Prof. Dr. Rüdiger Ehlers Francisco Palau, M.Sc.

Applied Computational Engines 2018 – Assignment Sheet 10

Due: **Monday, 25th June 2018, 9:30am**

Please indicate your **name** and **email address**. You can work in **groups** of up to **three** students. Only one submission per group is necessary. However, in the tutorials every group member must be able to present the solutions to each problem solved by your group.

Please submit your solutions either

- by e-mail to `fpalau@uni-bremen.de` **and** `rehlers@uni-bremen.de`, or
- on paper in **letter box 52** (Francisco Palau-Romero) on floor 6 of the MZH building.

Note that you will need 50% of the points on all exercise sheets in order to take the "Fachgespräch" OR the oral exam. We may ask you to present your solutions in the tutorial, especially if you work in a group. We aim for asking everyone taking part in the course to present at least twice during the course of the semester.

SMT (50 pts.)

Model the following problem as an SMT instance and solve it, for example using the SMT solver `Z3` from Microsoft research. You can either download/compile it on your own machine, or you can just use the on-line version at `http://rise4fun.com/Z3` to solve the problem (Note: that website seems to use Google Analytics. Consider using a web browser in incognito mode if you are concerned about that).

Your company plans to hire 5 *sales persons* (*SPs*) to sell paint and your role is to assess whether this plan can actually work out in the best case.

The continent in which the sales are to be performed is divided into 5 *regions*. Each salesperson is assigned to exactly one of them. Not all regions have to be covered – but then, no sales will be made in the uncovered region(s). It is known that the *overall size of the market* is exactly 30 million gallons of paint. But how these 30 millions divide across the regions is not exactly known. We do however know some limits:

Region	Minimum	Maximum
0	1000000	10000000
1	1000000	8000000
2	2000000	8000000
3	4000000	5000000
4	3000000	4000000

The *real market sizes in the regions*, namely r_0, r_1, r_2, r_3, and r_4 are somewhere in between the minimum and maximum values (where being exactly at a minimum or maximum value is possible as well).

The SPs have different properties. First of all, they have been promised to work in regions in market sizes above certain thresholds (as the SPs work on commission). Also, the SPs have different levels of skill, which determine what market share the SP can achieve:

Sales person	Achievable market share	Minimum real region market size
0	20%	10000000
1	15%	7000000
2	15%	3000000
3	10%	7000000
4	8%	5000000

As an example, when assigning SP 4 to region i with a market size of r_i gallons of paint, the SP will be able to sell $0.08 \cdot r_i$ many gallons. If multiple SPs are assigned to the same region, their sales add up. However, no matter how many SPs are assigned to a region, our company will never achieve an overall market share of more than 25% in any region. So, for example, when assigning both SP 0 and SP1 to the same region with real paint market size r_i, both SPs together will sell only $0.25 \cdot r_i$ many gallons of paint.

All SPs together must sell 6.3 million gallons of paint.

The question is now if this plan is realistic in the *best case*, i.e., if there are values $r_0, \ldots, r_4$ and an assignment of the SPs to the regions such that all constraints given above are satisfied.

Note that part of this exercise is to make use of the SMT solver's rich input language, including the `ite` command and uninterpreted functions. Not all constraints can be written down in a very elegant way, but your choice of variables should enable some of the constraints to be written down in an elegant way. Note that using $5 \cdot 5$ Boolean variables for encoding the assignment of the SPs to the regions is definitely not elegant.

Grading

25 points are awarded for modelling the problem **correctly** (i.e., such that the SMT solver computes a correct result without hard-coding the result or part of the result in the instance), and 25 points are awarded for modelling the problem **elegantly**. You should be able to write down the problem instance in the SMT2LIB format directly, without writing a program to build the problem instance. The copy&paste functionality of your text editor will sometimes come in handy, though.

Notes

The lecture notes contain an example for using an SMT solver in practice, which should be useful to solve this task. Give Z3's output for the problem instance and mark where the values for $r_0, \ldots, r_4$ can be found as well as where the assignment of the SPs to the regions can be found in the output.

Mentioned in the text above is the `ite` command, which corresponds to the ·?·:· operator in Java or C. The parameters and the order of them is the same in all cases – the main difference in SMT2LIB is that the operator comes first and how braces have to be used. As an example, the following constraint encodes for some integer or real constants x and y that $|x| > |y|$:

```
(assert (> (ite (< x 0) (* -1 x) x) (ite (< y 0) (* -1 y) y)))
```