21.12 For which one of the following values of chip thickness before the cut t_o would you expect the specific energy in machining to be the greatest: (a) 0.010 inch, (b) 0.025 inch, (c) 0.12 mm, or (d) 0.50 mm?

Answer. (c).

21.13 Which of the following cutting conditions has the strongest effect on cutting temperature: (a) feed or (b) speed?

Answer. (b).

Problems

Chip Formation and Forces in Machining

21.1 In an orthogonal cutting operation, the tool has a rake angle = 15°. The chip thickness before the cut = 0.30 mm and the cut yields a deformed chip thickness = 0.65 mm. Calculate (a) the shear plane angle and (b) the shear strain for the operation.

Solution: (a) $r = t_o/t_c$ = 0.30/0.65 = 0.4615
$\phi = \tan^{-1}(0.4615 \cos 15/(1 - 0.4615 \sin 15)) = \tan^{-1}(0.5062) = \mathbf{26.85°}$

(b) Shear strain $\gamma = \cot 26.85 + \tan (26.85 - 15) = 1.975 + 0.210 = \mathbf{2.185}$

21.2 In Problem 21.1, suppose the rake angle were changed to 0°. Assuming that the friction angle remains the same, determine (a) the shear plane angle, (b) the chip thickness, and (c) the shear strain for the operation.

Solution: From Problem 21.1, α = 15° and ϕ = 26.85°. Using the Merchant Equation, Eq. (21.16):
$\phi = 45 + \alpha/2 - \beta/2$; rearranging, $\beta = 2(45) + \alpha - 2\phi$
$\beta = 2(45) + \alpha - 2(\phi) = 90 + 15 – 2(26.85) = 51.3°$
Now, with $\alpha = 0$ and β remaining the same at 51.3°, $\phi = 45 + 0/2 – 51.3/2 = \mathbf{19.35°}$

(b) Chip thickness at $\alpha = 0$: $t_c = t_o/\tan \phi = 0.30/\tan 19.35 = \mathbf{0.854\ mm}$

(c) Shear strain $\gamma = \cot 19.35 + \tan (19.35 - 0) = 2.848 + 0.351 = \mathbf{3.199}$

21.3 In an orthogonal cutting operation, the 0.250 in wide tool has a rake angle of 5°. The lathe is set so the chip thickness before the cut is 0.010 in. After the cut, the deformed chip thickness is measured to be 0.027 in. Calculate (a) the shear plane angle and (b) the shear strain for the operation.

Solution: (a) $r = t_o/t_c$ = 0.010/0.027 = 0.3701
$\phi = \tan^{-1}(0.3701 \cos 5/(1 - 0.3701 \sin 5)) = \tan^{-1}(0.3813) = \mathbf{20.9°}$

(b) Shear strain $\gamma = \cot 20.9 + \tan (20.9 – 5) = 2.623 + 0.284 = \mathbf{2.907}$

21.4 In a turning operation, spindle speed is set to provide a cutting speed of 1.8 m/s. The feed and depth of cut of cut are 0.30 mm and 2.6 mm, respectively. The tool rake angle is 8°. After the cut, the deformed chip thickness is measured to be 0.49 mm. Determine (a) shear plane angle, (b) shear strain, and (c) material removal rate. Use the orthogonal cutting model as an approximation of the turning process.

Solution: (a) $r = t_o/t_c$ = 0.30/0.49 = 0.612
$\phi = \tan^{-1}(0.612 \cos 8/(1 – 0.612 \sin 8)) = \tan^{-1}(0.6628) = \mathbf{33.6°}$

(b) $\gamma = \cot 33.6 + \tan (33.6 - 8) = 1.509 + 0.478 = \mathbf{1.987}$

(c) R_{MR} = (1.8 m/s x 10^3 mm/m)(0.3)(2.6) = **1404 mm^3/s**

21.5 The cutting force and thrust force in an orthogonal cutting operation are 1470 N and 1589 N, respectively. The rake angle = 5°, the width of the cut = 5.0 mm, the chip thickness before the cut = 0.6, and the chip thickness ratio = 0.38. Determine (a) the shear strength of the work material and (b) the coefficient of friction in the operation.

Solution: (a) $\phi = \tan^{-1}(0.38 \cos 5/(1 - 0.38 \sin 5)) = \tan^{-1}(0.3916) = 21.38°$
$F_s = 1470 \cos 21.38 – 1589 \sin 21.38 = 789.3$ N
$A_s = (0.6)(5.0)/\sin 21.38 = 3.0/.3646 = 8.23$ mm^2
$S = 789.3/8.23 = 95.9$ N/mm^2 = **95.9 MPa**

(b) $\phi = 45 + \alpha/2 - \beta/2$; rearranging, $\beta = 2(45) + \alpha - 2\phi$
$\beta = 2(45) + \alpha - 2(\phi) = 90 + 5 – 2(21.38) = 52.24°$
$\mu = \tan 52.24 =$ **1.291**

21.6 The cutting force and thrust force have been measured in an orthogonal cutting operation to be 300 lb and 291 lb, respectively. The rake angle = 10°, width of cut = 0.200 in, chip thickness before the cut = 0.015, and chip thickness ratio = 0.4. Determine (a) the shear strength of the work material and (b) the coefficient of friction in the operation.

