Year 10 Mathematics

Solutions

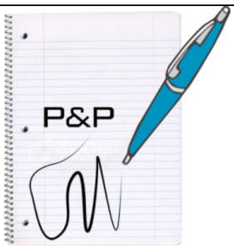
Learning Strategies

Mathematics is often the most challenging subject for students. Much of the trouble comes from the fact that mathematics is about logical thinking, not memorizing rules or remembering formulas. It requires a different style of thinking than other subjects. The students who seem to be “naturally” good at math just happen to adopt the correct strategies of thinking that math requires – often they don’t even realise it. We have isolated several key learning strategies used by successful maths students and have made icons to represent them. These icons are distributed throughout the book in order to remind students to adopt these necessary learning strategies:

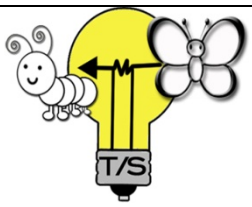
Talk Aloud Many students sit and try to do a problem in complete silence inside their heads. They think that solutions just pop into the heads of ‘smart’ people. You absolutely must learn to talk aloud and listen to yourself, literally to talk yourself through a problem. Successful students do this without realising. It helps to structure your thoughts while helping your tutor understand the way you think.

BackChecking This means that you will be doing every step of the question twice, as you work your way through the question to ensure no silly mistakes. For example with this question: $3 \times 2 - 5 \times 7$ you would do “3 times 2 is 5 ... let me check – no 3×2 is 6 ... minus 5 times 7 is minus 35 ... let me check ... minus 5×7 is minus 35. Initially, this may seem time-consuming, but once it is automatic, a great deal of time and marks will be saved.

Avoid Cosmetic Surgery Do not write over old answers since this often results in repeated mistakes or actually erasing the correct answer. When you make mistakes just put one line through the mistake rather than scribbling it out. This helps reduce silly mistakes and makes your work look cleaner and easier to backcheck.



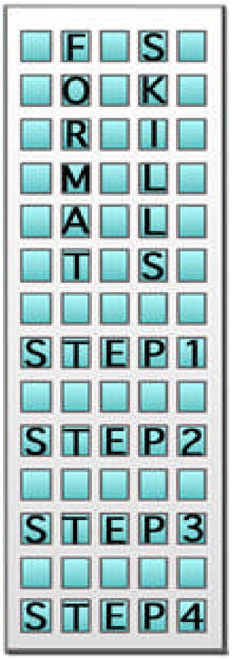
Pen to Paper It is always wise to write things down as you work your way through a problem, in order to keep track of good ideas and to see concepts on paper instead of in your head. This makes it easier to work out the next step in the problem. Harder maths problems cannot be solved in your head alone – put your ideas on paper as soon as you have them – always!



Transfer Skills This strategy is more advanced. It is the skill of making up a simpler question and then transferring those ideas to a more complex question with which you are having difficulty.

For example if you can’t remember how to do long addition because you can’t recall exactly how to carry the one: $\begin{array}{r} 5889 \\ + 4587 \\ \hline \end{array}$ then you may want to try adding numbers which you do know how to calculate that also involve carrying the one: $\begin{array}{r} 5 \\ + 9 \\ \hline \end{array}$

This skill is particularly useful when you can’t remember a basic arithmetic or algebraic rule, most of the time you should be able to work it out by creating a simpler version of the question.

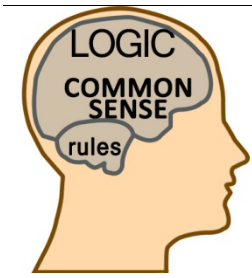


Format Skills These are the skills that keep a question together as an organized whole in terms of your working out on paper. An example of this is using the "=" sign correctly to keep a question lined up properly. In numerical calculations format skills help you to align the numbers correctly.

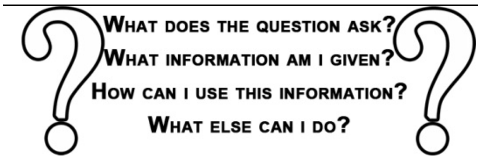
This skill is important because the correct working out will help you avoid careless mistakes. When your work is jumbled up all over the page it is hard for you to make sense of what belongs with what. Your "silly" mistakes would increase. Format skills also make it a lot easier for you to check over your work and to notice/correct any mistakes.

Every topic in math has a way of being written with correct formatting. You will be surprised how much smoother mathematics will be once you learn this skill. Whenever you are unsure you should always ask your tutor or teacher.

Its Ok To Be Wrong Mathematics is in many ways more of a skill than just knowledge. The main skill is problem solving and the only way this can be learned is by thinking hard and making mistakes on the way. As you gain confidence you will naturally worry less about making the mistakes and more about learning from them. Risk trying to solve problems that you are unsure of, this will improve your skill more than anything else. It's ok to be wrong – it is NOT ok to not try.



Avoid Rule Dependency Rules are secondary tools; common sense and logic are primary tools for problem solving and mathematics in general. Ultimately you must understand <u>Why</u> rules work the way they do. Without this you are likely to struggle with tricky problem solving and worded questions. Always rely on your logic and common sense first and on rules second, always ask <u>Why?</u>



Self Questioning This is what strong problem solvers do naturally when they get stuck on a problem or don't know what to do. Ask yourself these questions. They will help to jolt your thinking process; consider just one question at a time and <u>Talk Aloud</u> while putting <u>Pen To Paper</u>.

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Year 10 Mathematics

Number

Exercise 1

Surds & Indices

1) Simplify the following, leaving your answers in index form

a) $(3^3)^4$

$= 3^{2\times 4} = 3^8$

b) $(2^5)^2$

$= 2^{5\times 2} = 2^{10}$

c) $(5^3)^5$

$= 5^{3\times 5} = 5^{15}$

d) $(10^4)^2$

$= 10^{4\times 2} = 10^8$

2) Evaluate the following

a) $(2^4)^{1/2}$

$= 2^{4\times\frac{1}{2}} = 2^2 = 4$

b) $\left(3^{1/3}\right)^3$

$= 3^{\frac{1}{3}\times 3} = 3^1 = 3$

c) $(2^9)^{1/3}$

$= 2^{9\times\frac{1}{3}} = 2^3 = 8$

d) $(10^6)^{1/3}$

$= 10^{6\times\frac{1}{3}} = 10^2 = 100$

e) $\left(6^{1/4}\right)^4$

$= 6^{\frac{1}{4}\times 4} = 6^1 = 6$

3) Simplify and evaluate the following

a) $8^{1/3}$

2

b) $4^{1/2}$

2

c) $16^{1/4}$

2

d) $125^{1/3}$

5

4) Simplify and evaluate the following

a) $\left(\frac{1}{8}\right)^{-2}$

$= 8^2 = 64$

b) $\left(\frac{1}{8}\right)^{\frac{1}{3}}$

$= \frac{1}{2}$

c) $\left(\frac{1}{8}\right)^{-\frac{1}{3}}$

$= 8^{\frac{1}{3}} = 2$

d) $8^{-\frac{1}{3}}$

$= \frac{1}{8^{\frac{1}{3}}} = \frac{1}{2}$

5) Convert the following to index form

a) $\sqrt{2}$

$2^{\frac{1}{2}}$

b) $\sqrt{x}$

$x^{\frac{1}{2}}$

c) $\sqrt[3]{8}$

$8^{\frac{1}{3}}$

d) $\sqrt[3]{5^2}$

$5^{\frac{2}{3}}$

e) $\sqrt[5]{x^3}$

$x^{\frac{3}{5}}$

f) $\sqrt[3]{6^{-2}}$

$6^{-\frac{2}{3}}$

g) $\sqrt[-3]{6^2}$

$6^{-\frac{2}{3}}$

6) Convert the following to surd form

a) $(6)^{\frac{1}{2}}$

$\sqrt{6}$

b) $(5)^{\frac{1}{3}}$

$\sqrt[3]{5}$

c) $(7)^{\frac{2}{3}}$

$\sqrt[3]{7^2}$

d) $(3)^{-\frac{2}{5}}$

$\sqrt[5]{3^{-2}}$

e) $(x)^{\frac{a}{b}}$

$\sqrt[b]{x^a}$

7) Simplify the following

a) $\sqrt{4} + \sqrt{9}$

$= 2 + 3 = 5$

b) $\sqrt{4} \times \sqrt{9}$

$= 2 \times 3 = 6$

c) $\sqrt{4 \times 9}$

$= \sqrt{36} = 6$

d) $\frac{\sqrt{64}}{\sqrt{4}}$

$\frac{8}{2} = 4$

e) $\sqrt{\frac{64}{4}}$

$= \sqrt{16} = 4$

8) Expand and simplify

a) $(3 - \sqrt{5})(3 + \sqrt{5})$

$= 9 - 3\sqrt{5} + 3\sqrt{5} - 5$

$= 4$

b) $(\sqrt{2} + 3)(\sqrt{2} - 3)$

$= 2 - 3\sqrt{2} + 3\sqrt{2} - 9$

$= -7$

c) $(1 - \sqrt{7})(\sqrt{7} + 1)$

$\sqrt{7} - 7 + 1 - \sqrt{7}$

$= -6$

d) $(a + \sqrt{b})(a - \sqrt{b})$

$= a^2 - b$

9) Rationalise the denominator

a) $\frac{4}{\sqrt{5}-3}$

$= \left(\frac{4}{\sqrt{5} - 3}\right) \times \left(\frac{\sqrt{5} + 3}{\sqrt{5} + 3}\right)$

$= \frac{4\sqrt{5} + 12}{-4}$

$= -\sqrt{5} - 3$

b) $\frac{1}{2+\sqrt{3}}$

$= \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$

$= \frac{2 - \sqrt{3}}{1}$

$2 - \sqrt{3}$

c) $\frac{1+\sqrt{5}}{1-\sqrt{5}}$

$= \frac{1 + \sqrt{5}}{1 - \sqrt{5}} \times \frac{1 + \sqrt{5}}{1 + \sqrt{5}}$

$= \frac{6 + 2\sqrt{5}}{-4}$

$= -\frac{3}{2} - \frac{\sqrt{5}}{2}$

d) $\frac{2-\sqrt{7}}{2+\sqrt{7}}$

$= \frac{2 - \sqrt{7}}{2 + \sqrt{7}} \times \frac{2 - \sqrt{7}}{2 - \sqrt{7}}$

$$= \frac{11 - 4\sqrt{7}}{-3}$$

10) Show the following in simplest form

a) $\sqrt{20}$

$= \sqrt{5 \times 4}$

$= 2\sqrt{5}$

b) $\sqrt{8}$

$= \sqrt{2 \times 4}$

$= 2\sqrt{2}$

c) $\sqrt{45}$

$= \sqrt{5 \times 9}$

$= 3\sqrt{5}$

d) $\sqrt{8}\sqrt{8}$

$= \sqrt{8 \times 8}$

$= 8$

e) $\sqrt{75}$

$= \sqrt{3 \times 25}$

$5\sqrt{3}$

11) Simplify the following

a) $\frac{\sqrt{12}}{\sqrt{75}}$

$$= \sqrt{\frac{12}{75}}$$

$$= \sqrt{\frac{3 \times 4}{3 \times 25}}$$

$$= \frac{2\sqrt{3}}{5\sqrt{3}} = \frac{2}{5}$$

b) $\frac{\sqrt{50}}{3\sqrt{8}}$

$$= \frac{\sqrt{25 \times 2}}{3\sqrt{4 \times 2}}$$

$$= \frac{5\sqrt{2}}{6\sqrt{2}} = \frac{5}{6}$$

c) $\frac{3\sqrt{24}}{2\sqrt{2}}$

$$= \frac{3\sqrt{4 \times 6}}{2\sqrt{2}}$$

$$= \frac{6\sqrt{6}}{2\sqrt{2}}$$

$$= 3\sqrt{3}$$

d) $\sqrt{27} - 2\sqrt{3}$

$= \sqrt{3 \times 9} - 2\sqrt{3}$

$= 3\sqrt{3} - 2\sqrt{3} = \sqrt{3}$

Exercise 2

Consumer Arithmetic

1) Calculate the total amount payable over the course of a loan under the following conditions

$A = P(1+r)^n$

a) Principal of $20,000 at 12% per annum compound interest calculated yearly for 2 years

$A = 20000(1+0.12)^2$

$= 20000 \times 1.12^2$

$= \$25088$

b) Principal of $10,000 at 10% per annum compound interest calculated yearly for 4 years

$A = 10000(1+0.10)^4$

$= 10000 \times 1.1^4$

$= \$14641$

c) Principal of $30,000 at 5% per annum compound interest calculated yearly for 3 years

$A = 30000(1+0.05)^3$

$= 30000 \times 1.05^3$

$= \$34728.75$

d) Principal of $8,000 at 15% per annum compound interest calculated yearly for 5 years

$A = 8000(1+0.15)^5$

$= 8000 \times 1.15^5$

$= \$16090.86$

2) Calculate the total amount payable over the course of a loan under the following conditions

a) Principal of $20,000 at 15% per annum compound interest calculated yearly for 10 years

$A = 20000(1+0.15)^3$

$= 20000 \times 1.15^3$

$= \$30417.50$

b) Principal of $20,000 at 10% per annum compound interest calculated quarterly for 3 years

$A = 20000\left(1+\frac{0.10}{4}\right)^{12}$

$= 20000 \times 1.025^{12}$

$= \$26897.78$

c) Principal of \$20,000 at 12% per annum compound interest calculated monthly for 2 years

$$A = 20000(1 + 0.01)^{24}$$

$$= 20000 \times 1.01^{24}$$

$$= \$25394.69$$

d) Principal of \$20,000 at 18% per annum compound interest calculated monthly for 4 years

$$A = 20000\left(1 + \frac{0.18}{12}\right)^{48}$$

$$= 20000 \times 1.015^{48}$$

$$= \$40869.57$$

3) Calculate the total **interest** payable over the course of a loan under the following conditions

a) Principal of \$10,000 at 24% per annum compound interest calculated monthly for 2 years

$$A = 10000\left(1 + \frac{0.24}{12}\right)^{24}$$

$$= 10000 \times 1.02^{24}$$

$$= \$16084.37$$

Interest $= A - P$

$$= 16084.37 - 10000$$

$$= \$6084.37$$

b) Principal of \$15,000 at 9% per annum compound interest calculated monthly for 2 years

$$A = 15000\left(1 + \frac{0.09}{12}\right)^{24}$$

$$= 15000 \times 1.0075^{24}$$

$$= \$17946.20$$

Interest $= A - P$

$$= 17946.20 - 15000$$

$$\$2946.20$$

c) a Principal of \$20,000 at 5.25% per annum compound interest calculated weekly for 2 years

$$A = 20000\left(1 + \frac{0.0525}{52}\right)^{104}$$

$$= 10000 \times 1.001^{104}$$

$$= \$11106.52$$

Interest $= A - P$

$$= \$1106.52$$

4) Alan takes a loan to purchase a car, and agrees to pay simple interest on the loan at the rate of 10% per annum. If the loan is for $40,000 how much should he repay per month to have the loan fully repaid in 5 years?

Interest = P x R x T

$= 40000 \times 0.1 \times 5$

$= \$20000$

$(20000 + 40000) \div 60 = \$1000/$month

5) If the above loan was calculated using compound interest, and Alan made no repayments, calculate how much would he owe on the loan after 1, 2 and 3 years.

$A = P(1 + r)^n$

After one year

$A = 40000 \times 1.1^1$

$= \$44000$

After 2 years

$A = 40000 \times 1.1^2$

$\$48400$

After 3 years

$A = 40000 \times 1.1^3$

$= \$53240$

6) A man invests $25,000 at a rate of 5% compound interest paid annually. How much would his investment be worth after 1, 2, and 3 years?

After 1 year

$A = 25000 \times 1.05^1$

$= \$26250$

After 2 years

$A = 25000 \times 1.05^2$

$= \$27562.50$

After 3 years

$A = 25000 \times 1.05^3$

$= \$28940.62$

7) Calculate the book value of the following assets (use compound interest formulae)

a) A motor vehicle that originally cost $50,000 that depreciates at a rate of 10% of book value per year for 5 years

$A = 50000(1 - r)^n$

$= 50000 \times 0.9^5$

$= \$29524.50$

b) A computer system that originally cost $20,000 that depreciates at a rate of 5% of book value per year for 10 years

$A = 20000 \times 0.95^{10}$

$= \$11974.74$

c) A truck that originally cost $80,000 that depreciates at a rate of 15% of book value per year for 4 year

$A = 80000 \times 0.85^{4}$

$= \$41760.50$

8) Which investment is worth more?

a) An initial deposit of $100,000 that has simple interest paid at a rate of 10% per annum

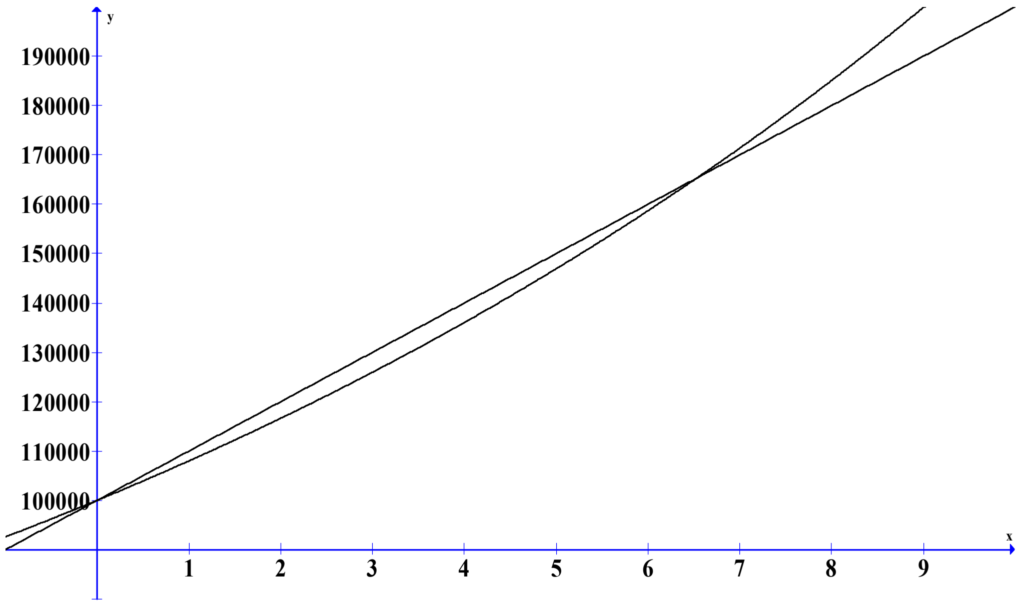
b) An initial deposit of $100,000 that has compound interest paid at a rate of 8% per annum

It depends on how long the investment is for

Years	10% Simple Interest	8% Compound Interest
1	110000	108000
2	120000	116640
3	130000	125971.20
4	140000	136048.90
5	150000	146932.81
6	160000	158687.43
7	170000	171382.43

For an investment period less than 7 years, simple interest returns the most; after this time compound interest returns more

As a note of interest, compare the graphs of the two investments: the straight line represents the simple interest



9) Alan takes a loan to purchase a car, and agrees to pay simple interest on the loan at the rate of 5% per annum. If the loan is for $40,000 how much should he repay per month to have the loan fully repaid in 10 years?

Interest $= 40000 \times 0.05 \times 10 = \$20000$

Total amount payable $= 40000 + 20000 = \$60000$

Amount payable per month $= 60000 \div 120 = \$500$

10) If the above loan was calculated using compound interest, and Alan made no repayments, calculate how much would he owe on the loan after 1, 2 and 3 years.

After 1 year

$A = 40000 \times 1.05 = \$42000$

After 2 years

$A = 40000 \times 1.05^2 = \$44100$

After 3 years

$A = 40000 \times 1.05^3 = \$46305$

Exercise 3

Recurring Decimals & Rates

1) Convert the following recurring decimals to fractions

a) $0.\dot{1}$

$x = 0.\dot{1}$

$10x = 1.\dot{1} = 1 + x$

$9x = 1$

$x = \frac{1}{9}$

b) $0.\dot{4}$

$x = 0.\dot{4}$

$10x = 4.\dot{4} = 4 + x$

$9x = 4$

$x = \frac{4}{9}$

c) $0.\dot{5}$

$x = 0.\dot{5}$

$10x = 5.\dot{5} = 5 + x$

$9x = 5$

$x = \frac{5}{9}$

d) $0.\dot{7}$

$x = 0.\dot{7}$

$10x = 7.\dot{7} = 7 + x$

$9x = 7$

$x = \frac{7}{9}$

e) $0.\dot{8}$

$x = 0.\dot{8}$

$10x = 8.\dot{8} = 8 + x$

$9x = 8$

$x = \frac{8}{9}$

f) $0.\dot{9}$

$x = 0.\dot{9}$

$10x = 9.\dot{9} = 9 + x$

$9x = 9$

$x = 1$

2) Convert the following recurring decimals to fractions

a) $0.\dot{3}\dot{4}$

$x = 0.\dot{3}\dot{4}$

$100x = 34.\dot{3}\dot{4} = 34 + x$

$99x = 34$

$x = \frac{34}{99}$

Similarly:

b) $0.\dot{2}\dot{6} = \frac{26}{99}$

c) $0.\dot{1}\dot{5} = \frac{15}{99}$

d) $0.\dot{6}\dot{1} = \frac{61}{99}$

e) $0.\dot{0}\dot{5} = \frac{5}{99}$

f) $0.\dot{5}\dot{1} = \frac{51}{99} = \frac{17}{33}$

3) Convert the following recurring decimals to fractions

a) $0.4\dot{2}$

$x = 0.4\dot{2}$

$10x = 4.\dot{2}$

$10x = 4 + \frac{2}{9} = \frac{38}{9}$

$x = \frac{38}{90} = \frac{19}{45}$

b) $0.1\dot{5}$

$x = 0.1\dot{5}$

$10x = 1.\dot{5}$

$10x = 1 + \frac{5}{9} = \frac{14}{9}$

$x = \frac{14}{90} = \frac{7}{45}$

c) $0.6\dot{3}$

$x = 0.6\dot{3}$

$10x = 6.\dot{3}$

$10x = 6 + \frac{3}{9} = \frac{57}{9}$

$x = \frac{57}{90} = \frac{19}{30}$

d) $0.3\dot{5}$

$x = 0.3\dot{5}$

$10x = 3.\dot{5}$

$10x = 3 + \frac{5}{9} = \frac{32}{9}$

$x = \frac{32}{90} = \frac{16}{45}$

e) $0.7\dot{2}$

$x = 0.7\dot{2}$

$10x = 7.\dot{2}$

$10x = 7 + \frac{2}{9} = \frac{65}{9}$

$x = \frac{65}{90} = \frac{13}{18}$

f) $0.0\dot{2}$

$x = 0.0\dot{2}$

$10x = 0.\dot{2} = \frac{2}{9}$

$x = \frac{2}{90} = \frac{1}{45}$

4) Convert the following to metres per second

a) 60 m per minute

$60 \div 60 = 1\ m\ per\ second$

b) 1 km per second

1×1000
$= 1000\ m\ per\ second$

c) 10 km per hour

10×1000
$= 10000\ m\ per\ hour$

$10000 \div 3600$
$= 2.78\ m\ per\ second$

d) 3 km per minute

3×1000
$= 3000\ m\ per\ minute$

$3000 \div 60$
$= 50\ m\ per\ second$

e) 0.2 km per hour

0.2×1000
$= 200\ m\ per\ hour$

$200 \div 3600$
$= 0.0\dot{5}m\ per\ second$

f) 3600 km per day

$3600 \div 24$
$= 150\ km\ per\ hour$

150×1000
$= 150000\ m\ per\ hour$

$150000 \div 3600$
$= 41.\dot{6}m\ per\ second$

5) Convert the following to km per hour

a) 3 m per second

3×3600
$= 10800\ m\ per\ hour$

$10800 \div 1000$
$= 10.8\ km\ per\ hour$

b) 10 m per minute

10×60
$= 600\ m\ per\ hour$

$600 \div 1000$
$= 0.6\ km\ per\ hour$

c) 240 km per day

$240 \div 24$
$= 10\ km\ per\ hour$

d) 260 m per hour

$260 \div 1000$
$= 0.26\ km\ per\ hour$

e) 10 km per second

10×3600
$= 36000\ km\ per\ hour$

6) Convert the following simple interest rates to a monthly interest rate

a) 12% per annum

$12 \div 12 = 1\%\ per\ month$

b) 6% per quarter

$6 \div 3 = 2\%\ per\ month$

c) 8% per half year

$8 \div 6 = 1.\dot{3}\%\ per\ month$

d) 50% per decade

$50 \div 120$
$= 0.41\dot{6}\%\ per\ month$

e) 0.5% per day

0.5×365
$= 182.5\%\ per\ annum$

$182.5 \div 12$
$= 15.208\dot{3}\%\ per\ month$

7) Tony wants to buy some meat and checks the prices at three supermarkets

- Scoles Supermarket has meat at \$18 per kg
- Bullworths price is \$2 per 100 grams
- PGA Supermarket is selling the meat for \$4 per 250 grams

Which supermarket is cheapest?

Convert all prices to same units

\$2 per 100 grams $= 2 \times 10 = \$20\ per\ kg$

\$4 per 250 grams $= 4 \times 4 = \$16\ per\ kg$

PGA has the cheapest meat

8) Put the following speeds in order from slowest to fastest

50 m per second
1000 km per minute
500000 cm per minute
500 km per hour

17500 m per hour
10000 km per day

Convert speeds to km per hour

50 m per second $= 50 \times 3600 = 180000\ m\ per\ hour = 180\ km\ per\ hour$

1000 km per minute $= 1000 \times 60 = 60000\ km\ per\ hour$

500000 cm per minute $= 5\ km\ per\ minute = 5 \times 60 = 300\ km\ per\ hour$

17500 m per hour $= 17.5\ km\ per\ hour$

10000 km per day $= 10000 \div 24\ km\ per\ hour = 41.\dot{6}\ km\ per\ hour$

9) Put the following hire charges in order from highest to lowest

$ 864 per day

$36 per hour

1 cent per second

60 cents per minute

$25920 for the month of September

Change all to $ per day

$36 per hour $= \$36 \times 24\ per\ day = \$864\ per\ day$

1 cent per second$= 1 \times 86400\ cents\ per\ day = \$864\ per\ day$

60 cents per minute$= 60 \times 1440\ cents\ per\ day = 86400\ cents\ per\ day = \$864\ per\ day$

$25920 for 30 days (September)$= \$25920 \div 30 = \$864\ per\ day$

All hire charges are the same

10) The speed of light in a vacuum is 3×10^8 metres per second

a) How far does light travel in 10 seconds?

$3 \times 10^8 \times 10 = 3 \times 10^9 metres\ in\ 10\ seconds$

b) How long does light take to travel 90000 metres?

$$Time = \frac{Distance}{Speed}$$

$$\frac{90000}{3 \times 10^8} = \frac{30000}{10^8} = 3 \times 10^4 \times 10^{-8} = 3 \times 10^{-4}\ seconds$$

c) There are approximately 31.5 million seconds in a year. How far does light travel in one year?

$$Distance = Speed \times Time$$

$$3 \times 10^8 \times 31.5 \times 10^6 = 94.5 \times 10^{14} metres = 9.45 \times 10^{15} metres$$
$$= 9.45 \times 10^{12} km$$

d) The closest star to Earth is approximately 4.3 light years away. How long would it take a rocket travelling at 40,000 km per hour to reach it?

$$4.3 \times 9.45 \times 10^{12} = 40.635 \times 10^{12} km$$

$$(40.635 \times 10^{12}) km \div 40000 km\ per\ hour \cong 0.001 \times 10^{12} hours$$
$$= 10^9 hours$$

This is approximately equal to 114000 years

In all the above, use your knowledge of index laws

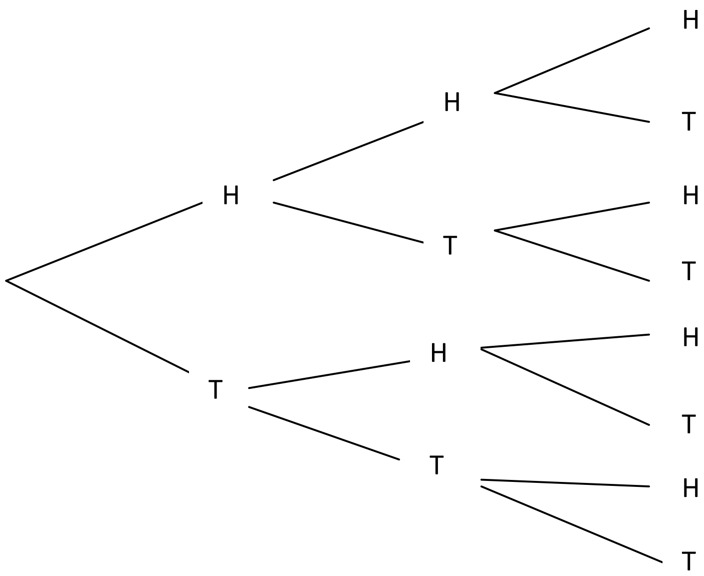
Year 10 Mathematics

Chance & Data

Exercise 1

Probability

1) Construct a tree diagram that details the possible outcomes of tossing a coin three times. From your tree diagram determine the probability of:



Sample set has 8 outcomes

a) Three heads

$\frac{1}{8}$

b) Two heads and a tail

$\frac{3}{8}$

c) A head on the third throw given that the first two throws were tails

The result of the first two throws does not alter the probability of the third throw, so probability $= \frac{1}{2}$

d) If the first three throws were tails, what is the probability that a fourth throw is a head?

As per part c, the probability $= \frac{1}{2}$

2) A pizza shop has the following choices of ingredients:

- Ham
- Onion
- Tomato
- Pineapple

If you are allowed three toppings only, construct a list of all possible pizzas you could make. From this table:

a) How many possible pizzas can be made?

HOT, HOP, OTP, HTP

b) What is the probability that one of these pizzas contains ham?

$\frac{3}{4}$

c) What is the probability that one of these pizzas contains ham and onion?

$\frac{2}{4} = \frac{1}{2}$

d) What is the probability that one of these pizzas does not contain Tomato?

$\frac{1}{4}$

3) The table below shows the distribution of cars at a car yard. For example, there are 200, 6 cylinder black cars.

	BLACK	WHITE	
6 CYLINDER	200	100	300
8 CYLINDER	150	350	500

	350	450	800

From this table:

a) What is the probability that a car chosen at random will be white?

$$\frac{450}{800} = \frac{9}{16}$$

b) What is the probability that a car chosen at random will be a 6 cylinder?

$$\frac{300}{800} = \frac{3}{8}$$

c) What is the probability that a car chosen at random will be an 8 cylinder black car?

$$\frac{150}{800} = \frac{3}{16}$$

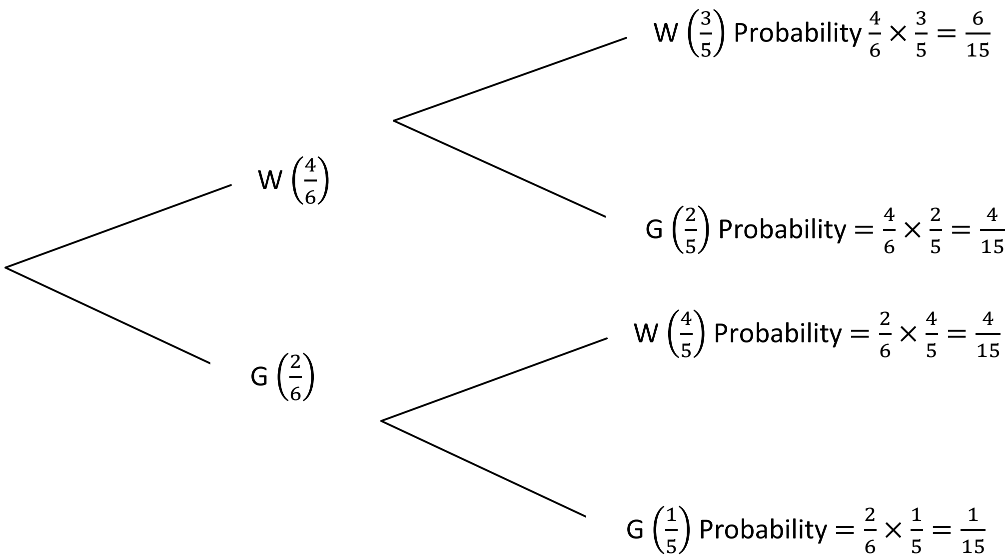
d) What is the probability that a black car chosen at random will be a 6 cylinder?

$$\frac{200}{350} = \frac{4}{7}$$

e) What is the probability that a white car chosen at random will be an 8 cylinder?

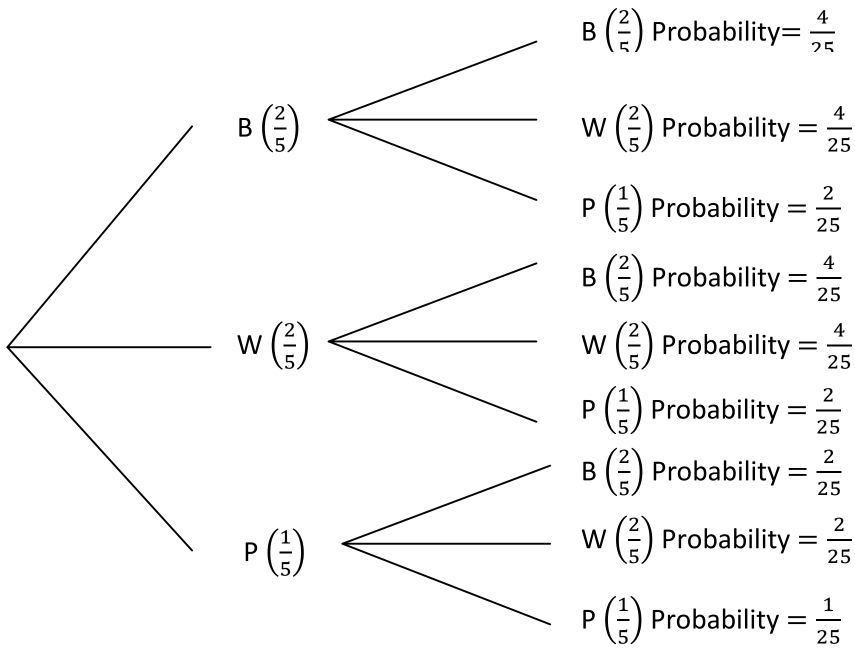
$$\frac{350}{450} = \frac{7}{9}$$

4) There are 4 white and 2 green shirts in a draw. Construct a tree diagram that shows the possible combinations of 2 shirts that can be taken at random, and show the probabilities.



5) A box contains 2 black, 2 white and 1 pink balls. A ball is chosen at random from the box, replaced, and a second ball is then chosen at random.

a) Construct a probability tree diagram, and use it to answer the following:



b) What is the probability that both balls are black?

$$\frac{4}{25}$$

c) What is the probability that one ball is white and the other is pink?

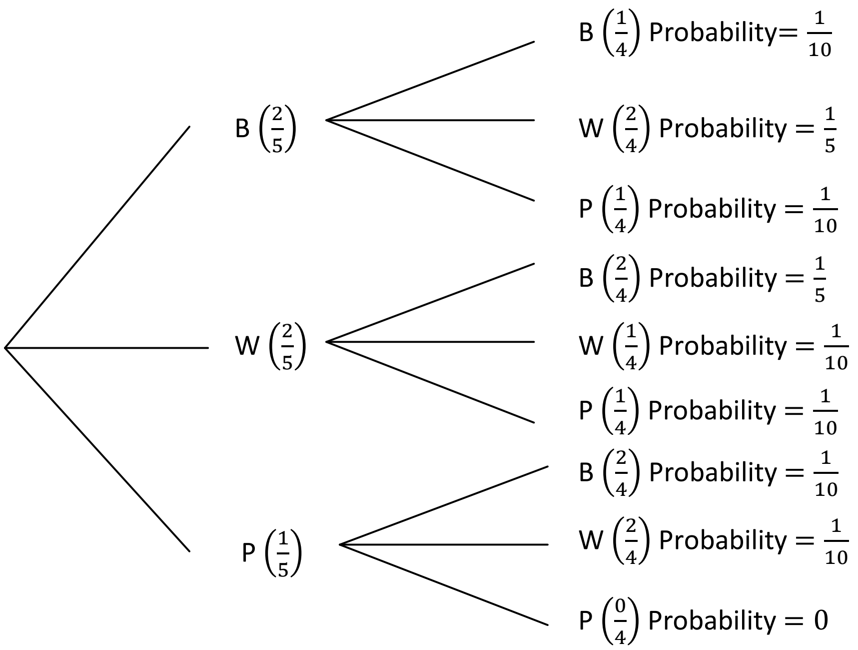
$$\frac{2}{25}+\frac{2}{25}=\frac{4}{25}$$

d) What is the probability that both balls are pink?

$$\frac{1}{25}$$

6) Repeat question 5, but assume the first ball is not replaced

a)



b) $\frac{1}{10}$

c) $\frac{1}{10}+\frac{1}{10}=\frac{1}{5}$

d) 0

7) A survey was taken to investigate possible links between eating excessive fast food and diabetes. The results are shown in the table below

	Diabetic	Non-diabetic	
Fast food more than 2 times a week	500	100	600
Fast food less than 3 times a week	200	400	600
	700	500	

a) How many people were studied?

