

10 FUNDAMENTALS OF METAL CASTING

Review Questions

10.1 Identify some of the important advantages of shape-casting processes.

Answer. Advantages include (1) complex part geometries are possible; (2) some casting operations are net shape processes, meaning that no further manufacturing operations are needed to accomplish the final part geometry; (3) very large parts are possible; (4) casting is applicable to any metal that can be melted; and (5) some casting processes are suited to mass production.

10.2 What are some of the disadvantages of casting?

Answer. Disadvantages include (1) limitations on mechanical strength properties; (2) porosity; (3) poor dimensional accuracy and surface finish for some casting processes; (4) safety hazards due to handling of hot metals; and (5) environmental problems.

10.3 What is a factory that performs casting operations called?

Answer. A foundry.

10.4 What is the difference between an open mold and a closed mold?

Answer. An open mold is open to the atmosphere at the top; it is an open container in the desired shape which must be flat at the top. A closed mold has a cavity that is entirely enclosed by the mold, with a passageway (called the gating system) leading from the outside into the cavity. Molten metal is poured into this gating system to fill the mold.

10.5 Name the two basic mold types that distinguish casting processes.

Answer. The two mold types are (1) expendable molds and (2) permanent molds.

10.6 Which casting process is the most important commercially?

Answer. Sand casting.

10.7 What is the difference between a *pattern* and a *core* in sand molding?

Answer. The pattern determines the external shape of the cast part, while a core determines its internal geometry if the casting includes a cavity.

10.8 The heat required to raise the temperature of a metal to the required level for casting is the sum of what three energy components?

Answer. The heat energy required is the sum of (1) the heat to raise the temperature to the melting point, (2) the heat of fusion to convert it from solid to liquid, and (3) the heat to raise the molten metal to the desired temperature for pouring..

10.9 What is meant by the term *superheat*?

Answer. Superheat is the temperature difference above the melting point at which the molten metal is poured. The term also refers to the amount of heat that is removed from the molten metal between pouring and solidification.

10.10 Why should turbulent flow of molten metal into the mold be avoided?

Answer. Turbulence causes the following problems: (1) it accelerates formation of oxides in the solidified metal, and (2) it causes mold erosion or gradual wearing away of the mold due to impact of molten metal.

10.11 What is *Bernoulli's theorem*?

Answer. Bernoulli's theorem states that the sum of the energies (head, pressure, kinetic, and friction) at any two points in a flowing liquid are equal.

10.12 What is the *continuity law* as it applies to the flow of molten metal in casting?

Answer. The continuity law, or continuity equation, states that the volumetric flow rate remains constant throughout the liquid.

10.13 What are some of the factors that affect the fluidity of a molten metal during pouring into a mold cavity?

Answer. The factors include (1) pouring temperature above the melting point, (2) metal alloy composition, (3) viscosity of the liquid metal, and (4) heat transfer to the surroundings.

10.14 What does *heat of fusion* mean in casting?

Answer. Heat of fusion is the amount of heat energy required to transform the metal from solid state to liquid state.

10.15 How does solidification of alloys differ from solidification of pure metals?

Answer. Pure metals solidify at a single temperature equal to the melting point. Most alloys (exceptions are eutectic alloys) start to solidify at the liquidus and complete solidification occurs at the solidus, where the liquidus is a higher temperature than the solidus.

10.16 What is a *eutectic alloy*?

Answer. A eutectic alloy is a particular composition in an alloy system for which the solidus and liquidus temperatures are equal. The temperature is called the eutectic temperature. Hence, solidification occurs at a single temperature, rather than over a temperature range.

10.17 What is meant by the term *mushy zone* during solidification of a metal alloy?

Answer. The mushy zone is a mixture of solid and liquid metal that occurs while the temperature of the metal is between the liquidus and solidus for the alloy.

10.18 What is the relationship known as *Chvorinov's rule* in casting?

Answer. Chvorinov's rule is summarized: $T_{TS} = C_m(V/A)^2$, where T_{TS} = total solidification time, C_m = mold constant, V = volume of casting, and A = surface area of casting.

