Year 9 Mathematics

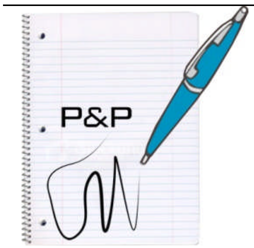
Learning Strategies

Mathematics is often the most challenging subject for students. Much of the trouble comes from the fact that mathematics is about logical thinking, not memorizing rules or remembering formulas. It requires a different style of thinking than other subjects. The students who seem to be "naturally" good at math just happen to adopt the correct strategies of thinking that math requires – often they don't even realise it. We have isolated several key learning strategies used by successful maths students and have made icons to represent them. These icons are distributed throughout the book in order to remind students to adopt these necessary learning strategies:

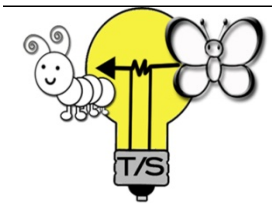
Talk Aloud Many students sit and try to do a problem in complete silence inside their heads. They think that solutions just pop into the heads of 'smart' people. You absolutely must learn to talk aloud and listen to yourself, literally to talk yourself through a problem. Successful students do this without realising. It helps to structure your thoughts while helping your tutor understand the way you think.

BackChecking This means that you will be doing every step of the question twice, as you work your way through the question to ensure no silly mistakes. For example with this question: $3 \times 2 - 5 \times 7$ you would do "3 times 2 is 5 ... let me check – no 3×2 is 6 ... minus 5 times 7 is minus 35 ... let me check ... minus 5×7 is minus 35. Initially, this may seem time-consuming, but once it is automatic, a great deal of time and marks will be saved.

Avoid Cosmetic Surgery Do not write over old answers since this often results in repeated mistakes or actually erasing the correct answer. When you make mistakes just put one line through the mistake rather than scribbling it out. This helps reduce silly mistakes and makes your work look cleaner and easier to backcheck.



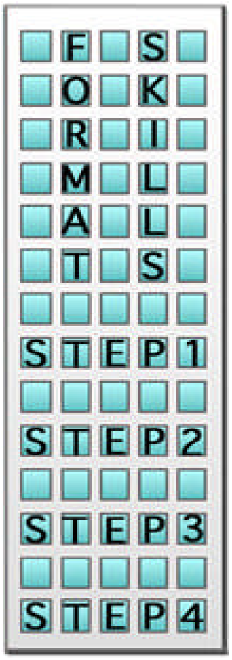
Pen to Paper It is always wise to write things down <u>as</u> you work your way through a problem, in order to keep track of good ideas and to see concepts on paper instead of in your head. This makes it easier to work out the next step in the problem. Harder maths problems cannot be solved in your head alone – put your ideas on paper as soon as you have them – always!



Transfer Skills This strategy is more advanced. It is the skill of making up a simpler question and then transferring those ideas to a more complex question with which you are having difficulty.

For example if you can't remember how to do long addition because you can't recall exactly how to carry the one: $\underline{{}^{+}{}^{5889}_{4587}}$ then you may want to try adding numbers which you do know how to calculate that also involve carrying the one: $\underline{{}^{+}{}^{5}_{9}}$

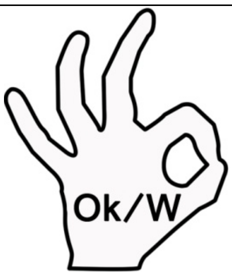
This skill is particularly useful when you can't remember a basic arithmetic or algebraic rule, most of the time you should be able to work it out by creating a simpler version of the question.



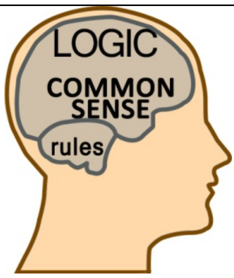
Format Skills These are the skills that keep a question together as an organized whole in terms of your working out on paper. An example of this is using the "=" sign correctly to keep a question lined up properly. In numerical calculations format skills help you to align the numbers correctly.

This skill is important because the correct working out will help you avoid careless mistakes. When your work is jumbled up all over the page it is hard for you to make sense of what belongs with what. Your "silly" mistakes would increase. Format skills also make it a lot easier for you to check over your work and to notice/correct any mistakes.

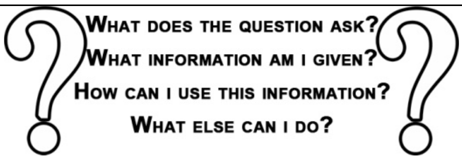
Every topic in math has a way of being written with correct formatting. You will be surprised how much smoother mathematics will be once you learn this skill. Whenever you are unsure you should always ask your tutor or teacher.



Its Ok To Be Wrong Mathematics is in many ways more of a skill than just knowledge. The main skill is problem solving and the only way this can be learned is by thinking hard and making mistakes on the way. As you gain confidence you will naturally worry less about making the mistakes and more about learning from them. Risk trying to solve problems that you are unsure of, this will improve your skill more than anything else. It's ok to be wrong – it is NOT ok to not try.



Avoid Rule Dependency Rules are secondary tools; common sense and logic are primary tools for problem solving and mathematics in general. Ultimately you must understand Why rules work the way they do. Without this you are likely to struggle with tricky problem solving and worded questions. Always rely on your logic and common sense first and on rules second, always ask Why?



Self Questioning This is what strong problem solvers do naturally when they get stuck on a problem or don't know what to do. Ask yourself these questions. They will help to jolt your thinking process; consider just one question at a time and Talk Aloud while putting Pen To Paper.

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Number

Useful formulae and hints

Index laws of multiplication and division are valid for the same base

$2^a \times 2^b = 2^{a+b}$

$2^a \times 3^b$ cannot be simplified since the bases are different

Different bases can be used if they are both raised to the same power

$6^2 = (3 \times 2)^2 = 3^2 \times 2^2$

If a number has a negative power, it can be converted to a positive power

$2^{-2} = \frac{1}{2^2}$

Any number to the power zero is 1

$2^0 = 1$

$23134577^0 = 1$

Any number to the power 1 is itself

$2^1 = 2$

$234876^1 = 234876$

A number is in scientific notation when it is on the form:

$a.bc \ldots \times 10^x$

To convert a number to scientific notation

Write it in the form $a.bc$ ... (that is one digit then the decimal point)

Count how many places you would have to move the decimal point to get back to the original number; to the left is negative, to the right is positive. This number is the power of 10

Example

$3125 = 3.125 \times 10^{3}$

$0.24376 = 2.4376 \times 10^{-1}$

Significant figures are the number of digits from the left of a number

Zeroes are only counted as significant when they surround non-zero digits; in other cases they are just used to keep the other digits in their correct place value. The digits are rounded to the nearest whole number

Example

52347

To one significant figure is 50000

To two significant figures is 52000

To three significant figures is 52300

To four significant figures is 52350

0.004367

To one significant figure is 0.004000 (the first zeroes are not significant

0.004062

To 3 significant figures is 0.00406 (the 0 between the 4 and the 6 is significant)

Estimating products by using index laws and rounding example:

$$(0.0312 \times 10^6) \times (48.7 \times 10^{-2}) \cong (3.12 \times 10^4) \times (4.87 \times 10^{-1})$$
$$\cong (3 \times 10^4) \times (5 \times 10^{-1}) \cong 15 \times 10^3$$

There are 52 weeks, or 26 fortnights, or 12 months, or 365 days in a year (assuming not a leap year)

Salary is an amount that is paid to a person regardless of the hours they work

Taxation and other deductions are amounts taken from peoples' wages and paid to the government to help fund such things as education, roads, hospitals etc. They are usually taken as a percentage of one's pay

Example: 8% tax on \$35000= $\frac{8}{100} \times \$35000 = \$2800$

Simple interest =Principal x rate (per annum, expressed as a fraction) x time

$I = PRT$

Example: Interest on a principal of \$20000 at an interest rate of 4% per annum for 5 years $= 20000 \times \frac{4}{100} \times 5 = \$4000$

If interest is not per annum, convert it first. For example, 3% per six months = 6% per annum

To calculate discount of a price, express the percent discount as a fraction and multiply the price by this fraction

Example: 5% discount on \$200 $= \frac{5}{100} \times 200 = \$10$

Exercise 1

Indices

1) Evaluate the following

a) 2^3

b) 3^2

c) 1^6

d) 4^3

e) 5^0

2) Express the following as powers of 2

a) 16

b) 2

c) 1

d) 64

e) 32

3) Express the following as powers of 3

a) 9

b) 81

c) 3

d) 27

e) 1

4) Evaluate the following

a) 2^{-1}

b) 3^{-2}

c) 2^{-3}

d) 4^{-2}

e) 2^{-4}

5) Evaluate the following

a) $2^3 \times 2^2$

b) $3^1 \times 3^2$

c) $4^1 \times 4^1$

d) $3^3 \times 3^0$

e) $5^2 \times 5^2$

6) Evaluate the following

a) $2^5 \times 2^{-3}$

b) $3^{-5} \times 3^6$

c) $2^{-1} \times 2^{-2}$

d) $4^{-4} \times 4^6$

e) $13^6 \times 13^{-6}$

7) Evaluate the following

a) $3^5 \div 3^3$

b) $2^{10} \div 2^8$

c) $4^4 \div 4^4$

d) $10^3 \div 10^2$

e) $1^{10} \div 1^6$

8) Evaluate the following

a) $3^2 + 2^2$

b) 5^2

c) $3^2 \times 2^2$

d) $(3 \times 2)^2$

e) $4^2 + 5^2$

f) 9^2

g) $4^2 \times 5^2$

h) $(4 \times 5)^2$

9) From your answers to question 8, which of the following statements are true and which are false?

a) $a^2 + b^2 = (a + b)^2$

b) $(a + b)^2 = a^2 \times b^2$

c) $(a \times b)^2 = a^2 \times b^2$

d) $(a \times b)^2 = (a + b)^2$

10) Find the values of x and y in the following

$2^6 = 8^x = y^3$

Exercise 2

Scientific Notation & Significant Figures

1) Express the following in scientific notation

a) 3125

b) 1000

c) 14250

d) 105000

e) 775

f) 7777

2) Convert the following to decimal form

a) 3.233×10^2

b) 4.1002×10^3

c) 7.06×10^4

d) 5.007×10^2

e) 3.0207×10^3

f) 1.00001×10^5

3) Express the following in scientific notation

a) 0.1005

b) 0.0514

c) 0.75

d) 0.000523123

e) 0.0554

f) 6.5121

4) Convert the following to decimal form

a) 3.452×10^{-2}

b) 2.6552×10^{-1}

c) 7.5×10^{-3}

d) 1.423×10^{-4}

5) Use your knowledge of index laws and scientific notation to estimate the following products (see example at beginning of chapter)

a) $(3.15 \text{ x } 10^3) \text{ x } (5.22 \text{ x } 10^4)$

b) $(4.85 \text{ x} 10^2) \text{ x } (6.33 \text{ x } 10^5)$

c) $(2.96 \text{ x} 10^4) \text{ x } (4.98 \text{ x } 10^6)$

d) $(6.05 \text{ x } 10^7 \div 3.11 \text{ x } 10^3)$

6) Round the following to 4 significant figures

a) 42.7567

b) 0.39848

c) 17152.54

d) 11.111111

7) Round the following to 3 significant figures

a) 19.672

b) 555.55

c) 1012

d) 0.82556

e) 212.75

f) 10001

8) The speed of light in a vacuum is 3×10^8 metres per second

a) How far does light travel in 10 seconds?

b) How long does light take to travel 90000 metres?

c) There are approximately 31.5 million seconds in a year. How far does light travel in one year?

9) The closest star to Earth is approximately 4.3 light years away. How long would it take a rocket travelling at 40,000 km per hour to reach it?

Exercise 3

Consumer Arithmetic

1) You get a part time job that pays a wage of $15 per hour. How much would you earn (before tax) in each of the following weeks?

a) Worked 10 hours

b) Worked 12 hours

c) Worked 32 hours

d) Worked 15 hours

e) Worked 9 hours

f) Worked 36 hours

2) In your next job you are given a salary of $38,000 per annum. How much would you be paid:

a) Per month

b) Per week

c) Per fortnight

d) Per day

(Assume an equal payment per month regardless of number of days, and assume you work 5 days a week, ignore public holidays)

3) In your next job you are given $200 per week plus a commission of 5% of all your sales. How much would you earn in each of the following weeks?

a) Sales of $500

b) Sales of $1000

c) Sales of $5000

d) Sales of $20000

e) What must the value of your sales be if you needed to earn $2200 for a week?

4) In your next job you are required to work overtime, but get paid more for doing so. The agreement is:

The first 35 hours work are paid at the rate of $20 per hour

The next 5 hours work are paid at one and a half times your normal rate

All hours worked above this are paid at twice your normal rate

How much would you earn for working the following hours per week?

a) 30 hours

b) 35 hours

c) 37 hours

d) 40 hours

e) 46 hours

f) 50 hours

5) Assume you are working a job that pays a salary of $52,000 per annum

a) If taxation is deducted at the rate of 20%, how much would you actually receive per week?

b) If taxation is deducted at the rate of 15%, superannuation at the rate of 2%, and Medicare levy at the rate of 1.5%, how much would you actually receive per week?

c) If the first $12,000 of your income is not taxed, but the remainder is taxed at 20%, how much would you actually receive per week?

d) If the first $6,000 of your income is not taxed, but the remainder is taxed at 15%, and you have to pay a Medicare levy of 2% on your whole income, how much would you actually receive per week?

