Year 9 Mathematics Solutions

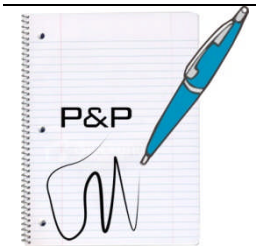
Learning Strategies

Mathematics is often the most challenging subject for students. Much of the trouble comes from the fact that mathematics is about logical thinking, not memorizing rules or remembering formulas. It requires a different style of thinking than other subjects. The students who seem to be "naturally" good at math just happen to adopt the correct strategies of thinking that math requires – often they don't even realise it. We have isolated several key learning strategies used by successful maths students and have made icons to represent them. These icons are distributed throughout the book in order to remind students to adopt these necessary learning strategies:

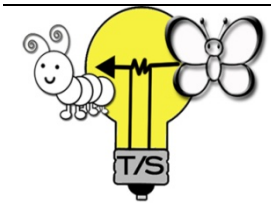
Talk Aloud Many students sit and try to do a problem in complete silence inside their heads. They think that solutions just pop into the heads of 'smart' people. You absolutely must learn to talk aloud and listen to yourself, literally to talk yourself through a problem. Successful students do this without realising. It helps to structure your thoughts while helping your tutor understand the way you think.

BackChecking This means that you will be doing every step of the question twice, as you work your way through the question to ensure no silly mistakes. For example with this question: $3 \times 2 - 5 \times 7$ you would do "3 times 2 is 5 ... let me check – no 3×2 is 6 ... minus 5 times 7 is minus 35 ... let me check ... minus 5×7 is minus 35. Initially, this may seem time-consuming, but once it is automatic, a great deal of time and marks will be saved.

Avoid Cosmetic Surgery Do not write over old answers since this often results in repeated mistakes or actually erasing the correct answer. When you make mistakes just put one line through the mistake rather than scribbling it out. This helps reduce silly mistakes and makes your work look cleaner and easier to backcheck.



Pen to Paper It is always wise to write things down as you work your way through a problem, in order to keep track of good ideas and to see concepts on paper instead of in your head. This makes it easier to work out the next step in the problem. Harder maths problems cannot be solved in your head alone – put your ideas on paper as soon as you have them – always!



Transfer Skills This strategy is more advanced. It is the skill of making up a simpler question and then transferring those ideas to a more complex question with which you are having difficulty.

For example if you can't remember how to do long addition because you can't recall exactly how to carry the one: $\begin{array}{r} 5889 \\ +\,4587 \\ \hline \end{array}$ then you may want to try adding numbers which you do know how to calculate that also involve carrying the one: $\begin{array}{r} 5 \\ +\,9 \\ \hline \end{array}$

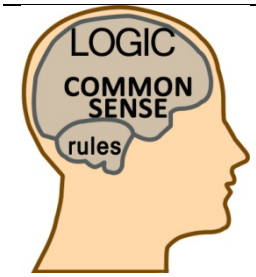
This skill is particularly useful when you can't remember a basic arithmetic or algebraic rule, most of the time you should be able to work it out by creating a simpler version of the question.

Format Skills These are the skills that keep a question together as an organized whole in terms of your working out on paper. An example of this is using the "=" sign correctly to keep a question lined up properly. In numerical calculations format skills help you to align the numbers correctly.

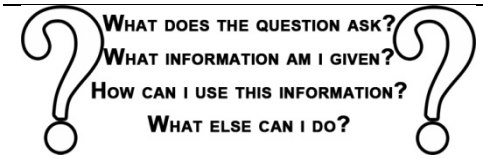
This skill is important because the correct working out will help you avoid careless mistakes. When your work is jumbled up all over the page it is hard for you to make sense of what belongs with what. Your "silly" mistakes would increase. Format skills also make it a lot easier for you to check over your work and to notice/correct any mistakes.

Every topic in math has a way of being written with correct formatting. You will be surprised how much smoother mathematics will be once you learn this skill. Whenever you are unsure you should always ask your tutor or teacher.

Its Ok To Be Wrong Mathematics is in many ways more of a skill than just knowledge. The main skill is problem solving and the only way this can be learned is by thinking hard and making mistakes on the way. As you gain confidence you will naturally worry less about making the mistakes and more about learning from them. Risk trying to solve problems that you are unsure of, this will improve your skill more than anything else. It's ok to be wrong – it is NOT ok to not try.



Avoid Rule Dependency Rules are secondary tools; common sense and logic are primary tools for problem solving and mathematics in general. Ultimately you must understand Why rules work the way they do. Without this you are likely to struggle with tricky problem solving and worded questions. Always rely on your logic and common sense first and on rules second, always ask Why?



Self Questioning This is what strong problem solvers do naturally when they get stuck on a problem or don't know what to do. Ask yourself these questions. They will help to jolt your thinking process; consider just one question at a time and Talk Aloud while putting Pen To Paper.

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Year 9 Mathematics

Number

Exercise 1

Indices

1) Evaluate the following

a) 2^3

$= 2 \times 2 \times 2 = 8$

b) 3^2

$= 3 \times 3 = 9$

c) 1^6

$= 1 \times 1 \times 1 \times 1 \times 1 \times 1$
$= 1$

d) 4^3

$= 4 \times 4 \times 4 = 64$

e) 5^0

$= 1$

2) Express the following as powers of 2

a) 16

$= 2 \times 2 \times 2 \times 2 = 2^4$

b) 2

$= 2^1$

c) 1

$= 2^0$

d) 64

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 2^6$

e) 32

$= 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

3) Express the following as powers of 3

a) 9

$= 3 \times 3 = 3^2$

b) 81

$= 3 \times 3 \times 3 \times 3 = 3^4$

c) 3

$= 3^1$

d) 27

$= 3 \times 3 \times 3 = 3^3$

e) 1

$= 3^0$

4) Evaluate the following

a) 2^{-1}

$= \frac{1}{2^1} = \frac{1}{2}$

b) 3^{-2}

$= \frac{1}{3^2} = \frac{1}{9}$

c) 2^{-3}

$= \frac{1}{2^3} = \frac{1}{8}$

d) 4^{-2}

$= \frac{1}{4^2} = \frac{1}{16}$

e) 2^{-4}

$= \frac{1}{2^4} = \frac{1}{16}$

5) Evaluate the following

a) $2^3 \times 2^2$

$= 2^{3+2} = 2^5 = 32$

b) $3^1 \times 3^2$

$= 3^{1+2} = 3^3 = 27$

c) $4^1 \times 4^1$

$= 4^{1+1} = 4^2 = 16$

d) $3^3 \times 3^0$

$= 3^{3+0} = 3^3 = 27$

e) $5^2 \times 5^2$

$= 5^{2+2} = 5^4 = 625$

6) Evaluate the following

a) $2^5 \times 2^{-3}$

$= 2^{5-3} = 2^2 = 4$

b) $3^{-5} \times 3^6$

$= 3^{-5+6} = 3^1 = 3$

c) $2^{-1} \times 2^{-2}$

$= 2^{-1-2} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

d) $4^{-4} \times 4^6$

$= 4^{-4+6} = 4^2 = 16$

e) $13^6 \times 13^{-6}$

$= 13^{6-6} = 13^0 = 1$

7) Evaluate the following

a) $3^5 \div 3^3$

$= 3^{5-3} = 3^2 = 9$

b) $2^{10} \div 2^8$

$2^{10-8} = 2^2 = 4$

c) $4^4 \div 4^4$

$= 4^{4-4} = 4^0 = 1$

d) $10^3 \div 10^2$

$= 10^{3-2} = 10^1 = 10$

e) $1^{10} \div 1^6$

$= 1^{10-6} = 1^4 = 1$

8) Evaluate the following

a) $3^2 + 2^2$

$= 9 + 4 = 13$

b) 5^2

$= 5 \times 5 = 25$

c) $3^2 \times 2^2$

$9 \times 4 = 36$

d) $(3 \times 2)^2$

$= 6^2 = 36$

e) $4^2 + 5^2$

$16 + 25 = 41$

f) 9^2

$= 9 \times 9 = 81$

g) $4^2 \times 5^2$

$= 16 \times 25 = 400$

h) $(4 \times 5)^2$

$= 9^2 = 81$

9) From your answers to question 8, which of the following statements are true and which are false?

a) $a^2 + b^2 = (a + b)^2$

False

b) $(a + b)^2 = a^2 \times b^2$

False

c) $(a \times b)^2 = a^2 \times b^2$

True

d) $(a \times b)^2 = (a + b)^2$

False

10) Find the values of x and y in the following

$2^6 = 8^x = y^3$

$8 = 2^3$

Therefore, $8^x = 2^{3x}$

$2^{3x} = 2^6$

$x = 2$

So $8^2 = y^3$

$y^3 = 64$

$y = 4$

Exercise 2

Scientific Notation & Significant Figures

1) Express the following in scientific notation

a) 3125

3.125×10^3

b) 1000

1×10^3

c) 14250

1.425×10^4

d) 105000

1.05×10^5

e) 775

7.75×10^2

f) 7777

7.777×10^3

2) Convert the following to decimal form

a) 3.233×10^2

323.3

b) 4.1002×10^3

4100.2

c) 7.06×10^4

70600

d) 5.007×10^2

500.7

e) 3.0207×10^3

3020.7

f) 1.00001×10^5

100001

3) Express the following in scientific notation

a) 0.1005

1.005×10^{-1}

b) 0.0514

5.14×10^{-2}

c) 0.75

7.5×10^{-1}

d) 0.000523123

5.23123×10^{-4}

e) 0.0554

5.54×10^{-2}

f) 6.5121

6.5121×10^0

4) Convert the following to decimal form

a) 3.452×10^{-2}

0.03452

b) 2.6552×10^{-1}

0.26552

c) 7.5×10^{-3}

0.0075

d) 1.423×10^{-4}

0.0001423

5) Use your knowledge of index laws and scientific notation to estimate the following products (see example at beginning of chapter)

a) (3.15 x 10^3) x (5.22 x 10^4)

$\cong 3 \times 5 \times 10^3 \times 10^4$

$= 15 \times 10^7$

$= 1.5 \times 10^8$

b) (4.85 x10^2) x (6.33 x 10^5)

$\cong 5 \times 6 \times 10^2 \times 10^5$

$= 30 \times 10^7 = 3 \times 10^8$

c) (2.96 x10^4) x (4.98 x 10^6)

$\cong 3 \times 5 \times 10^4 \times 10^6$

$= 15 \times 10^{10}$

$= 1.5$

$\times 10^{11}$

d) (6.05 x10^7 ÷3.11 x 10^3)

$\cong (6 \div 3) \times (10^7 \div 10^3)$

$= 2 \times 10^4$

6) Round the following to 4 significant figures

a) 42.7567

42.76

b) 0.39848

0.3985

c) 17152.54

17150

d) 11.111111

11.11

7) Round the following to 3 significant figures

a) 19.672

19.7

b) 555.55

556

c) 1012

1010

d) 0.82556

0.826

e) 212.75

213

f) 10001

10000

8) The speed of light in a vacuum is 3 x 10^8 metres per second

a) How far does light travel in 10 seconds?

$3 \times 10^8 \times 10 = 3 \times 10^9\ m$

b) How long does light take to travel 90000 metres?

$90000 = 9 \times 10^4\ m$

Time $= Distance \div speed$

Time $= \frac{9 \times 10^4}{3 \times 10^8}$ seconds$= 3 \times 10^{-4}$ seconds$= 0.0003$ seconds

c) There are approximately 31.5 million seconds in a year. How far does light travel in one year?

31.5 million $= 3.15 \times 10^7$

Distance $= Speed \times time$

Distance $= 3 \times 10^8 \times 3.15 \times 10^7 = 9.45 \times 10^{15} metres = 9.45 \times 10^{12} km$

9) The closest star to Earth is approximately 4.3 light years away. How long would it take a rocket travelling at 40,000 km per hour to reach it?

4.3 light years $= 4.3 \times 9.45 \times 10^{12} = 40.635 \times 10^{12} km \cong 40 \times 10^{12} km$

Time = Distance ÷speed

Time $\frac{40 \times 10^{12}}{40 \times 10^{3}} = 1 \times 10^{9}\ hours$

There are approximately 8760 hours in a year

$$\frac{1 \times 10^{9}}{8760} = 1.14 \times 10^{-4} \times 10^{9} = 1.14 \times 10^{5} years$$

Exercise 3

Consumer Arithmetic

1) You get a part time job that pays a wage of $15 per hour. How much would you earn (before tax) in each of the following weeks?

a) Worked 10 hours

$10 \times 15 = \$150$

b) Worked 12 hours

$12 \times 15 = \$180$

c) Worked 32 hours

$32 \times 15 = \$480$

d) Worked 15 hours

$15 \times 15 = \$225$

e) Worked 9 hours

$9 \times 15 = \$135$

f) Worked 36 hours

$36 \times 15 = \$540$

2) In your next job you are given a salary of $38,000 per annum. How much would you be paid:

a) Per month

$38000 \div 12 = \$3166.67$

b) Per week

$38000 \div 52 = \$730.77$

c) Per fortnight

$38000 \div 26 = \$1461.54$

d) Per day

There are 260 working days assumed in the year

$38000 \div 260 = \$146.15$

(Assume an equal payment per month regardless of number of days, and assume you work 5 days a week, ignore public holidays)

3) In your next job you are given $200 per week plus a commission of 5% of all your sales. How much would you earn in each of the following weeks?

a) Sales of $500

$500 \times 5\% = \$25$

$\$200 + \$25 = \$225$

b) Sales of $1000

$1000 \times 5\% = \$50$

$\$200 + \$50 = \$250$

c) Sales of $5000

$5000 \times 5\% = \$250$

$\$200 + \$250 = \$450$

d) Sales of \$20000

$20000 \times 5\% = \$1000$

$\$200 + \$1000 = \$1200$

e) What must the value of your sales be if you needed to earn \$2200 for a week?

Commission must be
$2200 - 200 = \$2000$

\$2000 is 5% of \$40000

4) In your next job you are required to work overtime, but get paid more for doing so. The agreement is:

The first 35 hours work are paid at the rate of \$20 per hour

The next 5 hours work are paid at one and a half times your normal rate

All hours worked above this are paid at twice your normal rate

How much would you earn for working the following hours per week?

a) 30 hours

$30 \times \$20 = \$600$

b) 35 hours

$35 \times \$20 = \$700$

c) 37 hours

$35 \times \$20 = \$700$

$2 \times \$30 = \$60$

Total pay $= \$760$

d) 40 hours

$35 \times \$20 = \$700$

$5 \times \$30 = \$150$

Total pay $= \$850$

e) 46 hours

$35 \times \$20 = \$700$

$5 \times \$30 = \$150$

$6 \times \$40 = \$240$

Total pay $= \$1090$

f) 50 hours

$35 \times \$20 = \$700$

$5 \times \$30 = \$150$

$10 \times \$40 = \$400$

Total pay $= \$1250$

5) Assume you are working a job that pays a salary of $52,000 per annum

a) If taxation is deducted at the rate of 20%, how much would you actually receive per week?

$20\% \times \$52000 = \$10400$

Would actually receive $\$52000 - 10400 = \$41600$ per annum

$= \frac{41600}{52} = \$800$ per week

b) If taxation is deducted at the rate of 15%, superannuation at the rate of 2%, and Medicare levy at the rate of 1.5%, how much would you actually receive per week?

Total deductions are $(15\% + 2\% + 1.5\%) \times \$52000 = 18.5\% \times \$52000 = \$9620$

Would actually receive $\$52000 - \$9620 = \$42380$ per annum

$= \frac{42380}{52} = \$815$ per week

c) If the first $12,000 of your income is not taxed, but the remainder is taxed at 20%, how much would you actually receive per week?

$\$52000 - \$12000 = \$40000$

$20\% \times \$40000 = \$8000$

Would receive $\$52000 - \$8000 = \$44000$ per annum

$= \frac{44000}{52} = \$846.15$ per week

d) If the first $6,000 of your income is not taxed, but the remainder is taxed at 15%, and you have to pay a Medicare levy of 2% on your whole income, how much would you actually receive per week?

$\$52000 - \$6000 = \$46000$

$15\% \times \$46000 = \$6900$

Medicare levy $= 2\% \times 52000 = \$1040$

Would receive $\$52000 - \$6900 - \$1040 = \$44060$ per annum

$= \$847.31$ per week

6) Complete the following table, assuming interest is simple

Principal (P)	Interest Rate (R)	Time (T) (in years)	Total Interest (I)
$20,000	5%	5	**$5000**
$10,000	8%	4	**$3200**
$7,500	10%	10	**$7500**
$32000	5%	5	$8,000
$2,000	**25%**	8	$4,000
$2,500	10%	**3**	$750

7) Which of the following pairs of options costs the least, and by how much?

a) Buying petrol at $1.40 per litre, or at $1.50 per litre with a 5% discount

$\$1.50 \times 5\% = \$0.075 = 7.5$ cents

Cost with discount $= \$1.425$ per litre

Buying without discount is cheaper

b) Buying a lounge suite for $1200 cash, or a $900 lounge suite on credit with a simple interest rate of 10% per annum for 3 years

Total interest $\$900 \times 10\% \times 3 = \$270$

Total payable $= \$900 + \$270 = \$1170$

Buying on credit is cheaper in this instance

c) Buying a new TV for $2000 cash, or lay-by of 5 payments of $400

Since lay by attracts no interest, the amounts paid would be equal

d) A company offers an exercise system, for 6 equal monthly instalments of $49.95, plus postage & handling of $19.95, or a one off cash payment of $300 with no postage or handling fee. You would have to borrow the $300 at a simple interest rate of 8% per annum, for 6 months.

Cost of instalments

$6 \times \$49.95 = \$299.70$

$\$299.70 + \$19.95 = \$319.65$

Cost of financing

Total interest $\$300 \times 8\% \times 0.5 = \$12$

$\$300 + \$12 = \$312$

Cost of financing is cheaper in this instance

8) A store advertises a discount of 10% off all marked prices. You also have a discount card that entitles you to 15% off your purchases. Calculate the price you pay for each of the following items

a) A pair of shoes with a marked price of $120

$10\% \times \$120 = \$12$

New marked price $= \$108$

$15\% \times \$108 = \$16.20$

New price $= \$91.80$

b) A dining set with a marked price of $200

$10\% \times \$200 = \$20$

New marked price $= \$180$

$15\% \times \$180 = \$27$

New price $= \$153$

c) A roll of carpet of 20 metres in length with a marked price of $15 per metre

$10\% \times \$15 = \$1.50$

New marked price $= \$13.50$

$15\% \times \$13.50 = \$2.025$

New price $= \$11.475$ per meter

Total cost of 20 meters is $20 \times 11.475 = \$229.50$

d) A stereo system with a marked price of $567

$10\% \times \$567 = \$56.70$

New marked price $= \$510.30$

$15\% \times \$510.30 = \$76.55$

New price $= \$433.75$

9) Answer the following using the information from question 8

a) Is it cheaper to receive the 15% card discount, then the 10% store discount, or the other way around? Prove your answer with at least two examples.

From Q 8 part b, getting the 10% store discount then 15% card discount on a $200 item reduces its prices to $153

$15\% \times \$200 = \$30$

New price $= \$170$

$10\% \times \$170 = \$17$

New price $= \$153$

From Q 8 part a, getting the 10% store discount then 15% card discount on a $120 item reduces its prices to $91.80

$15\% \times \$120 = \$18$

New price $= \$102$

$10\% \times \$102 = \$10.20$

New price $= \$91.80$

It doesn't matter which way round the discounts are applied; the final price is the same

b) Is it cheaper to receive the 10% store discount and 15% card discount, or to receive 25% discount (10 + 15) immediately off the purchase price? Prove your answer.

From above, a $200 item after two discounts reduces to $153

$\$200 \times 25\% = \$50$

New price after 25% discount is $150

Therefore cheaper to get full 25% discount all at once

10) (Challenge question) The marked price of a bike has fallen off, and when you take it to the register you are charged $229.50 after both discounts. How much was the original marked price? (Use guess check and improve)

Try a guess of $350

$\$350 \times 10\% = \$35$

New ticket price $= \$315$

$\$315 \times 15\% = \$47.25$

Final price $= \$267.75$

Guess is too high

Try a guess of \$250

$\$250 \times 10\% = \$25$

New ticket price $= \$225$

$\$225 \times 15\% = \$33.75$

Final price $= \$181.25$

Guess is too low

Try a guess of \$300

$\$300 \times 10\% = \$30$

New ticket price $= \$270$

$\$270 \times 15\% = \$40.50$

Final price $= \$229.50$

Initial price was \$300

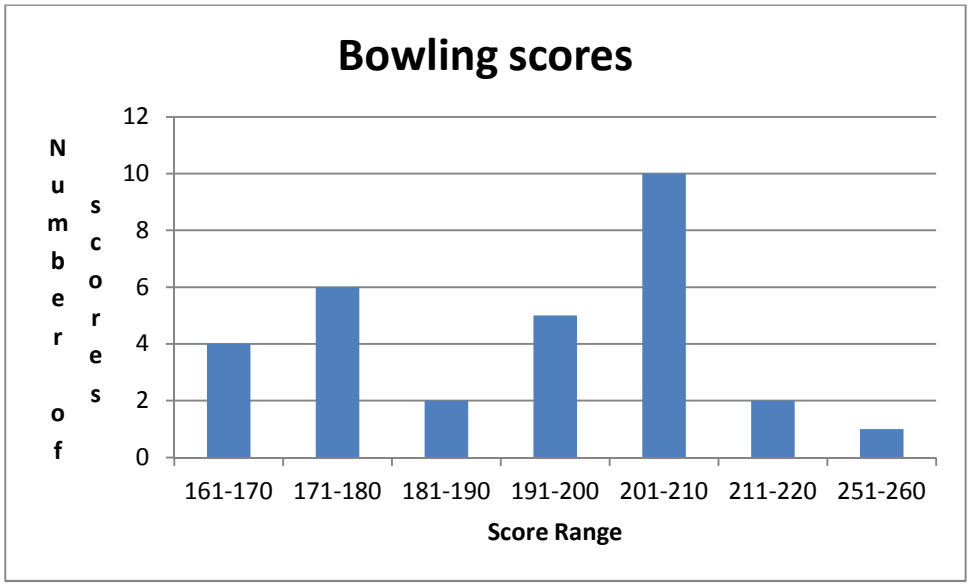
Year 9 Mathematics

Chance & Data

Exercise 1

Simple Probability

1) Peter plays ten pin bowling; his last 30 scores have been graphed in a frequency chart, shown here



Basing you answers on the chart data

a) Is Peter more likely to score 205 or 185 when he next bowls?

205 (there are many more scores in the 201-210 range)

b) Is he more or less likely to score over 200 when he next bowls?

There are more scores under 200, so based on his previous scores he is more likely to score less than 200

c) What would be his probability of scoring over 250 when next he bowls?

$\frac{1}{30}$

d) What would be his probability of scoring between 201 and 210 when next he bowls?

$\frac{10}{30}$

e) Discuss a major drawback with using this chart to predict the probabilities of future scores

Although the scores reflect Peter's ability in bowling, they do not take any factors into account, such as

- Illness or injury during any past games
- How long ago the games were played or how long between games
- Whether Peter has had any training before his next game
- Past scores cannot influence future scores

2) Craig rolled a pair of dice 360 times and recorded the sum of the two each time. He summarized his results in the table below

SUM of TWO DICE	Frequency
2	8
3	21
4	30
5	42
6	49
7	62
8	51
9	41
10	28
11	21
12	7

Based on his table:

a) What total is most likely to be rolled by two dice?

7

b) What is the most likely double?

Double 4

c) What total is least likely to be rolled by two dice

12

d) Is he more likely to roll a sum of 10 or a sum of 6 with two dice?

A sum of 6

e) Is this data more reliable than that of Q1? Give two reasons to support your answer

Yes

The results have no external factors affecting them (assuming the dice are fair)

The results can be tested against theoretical data

3) What is the theoretical probability of each of the following?

a) A head being thrown when a coin is tossed

$\frac{1}{2}$

b) A blue sock being taken from a draw containing 3 blue and 5 red socks

$\frac{3}{8}$

c) The number 2 being rolled on a dice

$$\frac{1}{6}$$

d) An even number being rolled on a dice

$$\frac{3}{6} = \frac{1}{2}$$

4) A card is drawn from a standard pack of 52 cards. What is the probability of the card being:

a) A black card

$$\frac{26}{52} = \frac{1}{2}$$

b) A club

$$\frac{13}{52} = \frac{1}{4}$$

c) An ace

$$\frac{4}{52} = \frac{1}{13}$$

d) A black 2

$$\frac{2}{52} = \frac{1}{26}$$

e) A picture card

$$\frac{12}{52} = \frac{3}{13}$$

f) The 2 of diamonds

$$\frac{1}{52}$$

5) A man throws two coins into the air

a) List the possible combinations, and from this table:

Head on first coin; and head on second

Head on first coin; and tail on the second

Tail on first coin; and head on second

Tail on first coin; and tail on second

b) What is the probability of throwing two heads?

$$\frac{1}{4}$$

c) What is the probability of throwing a head and a tail?

$$\frac{2}{4} = \frac{1}{2}$$

d) If the first coin lands on a head, is the second coin more likely or less likely to be a head?

There are two ways to look at this:

Firstly what is probability that if the first coin is a head the second is a head? There are two possibilities for the second coin if the first is a head; therefore the probability is $\frac{1}{2}$ that the second coin will be a head

Secondly, it doesn't matter what the first coin lands on, the probability of the second coin being a head is still $\frac{1}{2;}$; it is independent of the first throw

6) A coin is tossed and a dice is rolled

a) List the possible combinations of the coin and dice, and from this table:

	1	2	3	4	5	6
Head	H1	H2	H3	H4	H5	H6
Tail	T1	T2	T3	T4	T5	T6

b) What is the probability of throwing a six and a head?

$$\frac{1}{12}$$

c) What is the probability of throwing an odd number and a tail?

$$\frac{3}{12} = \frac{1}{4}$$

d) What is the probability of throwing a number higher than 4 and a head?

$$\frac{2}{12} = \frac{1}{6}$$

e) What is the probability of throwing a head and a 2 or a head and a 4?

$$\frac{2}{12} = \frac{1}{6}$$

7) A card is drawn from a normal pack. It is not replaced and a second card is drawn.

a) If the first card is red, what is the probability that the second card is also red?

The sample space has now been reduced to 51 cards

There are now 25 red cards left

Probability $= \frac{25}{51}$

b) If the first card is red, what is the probability that the second card is black?

Still 26 black cards left

Probability $= \frac{26}{51}$

c) If the first card is an ace, what is the probability that the second card is also an ace?

Only 3 aces left

Probability $= \frac{3}{51} = \frac{1}{51}$

d) If the first card is the jack of clubs, what is the probability that the second card is the jack of clubs?

There are no Jacks of clubs left

Probability $= 0$

8) A set of cards consists of 10 red cards, numbered 1 to 10 and 10 black cards numbered 1 to 10

a) What is the probability of pulling a 10 at random?

$\frac{2}{20} = \frac{1}{10}$

b) What is the probability of pulling a black card at random?

$\frac{10}{20} = \frac{1}{2}$

c) What is the probability of pulling a red 2 at random?

$\frac{1}{20}$

d) What is the probability of pulling a red 2 on the second draw if the first card is a black 2, and it is not replaced?

Sample space is now 19 cards

Probability $= \frac{1}{19}$

e) What is the probability of pulling an 8 on the second draw if the first card is an 8, and it is not replaced?

$\frac{1}{19}$

9) Consider the word ANATOMICALLY

a) What is the probability that a randomly chosen letter from this word will be an L?

$\frac{2}{12} = \frac{1}{6}$

b) What is the probability that a randomly chosen letter from this word will be an A?