Solution: $\phi = \tan^{-1}(0.4 \cos 10/(1 - 0.4 \sin 10)) = \tan^{-1}(0.4233) = 22.94°$
$F_s = 300 \cos 22.94 - 291 \sin 22.94 = 162.9$ lb.
$A_s = (0.015)(0.2)/\sin 22.94 = 0.0077$ in^2
$S = 162.9/0.0077 =$ **21,167 lb/in^2**
$\beta = 2(45) + \alpha - 2(\phi) = 90 + 10 - 2(22.94) = 54.1°$
$\mu = \tan 54.1 =$ **1.38**

21.7 An orthogonal cutting operation is performed using a rake angle of 15°, chip thickness before the cut = 0.012 in and width of cut = 0.100 in. The chip thickness ratio is measured after the cut to be 0.55. Determine (a) the chip thickness after the cut, (b) shear angle, (c) friction angle, (d) coefficient of friction, and (e) shear strain.

Solution: (a) $r = t_o/t_c$, $t_c = t_o/r = 0.012/0.55 =$ **0.022 in**

(b) $\phi = \tan^{-1}(0.55 \cos 15/(1 - 0.55 \sin 15)) = \tan^{-1}(0.6194) =$ **31.8°**

(c) $\beta = 2(45) + \alpha - 2(\phi) = 90 + 15 - 2(31.8) =$ **41.5°**

(d) $\mu = \tan 41.5 =$ **0.88**

(e) $\gamma = \cot 31.8 + \tan(31.8 - 15) = 1.615 + 0.301 =$ **1.92**

21.8 The orthogonal cutting operation described in previous Problem 21.7 involves a work material whose shear strength is 40,000 lb/in^2. Based on your answers to the previous problem, compute (a) the shear force, (b) cutting force, (c) thrust force, and (d) friction force.

Solution: (a) $A_s = (0.012)(0.10)/\sin 31.8 = 0.00228$ in^2.
$F_s = A_sS = 0.00228(40{,}000) =$ **91.2 lb**

(b) $F_c = 91.2 \cos (41.5 - 15)/\cos (31.8 + 41.5 -15) =$ **155 lb**

(c) $F_t = 91.2 \sin (41.5 - 15)/\cos (31.8 + 41.5 -15) =$ **77.2 lb**

(d) $F = 155 \sin 15 - 77.2 \cos 15 =$ **115 lb**

21.9 In an orthogonal cutting operation, the rake angle = -5°, chip thickness before the cut = 0.2 mm and width of cut = 4.0 mm. The chip ratio = 0.4. Determine (a) the chip thickness after the cut, (b) shear angle, (c) friction angle, (d) coefficient of friction, and (e) shear strain.

Solution: (a) $r = t_o/t_c$, $t_c = t_o/r = 0.2/.4 =$ **0.5 mm**

(b) $\phi = \tan^{-1}(0.4 \cos(-5)/(1 - 0.4 \sin(-5))) = \tan^{-1}(0.3851) =$ **21.1°**

(c) $\beta = 2(45) + \alpha - 2(\phi) = 90 + (-5) - 2(21.8) =$ **42.9°**

(d) $\mu = \tan 42.9 =$ **0.93**

(e) $\gamma = \cot 31.8 + \tan(31.8 - 15) = 2.597 + 0.489 =$ **3.09**

21.10 The shear strength of a certain work material = 50,000 lb/in^2. An orthogonal cutting operation is performed using a tool with a rake angle = 20° at the following cutting conditions: cutting speed = 100 ft/min, chip thickness before the cut = 0.015 in, and width of cut = 0.150 in. The resulting chip thickness ratio = 0.50. Determine (a) the shear plane angle, (b) shear force, (c) cutting force and thrust force, and (d) friction force.

Solution: (a) $\phi = \tan^{-1}(0.5 \cos 20/(1 - 0.5 \sin 20)) = \tan^{-1}(0.5668) =$ **29.5°**

(b) $A_s = (0.015)(0.15)/\sin 29.5 = 0.00456$ in^2.
$F_s = A_s S = 0.00456(50{,}000) =$ **228 lb**

(c) $\beta = 2(45) + \alpha - 2(\phi) = 90 + 20 - 2(29.5) = 50.9°$
$F_c = 228 \cos (50.9 - 20)/\cos (29.5 + 50.9 - 20) =$ **397 lb**
$F_t = 228 \sin (50.9 - 20)/\cos (29.5 + 50.9 - 20) =$ **238 lb**

(d) $F = 397 \sin 20 + 238 \cos 20 =$ **359 lb**

21.11 Consider the data in Problem 21.10 except that rake angle is a variable, and its effect on the forces in parts (b), (c), and (d) is to be evaluated. (a) Using a spreadsheet calculator, compute the values of shear force, cutting force, thrust force, and friction force as a function of rake angle over a range of rake angles between the high value of 20° in Problem 21.10 and a low value of -10°. Use intervals of 5° between these limits. The chip thickness ratio decreases as rake angle is reduced and can be approximated by the following relationship: $r = 0.38 + 0.006\alpha$, where r = chip thickness and α = rake angle. (b) What observations can be made from the computed results?

Solution: (a) The author's spreadsheet calculations (using Excel) returned the following results:

α	r	F_s (lb)	F_c (lb)	F_t (lb)	F (lb)
20	0.5	228	397	238	359
15	0.47	245	435	309	411
10	0.44	265	480	399	476
5	0.41	288	531	515	559
0	0.38	317	592	667	667
-5	0.35	351	665	870	809
-10	0.32	393	754	1150	1001

(b) Observations: (1) All forces increase as rake angle decreases. (2) The change in forces with rake angle increases at an accelerating rate as rake angle is reduced. (3) Thrust force is less than cutting force at high rake angles (e.g., 20°) but increases at a faster rate than cutting force so that its value is greater at low and negative rake angles. (4) Chip thickness ratio decreases with decreasing rake angle, as indicated by the given relationship $r = 0.38 + 0.006\alpha$.

