on a bearing of 050° at a speed of 4.5 kilometres per hour. Ron walks on a bearing of 110° at a speed of 5 kilometres per hour. If both walk at steady speeds, how far apart will they be after 2 hours??

8) A ship leaves a harbour on a bearing of 50° and sails 50km. It then turns on a bearing of 120° and sails for another 40km. How far is the ship from its starting point?

9) Two ships A and B are anchored at sea. B is 75km due east of A. A lighthouse is positioned on a bearing of 045° from A and on a bearing of 320° from B. Calculate how far the lighthouse is from the ships

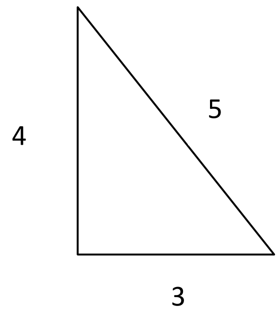
Year 10 Mathematics

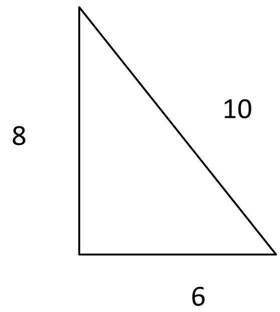
Space

Useful formulae and hints

Triangles are similar if corresponding sides of them have a common ratio

For example





Are similar triangles since each side in the second triangle is double the length of the corresponding side in the first triangle.

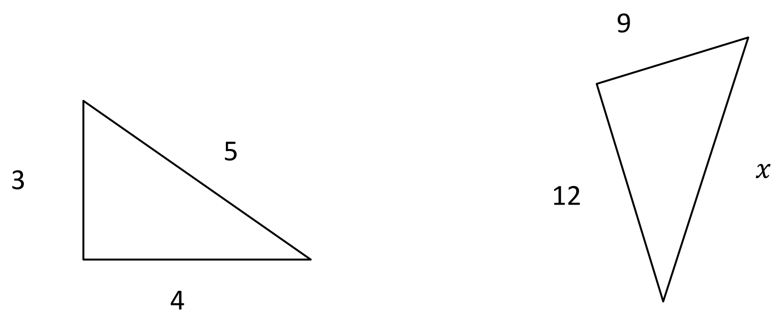
$$\frac{3}{6} = \frac{4}{8} = \frac{5}{10}$$

Also the ratios between corresponding pairs of sides in both triangles are equal

That is $\frac{3}{4} = \frac{6}{8} = \frac{5}{10}$

Missing side lengths can be found by using the above formula, but the corresponding sides must be matched

Example



To solve for x the corresponding sides must be matched

$\frac{3}{12} \neq \frac{4}{9}$ so these sides do not match

$\frac{3}{9} = \frac{4}{12}$, so these are the matching sides

$$\frac{3}{9} = \frac{5}{x}$$

$$x = 5 \times \frac{9}{3} = 15$$

There are tests between triangles that are sufficient to show similarity

These are:

- Three pairs of corresponding angles are the same size (AAA). Note if 2 pairs of angles are equal then the third pair must also be equal
- Three pairs of corresponding sides are the same size (SSS)
- Two pairs of corresponding sides and the angle between them are the same size (SAS)

Triangles are congruent if each corresponding side and angle are the same size

There are also tests that are sufficient to show congruence between triangles

- All three corresponding sides are equal in length (SSS)
- A pair of corresponding sides and the angles between them are equal (SAS)
- A pair of corresponding angles and the sides between them are equal(ASA)
- A pair of corresponding angles and a side are equal (AAS)
- The hypotenuse and one pair of sides are equal (HL)

Note that AAA is not sufficient to prove congruence as two triangles with corresponding angle sizes may have different lengths (they would be similar but not congruent)

Note that SSA is not sufficient to prove congruence (see question in exercises for proof)

Quadrilaterals are 4 sided two dimensional figures. Some quadrilaterals are merely special versions of others

All parallelograms are quadrilaterals that have opposite pairs of sides equal in length and parallel

Rectangles are parallelograms that have a right angle at each vertex

A square is a rectangle (and hence a parallelogram) that has all side lengths equal