$\frac{3}{12} = \frac{1}{4}$

c) What is the probability that a randomly chosen letter from this word will not be a vowel

$\frac{7}{12}$

d) What is the probability that a randomly chosen letter from this word will be a Z?

There are no Z's in the word, so probability $= 0$

10) What is the probability that a digit chosen randomly from all digits (0- 9) is:

a) A prime number?

Note: 1 is NOT a prime number, 2 IS a prime number

$$\frac{4}{10} = \frac{2}{5}$$

b) An even number?

$$\frac{4}{10} = \frac{2}{5}$$

c) Not 7?

$$\frac{9}{10}$$

d) Greater than 4?

$$\frac{5}{10} = \frac{1}{2}$$

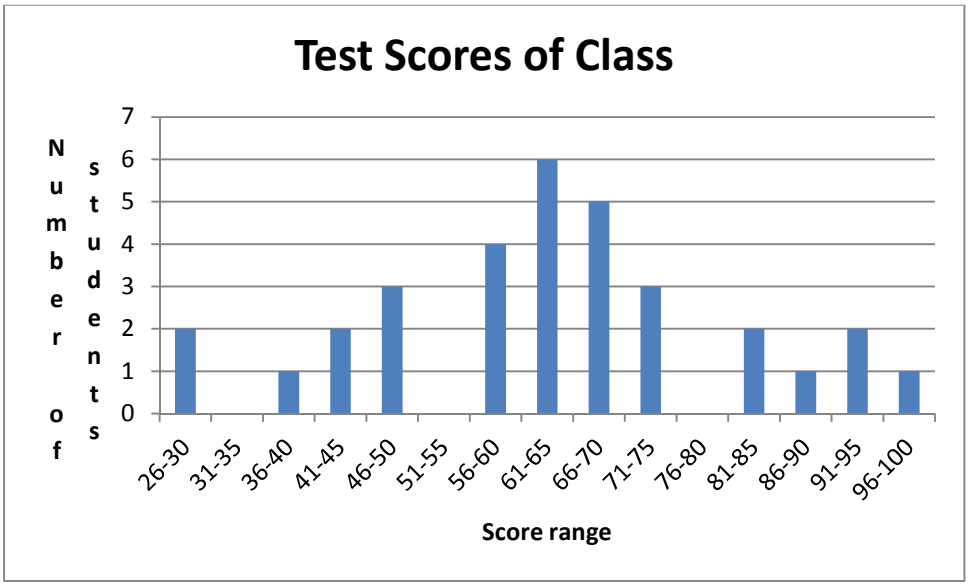
e) Less than 10?

All the digits are less than 10, so probability $= 1$ (certain)

Exercise 2

Data Representation & Analysis

1) Construct a cumulative frequency table from the following bar graph



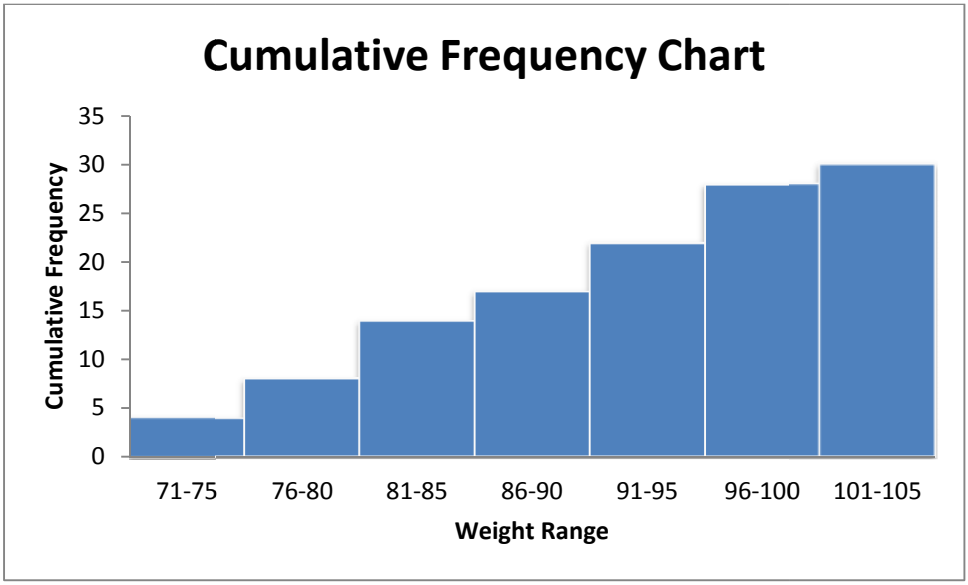
Range of scores	Frequency	Cumulative frequency
26-30	2	2
31-35	0	2
36-40	1	3
41-45	2	5
46-50	3	8
51-55	0	8
56-60	4	12
61-65	6	18
66-70	5	23
71-75	3	26
76-80	0	26
81-85	2	28
86-90	1	29
91-95	2	31
96-100	1	32

How many students sat the test, and how many passed?

32 students sat the test; the cumulative frequency under 50% was 8; therefore 24 students passed the test

2) Construct a cumulative frequency histogram from the following data of the weights of 30 people in a group (in kgs)

72, 73, 73, 75, 77, 77, 78, 80, 83, 84, 84, 84, 85, 85, 88, 88, 90, 92, 92, 93, 95, 95, 96, 97, 97, 98, 98, 100, 103, 104

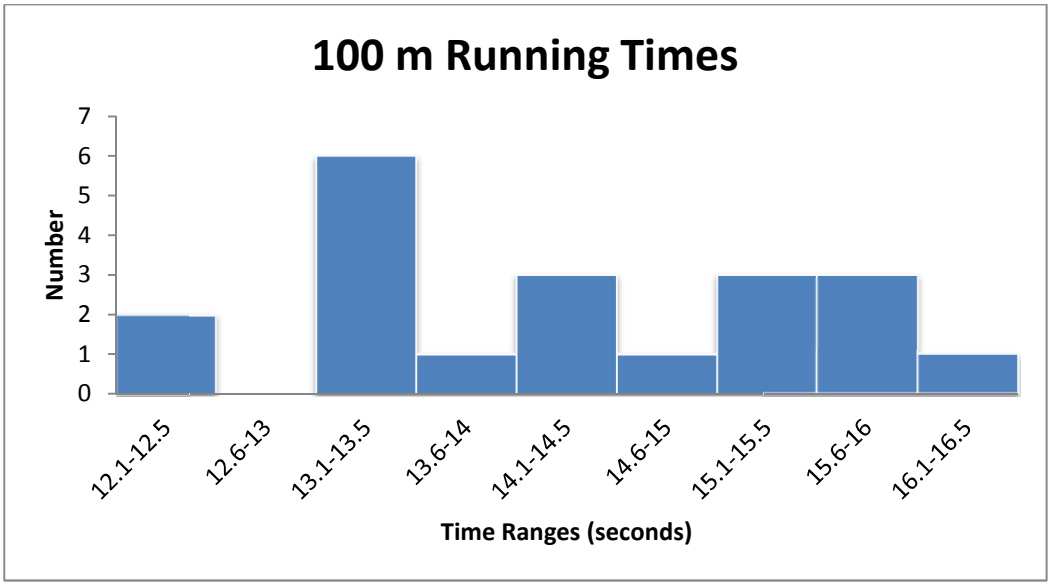


3) The following data shows the time taken for the members of an athletic club to run 100 metres

12.2, 12.4, 13.1, 13.2, 13.3, 13.4, 13.4, 13.5, 13.8, 14.1, 14.2, 14.3, 15, 15.2, 15.5, 15.5, 15.7, 15.8, 16, 16.2

a) Group the data into class intervals

b) Construct a histogram of the grouped data



c) Find the mean of the grouped data

Sum of class midpoints $= (2 \times 12.3) + (6 \times 13.3) + (1 \times 13.8) + (3 \times 14.3) + (1 \times 14.8) + (3 \times 15.3) + (3 \times 15.8) + (1 \times 16.3) = 286.7$

Mean $= \frac{286.7}{20} = 14.335$

d) Find the modal class of the grouped data

The modal class is the range $13.1 - 13.5$

4) The set of weights (in kg) from Q2 are repeated here:

72, 73, 73, 75, 77, 77, 78, 80, 83, 84, 84, 84, 85, 85, 88, 88, 90, 92, 92, 93, 95, 95, 96, 97, 97, 98, 98, 100, 103, 104

Determine:

a) The range of the data

Range $= 104 - 72 = 32$

b) The median of the data

Middle of 15^{th} and 16^{th} scores $= 88$

c) The upper and lower quartiles of the data

Upper quartile $= 80$

Lower quartile $= 96$

d) The inter-quartile range

IQR $= 96 - 80 = 16$

5) Calculate the mean and standard deviation (using a calculator) for the following sets of data

a) 1, 2, 3, 4, 5, 6, 7, 8, 9

Mean $= \frac{45}{9} = 5$

Standard Deviation $= 2.58$

b) 1, 1, 1, 1, 1, 1, 1, 1

Mean $= \frac{8}{8} = 1$

Standard deviation $= 0$

c) 2, 4, 6, 8, 10, 100

Mean $= \frac{130}{6} = 21.67$

Standard deviation $= 35.13$

d) 1, 22, 30, 40, 75, 90

Mean $= \frac{258}{6} = 43$

Standard deviation $= 30.59$

6) A year 7 class takes a test and receives the following marks

35, 42, 48, 51, 54, 56, 60, 65, 66, 68, 70, 70, 72, 73, 75, 77, 80, 85, 87, 90, 94, 94, 97, 99

To get an A on the test a student must score more than the mean plus one standard deviation.

How many students got an A on the test?

Mean $= \frac{1708}{24} = 71.17$

Standard deviation $= 17.37$

A student get an A if his score is greater than $71.17 + 17.37 = 88.54$

5 students get an A

7) The scores for a test to two different classes are shown under

Class 1: 85, 96, 75, 84, 65, 91, 78, 82, 80, 70, 80, 58, 71, 78, 98, 99, 75, 62, 75

Class 2: 61, 53, 54, 75, 99, 98, 98, 96, 78, 57, 90, 75, 93, 51, 75, 96, 99, 59, 95

a) Calculate the mean and standard deviation for the two classes

Class 1

Mean $= \frac{1502}{19} = 79.05$

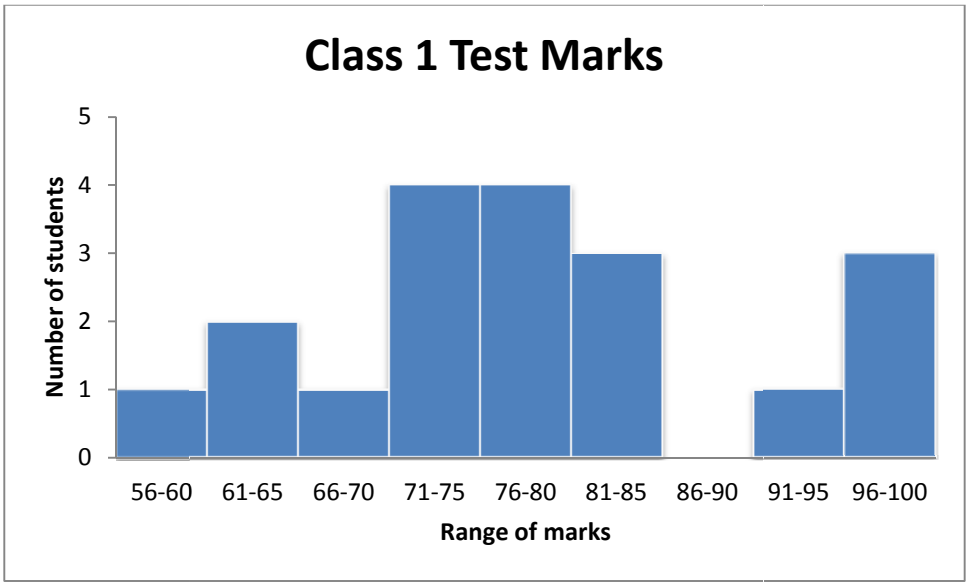
Standard deviation $= 11.21$

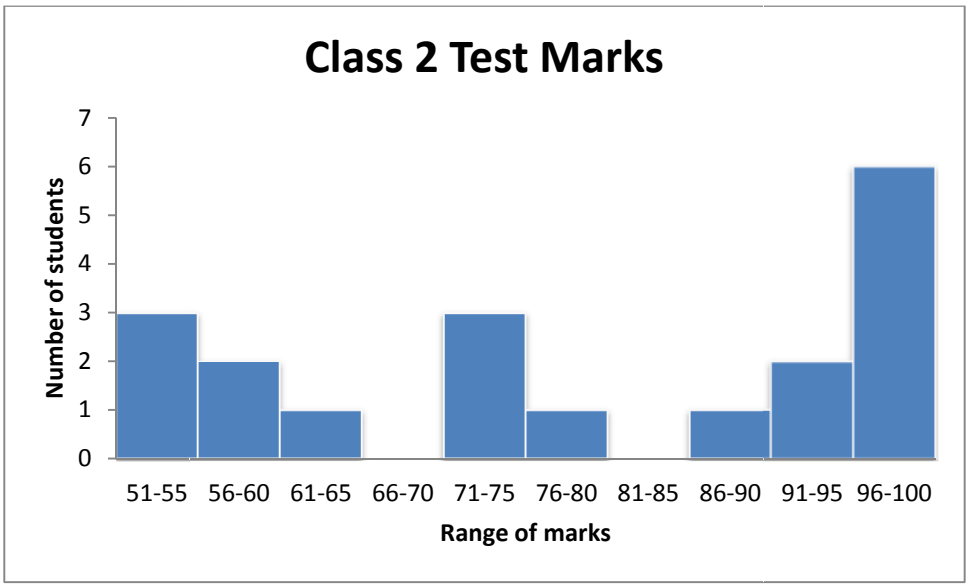
Class 2

Mean $= \frac{1502}{19} = 79.05$

Standard deviation $= 17.79$

b) Draw a histogram for each data set





c) Comment on the relationship between the histograms and the standard deviations

Although the means are the same, the distribution of the scores in the 2 classes is different. Class 2 has more students in the higher and lower ranges than class 1. The distribution and histogram of class 1's scores indicate that the students are generally similar in ability, with only relatively few in the upper and lower ranges. The class 2 deviation and histogram indicates a wider diversity in the abilities of the students, with more higher scoring and lower scoring students than class 1

8) The mean of the following data set is 12. What is the value of x?

8, 10, 7, 4, 20, x, 12, 14, 15, 25

Number of scores $= 10$

Mean $= 12$, therefore sum of scores is 120

Known scores add to 115

So missing score is 5

9) The mean of a set of 9 scores is 7. After another score is added the mean drops to 6.5. What was the added score?

Mean $= 7$ for 9 scores, therefore sum of scores $= 63$

Adding another score makes 10 scores, mean $= 6.5$, therefore sum of scores $= 65$

The added score was $65 - 63 = 2$

Year 9 Mathematics

Algebraic Expressions

Exercise 1

Simplifying Expressions Using Index Laws

1) Simplify the following using index laws

a) $x^a \times x^b$

x^{a+b}

b) $x^a \div x^b$

x^{a-b}

c) $(x^a \times x^b) \div x^c$

x^{a+b-c}

d) $(x^a)^b$

x^{ab}

e) $(x^a)^b \div x^c$

x^{ab-c}

2) Simplify the following

a) $x^2 \div x^2$

$= x^{2-2} = x^0 = 1$

b) $x^4 \times x^{-4}$

$= x^{4-4} = x^0 = 1$

c) $b^0 \times b^3$

$= b^{0+3} = b^3$

d) $a^0 \times a^0$

$= a^{0+0} = a^0 = 1$

e) $x^0 + x^0$

$= 1 + 1 = 2$

3) Simplify the following

a) $2x^3 \times 5x^2$

$= 10x^{3+2} = 10x^5$

b) $5a^3 \times 3a^3$

$= 15a^{3+3} = 15a^6$

c) $(2x^4)^2$

$= 2^2 x^{4\times 2} = 4x^8$

d) $3a^4 + 3a^4$

$= 2 \times (3a^4)$

$= 6a^4$

e) $2x^2 + 3x^3$

$= 6 \times x^{2+3} = 6x^5$

4) Simplify the following

a) $6x^6 \div 3x^3$

$= 2 \times x^{6-3} = 2x^3$

b) $20a^8 \div 5a^2$

$= 4a^{8-2} = 4a^6$

c) $8c^5 \div 4c^5$

$= 2c^{5-5}$

d) $5x^6 - 4x^3$

Cannot be simplified

Can be factorized

$= x^3(5x^3 - 4)$

e) $(x)^{0.5} \times (x)^{0.5}$

$= x^{0.5+0.5} = x^1 = x$

5) Simplify

a) $2x(x^2 + 3)$

$= 2x^3 + 6x$

b) $x^2(x + 1)$

$= x^3 + x$

c) $ab(a + b)$

$= a^2b + ab^2$

d) $x^0(a + b)$

$= x^0 = 1$

$x^0(a + b) = a + b$

e) $a(x^0 + 4)$

$x^0 = 1$
=0

$a(x^0 + 4) = 5a$

6) Simplify the following

a) $\frac{3x^2 \times 4x^4}{2x^6}$

$= 6x^{2+4-6} = 6x^0 = 6$

b) $\frac{5a^3 \times 4a^5}{10a^2}$

$= 2a^{3+5=2} = 2a^6$

c) $\frac{4x^5 \times 3x^3}{12x^8}$

$= 1x^{5+3-8} = 1x^0 = 1$

d) $\frac{x^{12}}{x^4} - x^8$

$x^{12-4} - x^8 = x^8 - x^8 = 0$

7) Put the following in order from smallest to largest for x>1

$5x, 5^0x, 5^0x^0, (5x)^0, x^0, 5^0$

$, 5^0x^0, (5x)^0, x^0, 5^0$ are all equal to 1

$5x$ and 5^0x are both > 1

8) James asks Alan how far it is from his house to school. Alan replies:

"If you square the distance and multiply it by the distance to the power of 3 you get 32"

How far is Alan's house from school (in kilometres)?

Let distance be represented by x

$x^2 \times x^3 = 32$

$x^5 = 32$

$x = 2$

Alan's house is 2 km from school

Exercise 2

Expressions Involving Fractions

1) Simplify the following

a) $\frac{x}{3}+\frac{x}{3}$

$=\frac{2x}{3}$

b) $\frac{2x}{5}+\frac{x}{5}$

$=\frac{3x}{5}$

c) $\frac{5x}{7}-\frac{2x}{7}$

$\frac{3x}{7}$

d) $\frac{3y}{4}-\frac{2y}{4}$

$=\frac{y}{4}$

e) $\frac{8t}{5}-\frac{8t}{5}$

$=\frac{0}{5}=0$

2) Simplify the following

a) $\frac{x}{2}+\frac{x}{4}$

$=\frac{2x}{4}+\frac{x}{4}=\frac{3x}{4}$

b) $\frac{4t}{9}-\frac{t}{3}$

$=\frac{4t}{9}-\frac{3t}{9}=\frac{t}{9}$

c) $\frac{3x}{4}+\frac{x}{8}$

$\frac{6x}{8}+\frac{x}{8}=\frac{7x}{8}$

d) $\frac{5y}{12}-\frac{y}{6}$

$=\frac{5y}{12}-\frac{2y}{12}=\frac{3y}{12}=\frac{y}{4}$

e) $\frac{x}{7}+\frac{2x}{21}$

$=\frac{3x}{21}+\frac{2x}{21}=\frac{5x}{21}$

3) Simplify the following

a) $\frac{x}{3}+\frac{x}{5}$

$=\frac{5x}{15}+\frac{3x}{15}=\frac{8x}{15}$

b) $\frac{y}{2}+\frac{2y}{3}$

$=\frac{3y}{6}+\frac{4y}{6}=\frac{7y}{6}$

c) $\frac{3t}{2}+\frac{t}{7}$

$=\frac{21t}{14}+\frac{2t}{14}=\frac{23t}{14}$

d) $\frac{3x}{5}+\frac{5x}{3}$

$=\frac{9x}{15}+\frac{25x}{15}=\frac{34x}{15}$

e) $x + \frac{5x}{11}$

$$= \frac{11x}{11} + \frac{5x}{11} = \frac{16x}{11}$$

4) Simplify the following

a) $\frac{3t}{2} - \frac{2t}{3}$

$$= \frac{9t}{6} - \frac{4t}{6} = \frac{5t}{6}$$

b) $\frac{5x}{2} - \frac{2x}{7}$

$$= \frac{35x}{14} - \frac{4x}{14} = \frac{31x}{14}$$

c) $\frac{4x}{5} - \frac{x}{2}$

$$= \frac{8x}{10} - \frac{5x}{10} = \frac{3x}{10}$$

d) $\frac{4y}{15} - \frac{y}{6}$

$$= \frac{9t}{6} - \frac{4t}{6} = \frac{5t}{6}$$

e) $\frac{16a}{20} - \frac{4a}{5}$

$$= \frac{16a}{20} - \frac{16a}{20} = 0$$

5) Simplify the following

a) $\frac{2x}{3} \div \frac{x}{6}$

$$= \frac{2x}{3} \times \frac{6}{x} = \frac{12x}{3x} = 4$$

b) $\frac{3a}{4} \div \frac{a}{6}$

$$= \frac{3a}{4} \times \frac{6}{a} = \frac{18a}{4a} = \frac{9}{2}$$

c) $\frac{5x}{9} \div \frac{x}{18}$

$$= \frac{5x}{9} \times \frac{18}{x} = \frac{90x}{9x} = 10$$

d) $\frac{y}{5} \div \frac{2y}{15}$

$$= \frac{y}{5} \times \frac{15}{2y} = \frac{15y}{10y} = \frac{3}{2}$$

e) $\frac{3x}{2} \div \frac{6x}{5}$

$$= \frac{3x}{2} \times \frac{5}{6x} = \frac{15x}{12x} = \frac{5}{4}$$

6) Simplify the following

a) $\frac{3xy}{4} \times \frac{8}{x}$

$$= \frac{24xy}{4x} = 6y$$

b) $\frac{5at}{3} \times \frac{9}{10t} \frac{6abc}{9} \times \frac{2}{4ac}$

$$= \frac{540a^2bct}{1080act} = \frac{ab}{2}$$

c) $\frac{xy}{5} \times \frac{35}{x} \times \frac{1}{7y}$

$= \frac{35xy}{35xy} = 1$

$= \frac{3xy}{10ab}$

d) $\frac{xy}{10} \times \frac{3}{ab}$

7) Three friend, Alan, Colin and William share a pizza. Alan eats half of the pizza, and Colin eats a third of the pizza. What fraction of the original pizza is left for William?

Let full pizza $= x$

William's share $= x - \frac{1}{2}x - \frac{1}{3}x = \frac{6}{6}x - \frac{3}{6}x - \frac{2}{6} = \frac{1}{6}x$

There is $\frac{1}{6}$ of the pizza left for William

8) Some students ask their maths teacher how old he is. The teacher replies "Half of my age subtract one-third of my age equals 7." How old is the maths teacher?

Let age of teacher be represented by t

$\frac{1}{2}t - \frac{1}{3}t = 7$

$\frac{3}{6}t - \frac{2}{6}t = 7$

$\frac{1}{6}t = 7$

$t = 42$

The teacher is 42 years old

9) Pocket money is divided between three brothers according to their ages. Tony receives half of the total pocket money paid out, while Michael receives one-fifth of the total. What fraction of the pocket money does the middle child, Peter, receive?

Let total pocket money be represented by f

Peter's share $= f - \frac{1}{2}f - \frac{1}{5}f = \frac{10}{10}f - \frac{5}{10}f - \frac{2}{10}f = \frac{3}{10}f$

Peter receives $\frac{3}{10}$ of the pocket money

Exercise 3

Solving Equations

1) Solve each of the following equations

a) $x + 3 = 5$

$x = 5 - 3 = 2$

b) $2x - 4 = 6$

$2x = 6 + 4 = 10$

$x = \frac{10}{2} = 5$

c) $\frac{1}{2}x - 6 = 8$

$\frac{1}{2}x = 8 + 6 = 14$

$x = 2 \times 14 = 28$

d) $\frac{x}{2} + \frac{x}{3} = 4$

$\frac{3x}{6} + \frac{2x}{6} = 4$

$\frac{5x}{6} = 4$

$x = \frac{4 \times 6}{5} = \frac{24}{5}$

e) $\frac{2x-4}{3} = 6$

$2x - 4 = 6 \times 3 = 18$

$2x = 18 + 4 = 22$

$x = \frac{22}{2} = 11$

f) $\frac{-2x-3}{2} + 5 = 3$

$\frac{-2x + 3}{2} = 3 - 5 = -2$

$-2x + 3 = -2 \times 2 = -4$

$-2x = -4 - 3 = -7$

$x = \frac{7}{2}$

2) Solve each of the following equations

a) $2(x + 1) = 10$

$x + 1 = \frac{10}{2} = 5$

$x = 5 - 1 = 4$

b) $\frac{1}{2}(3x - 2) = 8$

$3x - 2 = 2 \times 8 = 16$

$3x = 16 + 2 = 18$

$x = \frac{18}{3} = 6$

c) $4(x + 2) + 2(x + 1) = 0$

$4x + 8 + 2x + 2 = 0$

$6x + 10 = 0$

$6x = -10$

$x = -\frac{10}{6} = -\frac{5}{3}$

d) $2(2x-1)-3(x-3)=4$

$4x-2-3x+9=4$

$x+7=4$

$x=4-7=-3$

e) $3(2-3x)-(1-x)=-2$

$6-9x-1+x=-2$

$5-8x=-2$

$-8x=-2-5=-7$

$x=\frac{-7}{-8}=\frac{7}{8}$

3) A man declares "If you add 4 to my age and double the result, you will get 3 times my age less 22." How old is the man?

Let x represent the man's age

Then $2(x+4)=3x-22$

$2x+8=3x-22$

$2x-3x=-22-8$

$-x=-30$

$x=30$

The man is 30 years old

4) Half of a number equals twice that number plus 6. What is the number?

Let y represent the number

$\frac{1}{2}y=2y+6$

$\frac{1}{2}y-2y=6$

$-\frac{3}{2}y=6$

$$y = 6 \times \left(-\frac{2}{3}\right) = -4$$

5) If you subtract 4 from a number and halve the result, you will get twice the same number less 8. What is the number?

Let t represent the number

$$\frac{1}{2}(t-4) = 2t - 8$$

$$t - 4 = 2(2t - 8)$$

$$t - 4 = 4t - 16$$

$$t - 4t = -16 + 4$$

$$-3t = -12$$

$$t = 4$$

6) How many solutions does each of the following equations have?

a) $x^2 = 4$

Two $(-2, 2)$

b) $x^2 = 9$

Two $(-3, 3)$

c) $x^2 = 0$

One (0)

d) $x^2 = 16$

Two $(-4, 4)$

e) $x^2 = 3$

Two $\left(-\sqrt{3}, \sqrt{3}\right)$

f) $x^2 = -4$

No solutions

7) From your answers to question 6, how does the value of c in the equation $x^2 = c$ affect the number of solutions of the equation?