21.12 Solve previous Problem 21.10 except that the rake angle has been changed to -5° and the resulting chip thickness ratio = 0.35.

Solution: (a) $\phi = \tan^{-1}(0.35 \cos(-5)/(1 - 0.35 \sin(-5))) = \tan^{-1}(0.3384) =$ **18.7°**

(b) $A_s = (0.015)(0.15)/\sin 18.7 = 0.00702$ in^2.
$F_s = A_s S = 0.00702(50{,}000) =$ **351 lb**

(c) $\beta = 2(45) + \alpha - 2(\phi) = 90 + (-5) - 2(18.7) = 47.6°$
$F_c = 351 \cos(47.6 - (-5))/\cos(18.7 + 47.6 - (-5)) =$ **665 lb**
$F_t = 351 \sin(47.6 - (-5))/\cos(18.7 + 47.6 - (-5)) =$ **870 lb**

(d) $F = 665 \sin(-5) + 870 \cos(-5) =$ **809 lb**

21.13 A carbon steel bar with 7.64 in diameter has a tensile strength of 65,000 lb/in^2 and a shear strength of 45,000 lb/in^2. The diameter is reduced using a turning operation at a cutting speed of 400 ft/min. The feed is 0.011 in/rev and the depth of cut is 0.120 in. The rake angle on the tool in the direction of chip flow is 13°. The cutting conditions result in a chip ratio of 0.52. Using the orthogonal model as an approximation of turning, determine (a) the shear plane angle, (b) shear force, (c) cutting force and feed force, and (d) coefficient of friction between the tool and chip.

Solution: (a) $\phi = \tan^{-1}(0.52 \cos 13/(1 - 0.52 \sin 13)) = \tan^{-1}(0.5738) =$ **29.8°**

(b) $A_s = t_o w/\sin \phi = (0.011)(0.12)/\sin 29.8 = 0.00265$ in^2.
$F_s = A_s S = 0.00587(40{,}000) =$ **119.4 lb**

(c) $\beta = 2(45) + \alpha - 2(\phi) = 90 + 10 - 2(29.8) = 43.3°$
$F_c = F_s \cos(\beta - \alpha)/\cos(\phi + \beta - \alpha)$
$F_c = 264.1 \cos(43.3 - 13)/\cos(29.8 + 43.3 - 13) =$ **207 lb**
$F_t = F_s \sin(\beta - \alpha)/\cos(\phi + \beta - \alpha)$
$F_t = 264.1 \sin(43.3 - 13)/\cos(29.8 + 43.3 - 13) =$ **121 lb**
(d) $\mu = \tan \beta = \tan 43.3 =$ **0.942**

21.14 Low carbon steel having a tensile strength of 300 MPa and a shear strength of 220 MPa is cut in a turning operation with a cutting speed of 3.0 m/s. The feed is 0.20 mm/rev and the depth of cut is 3.0 mm. The rake angle of the tool is 5° in the direction of chip flow. The resulting chip ratio is 0.45. Using the orthogonal model as an approximation of turning, determine (a) the shear plane angle, (b) shear force, (c) cutting force and feed force.

Solution: (a) $\phi = \tan^{-1}(0.45 \cos 5/(1 - 0.45 \sin 5)) = \tan^{-1}(0.4666) =$ **25.0°**

(b) $A_s = t_o w/\sin \phi = (0.2)(3.0)/\sin 25.0 = 1.42$ mm^2.
$F_s = A_s S = 1.42(220) =$ **312 N**

(c) $\beta = 2(45) + \alpha - 2(\phi) = 90 + 5 - 2(25.0) = 45.0°$
$F_c = F_s \cos(\beta - \alpha)/\cos(\phi + \beta - \alpha)$
$F_c = 312 \cos(45 - 5)/\cos(25.0 + 45.0 - 5) =$ **566 N**
$F_t = F_s \sin(\beta - \alpha)/\cos(\phi + \beta - \alpha)$
$F_t = 312 \sin(45 - 5)/\cos(25.0 + 45.0 - 5) =$ **474 N**

21.15 A turning operation is made with a rake angle of 10°, a feed of 0.010 in/rev and a depth of cut = 0.100 in. The shear strength of the work material is known to be 50,000 lb/in^2, and the chip thickness ratio is measured after the cut to be 0.40. Determine the cutting force and the feed force. Use the orthogonal cutting model as an approximation of the turning process.

Solution: $\phi = \tan^{-1}(0.4 \cos 10/(1 - 0.4 \sin 10)) = \tan^{-1}(0.4233) =$ **22.9°**
$A_s = (0.010)(0.10)/\sin 22.9 = 0.00257$ in^2
$F_s = A_s S = 0.00256(50{,}000) =$ **128 lb**
$\beta = 2(45) + \alpha - 2(\phi) = 90 + 10 - 2(22.9) = 54.1°$
$F_c = 128 \cos(54.1 - 10)/\cos(22.9 + 54.1 - 10) =$ **236 lb**
$F_t = 128 \sin(54.1 - 10)/\cos(22.9 + 54.1 - 10) =$ **229 lb**

21.16 Show how Eq. (21.3) is derived from the definition of chip ratio, Eq. (21.2), and Figure 21.5(b).

Solution: Begin with the definition of the chip ratio, Eq. (21.2): $r = t_o/t_c = \sin \phi/\cos(\phi - \alpha)$

Rearranging, $r \cos(\phi - \alpha) = \sin\phi$
Using the trigonometric identity $\cos(\phi - \alpha) = \cos\phi \cos\alpha + \sin\phi \sin\alpha$
$r(\cos\phi \cos\alpha + \sin\phi \sin\alpha) = \sin\phi$
Dividing both sides by $\sin\phi$, we obtain $r\cos\alpha/\tan\phi + r\sin\alpha = 1$
$r\cos\alpha/\tan\phi = 1 - r\sin\alpha$
Rearranging, $\tan\phi = r\cos\alpha/(1 - r\sin\alpha)$ Q.E.D.