6) Complete the following table, assuming interest is simple

Principal (P)	Interest Rate (R)	Time (T) (in years)	Total Interest (I)
$20,000	5%	5	
$10,000	8%	4	
$7,500	10%	10	
	5%	5	$8,000
$2,000		8	$4,000
$2,500	10%		$750

7) Which of the following pairs of options costs the least, and by how much?

a) Buying petrol at $1.40 per litre, or at $1.50 per litre with a 5% discount

b) Buying a lounge suite for $1200 cash, or a $900 lounge suite on credit with a simple interest rate of 10% per annum for 3 years

c) Buying a new TV for $2000 cash, or lay-by of 5 payments of $400

d) A company offers an exercise system, for 6 equal monthly instalments of $49.95, plus postage & handling of $19.95, or a one off cash payment of $300 with no postage or handling fee. You would have to borrow the $300 at a simple interest rate of 8% per annum, for 6 months.

8) A store advertises a discount of 10% off all marked prices. You also have a discount card that entitles you to 15% off your purchases. Calculate the price you pay for each of the following items

a) A pair of shoes with a marked price of $120

b) A dining set with a marked price of $200

c) A roll of carpet of 20 metres in length with a marked price of $15 per metre

d) A stereo system with a marked price of $567

9) Answer the following using the information from question 8

a) Is it cheaper to receive the 15% card discount, then the 10% store discount, or the other way around? Prove your answer with at least two examples.

b) Is it cheaper to receive the 10% store discount and 15% card discount, or to receive 25% discount (10 + 15) immediately off the purchase price? Prove your answer.

10) (Challenge question) The marked price of a bike has fallen off, and when you take it to the register you are charged $229.50 after both discounts. How much was the original marked price? (Use guess check and improve)

Year 9 Mathematics

Chance & Data

Useful formulae and hints

The probability of an event occurring can be calculated in two ways

- Theoretical probability ($\frac{Number\ of\ desired\ outcomes}{Total\ number\ of\ possible\ outcomes}$)

- Predictions based on previous data or trials

If using the latter, the number of trials should be large, and the trials repeatable under the same circumstances to give the data validity. The more external factors affecting the trials, the less valid the results and hence predictions made from those results. Examples may include scores when throwing darts, running times, test scores, where other factors may influence the results

If predicting probability based on theory, for 2 events occurring at the same time, the sample space must include all possible combinations of the events. For example, if two coins are tossed there are 4 possible outcomes (HH, HT, TH, TT)

When calculating probabilities for repeated events, the replacement or non replacement of an item (for example a card) before the second event will affect the sample space, and hence the probability.

Example: If a red card is drawn from a pack (probability $\frac{26}{52} = \frac{1}{2}$) and not replaced, the probability of drawing another red card is now $\frac{25}{51}$, since there is one less red card that can be drawn, and one less card in the pack

A cumulative frequency table or graph is a "running total" of the number of scores in a set.

Example

Score	Frequency	Cumulative frequency
1	5	5
2	2	5+2=7
3	3	7+3=10
4	6	10+6=16

The final cumulative frequency should equal the total number of individual scores

When dealing with grouped data, the exact median cannot be found, since the individual scores are not known (only the range of each group is known), therefore a modal class is found. This is the group that would contain the median

Example

Group	Frequency
10-20	5
21-30	2
31-40	5
41-50	7

There are 19 scores, so the median would be the 10^{th} score. This score occurs in the class 31-40; this is the modal class. Without further information the exact value of the median cannot be calculated

The range of a set of scores is the difference between the highest and lowest scores

The upper quartile is the median of the upper half of the scores in the data set

The lower quartile is the median of the lower half of the scores in the data set

The inter-quartile range is the difference between the upper and lower quartiles.

Example: For the data set 1, 3, 5, 6, 7, 8, 9, 15, 22, 31, 35

The median is the 6^{th} score, which is 8

The lower quartile is 5 (the median of the subset 1, 3, 5, 6, 7)

The upper quartile is 22 (the median of the subset 9, 15, 22, 31, 35)

The inter-quartile range is 17 (22-5)

The range is 34 (35-1)

At this level standard deviation is calculated with the aid of a calculator

The mean of a set of a data $= \frac{Sum\ of\ the\ scores}{Number\ of\ scores}$

Example: Mean of 2, 4, 5, 10, 12, 21 $= \frac{54}{6} = 9$

To calculate the missing value from a set of scores given the mean, calculate the sum of the scores (=mean x number of scores), and use it to find the missing scores

Example: A data set has a mean of 5; its scores are 1, 3, 4, 4, x, 7, 10

There are seven scores, so the sum of the scores is 5 x 7 = 35

The known scores add to 29, so the score that makes the set add to 35 (x) is 6

Graphs can show

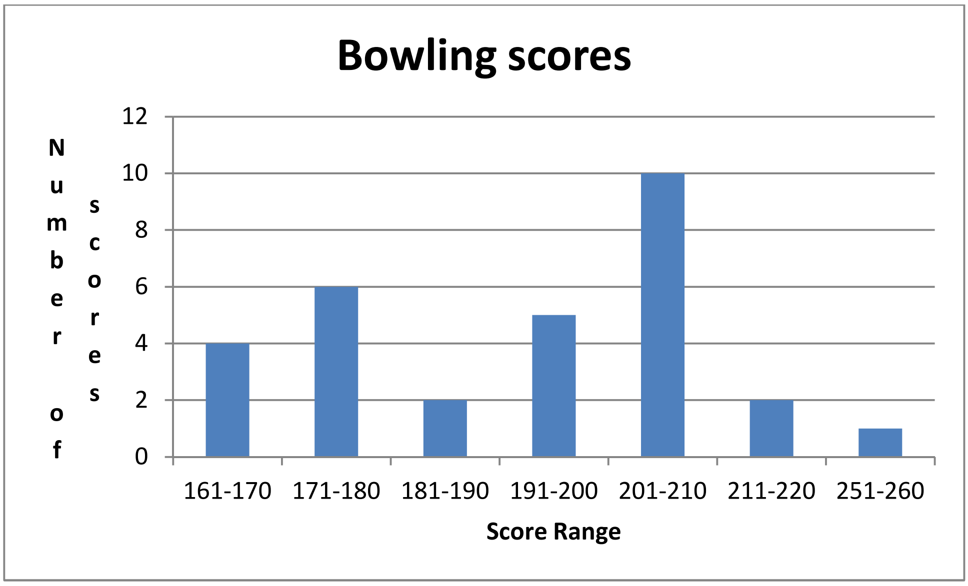
- Changes over time
- Records of certain events (for example number of students getting 60% on a test)
- Quantities at a point in time

Different types of graphs are more suitable than others depending on the information to be shown

Exercise 1

Simple Probability

1) Peter plays ten pin bowling; his last 30 scores have been graphed in a frequency chart, shown here



Basing you answers on the chart data

a) Is Peter more likely to score 205 or 185 when he next bowls?

b) Is he more or less likely to score over 200 when he next bowls?

c) What would be his probability of scoring over 250 when next he bowls?

d) What would be his probability of scoring between 201 and 210 when next he bowls?

e) Discuss a major drawback with using this chart to predict the probabilities of future scores

2) Craig rolled a pair of dice 360 times and recorded the sum of the two each time. He summarized his results in the table below

SUM of TWO DICE	Frequency
2	8
3	21
4	30
5	42
6	49
7	62
8	51
9	41
10	28
11	21
12	7

Based on his table:

a) What total is most likely to be rolled by two dice?

b) What is the most likely double?

c) What total is least likely to be rolled by two dice

d) Is he more likely to roll a sum of 10 or a sum of 6 with two dice?

e) Is this data more reliable than that of Q1? Give two reasons to support your answer

3) What is the theoretical probability of each of the following?

a) A head being thrown when a coin is tossed

b) A blue sock being taken from a draw containing 3 blue and 5 red socks

c) The number 2 being rolled on a dice

d) An even number being rolled on a dice

4) A card is drawn from a standard pack of 52 cards. What is the probability of the card being:

a) A black card

b) A club

c) An ace

d) A black 2

e) A picture card

f) The 2 of diamonds

5) A man throws two coins into the air

a) List the possible combinations, and from this table:

b) What is the probability of throwing two heads?

c) What is the probability of throwing a head and a tail?

d) If the first coin lands on a head, is the second coin more likely or less likely to be a head?

6) A coin is tossed and a dice is rolled

a) List the possible combinations of the coin and dice, and from this table:

b) What is the probability of throwing a six and a head?

c) What is the probability of throwing an odd number and a tail?

d) What is the probability of throwing a number higher than 4 and a head?

e) What is the probability of throwing a head and a 2 or a head and a 4?

7) A card is drawn from a normal pack. It is not replaced and a second card is drawn.

a) If the first card is red, what is the probability that the second card is also red?

b) If the first card is red, what is the probability that the second card is black?

c) If the first card is an ace, what is the probability that the second card is also an ace?

d) If the first card is the jack of clubs, what is the probability that the second card is the jack of clubs?

8) A set of cards consists of 10 red cards, numbered 1 to 10 and 10 black cards numbered 1 to 10

a) What is the probability of pulling a 10 at random?

b) What is the probability of pulling a black card at random?

c) What is the probability of pulling a red 2 at random?

d) What is the probability of pulling a red 2 on the second draw if the first card is a black 2, and it is not replaced?

e) What is the probability of pulling an 8 on the second draw if the first card is an 8, and it is not replaced?

9) Consider the word ANATOMICALLY

a) What is the probability that a randomly chosen letter from this word will be an L?

b) What is the probability that a randomly chosen letter from this word will be an A?

c) What is the probability that a randomly chosen letter from this word will not be a vowel

d) What is the probability that a randomly chosen letter from this word will be a Z?

10) What is the probability that a digit chosen randomly from all digits (0- 9) is:

a) A prime number?

b) An even number?

c) Not 7?

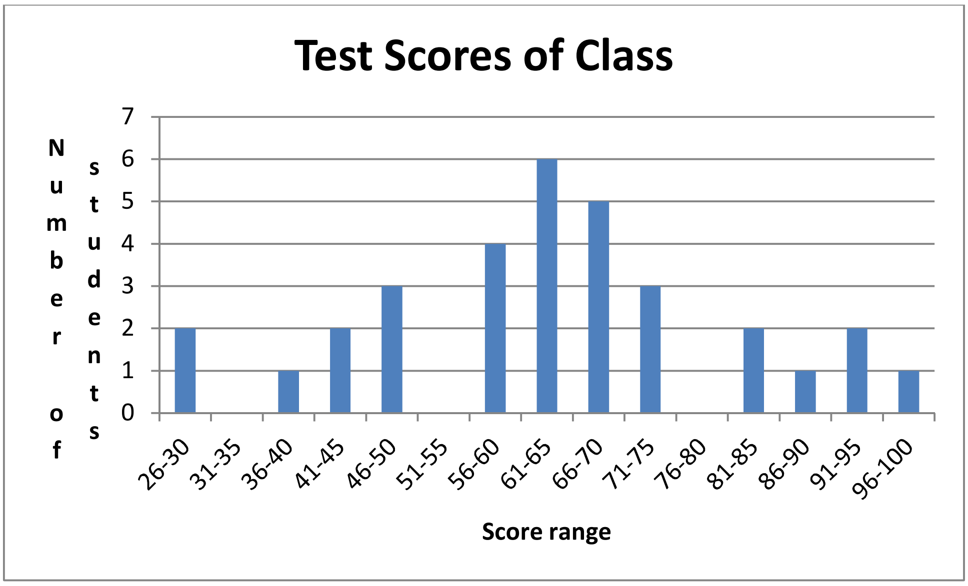
d) Greater than 4?

e) Less than 10?

Exercise 2

Data Representation & Analysis

1) Construct a cumulative frequency table from the following bar graph



How many students sat the test, and how many passed?

2) Construct a cumulative frequency histogram from the following data of the weights of 30 people in a group (in kgs)

72, 73, 73, 75, 77, 77, 78, 80, 83, 84, 84, 84, 85, 85, 88, 88, 90, 92, 92, 93, 95, 95, 96, 97, 97, 98, 98, 100, 103, 104

3) The following data shows the time taken for the members of an athletic club to run 100 metres

12.2, 12.4, 13.1, 13.2, 13.3, 13.4, 13.4, 13.5, 13.8, 14.1, 14.2, 14.3, 15, 15.2, 15.5, 15.5, 15.7, 15.8, 16, 16.2

a) Group the data into class intervals

b) Construct a histogram of the grouped data

c) Find the mean of the grouped data

d) Find the modal class of the grouped data

4) The set of weights (in kg) from Q2 are repeated here:

72, 73, 73, 75, 77, 77, 78, 80, 83, 84, 84, 84, 85, 85, 88, 88, 90, 92, 92, 93, 95, 95, 96, 97, 97, 98, 98, 100, 103, 104

Determine:

a) The range of the data

b) The median of the data

c) The upper and lower quartiles of the data

d) The inter-quartile range

5) Calculate the mean and standard deviation (using a calculator) for the following sets of data

a) 1, 2, 3, 4, 5, 6, 7, 8, 9

b) 1, 1, 1, 1, 1, 1, 1, 1

c) 2, 4, 6, 8, 10, 100

d) 1, 22, 30, 40, 75, 90

6) A year 7 class takes a test and receives the following marks

35, 42, 48, 51, 54, 56, 60, 65, 66, 68, 70, 70, 72, 73, 75, 77, 80, 85, 87, 90, 94, 94, 97, 99

To get an A on the test a student must score more than the mean plus one standard deviation.

How many students got an A on the test?