10.19 Identify the three sources of contraction in a metal casting after pouring.

Answer. The three contractions occur due to (1) contraction of the molten metal after pouring, (2) solidification shrinkage during transformation of state from liquid to solid, and (3) thermal contraction in the solid state.

Problems

Answers to problems labeled (A) are listed in an Appendix at the back of the book.

Heating and Pouring

- 10.1 (SI units) A casting made of cast iron weighs 4.7 kg. The riser and gating system add another 2.5 kg to the total metal poured during the operation. The melting temperature of this grade of cast iron = 1180°C, its density = 7.6 g/cm³, specific heat = 0.46 J/g°C, and heat of fusion = 126 J/g. Assume the specific heat has the same value for solid and molten metal. Compute (a) the unit energy for melting and pouring and (b) the total energy to heat the metal to a pouring temperature of 1300°C from a starting temperature of 25°C. (c) What is the volume of the casting?

Solution: (a) Unit energy $U_{mp} = 0.46(1180 - 25) + 126 + 0.46(1300 - 1180)$
 $U_{mp} = 531 + 126 + 55 = \mathbf{712 \text{ J/g}}$

(b) Total heat required $H_{mp} = (4.7 + 2.5)(10^3)(712) = \mathbf{5,126,000 \text{ J}}$

(c) Volume of casting $V = W/\rho = 4.7(10^3)/7.6 = \mathbf{618 \text{ cm}^3}$

- 10.2 (A) (SI units) A disk with diameter = 40 cm and thickness = 5 cm is cast of pure aluminum in an open mold casting operation (no gating system). Aluminum melts at 660°C, but the pouring temperature will be 800°C. The heat of fusion of aluminum = 398 J/g. Other properties can be found in Tables 4.1 and 4.2. Assume the specific heat has the same value for solid and molten aluminum. Compute (a) the unit energy for melting and pouring and (b) the total energy to heat the metal to the pouring temperature starting from a room temperature of 25°C.

Solution: (a) From Table 4.2, specific heat $C = 0.21 \text{ Cal/g}^\circ\text{C} = 0.879 \text{ J/g}^\circ\text{C}$
 Unit energy $U_{mp} = 0.879(660 - 25) + 398 + 0.879(800 - 660)$
 $U_{mp} = 558 + 398 + 123 = \mathbf{1079 \text{ J/g}}$

(b) Volume $V = \pi D^2 h / 4 = \pi(40)^2(5) / 4 = 6283 \text{ cm}^3$

From Table 4.1, density $\rho = 2.70 \text{ g/cm}^3$

Total heat required $H_{mp} = 2.7(6283)(1079) = \mathbf{18,304,264 \text{ J}}$

- 10.3 (USCS units) Pure copper is heated to cast a large rectangular plate in an open mold. The plate's length = 20 in, width = 10 in, and thickness = 2 in. The amount of metal heated will be 10% more than what is needed to fill the mold cavity. Density, melting point, and specific heat of the solid metal can be found in Tables 4.1 and 4.2. The specific heat of copper in the molten state = 0.090 Btu/lbm-F, and its heat of fusion = 88 Btu/lbm. Compute (a) the unit melting and pouring energy and (b) the total energy to heat the metal from ambient temperature (75°F) to a pouring temperature of 2100°F.

Solution: (a) Volume of rectangular plate $V = (20 \times 10 \times 2) = 400 \text{ in}^3$
 Volume of copper to be heated = $400(1 + 10\%) = 440 \text{ in}^3$
 $T_o = 75^\circ\text{F}$ and using Equation (10.1),

$U_{mp} = 0.092(1981 - 75) + 88 + 0.090(2100 - 1981)$

$U_{mp} = 175 + 88 + 11 = 274 \text{ Btu/lbm}$

(b) From Table 4.1, density $\rho = 0.324 \text{ lb/in}^3$

Total heat required $H_{mp} = 0.324(440)(274) = \mathbf{39,061 \text{ Btu}}$

- 10.4 (A) (SI units) The length of the downsprue leading into the runner of a mold = 200 mm. The cross-sectional area at its base = 400 mm^2 . Volume of the mold cavity = 0.0012 m^3 . Determine (a) velocity of the molten metal flowing through the base of the downsprue, (b) volume rate of flow, and (c) time required to fill the mold cavity.