1200

b) What is the probability that a person chosen at random from the group had diabetes?

$$\frac{700}{1200} = \frac{7}{12}$$

c) What is the probability that a person chosen at random from the group had fast food more than twice a week?

$$\frac{600}{1200} = \frac{1}{2}$$

d) What is the probability that a person chosen at random from the group who had fast food more than twice a week also had diabetes?

$$\frac{500}{600} = \frac{5}{6}$$

e) What is the probability that a person chosen at random from the group who did not have diabetes ate fast food more than twice a week?

$$\frac{100}{500} = \frac{1}{5}$$

8) One thousand students were surveyed regarding how many hours of homework they did each week

- Of the 400 girls surveyed, 100 said they did homework for more than 10 hours per week
- 400 boys did homework for less than 10 hours per week

Construct a probability table from the above information, and use it to determine the probability that a person who did homework for less than 10 hours a week was a girl

	Girls	Boys	
Homework more than 10 hours a week	100	200	300
Homework less than 10 hours a week	300	400	700
	400	600	1000

Probability $= \frac{300}{700} = \frac{3}{7}$

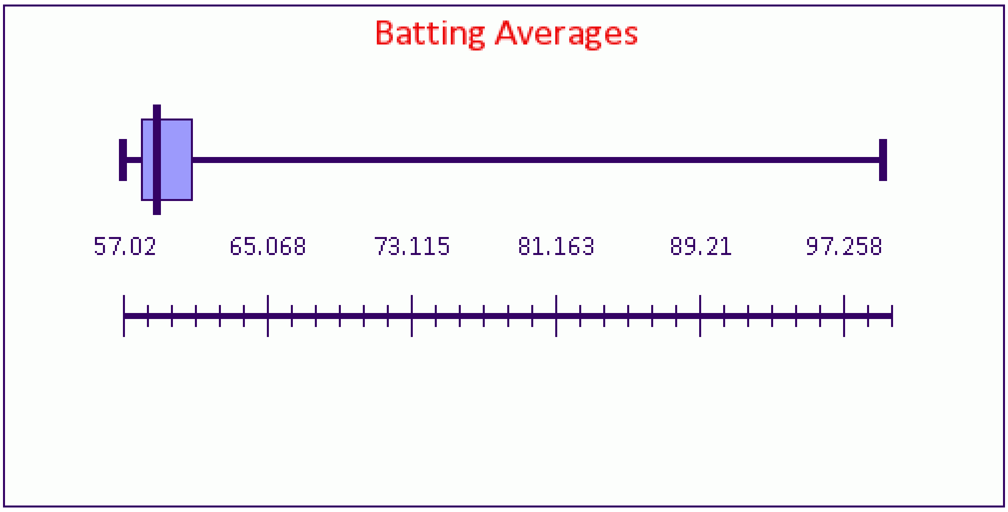
Exercise 2

Data Representation & Analysis

1) The top ten test batting averages in history are:

99.94, 60.97, 60.83, 60.73, 59.23, 58.67, 58.61, 58.45, 57.78, 57.02

a) Construct a box and whisker plot of the data



b) Predict if the mean will be higher or lower than the median and justify your answer

The mean will be higher, since the outlier (99.94) will skew the data, whereas it does not affect the median (the outlier could be one million and the median would still be the same)

c) Calculate the mean and standard deviation

Mean $= 63.22$
S.D. $=12.3$

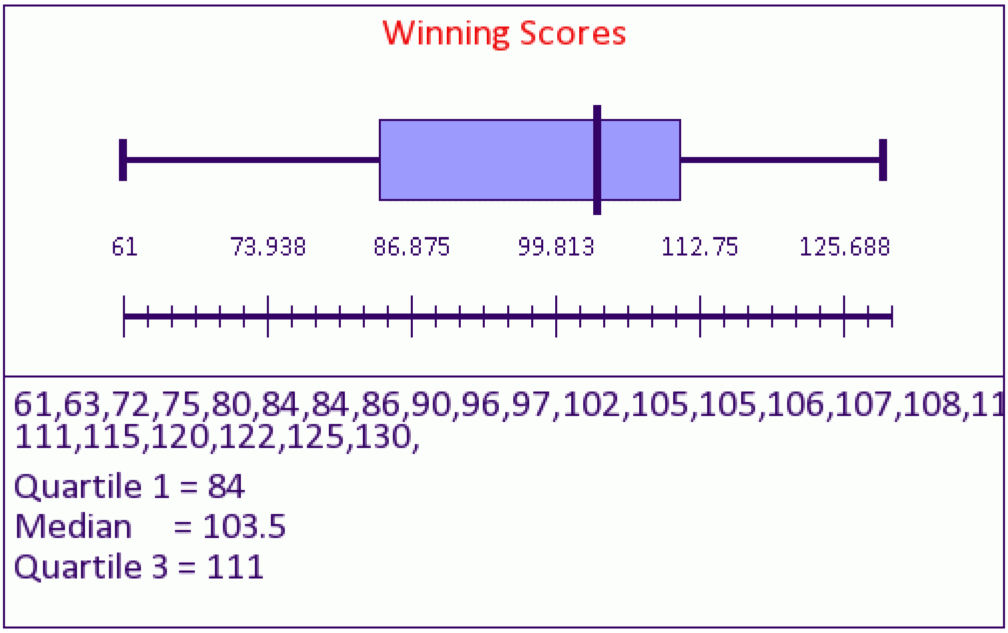
Is the range, inter-quartile range or standard deviation a more realistic measure of the spread of the data in this case? Justify your answer

The inter-quartile range, since the range and standard deviation are both affected by the outlier

2) The following data shows the score (in points) of the winning AFL football teams over three weeks

61, 63, 72, 75, 80, 84, 84, 86, 90, 96, 97, 102, 105, 105, 106, 107, 108, 110, 111, 115, 120, 122, 125, 130

a) Draw a box-and-whisker plot marking the 5 relevant points



b) What is the inter-quartile range?

27

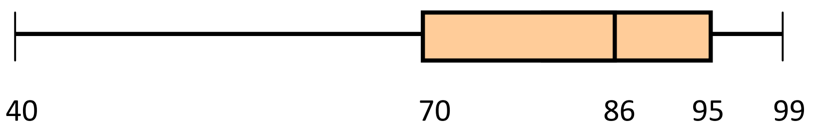
c) What is the median score?

103.5

d) Comment on the spread of the data

The data shows a large range compared to the IQR, indicating that there were a few high and low scores, but 50% of the scores fell between 84 and 111 points

3) The box and whisker plot below shows the distribution of students' maths test scores



a) .What was the lowest score in the test?

40

b) What percentage of the class scored above 70%?

75%

c) What was the median score?

86

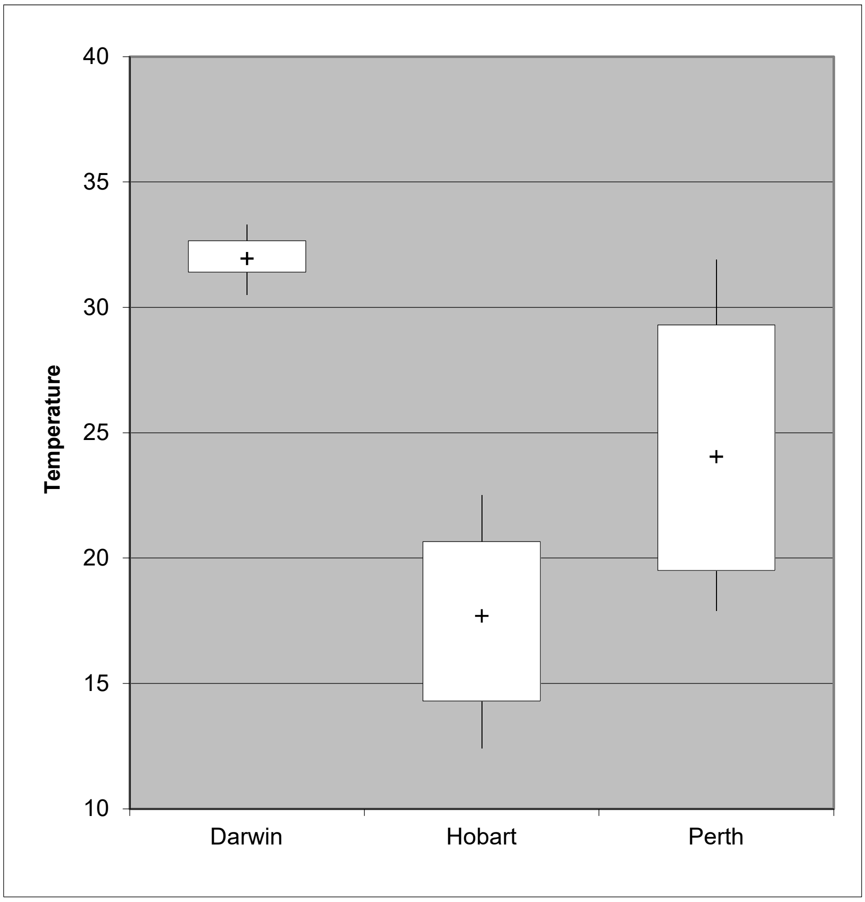
d) What percentage of the class scored between 70 and 86?

25%

e) Comment on the difficulty of the test

75% of the class scored 70 or higher, indicating that most students found the test easy. A quarter of the students scored in the high 90s, and half of them scored more than 86%

4) The following box plots show the distribution of the average monthly temperatures for a year for Hobart, Darwin and Perth. (the cross indicates the median for each data set)



Compare the three sets of data and comment on the similarities and differences in the distributions of average monthly temperatures for the three cities

Darwin has a much higher temperature than the other cities at all times

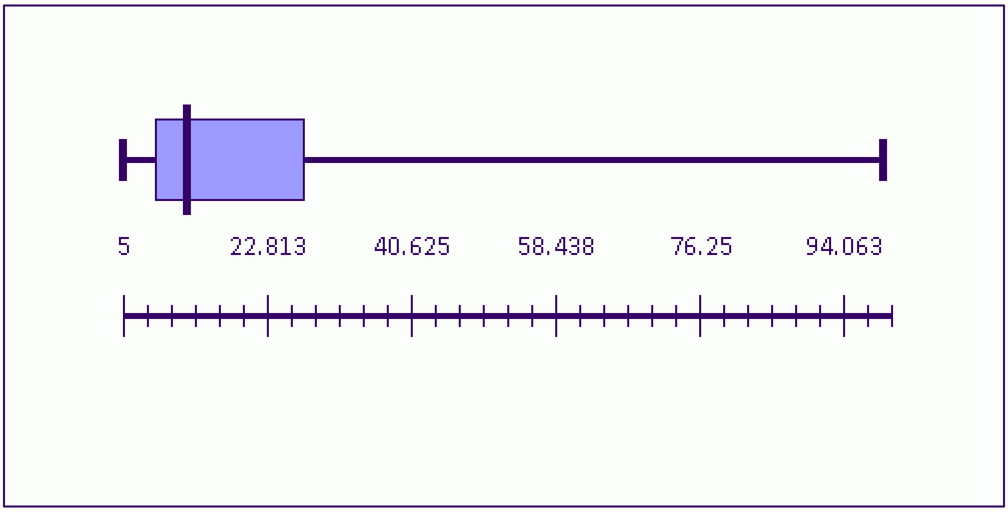
The spread of temperatures of Darwin is less than the other two cities

Perth and Hobart's temperatures are similarly distributed, but Hobart's temperatures are generally lower than Perth's

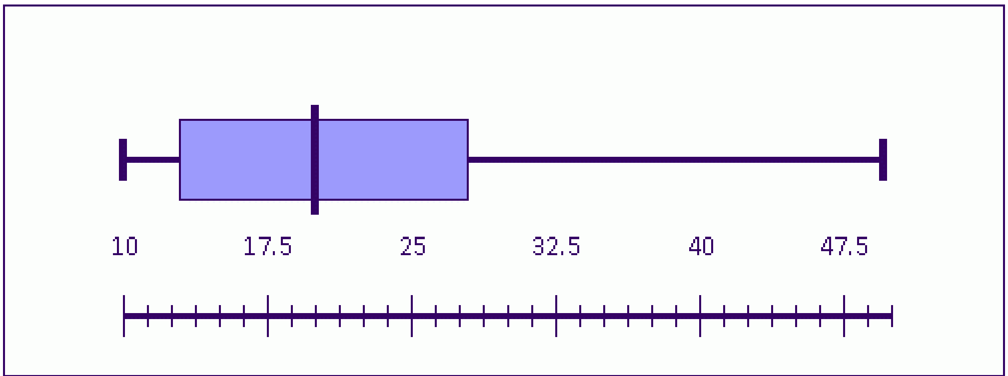
No city appears to experience great extremes from the norm, indicated by the relatively small length of the whiskers, and the location of the medians

5) Draw a box and whisker plot of the following data set

5, 9, 9, 12, 13, 22, 25, 30, 100



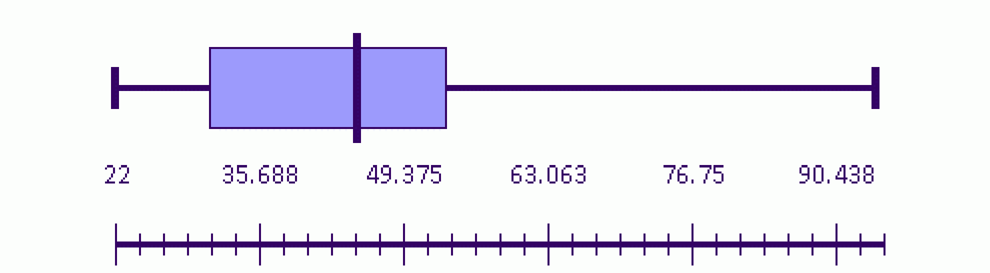
6) Draw a box and whisker plot for a set of data that has a median of 20, an inter quartile range of 15, and a range of 40



Numbers : 10,13,17,20,21,28,50
Quartile 1 = 13
Median = 20
Quartile 3 = 28

7) The following data is in stem and leaf form. Represent it as a box and whisker plot

Stem	**Leaf**
2	2 4 7 7 9
3	0 1 1 1 3 3 5 6
4	5 5 5 7 8 9
5	1 1 2 2 5
6	3 3 5 6 7
9	5



22,24,27,27,29,30,31,31,31,33,33,35,36,45,45,45,47,48,49,51,
51,52,52,55,63,63,65,66,67,95,

Quartile 1 = 31
Median = 45
Quartile 3 = 53.5

Year 10 Mathematics

Algebraic Techniques

Exercise 1

Simultaneous Equations

1) Solve the following simultaneous equations by using the guess check and improve method

a) $2x + y = 4 \; and \; x - y = 5$

$x = 0, y = 4 \; and \; y = -5$

$x = 1, y = 2 \; and \; y = -4$

As x guess increases, y values get closer

Try $x = 4, y = -4 \; and \; y = -1$

$x = 5, y = -6 \; and \; y = 0$

Further apart, so $x \; is \; either \; 2 \; or \; 3$

$x = 3, y = -2 \; and \; y = -2$

Solution is $x = 3, y = -2$

Following this format and the example in the hints section:

b) $x + y = 8 \; and \; 2x - y = 10$

$x = 6, y = 2$

c) $2x + y = 3 \; and \; 3x - 2y = 1$

$x = 1, y = 1$

d) $x - 3y = -4 \; and \; x + y = 4$

$x = 2, y = 2$

e) $x + 2y = 5 \; and \; x - 2y = 1$

$x = 3, y = 1$

f) $x + 3y = 7 \; and \; x - y = 3$

$x = 4, y = 1$

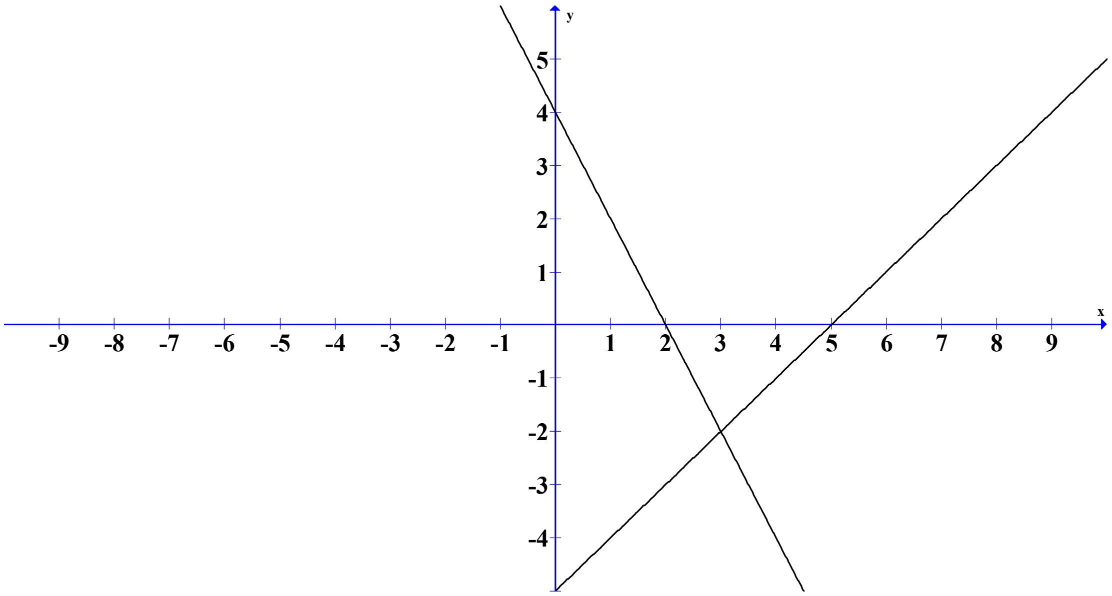
g) $2x - y = -1 \; and \; 5x + 7y = 7$

$x = 0, y = 1$

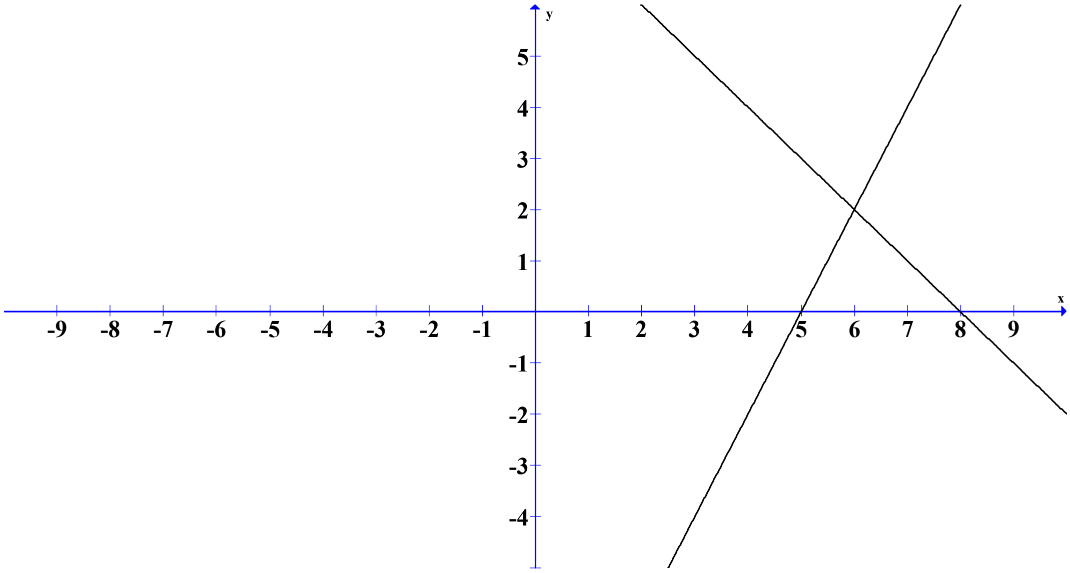
2) For each of the simultaneous equations in question 1, make a table of possible values and use it to check each of your solutions

3) Graph each pair of simultaneous equations from question 1, and use your graphs to check each of your solutions

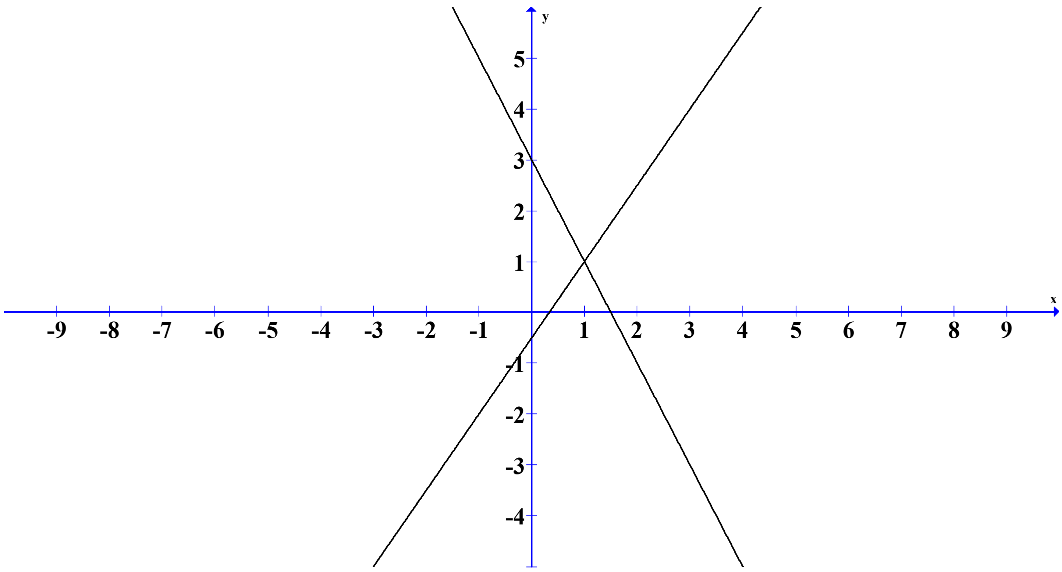
a)



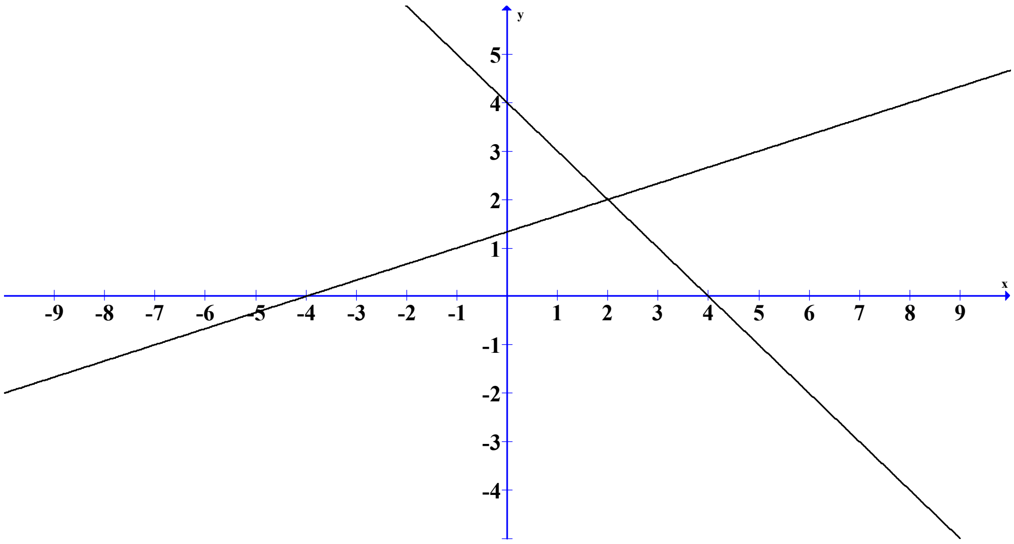
b)


y
5
4
3
2
1
-1
-2
-3
-4
x
-9 -8 -7 -6 -5 -4 -3 -2 -1 1 2 3 4 5 6 7 8 9

c)


y
5
4
3
2
1
-1
-2
-3
-4
x
-9 -8 -7 -6 -5 -4 -3 -2 -1 1 2 3 4 5 6 7 8 9

d)


y
5
4
3
2
1
-1
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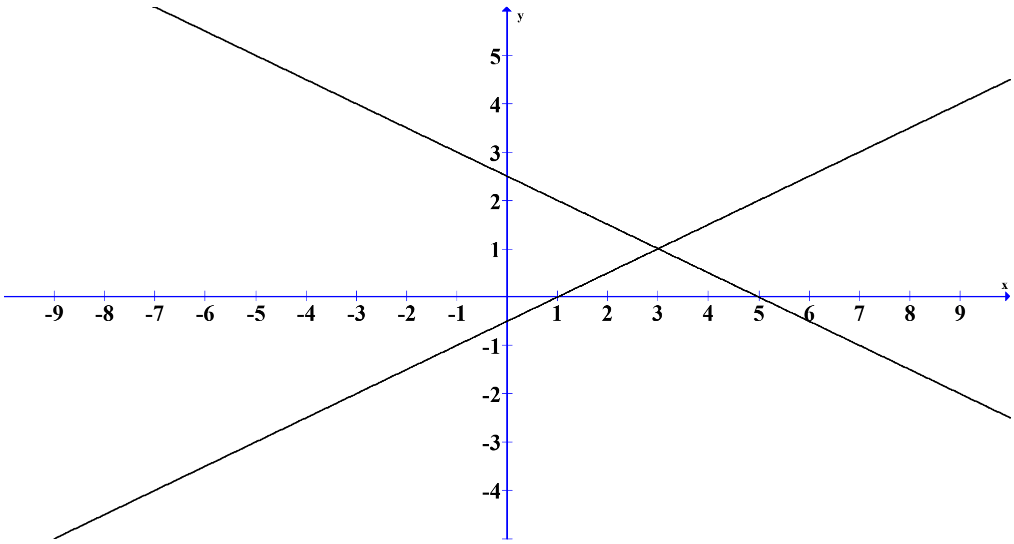
-3
-4
x
-9
-8
-7
-6
-5
-4
-3
-2
-1
1
2
3
4
5
6
7
8
9

e)


y
5
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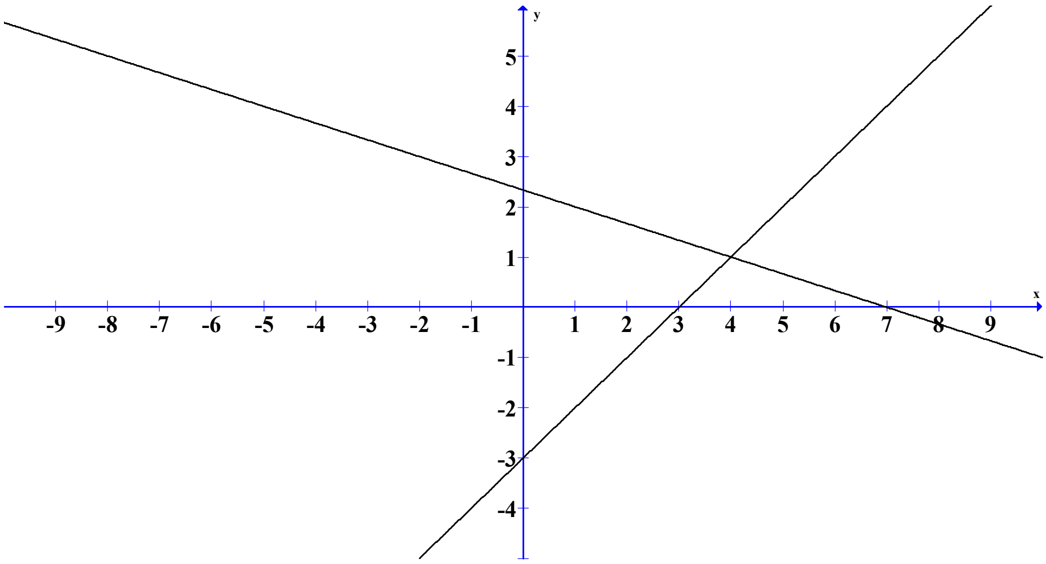
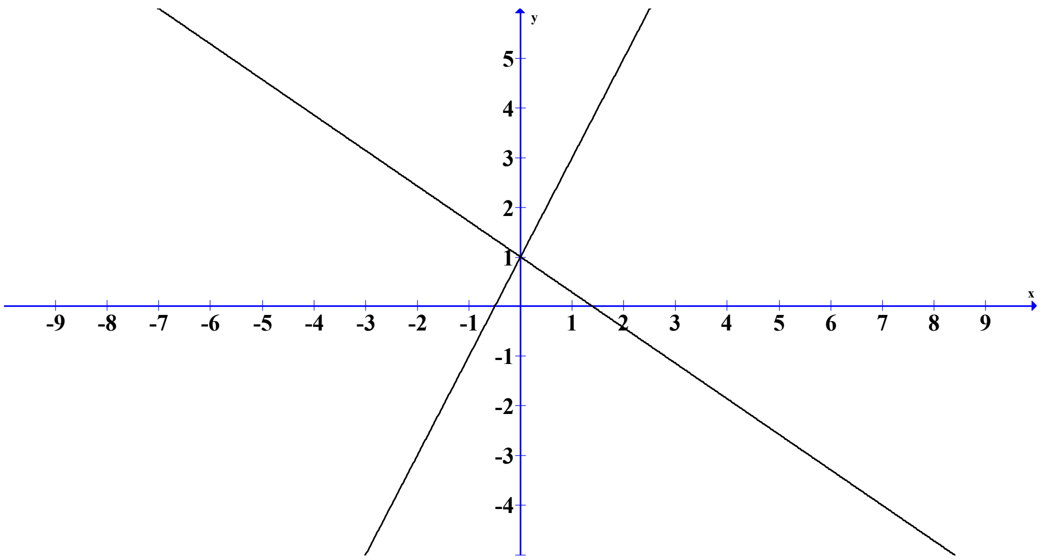
-3
-4
x
-9
-8
-7
-6
-5
-4
-3
-2
-1
1
2
3
4
5
6
7
8
9

f)



g)



4) Use an algebraic method (substitution, subtraction or addition of equations) to solve each pair of simultaneous equations from question 1.

a) Adding the equations gives

$2x + y + x - y = 4 + 5$

$3x = 9$

$x = 3, y = -2$

b) Equation 2 becomes $y = 2x - 10$

Substituting into equation 1 gives:

$x + (2x - 10) = 8$

$3x - 10 = 8$

$3x = 18$

$x = 6, y = 2$

c) Doubling equation 1 gives:

$4x + 2y = 6$

Adding the two equations gives:

$4x + 2y + 3x - 2y = 7$

$7x = 7$

$x = 1, y = 1$

d) From second equation, $y = 4 - x$

Substituting into first equation gives:

$x - 3(4 - x) = -4$

$4x - 12 = -4$

$4x = 8$

$x = 2, y = 2$

e) Subtracting equation 2 from equation 1 gives:

$$x + 2y - (x - 2y) = 4$$

$$4y = 4$$

$$y = 1, x = 3$$

f) From equation 2:

$$x = 3 + y$$

Substituting into equation 1 gives:

$$3 + y + 3y = 7$$

$$4y = 4$$

$$y = 1, x = 4$$

g) Multiplying equation 1 by 7 gives:

$$14x - 7y = -7$$

Adding the two equations gives:

$$19x = 0$$

$$x = 0, y = 1$$

5) Solve the following word problems by generating a pair of simultaneous equations and solving them by any of the methods used above. Check your solutions by substituting back into the original equations

a) The sum of two numbers is 8 and the difference is 4. Find the numbers.

Let the numbers be $x \ and \ y$

$x + y = 8 \text{ and } x - y = 4$

Adding the equations gives:

$x + y + x - y = 12$

$2x = 12$

$x = 6, y = 2$

The numbers are 6 and 2

b) The cost of two rulers and a pen is \$6.00. The difference of cost between 3 rulers and 2 pens is \$2.00. Find the cost of a ruler and a pen.

Let a be the cost of a ruler and b be the cost of a pen

$2a + b = 6$

$3a - 2b = 2$

Multiply the first equation by 2 gives:

$4a + 2b = 12$

Adding the equations gives:

$4a + 2b + 3a - 2b = 14$

$7a = 14$

$a = 2, b = 2$

A pen and a ruler each costs \$2

c) If I double two numbers and then add them together I get a total of 8. If I multiply the first number by 3, then subtract the second number I get 4. What are the two numbers?

Let $p \text{ and } q$ be the two numbers

$2p + 2q = 8$

$3p - q = 4$

Multiply the second equation by 2 gives:

$6p - 2q = 8$

Adding the equations gives:

$2p + 2q + 6p - 2q = 16$

$8p = 16$

$p = 2, q = 2$

Both numbers are 2

d) The average of two numbers is 9. The difference is 6. Find the numbers

Let $x\ and\ y$ be the two numbers

$\frac{x + y}{2} = 9$

$x - y = 6$

Doubling the first equation gives:

$x + y = 18$

Adding the equations gives:

$x + y + x - y = 24$

$2x = 24$

$x = 12, y = 6$

e) There are two angles on a straight line. One angle is 45 more than twice the other. Find the size of each angle.

Let size of the two angles be $t \text{ and } w$

$t + w = 180$ (a straight line is 180°)

$t - 2w = 45$

Subtract second equation from first gives:

$t + w - (t - 2w) = 135$

$3w = 135$

$w = 45$

Angles are 45° and 135°

f) The length of a rectangle is twice its width. The perimeter is 42. Find its dimensions

Let length of rectangle be z, and width be r

$z = 2r$

$z + z + r + r = 42$:
$2z + 2r = 42$

Substitute equation 1 into equation 2

$2z + z = 42$

$3z = 42$

$z = 14, r = 7$

6) One thousand tickets to a show were sold. Adult tickets cost \$8.50 and children's were \$4.50. \$7300 was raised from the sale of the tickets. How many of each type were sold?

Let k be the amount of adult tickets sold, and f be the amount of child tickets sold

$k + f = 1000$

$8.5k + 4.5f = 7300$

From equation 1:

$k = 1000 - f$

Substitute into equation 2 gives:

$8.5(1000 - f) + 4.5f = 7300$

$8500 - 8.5f + 4.5f = 7300$

$4f = 800$

$f = 200, k = 800$

There were 800 adult and 200 child tickets sold

7) Mrs. Brown. invested \$30,000; part at 5%, and part at 8%. The total interest on the investment was \$2,100. How much did she invest at each rate?

Let r be the amount @ 5%, and g be the amount at 8%

$a + g = 30000$

$\frac{5}{100}a + \frac{8}{100}g = 2100$

From first equation:

$a = 30000 - g$

Substitute into second equation gives:

$\frac{5}{100}(30000 - g) + \frac{8}{100}g = 2100$

$1500 - \frac{5}{100}g + \frac{8}{100}g = 2100$

$\frac{3}{100}g = 600$

$g = 20000, a = 10000$

She invested \$1000 @ 5% and \$20000 @8%

8) Tyler is catering a banquet for 250 people. Each person will be served either a chicken dish that costs \$5 each or a beef dish that costs \$7 each. Tyler spent \$1500. How many dishes of each type did Tyler serve?

Let m be the number of chicken dishes and y be the number of beef dishes

$m + y = 250$

$5m + 7y = 1500$

Multiply the first equation by 5 gives:

$5m + 5y = 1250$

Subtract second equation gives:

$5m + 7y - 5m - 5y = 1500 - 1250$

$2y = 250$

$y = 125, m = 125$

He serves 125 of each dish

9) Your teacher is giving you a test worth 100 points containing 40 questions. There are two-point and four-point questions on the test. How many of each type of question are on the test?