7) The scores for a test to two different classes are shown under

Class 1: 85, 96, 75, 84, 65, 91, 78, 82, 80, 70, 80, 58, 71, 78, 98, 99, 75, 62, 75

Class 2: 61, 53, 54, 75, 99, 98, 98, 96, 78, 57, 90, 75, 93, 51, 75, 96, 99, 59, 95

a) Calculate the mean and standard deviation for the two classes

b) Draw a histogram for each data set

c) Comment on the relationship between the histograms and the standard deviations

8) The mean of the following data set is 12. What is the value of x?

8, 10, 7, 4, 20, x, 12, 14, 15, 25

9) The mean of a set of 9 scores is 7. After another score is added the mean drops to 6.5. What was the added score?

Year 9 Mathematics

Algebraic Expressions

Useful formulae and hints

Index laws can be applied to operations involving variables. As long as the base (in this case the variable) is the same, the laws can be applied.

$x^a \times x^b = x^{a+b}$
$x^a \div x^b = x^{a-b}$
$(x^a)^b = x^{ab}$
$(xy)^a = x^a \times y^a$
$x^0 = 1$
$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$

Example:

$$\frac{(xy)^3 \times x^5}{(x^3)^2 \times (xy)^2 \times y} = \frac{x^3 \times y^3 \times x^5}{x^6 \times x^2 \times y^2 \times y} = \frac{x^8 \times y^3}{x^8 \times y^3}$$
$$= x^{8-8} \times y^{3-3} = x^0 \times y^0 = 1 \times 1 = 1$$

Remembering that $y^1 = y$

When adding or subtracting fractions containing variables, the same rules apply as with numerical fractions; find a common denominator and convert both fractions to an equivalent fraction in the same denominator, then simplify

Example
$$\frac{2x}{3} + \frac{3x}{5} = \frac{10x}{15} + \frac{9x}{15} = \frac{19x}{15}$$

When multiplying or dividing fractions containing variables, the same rules apply as with numerical fractions; multiply the numerators and multiply the denominators, then simplify the fraction. Use index laws when multiplying variables. For division remember to invert the second fraction and then multiply as above

Example

$$\frac{x^2}{3} \times \frac{6x^3}{5} \times \frac{1}{2x} = \frac{6x^5}{30x} = \frac{x^4}{5}$$

To solve equations involving addition/subtraction only, isolate the variable by adding or subtracting the non variable from both sides of the equation

Example

$x + 2 = 7$

$x + 2 - 2 = 7 - 2$

$x = 5$

To solve equations involving addition/subtraction and multiplication/division, first add or subtract the non variable as above, then multiply or divided both sides of the equation by the co-efficient of the variable, to isolate the variable

Example

$2x - 4 = 10$

$2x - 4 + 4 = 10 + 4$

$2x = 14$

$\frac{2x}{2} = \frac{14}{2}$

$x = 7$

If $x^2 = a$, then $x = a \; or - a$

For example, $x^2 = 9$, but $3^2 = 9, and \; (-3)^2 = 9$, so
$x = 3 \; or - 3$

Solve inequalities exactly the same way as solving equations, but keeping the inequality sign where the = sign would normally be. The exception is if multiplying or dividing by a negative number, flip the inequality sign

Examples

$2x - 2 > 10$

$2x > 12$

$x > 6$

$-2x + 2 < 8$

$-2x < 6$

$-x < 3$

$x > -3$

A negative index can be turned into a positive one by inverting the number

Example

$$\frac{x^{-2}}{2} = \frac{1}{2x^2}$$

$$\frac{3}{x^{-4}} = 3x^4$$

Note only the number with the index is inverted

To factorize an expression remove the highest common factor of all terms, place outside the bracket and divide each term by this factor to leave the remainders in the brackets

Examples

$2x + 4 = 2(x + 2)$
$x^2 + 3x = x(x + 3)$
$-4x - x^2 = -x(4 + x)$
$xy^2 + 2xy - x^3y = xy(y + 2 - x^2)$

To expand an expression, multiply every term in the bracket by the common factor. Note this method can be use to check the correctness of factorization

Examples

$2(x+2) = (2 \times x) + (2 \times 2) = 2x + 4$

$x(x+3) = (x \times x) + (x \times 3) = x^2 + 3x$

$-x(4+x) = (-x \times 4) + (-x \times x) = -4x - x^2$

$xy(y + 2 - x^2) = (xy \times y) + (xy \times 2) - (xy \times x^2)$

$\qquad = xy^2 + 2xy - x^3y$

Exercise 1

Simplifying Expressions Using Index Laws

1) Simplify the following using index laws

a) $x^a \times x^b$

b) $x^a \div x^b$

c) $(x^a \times x^b) \div x^c$

d) $(x^a)^b$

e) $(x^a)^b \div x^c$

2) Simplify the following

a) $x^2 \div x^2$

b) $x^4 \times x^{-4}$

c) $b^0 \times b^3$

d) $a^0 \times a^0$

e) $x^0 + x^0$

3) Simplify the following

a) $2x^3 \times 5x^2$

b) $5a^3 \times 3a^3$

c) $(2x^4)^2$

d) $3a^4 + 3a^4$

e) $2x^2 + 3x^3$

4) Simplify the following

a) $6x^6 \div 3x^3$

b) $20a^8 \div 5a^2$

c) $8c^5 \div 4c^5$

d) $5x^6 - 4x^3$

e) $(x)^{0.5} \times (x)^{0.5}$

5) Simplify

a) $2x(x^2 + 3)$

b) $x^2(x + 1)$

c) $ab(a + b)$

d) $x^0(a + b)$

e) $a(x^0 + 4)$

6) Simplify the following

a) $\frac{3x^2 \times 4x^4}{2x^6}$

b) $\frac{5a^3 \times 4a^5}{10a^2}$

c) $\frac{4x^5 \times 3x^3}{12x^8}$

d) $\frac{x^{12}}{x^4} - x^8$

7) Put the following in order from smallest to largest for x>1

$5x, 5^0x, 5^0x^0, (5x)^0, x^0, 5^0$

8) James asks Alan how far it is from his house to school. Alan replies:

"If you square the distance and multiply it by the distance to the power of 3 you get 32"

How far is Alan's house from school (in kilometres)?

Exercise 2

Expressions Involving Fractions

1) Simplify the following

a) $\frac{x}{3} + \frac{x}{3}$

b) $\frac{2x}{5} + \frac{x}{5}$

c) $\frac{5x}{7} - \frac{2x}{7}$

d) $\frac{3y}{4} - \frac{2y}{4}$

e) $\frac{8t}{5} - \frac{8t}{5}$

2) Simplify the following

a) $\frac{x}{2} + \frac{x}{4}$

b) $\frac{4t}{9} - \frac{t}{3}$

c) $\frac{3x}{4} + \frac{x}{8}$

d) $\frac{5y}{12} - \frac{y}{6}$

e) $\frac{x}{7} + \frac{2x}{21}$

3) Simplify the following

a) $\frac{x}{3} + \frac{x}{5}$

b) $\frac{y}{2} + \frac{2y}{3}$

c) $\frac{3t}{2} + \frac{t}{7}$

d) $\frac{3x}{5} + \frac{5x}{3}$

e) $x + \frac{5x}{11}$

4) Simplify the following

a) $\frac{3t}{2} - \frac{2t}{3}$

b) $\frac{5x}{2} - \frac{2x}{7}$

c) $\frac{4x}{5} - \frac{x}{2}$

d) $\frac{4y}{15} - \frac{y}{6}$

e) $\frac{16a}{20} - \frac{4a}{5}$

5) Simplify the following

a) $\frac{2x}{3} \div \frac{x}{6}$

b) $\frac{3a}{4} \div \frac{a}{6}$

c) $\frac{5x}{9} \div \frac{x}{18}$

d) $\frac{y}{5} \div \frac{2y}{15}$

e) $\frac{3x}{2} \div \frac{6x}{5}$

6) Simplify the following

a) $\frac{3xy}{4} \times \frac{8}{x}$

b) $\frac{5at}{3} \times \frac{9}{10t}$

c) $\frac{6abc}{9} \times \frac{2}{4ac}$

d) $\frac{xy}{5} \times \frac{35}{x} \times \frac{1}{7y}$

e) $\frac{xy}{10} \times \frac{3}{ab}$

7) Three friend, Alan, Colin and William share a pizza. Alan eats half of the pizza, and Colin eats a third of the pizza. What fraction of the original pizza is left for William?

8) Some students ask their maths teacher how old he is. The teacher replies "Half of my age subtract one-third of my age equals 7." How old is the maths teacher?

9) Pocket money is divided between three brothers according to their ages. Tony receives half of the total pocket money paid out, while Michael receives one-fifth of the total. What fraction of the pocket money does the middle child, Peter, receive?

Exercise 3

Solving Equations

1) Solve each of the following equations

a) $x + 3 = 5$

b) $2x - 4 = 6$

c) $\frac{1}{2}x - 6 = 8$

d) $\frac{x}{2} + \frac{x}{3} = 4$

e) $\frac{2x-4}{3}=6$

f) $\frac{-2x-3}{2} + 5 = 3$

2) Solve each of the following equations

a) $2(x + 1) = 10$

b) $\frac{1}{2}(3x - 2) = 8$

c) $4(x + 2) + 2(x + 1) = 0$

d) $2(2x - 1) - 3(x - 3) = 4$

e) $3(2 - 3x) - (1 - x) = -2$

3) A man declares "If you add 4 to my age and double the result, you will get 3 times my age less 22." How old is the man?

4) Half of a number equals twice that number plus 6. What is the number?

5) If you subtract 4 from a number and halve the result, you will get twice the same number less 8. What is the number?

6) How many solutions does each of the following equations have?

a) $x^2 = 4$

b) $x^2 = 9$

c) $x^2 = 0$

d) $x^2 = 16$

e) $x^2 = 3$

f) $x^2 = -4$

7) From your answers to question 6, how does the value of c in the equation $x^2 = c$ affect the number of solutions of the equation?

8) Solve the following equations

a) $x^2 = 4$

b) $2x^2 = 8$

c) $3x^2 = 27$

d) $x^2 = -6$

e) $-x^2 = 4$

f) $x^2 = 0$

9) Solve the following inequalities

a) $2x + 1 > 9$

b) $x - 3 < 6$

c) $2 - 3x > 4$

d) $-x - 5 < 10$

e) $2(x + 3) > 6$

f) $-(x - 4) < -2$

g) $\frac{x-4}{4} > 2$

h) $\frac{2-x}{2} < 6$

Exercise 4

Fractional & Negative Indices

1) Rewrite the following with positive indices

a) x^{-1}

b) x^{-4}

c) $2x^{-3}$

d) $(2x)^{-3}$

e) $\frac{1}{x^{-2}}$

f) $\frac{1}{2x^{-2}}$

2) Express the following using indices

a) $\sqrt{x}$

b) $\left(\sqrt{x}\right)^2$

c) $\left(x^{\frac{1}{2}}\right)^2$

d) $\sqrt{x} \times \sqrt{x}$

e) $\left(\sqrt{x}\right)^4$

3) Simplify the following, expressing your answer in positive indices

a) $x^{-4} \times x^2$

b) $(x^{-4})^2$

c) $(x^4)^{-2}$

d) $2x^{-3} \times 3x^{-2}$

e) $\frac{1}{2}x^{-2} \times 2x^2$

4) Simplify the following, expressing your answer in positive indices

a) $4x^2 \div 2x^{-4}$

b) $4x^{-2} \div 2x^4$

c) $4x^2 \div 2x^4$

d) $4x^{-2} \div 2x^{-4}$

e) $(4x)^{-2} \div (2x)^{-4}$

5) Simplify the following, expressing your answer with positive indices

a) $4(x)^{\frac{1}{4}} \div 2(x)^{\frac{1}{2}}$

b) $(4x)^{\frac{1}{4}} \div (2x)^{\frac{1}{2}}$

c) $(x)^{\frac{2}{3}} \div x$

d) $(2x)^{\frac{1}{2}} \div (2x)^2$

e) $\frac{1}{2}(x)^{\frac{1}{2}} \div 2x^2$

6) Simplify the following, expressing your answer in positive indices

a) $\left(x^{\frac{1}{2}}\right)^{-\frac{1}{2}}$

b) $\left(x^{-\frac{1}{2}}\right)^{\frac{1}{2}}$

c) $\frac{1}{\left(x^{\frac{1}{2}}\right)^{\frac{1}{2}}}$

d) $\left(\frac{1}{x^{\frac{1}{2}}}\right)^{\frac{1}{2}}$

e) $(x^2)^{-\frac{1}{2}}$

7) State whether the following statements are true of false. If false give the correct answer

a) $2x^{-5} = \frac{1}{2x^5}$

b) $\left(x^{-\frac{1}{2}}\right)^2 = x$

c) $x^2 \div x^4 = x^{-2}$

d) $\frac{1}{x^2} \div \frac{1}{x^4} = x^2$

e) $\frac{1}{x^{-2}} \times \frac{1}{x^{-2}} = \frac{1}{x^4}$

Exercise 5

Expanding & Factorizing

1) Expand the following by removing brackets and collecting and simplifying like terms where possible

a) $2x(x+3)+2(x+1)$

b) $3x(x-3)+(2x^2+x)$

c) $y(y-3)+y^2(y+1)$

d) $x(y+2)+2y(x+3)$

e) $x(1-x)+(x^2+x)$

2) Expand the following by removing brackets and collecting and simplifying like terms where possible

a) $x(x+2)+x^2(1-x)$

b) $2x(x+2)-x^2(2+x)$

c) $3x(2-x)+(x+2)$

d) $x(3+x)-2x(x-5)$

e) $4x(x-3)+x(3-x)$

f) $2x(x-4)-3x(3-2x)$

g) $x(2x-2)-2x(2-x)$

3) Expand the following by removing brackets and collecting and simplifying like terms where possible

a) $xy(x+y)+y(xy+x)$

b) $x^2(y-x)+x(x^2-y)$

c) $x^2y(2-y)-xy^2(1+x)$

d) $xy(2xy+x)-yx^2(2y+1)$

e) $(xy-y^2)-y(-x+y)$

4) Factorize the following expressions

a) $2x^2+4x$

b) $3y^3-y^2$

c) $4x^2-6x^4$

d) $8y-2y^2+6y^3$

e) $2x+4x^2-8y$

5) Factorize the following expressions

a) $8xy-4x^2y$

b) $5xy+10y^2$

c) $3x^2y^2-2xy$

d) $4x^2+8xy-8y^2$

e) $\frac{1}{2}x^2y-2xy^3$

6) Factorize the following expressions

a) $4x^2y^2 + 8xy + 12xy^2$

b) $xyz - x^2y^2z^2 - xy^2z$

c) $x(x - y) + (x - y)$

d) $x(x - y) + y(x - y)$

e) $y(y - 1) - x(1 - y)$

Year 9 Mathematics

Coordinate Geometry

Useful formulae and hints

An example showing how to calculate midpoint, gradient and length from a graph is given before exercise 1

