Prof. Dr. Rüdiger Ehlers Francisco Palau, M.Sc.

Applied Computational Engines 2018 – Assignment Sheet 5

Due: Monday, **21st May 2018**, 7:30am

Please indicate your **name** and **email address**. You can work in **groups** of up to **three** students. Only one submission per group is necessary. However, in the tutorials every group member must be able to present the solutions to each problem solved by your group.

Please submit your solutions either

- by e-mail to `fpalau@uni-bremen.de` **and** `rehlers@uni-bremen.de`, or
- on paper in **letter box 52** (Francisco Palau-Romero) on floor 6 of the MZH building.

Note that you will need 50% of the points on all exercise sheets in order to take the "Fachgespräch" OR the oral exam.

Phase transition encoding (30 pts.)

Let $\{y_0, \ldots, y_{n-2}\}$ be the variables of a phase transition encoding of a value $v \in \{0, \ldots, n-1\}$, and φ be the SAT constraints (in CNF) that encode that the valuation of $\{y_0, \ldots, y_{n-1}\}$ represents a valid ladder encoding for v.

Assuming that we do not want to introduce additional helper variables, how many CNF clauses are needed (to be used in addition of φ) in order to encode the constraint that v must be **even**?

Your answer should be a function that maps n to a number of clauses. For instance,

$$f(n) = \begin{cases} 3 \cdot n & \text{if } n \text{ is even} \\ 3 \cdot n - 2 & \text{if } n \text{ is odd} \end{cases}$$

would be such a function (but it is not the one that we are looking for). Observe that $v \in \{0, \ldots, n-1\}$, so for $n = 2$, there is one possible even values v, for $n = 3$, there are two possible even values, for $n = 4$, there are again two possible even values, and so on.

Don't forget to justify your answer, i.e., explain how you came to the definition of your function f.

Unit propagation and the phase transition encoding (20 pts.)

Reconsider the direct encoding of a value from a domain $V = \{0, \ldots, n-1\}$. For this encoding, we have seen in the lecture that if we learn clauses that rule out all values of V except for one, this allows the solver to deduce that the remaining value is the one to be chosen by unit propagation. In other words, the solver can immediately set the variable corresponding to the only remaining possible value in the encoding to **true** in such a case.

We have also seen that with the bitwise encoding, this did not work in general, i.e., even after clauses have been learned that exclude all valuations to the boolean variables except for one, the solver may still be unable to derive the values for all variables in the encoding by unit propagation.

But what about the phase transition encoding? Please perform the same analysis for this encoding.