Problem 1:

1. Classical Brownian Motion:
Given P_{t-1}:

$E[P_t] = E[P_{t-1}+r_t]$ $Std[P_t]= Std[P_{t-1}+r_t]$
$E[P_t] = E[P_{t-1}]$ $Std[P_t]= 0+Std[r_t]$
$\mathbf{E[P_t] = P_{t-1}}$ $\mathbf{Std[P_t]= \sigma}$

2. Arithmetic:
$E[P_t] = P_{t-1}E[r_t+1]$ $Std[P_t]= P_{t-1}Std[r_t+1]$
$E[P_t] = P_{t-1} * 1$ $Std[P_t]= P_{t-1}Std[r_t]$
$\mathbf{E[P_t] = P_{t-1}}$ $\mathbf{Std[P_t]= P_{t-1}\sigma}$

3.Log Return:
$E[\ln(P_t)] = \ln(E[P_{t-1}e^{\wedge}r_t])$ $Std[\ln(P_t)]= \ln(Std[P_{t-1}e^{\wedge}r_t])$
$E[\ln(P_t)]= \ln(P_{t-1}) * 1$ $Std[\ln(P_t)]= Std[r_t]$
$\mathbf{E[\ln(P_t)] = \ln(P_{t-1})}$ $\mathbf{Std[\ln(P_t)]=\sigma}$

$E[P_t] = E[P_{t-1}e^{\wedge}r_t]$ $Std[P_t]= Std[P_{t-1}e^{\wedge}r_t]$
$E[P_t]= P_{t-1}* 1$ $Std[P_t]= Std[r_t]$
$\mathbf{E[P_t]= P_{t-1}}$ $\mathbf{Std[P_t]= P_{t-1}\sigma}$

Suppose $\sigma=0.1$, $P_{t-1} =100$, $r\sim N(0,0.1)$:
Then we know:
1. Classical Brownian Motion:
Mean=100, Std=0.1

2. Arithmetic:
Mean=100, Std=10

3.Log Return:
ln(Mean)=ln(100)=4.6, ln(Std)=0.1
Mean =100, Std=10

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Classical Brownian Motion:
mean=  100.0  std=  0.1

Arithmetic:
mean=  100.02  std=  9.98

Log Return
mean=  100.52  std=  10.02
mean(ln(price3))=  4.61  std(ln(price3)=  0.1
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Comparing the calculated results(Mean and Std for each methods) from above with the results from simulations(as the figure shows), we noticed that they matched with each other(with some very small differences), which indicated that our mathematical proof is correct.

Problem 2:

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1. VaR using Normal Distribution is : 3.4079169845177866 %
2. VaR using Exp weighted Normal Distribution is : 2.616731388605204 %
3. VaR using MLE fitted T distribution is : 2.5374907663744968 %
4. VaR using Historical Simulation is : 2.072516857535336 %
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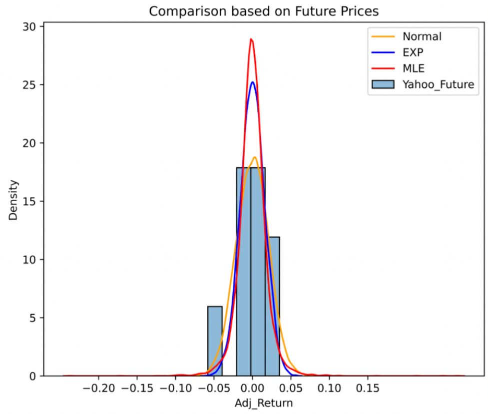
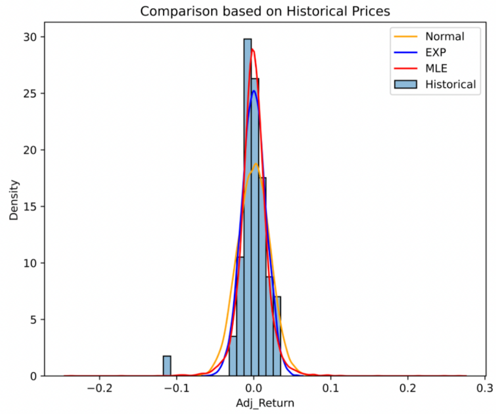
We see from the description above that MLE method performed the best on caching the empirical price pattern, and the Normal Distribution method did the worst work.





From the figures above we can see that Normal Distribution method matched the pattern of the future prices very well, it actually did the best job on predicting the future. However, it performed worst on matching with the empirical prices. Both MLE and EW methods perform bad on predicting the future price, where EW method did slightly better. On the other hand, both performed pretty well on matching with the empirical price, where MLE method did the best work.

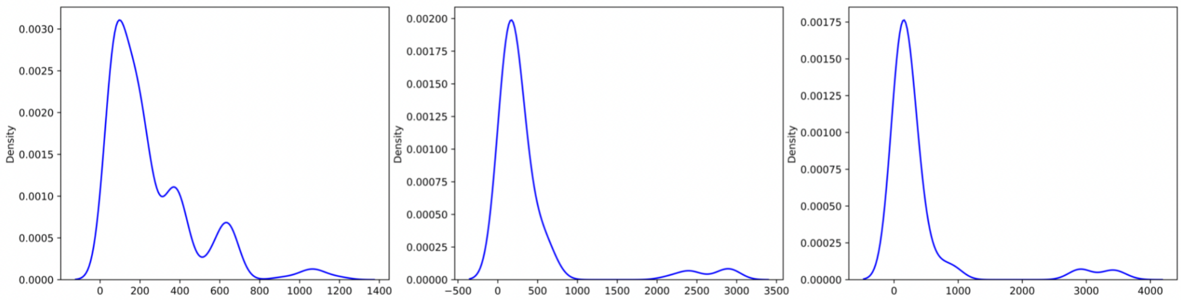
In conclusion, the MLE method has the best ability to describe the empirical risk in this stock, whereas Normal Distribution method has the worst. Meanwhile, the Normal Distribution method has the best ability to describe the future risk in this stock, whereas MLE method has the worst.

Problem 3:

VaR	Portfolio
5298.49	A
5576.13	B
3307.76	C
12460.9	Total

I chose to use historical simulation and the figure above shows the calculated VaR for each portfolio (and for the total portfolio). It is hard to tell whether the VaR is good or not without some more information, but I would say they are acceptable.

The major reason I chose historical simulation is that it holds less assumptions. It does not require normality and linearity.



The graph above shows the distribution of the return of the three portfolios. It is obvious from the graph that their returns are not normally distributed, as the distribution of their return clearly violated the 68-95-99 rule, over 90% of the data are concentrated on the left side of mean. There are huge left skewness This indicate that historical simulation method might be better than other methods.