To determine the above using formulae:

Using the points $(3, 2)\ and\ (1, -4)$ as an example

Midpoint of a line segment $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{3+1}{2}, \frac{2+-4}{2}\right) =$ $(2, -1)$

Make sure that you use the same point for the same subscript; here $x_1 = 3$, so $y_1 = 2$, NOT -4

Length of a line segment $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} =$ $\sqrt{(1-3)^2 + (-4-2)^2} = \sqrt{4 + 36} = \sqrt{40}$

Gradient of a line $= \frac{y_2-y_1}{x_2-x_1} = \frac{-4-2}{1-3} = 3$

When graphing lines only two points are necessary. If graphing from a table of values, choose two points

If graphing from an equation, set $y = 0$ and calculate the value of $x$at this point, then set $x = 0$ and calculate the value of y at this point

Example

$y = 2x - 4$

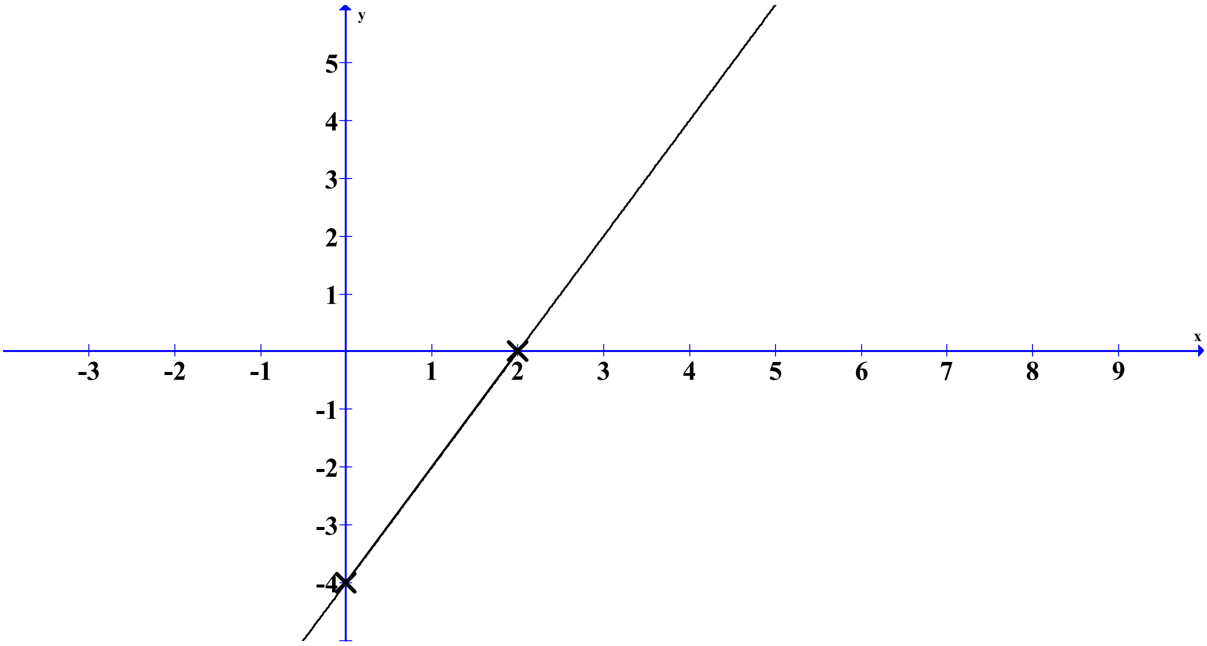
When $y = 0, 2x - 4 = 0, x = 2$

So $(2{,}0)$ is a point of the line

When $x = 0, y = -4$

So $(0, -4)$ is a point of the line

Plot these two points and join



By putting an equation into the form $y = mx + c$, the above points can be more easily calculated

Furthermore, the gradient can be easily determined: (its is the value of m in the above)

Example

$3y - 6x = 9$

$3y = 6x + 9$

$y = 2x + 3$

Gradient of the line is 2, and the graph crosses the y axis (when $x = 0$) at $y = 3$

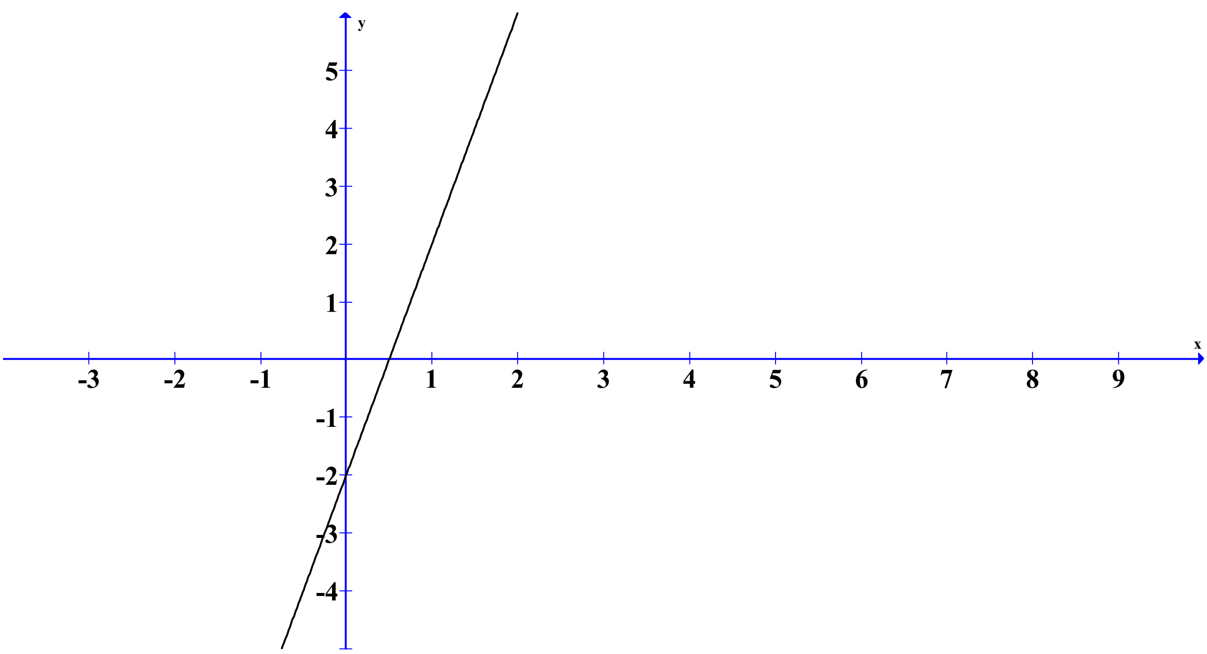
Two equations that have the same value of m when both are in the above format are parallel

To determine the equation of a graph

Find the value of c (it is the point where $x = 0$, that is where the graph crosses the y axis)

Substitute any other of point on the line into the part equation you now have

Example



When $x = 0,\ y = -2)$

The equation is now $y = mx - 2$

Another point is $x = \frac{1}{2}, y = 0$

So $0 = \frac{1}{2}m - 2$

$$\frac{1}{2}m = 2$$

$m = 4$

Equation is $y = 4x - 2$

Exercise 1

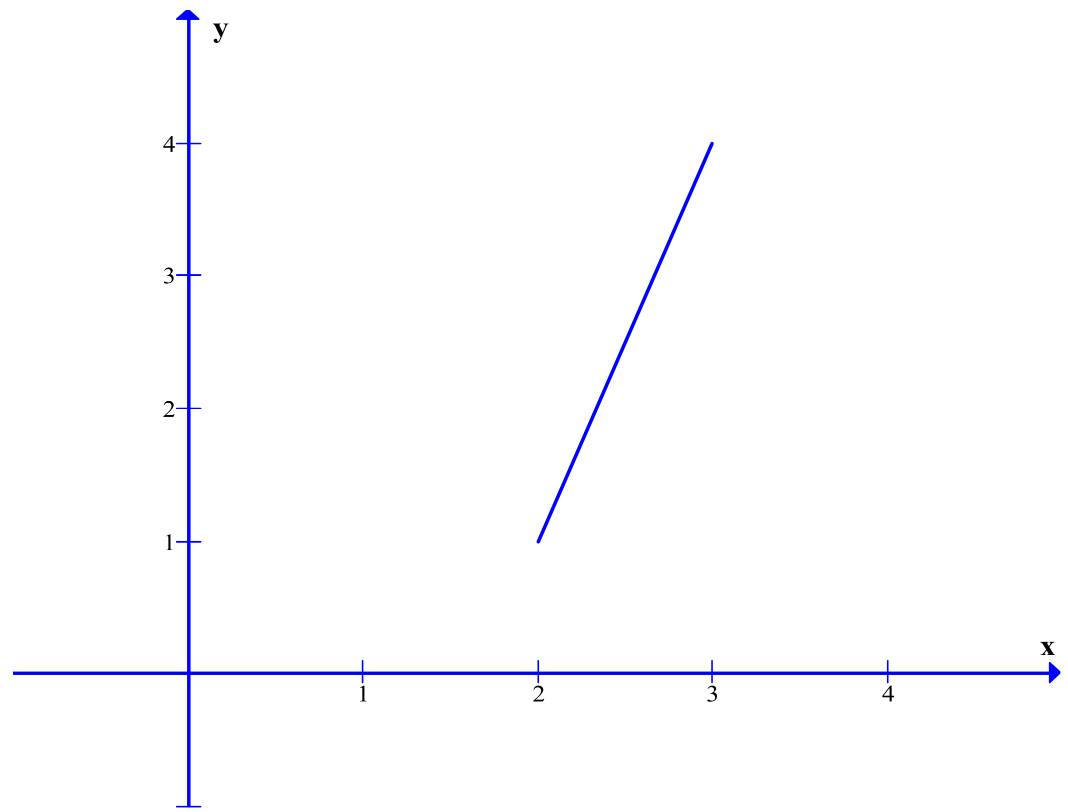
Determining Midpoint, Length & Gradient

1) Each part below lists a pair of coordinates. For each pair you are required to:

- Plot the points on a graph, and join to form a line segment
- Determine the midpoint of the line segment drawn from the diagram
- Using the line segment as the hypotenuse, construct a right angled triangle
- Use the above construction and Pythagoras' Theorem to determine the length of the line segment
- State whether the line segment has a positive or negative gradient (slope)
- Using the right angled triangle drawn to determine the value of the gradient of the line segment (gradient = rise/run)

Use the following example as a guide

For the points (2, 1) and (3, 4)

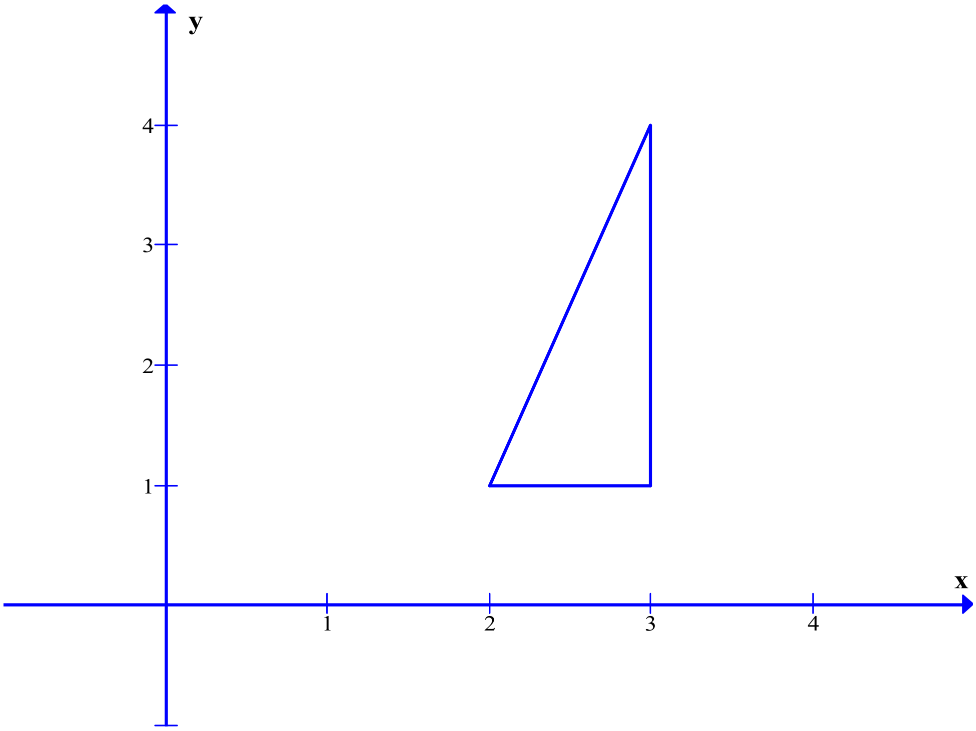


Midpoint is the (x, y) coordinate of the point halfway along line

Here midpoint is (2.5, 2.5)

