Machine Learning Homework Sheet 04

Linear Classification

1 Linear classification

Problem 1: We want to create a generative binary classification model for classifying *nonnegative* one-dimensional data. This means, that the labels are binary ($y \in \{0, 1\}$) and the samples are $x \in [0, \infty)$.

We place a uniform prior on y

$$p(y = 0) = p(y = 1) = \frac{1}{2}.$$

As our samples x are nonnegative, we use exponential distributions (and not Gaussians) as class conditionals:

$$p(x \mid y = 0) = \text{Expo}(x \mid \lambda_0) \qquad \text{and} \qquad p(x \mid y = 1) = \text{Expo}(x \mid \lambda_1),$$

where $\lambda_0 \neq \lambda_1$. Assume, that the parameters λ_0 and λ_1 are known and fixed.

a) What is the name of the posterior distribution $p(y \mid x)$? You only need to provide the name of the distribution (e.g., "normal", "gamma", etc.), not estimate its parameters.

b) What values of x are classified as class 1?
(As usual, we assume that the classification decision is $y_{predicted} = \arg\max_k p(y = k \mid x)$)

Problem 2: Assume you have a linearly separable data set. What properties does the maximum likelihood solution for the decision boundary $\boldsymbol{w}$ of a logistic regression model have? Assume that $\boldsymbol{w}$ includes the bias term.

What is the problem here and how do we prevent it?

Problem 3: Show that the softmax function is equivalent to a sigmoid in the 2-class case.

Upload a single PDF file with your solution to Moodle by 18.11.2018, 23:59 CET. We recommend to typeset your solution (using LaTeXor Word), but handwritten solutions are also accepted. If your handwritten solution is illegible, it won't be graded and you waive your right to dispute that.

Problem 4: Which basis function $\phi(x_1, x_2)$ makes the data in the example below linearly separable (crosses in one class, circles in the other)?