Solution: (a) Velocity $v = (2 \times 9810 \times 200)^{0.5} = (3,924,000)^{0.5} = \mathbf{1981 \text{ mm/s}}$

(b) Volume flow rate $Q = vA = 1981 \times 400 = \mathbf{594,273 \text{ mm}^3/\text{s}}$

(c) Time to fill cavity $T_{MF} = V/Q = 1,200,000/594,273 = \mathbf{2.02 \text{ s}}$

- 10.5 (USCS units) A mold has a downsprue length = 6.0 in. The cross-sectional area at the bottom of the sprue is 0.5 in^2 . The sprue leads into a horizontal runner that feeds the mold cavity, whose volume = 75 in^3 . Determine (a) velocity of the molten metal flowing through the base of the downsprue, (b) volume rate of flow, and (c) time required to fill the mold cavity.

Solution: (a) Velocity $v = (2 \times 32.2 \times 12 \times 6.0)^{0.5} = (4636.8)^{0.5} = \mathbf{68.1 \text{ in/sec}}$

(b) Volume flow rate $Q = vA = 68.1 \times 0.5 = \mathbf{34.05 \text{ in}^3/\text{sec}}$

(c) Time to fill cavity $T_{MF} = V/Q = 75.0/34.05 = \mathbf{2.2 \text{ sec}}$

- 10.6 (SI units) The flow rate of liquid metal into the downsprue of a mold = 1.0 L/sec . The cross-sectional area at the top of the sprue = 750 mm^2 , and its length = 200 mm. What area should be used at the base of the sprue to avoid aspiration of the molten metal?

Solution: Flow rate $Q = 1.0 \text{ L/s} = 10^6 \text{ mm}^3/\text{s}$

Velocity $v = (2 \times 9810 \times 200)^{0.5} = 1981 \text{ mm/s}$

Assuming volumetric continuity, area at base $A = 1,000,000/1981 = \mathbf{505 \text{ mm}^2}$

- 10.7 (SI units) Molten metal is poured into the pouring cup of a sand mold at a steady rate of $400 \text{ cm}^3/\text{s}$. The molten metal overflows from the pouring cup and flows into the downsprue. The cross section of the sprue is round, with a diameter at the top = 3.4 cm. If the sprue is 20 cm long, determine the proper diameter at its base so as to maintain the same volume flow rate.

Solution: Velocity at base $v = (2gh)^{0.5} = (2 \times 981 \times 20)^{0.5} = 198.1 \text{ cm/s}$

Assuming volumetric continuity, area at base $A = (400 \text{ cm}^3/\text{s})/(198.1 \text{ cm/s}) = 2.02 \text{ cm}^2$

Area of sprue $A = \pi D^2/4$; rearranging, $D^2 = 4A/\pi = 4(2.02)/\pi = 2.57 \text{ cm}^2$

$D = 1.60 \text{ cm}$

- 10.8 (A) (USCS units) The volume flow rate of molten metal into the downsprue from the pouring cup is $50 \text{ in}^3/\text{sec}$. At the top where the pouring cup leads into the downsprue, the cross-sectional area = 1.0 in^2 . Determine what the area should be at the bottom of the sprue if its length = 6.0 in. It is desired to maintain a constant flow rate, top and bottom, in order to avoid aspiration of the liquid metal.