Gradient is positive

Right angled triangle drawn



Length of hypotenuse (line segment) =c

From Pythagoras: $c^2 = a^2 + b^2$

$c^2 = 1^2 + 3^2$

$c^2 = 1 + 9 = 10$

$c = \sqrt{10}$

Note leave in square root form if cannot be simplified

Gradient = rise/run = $\frac{4-1}{3-2} = \frac{3}{1} = 3$

a) (1, 1) and (4, 5)

b) (1, 1) and (2, 2)

c) (-2, 10) and (4, 2)

d) (-1, -1) and (1, 3)

e) (2, 1) and (3, -2)

f) (3, -1) and (5, -4)

g) (-1, -2) and (2, 3)

h) (-3, 8) and (2, -4)

i) (0, -1) and (2, 1)

j) (0, 1) and 1, -4)

2) For each of the pairs of points given below, determine using the appropriate formula:

- The Midpoint of the line segment between the points
- The length of the line segment between the points
- The gradient of the line segment between the points

a) (0, 0) and (3, 3)

b) (2, 4) and (3, 6)

c) (-2, 4) and (0, 8)

d) (-1, 4) and (-4, -2)

e) (0, 2) and (0, 8)

f) (2, 0) and (2, 8)

g) (-10, 10) and (0, -10)

h) (3, -6) and (-6, 15)

i) (-1, -1) and (-5, 15)

j) (4, 10) and (-2, -2)

Exercise 2

Graphing Linear Relationships

For each of the equations below you are required to:

- Construct a table of values
- Plot the points tabled using a suitable scale
- Determine the x and y intercepts
- Determine if the point given lies on the line or curve by substituting the point into the equation, or plotting the point on the graph and seeing if it lies on the line/curve

a) $y = 2$ (1, 2)

b) $x = 3$ (1, 3)

c) $y = x + 2$ (2, 4)

d) $y = x - 3$ (4, -1)

e) $x + y = 4$ (0, 4)

f) $x - y = 2$ (-2, -4)

g) $y = \frac{x-2}{3}$ (8, 2)

h) $y = \frac{1}{2}x$ (5, 10)

i) $y = \frac{1}{2}x - 1$ (4, 1)

j) $y = 0$ (0, 0)

k) $x = 0$ (0, 0)

Exercise 3

Gradient/Intercept Form of Linear Equations

1) For each equation below state the value of the gradient, and the coordinate of the y-intercept

a) $y = 2x + 3$

b) $y = 3x - 1$

c) $y = \frac{1}{2}x + 5$

d) $y = 3x$

e) $y = 2$

f) $y = -3x - 4$

g) $y = -\frac{1}{2}x$

h) $y = -\frac{2}{3}x + 6$

2) Rearrange the following equations into the form $y = mx + b$

a) $x + y + 3 = 0$

b) $y - x - 4 = 0$

c) $2y + 4x - 6 = 0$

d) $\frac{1}{2}y - x - 2 = 0$

e) $x + y = 0$

f) $-y - 2x = 0$

g) $y - 2 = 0$

3) Draw graphs of the following equations given the gradient and the y-intercept. State the equation of the line

a) Gradient = 2, y-intercept = 1

b) Gradient = 1, y-intercept = -2

c) Gradient = -2, y-intercept = -4

d) Gradient = 0, y-intercept = 3

e) Gradient = $-\frac{1}{2}$, y-intercept = 0

f) Gradient = -3, y-intercept = $\frac{1}{2}$

g) Gradient = 1, y-intercept = 0

4) State if the following pairs of lines are parallel, showing your working

a) $y = 2x + 3, y = 2x - 1$

b) $y = x + 4, y = 2x - 2$

c) $2y = 4x - 5, y = 2x - 7$

d) $2x + 2y + 3 = 0, y = -x - 3$

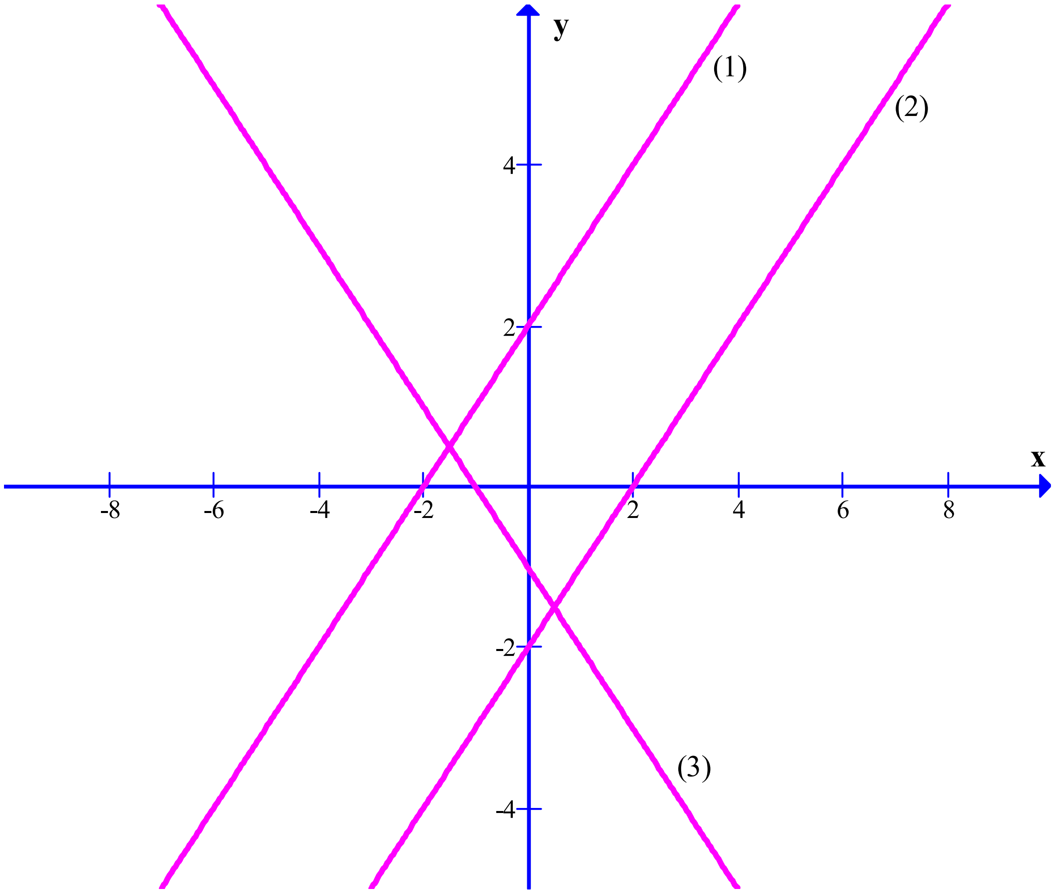
e) $2y + 4x - 4 = 0, y = -2x$

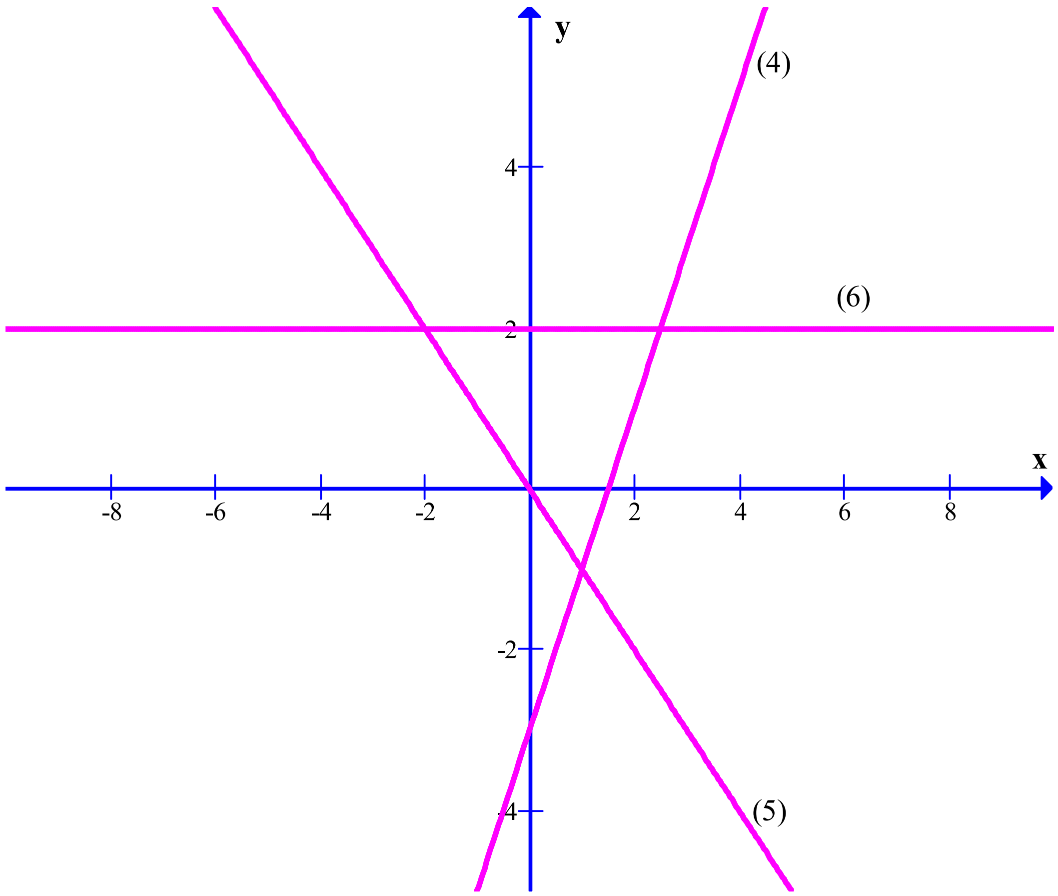
f) $3x - 6y + 3 = 0, y + \frac{1}{2}x + 4 = 0$

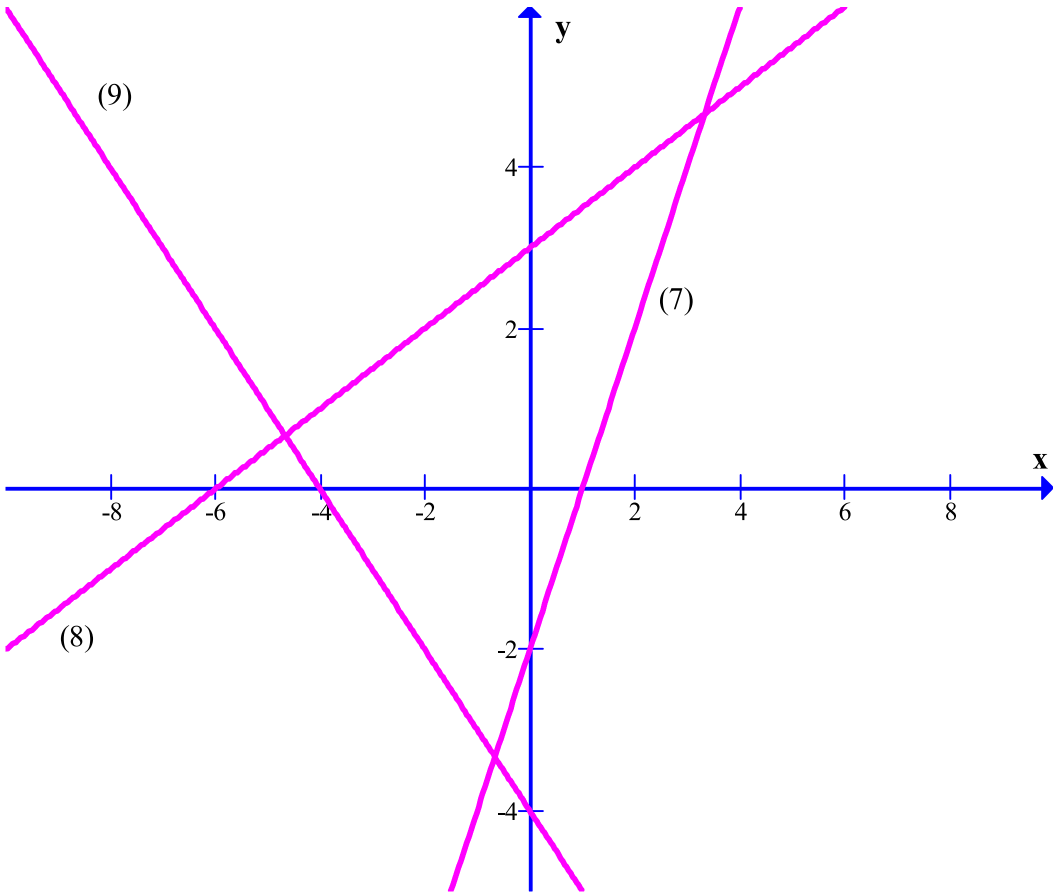
g) $x = 4, y = 4$

h) $y = 4, y = 2$

5) Determine the equations of the graphs drawn below (3 questions per graph)




y
x
(4)
(5)
(6)
4
2
-2
-4
-8
-6
-4
-2
2
4
6
8


y
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(9)
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-2
-4
-8
-6
-4
-2
2
4
6
8

Year 9 Mathematics

Measurement:

Useful formulae and hints

When determining the area of unusual shapes, initially see if the shape can be identified and if it has a formula

For example, the area of a kite can be found by using the formula $A = \frac{1}{2}(d_1 \times d_2)$, where d_1, d_2 are the diagonals of the kite