21.17 Show how Eq. (21.4) is derived from Figure 21.6.

Solution: In the figure, $\gamma = AC/BD = (AD + DC)/BD = AD/BD + DC/BD$
$AD/BD = \cot\phi$ and $DC/BD = \tan(\phi - \alpha)$
Thus, $\gamma = \cot\phi + \tan(\phi - \alpha)$ Q.E.D.

21.18 Derive the force equations for F, N, F_s, and F_n (Eqs. (21.9) through (21.12) in the text) using the force diagram of Figure 21.11.

Solution: Eq. (21.9): In Figure 23.11, construct a line starting at the intersection of $\boldsymbol{F}_t$ and $\boldsymbol{F}_c$ that is perpendicular to the friction force $\boldsymbol{F}$. The constructed line is at an angle α with $\boldsymbol{F}_c$. The vector $\boldsymbol{F}$ is divided into two line segments, one of which = $\boldsymbol{F}_c \sin\alpha$ and the other = $\boldsymbol{F}_t \cos\alpha$.
Thus, $\boldsymbol{F} = \boldsymbol{F}_c \sin\alpha + \boldsymbol{F}_t \cos\alpha$. Q.E.D.

Eq. (21.10): In Figure 23.11, translate vector $\boldsymbol{N}$ vertically upward until it coincides with the previously constructed line, whose length = $\boldsymbol{F}_c \cos\alpha$. Next, translate vector $\boldsymbol{F}_t$ to the right and downward at an angle α until its base is at the arrowhead of $\boldsymbol{F}$. $\boldsymbol{F}_t$ now makes an angle α with $\boldsymbol{F}$. The arrowhead of $\boldsymbol{F}_t$ will now be at the base of the translated base of **N**. The distance along the previously constructed line between the $\boldsymbol{F}_t$ arrowhead (base of translated $\boldsymbol{N}$ vector) and $\boldsymbol{F}$ is $\boldsymbol{F}_t \sin\alpha$.
Hence, $\boldsymbol{N} = \boldsymbol{F}_c \cos\alpha - \boldsymbol{F}_t \sin\alpha$ Q.E.D.

Eq. (21.11): In Figure 23.11, extend vector $\boldsymbol{F}_s$ in the opposite direction of its arrowhead, and from the intersection of $\boldsymbol{F}_t$ and $\boldsymbol{F}_c$ construct a line that is perpendicular to vector $\boldsymbol{F}_s$. A right triangle now exists in which $\boldsymbol{F}_c$ is the hypotenuse and the two sides are (1) the extended $\boldsymbol{F}_s$ vector and (2) the constructed line that runs between F_s and the intersection of $\boldsymbol{F}_c$ and $\boldsymbol{F}_t$. The extended $\boldsymbol{F}_s$ vector is related to $\boldsymbol{F}_c$ as $\boldsymbol{F}_c \cos\phi$. The length difference between the extended $\boldsymbol{F}_s$ vector and the original $\boldsymbol{F}_s$ vector is $\boldsymbol{F}_t \sin\phi$.
Thus $\boldsymbol{F}_s$ (original) = $\boldsymbol{F}_c \cos\phi - \boldsymbol{F}_t \sin\phi$ Q.E.D.

Eq. (21.12): In Figure 23.11, construct a line from the intersection of $\boldsymbol{F}_t$ and $\boldsymbol{F}_c$ that is perpendicular to and intersects with vector $\boldsymbol{F}_n$. Vector $\boldsymbol{F}_n$ is now divided into two line segments, one of which = $\boldsymbol{F}_t \cos\phi$ and the other = $\boldsymbol{F}_c \sin\phi$.
Hence, $\boldsymbol{F}_n = \boldsymbol{F}_c \sin\phi + \boldsymbol{F}_t \cos\phi$ Q.E.D.

Power and Energy in Machining

21.18 In a turning operation on stainless steel with hardness = 200 HB, the cutting speed = 200 m/min, feed = 0.25 mm/rev, and depth of cut = 7.5 mm. How much power will the lathe draw in performing this operation if its mechanical efficiency = 90%. Use Table 21.2 to obtain the appropriate specific energy value.

Solution: From Table 21.2, $U = 2.8$ N-m/mm^3 = 2.8 J/mm^3
$R_{MR} = vfd$ = (200 m/min)(10^3 mm/m)(0.25 mm)(7.5 mm) = 375,000 mm^3/min = 6250 mm^3/s
P_c = (6250 mm^3/s)(2.8 J/mm^3) = 17,500 J/s = 17,500 W = 17.5 kW
Accounting for mechanical efficiency, P_g = 17.5/0.90 = **19.44 kW**

21.19 In Problem 21.18, compute the lathe power requirements if feed = 0.50 mm/rev.

Solution: This is the same basic problem as the previous, except that a correction must be made for the "size effect." Using Figure 21.14, for $f = 0.50$ mm, correction factor = 0.85.
From Table 21.2, $U = 2.8$ J/mm^3. With the correction factor, $U = 2.8(0.85) = 2.38$ J/mm^3.
$R_{MR} = vfd = (200 \text{ m/min})(10^3 \text{ mm/m})(0.50 \text{ mm})(7.5 \text{ mm}) = 750{,}000 \text{ mm}^3\text{/min} = 12{,}500 \text{ mm}^3\text{/s}$
$P_c = (12{,}500 \text{ mm}^3\text{/s})(2.38 \text{ J/mm}^3) = 29{,}750 \text{ J/s} = 29{,}750 \text{ W} = 29.75 \text{ kW}$
Accounting for mechanical efficiency, $P_g = 29.75/0.90 =$ **33.06 kW**

21.20 In a turning operation on aluminum, cutting speed = 900 ft/min, feed = 0.020 in/rev, and depth of cut = 0.250 in. What horsepower is required of the drive motor, if the lathe has a mechanical efficiency = 87%? Use Table 21.2 to obtain the appropriate unit horsepower value.