If $c > 0$, there are two solutions

If $c = 0$, there is one solution

If $c < 0$, there are no solutions

8) Solve the following equations

a) $x^2 = 4$

$x = \sqrt{4}$

$x = 2, -2$

b) $2x^2 = 8$

$x^2 = \frac{8}{2} = 4$

$x = 2, -2$

c) $3x^2 = 27$

$x^2 = \frac{27}{3} = 9$

$x = -3, 3$

d) $x^2 = -6$

There are no solutions

e) $-x^2 = 4$

$x^2 = -4$

Type equation here. There are no solutions

f) $x^2 = 0$

$x = 0$

9) Solve the following inequalities

a) $2x + 1 > 9$

$2x > 9 - 1 = 8$

$x > 4$

b) $x - 3 < 6$

$x < 6 + 3$

$x < 9$

c) $2 - 3x > 4$

$2 - 4 - 3x > 0$

$-2 - 3x > 0$

$3x < -2$

$x < -\frac{2}{3}$

d) $-x - 5 < 10$

$-x < 10 + 5$

$-x < 15$

$x > -15$

e) $2(x + 3) > 6$

$2x + 6 > 6$

$2x > 6 - 6$

$x > 0$

f) $-(x - 4) < -2$

$-x + 4 < -2$

$-x < -2 - 4$

$-x < -6$

$x > 6$

g) $\frac{x-4}{4} > 2$

$x - 4 > 2 \times 4$

$x - 4 > 8$

$x > 8 + 4$

$x > 12$

h) $\frac{2-x}{2} < 6$

$2 - x < 6 \times 2$

$2 - x < 12$

$-x < 12 - 2$

$-x < 10$

$x > -10$

Exercise 4

Fractional & Negative Indices

1) Rewrite the following with positive indices

a) x^{-1}

$\frac{1}{x}$

b) x^{-4}

$\frac{1}{x^4}$

c) $2x^{-3}$

$\frac{2}{x^3}$

d) $(2x)^{-3}$

$\frac{1}{(2x)^3}$

e) $\frac{1}{x^{-2}}$

x^2

f) $\frac{1}{2x^{-2}}$

$\frac{x^2}{2}$

2) Express the following using indices

a) $\sqrt{x}$

$x^{\frac{1}{2}}$

b) $\left(\sqrt{x}\right)^2$

x

c) $\left(x^{\frac{1}{2}}\right)^2$

x

d) $\sqrt{x} \times \sqrt{x}$

$x^{\frac{1}{2}} \times x^{\frac{1}{2}} = x$

e) $\left(\sqrt{x}\right)^4$

$= \left(x^{\frac{1}{2}}\right)^4$

$= x^2$

3) Simplify the following, expressing your answer in positive indices

a) $x^{-4} \times x^2$

$= x^{-4+2} = x^{-2} = \frac{1}{x^2}$

b) $(x^{-4})^2$

$= x^{-4\times2} = x^{-8} = \frac{1}{x^8}$

c) $(x^4)^{-2}$

$= x^{4\times-2} = x^{-8} = \frac{1}{x^8}$

d) $2x^{-3} \times 3x^{-2}$

$= 6x^{-3-2} = 6x^{-5} = \frac{6}{x^5}$

e) $\frac{1}{2}x^{-2} \times 2x^2$

$= 1x^{-2+2} = 1 \times 1 = 1$

4) Simplify the following, expressing your answer in positive indices

a) $4x^2 \div 2x^{-4}$

$= \frac{4}{2}x^{2--4} = 2x^6$

b) $4x^{-2} \div 2x^4$

$= \frac{4}{2}x^{-2-4}$

$= 2x^{-6}$

$= \frac{2}{x^6}$

c) $4x^2 \div 2x^4$

$\frac{4}{2}x^{2-4}$

$= 2x^{-2}$

$= \frac{2}{x^2}$

d) $4x^{-2} \div 2x^{-4}$

$= \frac{4}{2}x^{-2--4}$

$= 2x^2$

e) $(4x)^{-2} \div (2x)^{-4}$

$= 4^{-2}x^{-2} \div 2^{-4}x^{-4}$

$\left(\frac{1}{16} \div \frac{1}{16}\right)x^{-2--4}$

$= x^2$

5) Simplify the following, expressing your answer with positive indices

a) $4(x)^{\frac{1}{4}} \div 2(x)^{\frac{1}{2}}$

$= 2(x)^{\frac{1}{4}-\frac{1}{2}}$

$= 2x^{-\frac{1}{4}}$

$= \frac{2}{x^{\frac{1}{4}}}$

b) $(4x)^{\frac{1}{4}} \div (2x)^{\frac{1}{2}}$

$= 4^{\frac{1}{4}}x^{\frac{1}{4}} \div 2^{\frac{1}{2}}x^{\frac{1}{2}}$

$= 2^{\frac{1}{2}}x^{\frac{1}{4}} \div 2^{\frac{1}{2}}x^{\frac{1}{2}}$

$= x^{\frac{1}{4}-\frac{1}{2}}$

$= \frac{1}{x^{\frac{1}{4}}}$

c) $(x)^{\frac{2}{3}} \div x$

$= x^{\frac{2}{3}-1}$

$= \frac{1}{x^{\frac{1}{3}}}$

d) $(2x)^{\frac{1}{2}} \div (2x)^2$

$= 2^{\frac{1}{2}-2} x^{\frac{1}{2}-2}$

$= \frac{1}{2^{\frac{3}{2}} x^{\frac{3}{2}}}$

$= \frac{1}{(2x)^{\frac{3}{2}}}$

e) $\frac{1}{2}(x)^{\frac{1}{2}} \div 2x^2$

$= \frac{1}{4} x^{\frac{1}{2}-2}$

$\frac{1}{4x^{\frac{3}{2}}}$

6) Simplify the following, expressing your answer in positive indices

a) $\left(x^{\frac{1}{2}}\right)^{-\frac{1}{2}}$

$= x^{-\frac{1}{4}}$

$= \frac{1}{x^{\frac{1}{4}}}$

b) $\left(x^{-\frac{1}{2}}\right)^{\frac{1}{2}}$

$= x^{-\frac{1}{4}}$

$= \frac{1}{x^{\frac{1}{4}}}$

c) $\frac{1}{\left(x^{\frac{1}{2}}\right)^{\frac{1}{2}}}$

$= \frac{1}{x^{\frac{1}{4}}}$

d) $\left(\frac{1}{x^{\frac{1}{2}}}\right)^{\frac{1}{2}}$

$= \frac{1^{\frac{1}{2}}}{x^{\frac{1}{4}}}$

$= \frac{1}{x^4}$

e) $(x^2)^{-\frac{1}{2}}$

$= x^{-1}$

$= \frac{1}{x}$

7) State whether the following statements are true of false. If false give the correct answer

a) $2x^{-5} = \frac{1}{2x^5}$

False

$2x^{-5} = \frac{2}{x^5}$

b) $\left(x^{-\frac{1}{2}}\right)^2 = x$

False

$$\left(x^{-\frac{1}{2}}\right)^2 = x^{-1} = \frac{1}{x}$$

c) $x^2 \div x^4 = x^{-2}$

True

d) $\frac{1}{x^2} \div \frac{1}{x^4} = x^2$

True

e) $\frac{1}{x^{-2}} \times \frac{1}{x^{-2}} = \frac{1}{x^4}$

False

$$\frac{1}{x^{-2}} \times \frac{1}{x^{-2}} = x^2 \times x^2 = x^4$$

Exercise 5

Expanding & Factorizing

1) Expand the following by removing brackets and collecting and simplifying like terms where possible

a) $2x(x+3)+2(x+1)$

$= 2x^2+6x+2x+2$

$= 2x^2+8x+2$

b) $3x(x-3)+(2x^2+x)$

$= 3x^2-9x+2x^2+x$

$5x^2-8x$

c) $y(y-3)+y^2(y+1)$

$= y^2-3y+y^3+y^2$

$= y^3+2y^2-3y$

d) $x(y+2)+2y(x+3)$

$= xy+2y+2xy+6y$

$= 3xy+8y$

e) $x(1-x)+(x^2+x)$

$= x-x^2+x^2+x$

$= 2x$

2) Expand the following by removing brackets and collecting and simplifying like terms where possible

a) $x(x+2)+x^2(1-x)$

$= x^2+2x+x^2-x^3$

$= -x^3+2x^2+2x$

b) $2x(x+2)-x^2(2+x)$

$2x^2+4x-2x^2-x^3$

$= -x^3+4x$

c) $3x(2-x)+(x+2)$

$6x-3x^2+x+2$

$= -3x^2+7x+2$

d) $x(3+x)-2x(x-5)$

$= 3x+x^2-2x^2+10x$

$= -x^2+13x$

e) $4x(x-3)+x(3-x)$

$= 4x^2-12x+3x-x^2$

$= 3x^2-9x$

f) $2x(x-4)-3x(3-2x)$

$= 2x^2-8x-9x+6x^2$

$= 8x^2-17x$

g) $x(2x-2)-2x(2-x)$

$= 2x^2-2x-4x+2x^2$

$= 4x^2 - 6x$

3) Expand the following by removing brackets and collecting and simplifying like terms where possible

a) $xy(x + y) + y(xy + x)$

$x^2y + xy^2 + xy^2 + xy$

$x^2y + 2xy^2 + xy$

b) $x^2(y - x) + x(x^2 - y)$

$= x^2y - x^3 + x^3 - xy$

$= x^2y - xy$

c) $x^2y(2 - y) - xy^2(1 + x)$

$= 2x^2y - x^2y^2 - xy^2 - x^2y$

$x^2y - x^2y^2 - xy^2$

d) $xy(2xy + x) - yx^2(2y + 1)$

$= 2x^2y^2 + x^2y - 2x^2y^2 - x^2y$

$= 0$

e) $(xy - y^2) - y(-x + y)$

$= xy - y^2 + xy - y^2$

$= 2xy - 2y^2$

4) Factorize the following expressions

a) $2x^2 + 4x$

$= 2x(x + 2)$

b) $3y^3 - y^2$

$= y^2(3y - 1)$

c) $4x^2 - 6x^4$

$= 2x^2(2 - 3x^2)$

d) $8y - 2y^2 + 6y^3$

$= 2y(4 - y + 3y^2)$

e) $2x + 4x^2 - 8y$

$= 2(x + 2x^2 - 4y)$

5) Factorize the following expressions

a) $8xy - 4x^2y$

$= 4xy(2 - x)$

b) $5xy + 10y^2$

$= 5y(x + 2y)$

c) $3x^2y^2 - 2xy$

$= xy(3xy - 2)$

d) $4x^2 + 8xy - 8y^2$

$= 4(x^2 + 2xy - 2y^2)$

e) $\frac{1}{2}x^2y - 2xy^3$

$= \frac{1}{2}xy(x - 4y^2)$

6) Factorize the following expressions

a) $4x^2y^2 + 8xy + 12xy^2$

$= 4xy(xy + 2 + 3y)$

b) $xyz - x^2y^2z^2 - xy^2z$

$= xyz(1 - xyz - y)$

c) $x(x - y) + (x - y)$

$= 2x(x - y)$

d) $x(x - y) + y(x - y)$

$= (x + y)(x - y)$

e) $y(y - 1) - x(1 - y)$

$= y(y - 1) + x(y - 1)$

$= (x + y)(y - 1)$

Year 9 Mathematics

Coordinate Geometry

Exercise 1

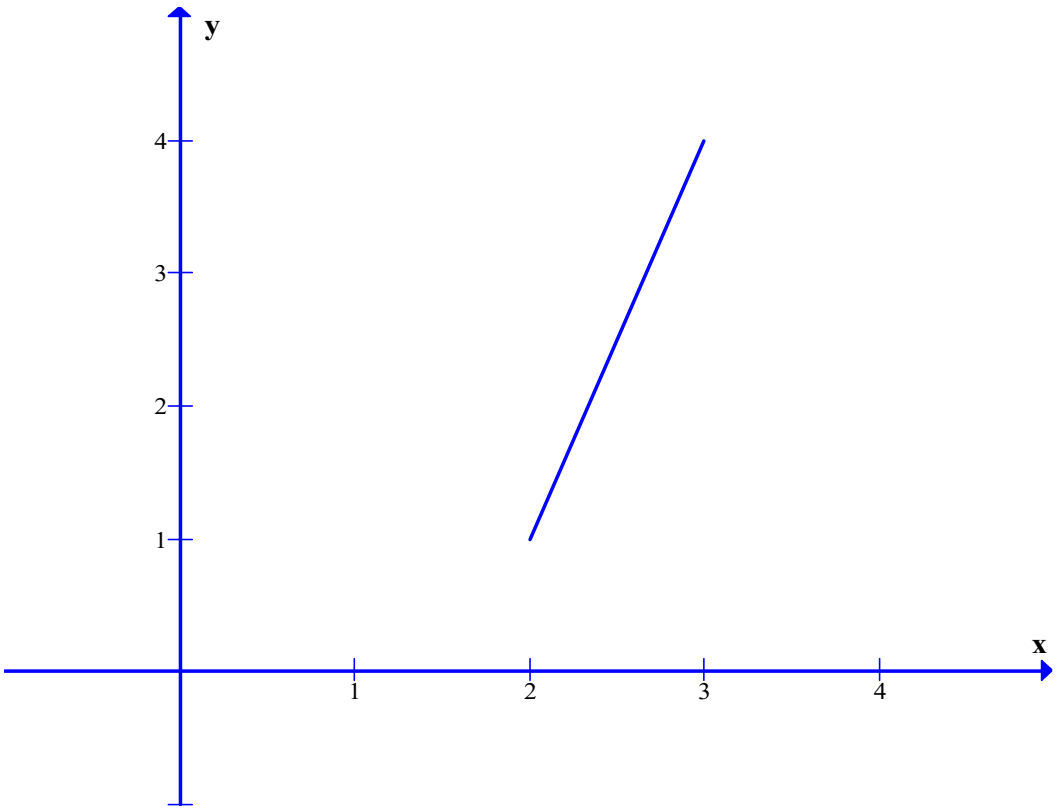
Determining Midpoint, Length & Gradient

1) Each part below lists a pair of coordinates. For each pair you are required to:

- Plot the points on a graph, and join to form a line segment
- Determine the midpoint of the line segment drawn from the diagram
- Using the line segment as the hypotenuse, construct a right angled triangle
- Use the above construction and Pythagoras' Theorem to determine the length of the line segment
- State whether the line segment has a positive or negative gradient (slope)
- Using the right angled triangle drawn to determine the value of the gradient of the line segment (gradient = rise/run)

Use the following example as a guide

For the points (2, 1) and (3, 4)

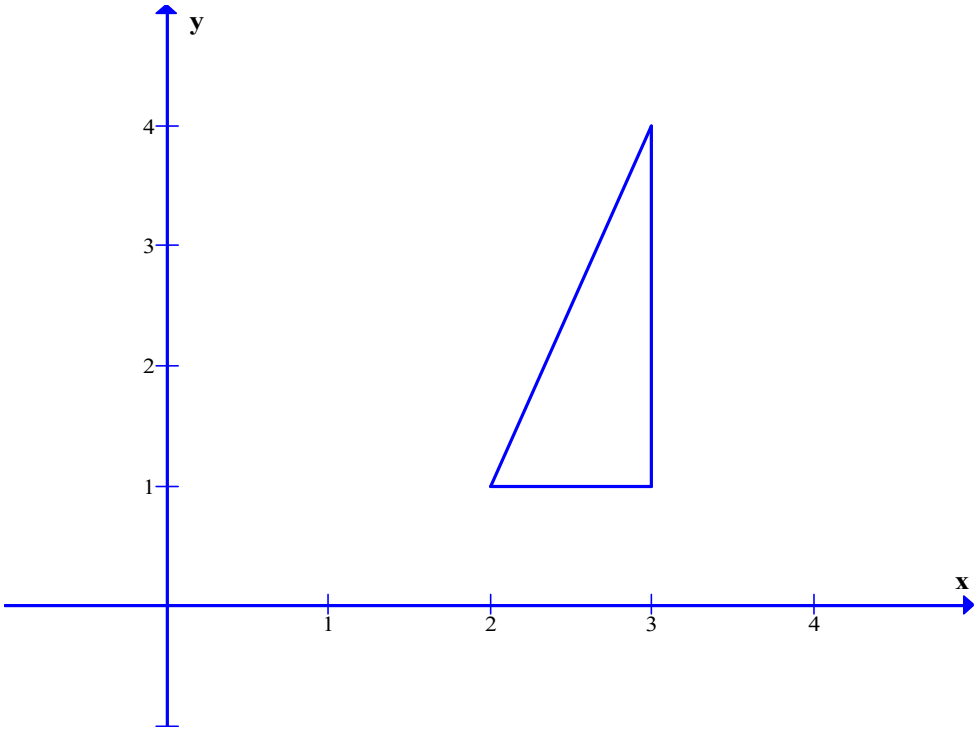


Midpoint is the (x, y) coordinate of the point halfway along line

Here midpoint is (2.5, 2.5)

Gradient is positive

Right angled triangle drawn



Length of hypotenuse (line segment) =c

From Pythagoras: $c^2 = a^2 + b^2$

$c^2 = 1^2 + 3^2$

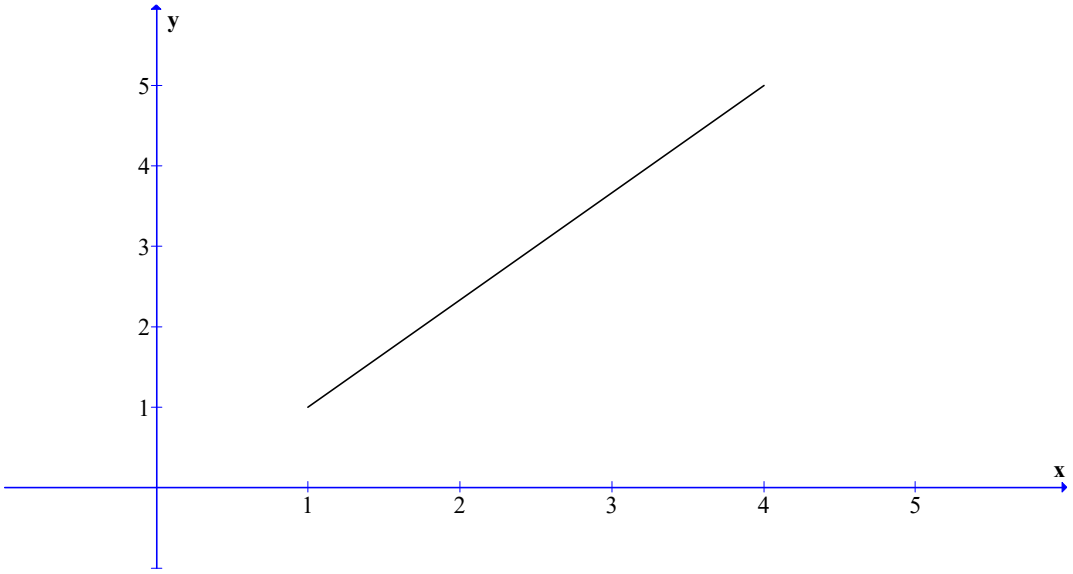
$c^2 = 1 + 9 = 10$

$c = \sqrt{10}$

Note leave in square root form if cannot be simplified

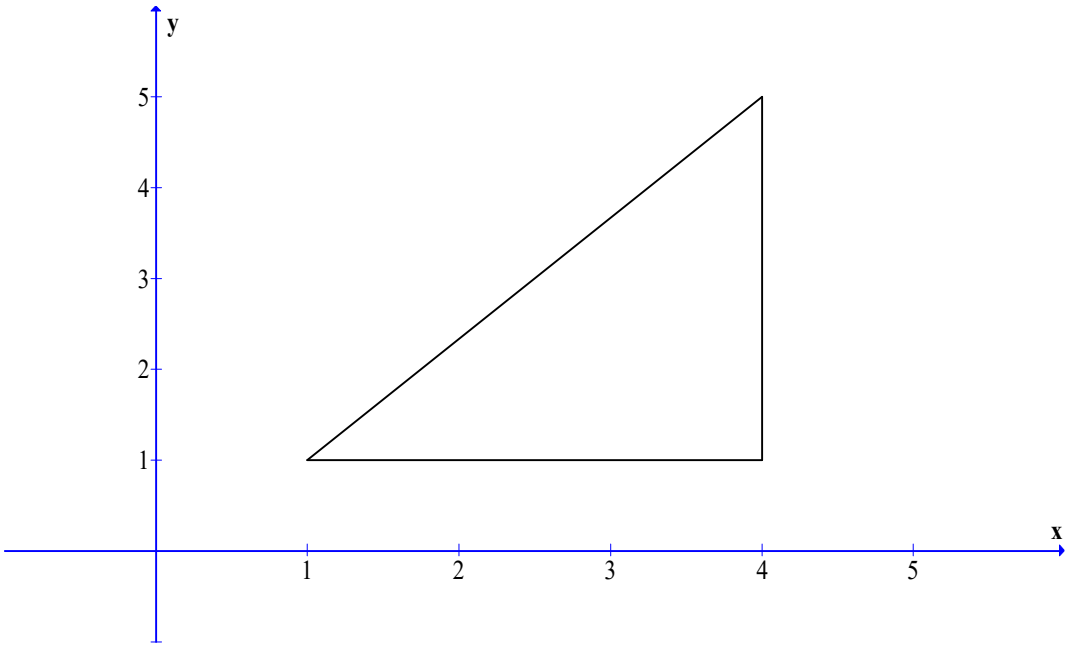
Gradient = rise/run = $\frac{4-1}{3-2} = \frac{3}{1} = 3$

a) (1, 1) and (4, 5)



Midpoint is $(2.5, 3)$

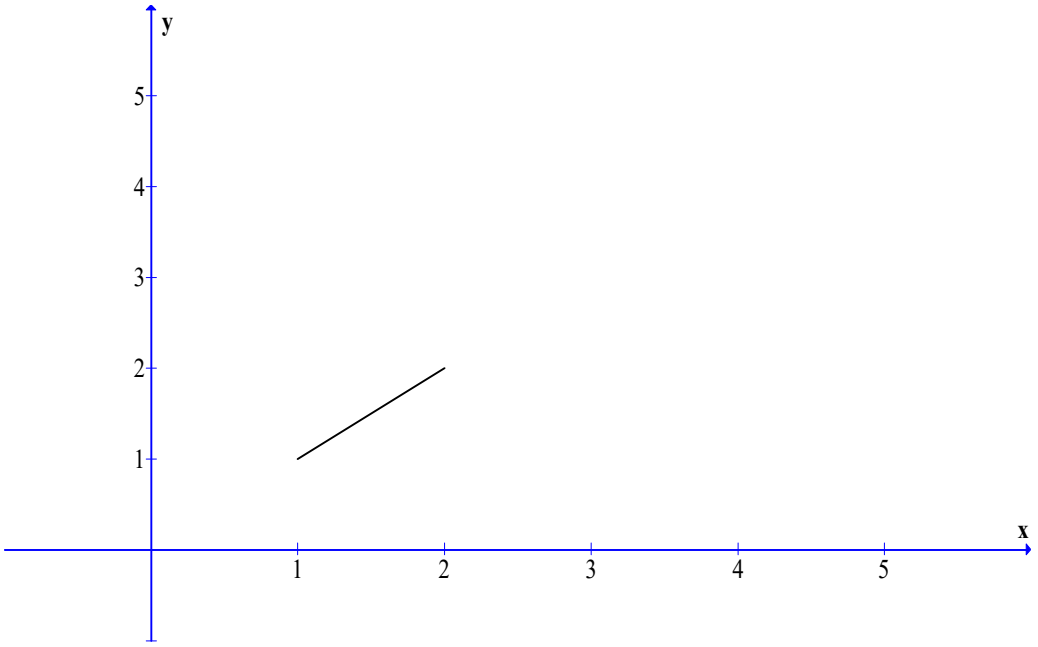
Gradient is positive



Length of hypotenuse $= \sqrt{4^2 + 3^2} = \sqrt{25} = 5$

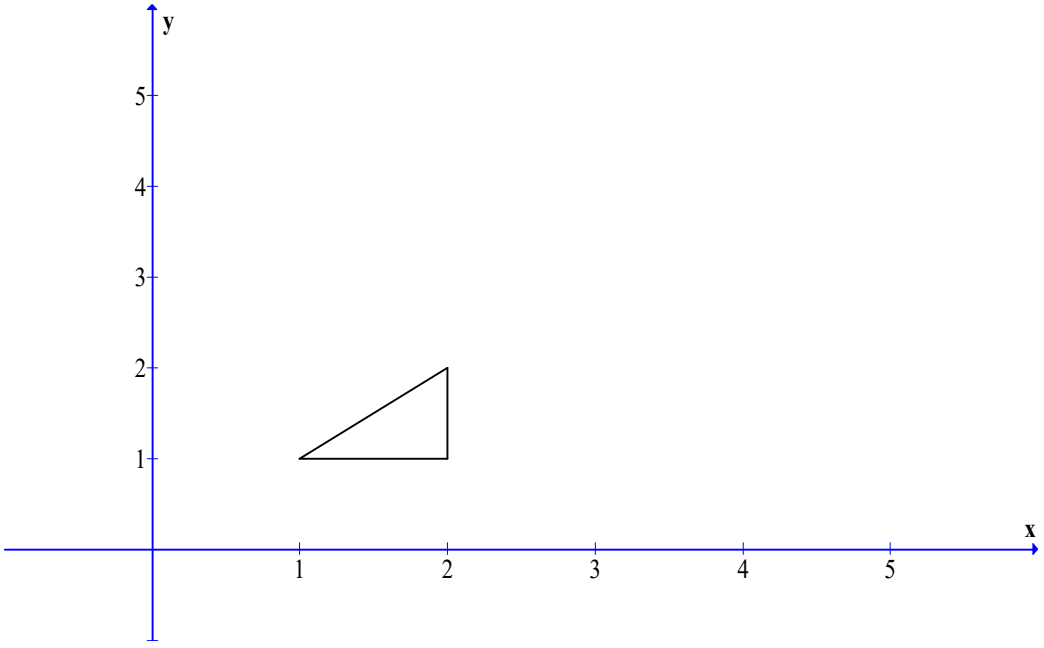
$$\text{Gradient} = \frac{5-1}{4-1} = \frac{4}{3}$$

b) (1, 1) and (2, 2)



