Compare Effective SNR and MATLAB 5G Toolbox-Defined SNR

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In MATLAB 5G Toolbox function hPrecodedSINR.m, the SNR is defined to be the sum of SNR's of each MIMO stream. We do not like this definition because the physical or algebraic meaning of adding two SNR's is not clear to us. We prefer another definition of SNR, which we will name effective SNR.

Definition 1. Let γ_i be the SNR of i-th stream in a MIMO channel, then the effective SNR γ_e is

$$\gamma_e = 2^C - 1 \tag{1}$$

where $C = \sum_i \log(1 + \gamma_i)$, the log is 2-based.

In other words, the effective SNR is defined to be the SNR of a virtual SISO channel as if the capacity of the MIMO channel under question were achieved upon. We like this definition because the summation of capacities makes good sense in information theory.

Here we also state the definition of SNR in MATLAB 5G Toolbox.

Definition 2. The SNR γ_M in MATLAB 5G Toolbox function hPrecodedSINR.m is defined by

$$\gamma_M = \sum_i \gamma_i$$

The aim of this article is to show the following inequality

$$\gamma_e > \gamma_M \tag{2}$$

Let $f(x) = \log(1 + x)$, then $f(x)$ is obviously invertible, concave, differentiable and also satisfies

$$f(0) = 0$$

Hence we can view equation (1) through another approach

$$\gamma_e = f^{-1}\left(\sum_i f(1 + \gamma_i)\right)$$

Since $f(x)$ is monotonically increasing, we can readily prove equation (2) by showing that

$$f\left(\sum_i \gamma_i\right) < \sum_i f(\gamma_i) \tag{3}$$

To prove equation (3), we first prove a lemma

Lemma 1. *If $f(x)$ is strictly concave, twice differentiable, monotonically increasing and satisfies $f(0) = 0$, then for any positive x, y*

$$f(x + y) < f(x) + f(y)$$

Proof. f concave and twice differentiable
$\Rightarrow f''(x) < 0$
$\Rightarrow f'(x)$ strictly decreasing
Also, $y > 0$
$\Rightarrow \forall t > 0,\ f'(t + y) < f'(t)$
$\Rightarrow \int_0^x f'(t + y)\mathrm{d}t < \int_0^x f'(t)\mathrm{d}t$
Note that $\int_0^x f'(t + y)\mathrm{d}t = \int_y^{x+y} f'(t)\mathrm{d}t = f(x + y) - f(y)$
and that $\int_0^x f'(t)\mathrm{d}t = f(x)$
$\Rightarrow f(x + y) - f(y) < f(x)$
which completes the proof. $\qquad\square$

Then equation (3) can be proved with lemma 1 by induction.