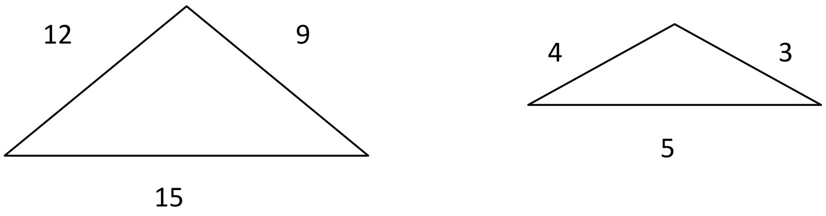
A rhombus is a parallelogram that has all sides equal in length. Hence a square is a special case of a rhombus, just as a rectangle is a special case of a parallelogram

Exercise 1

Congruence & Similarity

1) Decide if the following triangles are similar, and if so state the similarity conditions

a)



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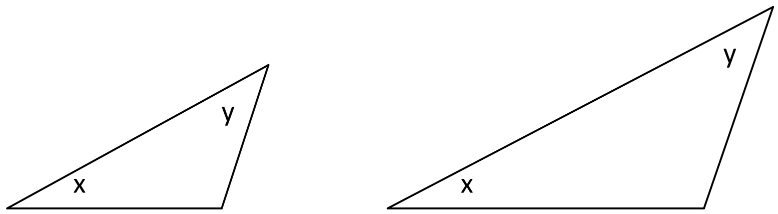

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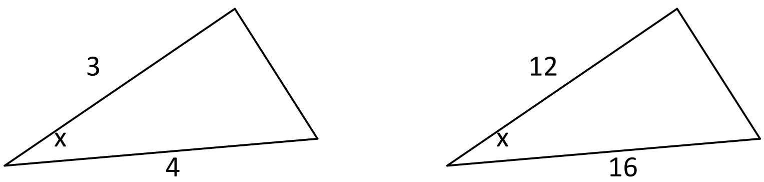

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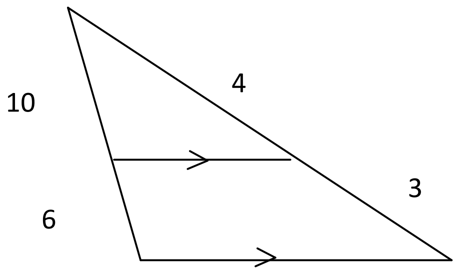
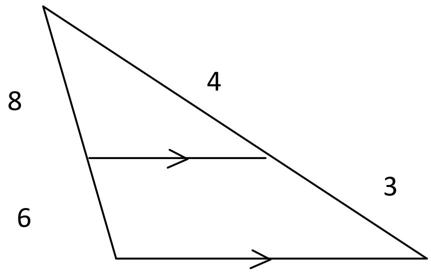
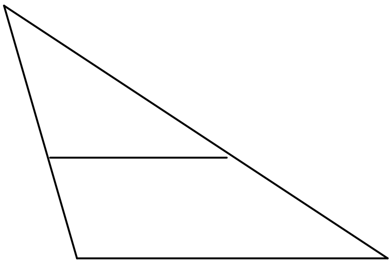


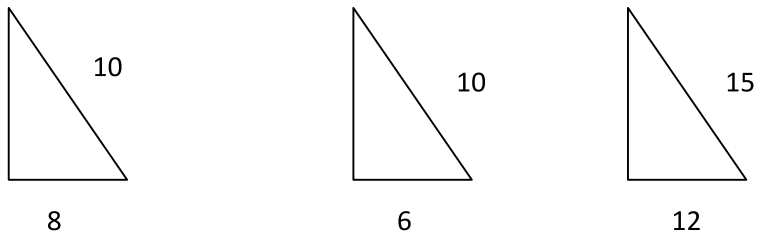
e)



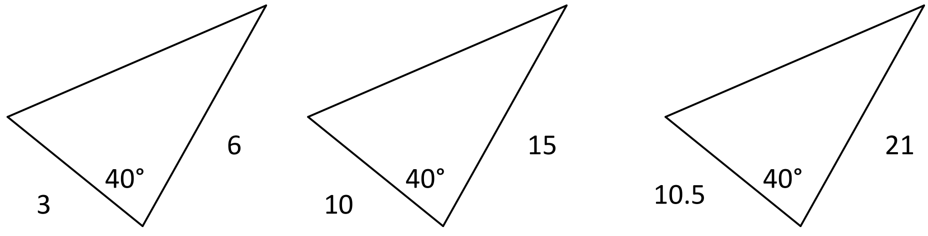
2) What additional information is needed to show that the two triangles are similar by AAA?