Midpoint $= (1.5, 1.5)$

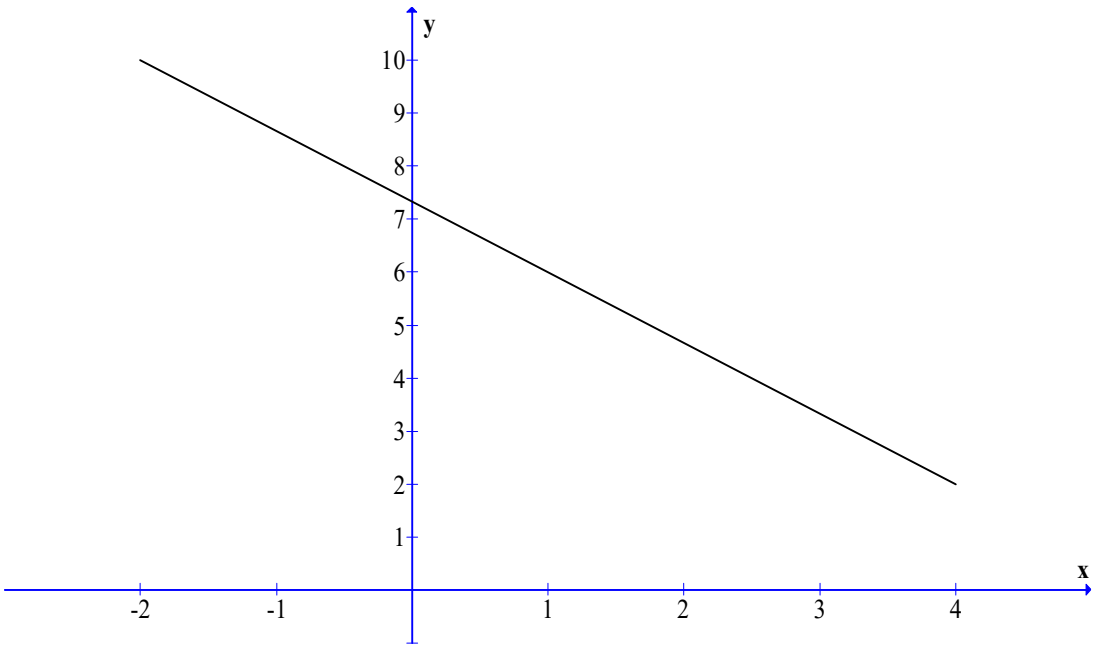
Gradient is positive



Length of hypotenuse $= \sqrt{1^2 + 1^2} = \sqrt{2}$

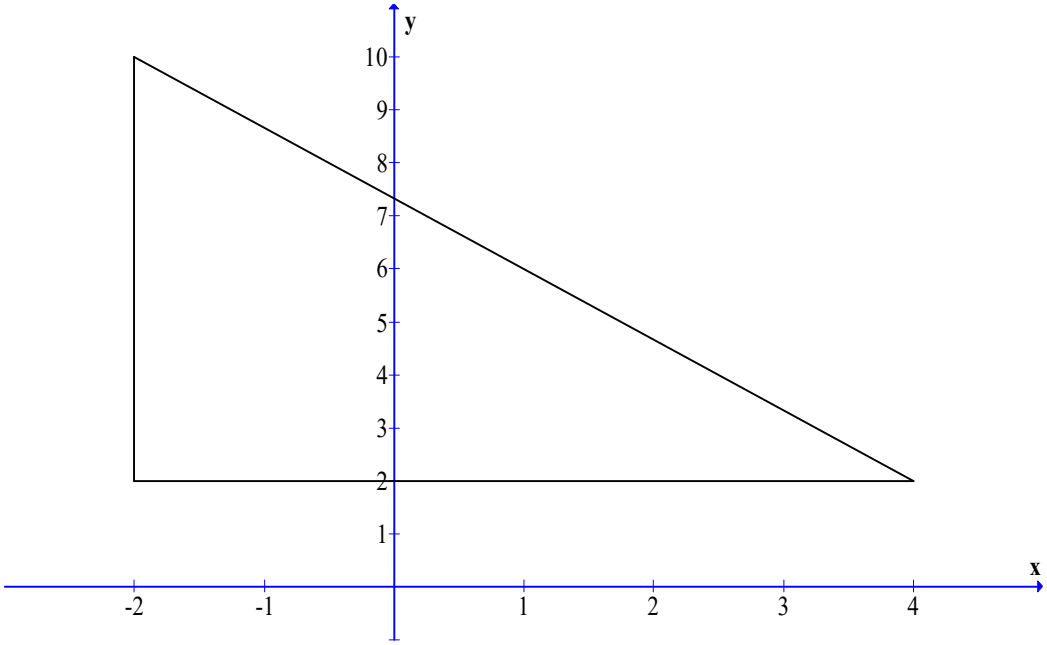
Gradient $= \frac{2-1}{2-1} = 1$

c) (-2, 10) and (4, 2)



Midpoint $= (1, 6)$

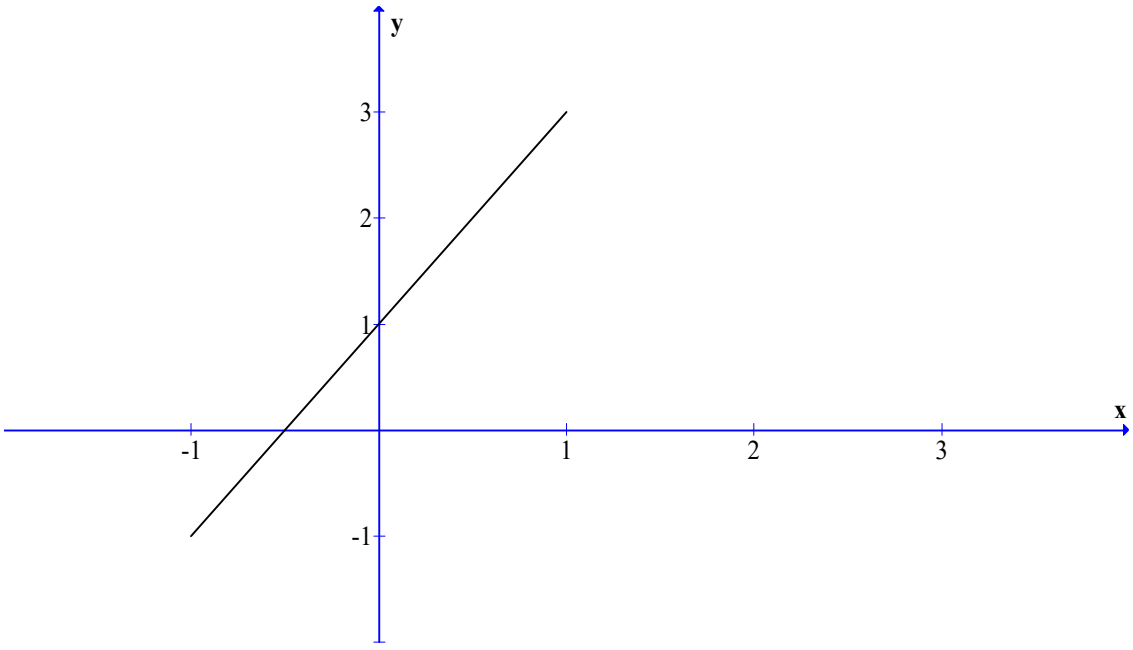
Gradient is negative



Length of hypotenuse $= \sqrt{8^2 + 6^2} = \sqrt{100} = 10$

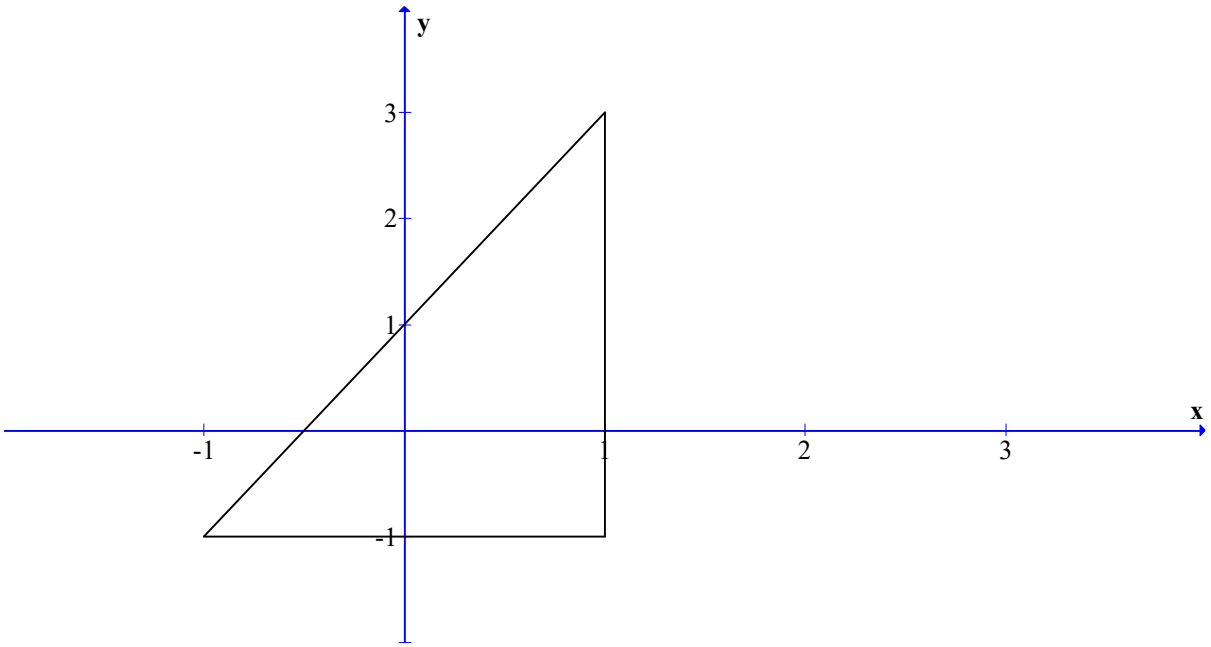
Gradient $= \frac{10-2}{-2-4} = -\frac{4}{3}$

d) (-1, -1) and (1, 3)



Midpoint $= (0, 1)$

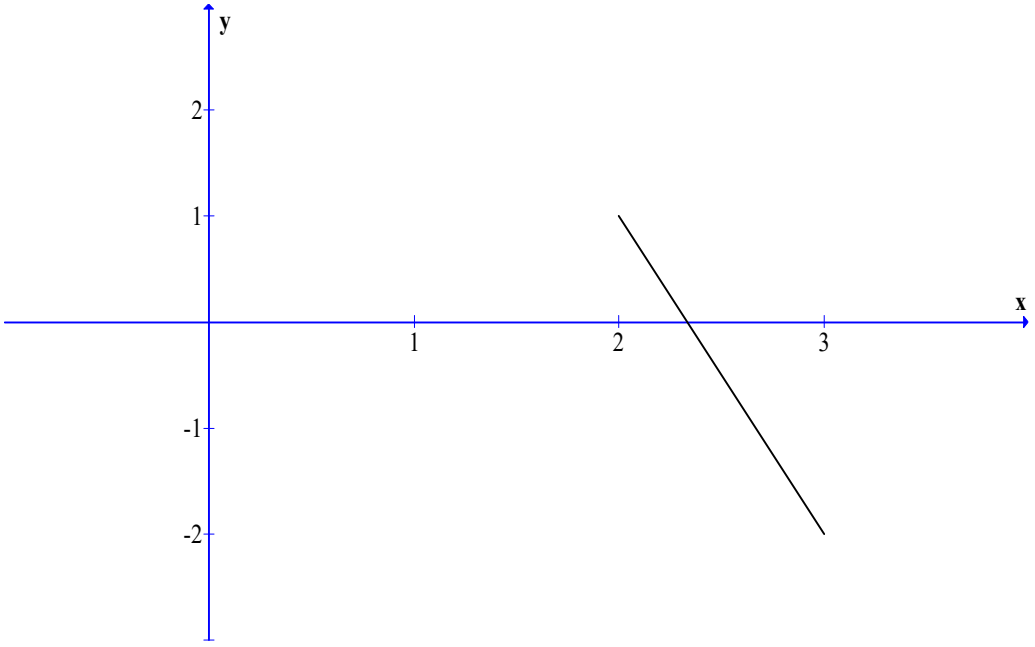
Gradient is positive



Length of hypotenuse $= \sqrt{2^2 + 4^2} = \sqrt{20}$

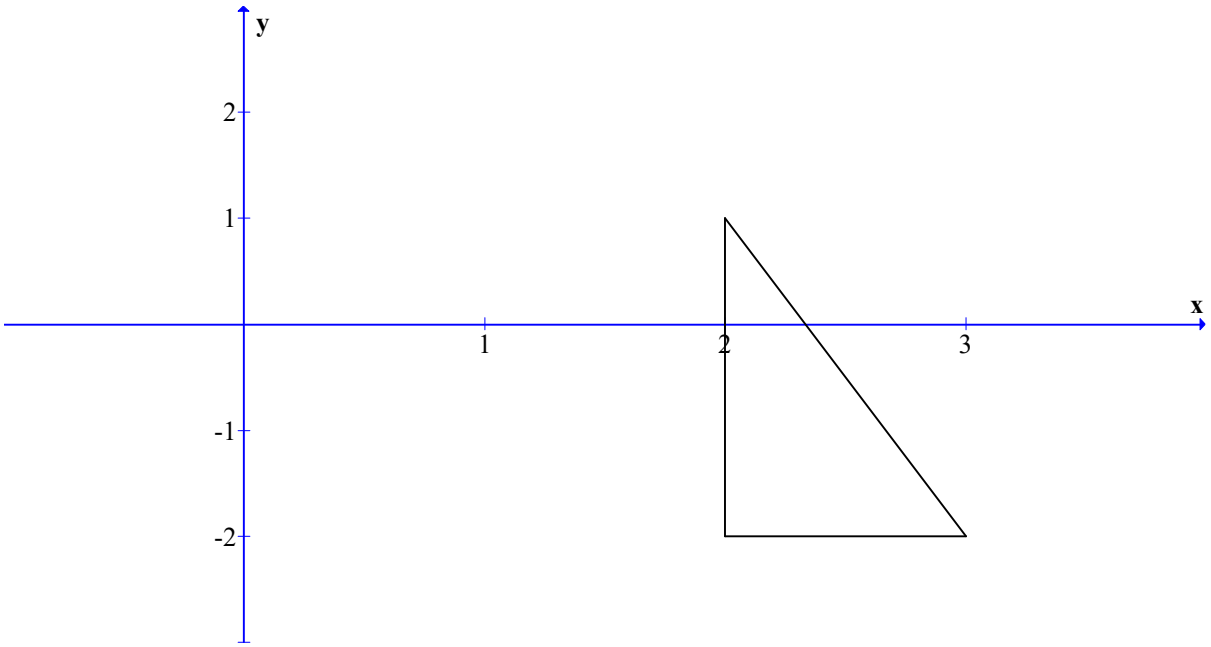
Gradient $= \frac{3--1}{1--1} = 2$

e) (2, 1) and (3, -2)



Midpoint $= (2.5, -0.5)$

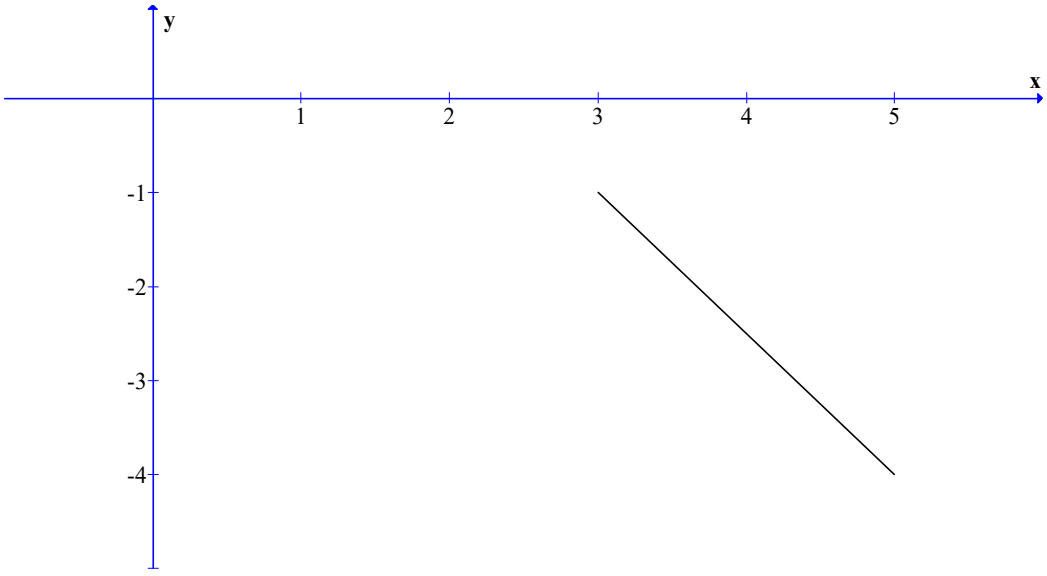
Gradient is negative



Length of hypotenuse $= \sqrt{3^2 + 1^2} = \sqrt{10}$

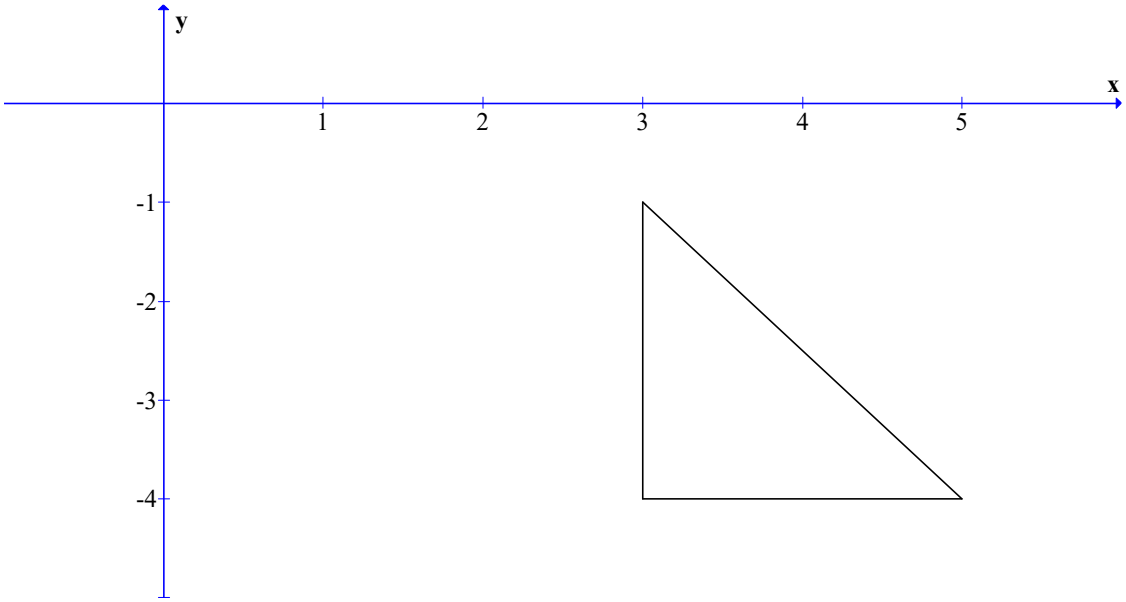
Gradient $= \frac{1--2}{2-3} = -3$

f) (3, -1) and (5, -4)



Midpoint $= (4, -2.5)$

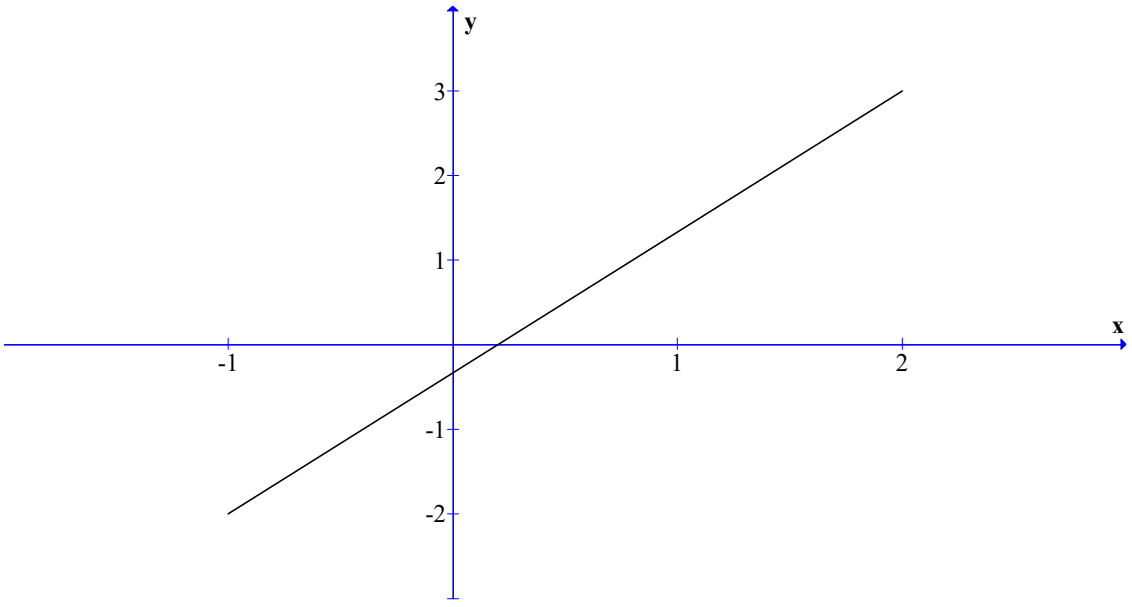
Gradient is negative



Length of hypotenuse $= \sqrt{3^2 + 2^2} = \sqrt{13}$

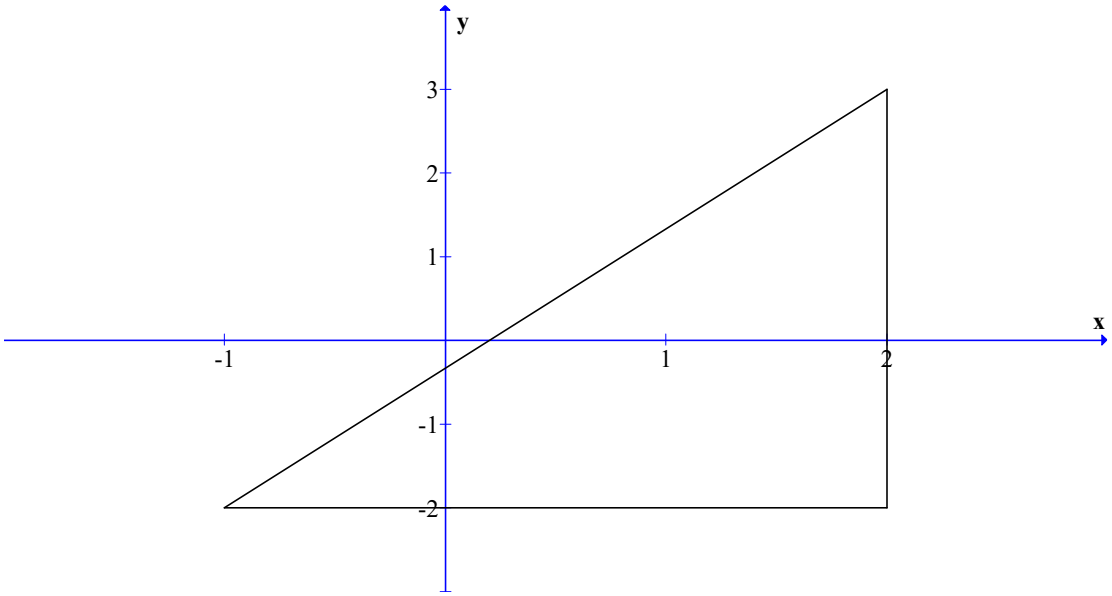
Gradient $= \frac{-1--4}{3-5} = -\frac{3}{2}$

g) (-1, -2) and (2, 3)



Midpoint $= (0.5, 0.5)$

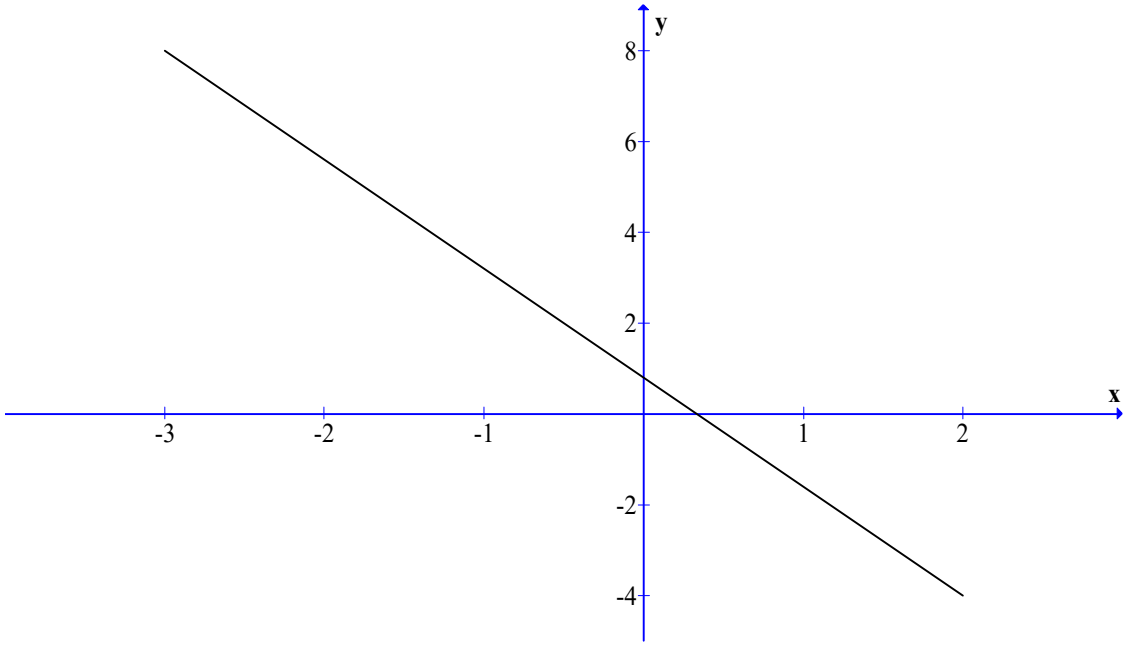
Gradient is positive



Length of hypotenuse $= \sqrt{3^2 + 5^2} = \sqrt{34}$

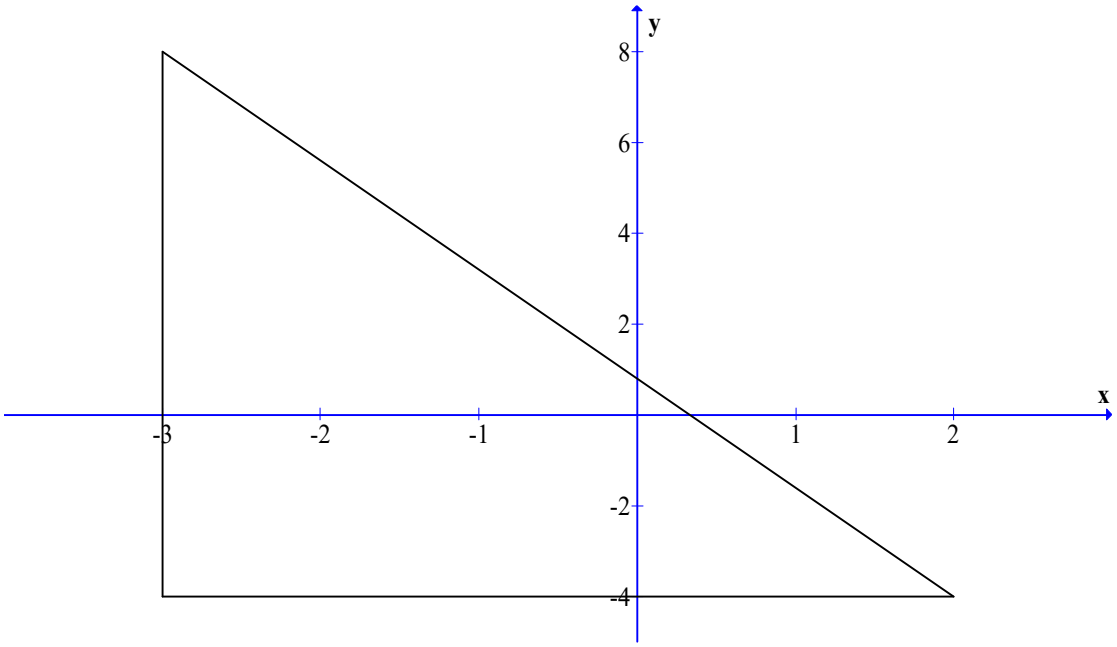
Gradient $= \frac{3--2}{2--1} = \frac{5}{3}$

h) (-3, 8) and (2, -4)