Let number of four point questions be p, and the number of two point questions be w

$p + w = 40$

$4p + 2w = 100$

From first equation:

$w = 40 - p$

Substitute into second equation gives:

$4p + 2(40 - p) = 100$

$4p + 80 - 2p = 100$

$2p = 20$

$p = 10, w = 30$

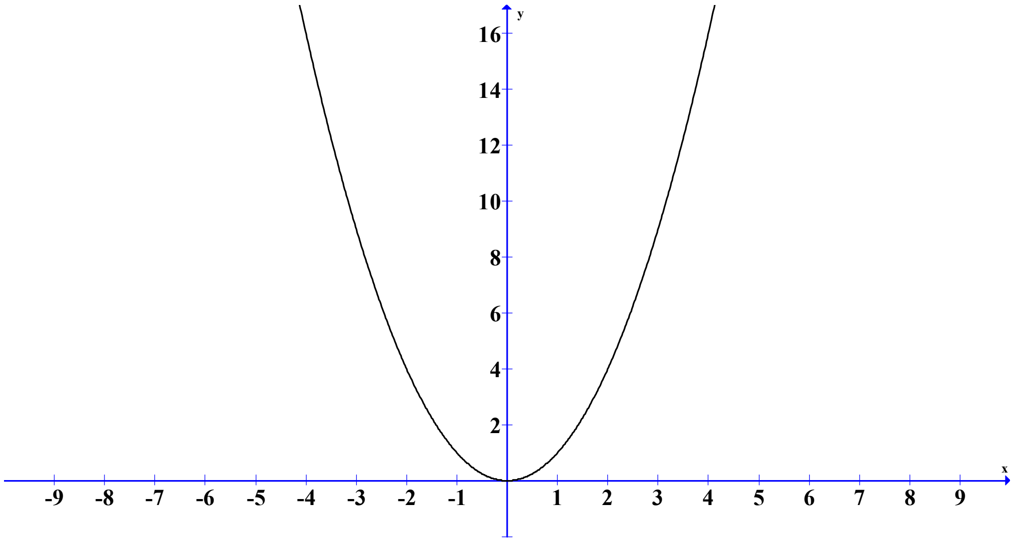
There are 10 four point questions and 30 two point questions

Exercise 2

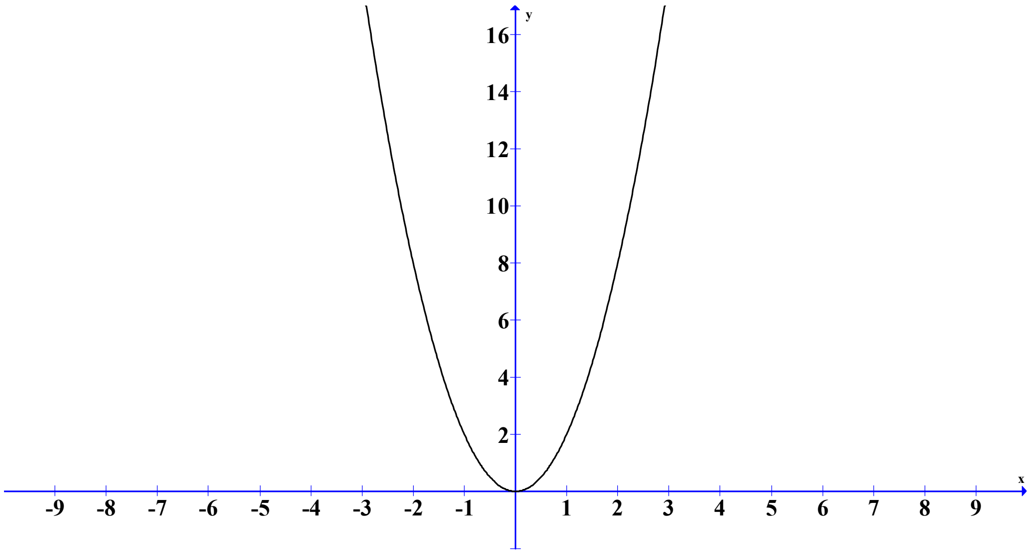
Parabolas & Hyperbolae

1) Graph each quadratic equation below, by first making a table of values

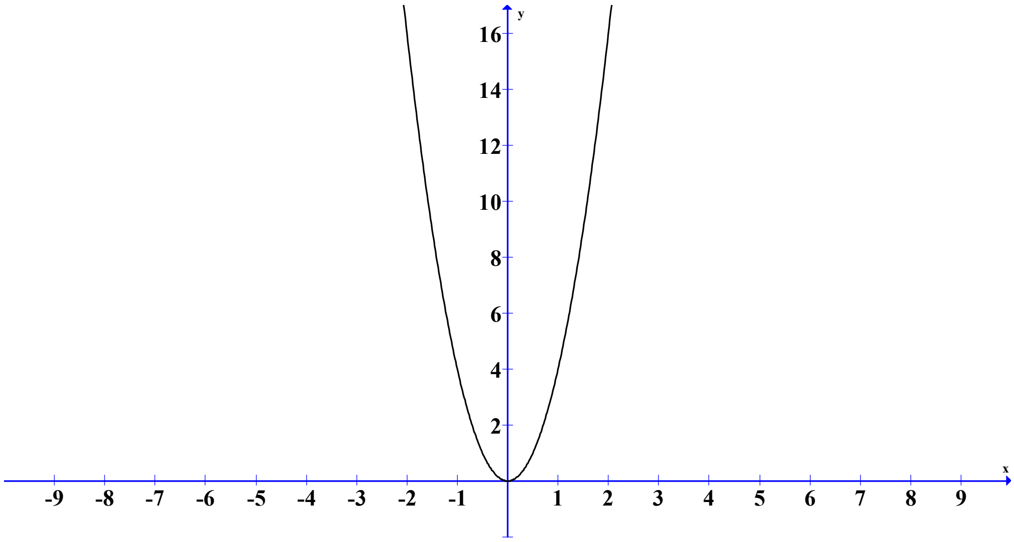
a) $y = x^2$



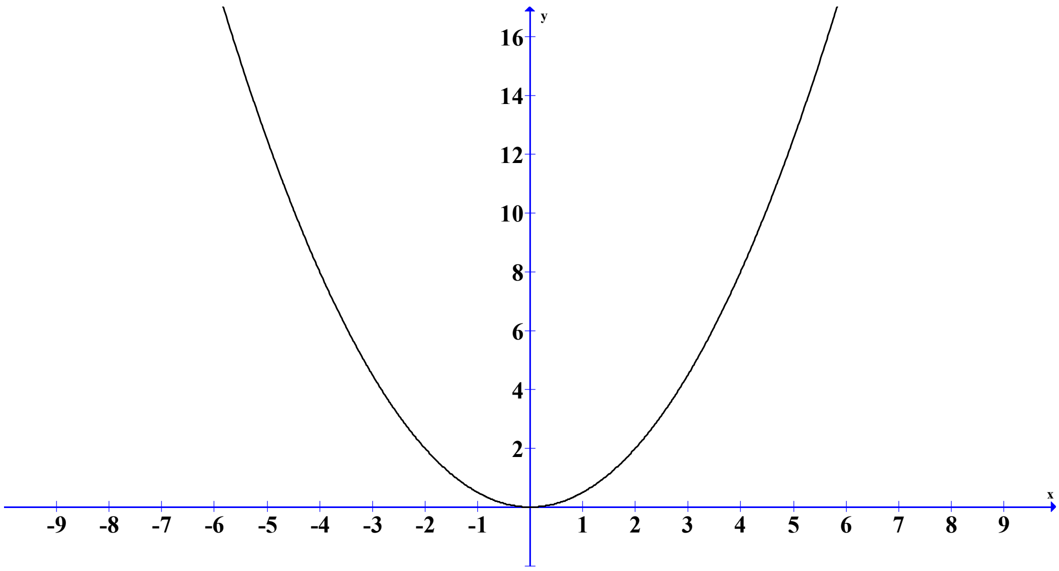
b) $y = 2x^2$



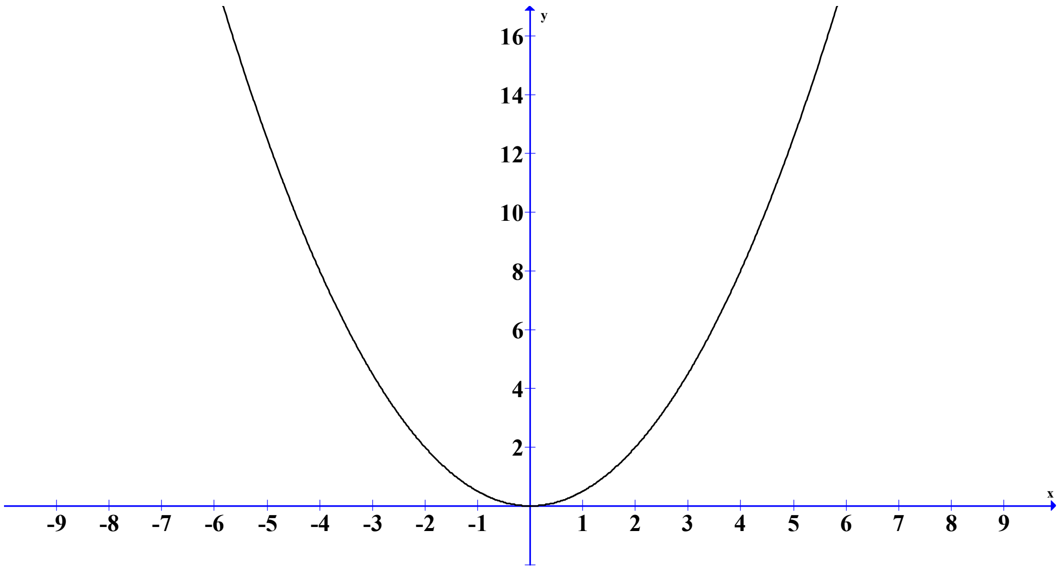
c) $y = 4x^2$



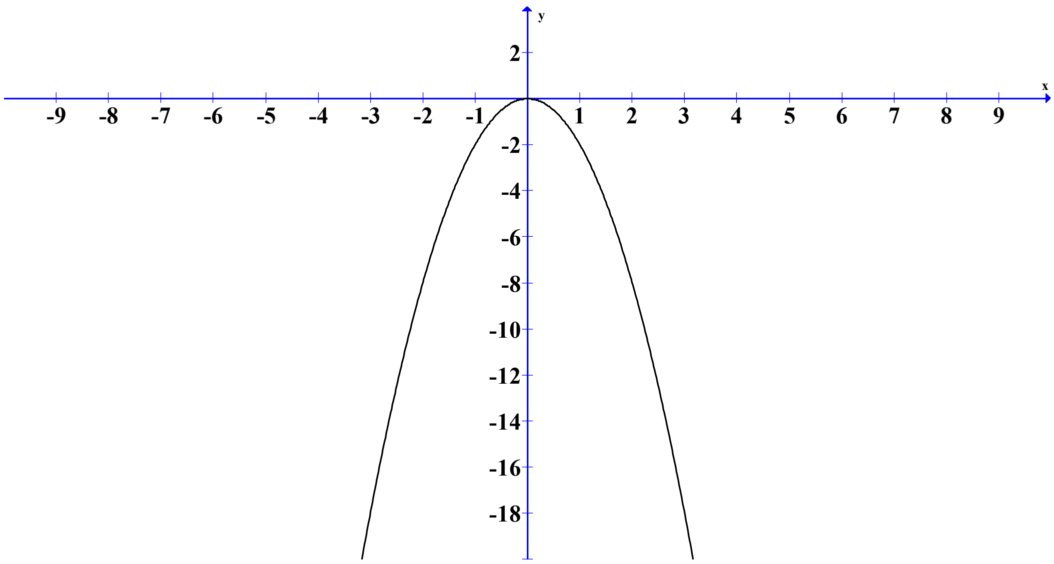
d) $y = 0.5x^2$



e) $y = \frac{1}{2}x^2$



f) $y = -2x^2$

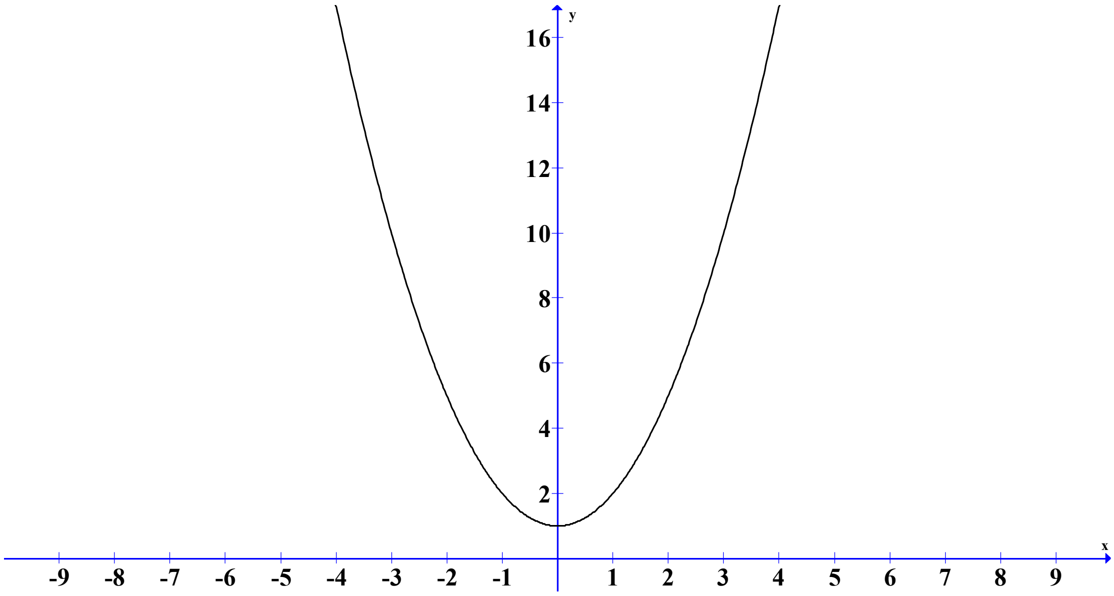


2) From your answers to question 1, comment on the effect of the value of the constant a on the graph of equations of the form $y = ax^2$

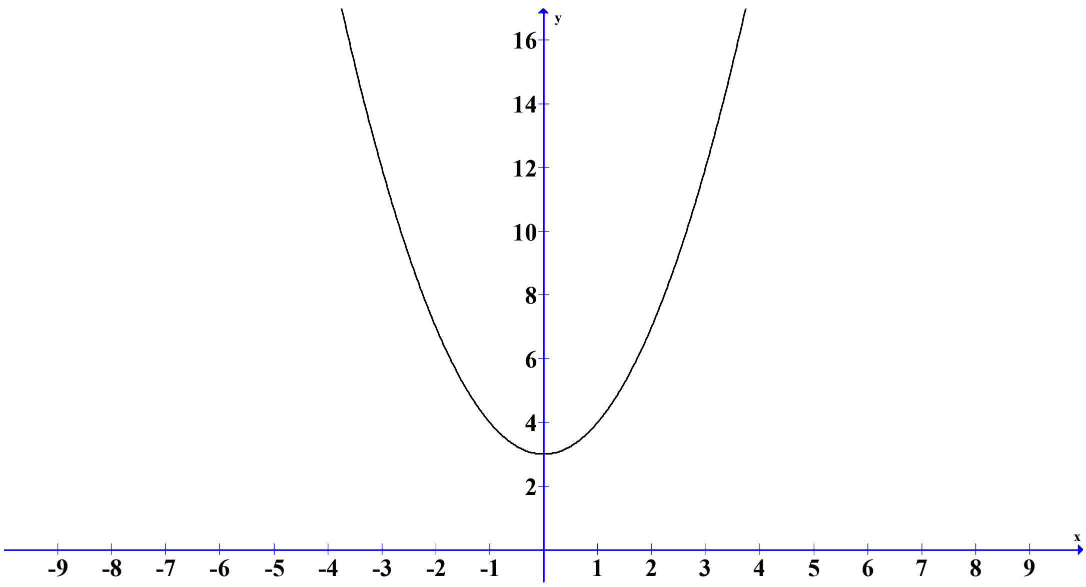
The higher the value of a, the faster y increases making for a sharper curve. A negative value of a inverts the graph

3) Graph each quadratic equation below, by first making a table of values

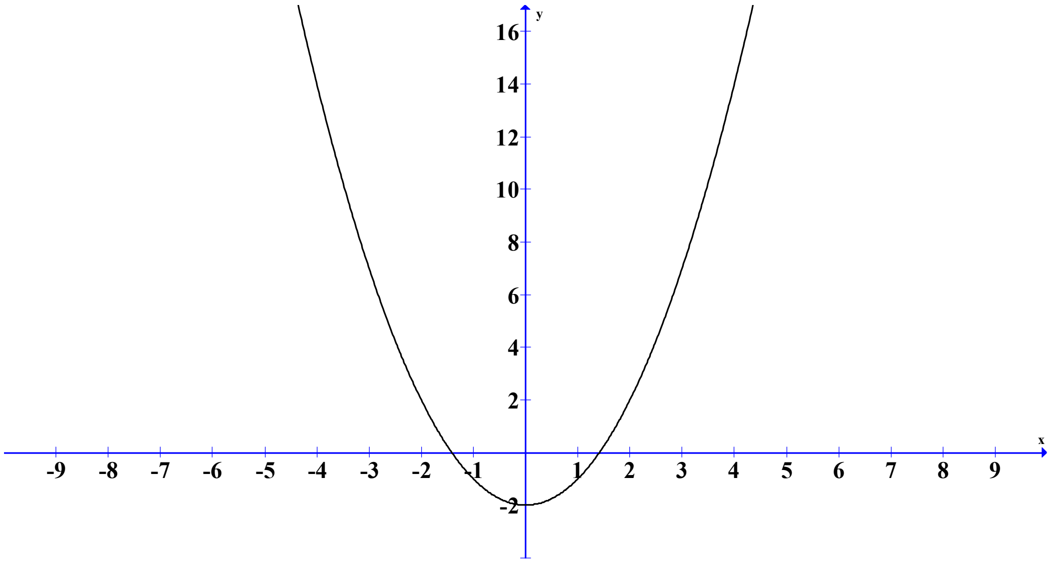
a) $y = x^2 + 1$



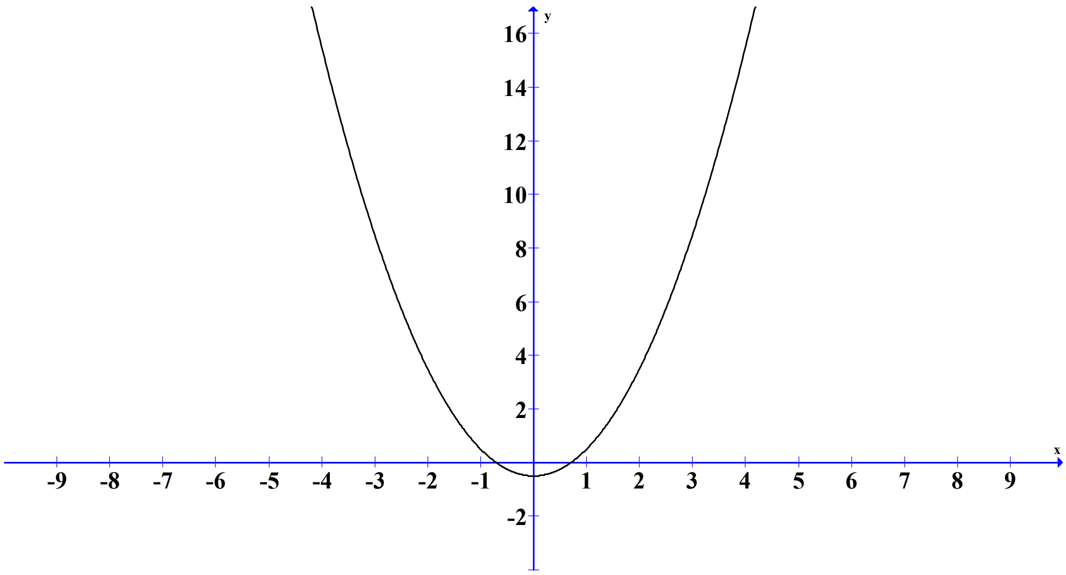
b) $y = x^2 + 3$



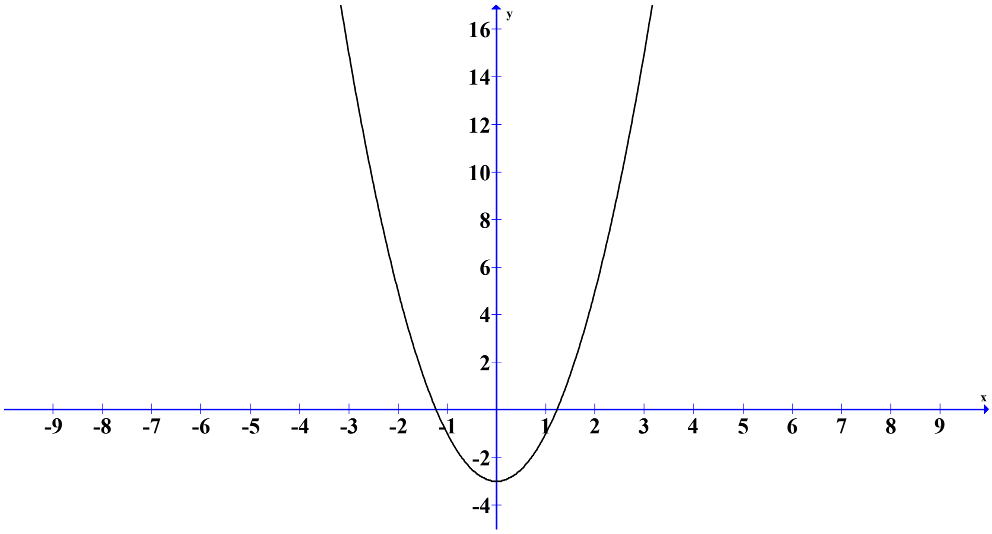
c) $y = x^2 - 2$



d) $y = x^2 - \frac{1}{2}$



e) $y = 2x^2 - 3$

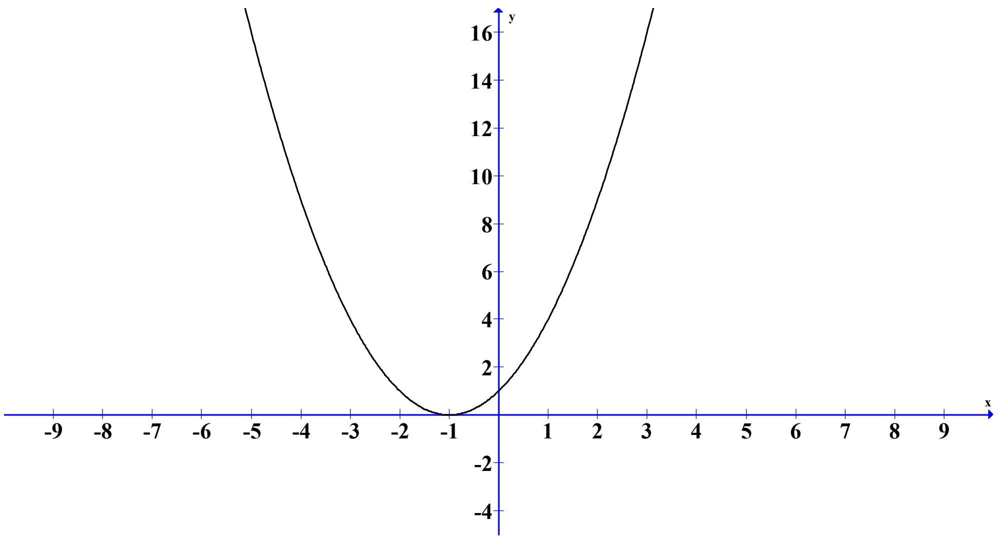


4) From your answers to question 3, comment on the effect of the value of the constant c on the graph of equations of the form $y = ax^2 + c$

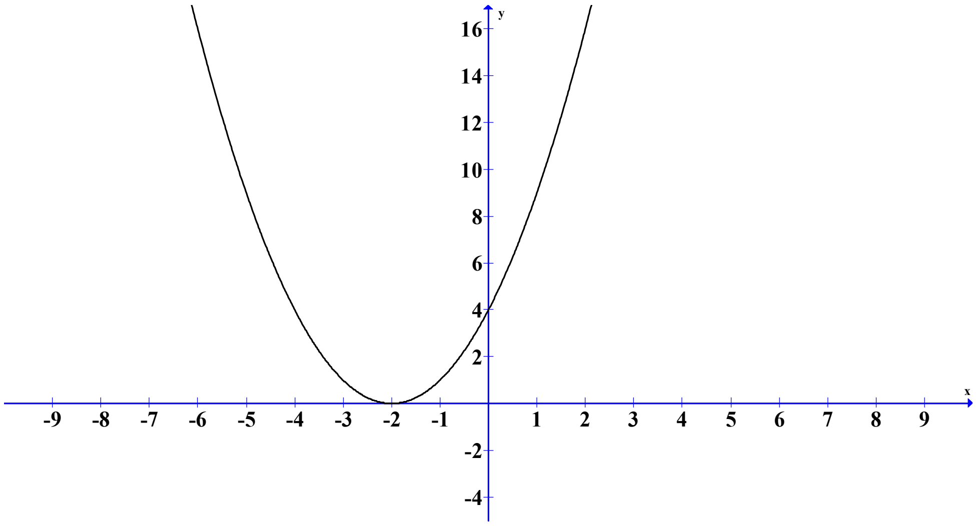
The curve shifts up and/or down the y axis c units away from the origin

5) Graph the following

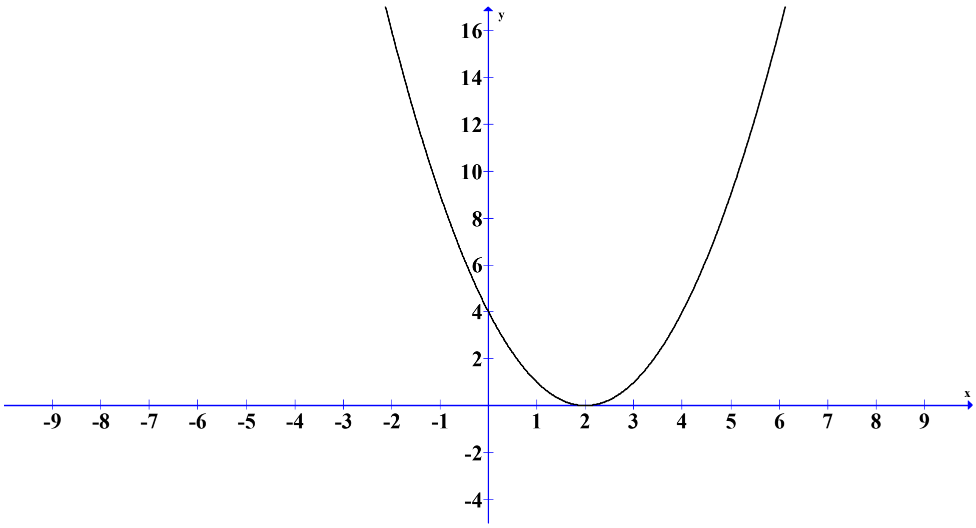
a) $y = (x + 1)^2$



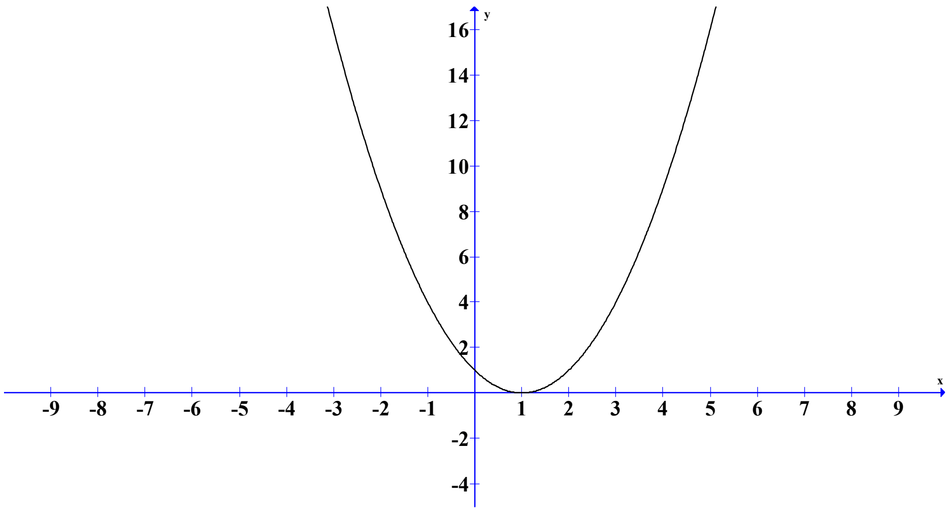
b) $y = (x+2)^2$



c) $y = (x-2)^2$



d) $y = (x-1)^2$

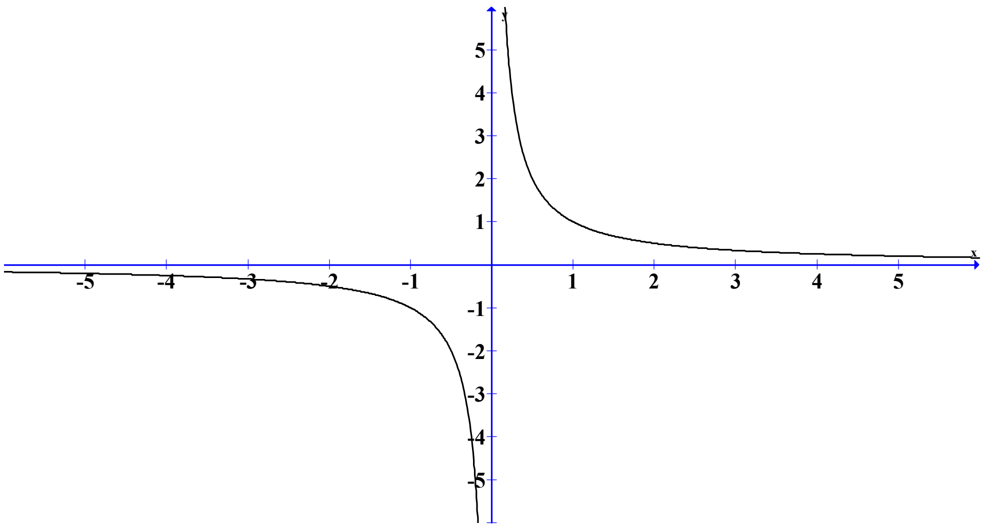


6) Comment on the effect of the value of the constant a on the graph of equations of the form $y = (x + a)^2$

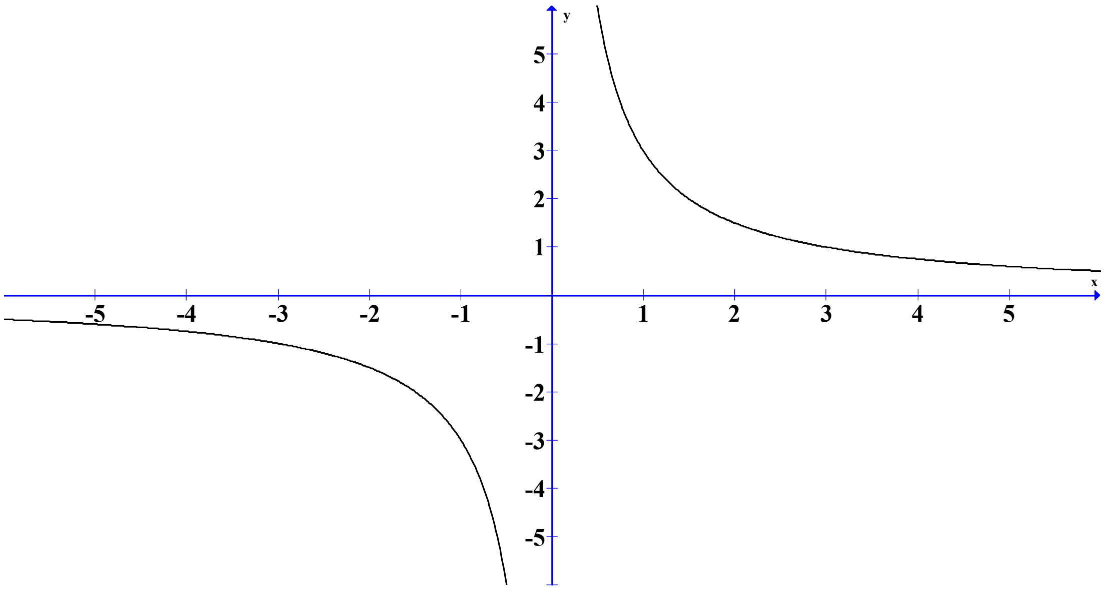
The curve is shifted a units to the right (if a is negative) or left (if a is positive) by a units

7) Graph each parabolic equation below, by first making a table of values

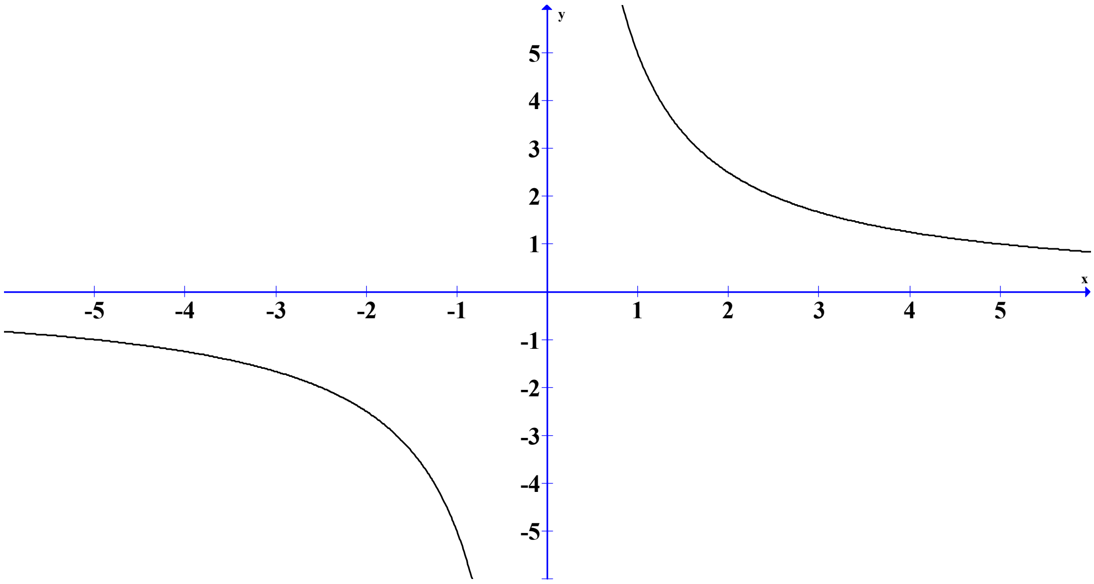
a) $y = \frac{1}{x}$



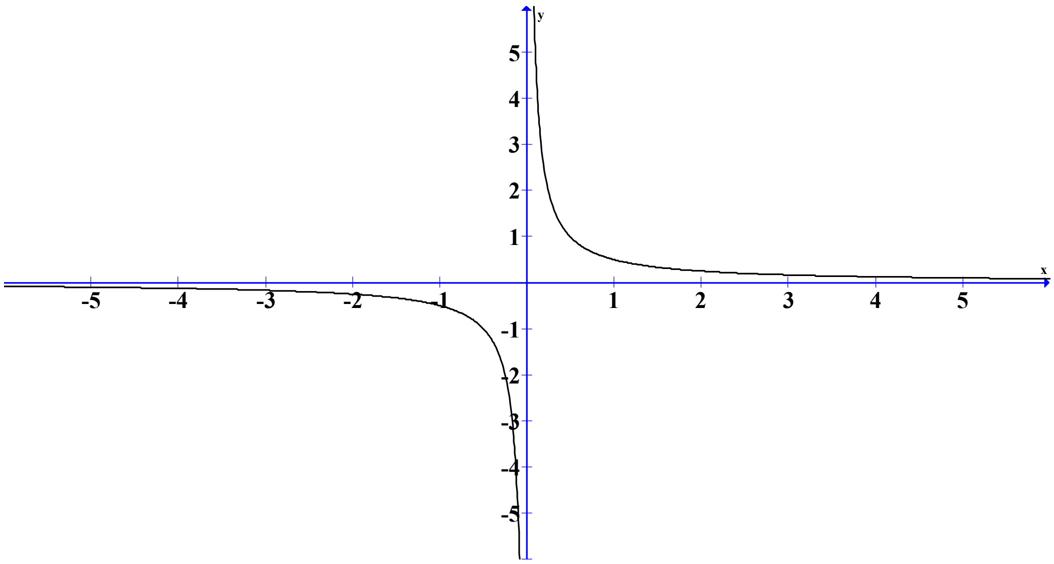
b) $y = \frac{3}{x}$



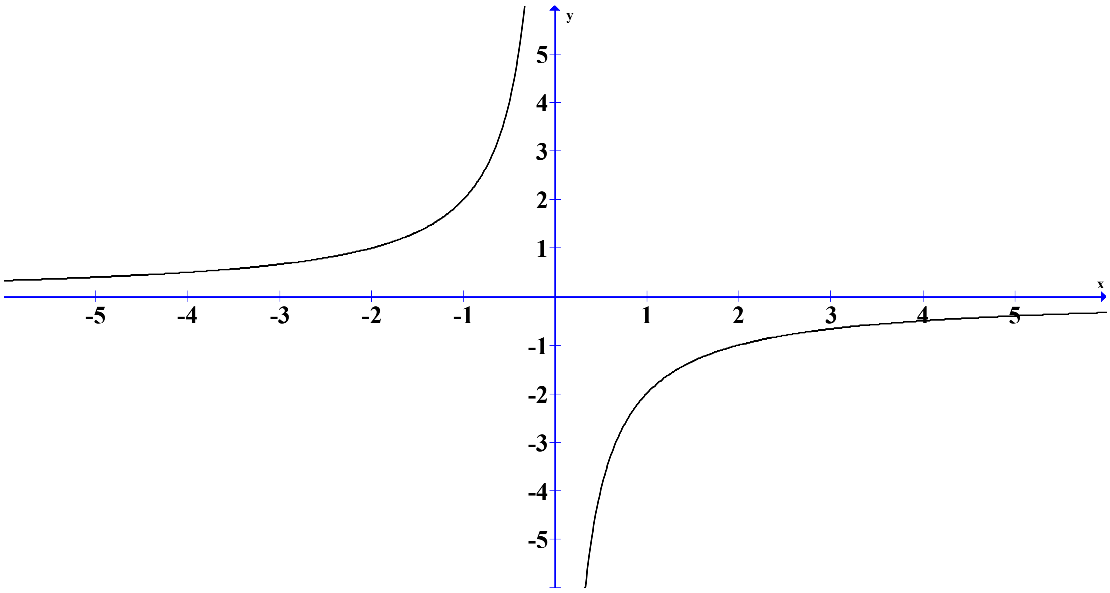
c) $y = \frac{5}{x}$



d) $y = \frac{1/2}{x}$



e) $y = \frac{-2}{x}$



8) From your answers to question 5, comment on the effect of the value of the constant k on the graph of equations of the form $y = \frac{k}{x}$