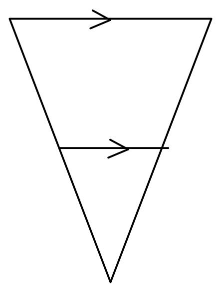
3) Of the following three right-angled triangles, which two are similar and why?



4) Of the following three triangles, which are similar and why?

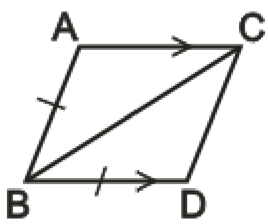


5) Prove that the two triangles in the diagram are similar



6) Prove that if two angles of a triangle are equal then the sides opposite those angles are equal

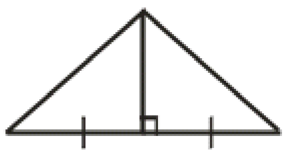
7) Is triangle ABC congruent DBC? If so, explain why.



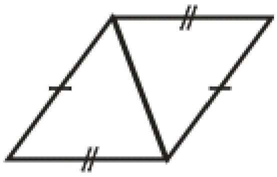
8) Which of the following is NOT a valid test for congruence?

SSA, ASA, AAS, SAS

9) State whether or not the following triangles are congruent. If so, state a reason



10) State whether or not the following triangles are congruent. If so, state a reason.



Exercise 2

Triangle proofs

1) Prove that the size of each angle of an equilateral triangle are equal

2) Using similar triangles prove Pythagoras' theorem

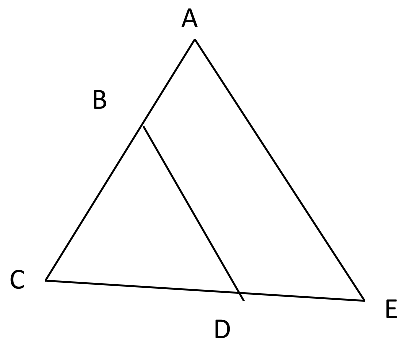
3) Triangles HIJ and MNO are similar. The perimeter of smaller triangle HIJ is 44. The lengths of two corresponding sides on the triangles are 13 and 26. What is the perimeter of MNO?

4) Prove that the sum of the internal angles of any triangle is 180°

5) Prove that the size of each exterior angle of a triangle is equal to the sum of the opposite interior angles

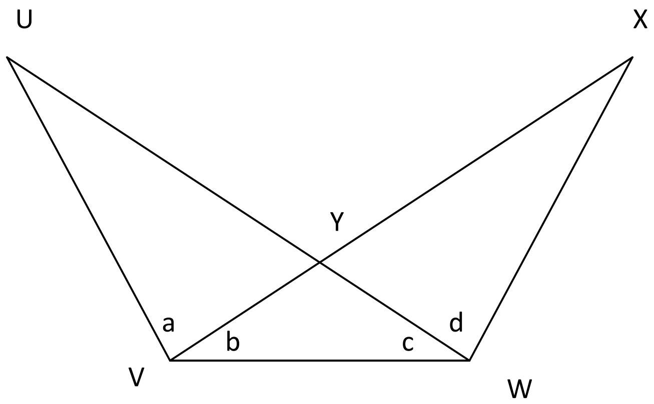
6) Prove that in an isosceles triangle, the base angles are congruent

7)

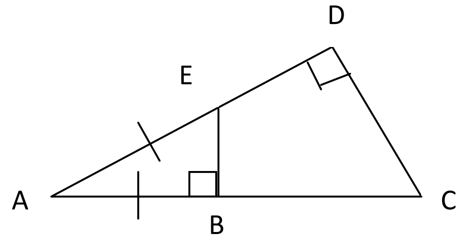


If AC = AE and AE || BD, prove that BD = BC

8) If for angles a, b, c, and d, a = d and b = c, prove that UW = XV



9)