If this does not work, deconstruct the shape into ones that have formulae. Use Pythagoras' Theorem to find unknown side lengths if right angled triangles are involved

The area of a sector (part of a circle $= \pi \times r^2 \times \frac{\theta}{360}$, where θ is the angle of the sector

This is the same as calculating the area of the whole circle and multiplying it by what fraction of the circle the sector takes up (since there are 360° in a circle

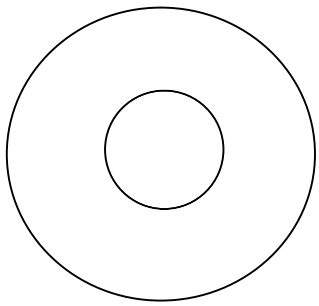
Example:

The area of a sector of radius 60 cm and subtending an angle of 60° is$A = \pi \times 60 \times 60 \times \frac{60}{360} = 3600 \times \frac{1}{6} \times \pi = 600\pi\ cm^2$

Note it is usual to leave the answer in terms of π

To find the area of an annulus (a doughnut), calculate the area of the whole circle and subtract the area of the inner circle

Example

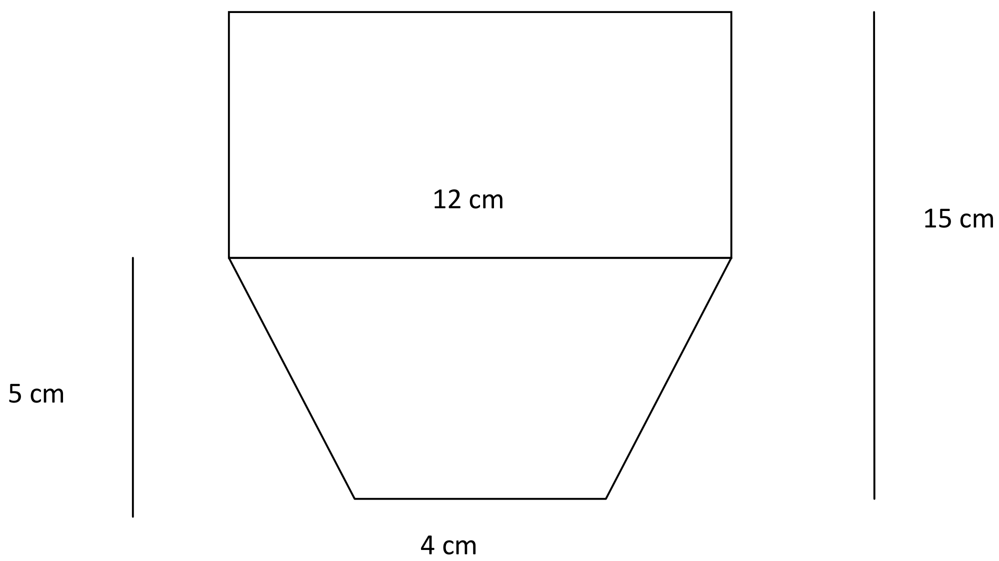


If the radius of the large circle is 8 cm and the radius of the small circle is 3 cm, then the area of the annulus is

$$A = \pi \times 8 \times 8 - \pi \times 3 \times 3 = 64\pi - 9\pi = 55\pi$$

Note again leave answers in terms of π

To find the area of compound shapes, break the shape up into known ones, and calculate the areas of each. The area of the compound shape is the sum of the area of the known shapes. Use the diagram, Pythagoras, or other techniques to find the lengths of required sides



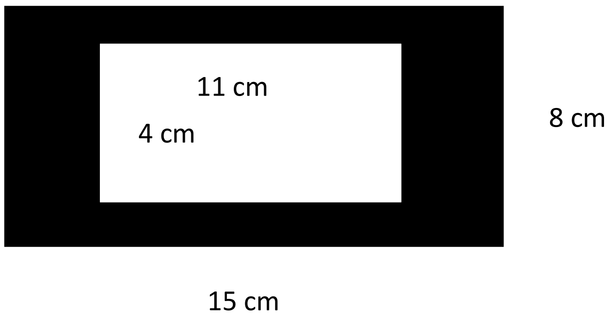
The above shape is a combination of a trapezium and a rectangle. The height of the whole shape is known (15 cm), and the height of the trapezium is known (5 cm) form the diagram. The height of the rectangle can be calculated as 15-5 = 10cm.

Area of rectangle = 10 x 12 = 120 cm^2

Area of trapezium = $\left(\frac{12+4}{2}\right) \times 5 = 40$ cm^2

Total area is 160 cm^2

To find the area of shaded regions, calculate the area of the whole shape and subtract the area of the non-shaded region



Area of whole rectangle = 15 x 8 = 120 cm^2

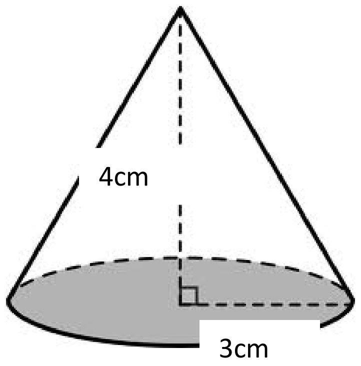
Area of small rectangle = 11 x 4 = 44 cm^2

Area of "path" = 120 – 44 = 76 cm^2

The surface area of a cylinder = $2\pi rh+2\pi r^2$, where r is the radius and h is the height

Surface area of a sphere = $4\pi r^2$, where r is the radius

The surface area of a cone = $\pi r^2 + \pi rs$, where r is the radius and s is the slope length (this can be calculated using Pythagoras' Theorem)



The slope is the hypotenuse of a right angled triangle with other sides of lengths 3 cm and 4 cm

To find the surface areas of pyramids, calculate the area of each face using known formulae (triangles, rectangles) and add these areas together

Volume of a cylinder = $\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of pyramid = $\frac{1}{3}$ x area of base x height. The base is either a square or a triangle; calculate these areas first as for any 2 dimensional problem, then multiply by $\frac{1}{3}$ x height

An easy way to remember the trigonometric ratios is the following saying

The **O**ld **A**nt: (Tangent = $\frac{Opposite}{Adjacent}$)

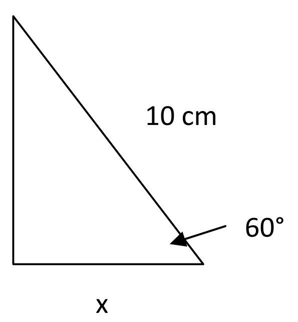
Sat **O**n **H**is: (Sine $= \frac{Opposite}{Hypotenuse}$)

Coat **A**nd **H**at: (Cosine $= \frac{Adjacent}{Hypotenuse}$)

To find a missing side in a right angled triangle

- Identify the known side length and the desired side length as opposite or adjacent to the given angle, or as the hypotenuse
- Use the appropriate ratio and write the formula substituting the known values, and using x for the unknown value
- Calculate the value of the sine, cosine or tangent (whichever is required) of the known angle; substitute into formula
- Solve for x

Example



The needed and known sides are the hypotenuse and the angle adjacent to the known angle; so cosine is used

$$\cos 60° = \frac{x}{10}$$

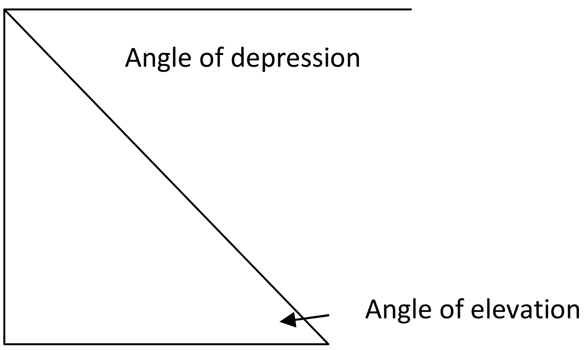
$$\frac{1}{2} = \frac{x}{10}$$

$$x = 5\ cm$$

To find a missing angle when two sides are known, use the arcsin (or arcos or arctan) button on your calculator

The angle of elevation is the angle measured from the ground to the top of the object being observed.

The angle of depression is the angle from the horizontal to the line going from the object to the ground



Bearings are measured from north in a clockwise direction.

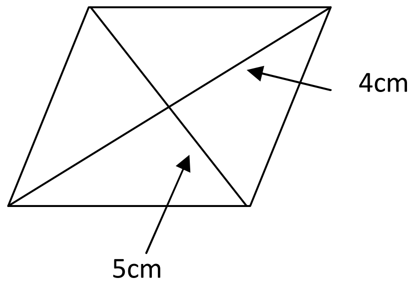
When problem solving using bearings, draw a diagram, construct or find right angled triangles and solve using methods outlined above

Exercise 1

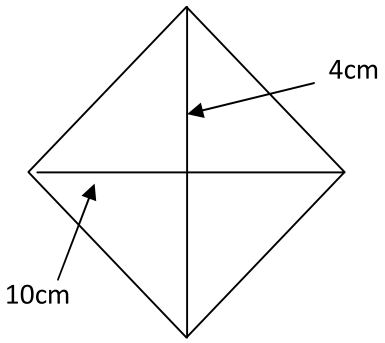
Area & Perimeter

1) .Calculate the area of the following

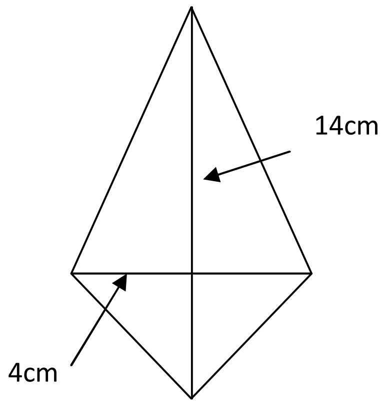
a)



b)



c)



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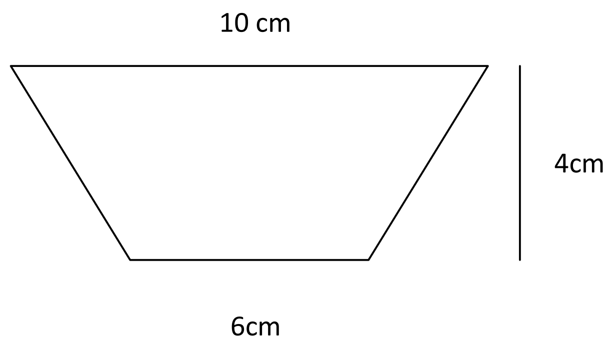
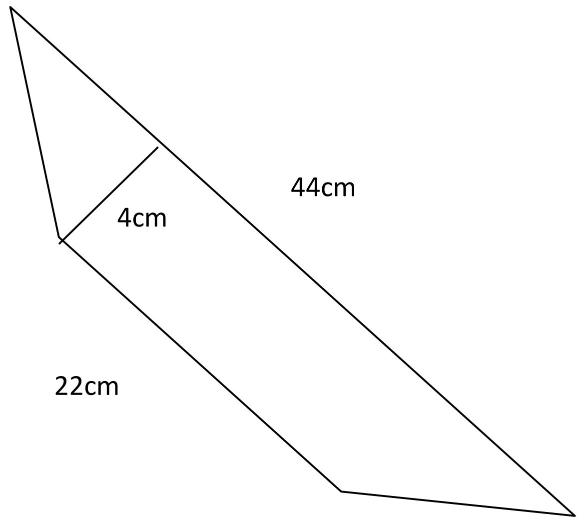
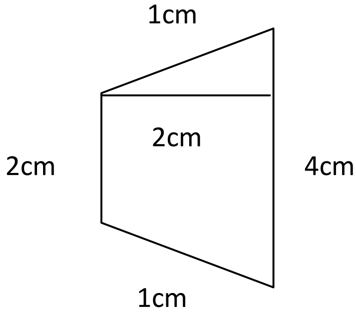


e)

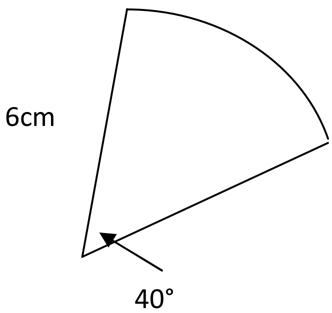


f)

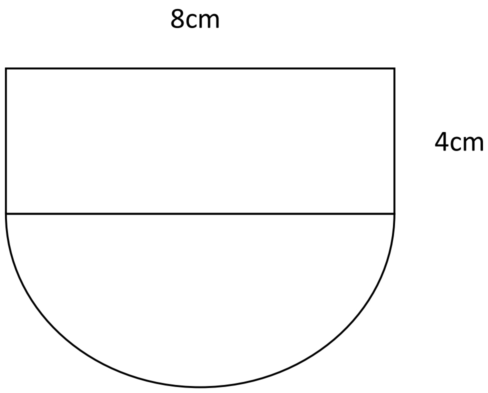


2) Calculate the area of each of the following

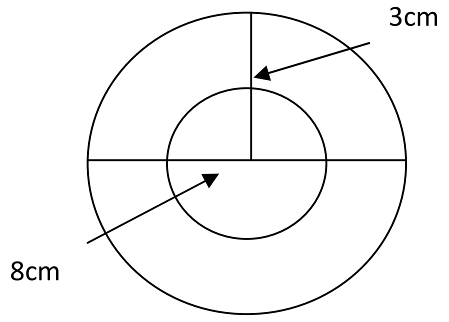
a)



b)