As the value of k increases, the two parts of the graph move further from the origin. A negative value of k rotates the curves into the next quadrants

9) Identify which of the following equations produce lines, parabolas or hyperbolae when graphed

a) $y = \frac{2}{x} + 4$

Parabola

b) $y = 3x - 6$

Line

c) $y = \frac{1/3}{x}$

Parabola

d) $y = \frac{1}{2}x^2 + 1$

Hyperbola

e) $y = \frac{1}{2}x$

Line

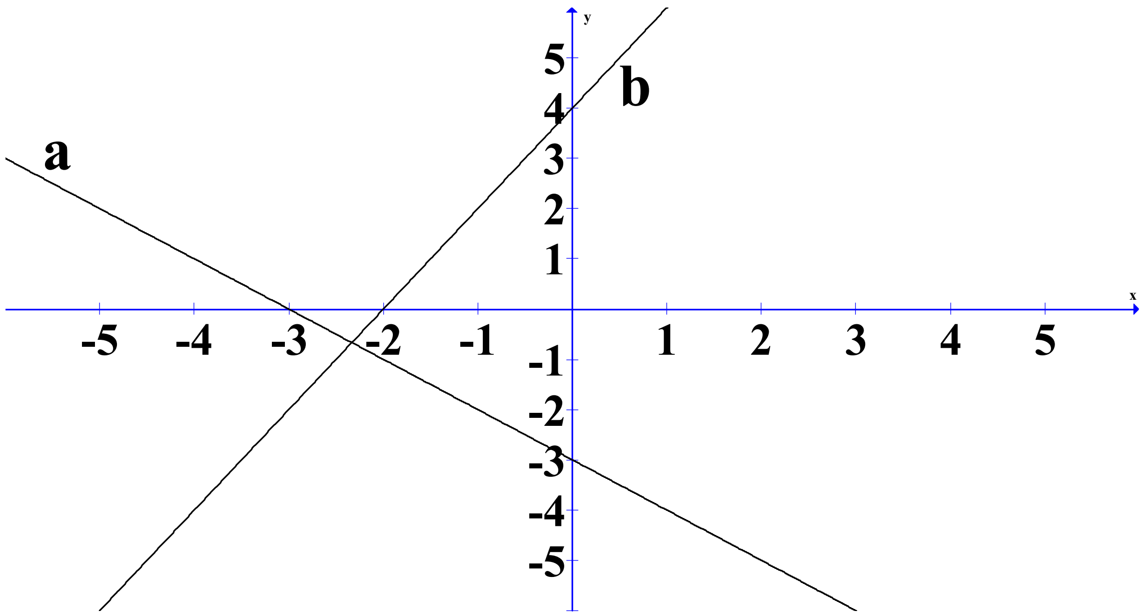
f) $y = -\frac{1}{2x}+2$

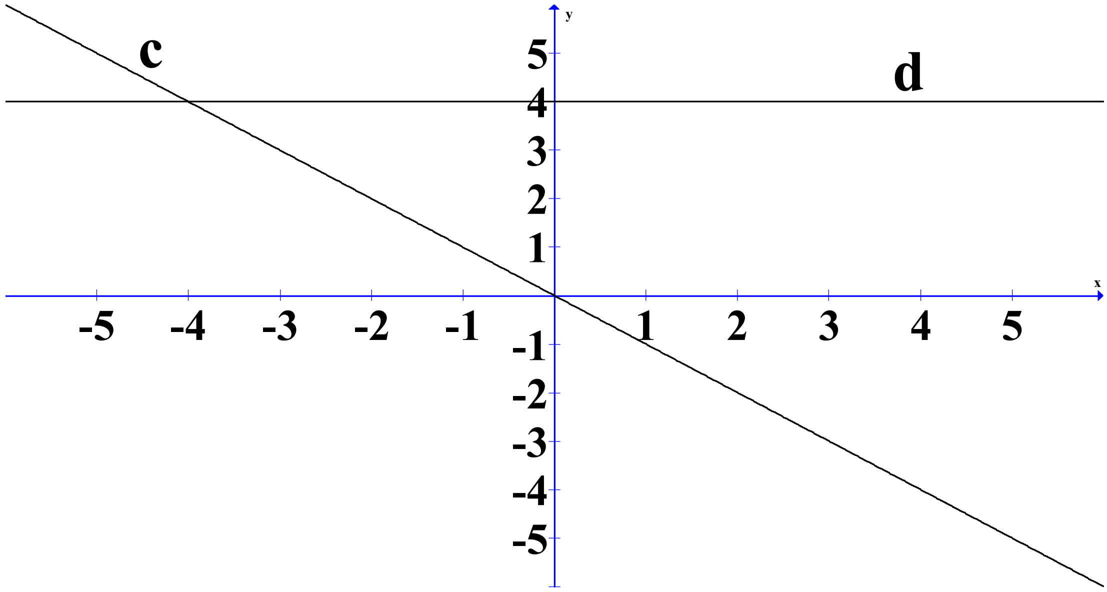
Parabola

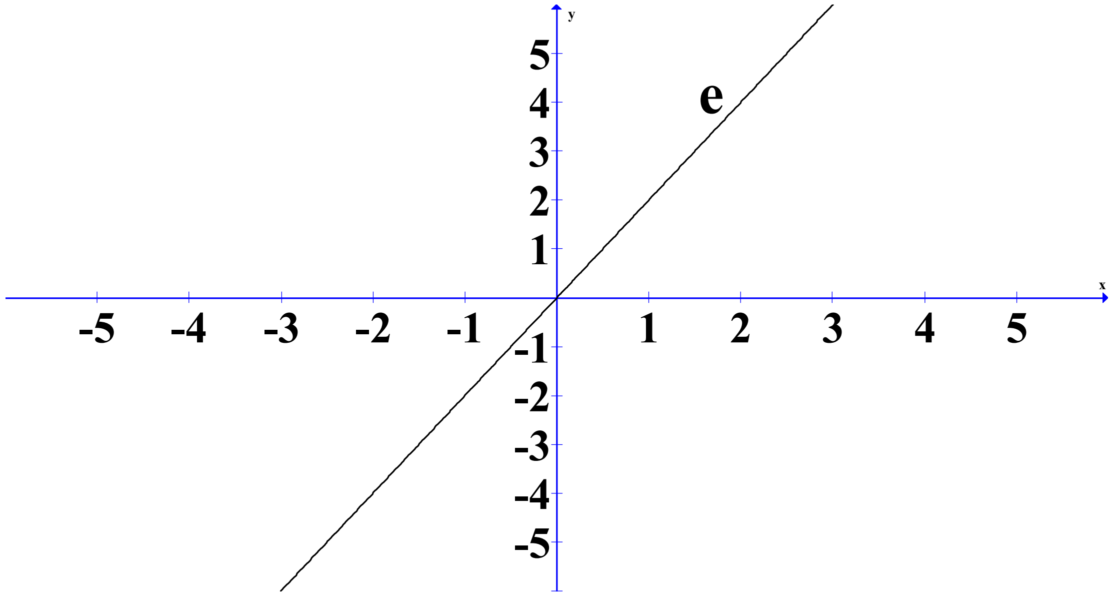
g) $y = -2 - \frac{1}{4}x^2$

Hyperbola

10) Match each of the following graphs to its correct equation

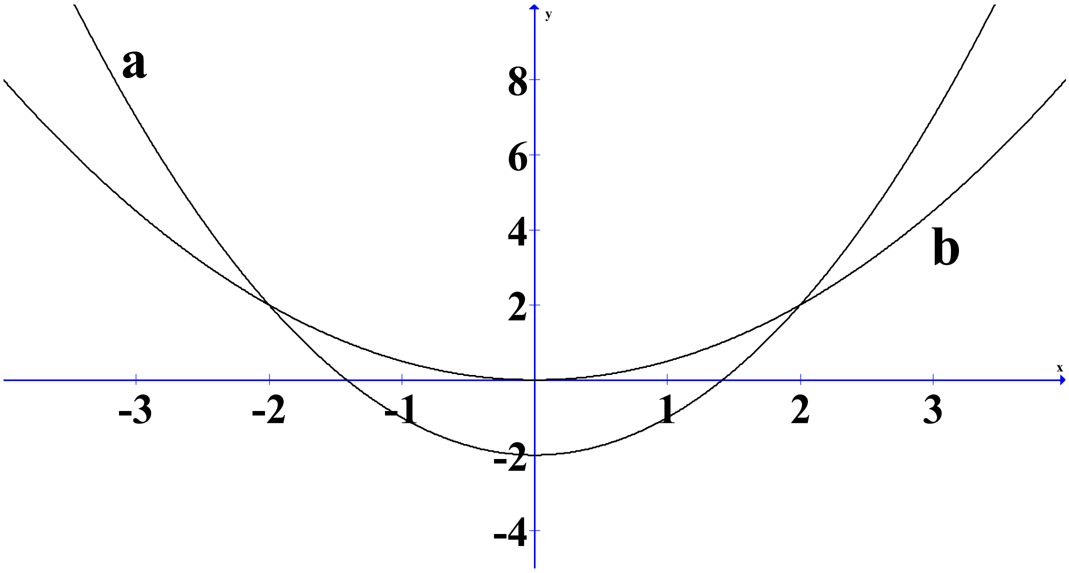


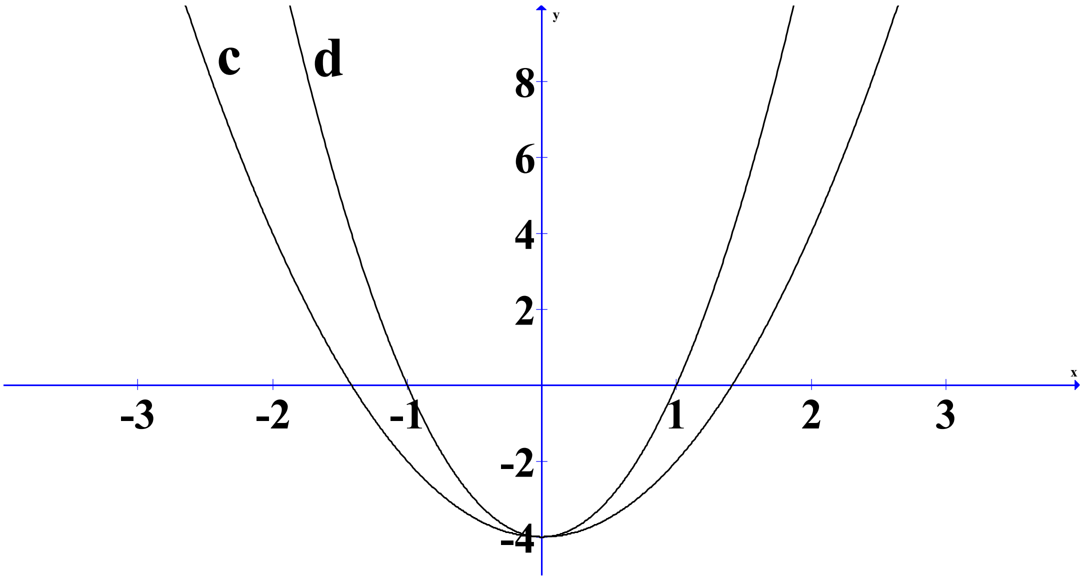


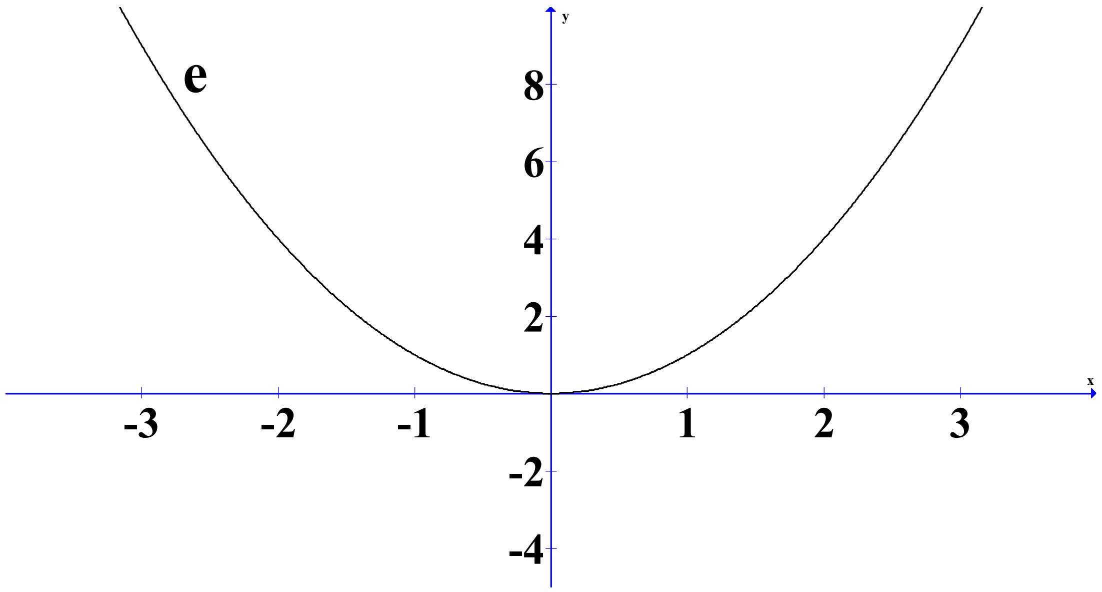


- $y = -x$ (c)
- $y = 2x + 4$ (b)
- $y = 4$ (d)
- $y = -x - 3$ (a)
- $y = 2x$ (e)

11) Match each of the following graphs to its correct equation

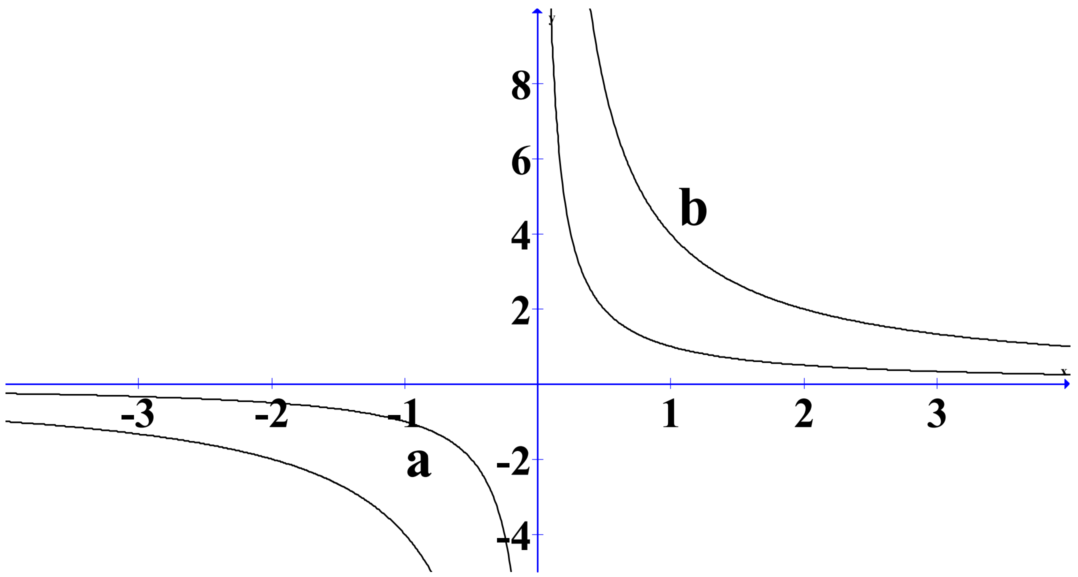


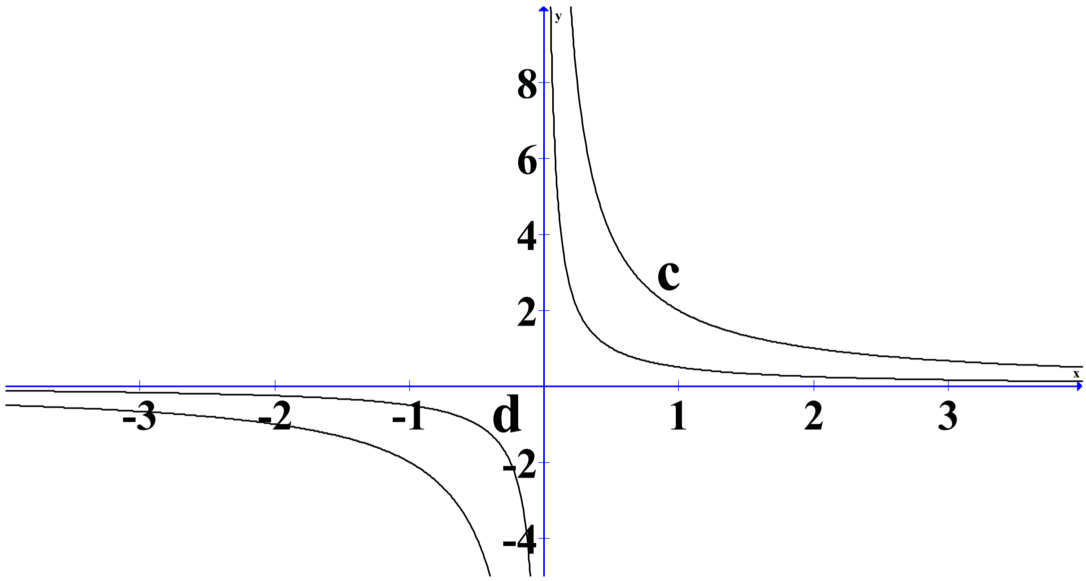




- $y = 4x^2 - 4$ (d)
- $y = \frac{1}{2}x^2$ (b)
- $y = x^2$ (e)
- $y = 2x^2 - 4$ (c)
- $y = x^2 - 2$ (a)

12) Match each of the following graphs to its correct equation

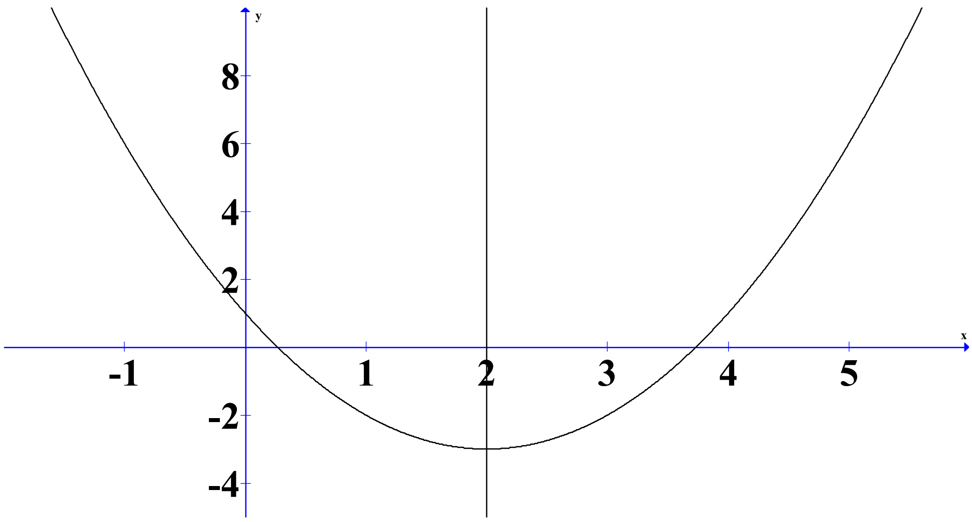




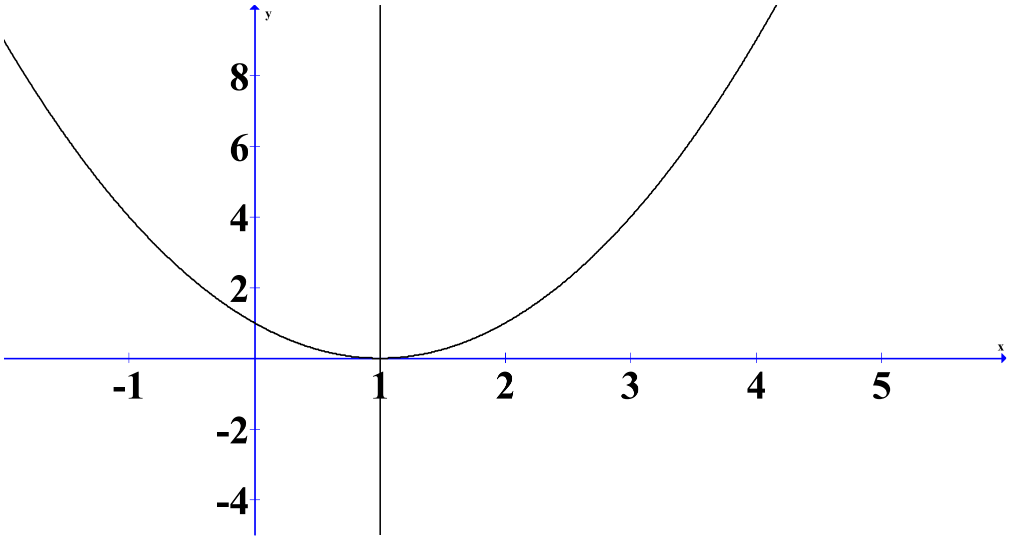
- $y = \frac{1/2}{x}$ (d)
- $y = \frac{4}{x}$ (b)
- $y = \frac{1}{x}$ (a)
- $y = \frac{2}{x}$ (c)

13) Graph each of the following and identify the line of symmetry

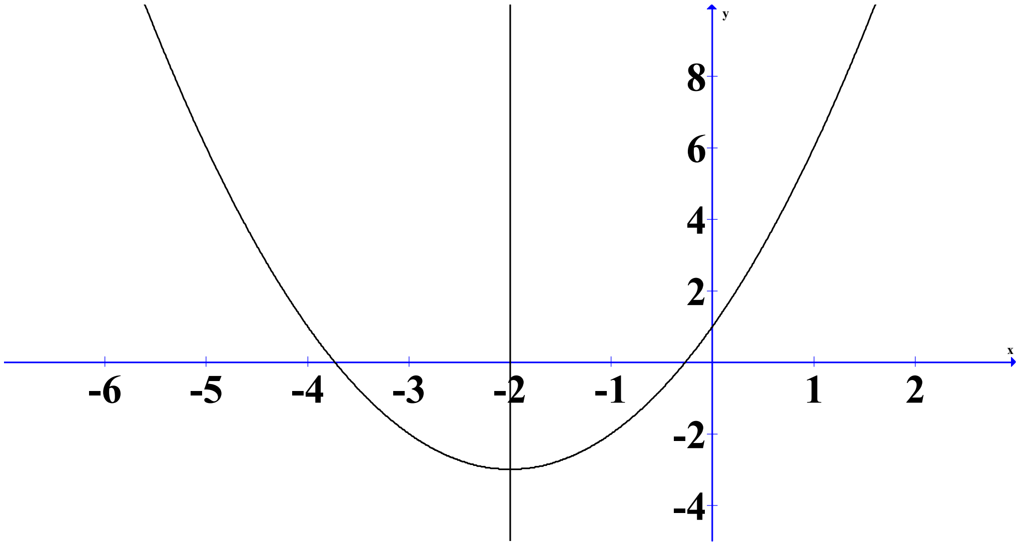
a) $y = x^2 - 4x + 1$



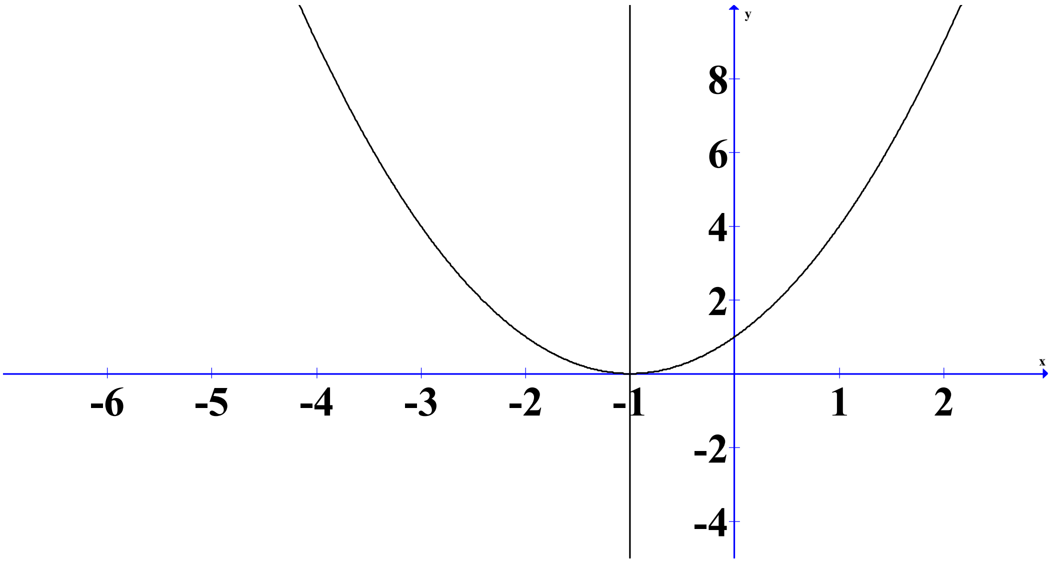
b) $y = x^2 - 2x + 1$



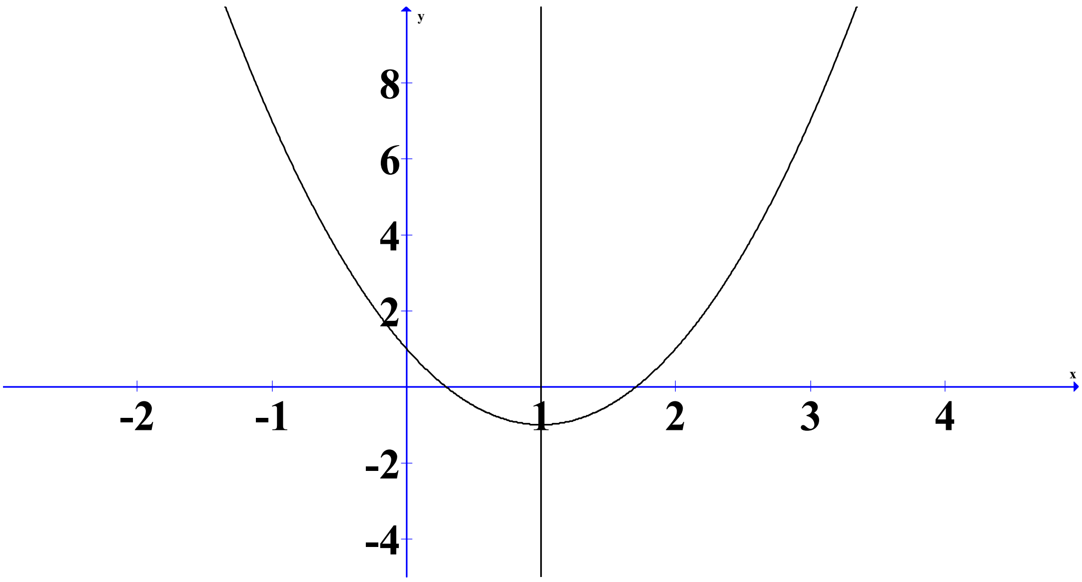
c) $y = x^2 + 4x + 1$



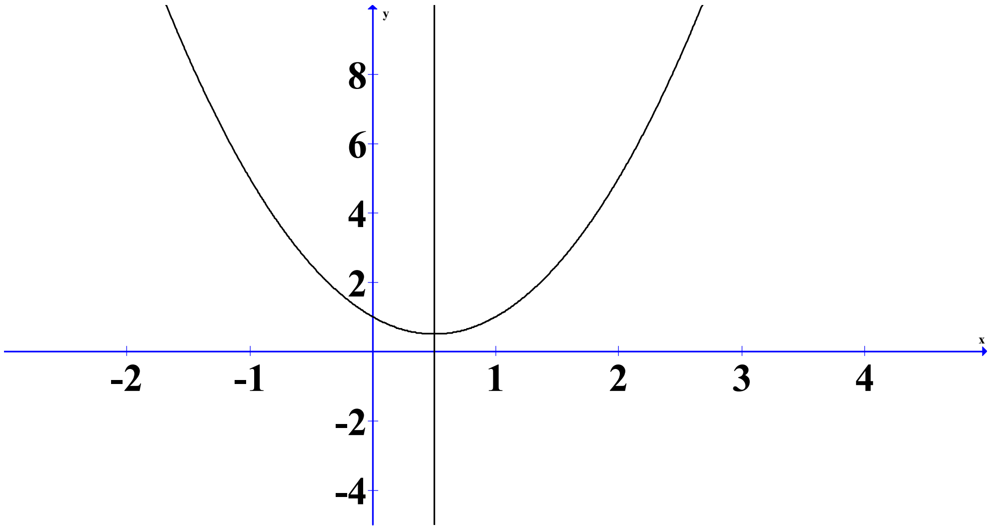
d) $y = x^2 + 2x + 1$



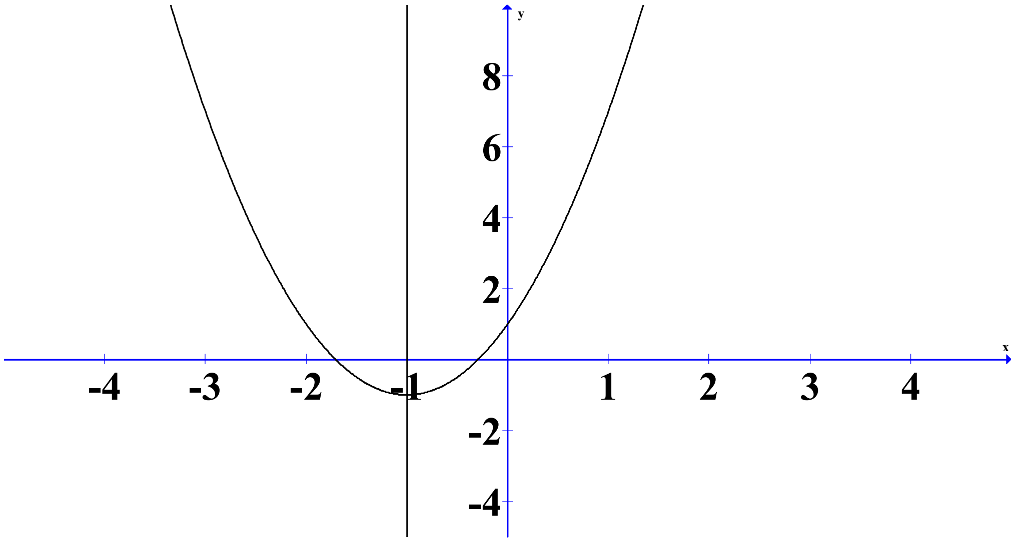
e) $y = 2x^2 - 4x + 1$



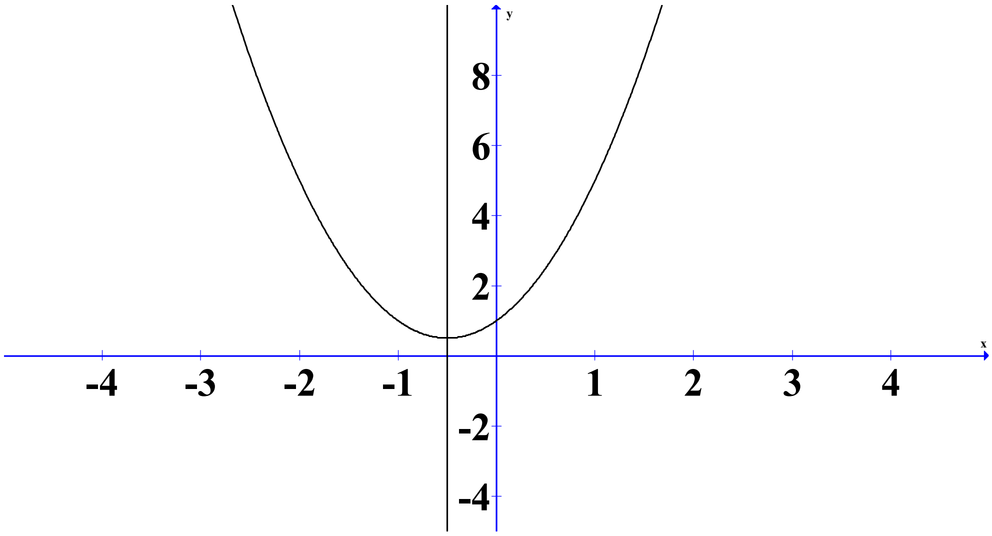
f) $y = 2x^2 - 2x + 1$



g) $y = 2x^2 + 4x + 1$



h) $y = 2x^2 + 2x + 1$



14) From your answers to question 13, or otherwise, comment on the relationship between the equation of the line of symmetry and the equation of the parabola of the form $y = ax^2 + bx$ +c

The equation of the line of symmetry is $y = -\frac{b}{2a}$

15) From your graphs in question 13, determine the co-ordinate of the turning point of the parabola and relate the co-ordinate to your answer to question 14

The x co-ordinate is $x = -\frac{b}{2a}$, and the y co-ordinate is the solution to the equation when the value is substituted into it

For example, for the equation $y = 2x^2 - 4x + 1$, the x co-ordinate is $-\frac{-4}{2\times 2} = 1$
The y co-ordinate is found by substituting $x = 1$ into the equation.
$y = 2(1^2) - 4(1) + 1 = -1$

16) Complete the square of the equations in question 13, and determine how the equations of the form $y = (x + a)^2 + c$ relate to the co-ordinates of the turning pint of the parabola

a) $(x - 2)^2 - 3$, turning point is $(2, -3)$

b) $(x - 1)^2$, turning point is $(1\ ,0)$

c) $(x+2)^2 - 3$, turning point is $(-2,-3)$

d) $(x+1)^2$, turning point is $(-1,0)$

e) $2(x-1)^2 - 1$, turning point is $(1,-1)$

f) $2\left(x-\frac{1}{2}\right)^2 + \frac{1}{2}$, turning point is $\left(\frac{1}{2},\frac{1}{2}\right)$

g) $2(x+1)^2 - 1$, turning point is $(-1,-1)$

h) $2\left(x+\frac{1}{2}\right)^2 + \frac{1}{2}$, turning point is $\left(-\frac{1}{2},\frac{1}{2}\right)$