Solution: From Table 21.2, $HP_u = 0.25$ hp/(in^3/min) for aluminum. Since feed is greater than 0.010 in/rev in the table, a correction factor must be applied from Figure 21.14. For $f = 0.020$ in/rev = t_o, correction factor = 0.9.
$HP_c = HP_u \text{ x } R_{MR}$, $HP_g = HP_c/E$
$R_{MR} = vfd = 900 \text{ x } 12(.020)(0.250) = 54 \text{ in}^3\text{/min}$
$HP_c = 0.9(0.25)(54) = 12.2$ hp
$HP_g = 12.2/0.87 =$ **14.0 hp**

21.21 In a turning operation on plain carbon steel whose Brinell hardness = 275 HB, the cutting speed is set at 200 m/min and depth of cut = 6.0 mm. The lathe motor is rated at 25 kW, and its mechanical efficiency = 90%. Using the appropriate specific energy value from Table 21.2, determine the maximum feed that can be set for this operation. Use of a spreadsheet calculator is recommended for the iterative calculations required in this problem.

Solution: From Table 21.2, $U = 2.8$ N-m/mm^3 = 2.8 J/mm^3
$R_{MR} = vfd = (200 \text{ m/min})(10^3 \text{ mm/m})(6 \text{ mm})f = 1200(10^3)f \text{ mm}^3\text{/min} = 20(10^3)f \text{ mm}^3\text{/s}$
Available power $P_c = P_g E = 25(10^3)(0.90) = 22.5\,(10^3) = 22{,}500\text{W} = 22{,}500$ N-m/s
Required power $P_c = (2.8 \text{ N-m/mm}^3)(20 \text{ x } 10^3)f = 56{,}000 f$ (units are N-m/s)
Setting available power = required power, $22{,}500 = 56{,}000 f$
$f = 22{,}500/56{,}000 = 0.402$ mm (this should be interpreted as mm/rev for a turning operation)
However, for this feed, correction factor in Figure 21.14 = 0.9. Thus $U = 2.8(0.90) = 2.52$ N-m/mm^3 and an iterative calculation procedure is required to match the unit power value with the feed, taking the correction factor into account.
Required $P_c = (2.52)(20 \text{ x } 10^3)f = 50{,}400 f$
Again setting available power = required power, $22{,}500 = 50{,}400 f$
$f = 22{,}500/50{,}400 = 0.446$ mm/rev
One more iteration using the correction factor yields a value around ***f* = 0.45 mm/rev**.

The author's spreadsheet calculations (using Excel) returned a value closer to ***f* = 0.46 mm/rev**. However, whether a spreadsheet is used or not, the difficulty that remains is reading the values of the feed and the correction factor in Figure 21.14.

21.22 A turning operation is to be performed on a 20 hp lathe that has an 87% efficiency rating. The roughing cut is made on alloy steel whose hardness is in the range 325 to 335 HB. The cutting speed is 375 ft/min, feed is 0.030 in/rev, and depth of cut is 0.150 in. Based on these values, can the job be performed on the 20 hp lathe? Use Table 21.2 to obtain the appropriate unit horsepower value.

Solution: From Table 21.2, $HP_u = 1.3$ hp/(in^3/min)
Since the uncut chip thickness (0.030 in) is different from the tabular value of 0.010, a correction factor must be applied. From Figure 21.14, the correction factor is 0.8. Therefore, the corrected HP_u $= 0.8*1.3 = 1.04$ hp/(in^3/min)
$R_{MR} = vfd = 375 \text{ ft/min}(12 \text{ in/ft})(0.03 \text{ in})(0.150 \text{ in}) = 20.25 \text{ in}^3\text{/min}$
$HP_c = (20.25 \text{ in}^3\text{/min})(1.04 \text{ hp/(in}^3\text{/min)}) =$ **21.06 hp** required.

At efficiency E = 87%, available horsepower = 0.87(20) = **17.4 hp**
Since required horsepower exceeds available horsepower, the job cannot be accomplished on the 20 hp lathe, at least not at the specified cutting speed of 375 ft/min.

21.23 Suppose the cutting speed in Problems 21.7 and 21.8 is 200 ft/min. From your answers to those problems, find (a) the horsepower consumed in the operation, (b) metal removal rate in in^3/min, (c) unit horsepower (hp-min/in^3), and (d) the specific energy (in-lb/in^3).

Solution: (a) From Problem 21.8, F_c = 155 lb. HP_c = 155(200)/33,000 = **0.94 hp**

(b) $R_{MR} = vfd$ = (200 x 12)(0.012)(0.100) = **2.88 in^3/min**

(c) HP_u = 0.94/2.88 = **0.326 hp/(in^3/min)**

(d) U = 155(200)/2.88 = **10,764 ft-lb/in^3 = 129,167 in-lb/in^3**

21.24 For Problem 21.12, the lathe has a mechanical efficiency = 0.83. Determine (a) the horsepower consumed by the turning operation; (b) horsepower that must be generated by the lathe; (c) unit horsepower and specific energy for the work material in this operation.