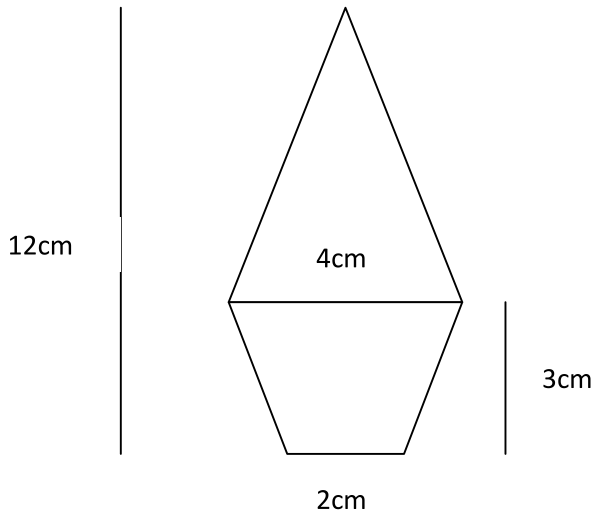


c) (Calculate area of outside ring)



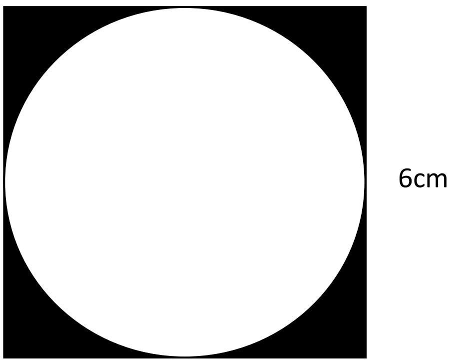
(Diameter of large circle is 8 cm; radius of small circle is 3 cm)

d)

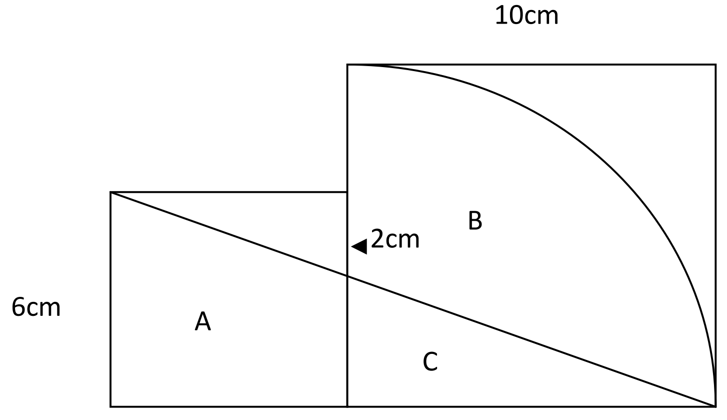


e) Calculate the perimeter of the shapes in parts a and b

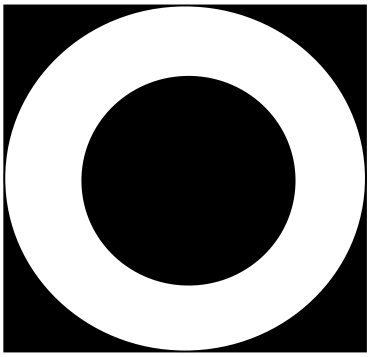
3) Calculate the area of the shaded region



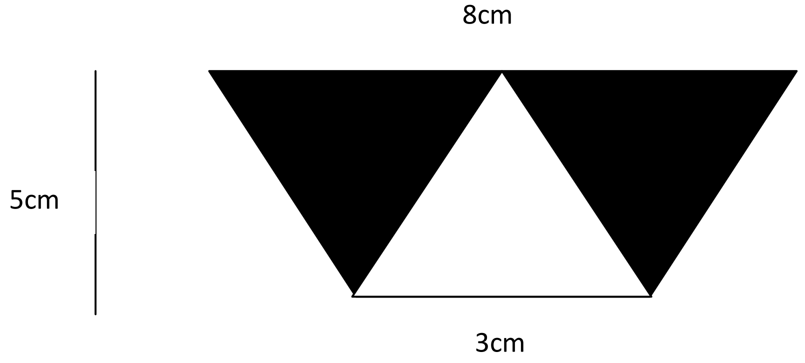
4) The figure shows two squares of side lengths 10cm and 6cm as shown in the diagram. Calculate the total area of the regions A + B + C.



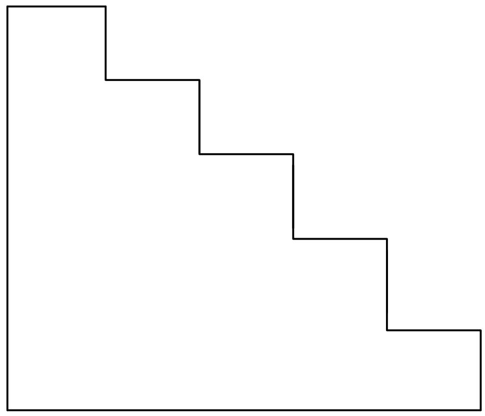
5) The diagram shows a square of side length 6cm with a donut inside it. If the small circle has a radius of 2cm, what is the total shaded area?



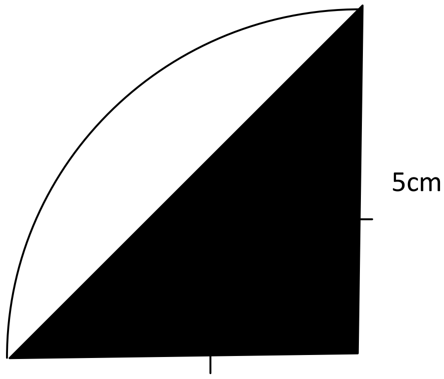
6) Calculate the shaded area



7) The diagram below shows the side view of a series of steps. Each step is 0.5 metres high and 0.5 metres wide. What is the area of the side of the block of steps?



8) Calculate the white area



Exercise 2

Volume & Surface Area

1) Calculate the surface area of the following right closed cylinders

a) Radius of 2mm, and a vertical height of 10mm

b) Radius of 1cm and a vertical height of 5mm

c) Vertical height of 500mm and a radius of 0.3m

d) Vertical height of 2.5m and a radius of 2000mm

2) Calculate the surface area of the following

a) A sphere having a radius of 10cm

b) A sphere having a diameter of 350mm

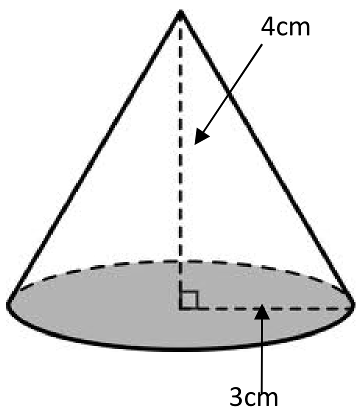
c) A hemisphere having a radius of 8m

d) A hemisphere having a diameter of 16mm

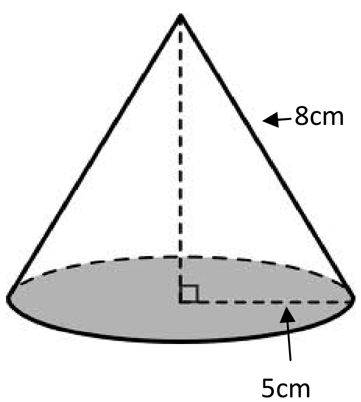
3

3) Calculate the surface area of the following.

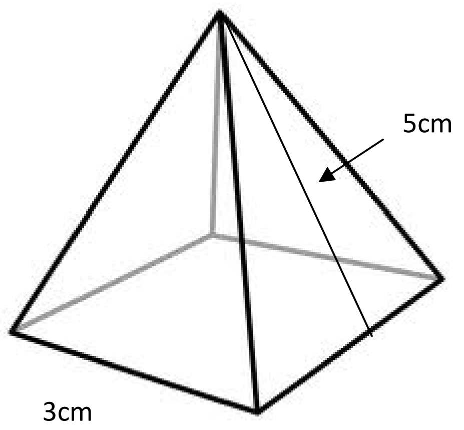
a)



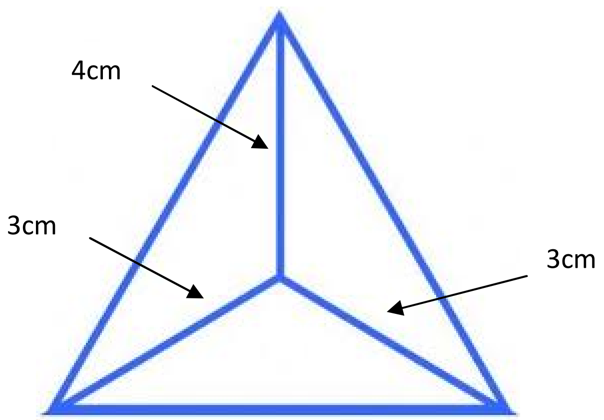
b)



c) A



d)



4) Calculate the volume of a right cone of radius 2cm and a vertical height of 5cm

5) Calculate the volume of a square pyramid of side length 2cm and vertical height 5cm.

6) A cone has a volume of $30\pi\ cm^3$. If the side length of a square pyramid is 6cm, what must its height be in order to have the same volume as the cone?

7) Calculate the volume of the following

a) A sphere having a radius of 2mm

b) A sphere having a diameter of 20cm

c) A hemisphere having a radius of 3.2m

d) A hemisphere having a diameter of 40mm

8) The volume of a sphere is $\frac{256}{3}\pi$ mm^3. What is its radius?

9) A square base pyramid has a perpendicular height of 10mm. What is the length of each side of the base if its volume is 270 mm^3?

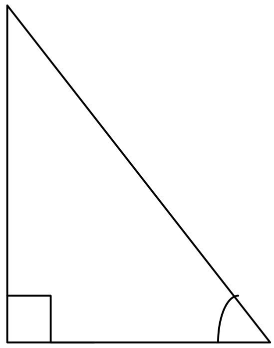
10) A sphere of volume $\frac{4}{3}\pi$ m^3 fits exactly inside a cube. What is the volume of the cube?

Exercise 3

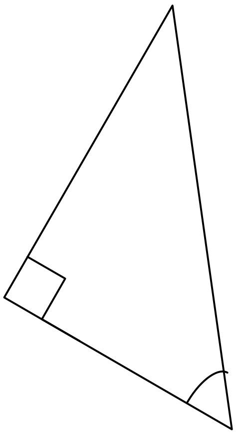
Trigonometry

1) In the following diagrams identify the angles adjacent and opposite the given angle, and also identify the hypotenuse.

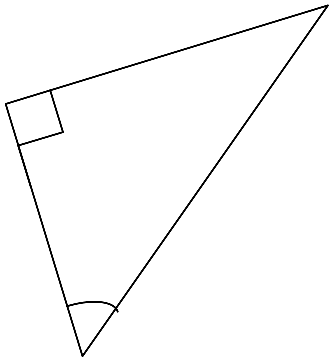
a)



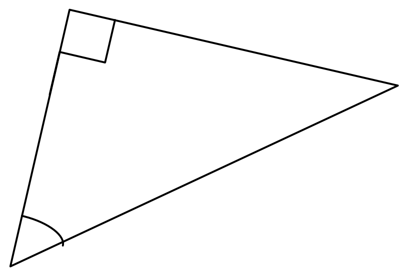
b)



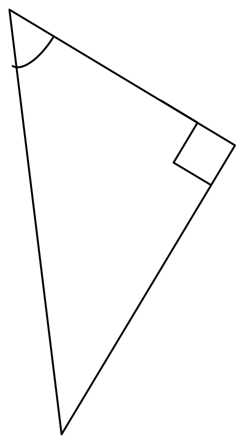
c)



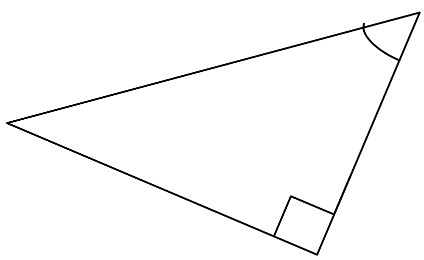
d)



e)

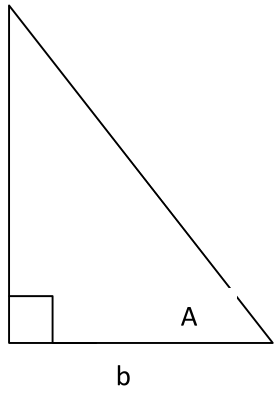


f)



2)

a) Complete the notation on the triangle sides and angles



b) Complete the following:

i. $\sin A =$

ii. $\cos B =$

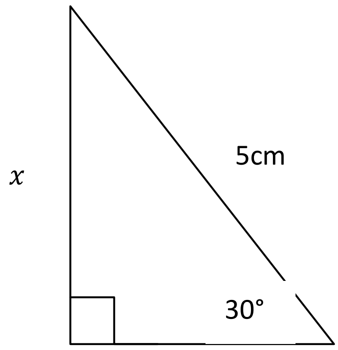
iii. $\tan A =$

c) Use a calculator to find the values from part (b) if the size of angle A = 55°

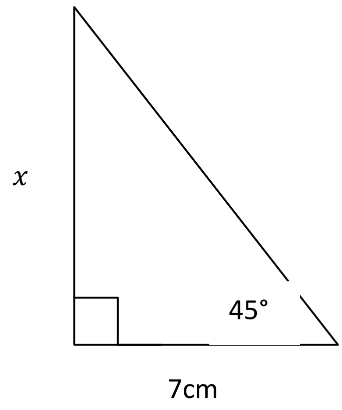
d) Use a calculator to find the size of angles A and B if the length of side a is 3cm, and the length of side b is 5 cm

3) Calculate the length of x in each of the diagrams below

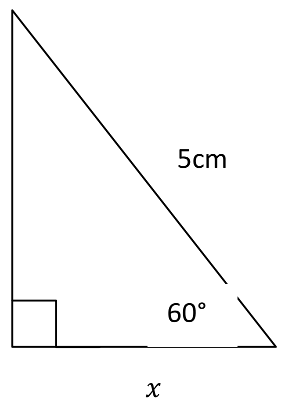
a)



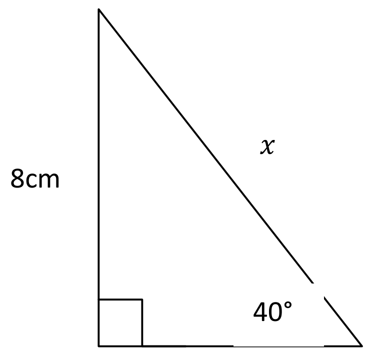
b)



c)

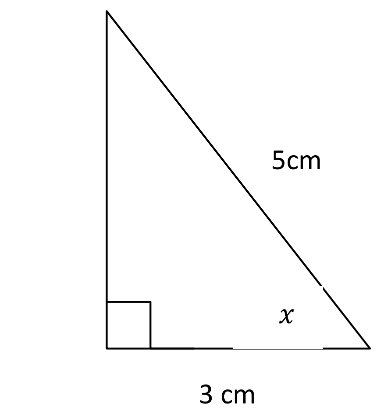


d)

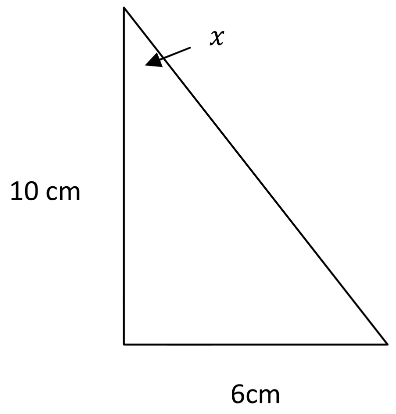


4) Calculate the size of angle x in the diagrams below, correct to the nearest degree.