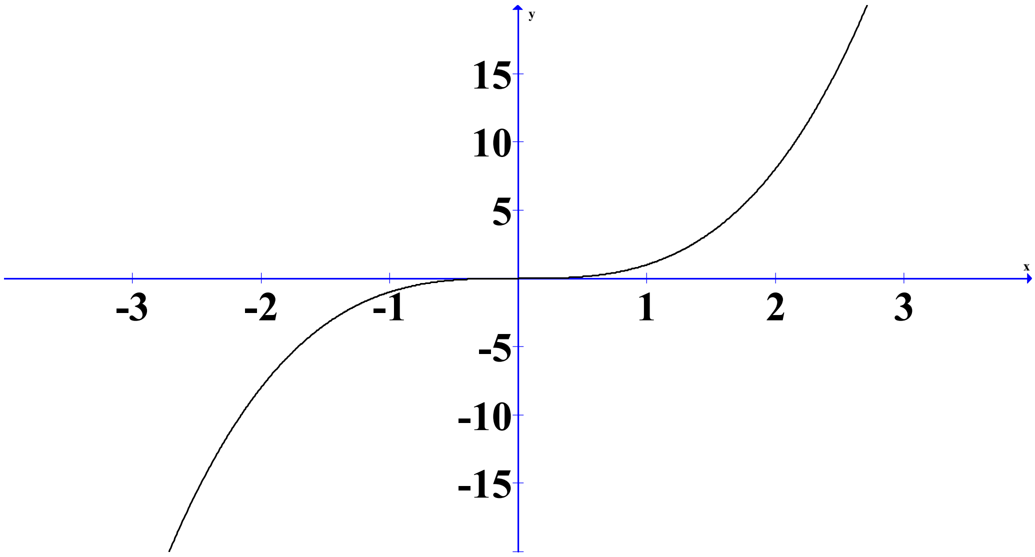
When a quadratic equation is put into the completed square form: $y = (x+h)^2 + k$, the turning point has the co-ordinates $(-h,k)$

Exercise 3

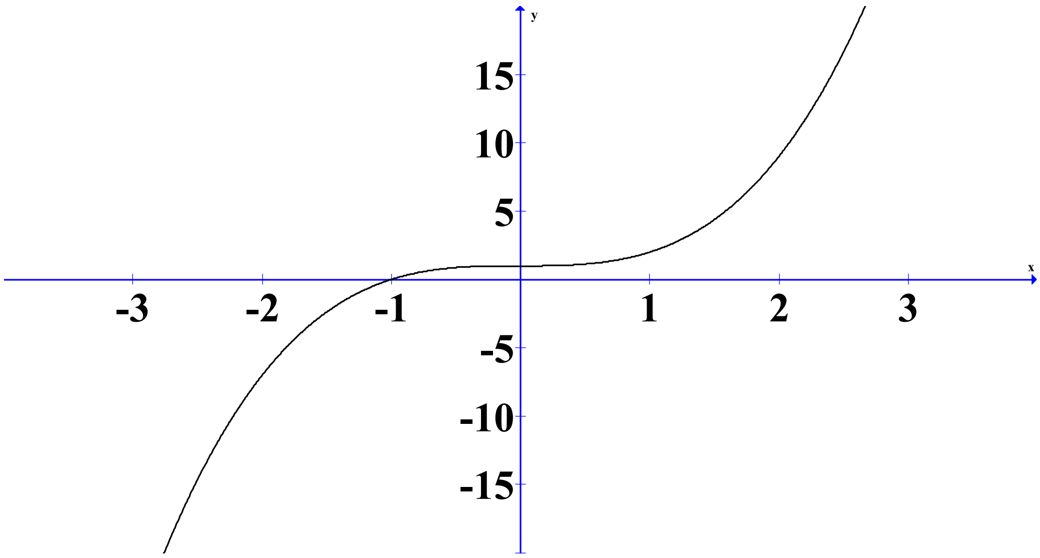
Cubics, Exponentials & Circles

1) Graph the following equations

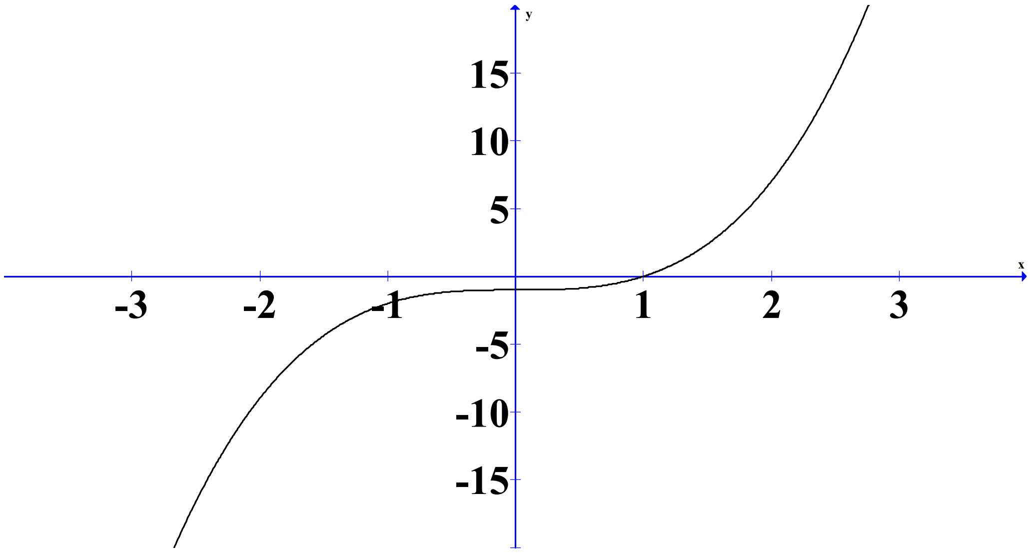
a) $y = x^3$



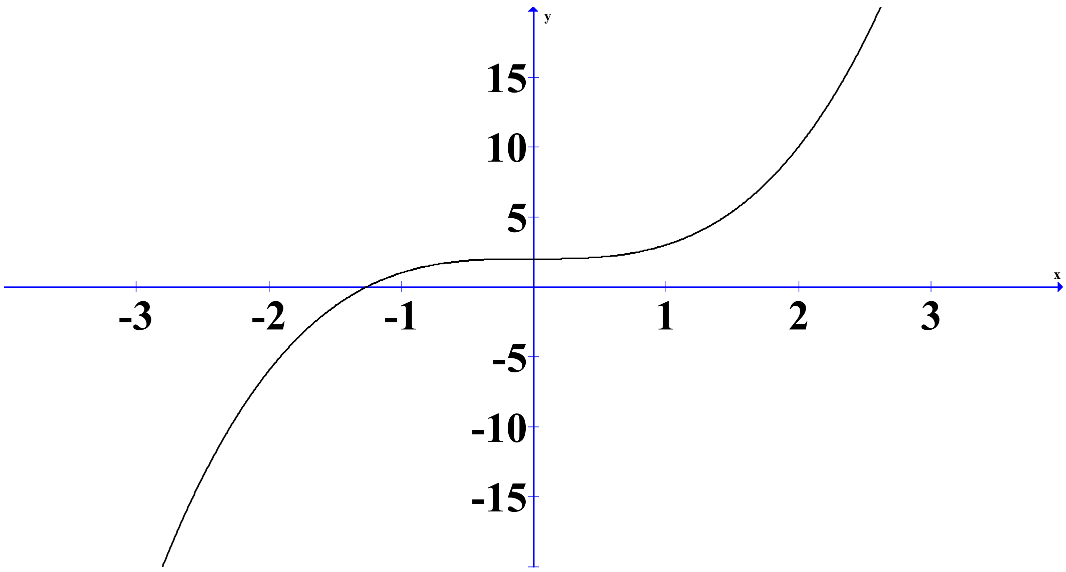
b) $y = x^3 + 1$



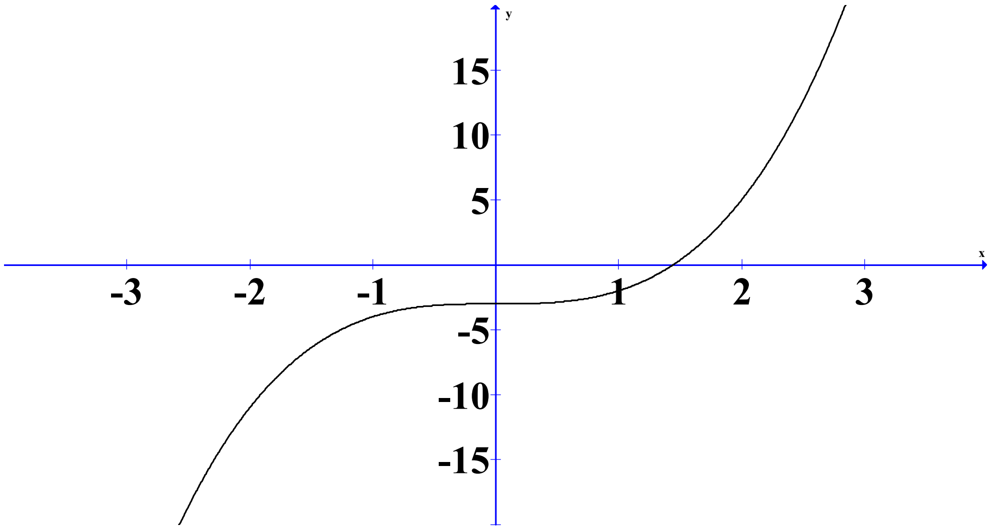
c) $y = x^3 - 1$



d) $y = x^3 + 2$



e) $y = x^3 - 3$

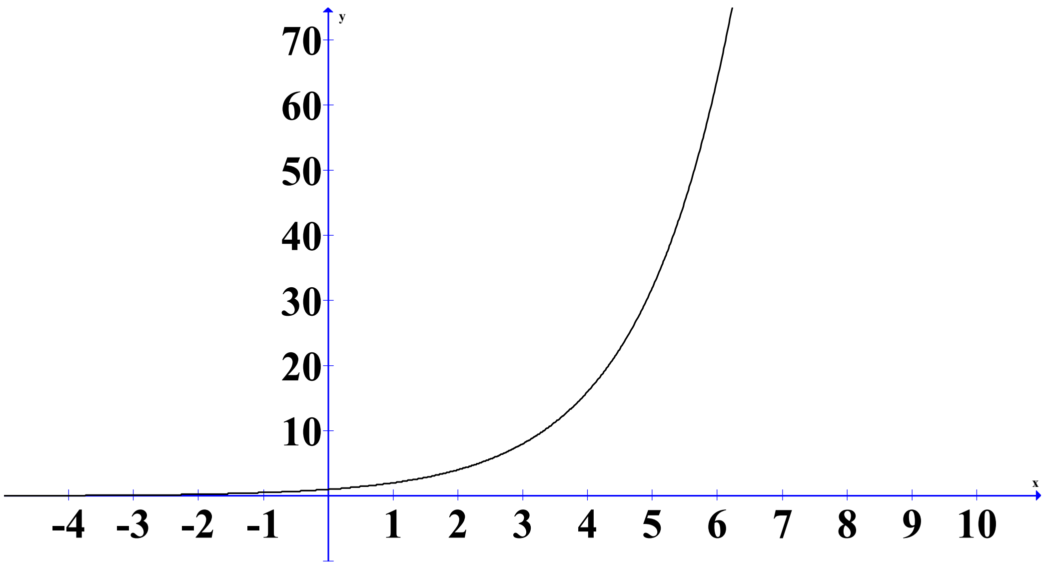


2) Comment on the effect of the value of the constant a on the graph of the form $y = x^3 + a$

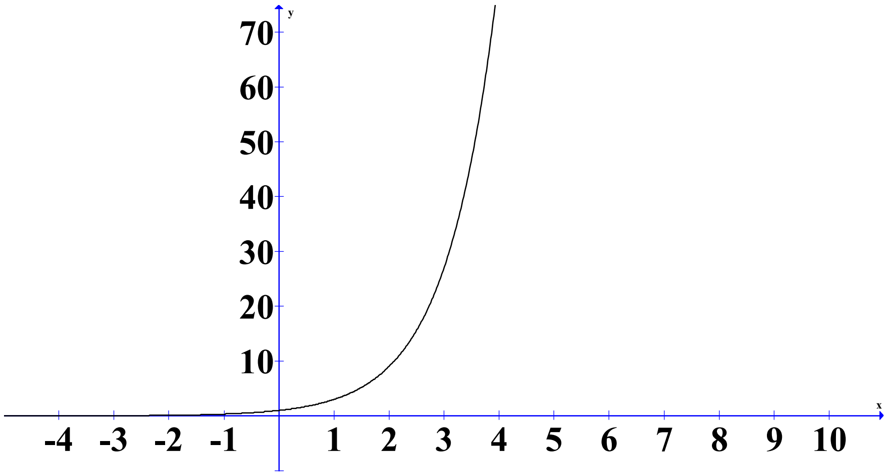
The curve $y = x^3$ is shifted by a units up or down the y axis

3) Graph the following equations

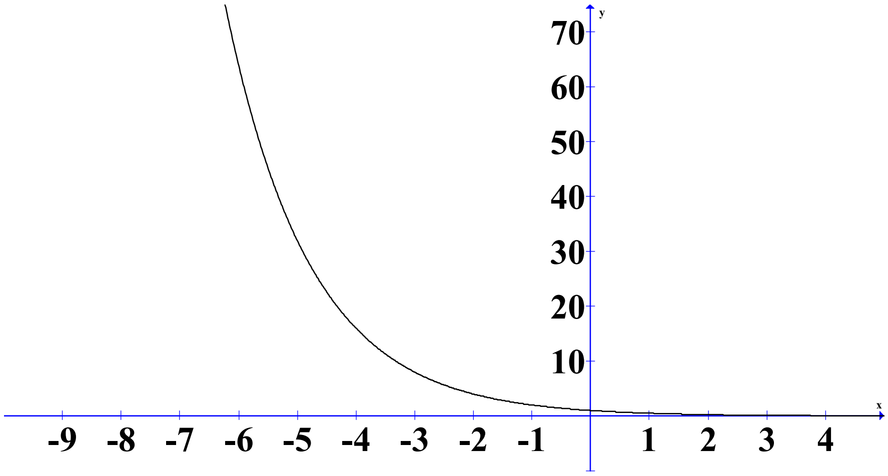
a) $y = 2^x$



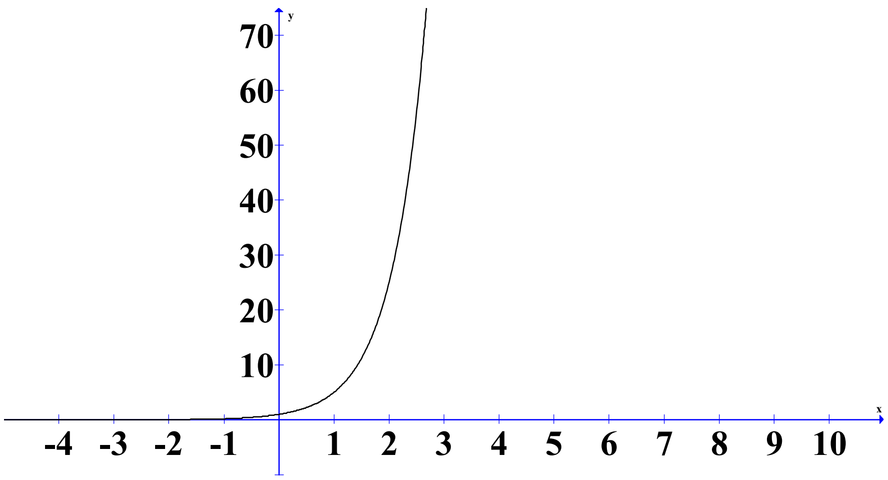
b) $y = 3^x$



c) $y = \left(\frac{1}{2}\right)^x$



d) $y = 5^x$

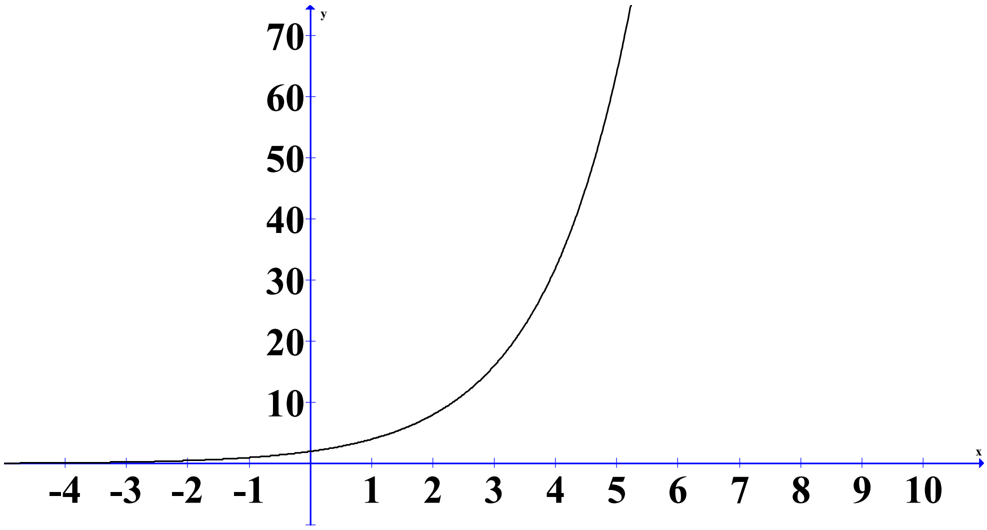


4) Comment on the effect of the value of the constant a on graphs of the form $y = a^x$

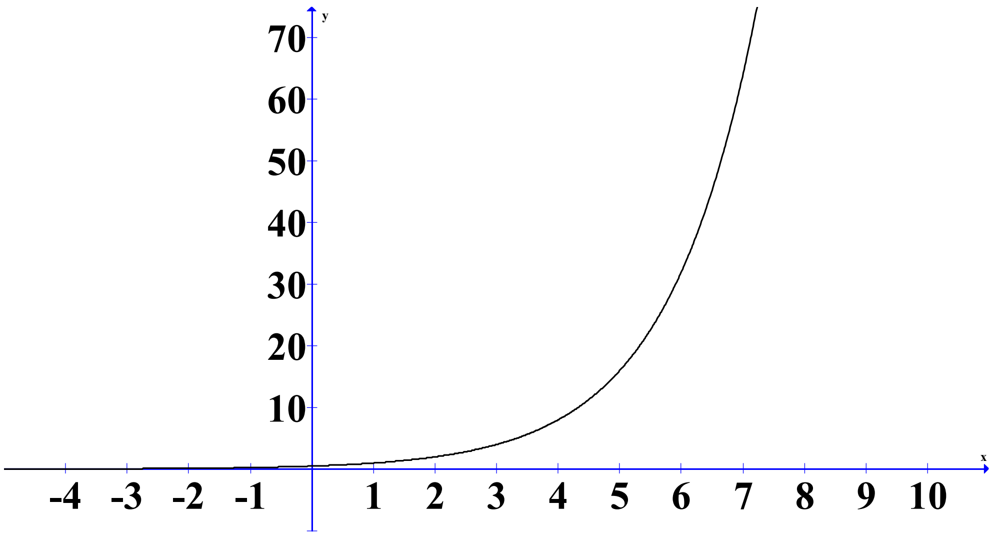
The greater the value of a the steeper the curve; if $a < 0$ the curve is reflected

5) Graph the following equations

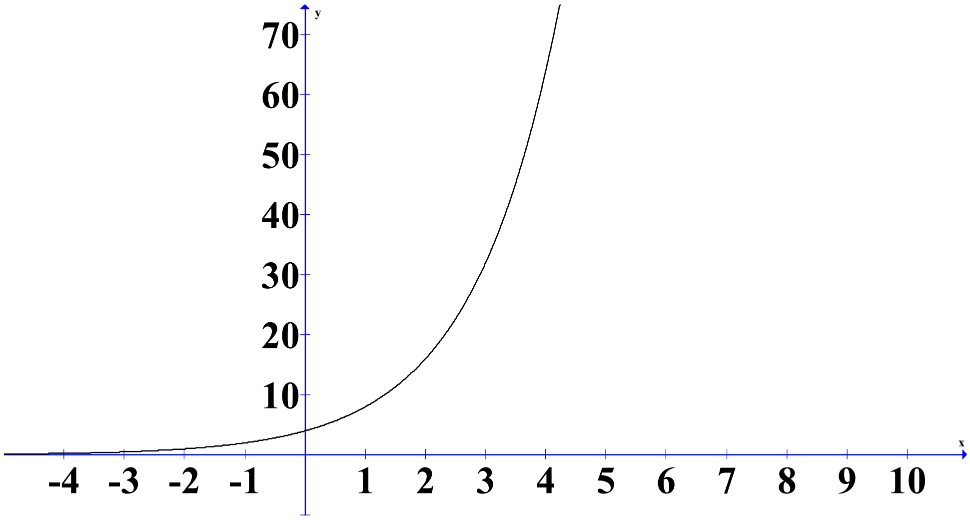
a) $y = 2^{x+1}$



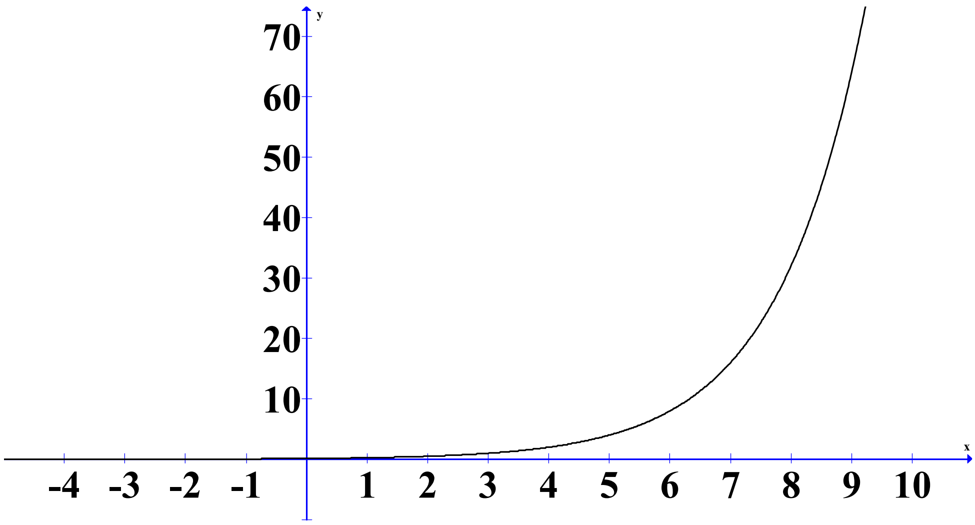
b) $y = 2^{x-1}$



c) $y = 2^{x+2}$



d) $y = 2^{x-3}$

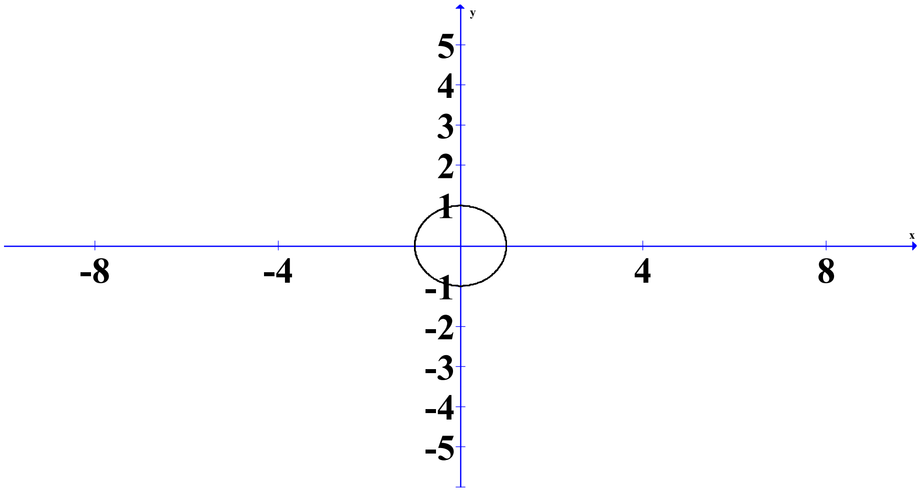


6) Comment on the effect of the value of the constant a on the graph of the form $y = 2^{x+a}$

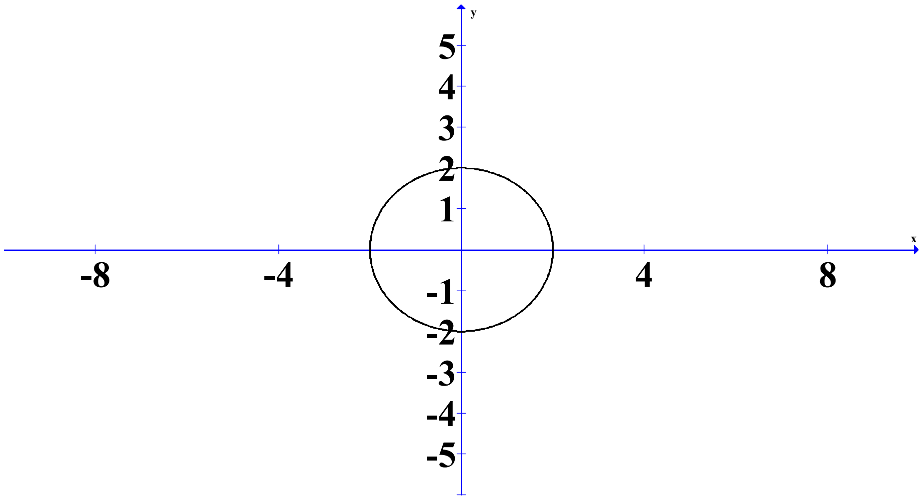
The greater the value of a the steeper the curve; negative values of a make the curve less steep

7) Graph the following

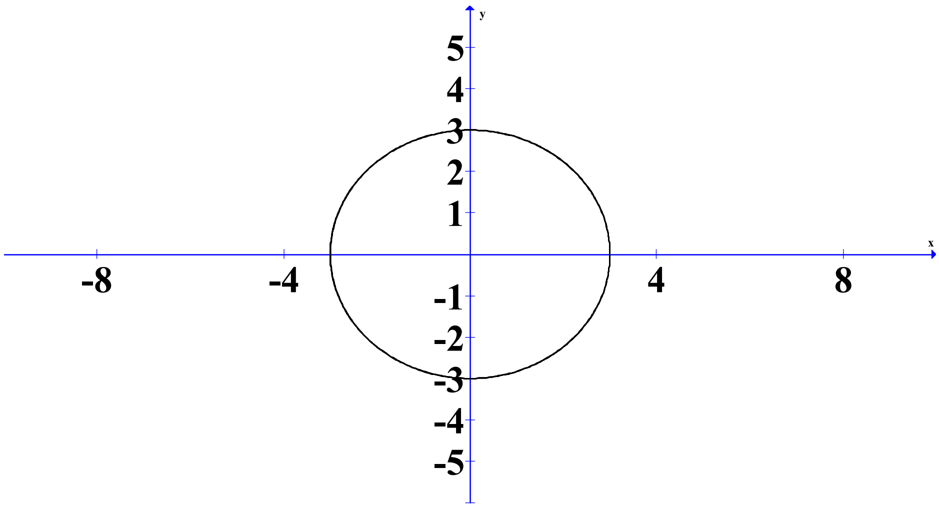
a) $x^2 + y^2 = 1$



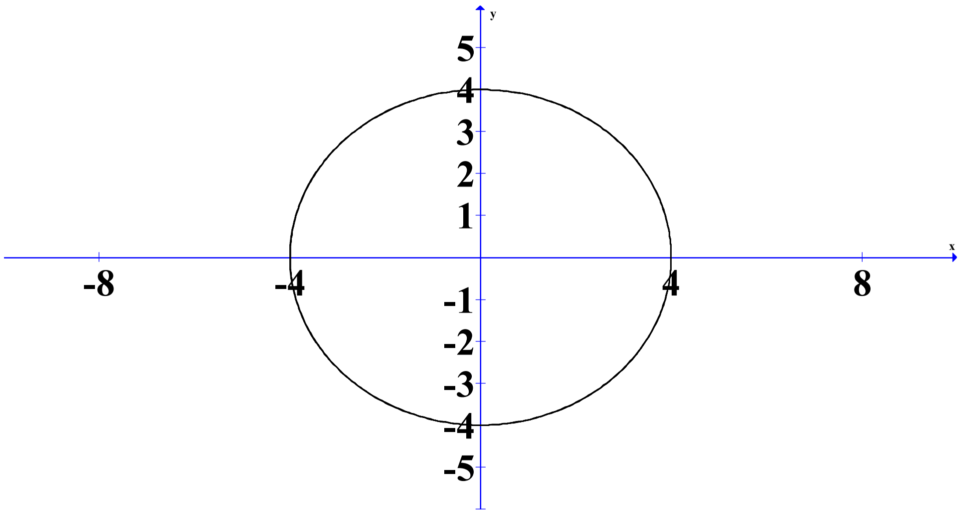
b) $x^2 + y^2 = 4$



c) $x^2 + y^2 = 9$



d) $x^2 + y^2 = 16$

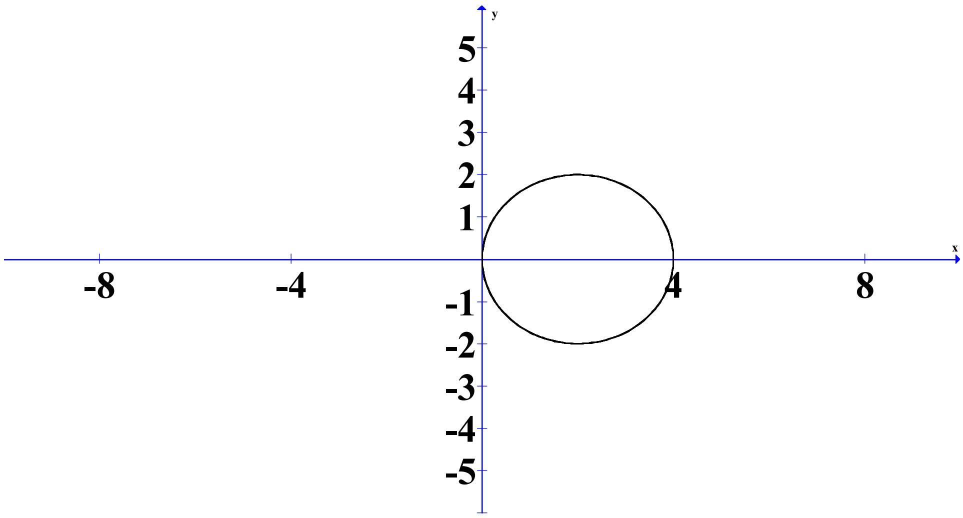


8) Comment on the effect of the value of the constant a on the graph of the form $x^2 + y^2 = a$

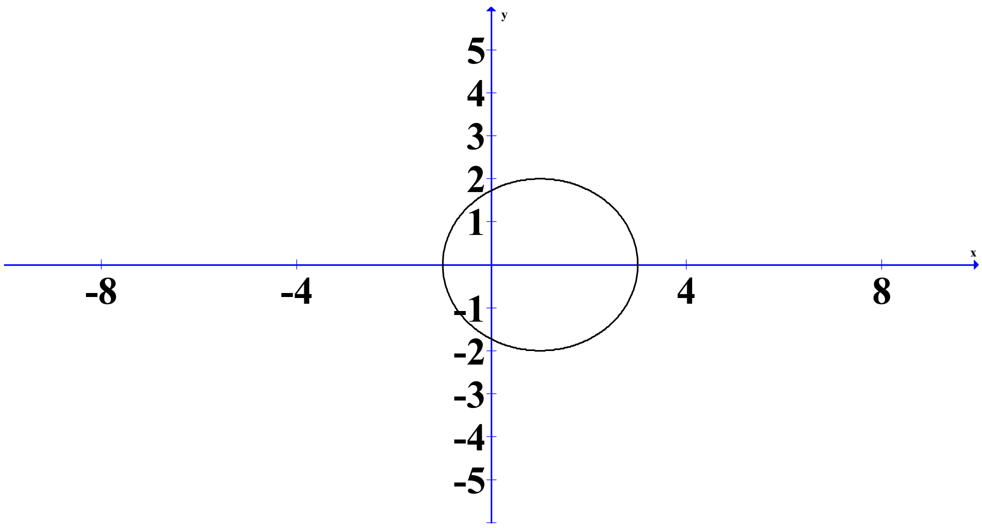
The larger the value of a the larger the circle; the radius of the circle is equal to $\sqrt{a}$