Midpoint $= (-0.5, 2)$

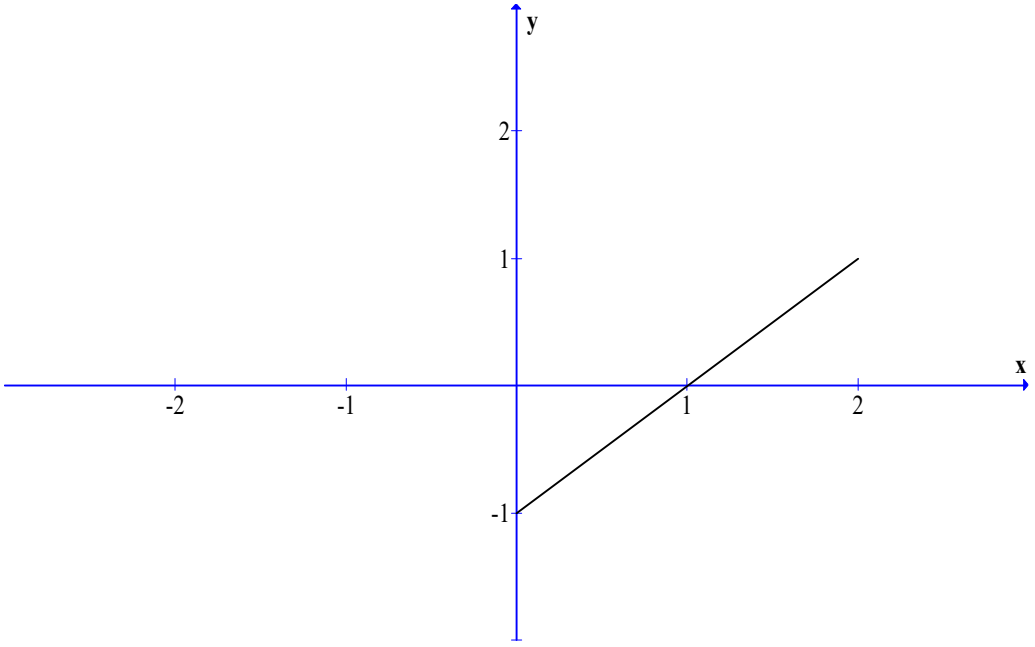
Gradient is negative



Length of hypotenuse $= \sqrt{5^2 + 12^2} = 13$

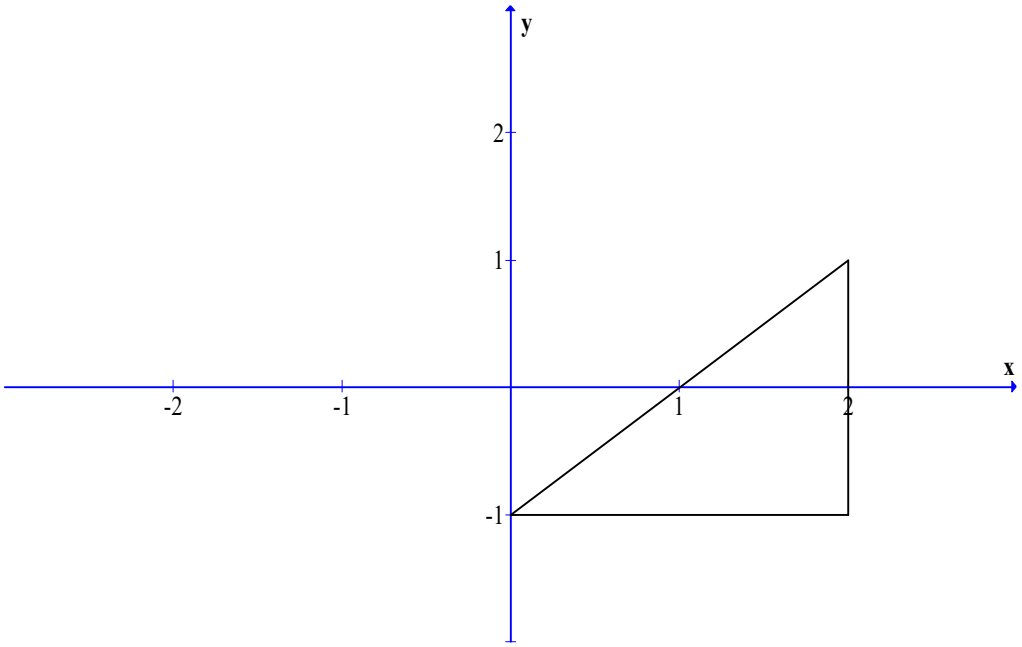
Gradient $= \frac{-4-8}{2--3} = -\frac{12}{5}$

i) (0, -1) and (2, 1)



Midpoint $= (1, 0)$

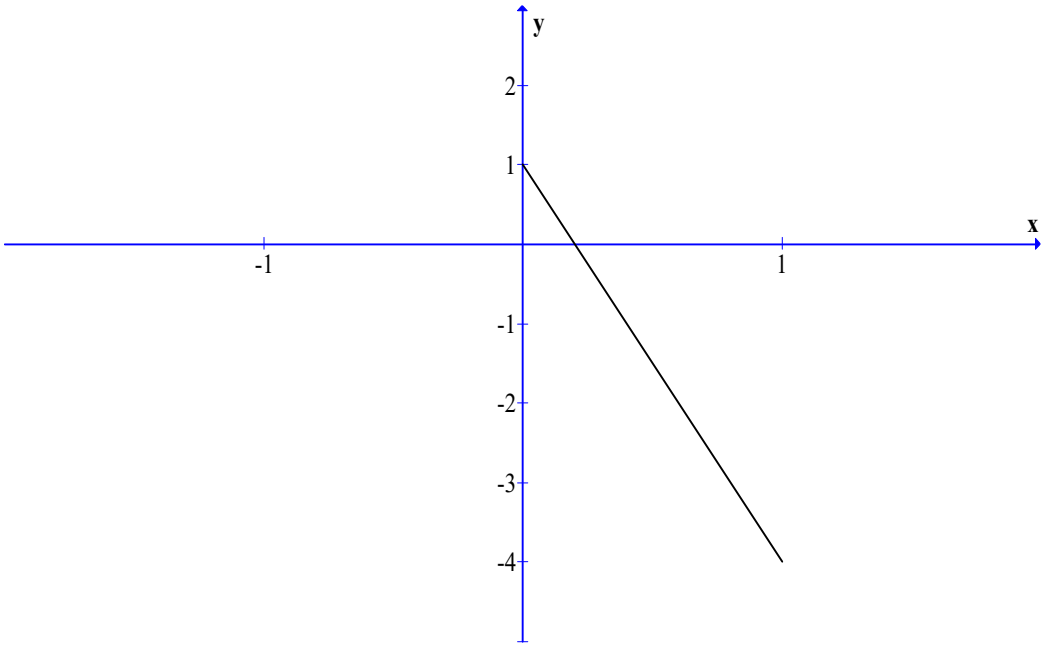
Gradient is positive



Length of hypotenuse $= \sqrt{1^2 + 2^2} = \sqrt{5}$

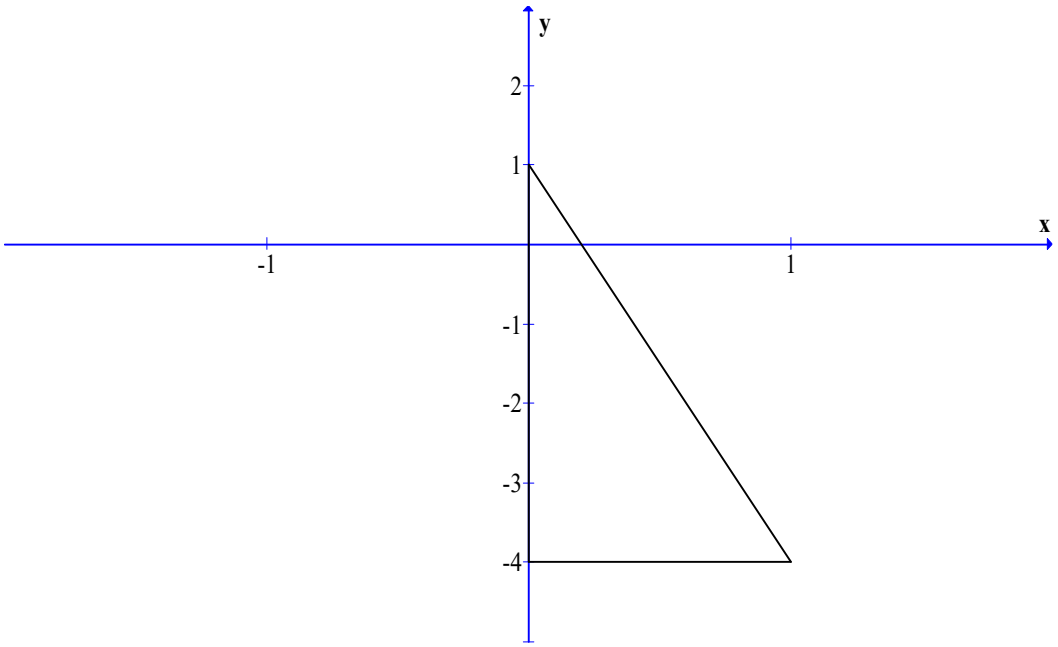
Gradient $= \frac{1--1}{2-0} = 1$

j) (0, 1) and 1, -4)



Midpoint $= (0.5, -1.5)$

Gradient is negative



Length of hypotenuse $= \sqrt{1^2 + 5^2} = \sqrt{26}$

Gradient $= \frac{-4-1}{1-0} = -5$

2) For each of the pairs of points given below, determine using the appropriate formula:

- The Midpoint of the line segment between the points
- The length of the line segment between the points
- The gradient of the line segment between the points

a) (0, 0) and (3, 3)

Midpoint $= (1.5, 1.5)$

Length of line segment $= \sqrt{3^2 + 3^2} = \sqrt{18}$

Gradient $= \frac{3-0}{3-0} = 1$

b) (2, 4) and (3, 6)

Midpoint $= (2.5, 5)$

Length of line segment $= \sqrt{2^2 + 1^2} = \sqrt{5}$

Gradient $= \frac{6-4}{3-2} = 2$

c) (-2, 4) and (0, 8)

Midpoint $= (-1, 6)$

Length of line segment $= \sqrt{4^2 + 2^2} = \sqrt{20}$

Gradient $= \frac{8-4}{0--2} = 2$

d) (-1, 4) and (-4, -2)

Midpoint $= (-2.5, 1)$

Length of line segment $= \sqrt{6^2 + 3^2} = \sqrt{45}$

Gradient $= \frac{-2-4}{-4--1} = 2$

e) (0, 2) and (0, 8)

Midpoint $= (0, 5)$

Length of line segment $= \sqrt{6^2 + 0^2} = 6$

Gradient $= \frac{8-2}{0-0}$ gradient does not exist (line is vertical)

f) (2, 0) and (2, 8)

Midpoint $= (0, 4)$

Length of line segment $= \sqrt{8^2 + 0^2} = 8$

Gradient $= \frac{8-0}{2-2}$ gradient does not exist (line is vertical)

g) (-10, 10) and (0, -10)

Midpoint $= (-5, 0)$

Length of line segment $= \sqrt{10^2 + 20^2} = \sqrt{500}$

Gradient $= \frac{-10-10}{0--10} = -2$

h) (3, -6) and (-6, 15)

Midpoint $= (-1.5, 4.5)$

Length of line segment $= \sqrt{9^2 + 21^2} = \sqrt{522}$

Gradient $= \frac{15--6}{-6-3} = -\frac{21}{9} = -\frac{7}{3}$

i) (-1, -1) and (-5, 15)

Midpoint $= (-3, 7)$

Length of line segment $= \sqrt{16^2 + 4^2} = \sqrt{272}$

Gradient $= \frac{15--1}{-5--1} = -4$

j) (4, 10) and (-2, -2)

Midpoint $= (1, 4)$

Length of line segment $= \sqrt{12^2 + 6^2} = \sqrt{180}$

Gradient $= \frac{-2-10}{-2-4} = 2$

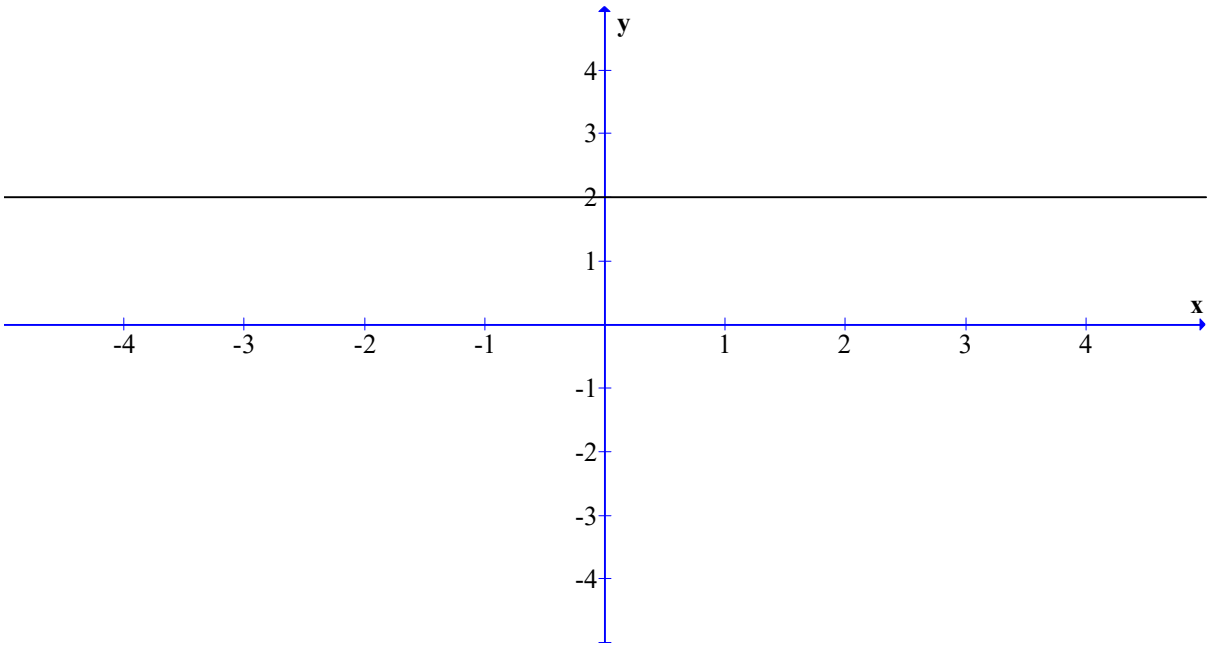
Exercise 2

Graphing Linear Relationships

For each of the equations below you are required to:

- Construct a table of values
- Plot the points tabled using a suitable scale
- Determine the x and y intercepts
- Determine if the point given lies on the line or curve by substituting the point into the equation, or plotting the point on the graph and seeing if it lies on the line/curve

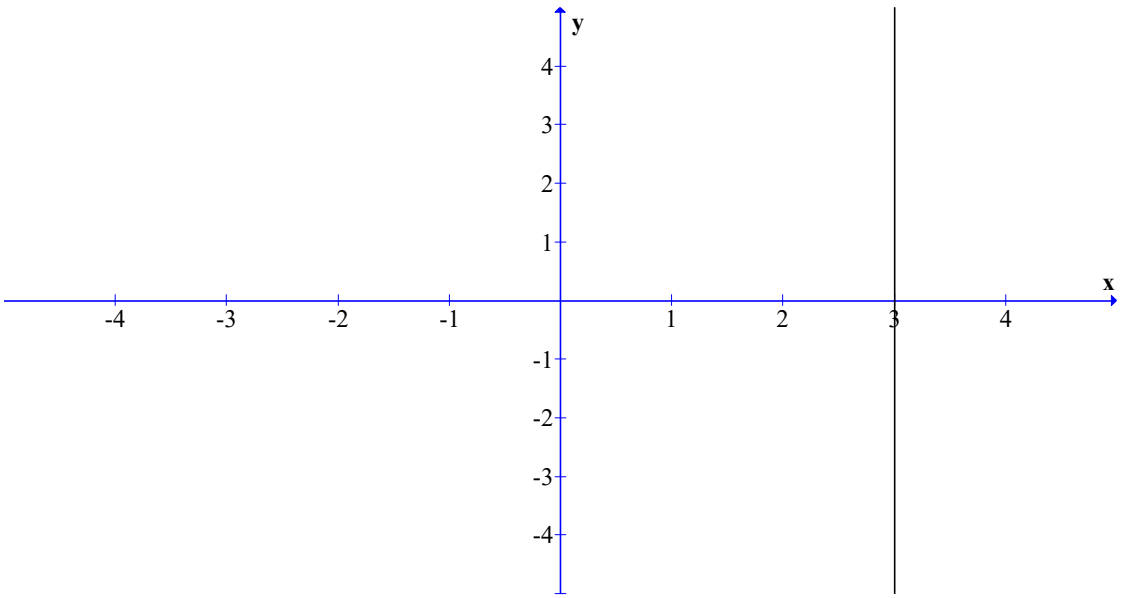
a) $y = 2$ (1, 2)



y intercept is 2, no x intercept

From the graph $(1, 2)$ lies on the line

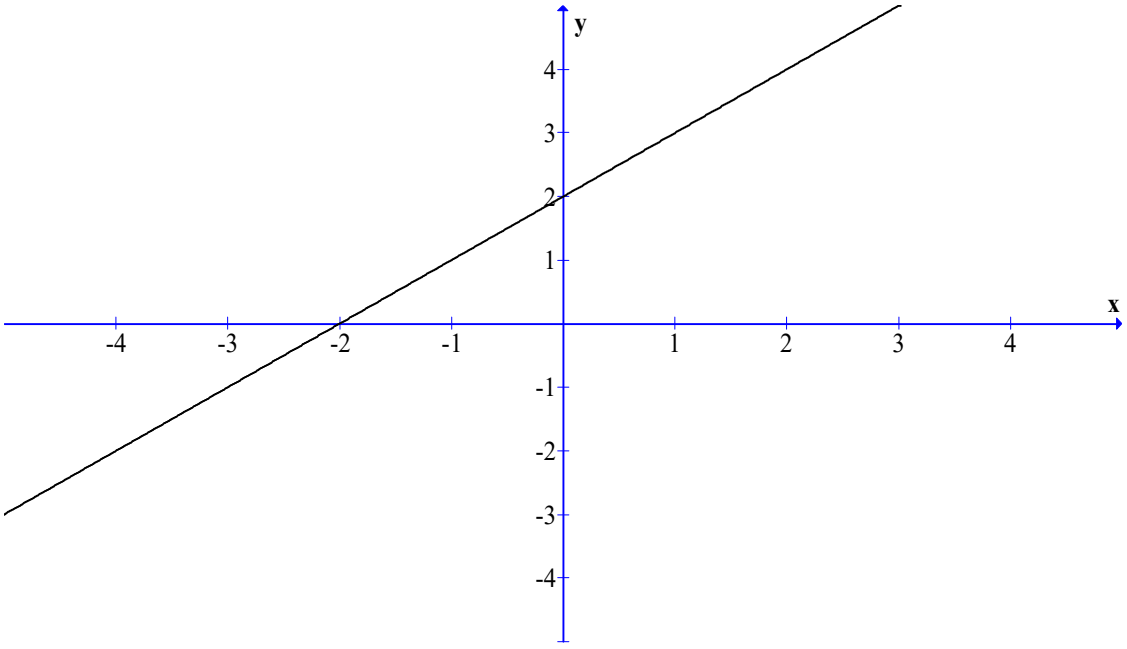
b) $x = 3$ (1, 3)



x intercept is 3, no y intercept

From graph, point does not lie on line

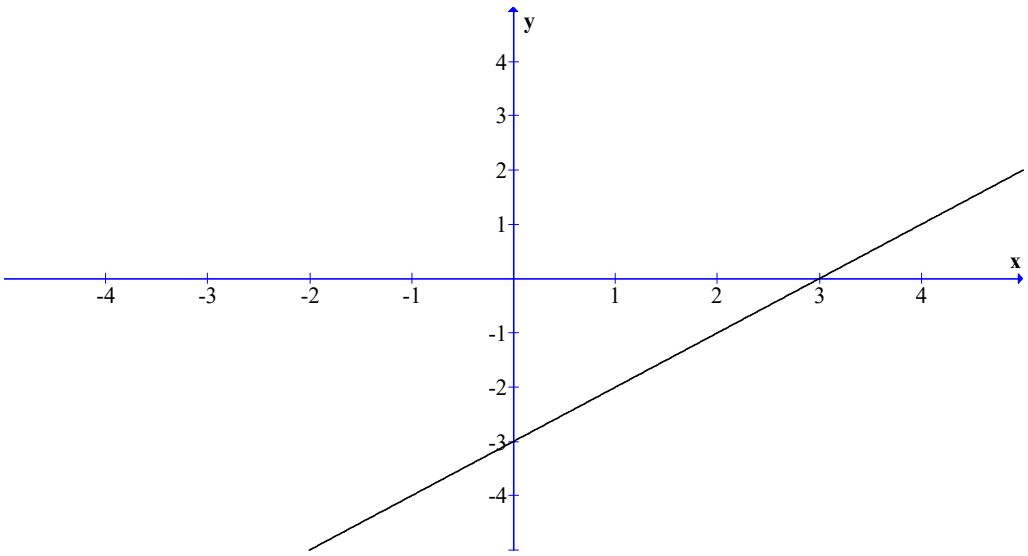
c) $y = x + 2$ (2, 4)



x intercept is -2, y intercept is 2

From equation, point lies on the line

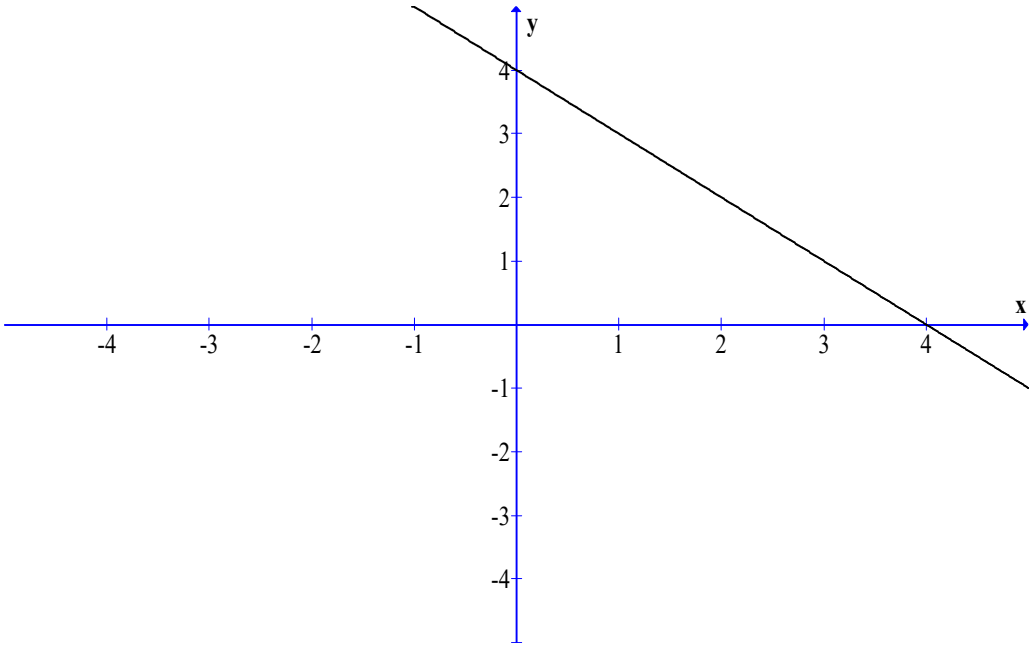
d) $y = x - 3$ (4, -1)



x intercept is 3, y intercept is -3

From graph, point does not lie on line

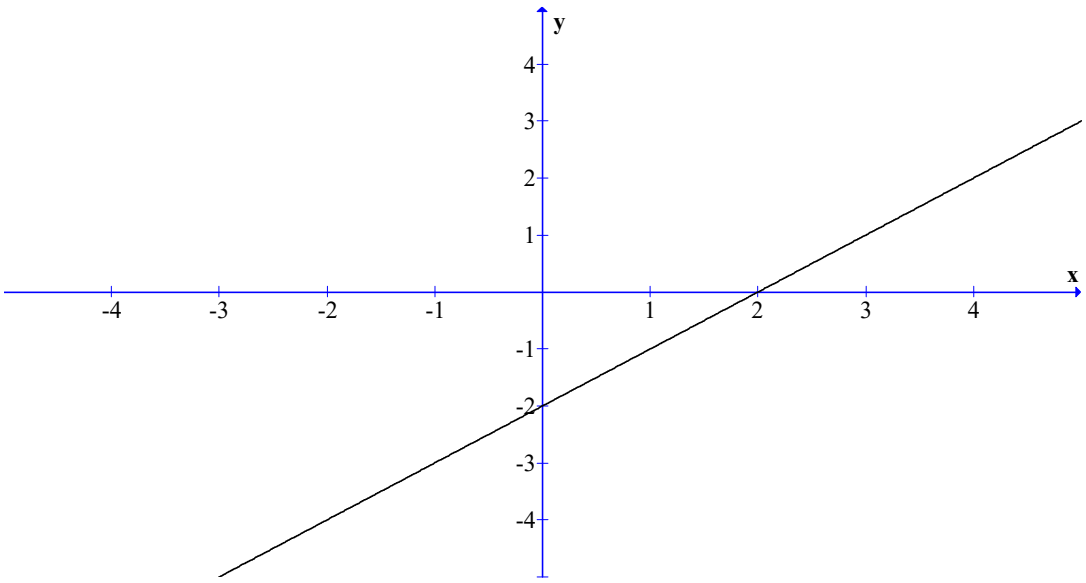
e) $x + y = 4$ (0, 4)



x intercept is 4, y intercept is 4

From graph, point lies on line

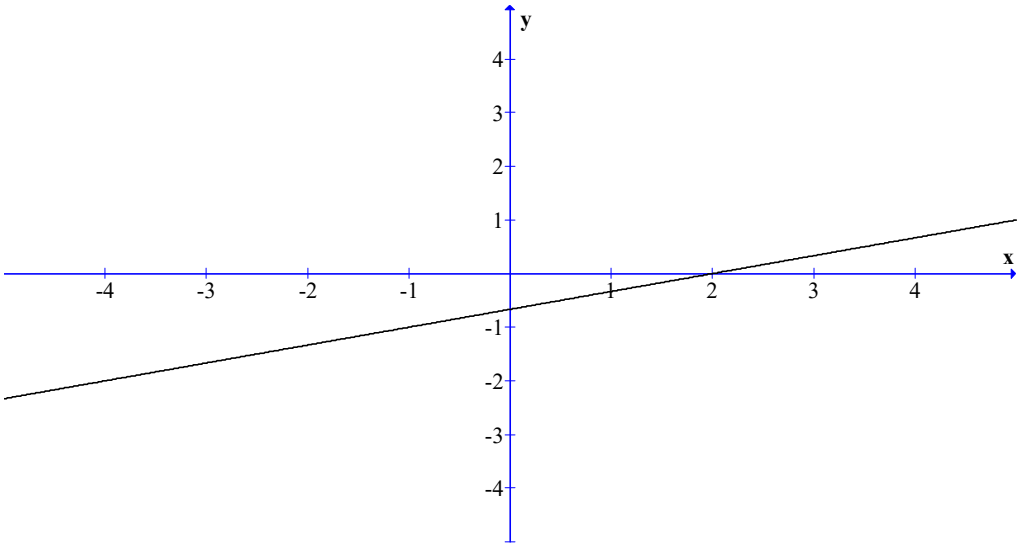
f) $x - y = 2$ (-2, -4)



x intercept is 2, y intercept is -2

From equation, point lies on the line

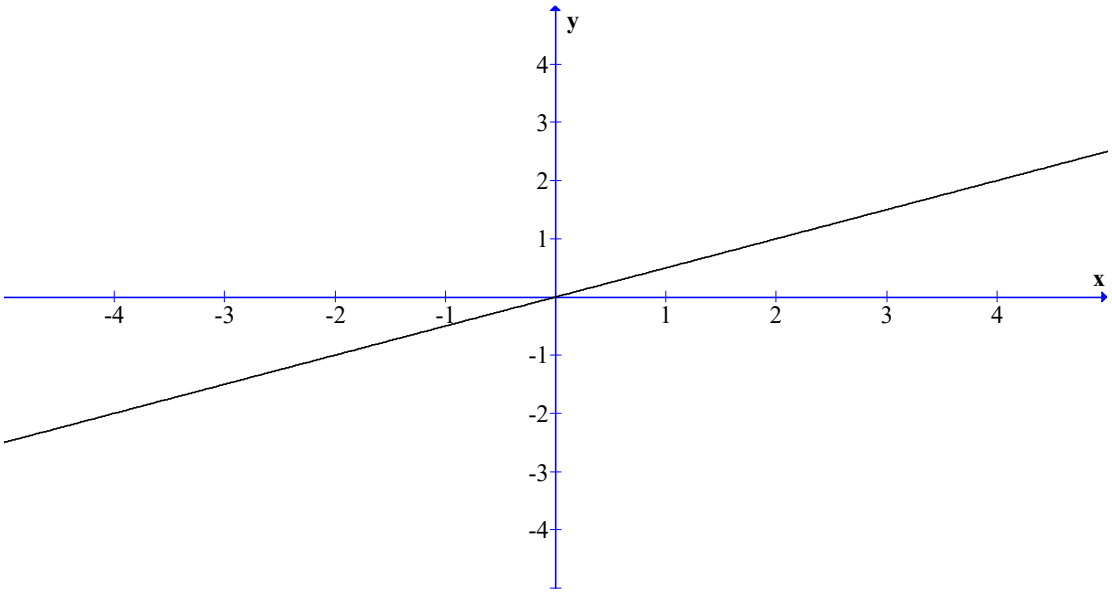
g) $y = \frac{x-2}{3}$ (8, 2)



