16.9 Which of the following processes combines pressing and sintering of the metal powders (three best answers): (a) hot isostatic pressing, (b) hot pressing, (c) metal injection molding, (d) pressing and sintering, and (e) spark sintering?

Answer. (a), (b), and (e).

16.10 Which of the following design features would be difficult or impossible to achieve by conventional pressing and sintering (three best answers): (a) outside rounded corners, (b) side holes, (c) threaded holes, (d) vertical stepped holes, and (e) vertical wall thickness of 1/8 inch (3 mm)?

Answer. (a), (b), and (c).

Problems

Characterization of Engineering Powders

16.1 A screen with 325 mesh count has wires with a diameter of 0.001377 in. Determine (a) the maximum particle size that will pass through the wire mesh and (b) the proportion of open space in the screen.

Solution: (a) By Eq. (16.1), particle size $PS = 1/MC - t_w = 1/325 - 0.001377$
$= 0.003077 - 0.001377 = \mathbf{0.00170\ in}$

(b) There are 325 x 325 = 105,625 openings in one square inch of the mesh. By inference from part (a), each opening is 0.00170 inch on a side, thus each opening is $(0.0017)^2 = 0.000002889$ in^2. The total open area in one square inch of mesh = 105,625(0.000002889 in^2) = 0.30523 in^2. This is total open space. Therefore, the percent open space in one square inch of mesh = **30.523%.**

16.2 A screen with 10 mesh count has wires with a diameter of 0.0213 in. Determine (a) the maximum particle size that will pass through the wire mesh and (b) the proportion of open space in the screen.

Solution: (a) By Eq. (16.1), particle size $PS = 1/MC - t_w = 1/10 - 0.0213 = \mathbf{0.0787\ in.}$

(b) There are 10 x 10 = 100 openings in one square inch of the mesh. By inference from part (a), each opening is 0.0787 inch on a side, thus each opening is $(0.0787)^2 = 0.00619$ in^2. The total open area in one square inch of mesh = 100(0.00619 in^2) = 0.619 in^2. This is total open space. Therefore, the percent open space in one square inch of mesh = **61.9%.**

16.3 What is the aspect ratio of a cubic particle shape?

Solution: The aspect ratio is the ratio of the maximum dimension to the minimum dimension of the particle shape. The minimum dimension is the edge of any face of the cube; call it L. The maximum dimension is the cube diagonal, which is given by $(L^2 + L^2 + L^2)^{0.5} = (3\,L^2)^{0.5} = (3)^{0.5}\,L = 1.732\,L$. Thus, the **aspect ratio = 1.732:1.**

16.4 Determine the shape factor for metallic particles of the following ideal shapes: (a) sphere, (b) cubic, (c) cylindrical with length-to-diameter ratio of 1:1, (d) cylindrical with length-to-diameter ratio of 2:1, and (e) a disk-shaped flake whose thickness-to-diameter ratio is 1:10.

Solution: (a) Sphere: $K_s = 6.0$ as shown in the text, Eq. (16.5).

(b) Cube: Let L = edge of one face. For a cube, $A = 6L^2$ and $V = L^3$
Find diameter D of a sphere of equivalent volume.
$V = \pi D^3/6 = L^3$
$D^3 = 6L^3/\pi = 1.90986\,L^3$
$D = (1.90986\,L^3)^{0.333} = 1.2407\,L$
$K_s = AD/V = (6L^2)(1.2407\,L)/L^3 = \mathbf{7.444}$

(c) Cylinder with $L/D = 1.0$. For this cylinder shape, $L = D$. Thus, $A = 2\pi D^2/4 + \pi DL = 0.5\pi L^2 + \pi L^2 = 1.5\pi L^2$, and $V = (\pi D^2/4)L = 0.25\pi L^3$.

Find diameter D of a sphere of equivalent volume.
$V = \pi D^3/6 = 0.25\pi L^3$
$D^3 = 6(0.25\pi L^3)/\pi = 1.5L^3$
$D = (1.5\,L^3)^{0.333} = 1.1447$ L
$K_s = AD/V = (1.5\pi L^2)(1.1447\,L)/0.25\pi L^3 =$ **6.868**