9) Graph the following

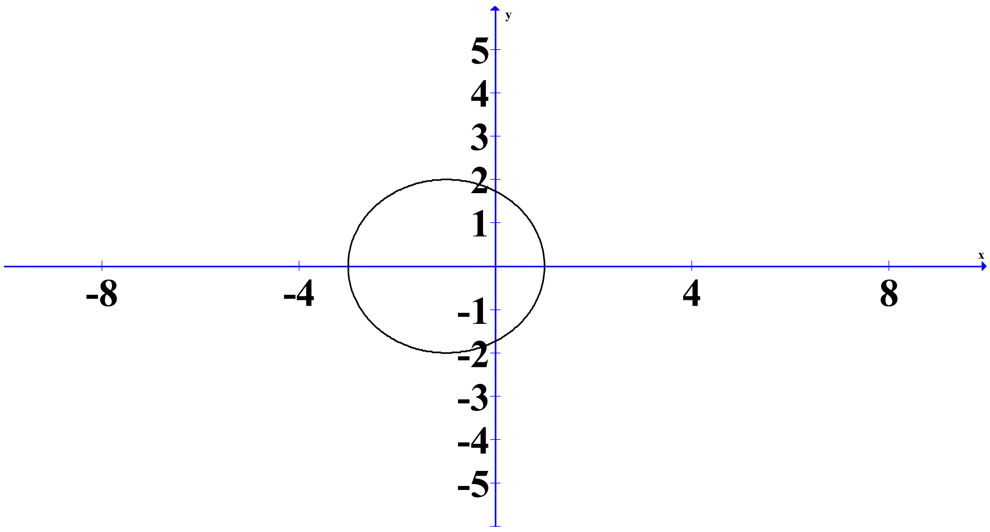
a) $(x-2)^2 + y^2 = 4$



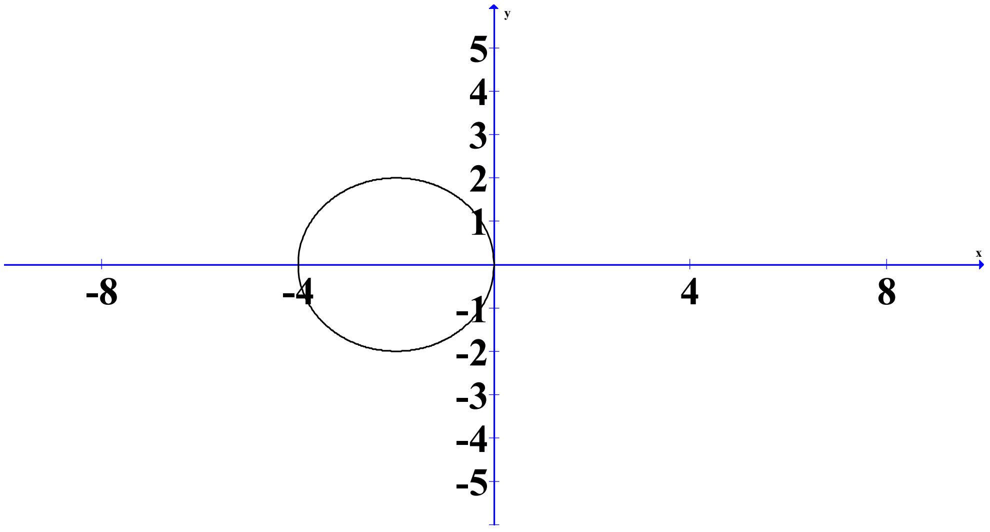
b) $(x-1)^2 + y^2 = 4$



c) $(x+1)^2 + y^2 = 4$

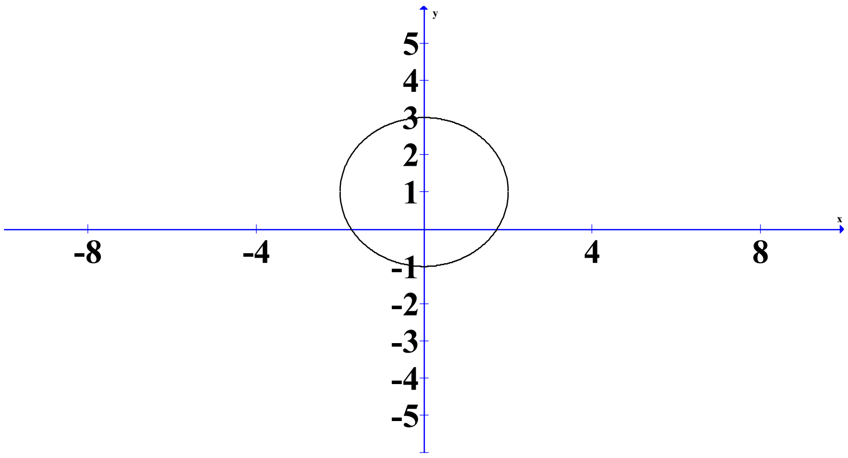


d) $(x+2)^2 + y^2 = 4$

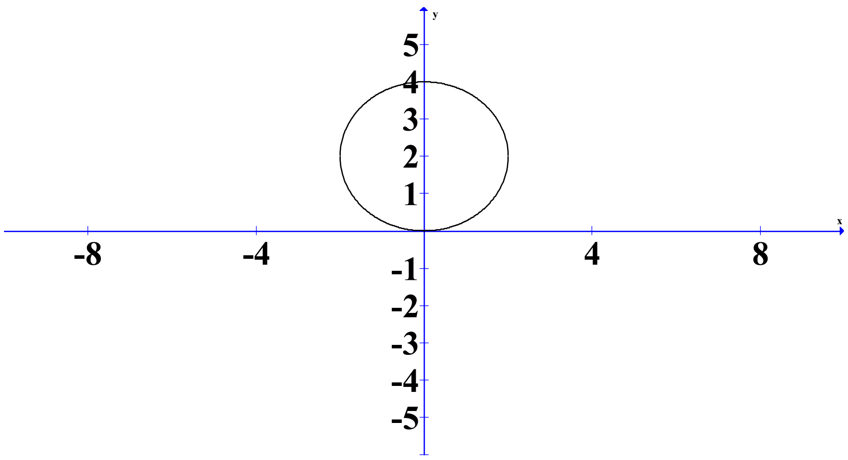


10) Graph the following

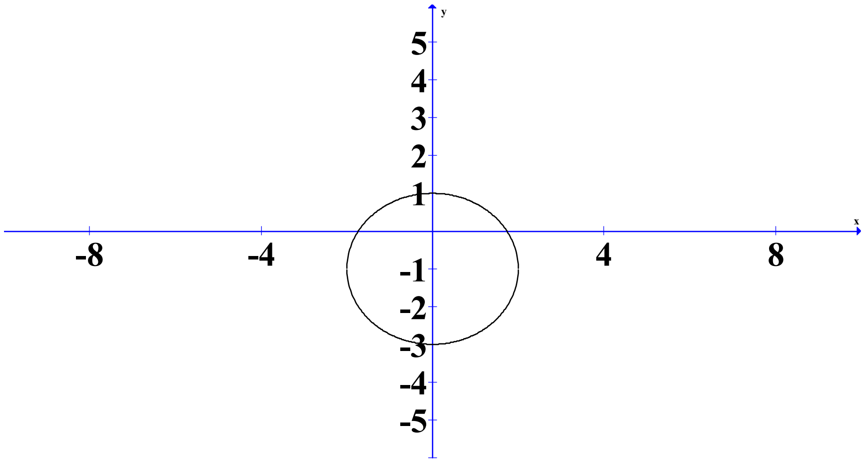
a) $x^2 + (y-1)^2 = 4$



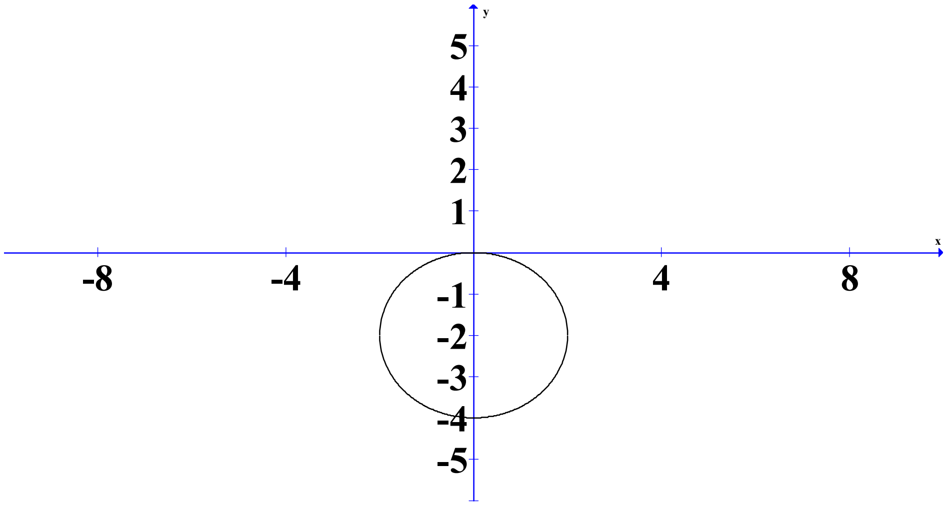
b) $x^2 + (y-2)^2 = 4$



c) $x^2 + (y+1)^2 = 4$



d) $x^2 + (y+2)^2 = 4$



11) Comment on the effect of the values of the constants a and b on graphs of the form $(x-a)^2 + (y-b)^2 = c$

The size of the circle stays the same, since its radius is determined by the value of c

Changes in the value of a shift the circle$a$ units in the direction of the x axis; left for positive values, right for negative values

Changes in the value of b shift the circle b units in the direction of the y axis; up for negative values of b, and down for positive values

Exercise 4

Solving Quadratic Equations

1) Solve the following by factorization

a) $x^2 + 2x - 3 = 0$

$(x + 3)(x - 1) = 0$

$x = -3 \; or \; x = 1$

b) $x^2 - 6x + 8 = 0$

$(x - 4)(x - 2) = 0$

$x = 4 \; or \; x = 2$

c) $x^2 + 6x + 5 = 0$

$(x + 5)(x + 1) = 0$

$x = -5 \; or \; x = -1$

d) $x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3 \; or \; x = -2$

e) $x^2 - 1 = 0$

$(x + 1)(x - 1) = 0$

$x = -1 \; or \; x = 1$

f) $x^2 - 2x + 10 = 0$

Cannot be factorized

2) Solve the equations from question 1 by completing the square

a) $(x + 1)^2 - 4 = 0$

$x + 1 = \pm 2$

$x = -3 \; or \; x = 1$

b) $(x - 3)^2 - 1 = 0$

$x - 3 = \pm 1$

$x = 4 \; or \; x = 2$

c) $(x + 3)^2 - 4 = 0$

$x + 3 = \pm 2$

$x = -5 \; or \; x = -1$

d) $\left(x - \frac{1}{2}\right)^2 - \frac{25}{4} = 0$

$x - \frac{1}{2} = \pm \frac{5}{2}$

$x = 3 \; or \; x = -2$

e) $(x + 0)^2 - 1 = 0$

$x = \pm 1$

$x = 1 \; or \; x = -1$

f) $(x - 1)^2 + 9 = 0$

$(x - 1)^2 = -9$

Cannot be solved

3) Solve the equations from question 1 by using the quadratic formula

a) $x = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-3)}}{2x1}$

$$x = \frac{-2 \pm \sqrt{16}}{2} = 1\ or - 3$$

b) $x = \frac{6 \pm \sqrt{-6^2 - 4 \times 1 \times (8)}}{2x1}$

$$x = \frac{6 \pm \sqrt{4}}{2} = 4\ or 2$$

c) $x = \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times (5)}}{2x1}$

$$x = \frac{-6 \pm \sqrt{16}}{2} = -1\ or - 5$$

d) $x = \frac{1 \pm \sqrt{(-1)^2 - 4 \times 1 \times (-6)}}{2x1}$

$$x = \frac{1 \pm \sqrt{25}}{2} = -2\ or 3$$

e) $x = \frac{0 \pm \sqrt{0^2 - 4 \times 1 \times (-1)}}{2x1}$

$$x = \frac{0 \pm \sqrt{4}}{2} = 1\ or - 1$$

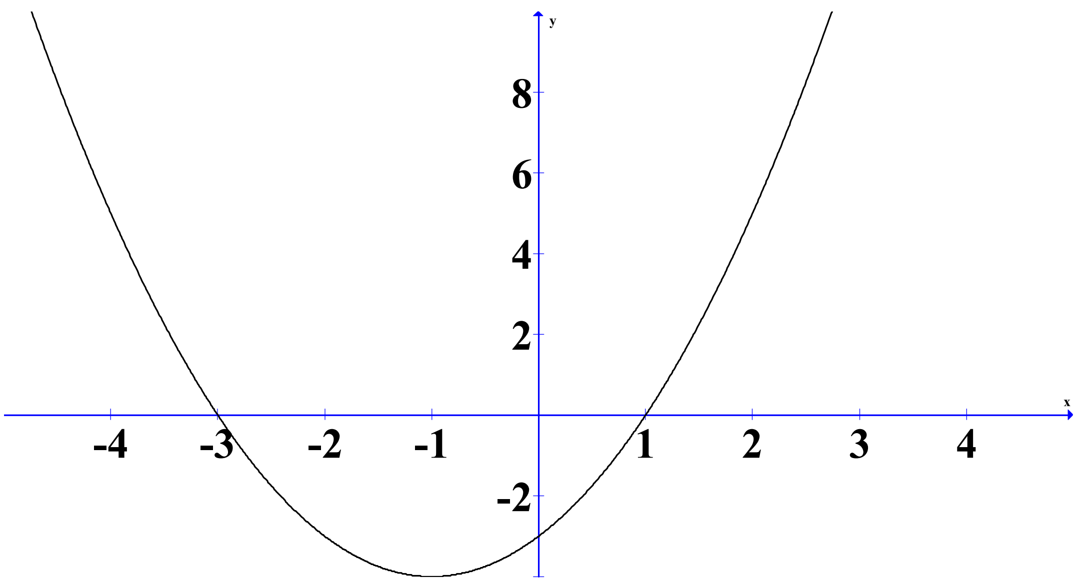
f) a $x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times (10)}}{2x1}$

$$x = \frac{2 \pm \sqrt{-36}}{2}$$

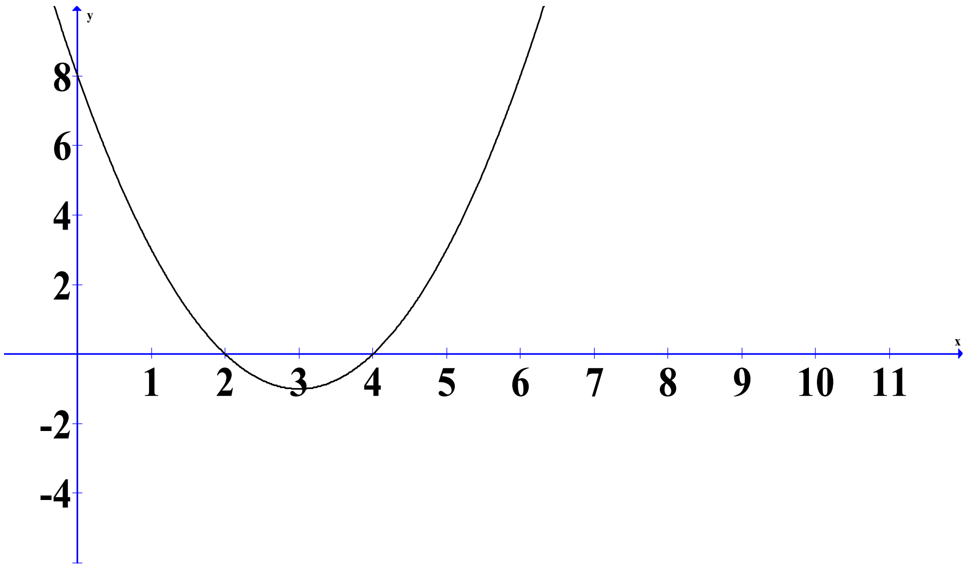
This cannot be simplified as $\sqrt{-36}$ is not real.
Therefore the equation has no real solutions

4) Graph the following equations, and relate your graph to the answers to questions 1 to 3

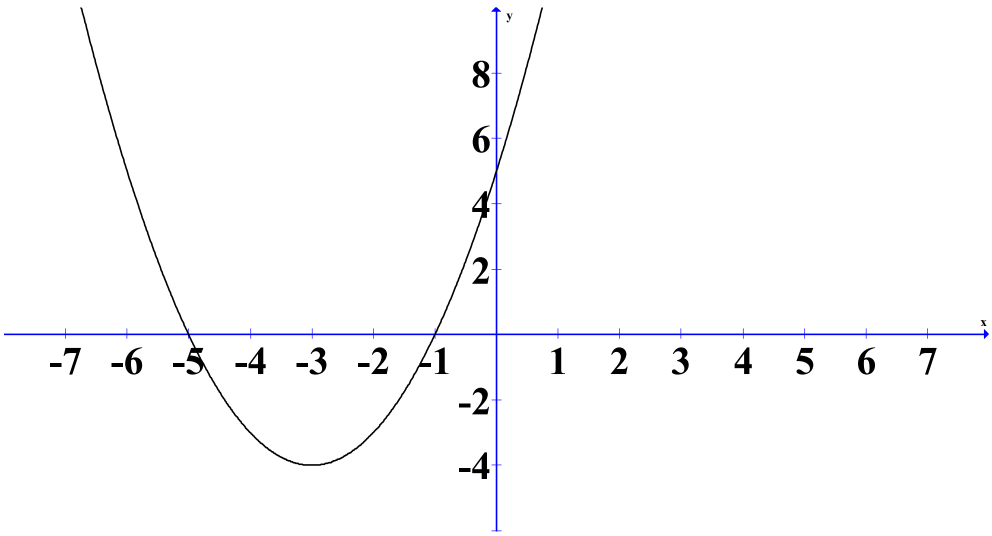
a) $y = x^2 + 2x - 3$



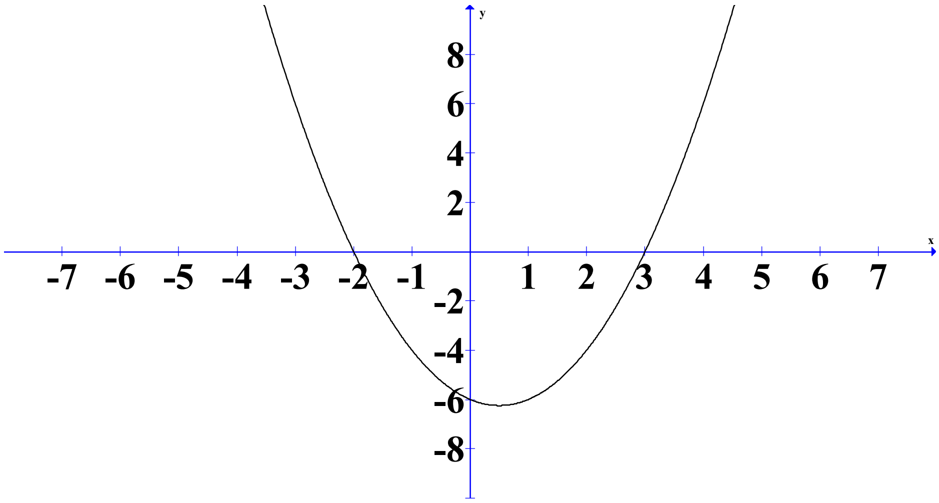
b) $y = x^2 - 6x + 8$



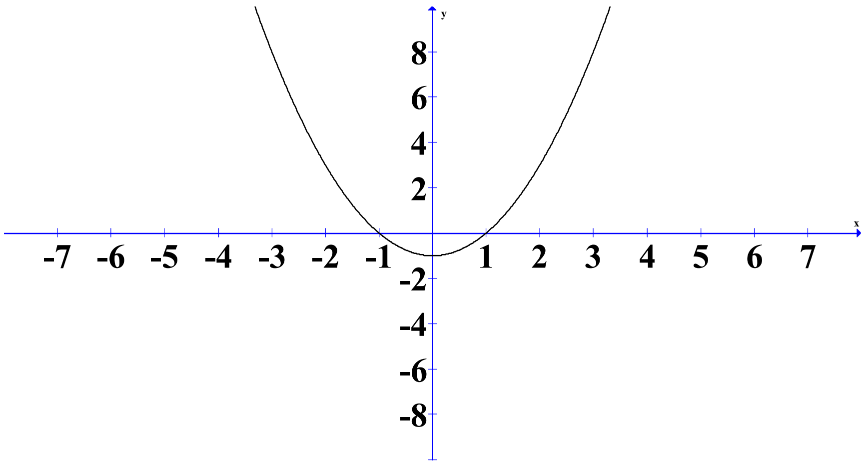
c) $y = x^2 + 6x + 5$



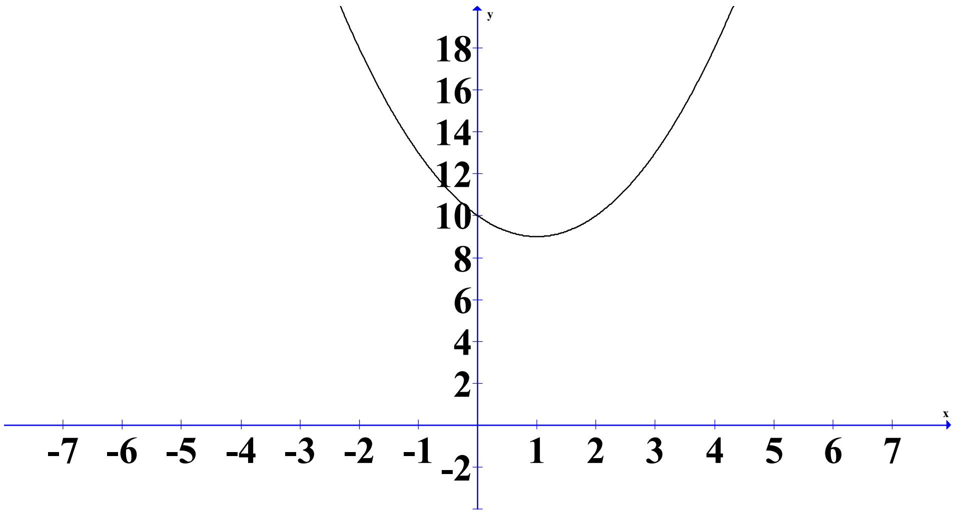
d) $y = x^2 - x - 6$



e) $y = x^2 - 1$



f) $y = x^2 - 2x + 10$



The solutions to the equations from the first 3 questions are equal to the points where the curve crosses the x axis. It can be seen from part f that the curve does not cross the x axis, which is consistent with its equation having no solution

5) Solve the following

a) $2x^2 + 5x - 3 = 0$

$(2x - 1)(x + 3) = 0$

$x = \frac{1}{2}$ or $x = -3$

b) $2x^2 - 7x - 4 = 0$

$(2x + 1)(x - 4) = 0$

$x = -\frac{1}{2}$ or $x = 4$

c) $4x^2 - 4x + 1 = 0$

$(2x - 1)(2x - 1) = 0$

$x = \frac{1}{2}$

d) $6x^2 - x - 2 = 0$

$(3x - 2)(2x + 1) = 0$

$x = \frac{2}{3}$ or $x = -\frac{1}{2}$

e) $6x^2 + x - 5 = 0$

$(6x - 5)(x + 1) = 0$

$x = \frac{5}{6}$ or $x = -1$

f) $3x^2 + x + 6 = 0$

From the quadratic equation

$b^2 - 4ac = -95$

Therefore the equation has no real solutions

6) Solve the following

a) $x^2 + 2x - 6 = 0$

$$x = \frac{-2 \pm \sqrt{4 - 4 \times 1 \times (-6)}}{2}$$

$$x = \frac{-2 \pm \sqrt{28}}{2}$$

$$x = -1 \pm \frac{\sqrt{4 \times 7}}{2}$$

$x = -1 + \sqrt{7}\ or\ x = -1 - \sqrt{7}$

b) $x^2 - 4 = 0$

$x^2 = 4$

$x = 2\ or\ x = -2$

c) $(x - 2)^2 = 9$

Substitute $y = x - 2$

$y^2 = 9$

$y = 3\ or\ y = -3$

So $x - 2 = 3\ or\ x - 2 = -3$

$x = 5\ or\ x = -1$

d) $x(x-4)=4$

$x^2-4x-4=0$

$x=\dfrac{4\pm\sqrt{16+16}}{2}$

$x=2\pm\dfrac{\sqrt{32}}{2}$

$x=2+\sqrt{8}\ or\ 2-\sqrt{8}$

e) $2x^2=9$

$2x^2-9=0$

$x=\dfrac{0\pm\sqrt{0-4\times2\times(-9)}}{4}$

$x=\dfrac{\sqrt{72}}{4}\ or\ x=-\dfrac{\sqrt{72}}{4}$

7) The product of two consecutive negative numbers is 110. What are the numbers?

Let the numbers be $x\ and\ x-1$

$x(x-1)=110$

$x^2-x-110=0$

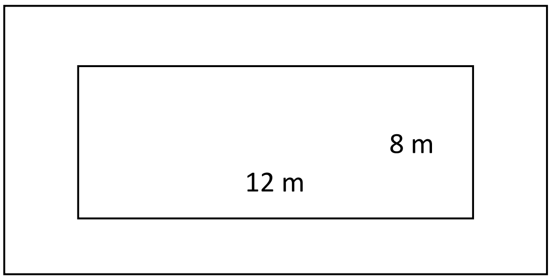
$(x-11)(x+10)=0$

$x=11\ orx=-10$

The required number is negative, so $x=-10$

The other number is -11

8) A pool measuring 8 by 12 meters is to have paving of equal width around it. When completed, the total area covered by the pool and the path will be 140 m^2. What will be the width of the path?



Let the width of the path be x

Then $(8 + 2x)(12 + 2x) = 140$

$96 + 24x + 16x + 4x^2 = 140$

$4x^2 + 40x + 96 = 140$

$4x^2 + 40x - 44 = 0$

$x^2 + 10x - 11 = 0$

$(x + 11)(x - 1) = 0$

$x = -11 \; or \; x = 1$

Since x must be positive, the path is 1 m wide

9) To fence a rectangular paddock takes 90 metres of fencing. If the area of the paddock is 450 m^2, what are its dimensions?

Let the length of the paddock be a and the width be b

Using the perimeter:

$2a + 2b = 90$

$$a = \frac{90 - 2b}{2} = 45 - b$$

Using the area:

$a \times b = 450$

Substituting:

$(45 - b) \times b = 450$

$45b - b^2 - 450 = 0$

$b^2 - 45b + 450 = 0$

$(b - 15)(b - 30) = 0$

$b = 15 \; or \; b = 30$

If $b = 15, a = 30$

If $b = 30, a = 15$

The paddock is 30 m x 15 m

10) When an integer is added to its inverse, the sum is $\frac{50}{7}$. Use a quadratic expression to calculate the value of the integer.

Let x be the integer

Then $x + \frac{1}{x} = \frac{50}{7}$

$$7x + \frac{7}{x} = 50$$

$7x^2 + 7 - 50x = 0$

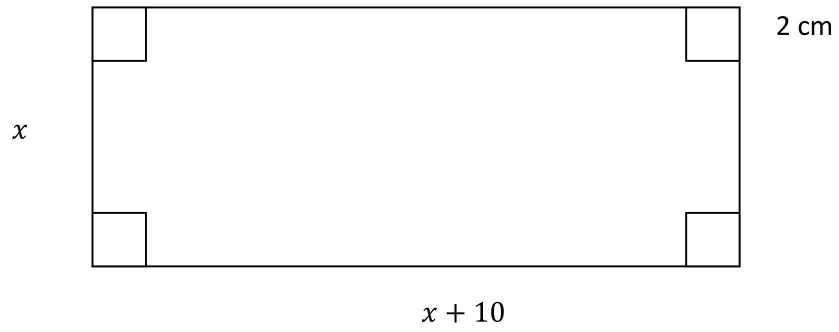
$(7x - 1)(x - 7) = 0$

$$x = \frac{1}{7} \; or \; x = 7$$

Since the required number is an integer, $x = 7$

11) The base of a box has a length 10 cm longer than its width. When a 2cm square is cut from each corner, the area of the cardboard left is 128 cm^2. What are the dimensions of the box before the corners are cut out?

Let x be the width of the base before the cut outs are made



The area cut out is $4 \times 4 = 16$ cm^2

Before the cut outs, the area of the base was $128 + 16 = 144$ cm^2

$x(x + 10) = 144$

$x^2 + 10x - 144 = 0$

$(x + 18)(x - 8) = 0$

$x = -18 \; or \; x = 8$

The width of the base is 8 cm, and its length is 18 cm

Exercise 5

Rearranging Equations

1) The formula relating the voltage (V), current (I) and resistance (R) is

$V = RI$

Rearrange the equation to make R the subject

$$R = \frac{V}{I}$$

2) Einstein's famous equation relating the mass of an object to the energy it could release is given by

$E = mc^2$, where c is the speed of light

Rearrange the equation so it is shown in terms of c

$$c^2 = \frac{E}{m}$$

$$C = \sqrt{\frac{E}{m}}$$

3) Pythagoras' theorem states

$a^2 + b^2 = c^2$, where a, b and c are the three side lengths of a triangle

Rearrange the equation to show the relationship in terms of b

$$b^2 = c^2 - a^2$$

$$b = \sqrt{c^2 - a^2}$$

4) The force (F) between two objects of mass m_1 and m_2 is given by the equation

$F = \gamma\frac{m_1 m_2}{r^2}$, where r is the distance between the objects and ɣ is a constant.

Express the relationship in terms of r

$$r^2 = \gamma\frac{m_1 m_2}{F}$$

$$r = \sqrt{\gamma\frac{m_1 m_2}{F}}$$

5) When two resistors are placed in a circuit in parallel, the total resistance R can be calculated from the values of the two resistors by the formula

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$$

Show how the value of one of the resistors can be calculated if the value of the other resistor and the total resistance is known.

$$\frac{1}{r_1} = \frac{1}{R} - \frac{1}{r_2}$$

$$r_1 = \frac{1}{\frac{1}{R} - \frac{1}{r_2}}$$

6) Find an expression for $y^2 - 2$ if $y = 3x$

$(3x)^2 - 2 = 9x^2 - 2$

7) Write the expression $x^2 - 2x + 3$ in terms of t if $x = t - 1$

$= (t-1)^2 - 2(t-1) + 3$

$= t^2 - 2t + 1 - 2t + 1 + 3$

$= t^2 - 4t + 5$

8) Substitute u for x^2 to solve

$x^4 + 4x^2 - 5 = 0$

$u^2 + 4u - 5 = 0$

$(u+5)(u-1) = 0$

$u = -5 \text{ or } u = 1$

$x^2 = -5 \text{ or } x^2 = 1$

$x = 1 \text{ or } -1$

9) Use the substitution $x = (r-1)^2$ to simplify and hence solve the equation $(r^2 - 2r - 3)^2 = 0$
Hint: Complete the square for r