Solution: (a) From Problem 21.12, F_c = 207 lb.
$HP_c = F_cv$/33,000 = 207(400)/33,000 = **2.51 hp**

(b) $HP_g = HP_c$/E = 2.51/0.83 = **3.02 hp**

(c) R_{MR} = 12 vfd = (400 x 12)(0.011)(0.120) = **6.336 in^3/min**
$HP_u = HP_c/R_{MR}$ = 2.51/6.336 = **0.396 hp/(in^3/min)**
$U = F_cv/R_{MR}$ = 207(400 x 12)/6.336 = **157,000 in-lb/in^3**

21.25 In a turning operation on low carbon steel (175 BHN), cutting speed = 400 ft/min, feed = 0.010 in/rev, and depth of cut = 0.075 in. The lathe has a mechanical efficiency = 0.85. Based on the unit horsepower values in Table 21.2, determine (a) the horsepower consumed by the turning operation and (b) the horsepower that must be generated by the lathe.

Solution: (a) From Table 21.2, HP_u = 0.6 hp/(in^3/min) for low carbon steel.
$HP_c = HP_u$ x R_{MR}
$R_{MR} = vfd$ = 400 x 12(.010)(0.075) = 3.6 in^3/min
HP_c = 0.6(3.6) = **2.16 hp**

(b) HP_g = 2.16/0.85 = **2.54 hp**

21.26 Solve Problem 21.25 except that the feed = 0.0075 in/rev and the work material is stainless steel (Brinell Hardness = 240 HB).

Solution: (a) From Table 21.2, HP_u = 1.0 hp/(in^3/min) for stainless steel. Since feed is lower than 0.010 in/rev in the table, a correction factor must be applied from Figure 21.14. For f = 0.0075 in/rev = t_o, correction factor = 1.1.
$HP_c = HP_u$ x R_{MR}
R_{MR} = 400 x 12(0.0075)(0.12) = 4.32 in^3/min
HP_c = 1.1(1.0)(4.32) = **4.75 hp**

(b) HP_g = 5.01/0.83 = **5.73 hp**

21.27 A turning operation is carried out on aluminum (100 BHN). Cutting speed = 5.6 m/s, feed = 0.25 mm/rev, and depth of cut = 2.0 mm. The lathe has a mechanical efficiency = 0.85. Based on the specific energy values in Table 21.2, determine (a) the cutting power and (b) gross power in the turning operation, in Watts.

Solution: (a) From Table 21.2, U = 0.7 N-m/mm^3 for aluminum.
$R_{MR} = vfd$ = 5.6(10^3)(.25)(2.0) = 2.8(10^3) mm^3/s.

$P_c = U R_{MR} = 0.7(2.8)(10^3) = 1.96(10^3)$ N-m/s = **1960 W**

(b) Gross power $P_g = 1960/0.85 =$ **2306 W**

21.28 Solve Problem 21.27 but with the following changes: cutting speed = 1.3 m/s, feed = 0.75 mm/rev, and depth = 4.0 mm. Note that although the power used in this operation is only about 10% greater than in the previous problem, the metal removal rate is about 40% greater.

Solution: (a) From Table 21.2, $U = 0.7$ N-m/mm^3 for aluminum. Since feed is greater than 0.25 mm/rev in the table, a correction factor must be applied from Figure 21.14. For $f = 0.75$ mm/rev = t_o, correction factor = 0.80.
$R_{MR} = vfd = 1.3(10^3)(.75)(4.0) = 3.9(10^3)$ mm^3/s.
$P_c = U R_{MR} = 0.8(0.7)(3.9)(10^3) = 2.184(10^3)$ N-m/s = **2184 W**

(b) Gross power $P_g = 2184/0.85 =$ **2569 W**

21.29 A turning operation is performed on an engine lathe using a tool with zero rake angle in the direction of chip flow. The work material is an alloy steel with hardness = 325 Brinell hardness. The feed is 0.015 in/rev, depth of cut is 0.125 in and cutting speed is 300 ft/min. After the cut, the chip thickness ratio is measured to be 0.45. (a) Using the appropriate value of specific energy from Table 21.2, compute the horsepower at the drive motor, if the lathe has an efficiency = 85%. (b) Based on horsepower, compute your best estimate of the cutting force for this turning operation. Use the orthogonal cutting model as an approximation of the turning process.

Solution: (a) From Table 21.2, $U = P_u = 520{,}000$ in-lb/in^3 for alloy steel of the specified hardness. Since feed is greater than 0.010 in/rev in the table, a correction factor must be applied from Figure 21.14. For $f = 0.015$ in/rev = t_o, correction factor = 0.95. Thus,
$U = 520{,}000(0.95) = 494{,}000$ in-lb/in^3 = 41,167 ft-lb/in^3.
$R_{MR} = 300 \times 12(.015)(0.125) = 6.75$ in^3/min
$P_c = U R_{MR} = 41{,}167(6.75) = 277{,}875$ ft-lb/min
$HP_c = 277{,}875/33{,}000 = 8.42$ hp
$HP_g = 8.42/0.85 =$ **9.9 hp**

(b) $HP_c = vF_c/33{,}000$. Rearranging, $F_c = 33{,}000\ (HP_c/v) = 33{,}000(8.42/300) =$ **926 lb.**

Check: Use unit horsepower from Table 21.2 rather than specific energy. $HP_u = 1.3$ hp/(in^3/min). Applying the correction factor correction factor = 0.95, $HP_u = 1.235$ hp/(in^3/min).
$R_{MR} = 300 \times 12(.015)(0.125) = 6.75$ in^3/min, same as before
$HP_c = 1.235(6.75) = 8.34$ hp
$HP_g = 8.34/0.85 =$ **9.8 hp**

(b) $F_c = 33{,}000\ (8.3/300) =$ **913 lb.**

21.30 A lathe performs a turning operation on a workpiece of 6.0 in diameter. The shear strength of the work is 40,000 lb/in^2 and the tensile strength is 60,000 lb/in^2. The rake angle of the tool is 6°. The cutting speed = 700 ft/min, feed = 0.015 in/rev, and depth = 0.090 in. The chip thickness after the cut is 0.025 in. Determine (a) the horsepower required in the operation, (b) unit horsepower for this material under these conditions, and (c) unit horsepower as it would be listed in Table 21.2 for a t_o of 0.010 in. Use the orthogonal cutting model as an approximation of the turning process.