Prove that triangle AEB is similar to triangle ACD

10) Prove that the exterior angles of a triangle add to 360°

Exercise 3

Properties of Quadrilaterals

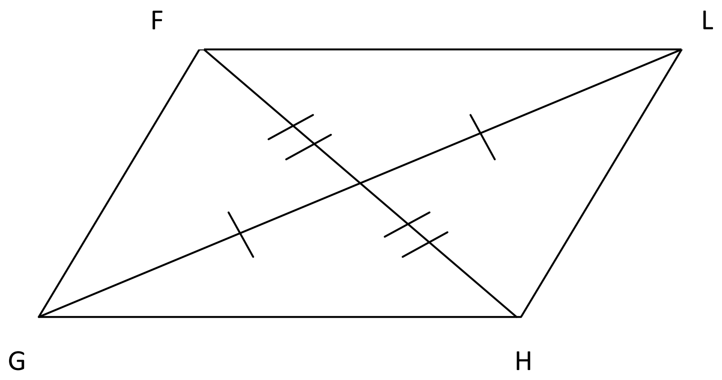
1) What is the definition of a parallelogram?

2) Prove that the exterior opposite angles of a parallelogram are equal

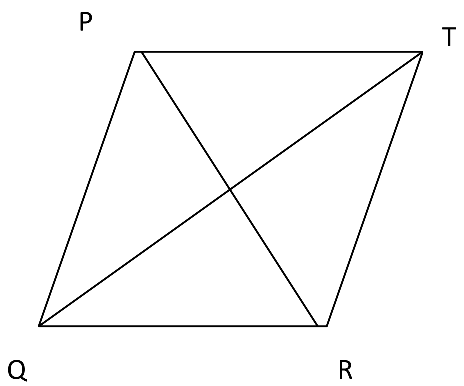
3) Show that the diagonals of a rectangle are equal

4) Prove that, if the opposite angles of a quadrilateral are equal, then the quadrilateral is a parallelogram.

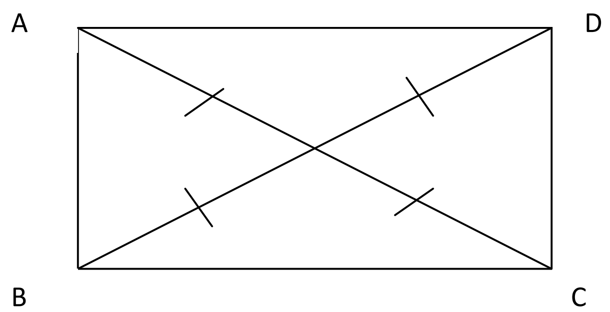
5) FGHL is a quadrilateral and the diagonals LG and FH bisect each other. Prove that FGHL is a parallelogram



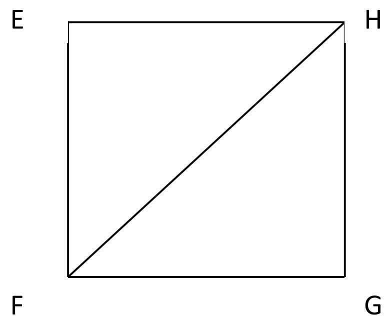
6) PQRT is a quadrilateral and the diagonals PR and QT bisect each other at right angles. Prove that PQRT is a rhombus



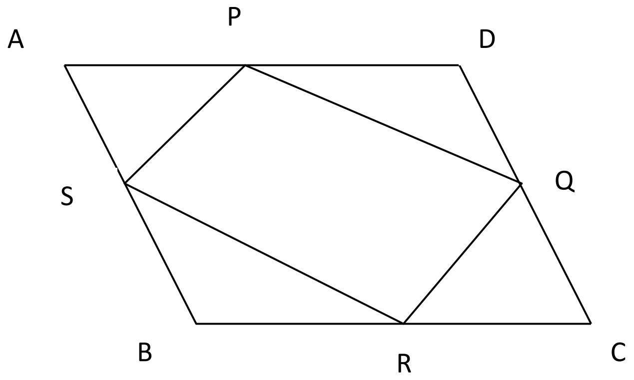
7) ABCD is a quadrilateral. The diagonals AC and BD are equal and bisect each other. Prove that ABCD is a rectangle



8) EFGH is a quadrilateral. The diagonals EG and FH are equal and bisect each other at right angles. Prove that EFGH is a square



9) ABCD is a parallelogram. AP = AS = CQ = CR. Prove that PQRS is a parallelogram



10) DEFG is a rectangle. W X Y and Z are the midpoints of the sides. Prove that WXYZ is a rhombus