X intercept is 2, y intercept is $-\frac{2}{3}$

From equation, point lies on line

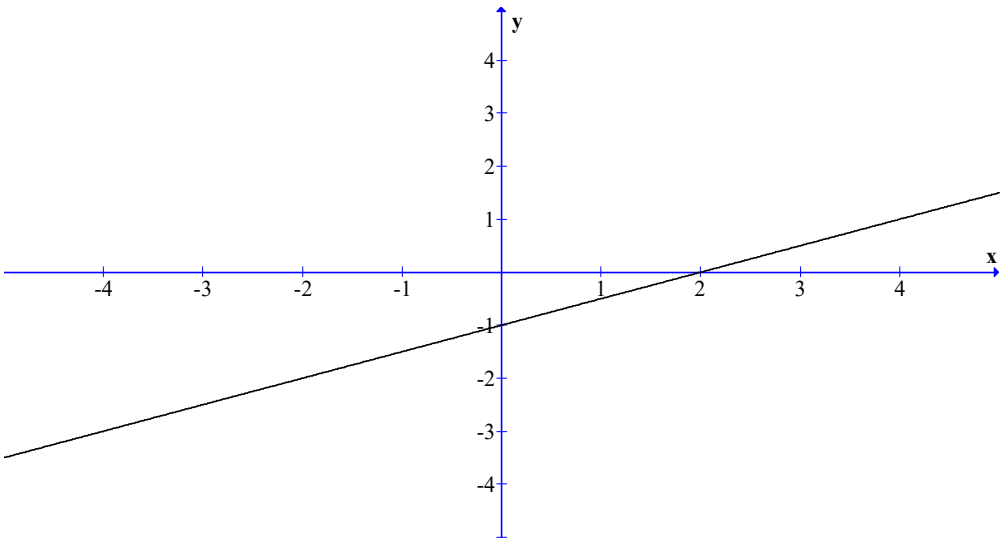
h) $y = \frac{1}{2}x$ (5, 10)



x intercept is 0, y intercept is 0

From equation, point lies on the line

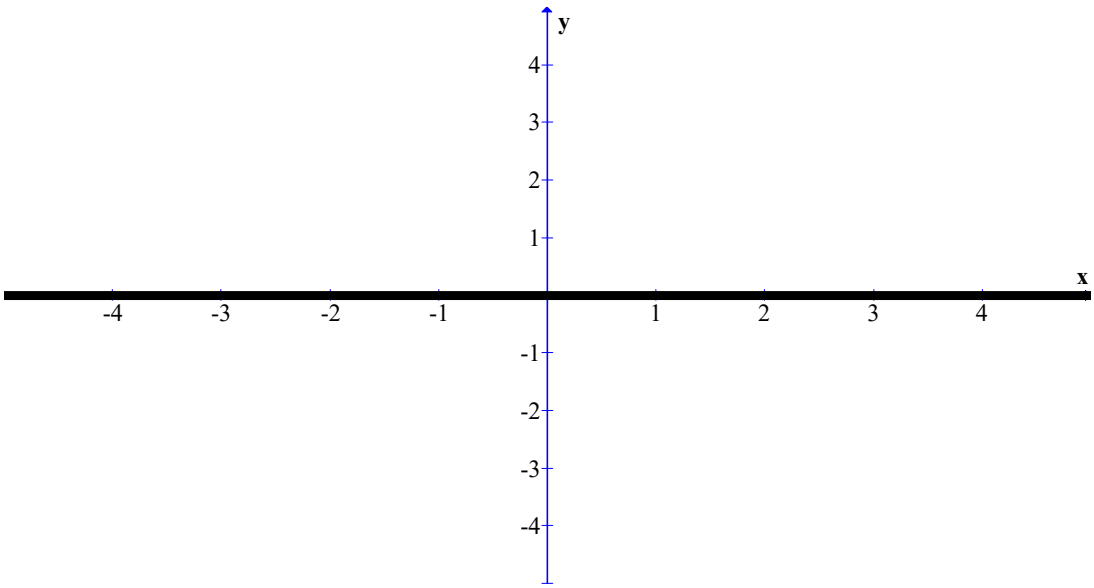
i) $y = \frac{1}{2}x - 1$ (4, 1)



x intercept is 2, y intercept is -1

From graph, point lies on line

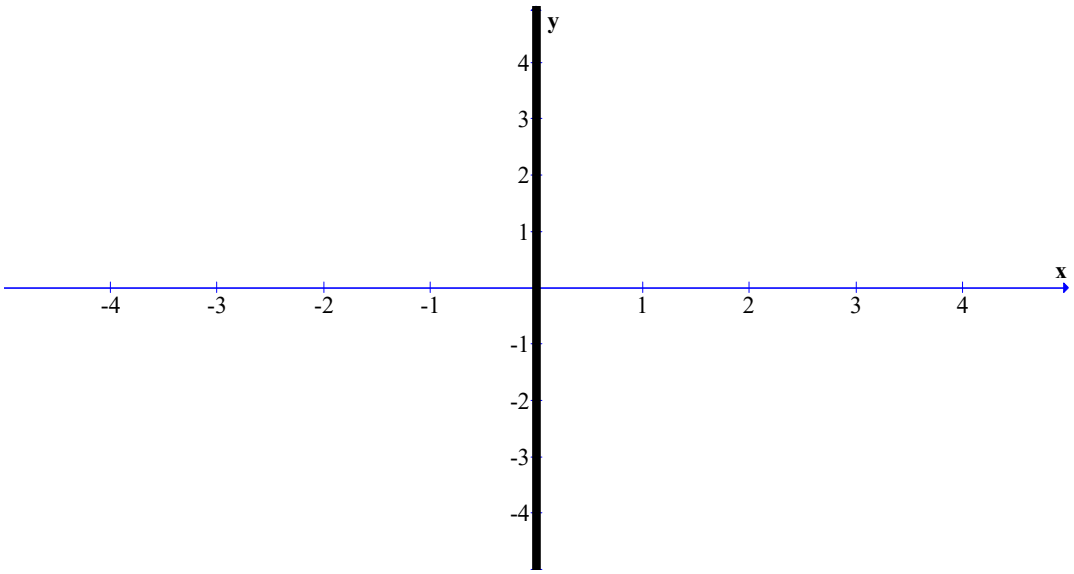
j) $y = 0$ (0, 0)



Line is x axis, y intercept is 0

From graph, point lies on line

k) $x = 0$ (0, 0)



X intercept is 0, line is y axis

From graph, point lies on line

Exercise 3

Gradient/Intercept Form of Linear Equations

1) For each equation below state the value of the gradient, and the coordinate of the y-intercept

a) $y = 2x + 3$

Gradient = 2

y intercept = 3

b) $y = 3x - 1$

Gradient = 3

y intercept = -1

c) $y = \frac{1}{2}x + 5$

Gradient = $\frac{1}{2}$

y intercept = 5

d) $y = 3x$

Gradient =3

y intercept = 0

e) $y = 2$

Gradient = 0

y intercept = 2

f) $y = -3x - 4$

Gradient = -3

y intercept = -4

g) $y = -\frac{1}{2}x$

Gradient = $-\frac{1}{2}$

y intercept = 0

h) $y = -\frac{2}{3}x + 6$

Gradient = $-\frac{2}{3}$

y intercept = 6

2) Rearrange the following equations into the form $y = mx + b$

a) $x + y + 3 = 0$

$y = -x - 3$

b) $y - x - 4 = 0$

$y = x + 4$

c) $2y + 4x - 6 = 0$

$2y = -4x + 6$

$y = -2x + 3$

d) $\frac{1}{2}y - x - 2 = 0$

$\frac{1}{2}y = x + 2$

$y = 2x + 4$

e) $x + y = 0$

$y = -x$

f) $-y - 2x = 0$

$y = -2x$

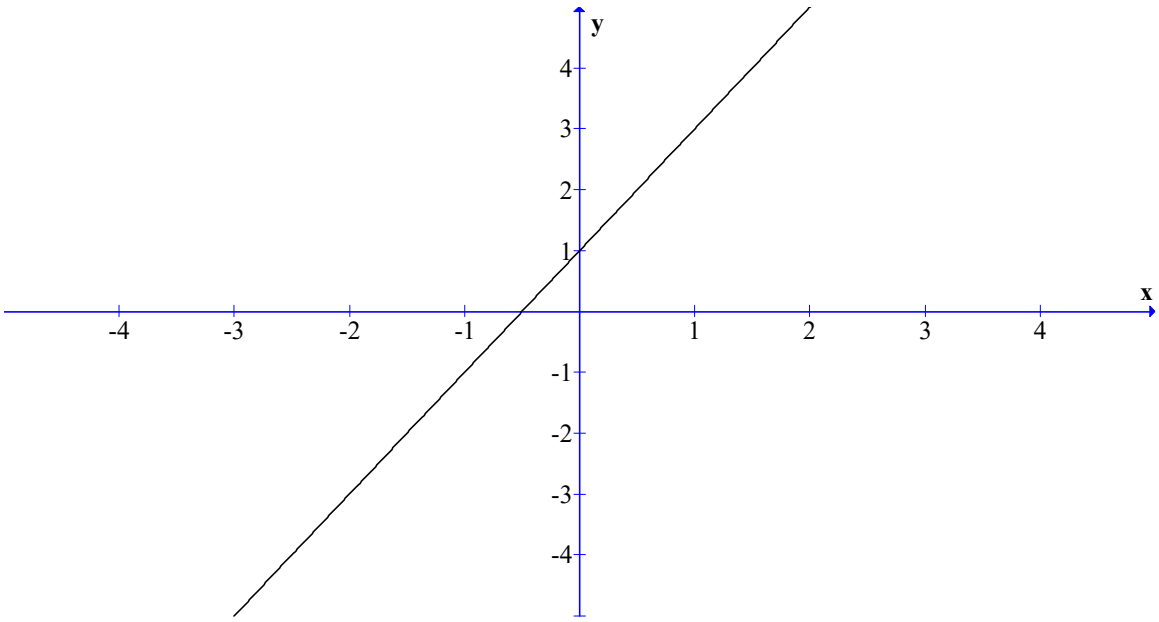
g) $y - 2 = 0$

$y = 2$

3) Draw graphs of the following equations given the gradient and the y-intercept. State the equation of the line

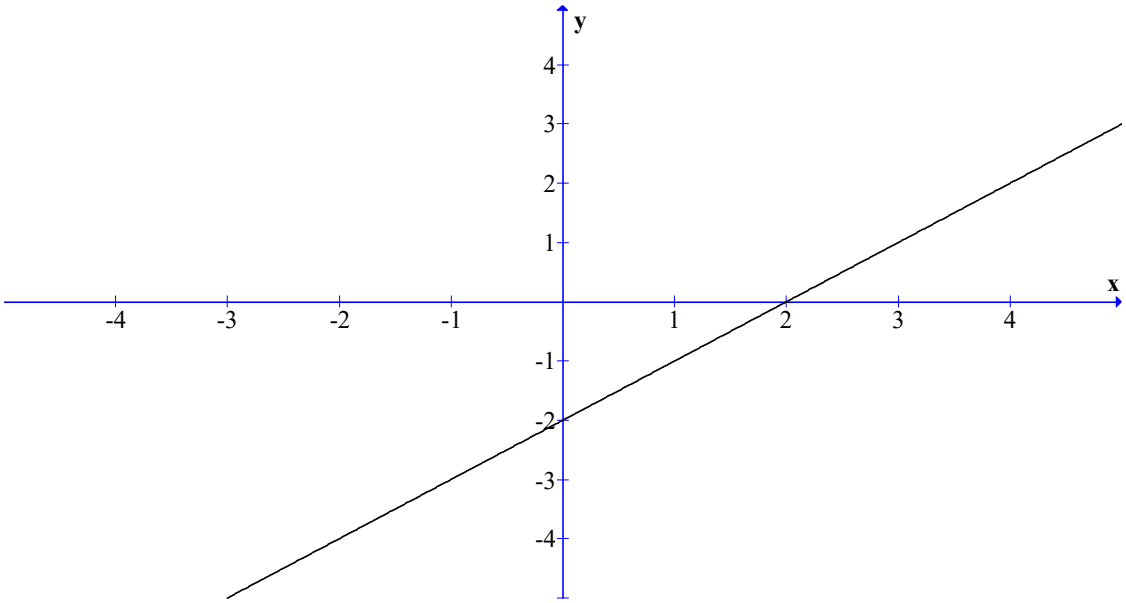
a) Gradient = 2, y-intercept = 1

$y = 2x + 1$



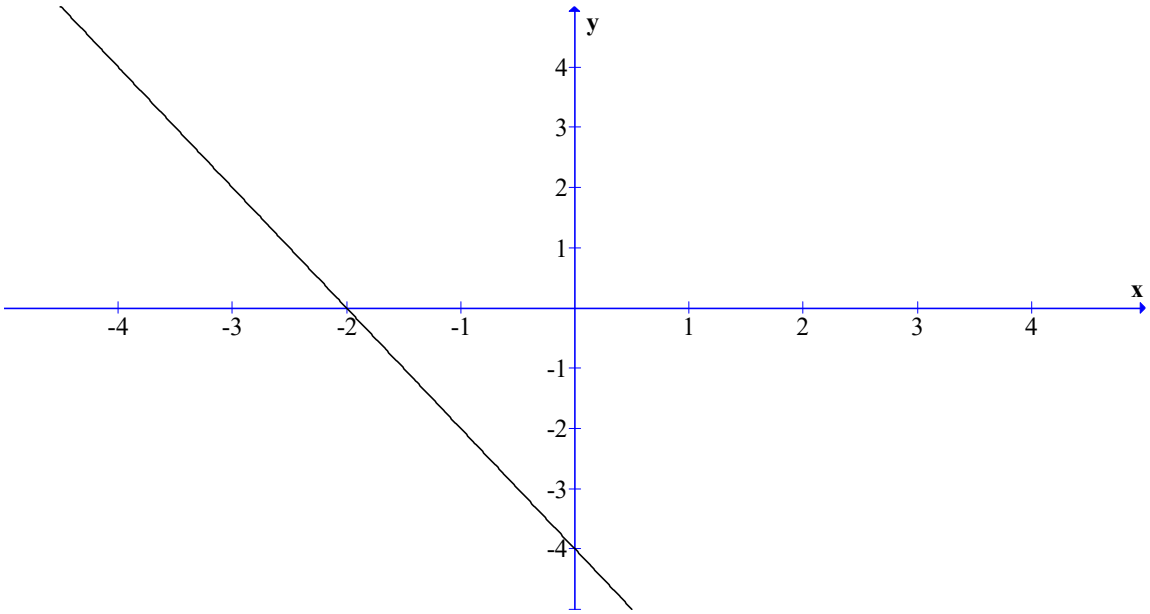
b) Gradient = 1, y-intercept = -2

$y = x - 2$



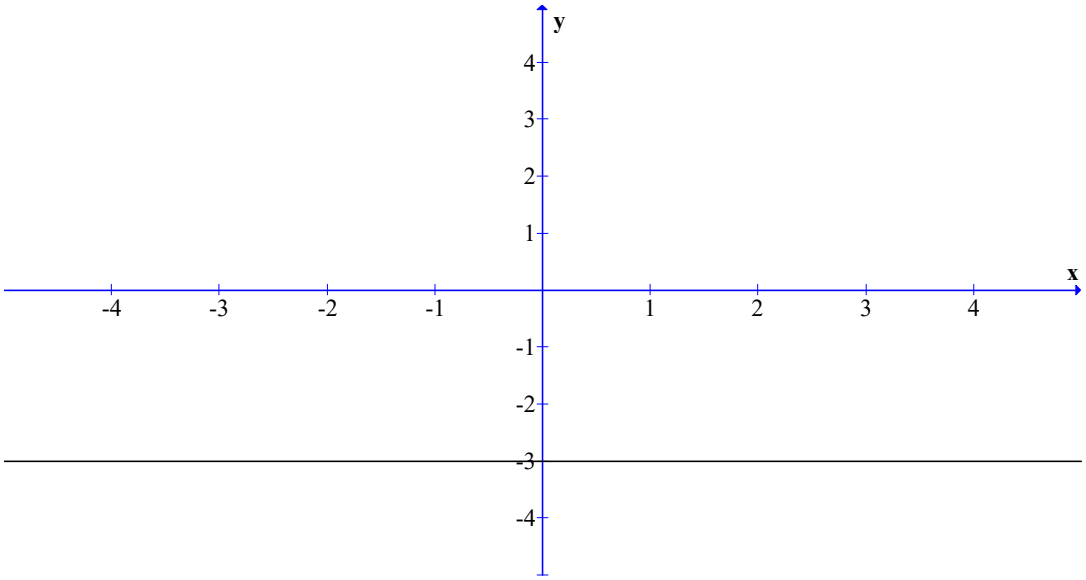
c) Gradient = -2, y-intercept = -4

$y = -2x - 4$



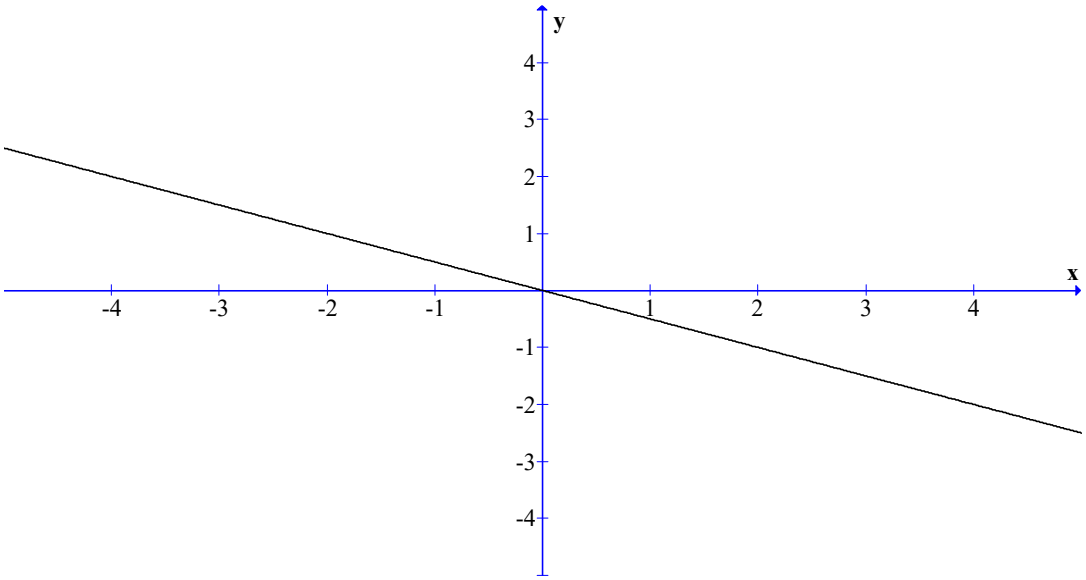
d) Gradient = 0, y-intercept = 3

$y = -3$



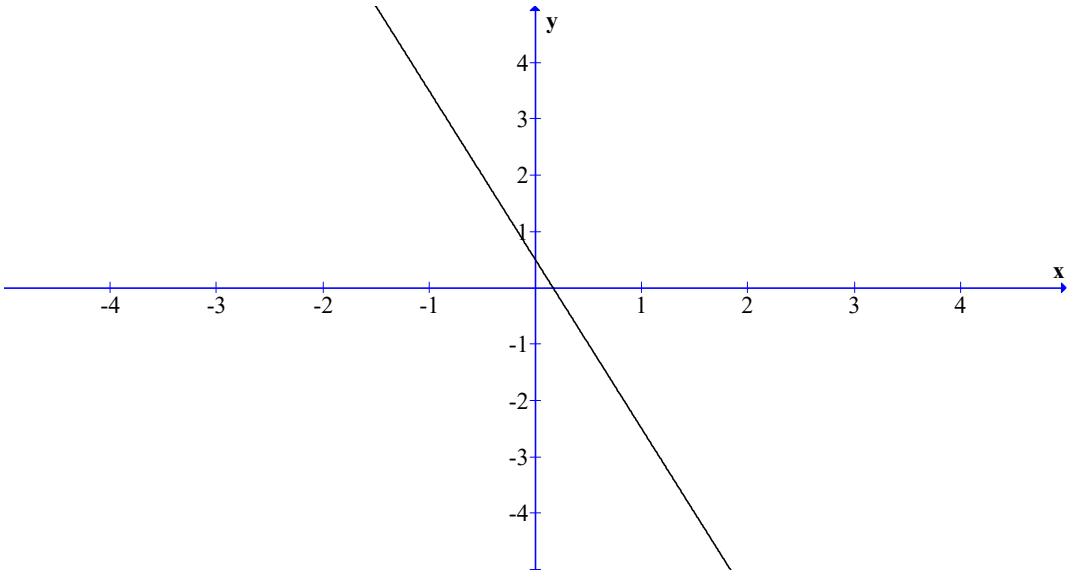
e) Gradient = $-\frac{1}{2}$, y-intercept = 0

$y = -\frac{1}{2}x$



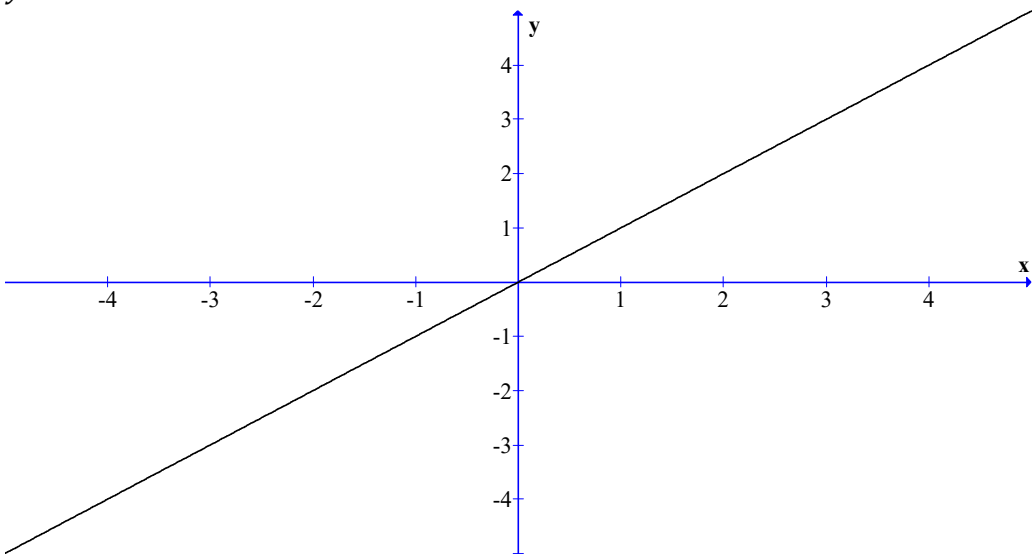
f) Gradient = -3, y-intercept = $\frac{1}{2}$

$$y = -3x + \frac{1}{2}$$



g) Gradient = 1, y-intercept = 0

$$y = x$$



4) State if the following pairs of lines are parallel, showing your working

a) $y = 2x + 3, y = 2x - 1$

Yes, the x coefficients and hence gradients are equal

b) $y = x + 4, y = 2x - 2$

No, the gradients are not equal

c) $2y = 4x - 5, y = 2x - 7$

First equation becomes $y = 2x - \frac{5}{2}$

Lines are parallel, gradients are equal

d) $2x + 2y + 3 = 0, y = -x - 3$

First equation becomes $y = -x - \frac{3}{2}$

Lines are parallel, gradients are equal

e) $2y + 4x - 4 = 0, y = -2x$

First equation becomes $y = -2x + 2$

Lines are parallel, gradients are equal

f) $3x - 6y + 3 = 0, y + \frac{1}{2}x + 4 = 0$

First equation becomes $y = \frac{1}{2}x + \frac{1}{2}$

Second equation becomes $y = -\frac{1}{2}x - 4$

Lines are not parallel

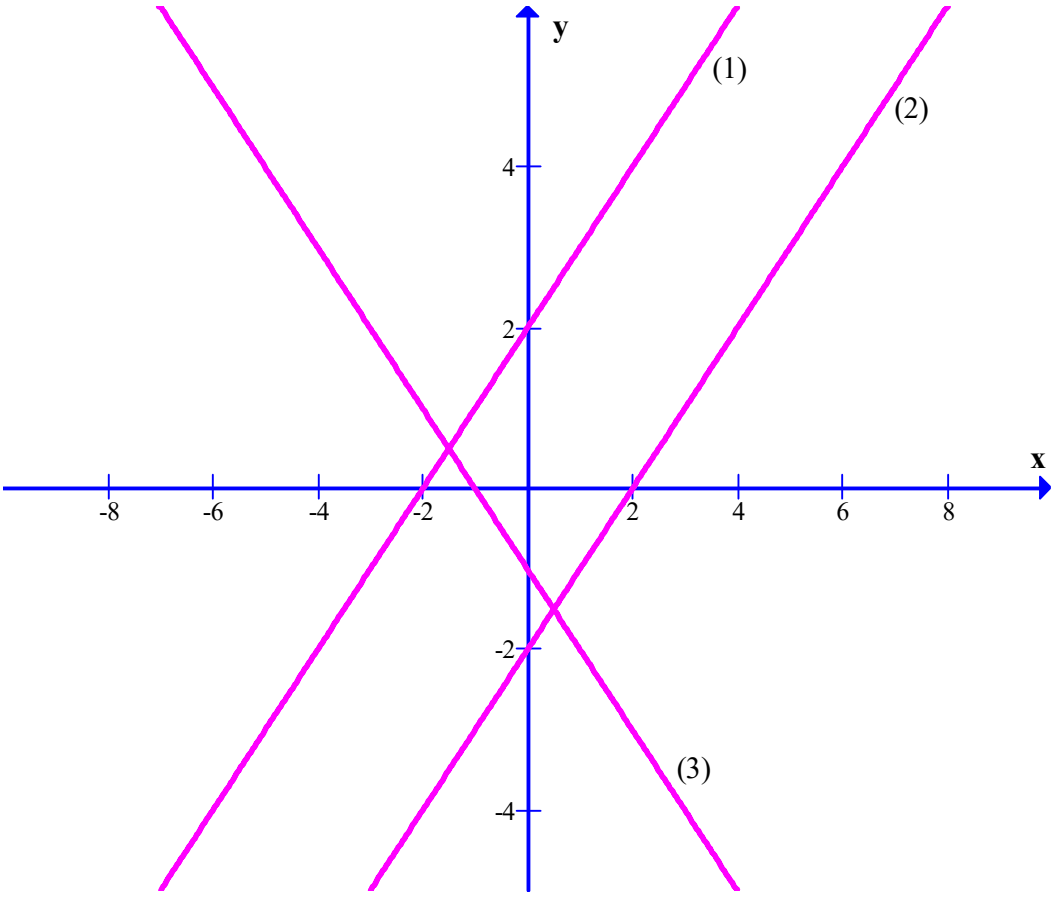
g) $x = 4, y = 4$

Lines are not parallel. One of them is parallel to the x axis, and the other is parallel to the y axis