Solution: (a) Must find F_c and v to determine HP.
$r = 0.015/0.025 = 0.6$
$\phi = \tan^{-1}(0.6 \cos 6/(1 - 0.6 \sin 6)) = \tan^{-1}(0.6366) = 32.5°$
$\beta = 2(45) + \alpha - 2(\phi) = 90 + 6 - 2(32.5) = 31.0°$
$A_s = t_o w/\sin \phi = (0.015)(0.09)/\sin 32.5 = 0.00251$ in^2
$F_s = SA_s = 40{,}000(0.00251) = 101$ lb.
$F_c = F_s \cos(\beta - \alpha)/\cos(\phi + \beta - \alpha)$

F_c = 101 cos(31 - 6)/cos(32.5 + 31.0 – 6) = 170 lb.
$HP_c = F_c v$/33,000 = 170(700)/33,000 = **3.61 hp.**

(b) R_{MR} = 700 x 12(0.0075)(0.075) = 11.3 in^3/min
$HP_u = HP_c / R_{MR}$ = 3.61/11.3 = **0.319 hp/(in^3/min)**

(c) Correction factor = 0.85 from Fig. 21.14 to account for the fact that f = 0.015 in/rev instead of 0.010 in/rev. Taking this correction factor into account, HP_u = 0.375/0.85 = **0.441 hp/(in^3/min)** as it would appear in Table 21.2 for a feed (t_o) = 0.010 in/rev.

21.31 In a turning operation on an aluminum alloy workpiece, the feed = 0.020 in/rev, and depth of cut = 0.250 in. The motor horsepower of the lathe is 20 hp and it has a mechanical efficiency = 92%. The unit horsepower value = 0.25 hp/(in^3/min) for this aluminum grade. What is the maximum cutting speed that can be used on this job?

Solution: From Table 21.3, HP_u = 0.25 hp/(in^3/min) for aluminum. Since feed is greater than 0.010 in/rev in the table, a correction factor must be applied from Figure 21.14. For f = 0.020 in/rev = t_o, correction factor = 0.9.
$HP_c = HP_u$ x R_{MR}, $HP_g = HP_c/E$
$R_{MR} = vfd = 12v(.020)(0.250) = 0.06\ v$ in^3/min
$HP_c = 0.9(0.25)(0.06\ v) = 0.0135\ v$ hp
$HP_g = 0.0135\ v/0.92 = 0.014674\ v = 20$ hp
v = 20/0.014674 = 1363 ft/min

Cutting Temperature

21.32 Orthogonal cutting is performed on a metal whose mass specific heat = 1.0 J/g-C, density = 2.9 g/cm^3, and thermal diffusivity = 0.8 cm^2/s. The cutting speed is 4.5 m/s, uncut chip thickness is 0.25 mm, and width of cut is 2.2 mm. The cutting force is measured at 1170 N. Using Cook's equation, determine the cutting temperature if the ambient temperature = 22°C.

Solution: ρC = (2.9 g/cm^3)(1.0 J/g-°C) = 2.90 J/cm^3-°C = (2.90x10^{-3}) J/mm^3-°C
K = 0.8 cm^2/s = 80 mm^2/s
$U = F_c v/R_{MR}$ = 1170 N x 4.5 m/s/(4500 mm/s x 0.25 mm x 2.2 mm) = 2.127 N-m/mm^3
$T = 0.4U/(\rho C)$ x $(vt_o/K)^{0.333}$
T = 22 + (0.4 x 2.127 N-m/mm^3/(2.90x10^{-3}) J/mm^3-C) [4500 mm/s x 0.25 mm/80 mm^2/s]$^{0.333}$
T = 22 + (0.2934 x 10^3 C)(14.06)$^{.333}$ = 22 + 293.4(2.41) = 22° + 707° = **729°C**

21.33 Consider a turning operation performed on steel whose hardness = 225 HB at a speed = 3.0 m/s, feed = 0.25 mm, and depth = 4.0 mm. Using values of thermal properties found in the tables and definitions of Section 4.1 and the appropriate specific energy value from Table 21.2, compute an estimate of cutting temperature using the Cook equation. Assume ambient temperature = 20°C.

