Machine Learning Homework Sheet 05

Optimization

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1 Convexity

Problem 1: Prove or disprove whether the following functions are convex on the given set D:

i) $f(x, y) = x^2 + 2y + \cos(\sin(\sqrt{\pi})) - \min\{-x^2, \log(y)\}$ and $D = (-100, 100) \times (1, 50)$

ii) $f(x) = \log(x) - x^3$ and $D = (1, \infty)$

iii) $f(x) = -\min\{\log(3x + 1), -x^4 - 3x^2 + 8x - 42\}$ and $D = \mathbb{R}^+$

iv) $f(x, y) = y \cdot x^3 - y \cdot x^2 + y^2 + y + 4$ and $D = (-10, 10) \times (-10, 10)$

Problem 2: Prove the following statement: Let $f_1 : \mathbb{R}^d \to \mathbb{R}$ and $f_2 : \mathbb{R}^d \to \mathbb{R}$ be convex functions, then $h(\boldsymbol{x}) := \max\{f_1(\boldsymbol{x}), f_2(\boldsymbol{x})\}$ is also a convex function.

Problem 3: Given two convex functions $f_1 : \mathbb{R} \to \mathbb{R}$ and $f_2 : \mathbb{R} \to \mathbb{R}$, prove or disprove that the function $g(x) = f_1(f_2(x))$ is also convex.

2 Minimization of convex functions

Problem 4: Prove that for convex functions each local minimum is a global minimum. More specifically, given a convex function $f : \mathbb{R}^N \to \mathbb{R}$, prove that if $\boldsymbol{\nabla} f(\boldsymbol{\theta}^*) = 0$ then $\boldsymbol{\theta}^*$ is a global minimum.

3 Gradient Descent

Problem 5: Load the notebook `homework_05_notebook.ipynb` from Piazza. Fill in the missing code and run the notebook. Convert the evaluated notebook to pdf and add it to the printout of your homework (instructions for this are provided within the notebook).

Upload a single PDF file with your solution to Moodle by 25.11.2018, 23:59 CET. We recommend to typeset your solution (using LaTeX or Word), but handwritten solutions are also accepted. If your handwritten solution is illegible, it won't be graded and you waive your right to dispute that.