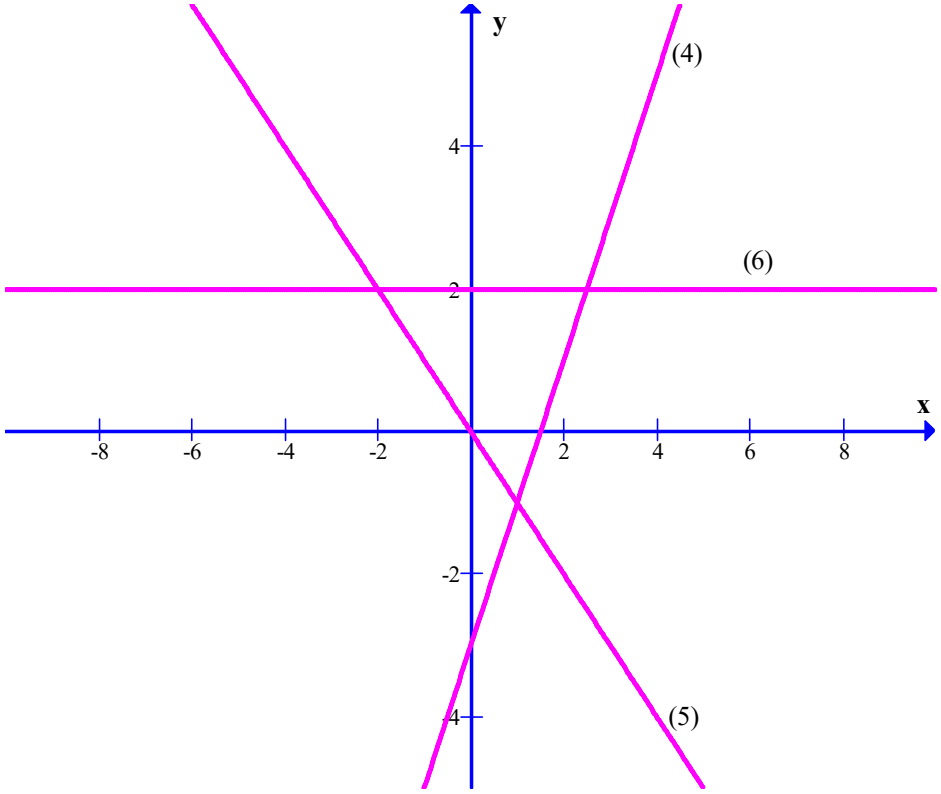
h) $y = 4, y = 2$

Both lines are parallel to the x axis and hence to each other

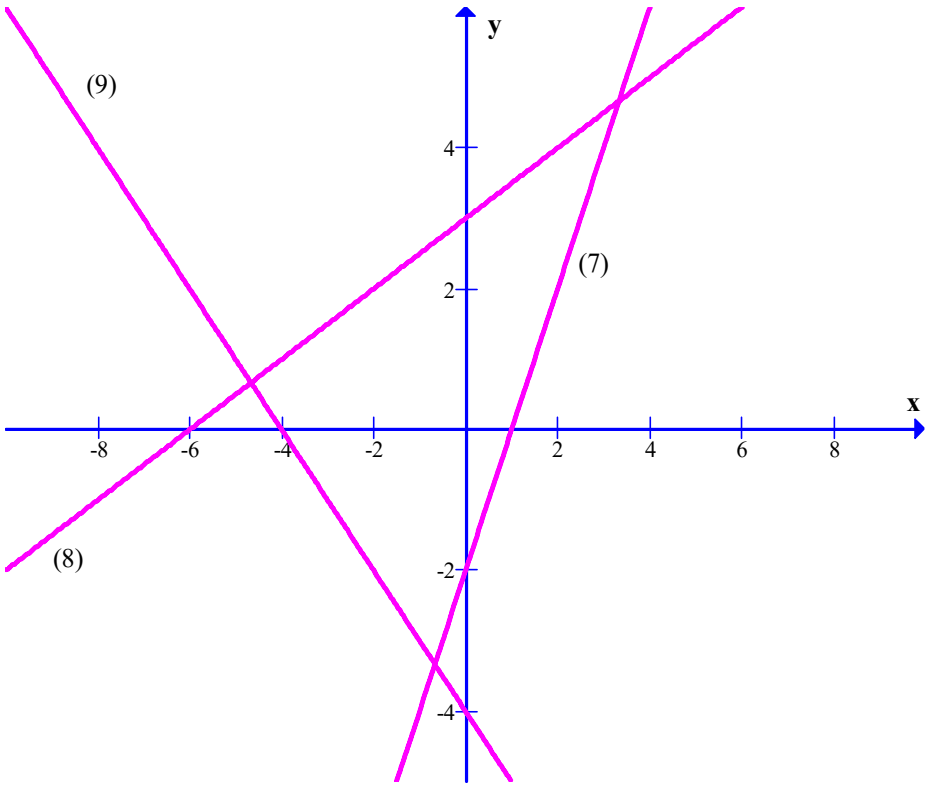
5) Determine the equations of the graphs drawn below (3 questions per graph)



1. $y = x + 2$
2. $y = x - 2$
3. $y = -x - 1$



4. $y = 2x - 3$
5. $y = -x$
6. $y = 2$



7. $y = 2x - 2$
8. $y = \frac{1}{2}x + 3$
9. $y = -x - 4$

Year 9 Mathematics

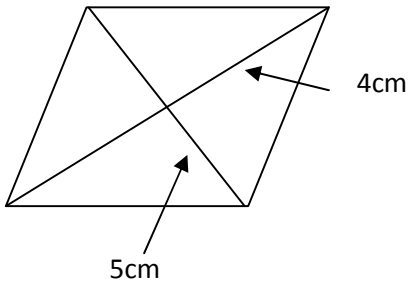
Measurement:

Exercise 1

Area & Perimeter

1) .Calculate the area of the following

a)



Area = 4 x area of triangle

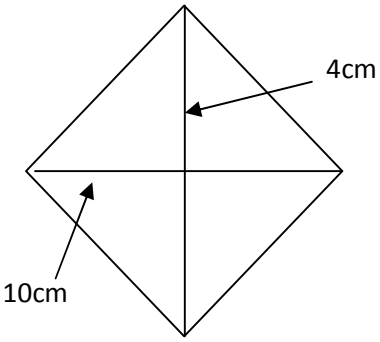
Diagonals are 5cm and 4 cm respectively

The triangles each have a base of 2.5 cm and a height of 2 cm (half the diagonal)

Area of each triangle $= \frac{1}{2} \times$ base x height$= \frac{1}{2} \times 2.5 \times 2 = 2.5$ cm^2

Area of shape = 10 cm^2

b)



Area = 4x area each triangle

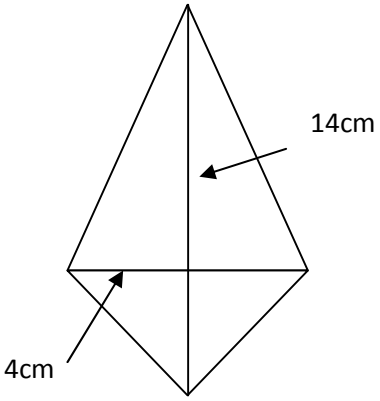
Diagonals are 10 cm and 4 cm respectively

The triangles each have a base of 5 cm and a height of 2 cm (half the diagonal)

Area of triangle $= \frac{1}{2}$ x base x height$= \frac{1}{2} \times 5 \times 2 = 5$ cm^2

Area of shape = 20 cm

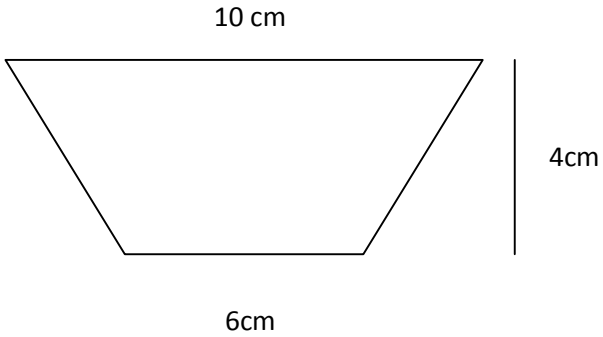
c)



Area of kite = $\frac{1}{2}xy$, where x and y are the lengths of the diagonals

Area $= \frac{1}{2} \times 14 \times 4 = 28$ cm^2

d)

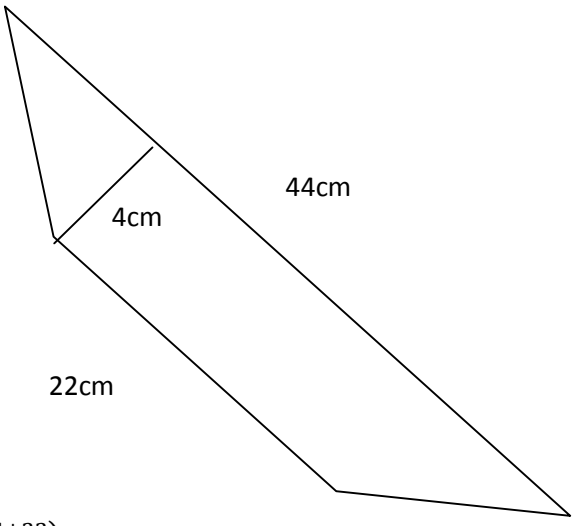


Area of trapezium $= \left(\frac{a+b}{2}\right) \times h$

$a = 6, b = 10, h = 4$

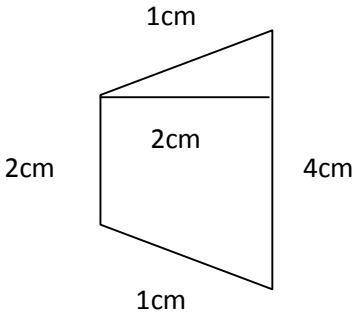
Area $= \left(\frac{6+10}{2}\right) \times 4 = 32$ cm^2

e)



Area $= \left(\frac{44+22}{2}\right) \times 4 = 132$ cm^2

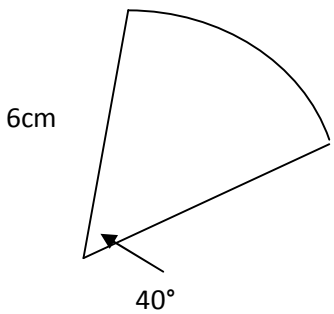
f)



Area $= \left(\frac{2+4}{2}\right) \times 2 = 6$ cm

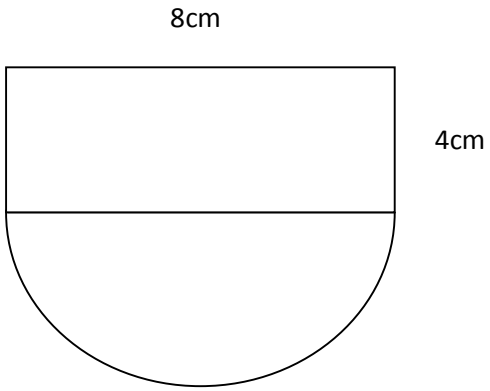
2) Calculate the area of each of the following

a)



Area of sector $= \pi r^2 \times \frac{\theta}{360} = \pi \times 6 \times 6 \times \frac{40}{360} = 4\pi \cong 12.56$ cm^2

b)



Total area = Area of rectangle + area of semicircle

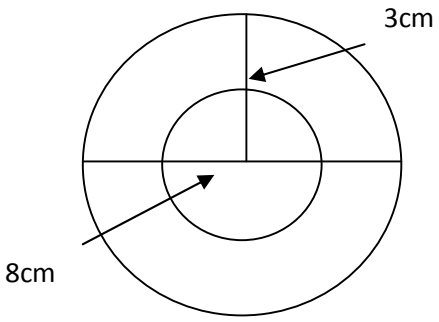
Area of rectangle $= 8 \times 4 = 32$ cm^2

Area of semicircle $= \frac{1}{2} \times \pi \times r \times r$

Diameter of circle $= 8$ cm

Radius $= 4$ cm

c) (Calculate area of outside ring)



(Diameter of large circle is 8 cm; radius of small circle is 3 cm)

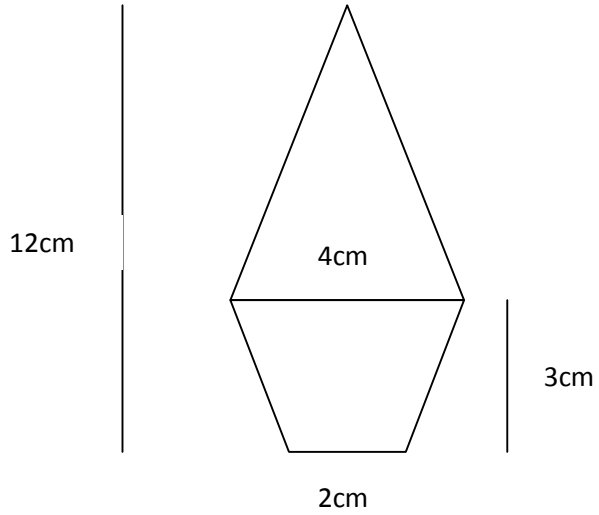
Area of ring (annulus) = Area of large circle – area of small circle

Area of large circle $= \pi \times r \times r = \pi \times 4 \times 4 = 16\pi$

Area of small circle $= \pi \times 3 \times 3 = 9\pi$

Area of annulus $= 16\pi - 9\pi = 7\pi \cong 21.98$ cm^2

d)



Area of shape = Area of trapezium + area of triangle

Area of trapezium $= \left(\frac{a+b}{2}\right) \times h = \left(\frac{4+2}{2}\right) \times 3 = 9$ cm^2

Area of triangle $= \frac{1}{2} \times$ base $\times$ height

Base $= 4$ cm

Height $= 12 - 3 = 9$ cm

Area of triangle $= \frac{1}{2} \times 4 \times 9 = 18$ cm^2

e) Calculate the perimeter of the shapes in parts a and b

For sector (part a), perimeter = length of radius + length of arc + length of radius

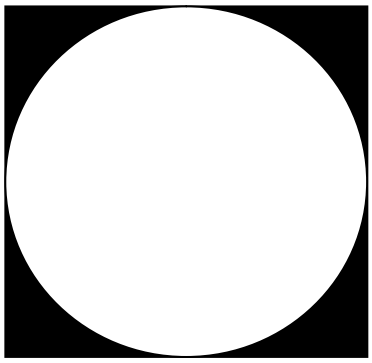
Length of arc $= 2 \times \pi \times r \times \frac{40}{360} = 2 \times \pi \times 6 \times \frac{40}{360} = \frac{4\pi}{3}$ cm

Perimeter $= \frac{4\pi}{3} + 6 + 6 \cong 16.19$ cm

For composite shape (part b), perimeter $= 8 + 4 +$length of arc

Length of arc $= \frac{1}{2} \times \pi \times d = \frac{1}{2} \times \pi \times 8 = 4\pi \cong 12.56$ cm

3) Calculate the area of the shaded region



Area of square $= 6 \times 6 = 36$ cm^2

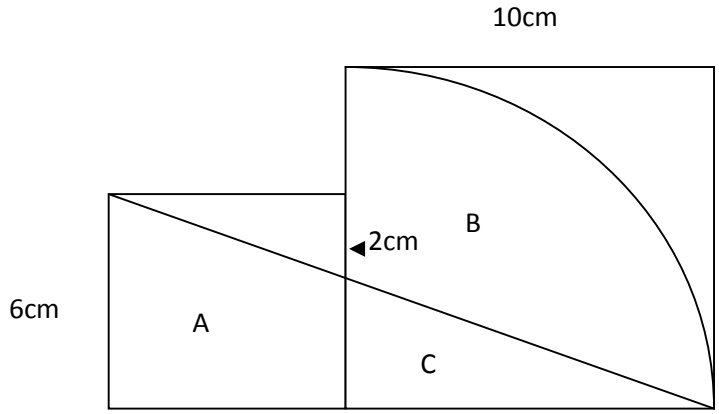
Area of circle $= \pi \times r \times r$

Diameter of circle $= 6$ cm, radius $= 3$ cm

Area of circle$= \pi \times 3 \times 3 = 9\pi$ cm^2

Area of shaded region $= 36 - 9\pi \cong 7.74$ cm^2

4) The figure shows two squares of side lengths 10cm and 6cm as shown in the diagram. Calculate the total area of the regions A + B +C.



Area of B + C = Area of quarter circle$= \pi \times r \times r \times \frac{1}{4} = \pi \times 10 \times 10 \times \frac{1}{4} = 25\pi$ cm^2

Area of A = Area of square – area of triangle of height 2 cm

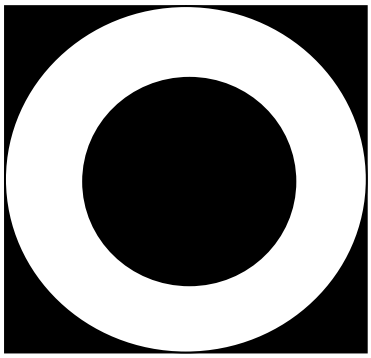
Area of square $= 6 \times 6 = 36$ cm^2

Area of triangle $= \frac{1}{2} \times 6 \times 2 = 6$ cm^2

Area of A $= 36 - 6 = 30$ cm^2

Area of A + B + C $= 30 + 25\pi \cong 108.5$ cm^2

5) The diagram shows a square of side length 6cm with a donut inside it. If the small circle has a radius of 2cm, what is the total shaded area?



Shaded area = Area of square – area of donut

Area of square $= 6 \times 6 = 36$ cm^2

Area of donut =Area of large circle – area of small circle

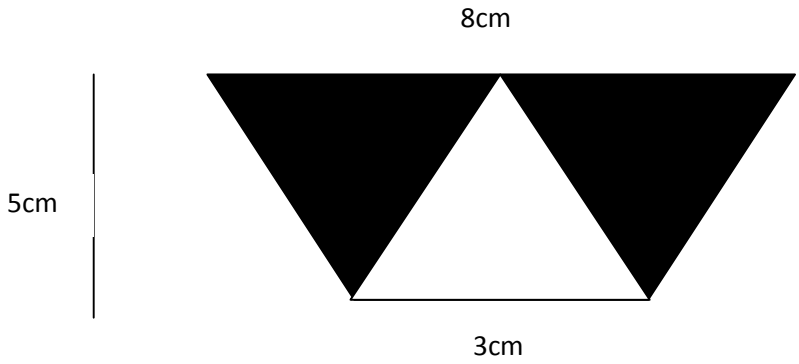
Area of large circle $= \pi \times 3 \times 3 = 9\pi$ cm^2

Area of small circle $= \pi \times 2 \times 2 = 4\pi$ cm^2

Area of donut $= 9\pi - 4\pi = 5\pi \cong 15.7$ cm^2

Shaded area $\cong 36 - 15.7 = 20.3$ cm^2

6) Calculate the shaded area



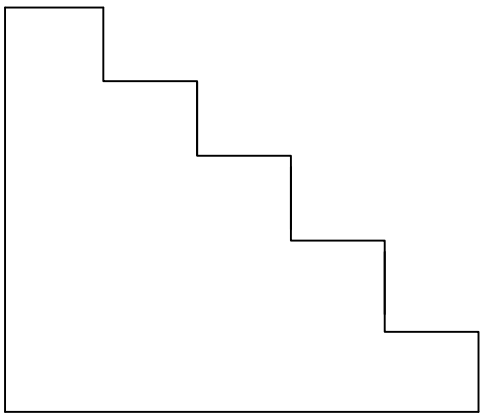
Shaded area = Area of trapezium – area of white triangle

Area of trapezium $= \left(\frac{8+3}{2}\right) \times 5 = 27.5$ cm^2

Area of white triangle $= \frac{1}{2} \times 3 \times 5 = 7.5$ cm^2

Shaded area $= 27.5 - 7.5 = 20$ cm^2

7) The diagram below shows the side view of a series of steps. Each step is 0.5 metres high and 0.5 metres wide. What is the area of the side of the block of steps?

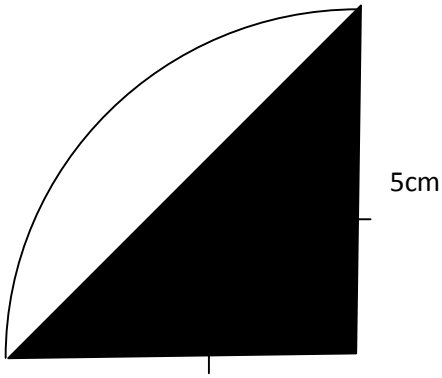


The structure can be broken into a series of blocks each 0.5 x 0.5 m of area $0.5 \times 0.5 = 0.25$ cm^2

The first row has 5 blocks; the second row has 4, the third 3, the fourth 2, and the top row has 1 block

Total blocks $= 15$; total area $= 15 \times 0.25 = 3.75$ m^2

8) Calculate the white area



Area of white = Area of sector – area of triangle

Area of sector $= \pi \times r \times r \times \frac{1}{4} = \pi \times 5 \times 5 \times \frac{1}{4} = 6.25\pi$ cm^2

Area of triangle $= \frac{1}{2} \times 5 \times 5 = 12.5$ cm^2

Area of white region $= 6.25\pi - 12.5 \cong 7.125$ cm^2

Exercise 2

Volume & Surface Area

1) Calculate the surface area of the following right closed cylinders

a) Radius of 2mm, and a vertical height of 10mm

V$= \pi r^2 h$
$= \pi \times 2 \times 2 \times 10$
$= 40\pi \cong 125.6$ mm^3

b) Radius of 1cm and a vertical height of 5mm

V$= \pi \times 10 \times 10 \times 5$
$= 500\pi \cong 1570$ mm^3

c) Vertical height of 500mm and a radius of 0.3m

V$= \pi \times 0.3 \times 0.3 \times 0.5$
$= 0.045\pi \cong 0.1413$ m^3

d) Vertical height of 2.5m and a radius of 2000mm

V$= \pi \times 2 \times 2 \times 2.5$
$= 10\pi \cong 31.4$ m^3

2) Calculate the surface area of the following

a) A sphere having a radius of 10cm

SA of sphere $= 4\pi r^2$

$= 4 \times \pi \times 10 \times 10$

$= 400\pi \cong 1256$ cm^2

b) A sphere having a diameter of 350mm

Radius =175 mm

SA $= 4 \times \pi \times 175 \times 175$

$= 122500\pi \cong 384650$ mm^2

c) A hemisphere having a radius of 8m

SA of sphere $= 4 \times \pi \times 8 \times 8$

$= 256\pi$

SA hemisphere $= \frac{1}{2} \times 256\pi$

$= 128\pi \cong 401.92$ m^2

d) A hemisphere having a diameter of 16mm

Radius $= 8$ mm

SA sphere $= 4 \times \pi \times 8 \times 8$

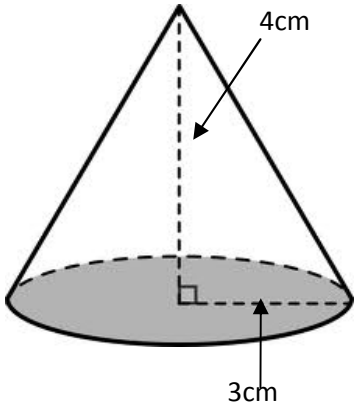
$= 256\pi$

SA hemisphere $= \frac{1}{2} \times 256$

$= 128$ mm^2

3) Calculate the surface area of the following.

a)



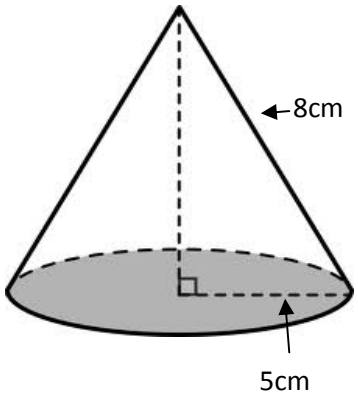
SA of cone$= (\pi rs) + \pi r^2$, where s is the length of the slope

From Pythagoras, $s = \sqrt{3^2 + 4^2} = 25$

$s = 5$ cm

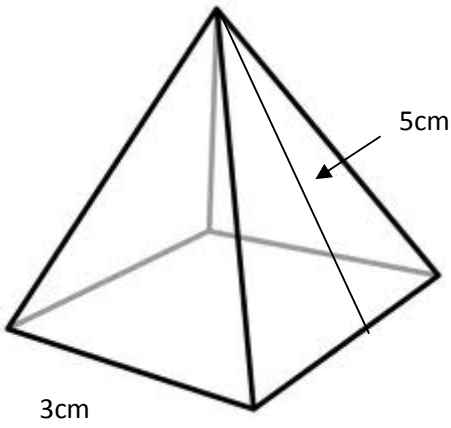
SA $= (\pi \times 3 \times 5) + (\pi \times 3 \times 3) = 24\pi \cong 75.36$ cm^2

b)



SA $= (\pi \times 5 \times 8) + (\pi \times 5 \times 5) = 65\pi \cong 204.1$ cm^2

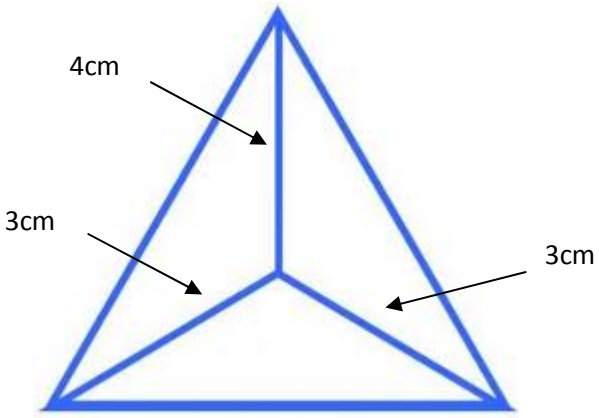
c)



SA of square pyramid$= 2bs + b^2$, where b is the length of the base, and s is the length of the slope

SA $= (2 \times 3 \times 5) + 3^2 = 39$ cm^2

d)



SA of triangular pyramid = Sum of areas of triangles

There are 4 right triangles:

Triangle 1 is a right triangle of base 3 cm and height 4 cm

Triangle 2 is equivalent to triangle 1

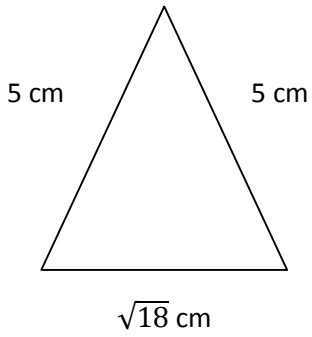
Triangle 3 is the base, and is a right triangle of base 3cm and height 3cm

Triangle 4 is an isosceles triangle; the side lengths need to be calculated

The equal sides are 5 cm each (using Pythagoras on triangles 1 and 2)

The other side is the unknown side of the base triangle. Using Pythagoras,

$x = \sqrt{3^2 + 3^2} = \sqrt{18}$



Need height of this triangle to calculate its area

Using Pythagoras: $x^2 + \left(\frac{\sqrt{18}}{2}\right)^2 = 5^2$

$x^2 = 25 - \frac{18}{4} = 20.5$

$x = \sqrt{20.5}$

Area $= \frac{1}{2} \times \sqrt{18} \times \sqrt{20.5} = \frac{1}{2} \times \sqrt{369} \cong 9.6$ cm^2

Area of triangle 1 $= \frac{1}{2} \times 3 \times 4 = 6$ cm^2
Area of triangle 2 $= 6$ cm^2
Area of triangle 3 $= \frac{1}{2} \times 3 \times 3 = 4.5$ cm^2

Total surface area $= 6 + 6 + 4.5 + 9.6 = 26.1$ cm^2

4) Calculate the volume of a right cone of radius 2cm and a vertical height of 5cm

Volume of cone $= \frac{1}{3} \times \pi \times r^2 \times h$

V $= \frac{1}{3} \times \pi \times 2 \times 2 \times 5 = \frac{20\pi}{3} \cong 20.93$ cm^3

5) Calculate the volume of a square pyramid of side length 2cm and vertical height 5cm.