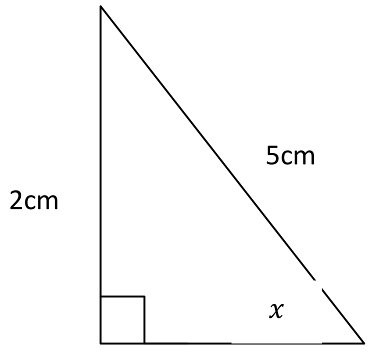
a)



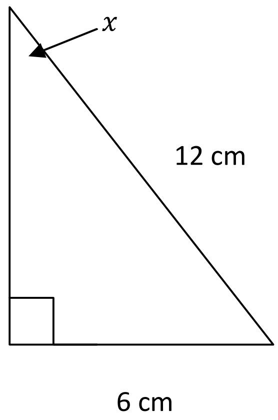
b)



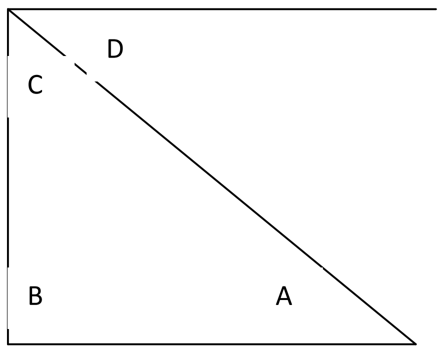
c)



d)

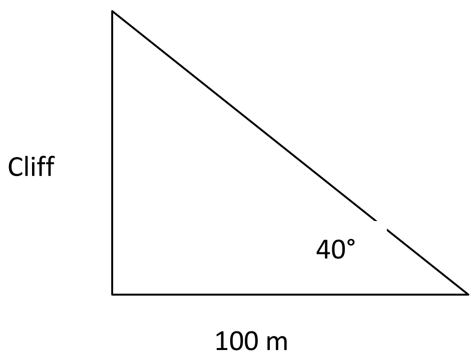


5) Identify the angles of elevation and depression in the diagram below

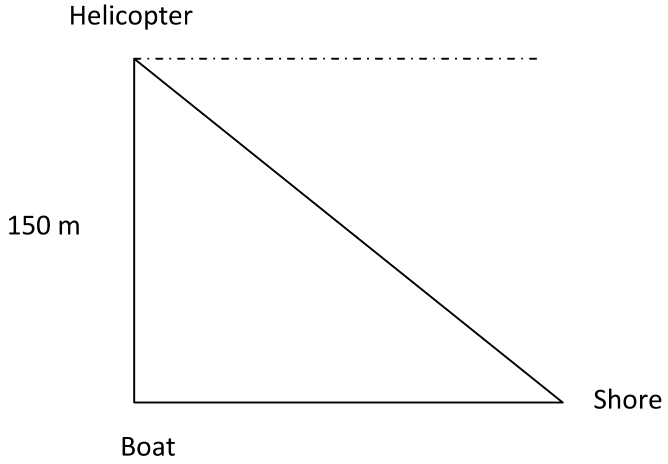


Complete the statement: The angle of elevation is .... the angle of depression

6) A man standing 100 metres away from the base of a cliff measures the angle of elevation to the top of the cliff to be 40 degrees. How high is the cliff?



7) A helicopter is hovering 150 metres above a boat in the ocean. From the helicopter, the angle of depression to the shore is measured to be 25 degrees. How far out to sea is the boat? (You need to fill in angle of depression on diagram)



(In questions 8 to 12 a diagram is required)

8) Two men walk from the same point. The first man heads on a bearing of 045°, whilst the second heads in the direction SE. After an hour they both stop. If the first man walked 4 km during this time, and the angle between their finishing points is 60 degrees, how far did the second man walk, and what is the shortest distance between them?

9) A ramp is built to allow wheelchair access to a lift. If the angle of elevation to the lift is 2 degrees, and the bottom of the lift is 50 cm above the ground how long is the ramp?

10) The angle of elevation to the top of a tree is 15 degrees. If the tree is 10 metres tall how far away from the base of the tree is the observer?

11) From the top of a tower a man sees his friend on the ground at an angle of depression of 30 degrees. If his friend is 80 metres from the base of the tower how tall is the tower?

12) A man walks on a bearing of 210 degrees for 2 km, and then walks due east until he is directly south of his starting position. How far south of his starting position is he at this time?

Year 9 Mathematics

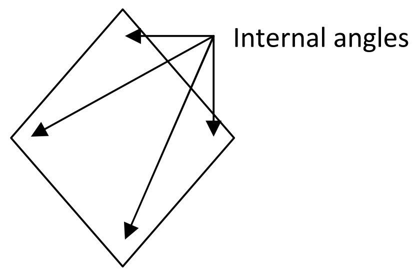
Space

Useful formulae and hints

Polygons are named according to the number of sides they have

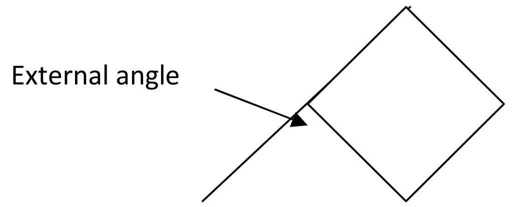
Name	Number of sides
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8

An internal angle is any angle bounded by two sides of a polygon



The number of internal angles equals the number of sides of a polygon

An external angle is any angle formed by one side of a polygon, and a line extending from an adjacent side



Two shapes are similar if they have the same shape, but not the same size. That is they are scaled versions of each other

Orientation does not affect similarity, but sides need to be matched to their similar counterparts, and the ratio of the side lengths preserved for all pairs

There exist conditions sufficient for proving that two triangles are similar

These are:

- AA(A): If any two sets of corresponding angles of two triangles are equal, then the third set must also be equal (since the sum of the angles of any triangle is 180°). If this is the case, them the three sets of sides must be in proportion, and hence the triangles are similar

- SSS: If all three sets of sides are in the same proportion to each other, then the triangles are similar

- SAS: If two sets of sides are in the same proportion, and the angle between them is the same for both triangles, then the triangles are similar

Two shapes are congruent if they have the same shape and size.

Again, orientation is irrelevant

Triangles can be said to be congruent if:

- SSS: The three pairs of sides are equal in the length.
- SAS: Two corresponding sides of a pair of triangles are qual in length and the corresponding angles that are between the sides are equal.
- ASA: Two corresponding angles of a pair of triangles are equal, and the corresponding sides that are enclosed by the angles are equal.
- AAS: Two corresponding angles of a pair of triangles are equal, and any other pair of sides are also equal
- HL: The hypotenuse of and any corresponding pair of sides are equal

Exercise 1

Properties of Polygons

1) Calculate the sum of the internal angles of the following

a) Triangle

b) Quadrilateral

c) Pentagon

d) Hexagon

e) Heptagon

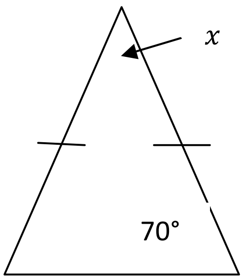
f) Octagon

2) From your answers to question 1, develop a formula that gives the sum of the internal angles of an n-sided polygon

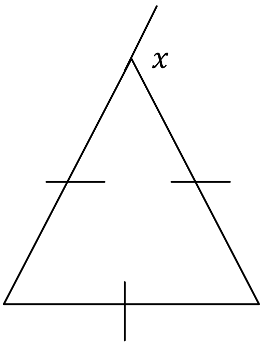
3) Similarly, develop a formula that gives the sum of the exterior angles of an n-sided convex polygon

4) Find the value of x in each of the following

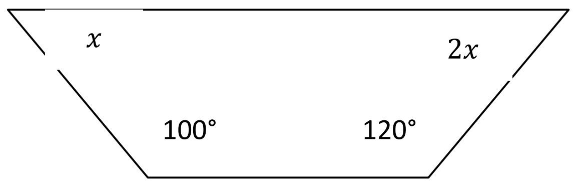
a)



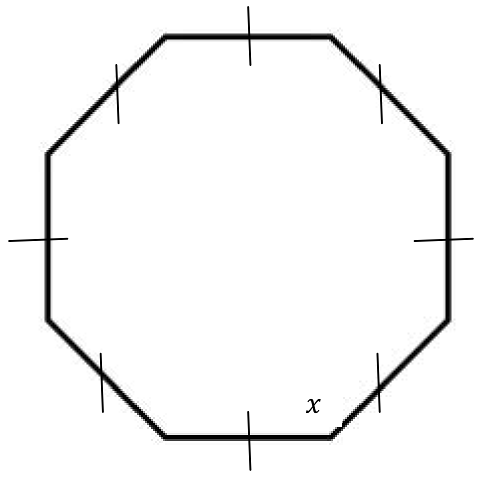
b)



c)

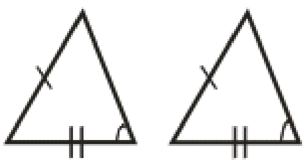


d)

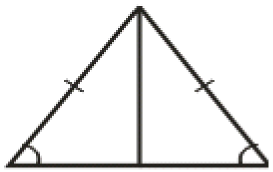


5)

a) State whether or not each of the following triangle pairs is congruent. If so, state a reason.



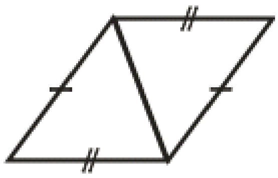
b) State whether or not each of the following triangle pairs is congruent. If so, state a reason.



6) Does AAA guarantee that two triangles are congruent? Why or why not?

7) Do congruent triangles have to be facing the same direction?

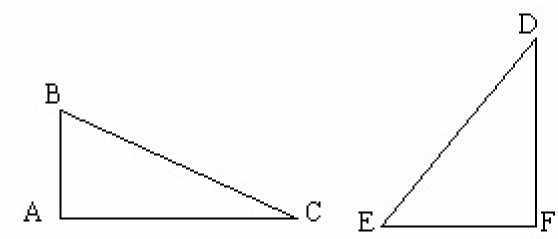
8) State whether or not each of the following triangle pairs is congruent. If so, state a reason.



9) List the methods of proving triangles congruent.

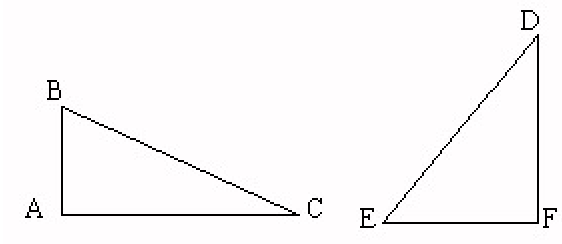
10) How is AAS different from ASA?

11)



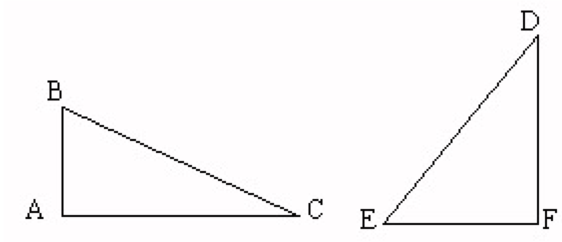
m∠B ≅ ∠E
m∠A ≅ ∠F
AC:FD = 6:12, and ED = 156, what is the length of BC?

12)



m∠A ≅ ∠F
m∠B ≅ ∠E
The length of the sides of ABC are 88, 156, and 100. The perimeter of FED is 172, what is the length of the longest side of FED

13)



m∠A ≅ ∠F
m∠B ≅ ∠E
The length of the sides of ABC are 60, 74, and 78. The length of the longest side of FED is 468, what is the perimeter of FED?

14) A tree 12 metres tall casts a 10 metre shadow. How much shadow does a 2metere tall man standing near the tree cast? (Draw a diagram)