$r^2 - 2r - 3 = (r-1)^2 - 4 = x - 4$

$(x-4)^2 = 0$

$x = 4$

$(r-1)^2 = 4$

$r - 1 = \pm 2$

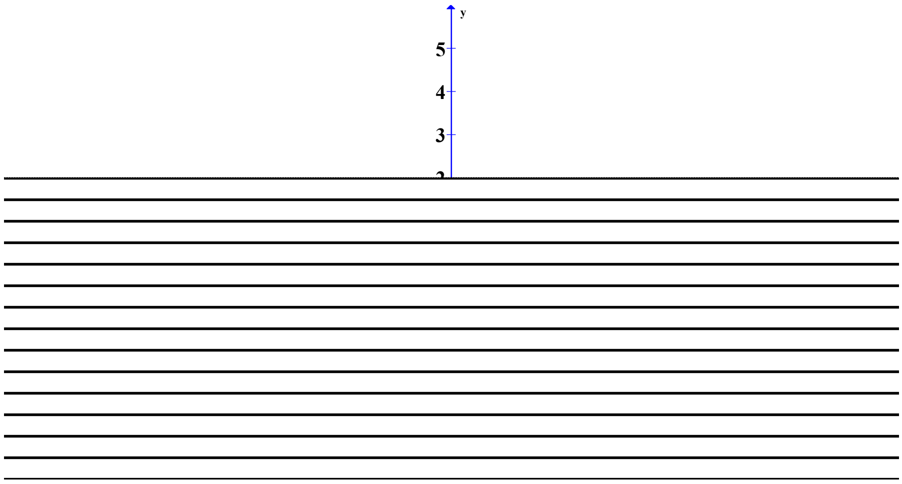
$r = 3 \text{ or } r = -1$

Exercise 6

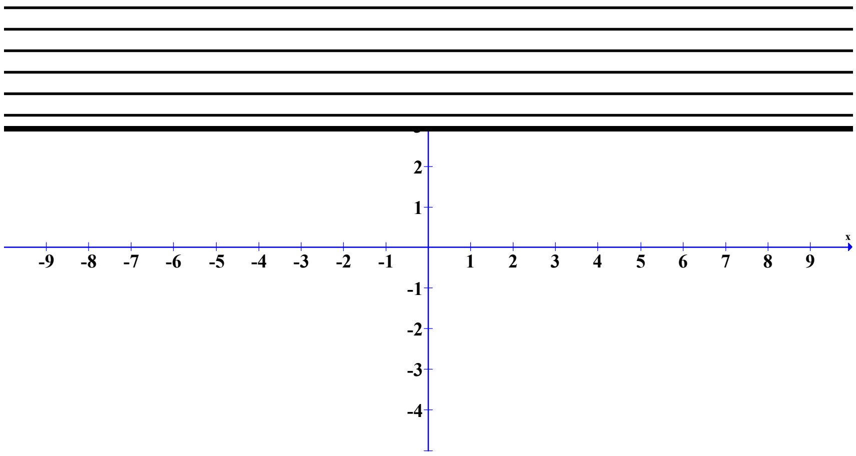
Graphing Regions

1) Graph the following regions

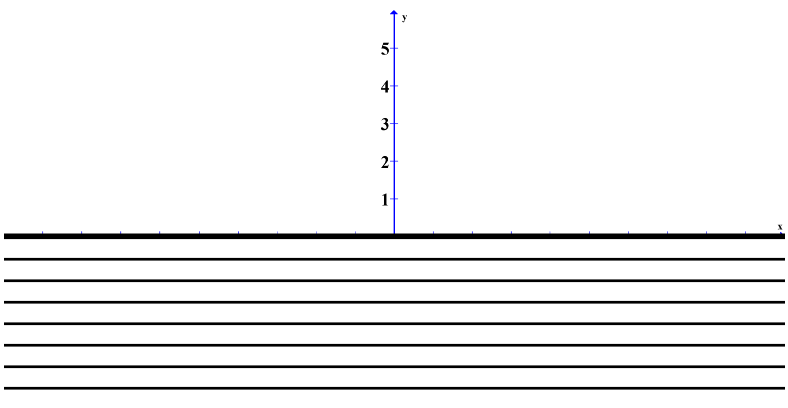
a) $y < 2$



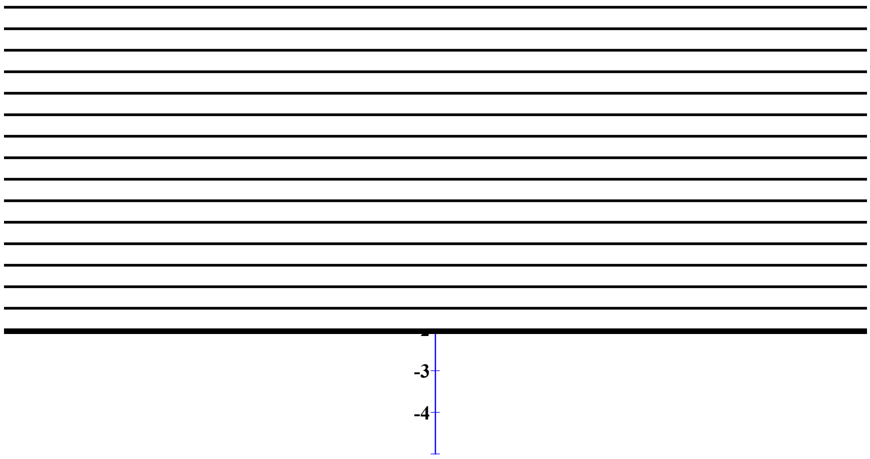
b) $y \geq 3$



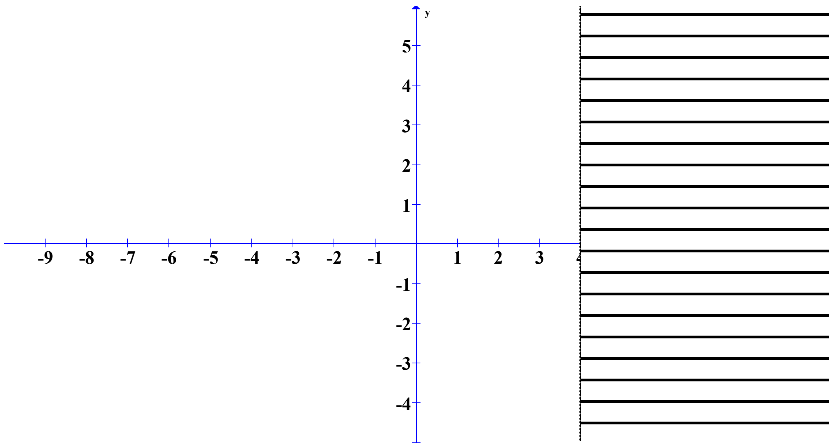
c) $y \leq 0$



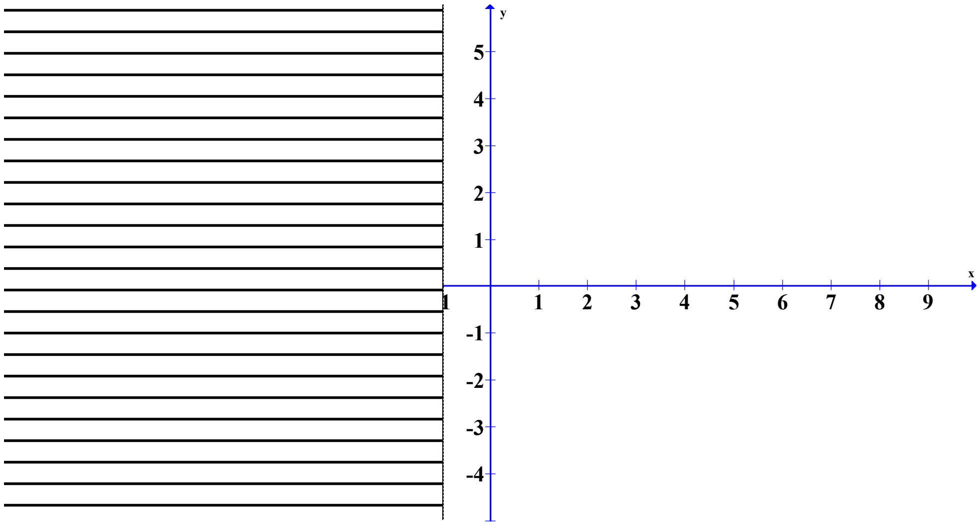
d) $y \geq -2$



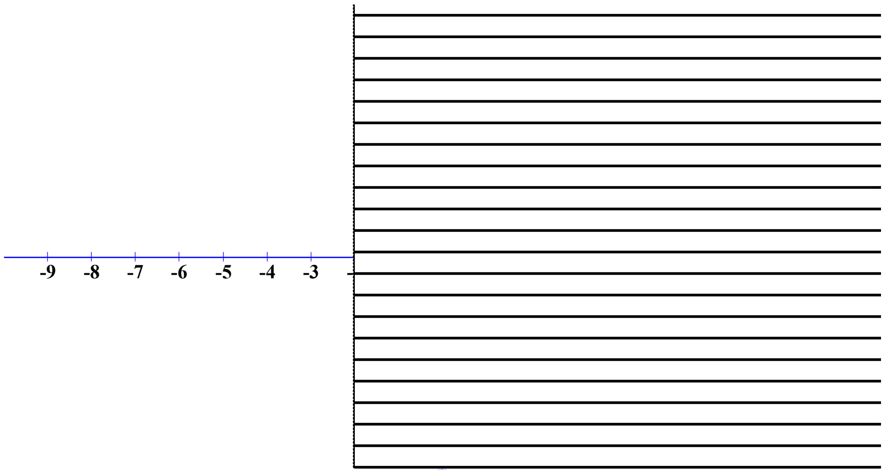
e) $x > 4$



f) $x < -1$

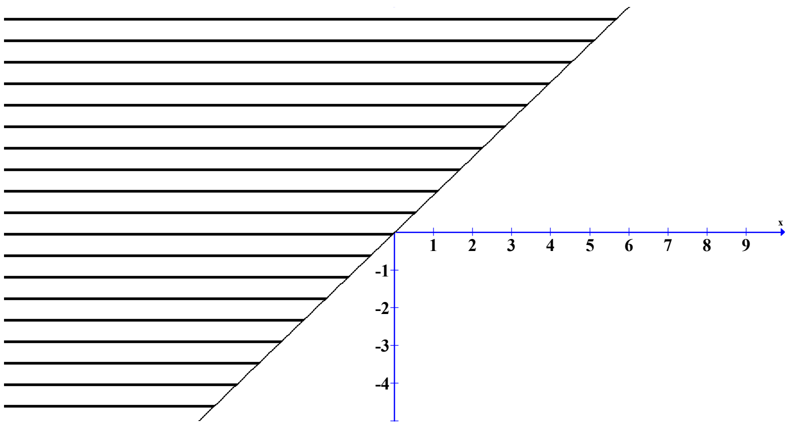


g) $x > -2$

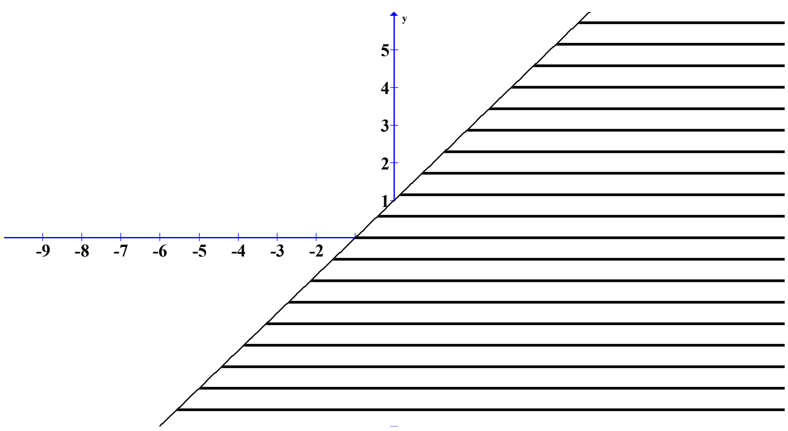


2) Graph the following regions

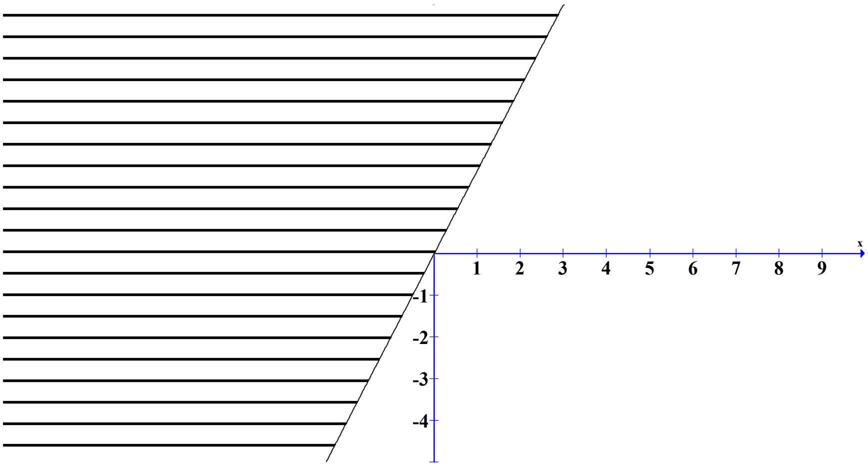
a) $y > x$



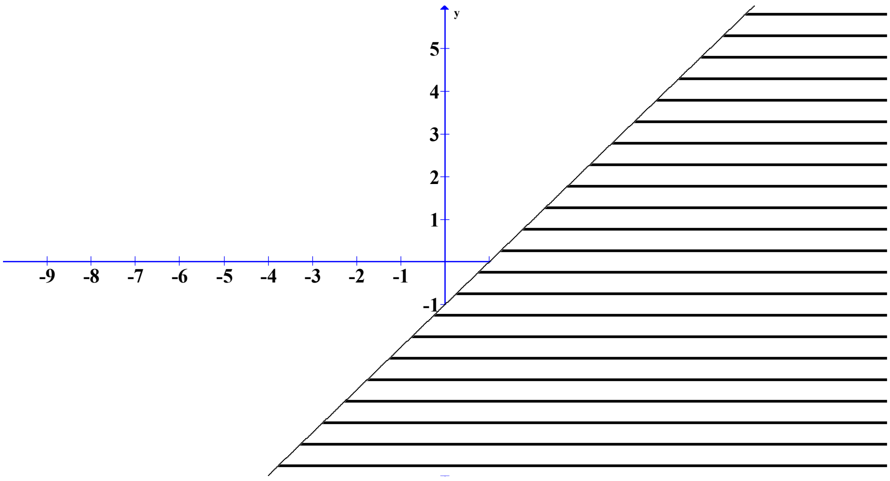
b) $y \leq x + 1$



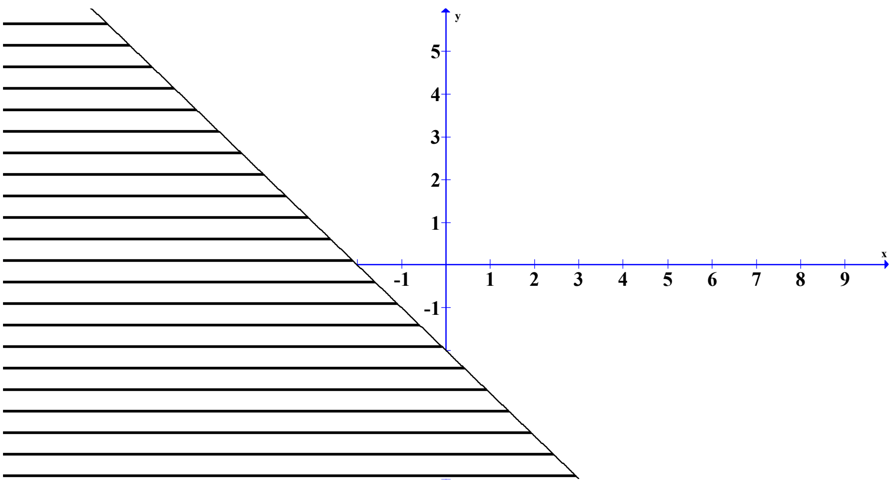
c) $y > 2x$



d) $y < x - 1$

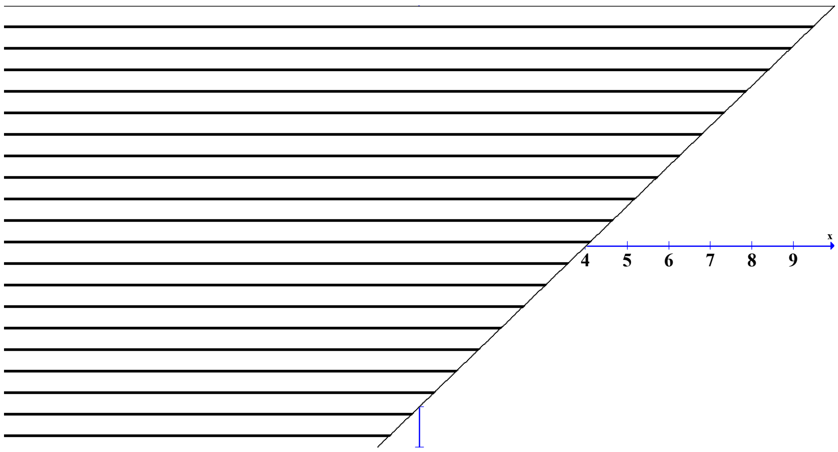


e) $y \geq -x - 2$

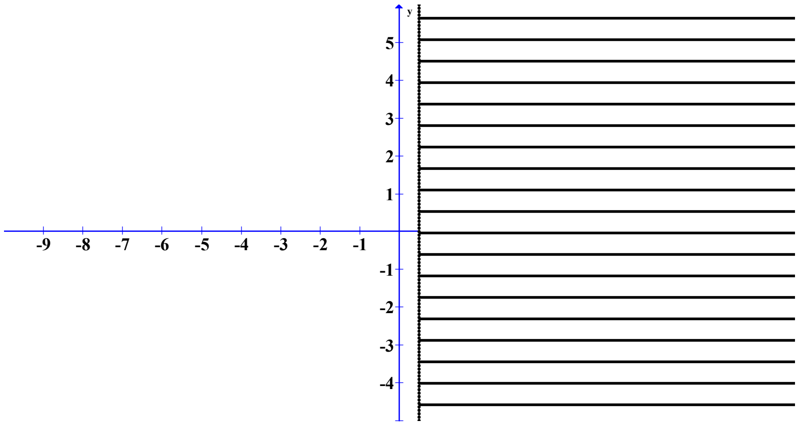


3) Graph the following regions

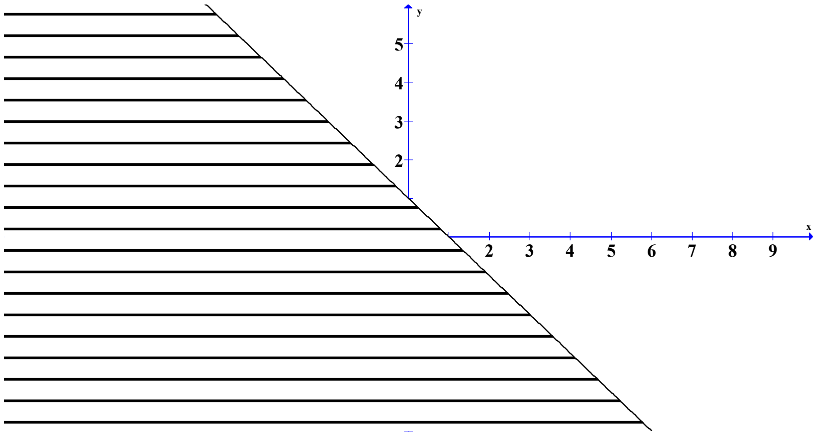
a) $x - y < 4$



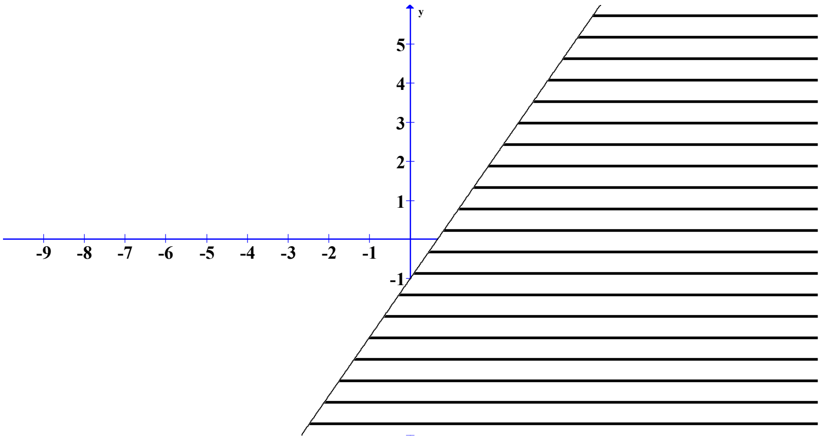
b) $2x + 1 > 2$



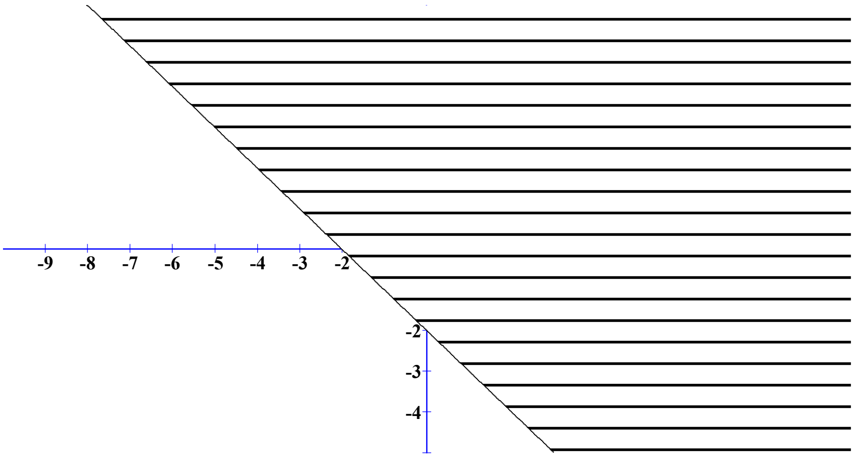
c) $x + y \leq 1$



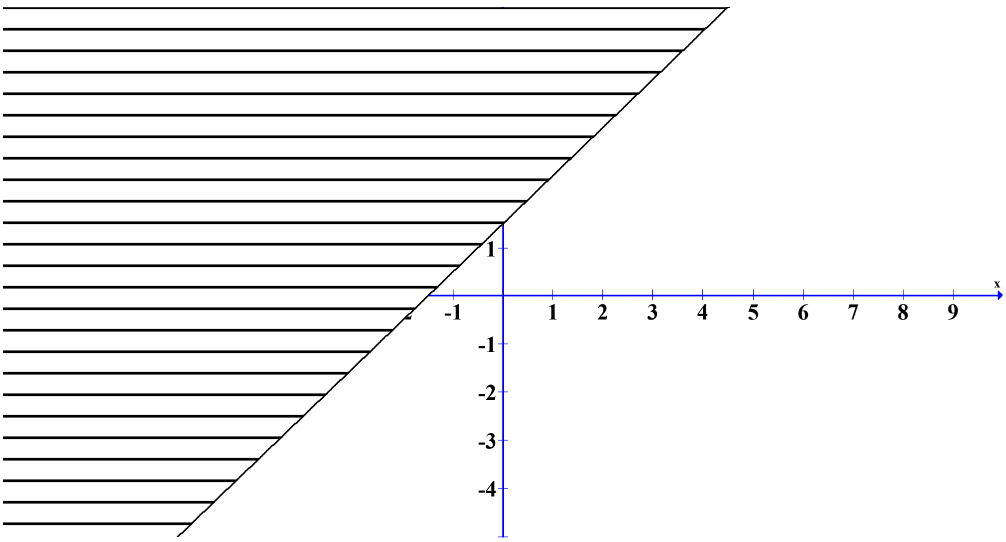
d) $3x - 2y > 2$



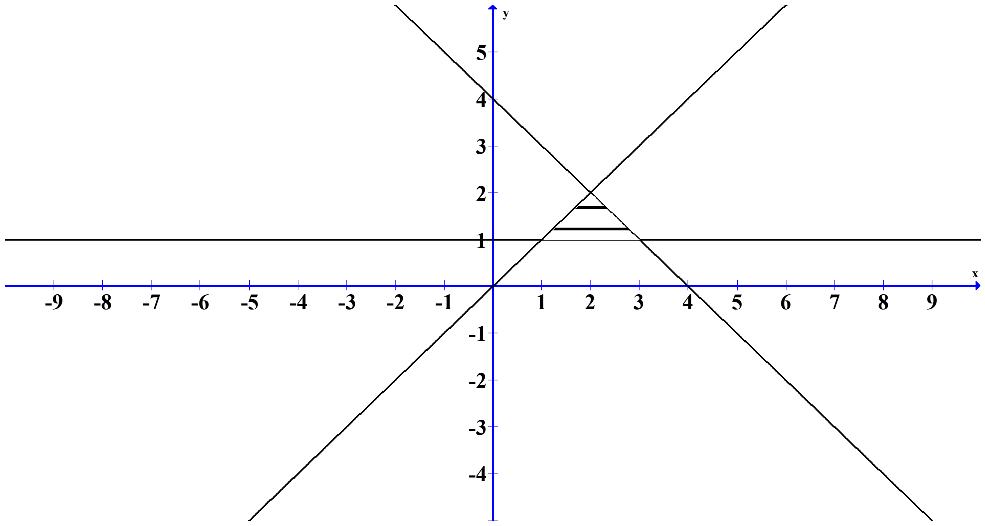
e) $-x - y < 2$



f) $-2x + 2y \geq 3$



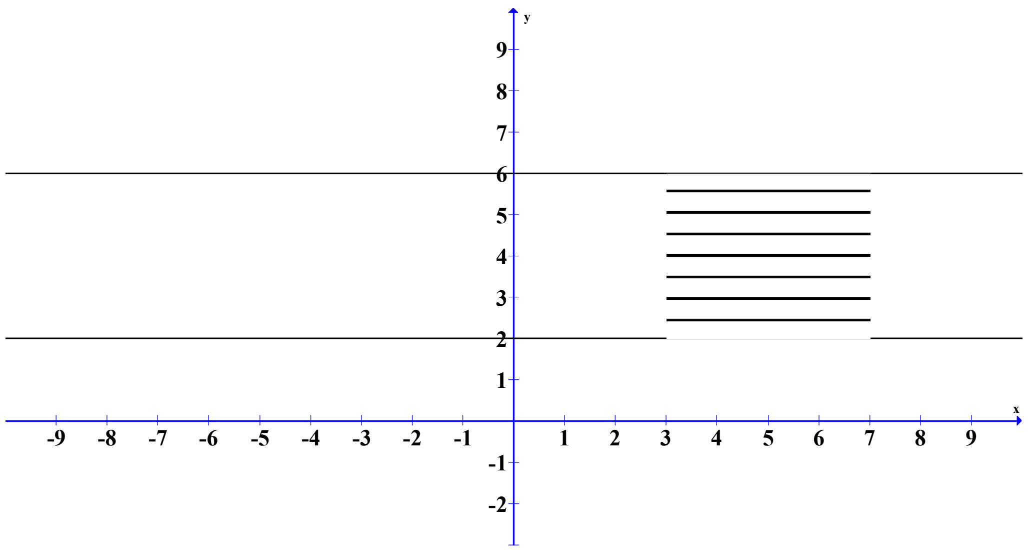
4) Find the area of the shape enclosed by the lines $y = 1, y = x$ and $y = -x + 4$



The shape is a triangle of base 2 and height 1 units

Area $= \frac{1}{2} \times 2 \times 1 = 1\ sq\ unit$

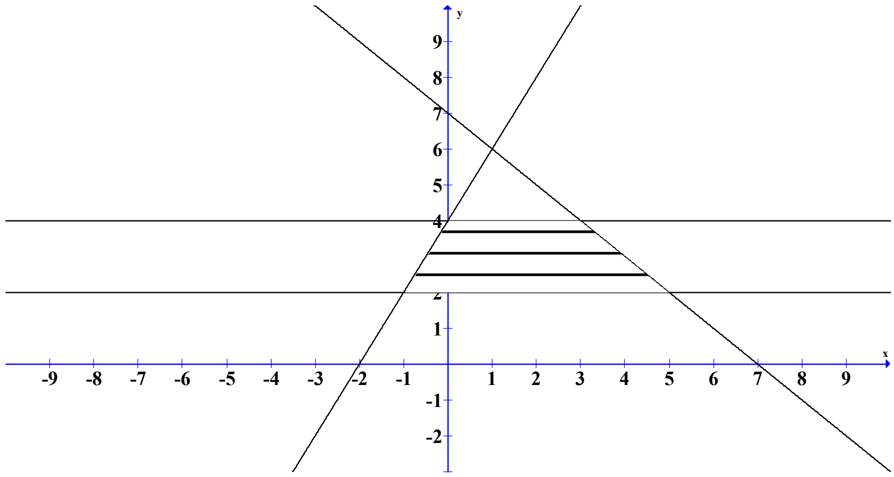
5) Find the area of the shape enclosed by the lines $y = 6$, $y = 2$, $x = 3$ and $x = 7$



Shape is a square of side length 4 units

Area $= 16\ square\ units$

6) Find the area of the shape enclosed by the lines $y = 4$, $y = 2$, $y = 2x + 4$ and $y = -x + 7$

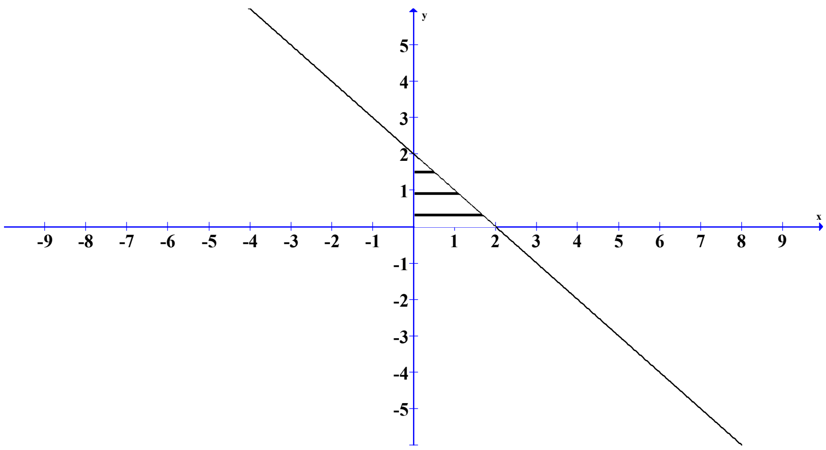


Shape is a trapezoid of height 2 and side lengths 4 and 6

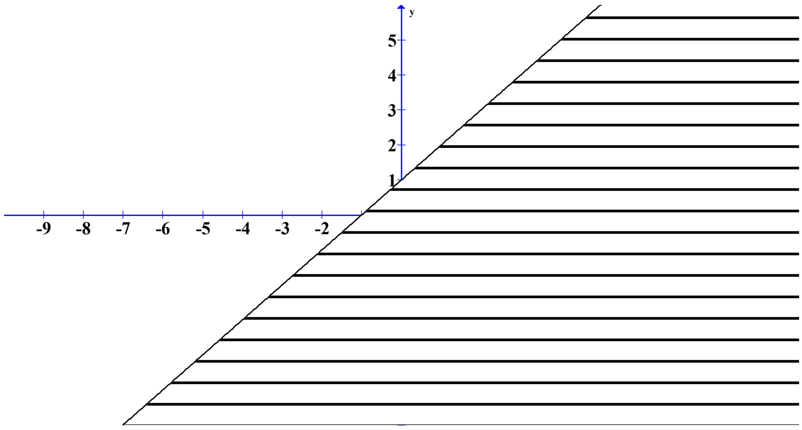
Area $= \frac{6+4}{2} \times 2 = 10\ square\ units$

7) Determine if the point (1, 1) lies within the regions bounded by

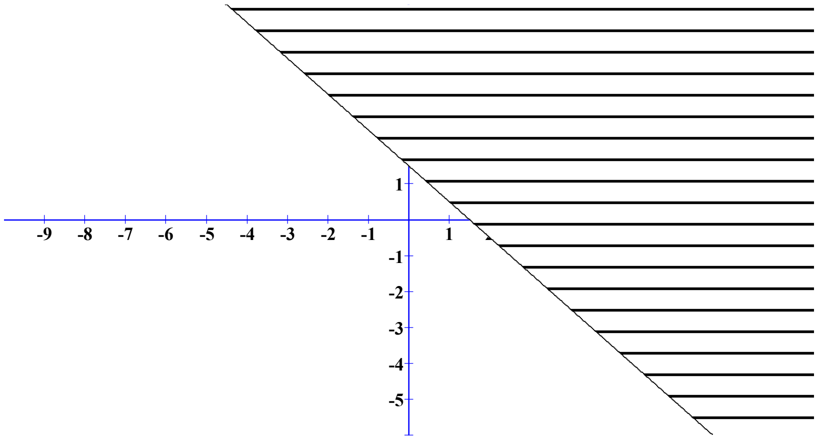
a) The x axis, the y axis, and the line $y = -x + 2$



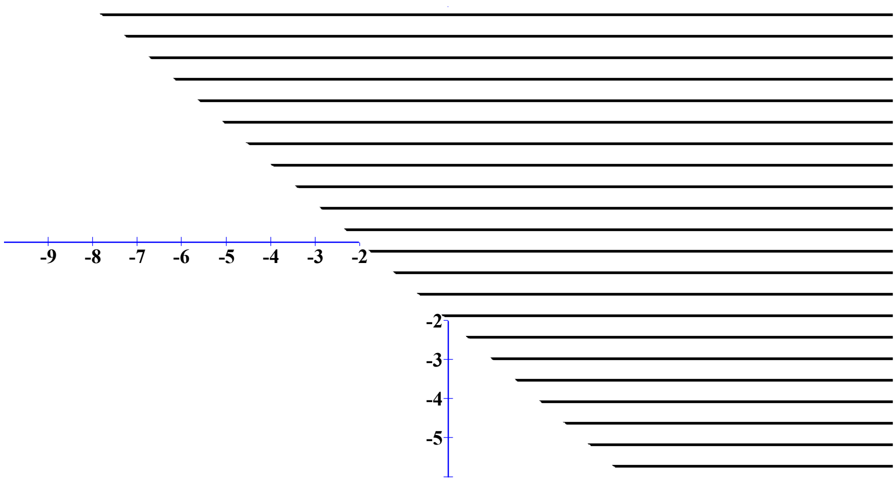
b) $y \leq x + 1$



c) $2x + 2y > 3$



d) $y > -x - 2$



Year 10 Mathematics

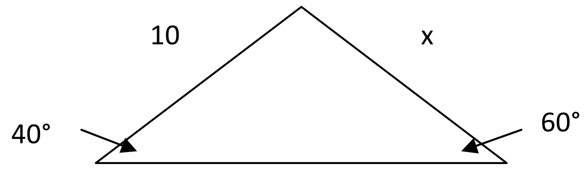
Measurement:

Exercise 1

Non Right Angled Triangles

1) .Use the sine rule to calculate the value of x

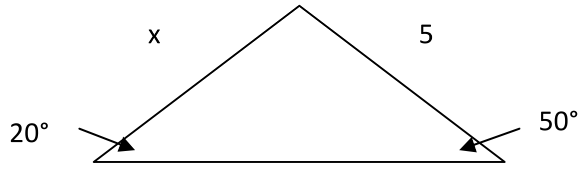
a)



$$\frac{x}{\sin 40°} = \frac{10}{\sin 60°}$$

$$x = \frac{10 \sin 40°}{\sin 60°} = \frac{10 \times 0.643}{0.866} \cong 7.42$$

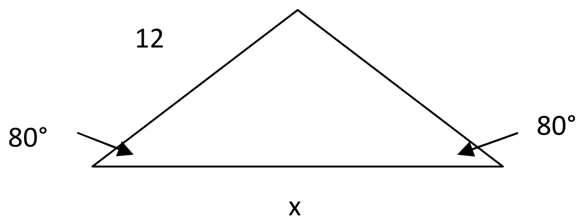
b)



$$\frac{x}{\sin 50°} = \frac{5}{\sin 20°}$$

$$x = \frac{5 \sin 50°}{\sin 20°} = \frac{5 \times 0.766}{0.342} \cong 11.2$$

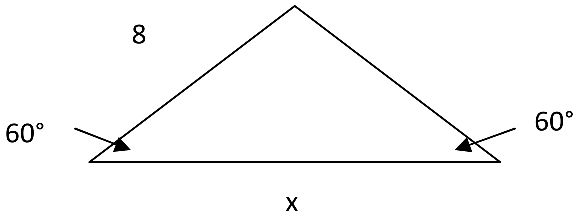
c)



$$\frac{x}{\sin 20°} = \frac{12}{\sin 80°}$$

$$x = \frac{12 \sin 20°}{\sin 80°} = \frac{12 \times 0.342}{0.985} \cong 4.16$$

d)

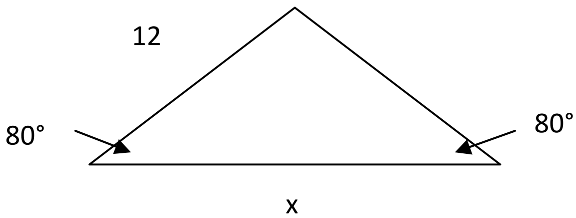


Missing angle is 60°

$$\frac{x}{\sin 60°} = \frac{8}{\sin 60°}$$

$$x = 8$$

e)

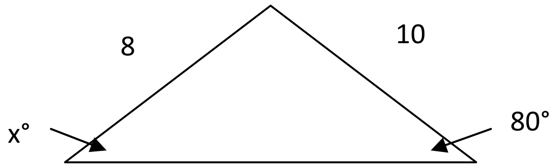


Missing angle is 20°

$$\frac{x}{\sin 20°} = \frac{12}{\sin 80°}$$

$$x = \frac{10 \sin 20°}{\sin 80°} = \frac{10 \times 0.342}{0.985} \cong 34.72$$

f)

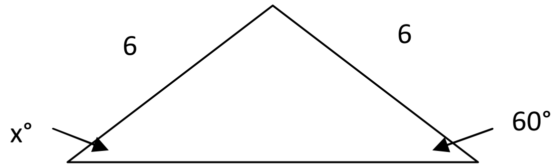


$$\frac{10}{\sin x} = \frac{8}{\sin 80°}$$

$$\sin x = \frac{10 \sin 80°}{8} = \frac{10 \times 0.985}{8} \cong 1.23$$

This triangle is not possible, since $\sin x$ cannot be greater than 1

g)

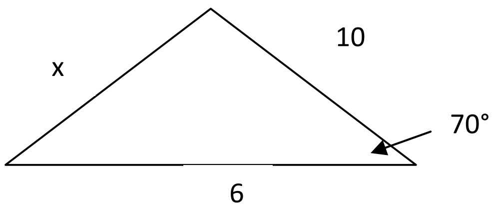


$$\frac{6}{\sin 60°} = \frac{6}{\sin x°}$$

$x = 60°$

2) Calculate the value of x by using the cosine rule

a)

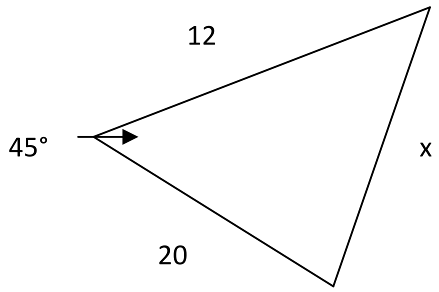


$x^2 = 10^2 + 6^2 - (2 \times 10 \times 6 \times \cos 70°)$

$x^2 = 100 + 36 - (120 \times 0.342) = 94.96$

$x \cong 9.74$

b)

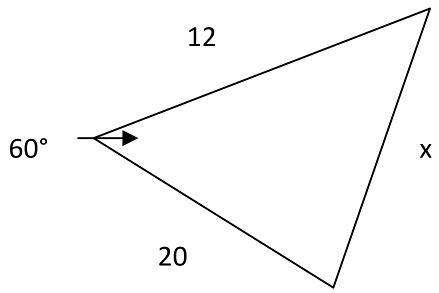


$$x^2 = 12^2 + 20^2 - (2 \times 12 \times 20 \times \cos 45°)$$

$$x^2 = 144 + 400 - (480 \times 0.707) = 204.59$$

$$x \cong 14.3$$

c)

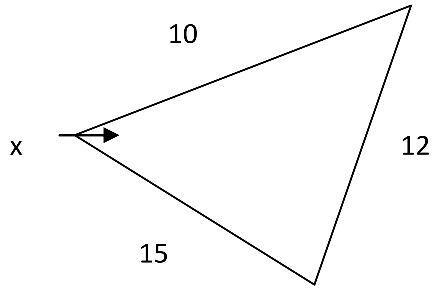


$$x^2 = 12^2 + 20^2 - (2 \times 12 \times 20 \times \cos 60°)$$

$$x^2 = 144 + 400 - (480 \times 0.5) = 344$$

$$x \cong 18.55$$

d)



$$12^2 = 10^2 + 15^2 - (2 \times 10 \times 15 \times \cos x)$$

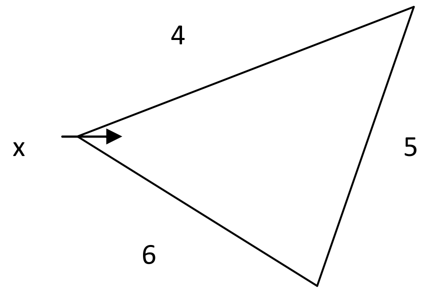
$$144 = 100 + 225 - (300 \times \cos x)$$

$$\cos x = \frac{144 - 325}{-300}$$

$\cos x = \frac{181}{300} \cong 0.603$

$x \cong 52.89°$

e)



$5^2 = 4^2 + 6^2 - (2 \times 4 \times 6 \times \cos x)$

$25 = 16 + 36 - (36 \times \cos x)$

$\cos x = \frac{25 - 52}{-36}$

$\cos x = \frac{27}{36} \cong 0.75$

$x \cong 41.41°$

3) Calculate the areas of the triangles from questions 1 and 2

1a: $A = \frac{1}{2} \times 10 \times 7.42 \times \sin 80° = 36.54$

1b: $A = \frac{1}{2} \times 11.2 \times 5 \times \sin 110° = 26.31$

1c: $A = \frac{1}{2} \times 12 \times 4.16 \times \sin 80° = 24.58$

1d: $A = \frac{1}{2} \times 8 \times 8 \times \sin 60° = 27.71$

1e: $A = \frac{1}{2} \times 12 \times 34.72 \times \sin 80° = 205.16$

1f:Triangle does not exist

1g: $A = \frac{1}{2} \times 6 \times 6 \times \sin 60° = 13.86$

2a: $A = \frac{1}{2} \times 10 \times 6 \times \sin 70° = 28.19$

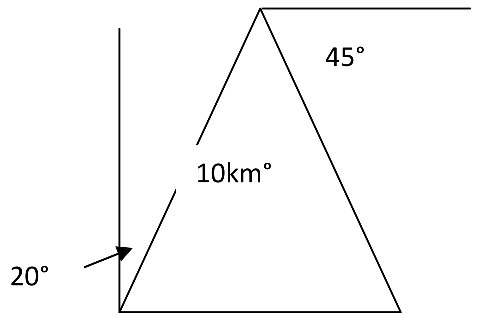
2b: $A = \frac{1}{2} \times 12 \times 20 \times \sin 45° = 84.85$

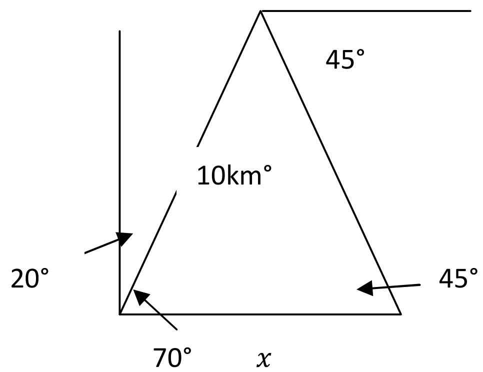
2c: $A = \frac{1}{2} \times 12 \times 20 \times \sin 60° = 103.92$

2d: $A = \frac{1}{2} \times 10 \times 15 \times \sin 52.89° = 59.81$

2e: $A = \frac{1}{2} \times 4 \times 6 \times \sin 41.41° = 7.94$

4) A man walks on a bearing of 20 degrees for 10km, then turns and walks on a bearing of 135 degrees until he is due east of his starting position. How far east of his starting position is he?



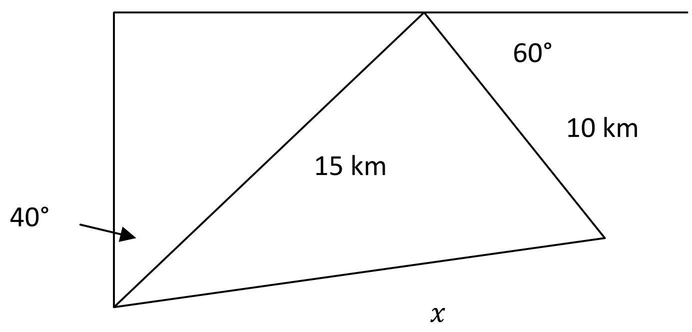


Missing angle in triangle is 65°

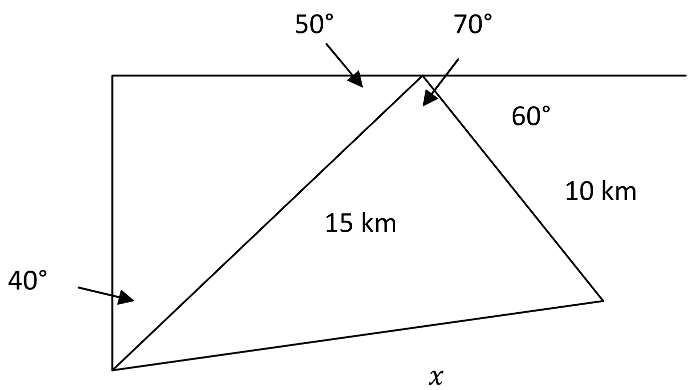
$$\frac{x}{\sin 65°} = \frac{10}{\sin 45°}$$

$$x = \frac{10 \sin 65°}{\sin 45°} \cong 12.81\ km$$

5) A man walks on a bearing of 40 degrees for 15km, then turns and walks on a bearing of 150 degrees for 10km. How far is he away from his starting position? (Use cosine rule)



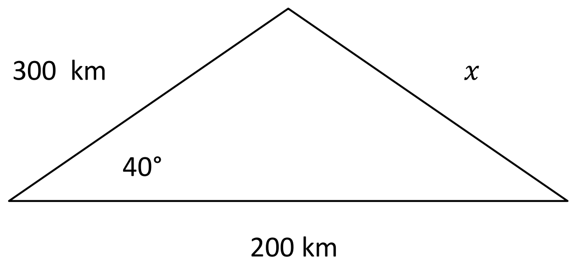
(A bearing of 150° is 60° south of east)



$$x^2 = 15^2 + 10^2 - 2 \times 15 \times 10 \times \cos 70° = 325 - 300 \times 0.342$$

$$x \cong \sqrt{222.4} \cong 14.91\ km$$

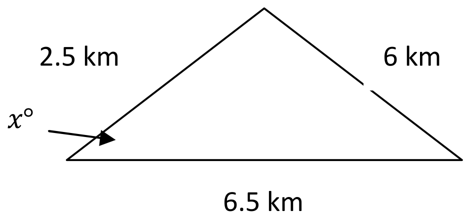
6) Two airplanes leave an airport, and the angle between their flight paths is 40º. An hour later, one plane has travelled 300km while the other has travelled 200km. How far apart are the planes at this time?



$$x^2 = 300^2 + 200^2 - 2 \times 300 \times 200 \times \cos 40° = 130000 - 120000 \times 0.766$$

$$x = \sqrt{38080} \cong 195.14\ km$$

7) A bicycle race follows a triangular course. The three legs of the race in order are 2.5km, 6km and 6.5km. Find the angle between the starting leg and the finishing leg to the nearest degree.