Solution: From Table 21.2, U = 2.2 N-m/mm^3 = 2.2 J/mm^3
From Table 4.1, ρ = 7.87 g/cm^3 = 7.87(10^{-3}) g/mm^3
From Table 4.1, C = 0.11 Cal/g-°C. From note "a" at the bottom of the table, 1 cal = 4.186 J.
Thus, C = 0.11(4.186) = 0.460 J/ g-°C
ρC = (7.87 g/cm^3)(0.46 J/g-°C) = 3.62(10^{-3}) J/mm^3-°C
From Table 4.2, thermal conductivity k = 0.046 J/s-mm-°C
From Eq. (4.3), thermal diffusivity $K = k/\rho C$
K = 0.046 J/s-mm-°C /[(7.87 x 10^{-3} g/mm^3)(0.46 J/g-°C)] = 12.7 mm^2/s
Using Cook's equation, $t_o = f$ = 0.25 mm
T = (0.4(2.2)/3.62(10^{-3}))[3(10^3)(0.25)/12.7]$^{0.333}$ = 0.2428(10^3)(59.06)$^{0.333}$
= 242.8(3.89) = 944.4 C°
Final temperature, taking ambient temperature in account T = 20 + 944 = **964°C**

21.34 An orthogonal cutting operation is performed on a certain metal whose volumetric specific heat = 110 in-lb/in^3-F, and thermal diffusivity = 0.140 in^2/sec. The cutting speed = 350 ft/min, chip thickness before the cut = 0.008 in, and width of cut = 0.100 in. The cutting force is measured at 200 lb. Using Cook's equation, determine the cutting temperature if the ambient temperature = 70°F.

Solution: v = 350 ft/min x 12 in/ft/60 sec/min = 70 in/sec.
$U = F_c v/vt_o w$ = 200(70)/(70 x 0.008 x 0.100) = 250,000 in-lb/in^3.
$T = 70 + (0.4U/\rho C)(vt_o/K)^{0.333}$ =
$T = 70 + (0.4 \times 250{,}000/110)[70 \times 0.008/0.14]^{0.333} = 70 + (909)(4)^{0.333} = 70 + 1436 =$ **1506°F**

21.35 It is desired to estimate the cutting temperature for a certain alloy steel whose hardness = 240 Brinell. Use the appropriate value of specific energy from Table 21.2 and compute the cutting temperature by means of the Cook equation for a turning operation in which the cutting speed is 500 ft/min, feed is 0.005 in/rev, and depth of cut is 0.070 in. The work material has a volumetric specific heat of 210 in lb/in^3-F and a thermal diffusivity of 0.16 in^2/sec. Assume ambient temperature = 88°F.

Solution: From Table 21.2, U for alloy steel (310 BHN) = 320,000 in-lb/in^3.
Since f = 0.005 in/rev, correction factor = 1.25.
Therefore U = 320,000(1.25) = 400,000 in-lb/in^3.
v = 500 ft/min x 12 in/ft/60 sec/min = 100 in/sec.
$T = T_a + (0.4U/\rho C)(vt_o/K)^{0.333} = 88 + (0.4 \times 400{,}000/210)(100 \times 0.005/0.16)^{0.333}$
$= 88 + (762)(3.125)^{0.333} = 88 + 1113 =$ **1201°F**

21.36 An orthogonal machining operation removes metal at 1.8 in^3/min. The cutting force in the process = 300 lb. The work material has a thermal diffusivity = 0.18 in^2/sec and a volumetric specific heat = 124 in-lb/in^3-F. If the feed $f = t_o$ = 0.010 in and width of cut = 0.100 in, use the Cook formula to compute the cutting temperature in the operation given that ambient temperature = 70°F.

Solution: $R_{MR} = vt_o w$, $v = R_{MR}/t_o w$ = 1.8/(0.01 x 0.100) = 1800 in/min = 30 in/sec
$U = F_c v/vt_o w$ = 300(30)/(30 x 0.010 x 0.100) = 300,000 in-lb/in^3.
$T = 70 + (0.4U/\rho C)(vt_o/K)^{0.333} = 70 + (0.4 \times 300{,}000/124)(30 \times 0.010/0.18)^{0.333}$
$= 70 + (968)(1.667)^{0.333} = 70 + 1147 =$ **1217°F**

21.37 A turning operation uses a cutting speed = 200 m/min, feed = 0.25 mm/rev, and depth of cut = 4.00 mm. The thermal diffusivity of the work material = 20 mm^2/s and the volumetric specific heat = 3.5 (10^{-3}) J/mm^3-C. If the temperature increase above ambient temperature (20°F) is measured by a tool-chip thermocouple to be 700°C, determine the specific energy for the work material in this operation.

Solution: Rearranging the Cook equation, $U = T(\rho C/0.4)(K/vt_o)^{0.333}$
$U = (700 - 20)(3.5 \times 10^{-3}/0.4)(20/\{(200/60)(10^3)(0.25)\})^{0.333}$
$U = 680(8.75 \times 10^{-3})(0.024)^{0.333} = 5.95(0.2888) =$ **1.72 N-m/mm^3**

21.38 During a turning operation, a tool-chip thermocouple was used to measure cutting temperature. The following temperature data were collected during the cuts at three different cutting speeds (feed and depth were held constant): (1) v = 100 m/min, T = 505°C, (2) v = 130 m/min, T = 552°C, (3) v = 160 m/min, T = 592°C. Determine an equation for temperature as a function of cutting speed that is in the form of the Trigger equation, Eq. (21.23).

Solution: Trigger equation $T = Kv^m$
Choose points (1) and (3) and solve simultaneous equations using $T = Kv^m$ as the model.
(1) $505 = K(100)^m$ and (3) $592 = K(160)^m$
(1) $\ln 505 = \ln K + m \ln 100$ and (3) $\ln 592 = \ln K + m \ln 160$

Combining (1) and (3): ln 505 - m ln 100 = ln 592 - m ln 160
6.2246 – 4.6052 m = 6.3835 – 5.0752 m
0.47 m = 0.1589 $\quad$ $\boldsymbol{m = 0.338}$
(1) $K = 505/100^{0.338} = 505/4.744 = 106.44$
(2) $K = 592/160^{0.338} = 592/5.561 = 106.45$ $\quad$ Use $\boldsymbol{K = 106.45}$

Check equation with data point (2): $T = 106.45(130)^{0.338} = 551.87$°C (pretty close to the given value of 552°C).