(d) Cylinder with $L/D = 2.0$. For this cylinder shape, $0.5L = D$. Thus, $A = 2\pi D^2/4 + \pi DL = 0.5\pi(0.5L)^2 + \pi(0.5L)L = 0.125\pi L^2 + 0.5\pi L^2 = 0.625\pi L^2$, and $V = (\pi D^2/4)L = 0.25\pi(0.5L)^2\,L = 0.0625\pi L^3$
Find diameter D of a sphere of equivalent volume.
$V = \pi D^3/6 = 0.0625\pi L^3$
$D^3 = 6(0.0625\pi L^3)/\pi = 0.375L^3$
$D = (0.375\,L^3)^{0.333} = 0.721\,L$
$K_s = A\,D/V = (0.625\pi L^2)(0.721\,L)/0.0625\pi L^3 =$ **7.211**

(e) Disk with $L/D = 0.10$. For this shape, $10L = D$. Thus, $A = 2\pi D^2/4 + \pi DL = 0.5\pi(10L)^2 + \pi(10L)L = 50\pi L^2 + 10\pi L^2 = 60\pi L^2$, and $V = (\pi D^2/4)L = 0.25\pi(10L)^2\,L = 25\pi L^3$
Find diameter D of a sphere of equivalent volume.
$V = \pi D^3/6 = 25\pi L^3$
$D^3 = 6(25\pi L^3)/\pi = 150L^3$
$D = (150\,L^3)^{0.333} = 5.313\,L$
$K_s = A\,D/V = (60\pi L^2)(5.313\,L)/25\pi L^3 =$ **12.75**

16.5 A pile of iron powder weighs 2 lb. The particles are spherical in shape and all have the same diameter of 0.002 in. (a) Determine the total surface area of all the particles in the pile. (b) If the packing factor = 0.6, determine the volume taken by the pile. Note: the density of iron = 0.284 lb/in^3.

Solution: (a) For a spherical particle of $D = 0.002$ in, $V = \pi D^3/6 = \pi(0.002)^3/6$
$= 0.00000000418 = 4.18 \times 10^{-9}$ in^3/particle
Weight per particle $W = \rho V = 0.284(4.18 \times 10^{-9}$ in$^3) = 1.19 \times 10^{-9}$ lb/particle
Number of particles in 2 lb = $2.0/(1.19 \times 10^{-9}) = 1.681 \times 10^9$
$A = \pi D^2 = \pi(0.002)^2 = 0.00001256$ in^2 = 12.56×10^{-6} in^2
Total surface area = $(1.681 \times 10^9)(12.56 \times 10^{-6})$ = **21.116 x 10^3 in^2**

(b) With a packing factor of 0.6, the total volume taken up by the pile = (2.0/0.284)/0.6 = **11.74 in^3**

16.6 Solve Problem 16.5, except that the diameter of the particles is 0.004 in. Assume the same packing factor.

Solution: (a) For a spherical particle of $D = 0.004$ in, $V = \pi D^3/6 = \pi(0.004)^3/6$
$= 0.00000003351 = 33.51 \times 10^{-9}$ in^3/particle
Weight per particle $W = \rho V = 0.284(33.51 \times 10^{-9}$ in$^3) = 9.516 \times 10^{-9}$ lb/particle
Number of particles in 2 lb = $2.0/(9.516 \times 10^{-9}) = 0.2102 \times 10^9$
$A = \pi D^2 = \pi(0.004)^2 = 0.00005027$ in^2 = 50.27×10^{-6} in^2
Total surface area = $(0.2102 \times 10^9)(50.27 \times 10^{-6})$ = **10.565 x 10^3 in^2**

(b) With a packing factor of 0.6, the total volume taken up by the pile = (2.0/0.284)/0.6 = **11.74 in^3**

16.7 Suppose in Problem 16.5 that the average particle diameter = 0.002 in; however, the sizes vary, forming a statistical distribution as follows: 25% of the particles by weight are 0.001 in, 50% are 0.002 in, and 25% are 0.003 in. Given this distribution, what is the total surface area of all the particles in the pile?

Solution: For a spherical particle of $D = 0.001$ in, $V = \pi D^3/6 = \pi(0.001)^3/6$
$= 0.5236 \times 10^{-9}$ in^3/particle

Weight per particle $W = \rho V = 0.284(0.5236 \times 10^{-9} \text{ in}^3) = 0.1487 \times 10^{-9}$ lb/particle
Particles of size $D = 0.001$ in constitute 25% of total 2 lb. = 0.5 lb
Number of particles in 0.5 lb = $0.5/(0.1487 \times 10^{-9}) = 3.362 \times 10^9$
$A = \pi D^2 = \pi(0.001)^2 = 3.142 \times 10^{-6}$ in^2/particle
Total surface area of particles of $D = 0.001$ in = $(3.362 \times 10^9)(3.142 \times 10^{-6}) = 10.563 \times 10^3$ in^2