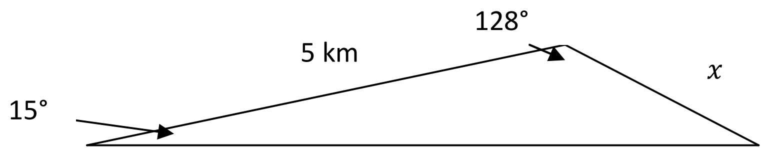


$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos x = \frac{2.5^2 + 6.5^2 - 6^2}{2 \times 2.5 \times 6.5} = \frac{6.25 + 42.25 - 36}{32.5} \cong 0.385$$

$$x \cong 67.3°$$

8) A straight road slopes upward at 15 degrees from the horizontal. The hill forms an angle of 128 degrees at its apex. If the distance up the hill is 5 km, how far is the distance down the hill?

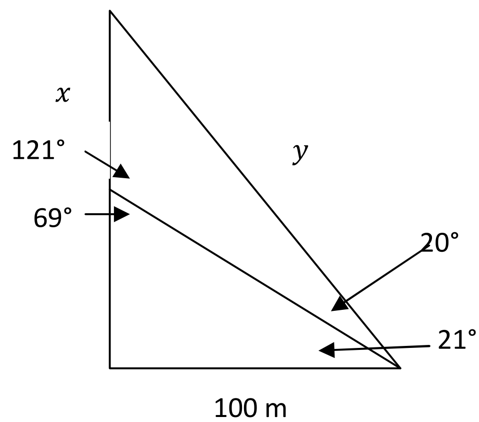


The missing angle in the triangle is $180 - 128 - 15 = 37°$

$$\frac{x}{\sin 15°} = \frac{5}{\sin 37°}$$

$$x = \frac{5 \times \sin 15°}{\sin 37°} = \frac{1.294}{0.602} \cong 2.15\ km$$

9) A building is of unknown height. At a distance of 100 metres away from the building, an observer notices that the angle of elevation to the top is 41º and that the angle of elevation to a poster on the side of the building is 21º. How far is the poster from the roof of the building



By first using right angled trigonometry, the length of the common hypotenuse (y) can be calculated

$$\cos 41° = \frac{100}{y}$$

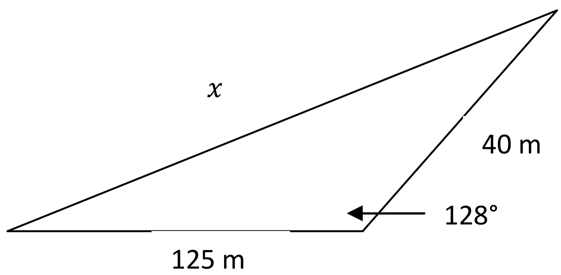
$$y = \frac{100}{\cos 41°} = 132.45\ m$$

Using the sine rule for the top triangle:

$$\frac{x}{\sin 20°} = \frac{132.45}{\sin 121°}$$

$$x = \frac{132.45 \times 0.342}{0.857} \cong 52.86\ m$$

10) An observer is near a river and wants to calculate the distance across the river to a point on the other side. He ties a rope to a point on his side of the river that is 125 metres away from him. The angle formed by him, the point on his side of the river, and the point on the opposite side of the river is 128°, and the distance from point to point is 40 metres. What is the distance from him to the point across the river?



$x^2 = 125^2 + 40^2 - 2 \times 125 \times 40 \times \cos 128°$

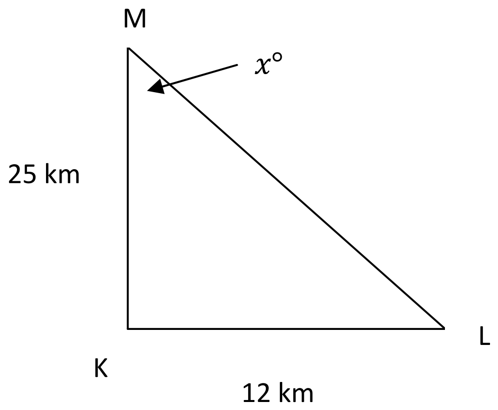
$x^2 = 15625 + 1600 - 10000 \times (-0.616) = 23385$

$x \cong 152.92\ m$

Exercise 2

Bearings

1) A point K is 12km due west of a second point L and 25km due south of a third point M. Calculate the bearing of L from M

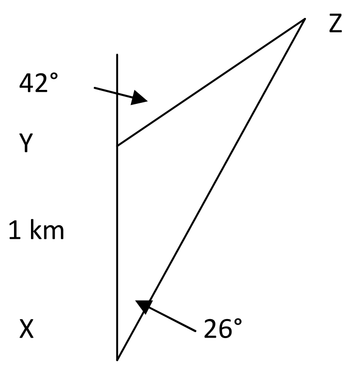


$$\tan x = \frac{12}{25}$$

$$x = 25.64°$$

$$\text{Bearing} = 180 - 25.64 = 154.36°$$

2) Point Y is 1km due north of point X. The bearings of point Z from X and Y are 26° and 42° respectively. Calculate the distance from point Y to point Z.

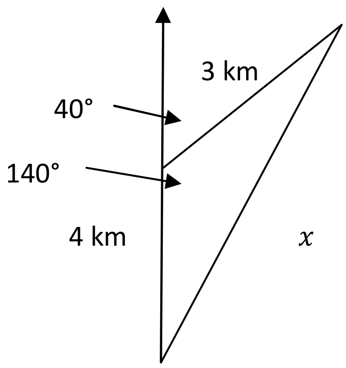


Angle Z is 16°

$$\frac{YZ}{\sin 26°} = \frac{1}{\sin 16°}$$

$$YZ = \frac{\sin 26°}{\sin 16°} = \frac{0.438}{0.276} \cong 1.59\ km$$

3) A ship steams 4km due north of a point then 3km on a bearing of 040°. Calculate the direct distance between the starting and finishing points.

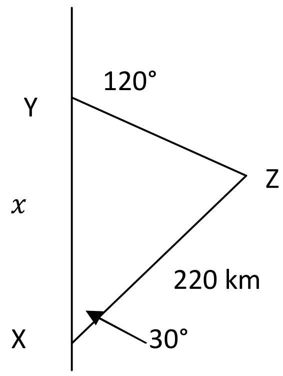


$x^2 = 3^2 + 4^2 - 2 \times 3 \times 4 \times \cos 140°$

$x^2 = 9 + 13 - 24 \times (-0.766) = 22 + 18.384 = 40.384$

$x = 6.35\ km$

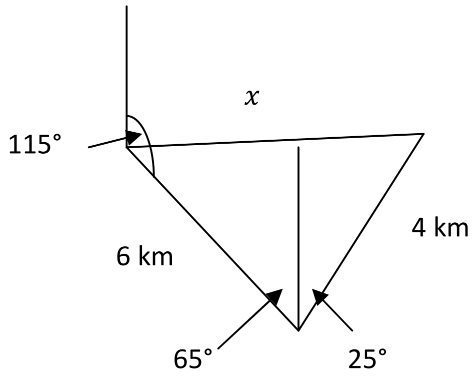
4) The bearings of a point Z from two points X and Y are 30° and 120°. The distance from X to Z is 220km. What is the distance of X from Y?



Z is a right angle, so $\cos 30° = \frac{220}{x}$

$$x = \frac{220}{\cos 30°} \cong 254\ km$$

5) A man walked along a road for 6km on a bearing of 115°. He then changed course to a bearing of 25° and walked a further 4km. Find the distance and bearing from his starting point



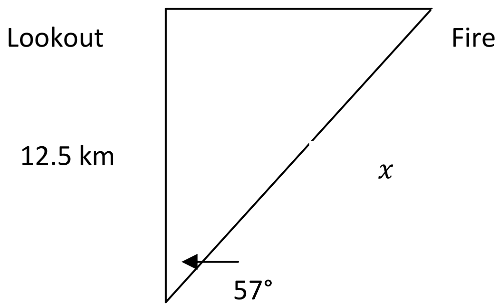
$x^2 = 4^2 + 6^2 = 52$

$x = 7.21\ km$

$\tan\theta = \frac{4}{6}$

$\theta = 33.69°$

6) Directly east of a lookout station, there is a small forest fire. The bearing of this fire from another station 12.5 km. south of the first is 57°. How far is the fire from the southerly lookout station?

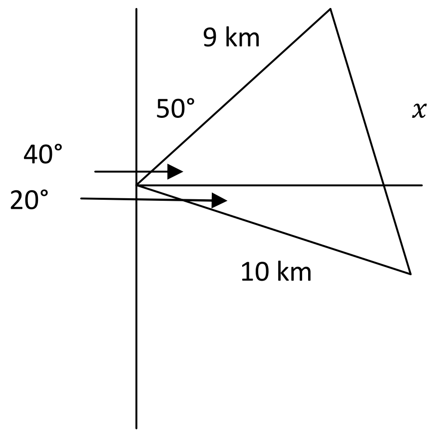


$\cos 57° = \frac{12.5}{x}$

$$x = \frac{12.5}{\cos 57°} = 22.95\ km$$

7) Mark and Ron leave a hostel at the same time. Mark walks on a bearing of 050° at a speed of 4.5 kilometres per hour. Ron walks on a bearing of 110° at a speed of 5 kilometres per hour. If both walk at steady speeds, how far apart will they be after 2 hours?

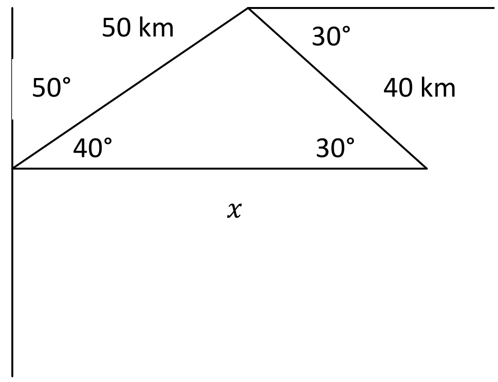
8)



$$x^2 = 9^2 + 10^2 - 2 \times 9 \times 10 \times \cos 60° = 181 - 180 \times \frac{1}{2}$$

$$x = \sqrt{91} \cong 9.54\ km$$

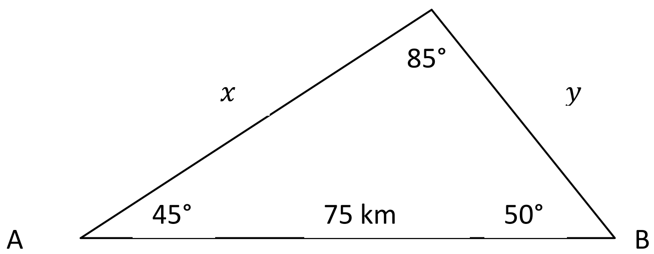
9) A ship leaves a harbour on a bearing of 50° and sails 50km. It then turns on a bearing of 120° and sails for another 40km. How far is the ship from its starting point?



$$x^2 = 50^2 + 40^2 - 2 \times 50 \times 40 \times \cos 110° = 4100 - 400 \times (-0.342)$$

$$x = \sqrt{4100 + 136.8} \cong 65.1\ km$$

10) Two ships A and B are anchored at sea. B is 75km due east of A. A lighthouse is positioned on a bearing of 045° from A and on a bearing of 320° from B. Calculate how far the lighthouse is from ship



$$\frac{75}{\sin 85°} = \frac{x}{\sin 50°} = \frac{y}{\sin 45°}$$

$$x = \frac{75 \times \sin 50°}{\sin 85°} \cong \frac{57.45}{0.996} \cong 57.68\ km$$

$$y = \frac{75 \times \sin 45°}{\sin 85°} \cong \frac{53.03}{0.996} \cong 53.25\ km$$

Year 10 Mathematics

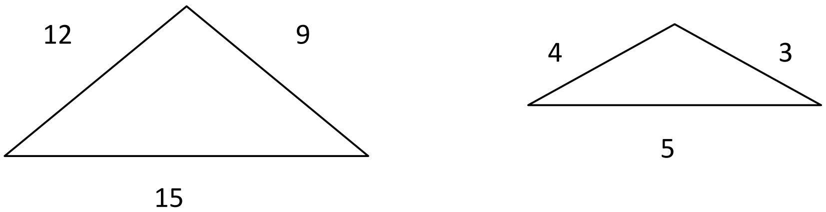
Space

Exercise 1

Congruence & Similarity

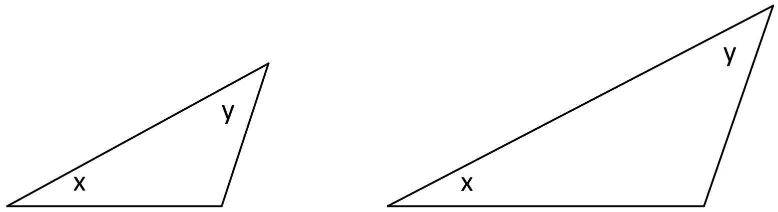
1) Decide if the following triangles are similar, and if so state the similarity conditions

a)



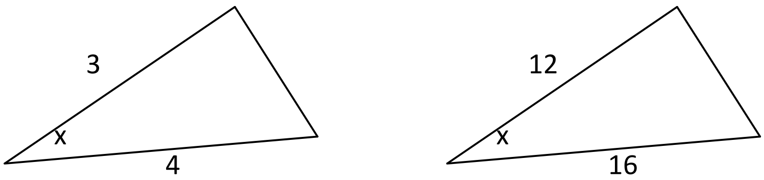
SSS

b)



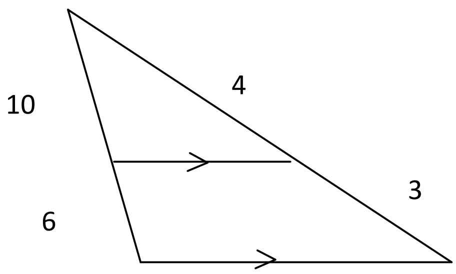
AA (and by extension AAA)

c)



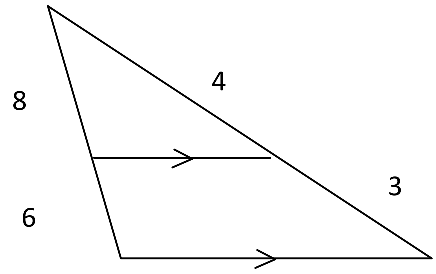
SAS

d)



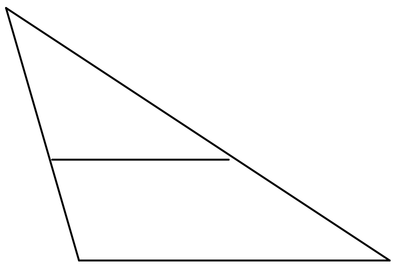
Not similar ($\frac{4}{7} \neq \frac{10}{16}$

e)



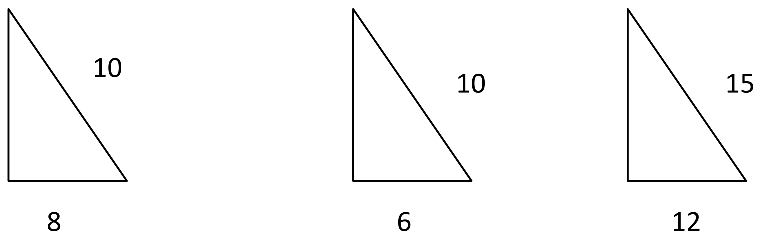
SAS (the sides are in proportion and they share a common angle

2) What additional information is needed to show that the two triangles are similar by AAA?



That the two horizontal lines are parallel; this would mean that the corresponding angles would be equal

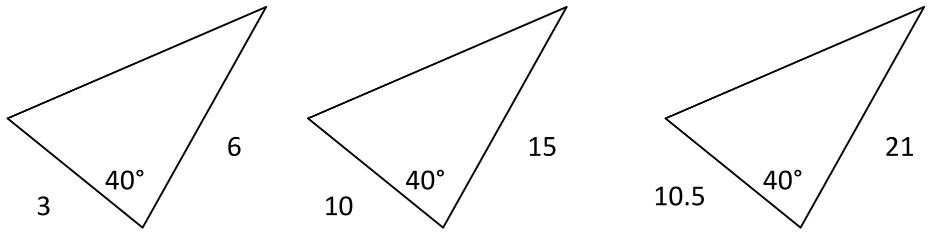
3) Of the following three right-angled triangles, which two are similar and why?



The lengths of the sides of the first and third triangles are in the ratio 4:5

They are similar under the SSS condition

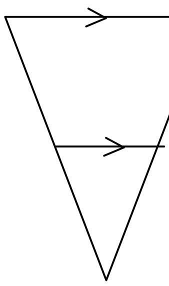
4) Of the following three triangles, which are similar and why?



The lengths of the sides of the first and third triangles are in the ratio 1:2

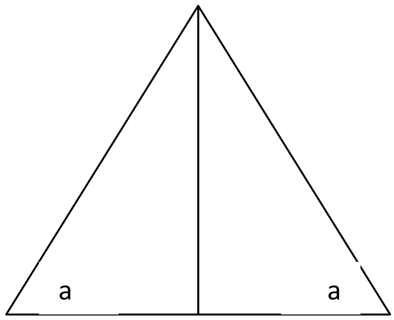
They are similar under the SAS condition

5) Prove that the two triangles in the diagram are similar



Since the bases of the triangles are parallel, their corresponding angles are equal. Since they also share a common angle, they are similar under the AAA condition

6) Prove that if two angles of a triangle are equal then the sides opposite those angles are equal



Draw a line from the apex of the triangle to the base

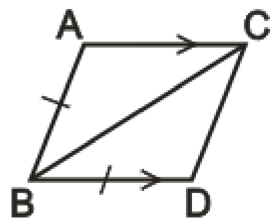
This line is a common line of the two triangles formed

Since the line is perpendicular to the base, each triangle has an angle of 90°

The triangles formed are congruent under the AAS condition

Therefore the corresponding sides (the sides opposite the equal angles a) are equal

7) Is triangle ABC congruent to DBC? If so, explain why.

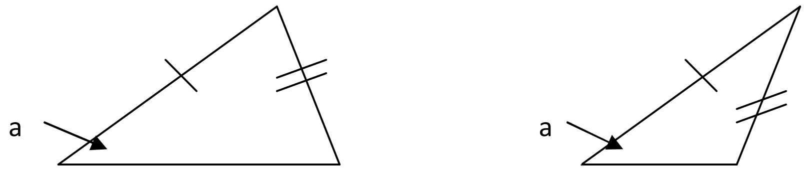


Yes: angle C is equal to angle B (alternate interior angles); they are congruent under condition SAS

8) Which of the following is NOT a valid test for congruence?

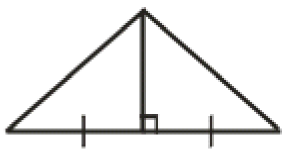
SSA, ASA, AAS, SAS

SSA is not sufficient



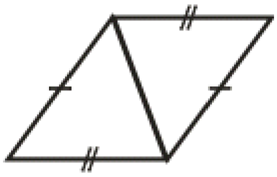
Angles a are equal as are the designated sides, but the triangles are not congruent

9) State whether or not the following triangles are congruent. If so, state a reason



SAS

10) State whether or not the following triangles are congruent. If so, state a reason.

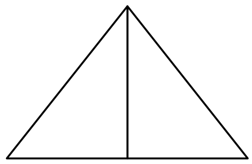


SSS

Exercise 2

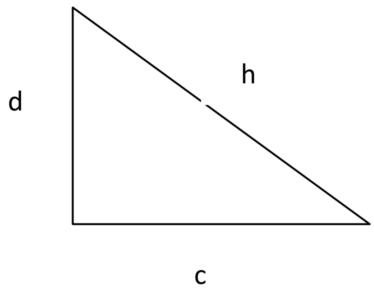
Triangle Proofs

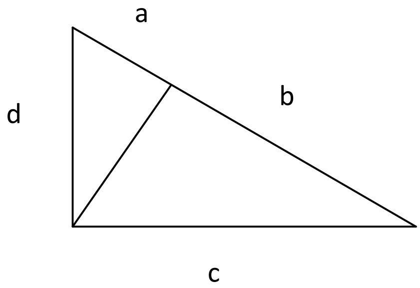
1) Prove that the size of each angle of an equilateral triangle are equal



Drawing a perpendicular line from the apex to the base produces two congruent triangles (HL). Therefore the corresponding angles are equal. Repeat this process from every angle to prove that all angles are equal

2) Using similar triangles prove Pythagoras' theorem





Divide the right angled triangle into two smaller right angled triangles by drawing a line perpendicular to the hypotenuse (see diagram), such that a + b = h

There are 3 similar triangles now shown: the original right angled triangle and the two smaller ones formed

Then $\frac{c}{a+b} = \frac{b}{c}$ and $\frac{d}{a+b} = \frac{a}{d}$

Rearranging the equations gives

$c^2 = b(a + b)$ and $d^2 = a(a + b)$

Adding the equations gives:$c^2 + d^2 = (a + b)(a + b) = h^2$

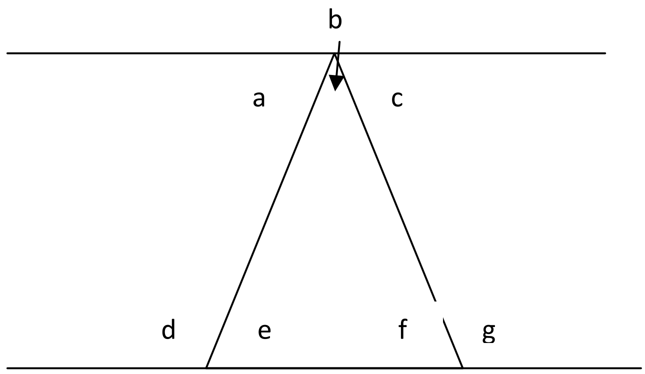
3) Triangles HIJ and MNO are similar. The perimeter of smaller triangle HIJ is 44. The lengths of two corresponding sides on the triangles are 13 and 26. What is the perimeter of MNO? Draw a diagram to help solve the problem

The sides are in the ratio 1:2

The sum of the two other sides of HIJ is 31, therefore the sum of the two other sides of MNO is 62

Therefore the perimeter of MNO is88

4) Prove that the sum of the internal angles of any triangle is 180°



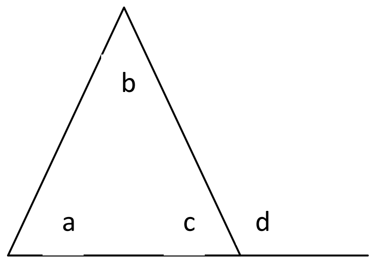
Since the lines at the top of the triangle and the base are parallel, angle a= angle e, and angle c = angle f

Since angles a, b, and c form a straight line, a + b + c = 180°

Substituting equal angles from above, e + b + f = 180°

e, b and f are the internal angles

5) Prove that the size of each exterior angle of a triangle is equal to the sum of the opposite interior angles



$c + d = 180°$

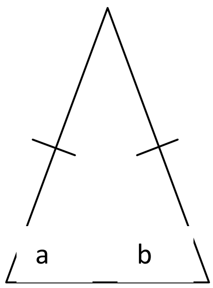
From question 4, $a + b + c = 180°$

So $a + b + c = c + d$

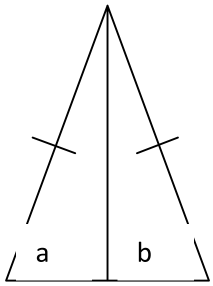
Therefore $a + b = d$

This can be similarly shown for all sets of angles in the triangle

6) Prove that in an isosceles triangle, the base angles are congruent

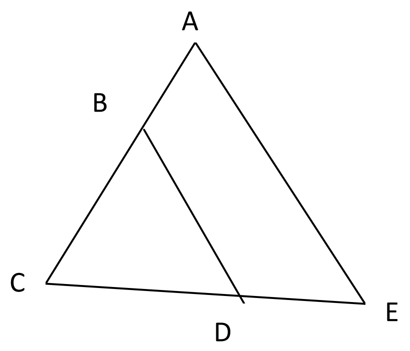


Draw a perpendicular line from the apex to the base



Then the triangles formed are congruent (HL), so angle a = angle b

7)



If AC = AE and AE || BD, prove that BD = BC

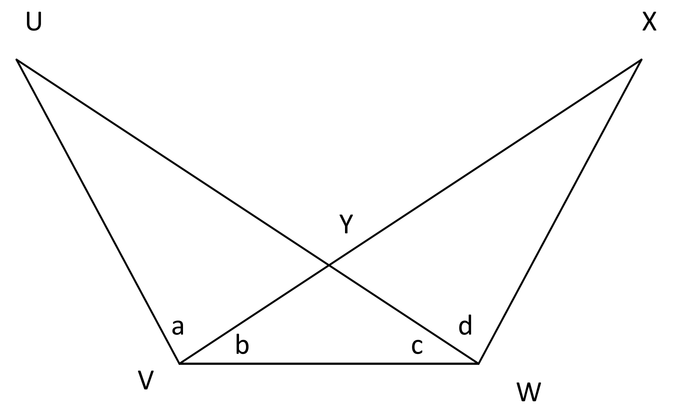
Angle C = Angle E (since ACE is isosceles)

The triangles are similar (AAA)

Therefore $\frac{AC}{AE} = \frac{BC}{BD}$

Since AC = AE, the ratio is 1:1, and so BC = BD

8) If for angles a, b, c, and d, a = d and b = c, prove that UW = XV



b = c

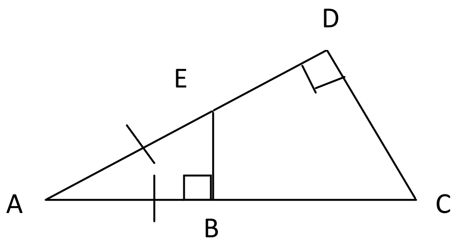
a + b = c + d

Then the triangles UVW and XWV are congruent (ASA)

The common side is VW

Therefore UW = XV

9)



Prove that triangle AEB is similar to triangle ACD

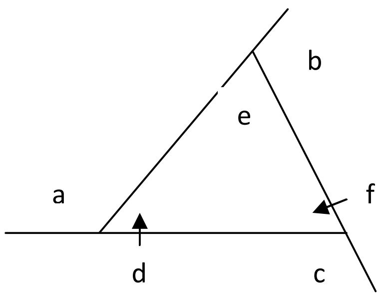
Since side AE = AB, angle EAB = angle AEB

Also angle DAC = angle EAB (they are the same angle)

Therefore angle DAC = AEB and angle DCA = BAE

The triangles are similar under condition AAA

10) Prove that the exterior angles of a triangle add to 360°



We know that d + e + f = °180

Also a + d = 180°, e + b = 180°, and c + f = 180°

So a + b + c + d + e + f = 540° = a + b + c + 180°

Therefore a + b + c = 360°

Exercise 3

Properties of Quadrilaterals

1) What is the definition of a parallelogram?

A four sided two dimensional shape with opposite sides parallel

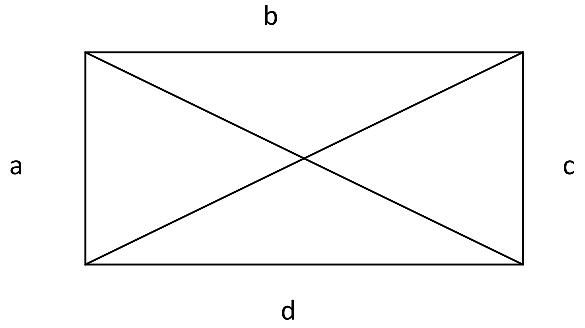
2) Prove that the exterior opposite angles of a parallelogram are equal

We know that angle b = angle c

Also a + b = c + d

Therefore angle a = angle d

3) Show that the diagonals of a rectangle are equal

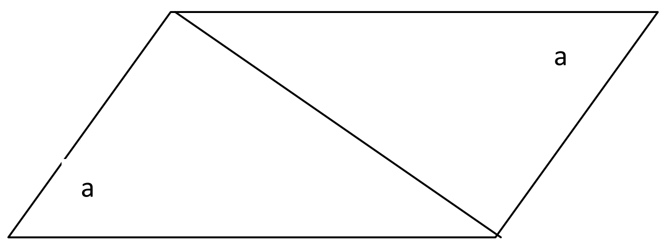


We know that a = c and b = d

So $a^2 + d^2 = b^2 + c^2$

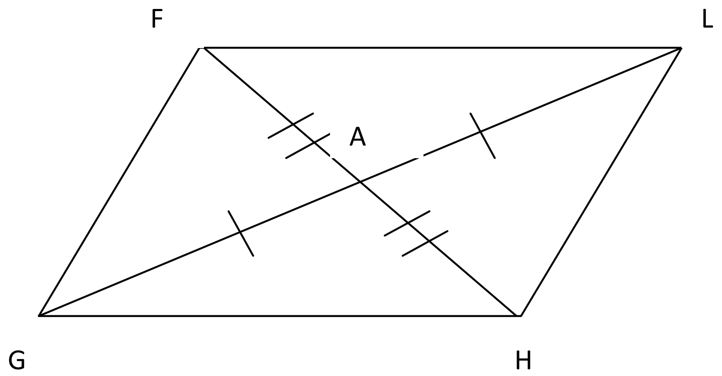
The diagonals are the hypotenuse of the two right angled triangles (one with sides a and, d, the other with sides b and c), and are therefore equal

4) Prove that, if the opposite angles of a quadrilateral are equal, then the quadrilateral is a parallelogram.



The triangles formed by the diagonal are congruent (ASA), therefore the opposite pairs of sides are congruent

5) FGHL is a quadrilateral and the diagonals LG and FH bisect each other. Prove that FGHL is a parallelogram

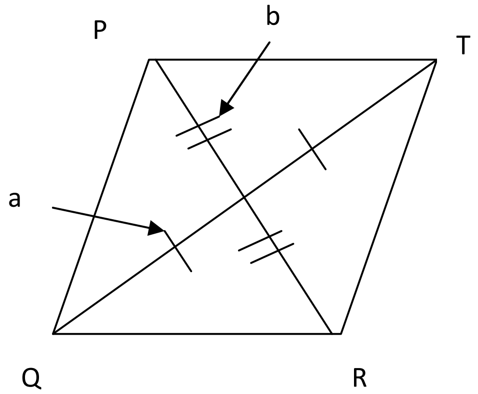


Triangles GAF and LAH are congruent (their angles are vertically opposite therefore equal), by SAS

Therefore FG is congruent to HL

Similarly FL is congruent to HG

6) PQRT is a quadrilateral and the diagonals PR and QT bisect each other at right angles. Prove that PQRT is a rhombus



$PQ^2 = a^2 + b^2$

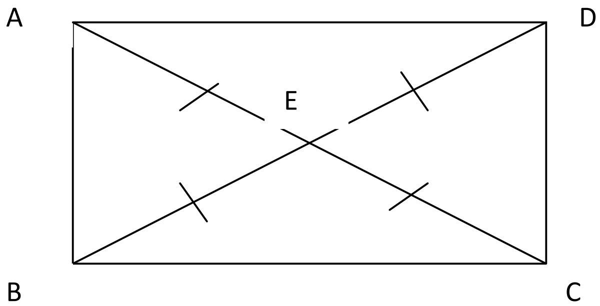
$PT^2 = a^2 + b^2$

$RT^2 = a^2 + b^2$

$RQ^2 = a^2 + b^2$

Therefore $PQ = PT = RT = RQ$

7) ABCD is a quadrilateral. The diagonals AC and BD are equal and bisect each other. Prove that ABCD is a rectangle



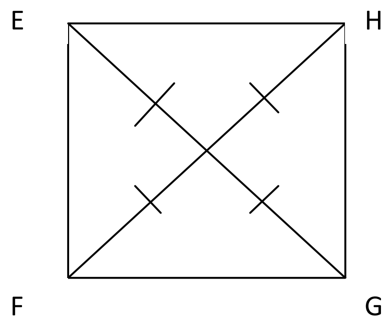
Angles EAD, EDA, EBC, ECB, EAB, EBA, EDC and ECD are all equal since they are each opposite the equal sides of an isosceles triangle

The sum of the internal angles of a quadrilateral is 360°

Therefore each angle is 45°

Therefore each pair of angles forms a right angle (e.g. EAD and EAB)

8) EFGH is a quadrilateral. The diagonals EG and FH are equal and bisect each other at right angles. Prove that EFGH is a square



Let the length of each half diagonal be a

Then $(EF)^2 = a^2 + a^2 = 2a^2$

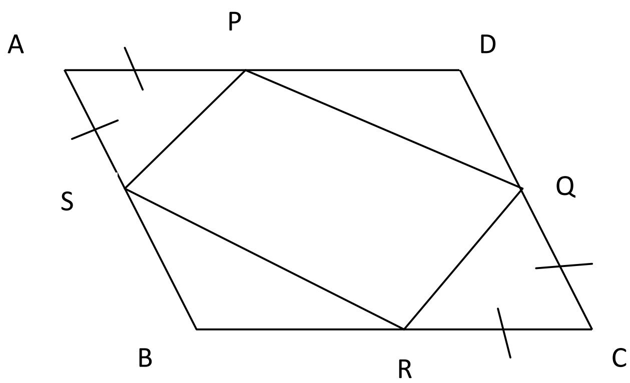
Similarly, $(EH)^2 = a^2 + a^2 = 2a^2$

$(HG)^2 = a^2 + a^2 = 2a^2$

$(FG)^2 = a^2 + a^2 = 2a^2$

Therefore EF = EH = HG = FG, and as per question 7, the angles formed at each vertex are right angles

9) ABCD is a parallelogram. AP = AS = CQ = CR. Prove that PQRS is a parallelogram



Since DC = AB, then if AS = CQ, DQ = BS

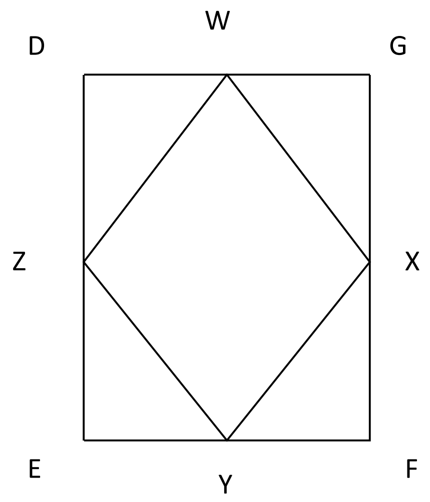
Similarly BR = DQ

Triangle APS is congruent to triangle RCQ (SAS)

Therefore PS = RQ

Similarly PQ = RS (since DQ = BS = DP =BR)

10) DEFG is a rectangle. W X Y and Z are the midpoints of the sides. Prove that WXYZ is a rhombus



DZ = EZ and DW = EY, therefore triangle EYZ is congruent to triangle DWZ (SAS)

Therefore WZ = YZ

Similarly WX = YX, WZ = WX, and YZ = YX

Therefore all sides of WZYX are equal in length