Volume of pyramid $= \frac{1}{3} \times$ area of base$\times$ height

$V = \frac{1}{3} \times (2 \times 2) \times 5 = \frac{20}{3} \cong 6.67 \text{ cm}^3$

6) A cone has a volume of 30π cm^3. If the side length of a square pyramid is 6cm, what must its height be in order to have the same volume as the cone?

$V = 30\pi = \pi \times 6 \times 6 \times h$

$h = \frac{30\pi}{36\pi} = \frac{5}{6} \cong 0.83 \text{ cm}$

7) Calculate the volume of the following

a) A sphere having a radius of 2mm

Volume sphere $= \frac{4}{3} \times \pi \times r^3 = \frac{4}{3} \times \pi \times 8 = \frac{32\pi}{3} \cong 33.49 \text{ cm}^3$

b) A sphere having a diameter of 20cm

Radius $= 10$ cm

Volume $= \frac{4}{3} \times \pi \times 1000 = \frac{4000\pi}{3} \cong 4186.67 \text{ cm}^3$

c) A hemisphere having a radius of 3.2m

Volume of sphere $= \frac{4}{3} \times \pi \times 3.2^3 = \frac{131.072\pi}{3} \cong 137.19 \text{ cm}^3$

Volume of hemisphere $= 137.19 \div 2 \cong 68.6 \text{ cm}^3$

d) A hemisphere having a diameter of 40mm

Radius $= 20$ mm

Volume of sphere $= \frac{4}{3} \times \pi \times 20^3 = \frac{32000\pi}{3} \cong 33493.33 \text{ mm}^3$

Volume of hemisphere $= 33493.33 \div 2 \cong 16746.66$ mm^3

8) The volume of a sphere is $\frac{256}{3}\pi$ mm^3. What is its radius?

V$= \frac{4}{3} \times \pi \times r^3 = \frac{256\pi}{3}$

$r^3 = \frac{256\pi}{3} \times \frac{3}{4\pi} = 64$

$r = 4$ mm

9) A square base pyramid has a perpendicular height of 10mm. What is the length of each side of the base if its volume is 270 mm^3?

V$= b^2 \times h$, where b is the base length and h is the height

V$= 270 = 10 \times b^2$

$b^2 = 27$

$b \cong 5.19$ mm

10) A sphere of volume $\frac{4}{3}\pi$ m^3 fits exactly inside a cube. What is the volume of the cube?

V$= \frac{4}{3} \times \pi \times r^3 = \frac{4}{3} \times \pi$

$r = 1$ m

Since the sphere fits inside the cube, the diameter of the sphere must equal the side length of the cube. Therefore the side length of the cube is 2 m

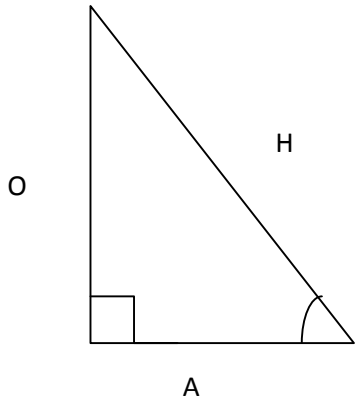
Volume of the cube $= 2 \times 2 \times 2 = 8$ m^3

Exercise 3

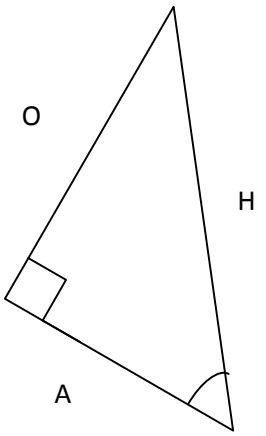
Trigonometry

1) In the following diagrams identify the angles adjacent and opposite the given angle, and also identify the hypotenuse.

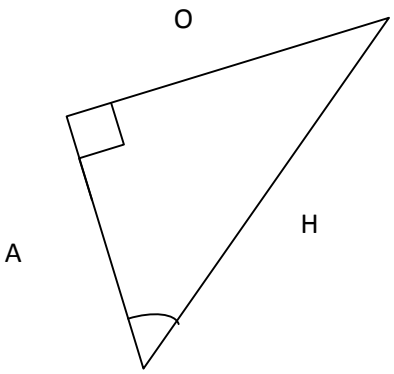
a)



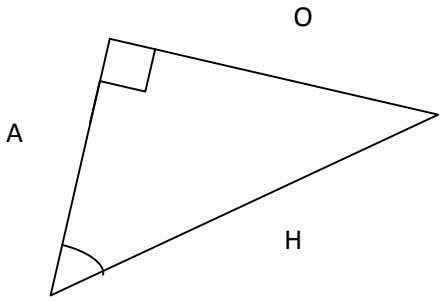
b)



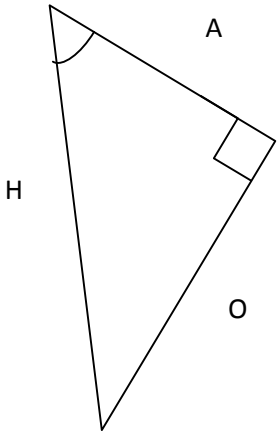
c)



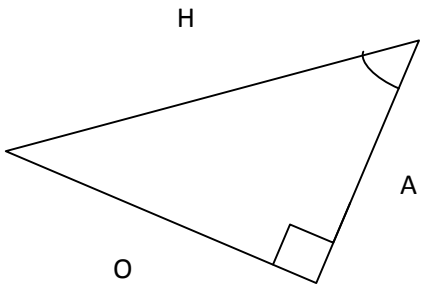
d)



e)

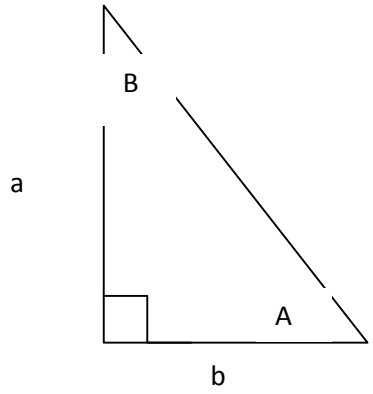


f)



2)

a) Complete the notation on the triangle sides and angles



b) Complete the following:

i. $\sin A = \frac{a}{H}$

ii. $\cos B = \frac{a}{H}$

iii. $\tan A = \frac{a}{b}$

c) Use a calculator to find the values from part (b) if the size of angle A = 55°

i. $\sin 55° \cong 0.82$

ii. $\cos 55° \cong 0.57$

iii. $\tan 55° \cong 1.43$

d) Use a calculator to find the size of angles A and B if the length of side a is 3cm, and the length of side b is 5 cm

$\tan A = \frac{a}{b} = \frac{3}{5}$

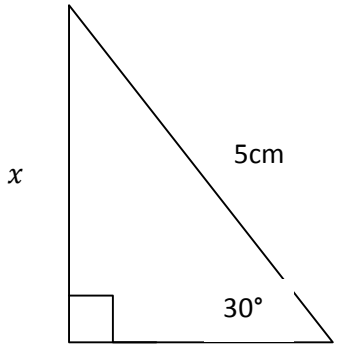
$A \cong 31°$

$$\tan B = \frac{b}{a} = \frac{5}{3}$$

$$B \cong 59°$$

3) Calculate the length of x in each of the diagrams below

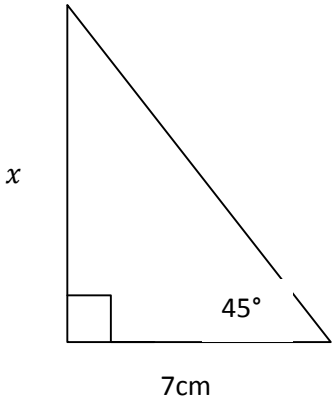
a)



$$\sin 30° = \frac{x}{5}$$

$$x = 5 \sin 30° = 2.5\ cm$$

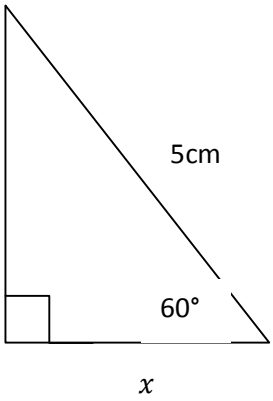
b)



$$\tan 45° = \frac{x}{7}$$

$$x = 7 \tan 45° = 7cm$$

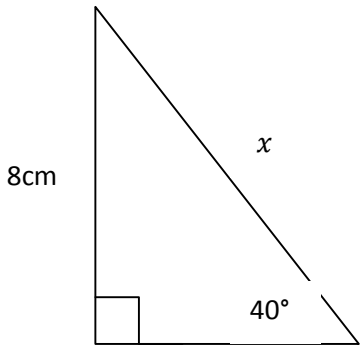
c)



$$\cos 60° = \frac{x}{5}$$

$$x = 5\cos 60° = 2.5cm$$

d)

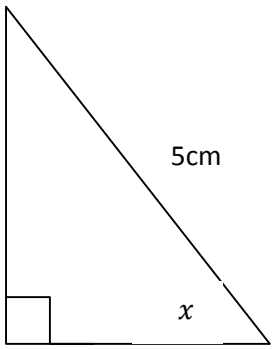


$$\sin 40° = \frac{8}{x}$$

$$x = \frac{8}{\sin 40°} \cong \frac{8}{0.64} = 12.5cm$$

4) Calculate the size of angle x in the diagrams below, correct to the nearest degree.

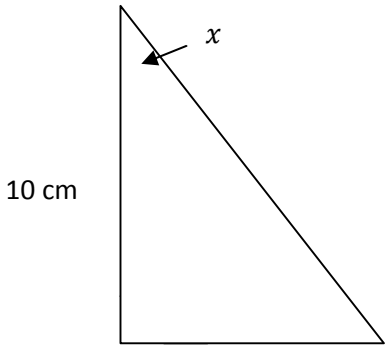
a)



$$\cos x = \frac{3}{5}$$

$$x \cong 53°$$

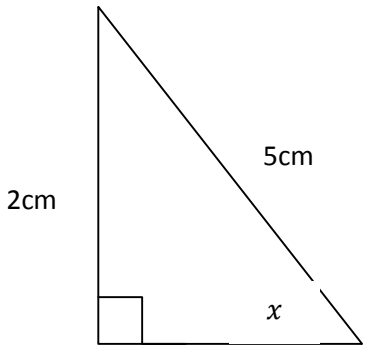
b)



$$\tan x = \frac{6}{10}$$

$$x \cong 31°$$

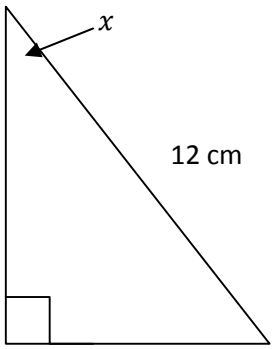
c)



$$\sin x = \frac{2}{5}$$

$$x \cong 24°$$

d)

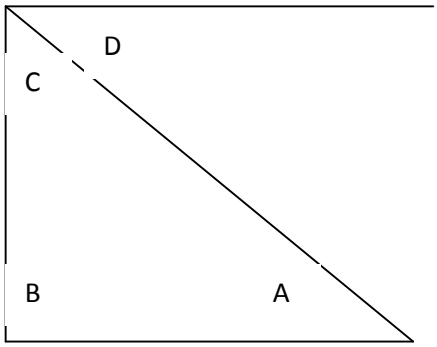


6 cm

$$\sin x = \frac{6}{12}$$

$$x = 30°$$

5) Identify the angles of elevation and depression in the diagram below

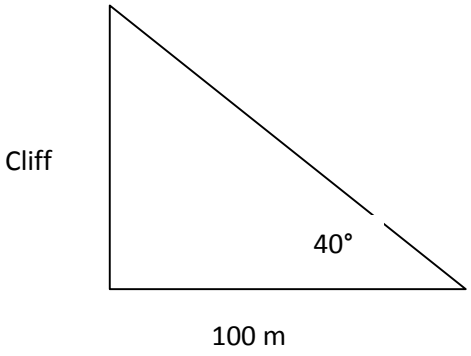


Angle of elevation is A

Angle of depression is D

Complete the statement: The angle of elevation is **EQUAL TO** the angle of depression

6) A man standing 100 metres away from the base of a cliff measures the angle of elevation to the top of the cliff to be 40 degrees. How high is the cliff?



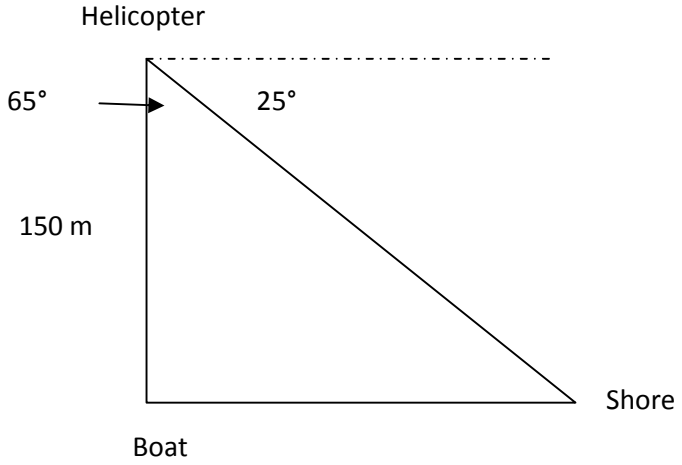
Let height of cliff be x

$$\tan 40° = \frac{x}{100}$$

$$x = 100 \tan 40° \cong 100 \times 0.8391 = 83.91\ m$$

The cliff is approximately 84 metres high

7) A helicopter is hovering 150 metres above a boat in the ocean. From the helicopter, the angle of depression to the shore is measured to be 25 degrees. How far out to sea is the boat? (You need to fill in angle of depression on diagram)



If angle of depression is 25°, complementary angle is 65°

Let distance to shore be x

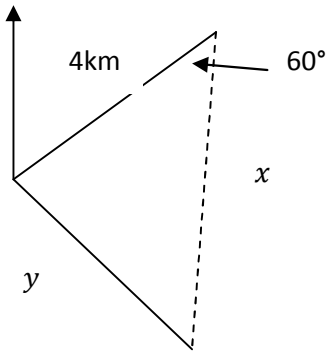
$$\tan 65° = \frac{x}{150}$$

$$x = 150 \tan 65°$$

$$x \cong 150 \times 2.1445 = 321.675$$

The boat is approximately 322 metres offshore

8) Two men walk from the same point. The first man heads on a bearing of 045°, whilst the second heads in the direction SE. After an hour they both stop. If the first man walked 4 km during this time, and the angle between their finishing points is 60 degrees, how far did the second man walk, and what is the shortest distance between them?



A right angled triangle is formed between the bearings of the two men

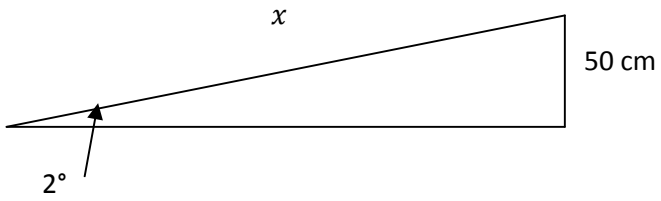
$$\cos 60° = \frac{4}{x}$$

$$x = \frac{4}{\cos 60°} = 8\ km$$

By using Pythagoras' Theorem or $\tan 60° = \frac{y}{4}$, the approximate value of y is 7 km

NOTE: It would be preferable to use the tangent, since it uses information already given in the question

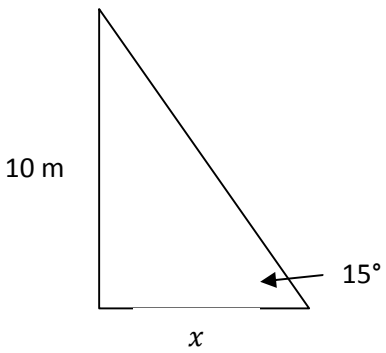
9) A ramp is built to allow wheelchair access to a lift. If the angle of elevation to the lift is 2 degrees, and the bottom of the lift is 50 cm above the ground how long is the ramp?



$$\sin 2° = \frac{50}{x}$$

$$x = \frac{50}{\sin 2°} \cong \frac{50}{0.035} \cong 1428.57\ cm \cong 14.3\ m$$

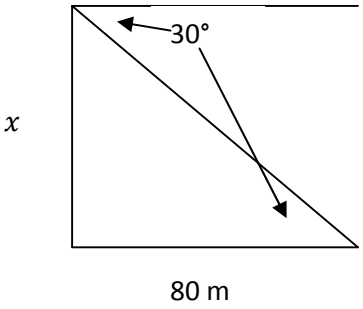
10) The angle of elevation to the top of a tree is 15 degrees. If the tree is 10 metres tall how far away from the base of the tree is the observer?



$$\tan 15° = \frac{10}{x}$$

$$x = \frac{10}{\tan 15°} \cong \frac{10}{0.268} \cong 37.3\ m$$

11) From the top of a tower a man sees his friend on the ground at an angle of depression of 30 degrees. If his friend is 80 metres from the base of the tower how tall is the tower?

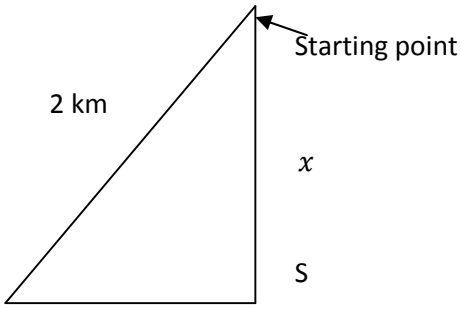


Angle of elevation = angle of depression = 30°

$$\tan 30° = \frac{x}{80}$$

$$x = 80 \tan 30° \cong 46.2\ m$$

12) A man walks on a bearing of 210 degrees for 2 km, and then walks due east until he is directly south of his starting position. How far south of his starting position is he at this time?



A bearing of 210 degrees is 30 degrees east of south

$$\cos 30° = \frac{x}{2}$$

$$x = 2 \cos 30° \cong 2 \times 0.866 = 1.73\ km$$

Year 9 Mathematics

Space

Exercise 1

Properties of Polygons

1) Calculate the sum of the internal angles of the following

a) Triangle

180°

b) Quadrilateral

360°

c) Pentagon

540°

d) Hexagon

720°

e) Heptagon

900°

f) Octagon

1080°

2) From your answers to question 1, develop a formula that gives the sum of the internal angles of an n-sided polygon

Difference between each result is 180; therefore equation is of the form

$s = 180n + c$

When $n = 4, s = 360$

Therefore $c = -360$

Equation is $s = 180n - 360$

Checking for $n = 8$

$s = 180 \times 8 - 360 = 1080$

Formula holds

3) Similarly, develop a formula that gives the sum of the exterior angles of an n-sided convex polygon

Using regular polygons for ease of calculation

The interior angle of each polygon is

Triangle: $\frac{180}{3} = 60°$

Quadrilateral: $\frac{360}{4} = 90°$

Pentagon: $\frac{540}{5} = 108°$

Hexagon: $\frac{720}{6} = 120°$

Heptagon: $\frac{900}{7}$

Octagon: $\frac{1080}{8} = 135°$

An exterior angle is the supplementary of the interior angle

So, the exterior angle of each polygon is

Triangle: 120°

Quadrilateral: 90°

Pentagon: 72°

Hexagon: $\frac{360}{7}$

Octagon: 45°

The sum of the exterior angles of each polygon is

Triangle: $120 \times 3 = 360°$

Quadrilateral: $90 \times 4 = 360°$

Pentagon: $72 \times 5 = 360°$

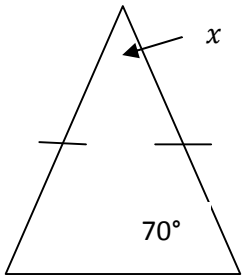
Hexagon: $\frac{360}{7} \times 7 = 360°$

Octagon: $45 \times 8 = 360°$

The sum of the exterior angles of any polygon is 360°

4) Find the value of x in each of the following

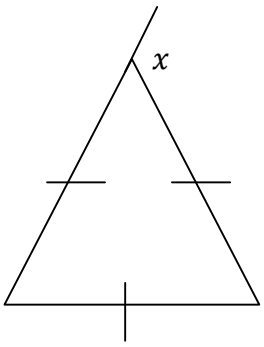
a)



The triangle is isosceles, so the other base angle is also 70°

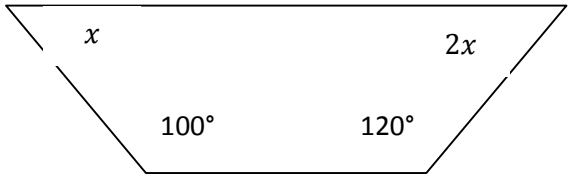
$x = 180 - 70 - 70 = 40°$

b)



The triangle is equilateral, so each internal angle is 60°. The supplementary angle, x, equals 120°

c)



The sum of the internal angles of a quadrilateral is 360°

$x + 2x + 100 + 120 = 360$

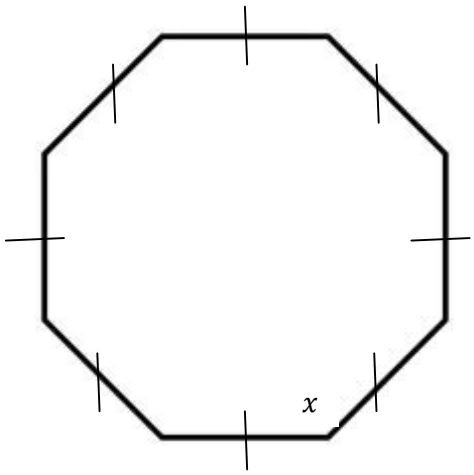
$3x + 220 = 360$

$3x = 140$

$x = 46.\dot{6}$

$2x = 93.\dot{3}$

d)

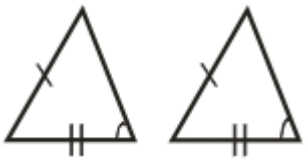


The sum of the internal angles of an octagon is 1080°

$x = \frac{1080}{8} = 135°$

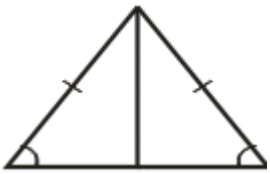
5)

a) State whether or not each of the following triangle pairs is congruent. If so, state a reason.



Not congruent: SSA is not sufficient to prove congruence

b) State whether or not each of the following triangle pairs is congruent. If so, state a reason.



Congruent: SSS. The triangles share a common side, and two other sides are equal, therefore all sides are equal

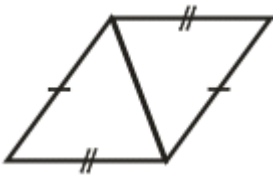
6) Does AAA guarantee that two triangles are congruent? Why or why not?

No: Two triangles can have equal angles but their side lengths could be scaled

7) Do congruent triangles have to be facing the same direction?

No, orientation is not required for congruence

8) State whether or not each of the following triangle pairs is congruent. If so, state a reason.



Congruent: SSS. Two sides are congruent and the third is shared

9) List the methods of proving triangles congruent.

SSS

SAS (the angle is between the two congruent sides)

ASA (the side is between the two angles)

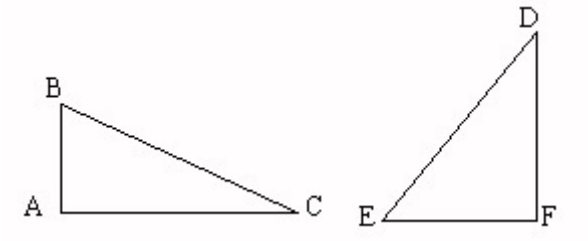
AAS

HL

10) How is AAS different from ASA?

In AAS the side is not included between the angles; in ASA it is

11)



m∠B ≅ ∠E

m∠A ≅ ∠F

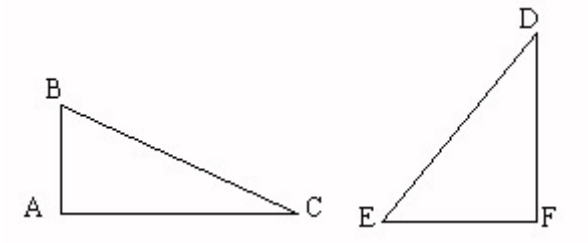
AC:FD = 6:12, and ED = 156, what is the length of BC?

The triangles are similar with a ratio of 1:2

BC is the side that corresponds to ED

So, the length of BC is $\frac{1}{2} \times 156 = 78$

12)



m∠A ≅ ∠F

m∠B ≅ ∠E

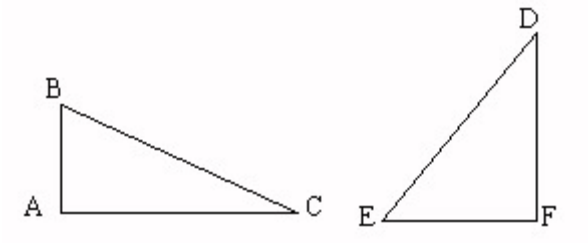
The length of the sides of ABC are 88, 156, and 100. The perimeter of FED is 172, what is the length of the longest side of FED

The perimeter of ABC is 344, which is twice the perimeter of FED

Therefore ED is half the length of BC, which is the longest side of ABC

ED = $\frac{1}{2} \times 156 = 78$

13)



m∠A ≅ ∠F

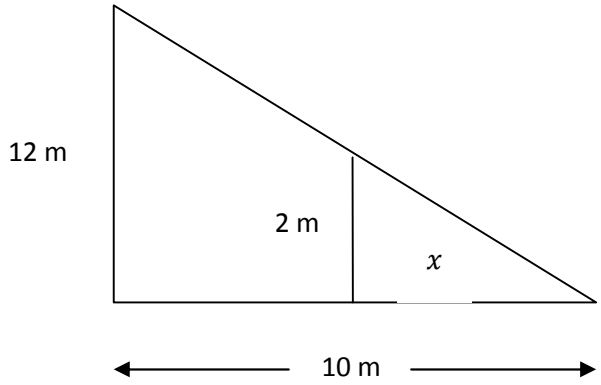
m∠B ≅ ∠E

The length of the sides of ABC are 60, 74, and 78. The length of the longest side of FED is 468, what is the perimeter of FED?

FED is similar to ABC with a scale factor of 6

The perimeter of ABC is 212, so the perimeter of FED $= 6 \times 212 = 1272$

14) A tree 12 metres tall casts a 10 metre shadow. How much shadow does a 2metere tall man standing near the tree cast? (Draw a diagram)



The triangles are similar. With a ratio of 2:12

Therefore $x = \frac{2}{12} \times 10 = 1.67\ m$