For a spherical particle of $D = 0.002$ in, $V = \pi(0.002)^3/6 = 4.18 \times 10^{-9}$ in^3/particle
Weight per particle $W = \rho V = 0.284(4.18 \times 10^{-9} \text{ in}^3) = 1.19 \times 10^{-9}$ lb/particle
Particles of size $D = 0.002$ in constitute 50% of total 2 lb. = 1.0 lb
Number of particles in 1 lb = $1.0/(1.19 \times 10^{-9}) = 0.8406 \times 10^9$
$A = \pi D^2 = \pi(0.002)^2 = 12.56 \times 10^{-6}$ in^2
Total surface area of particles of $D = 0.002$ in = $(0.8406 \times 10^9)(12.566 \times 10^{-6}) = 10.563 \times 10^3$ in^2

For a spherical particle of $D = 0.003$ in, $V = \pi(0.003)^3/6 = 14.137 \times 10^{-9}$ in^3/particle
Weight per particle $W = \rho V = 0.284(14.137 \times 10^{-9} \text{ in}^3) = 4.015 \times 10^{-9}$ lb/particle
Particles of size $D = 0.003$ in constitute 25% of total 2 lb. = 0.5 lb
Number of particles in 0.5 lb = $0.5/(4.015 \times 10^{-9}) = 0.124 \times 10^9$
$A = \pi D^2 = \pi(0.003)^2 = 28.274 \times 10^{-6}$ in^2
Total surface area of particles of $D = 0.003$ in = $(0.124 \times 10^9)(28.274 \times 10^{-6}) = 3.506 \times 10^3$ in^2

Total surface area of all particles = $10.563 \times 10^3 + 10.563 \times 10^3 + 3.506 \times 10^3 = 24.632 \times 10^3$ in^2.

16.8 A solid cube of copper with each side = 1.0 ft is converted into metallic powders of spherical shape by gas atomization. What is the percentage increase in total surface area if the diameter of each particle is 0.004 in (assume that all particles are the same size)?

Solution: Area of initial cube $A = 6(1 \text{ ft})^2 = 6 \text{ ft}^2 = 864$ in^2
Volume of cube $V = (1 \text{ ft})^3 = 1728$ in^3
Surface area of a spherical particle of $D = 0.004$ in is $A = \pi D^2 = \pi(0.004)^2$
$= 50.265 \times 10^{-6}$ in^3/particle
Volume of a spherical particle of $D = 0.004$ in is $V = \pi D^3/6 = \pi(0.004)^3/6$
$= 33.51 \times 10^{-9}$ in^3/particle
Number of particles in 1 ft^3 = $1728/33.51 \times 10^{-9} = 51.567 \times 10^9$
Total surface area = $(51.567 \times 10^9)(50.265 \times 10^{-6} \text{ in}^3) = 2{,}592 \times 10^3 = 2{,}592{,}000$ in^2
Percent increase = 100(2,592,000 - 864)/864 = **299,900%**

16.9 A solid cube of aluminum with each side = 1.0 m is converted into metallic powders of spherical shape by gas atomization. How much total surface area is added by the process if the diameter of each particle is 100 microns (assume that all particles are the same size)?

Solution: Area of starting cube $A = 6(1 \text{ m})^2 = 6$ m^2
Volume of starting cube $V = (1 \text{ m})^3 = 1$ m^3
$D = 100\ \mu\text{m} = 0.1 \text{ mm} = 0.1 \times 10^{-3}$ m
Surface area of a sphere of $D = 0.1 \times 10^{-3}$ m is $A = \pi D^2 = \pi(0.1 \times 10^{-3})^2$
$= 3.142 \times 10^{-8}$ m^3/particle
Volume of a sphere of $D = 0.1 \times 10^{-3}$ m is $V = \pi D^3/6 = \pi(0.1 \times 10^{-3})^3/6$
$= 0.5236 \times 10^{-12}$ m^3/particle
Number of particles in 1 m^3 = $1.0/0.5236 \times 10^{-12} = 1.91 \times 10^{12}$
Total surface area = $(1.91 \times 10^{12})(0.5236 \times 10^{-12} \text{ m}^3) = 5.9958 \times 10^4 = 59{,}958$ m^2
Added surface area = 59,958 - 6 = **59,952 m^2**

16.10 Given a large volume of metallic powders, all of which are perfectly spherical and having the same exact diameter, what is the maximum possible packing factor that the powders can take?

Solution: The maximum packing factor is achieved when the spherical particles are arranged as a face-centered cubic unit cell, similar to the atomic structure of FCC metals; see Figure 2.8(b). The unit cell of the FCC structure contains 8 spheres at the corners of the cube and 6 spheres on each face. Our approach to determine the packing factor will consist of: (1) finding the volume of the spheres and portions thereof that are contained in the cell, and (2) finding the volume of the unit cell cube. The ratio of (1) over (2) is the packing factor.

(1) Volume of whole and/or partial spheres contained in the unit cell. The unit cell contains 6 half spheres in the faces of the cube and 8 one-eighth spheres in corners. The equivalent number of whole spheres = 6(.5) + 8(.125) = 4 spheres. Volume of 4 spheres = $4\pi D^3/6 = 2.0944\ D^3$ where D = diameter of a sphere.

(2) Volume of the cube of one unit cell. Consider that the diagonal of any face of the unit cell contains one full diameter (the sphere in the center of the cube face) and two half diameters (the spheres at the corners of the face). Thus, the diagonal of the cube face = $2D$. Accordingly, the face is a square with each edge = $D\sqrt{2} = 1.414D$. The volume of the unit cell is therefore $(1.414D)^3 = 2.8284\ D^3$.

The packing factor = 2.0944/2.8284 = **0.7405 = 74.05%**

Compaction and Design Considerations

16.11 In a certain pressing operation, the metallic powder fed into the open die has a packing factor of 0.5. The pressing operation reduces the powders to 2/3 of their starting volume. In the subsequent sintering operation, shrinkage amounts to 10% on a volume basis. Given that these are the only factors that affect the structure of the finished part, determine its final porosity.

Solution: Packing factor = bulk density / true density
Density = (specific volume)$^{-1}$
Packing factor = true specific volume / bulk specific volume
Pressing reduces bulk specific volume to 2/3 = 0.667
Sintering further reduces the bulk specific volume to 0.90 of value after pressing.
Let true specific volume = 1.0
Thus for a packing factor of 0.5, bulk specific volume = 2.0.
Packing factor after pressing and sintering = 1.0/(2.0 x .667 x .90) = 1.0/1.2 = 0.833
By Eq. (18.7), porosity = 1 - 0.833 = **0.167**

16.12 A bearing of simple geometry is to be pressed out of bronze powders, using a compacting pressure of 207 MPa. The outside diameter = 44 mm, the inside diameter = 22 mm, and the length of the bearing = 25 mm. What is the required press tonnage to perform this operation?

Solution: Projected area of part $A_p = 0.25\pi(D_o^2 - D_i^2) = 0.25\pi(44^2 - 22^2) = 1140.4$ mm^2
$F = A_p p_c = 1140.4(207) =$ **236,062 N**

16.13 The part shown in Figure P16.13 is to be pressed of iron powders using a compaction pressure of 75,000 lb/in^2. Dimensions are inches. Determine (a) the most appropriate pressing direction, (b) the required press tonnage to perform this operation, and (c) the final weight of the part if the porosity is 10%. Assume shrinkage during sintering can be neglected.

Solution: (a) Most appropriate pressing direction is parallel to the part axis.

(b) Press tonnage $F = A_p p_c$
Projected area of part $A_p = 0.25\pi(D_o^2 - D_i^2) = 0.25\pi(2.8^2 - 0.875^2) = 5.556$ in^2
$F = A_p p_c = 5.556(75{,}000) =$ **416,715 lb = 208 tons.**

(c) $V = 0.25\pi(2.8^2 - 0.875^2)(0.5) + 0.25\pi(2.8^2 - 1.5^2)(1.25 - 0.5) = 0.25\pi(3.5372 + 4.1925)$
$= 6.071$ in^3

From Table 4.1, density of iron $\rho = 0.284$ lb/in^3.
At 10% porosity, part weight $W = 6.071(0.284)(0.90) =$ **1.55 lb**.

16.14 For each of the four part drawings in Figure P16.14, indicate which PM class the parts belong to, whether the part must be pressed from one or two directions, and how many levels of press control will be required? Dimensions are mm.

Solution: (a) Class II, 2 directions because of axial thickness, one level of press control.

(b) Class I, one direction part is relatively thin, one level of press control.

(c) Class IV, 2 directions of pressing, 3 levels of press control required.

(d) Class IV, 2 directions of pressing, 4 or 5 levels of press control due to multiple steps in part design.