Solution: Velocity at base $v = (2gh)^{0.5} = (2 \times 32.2 \times 12 \times 6)^{0.5} = 68.1 \text{ in/sec}$

Assuming volumetric continuity, area at base $A = (50 \text{ in}^3/\text{sec})/(68.1 \text{ in/sec}) = \mathbf{0.734 \text{ in}^2}$

Shrinkage

- 10.9 (A) (SI units) Determine the shrink rule to be used by mold makers for die casting of zinc. Using Table 10.1, express your answer in terms of decimal mm of elongation per 300 mm of length compared to a standard 300-mm scale.

Solution: For zinc, shrinkage = 1.3% from Table 10.1.

Thus, linear contraction = $1.0 - 0.013 = 0.987$

Shrink rule elongation = $(0.987)^{-1} = 1.0132$

For a 300-mm rule, $L = 1.0132(300) = 303.95$ mm

Elongation per 300 mm of length = **3.95 mm**

- 10.10 (USCS units) Determine the shrink rule to be used by pattern makers for yellow brass. Using Table 10.1, express your answer in terms of decimal fraction inches of elongation per foot of length compared to a standard 1-ft scale.

Solution: For yellow brass, shrinkage = 1.5% from Table 10.1.

Thus, linear contraction = $1.0 - 0.015 = 0.985$

Shrink rule elongation = $(0.985)^{-1} = 1.0152$

For a 12-inch rule, $L = 1.0152(12) = 12.183$ in

Elongation per foot of length = **0.183 in**

- 10.11 (SI units) A flat plate is to be cast in an open mold whose bottom has a square shape that is 200 mm by 200 mm. The mold is 40 mm deep. A total of 1,000,000 mm³ of molten aluminum is poured into the mold. Solidification shrinkage is known to be 6.0%, which is a volumetric contraction, not a linear contraction. Table 10.1 lists the linear shrinkage due to thermal contraction after solidification. If the availability of molten metal in the mold allows the square shape of the cast plate to maintain its 200 mm by 200 mm dimensions until solidification is completed, determine the final dimensions of the plate.

Solution: The initial volume of liquid metal = 1,000,000 mm³. When poured into the mold it takes the shape of the open mold, which is 200 mm by 200 mm square, or 40,000 mm². The starting height of the molten metal is $1,000,000 / 40,000 = 25$ mm. Volumetric solidification shrinkage is 6%, so when the aluminum solidifies its volume = $1,000,000(0.94) = 940,000$ mm³. Because its base still measures 200 mm by 200 mm due to the flow of liquid metal before solidification, its height has been reduced to $940,000 / 40,000 = 23.5$ mm. Thermal contraction causes a further shrinkage of 1.3%. Thus the final dimensions of the plate are $200(0.987)$ by $200(0.987)$ by $23.5(0.987) = \mathbf{197.40 \text{ mm by } 197.40 \text{ mm by } 23.195 \text{ mm}}$.

Solidification Time and Riser Design

- 10.12 (SI units) When casting low carbon steel under certain mold conditions, the mold constant in Chvorinov's rule = 4.0 min/cm². Determine how long solidification will take for a rectangular casting whose length = 30 cm, width = 15 cm, and thickness = 20 mm.

Solution: Volume $V = 30 \times 15 \times 2 = 900$ cm³

Area $A = 2(30 \times 15 + 30 \times 2 + 15 \times 2) = 1080$ cm²

Chvorinov's rule: $T_{TS} = C_m (V/A)^2 = 4(900/1080)^2 = \mathbf{2.431 \text{ min}}$

- 10.13 (SI units) In the previous problem, solve for total solidification time only using an exponent value of 1.9 in Chvorinov's rule instead of 2.0. What adjustment must be made in the units of the mold constant?

Solution: Chvorinov's rule: $T_{TS} = C_m (V/A)^{1.9} = 4(900/1080)^{1.9} = \mathbf{2.475 \text{ min}}$

The units for C_m become min/in^{1.9} - strange units but consistent with Chvorinov's empirical rule.

- 10.14 (A) (SI units) A disk-shaped part is cast out of aluminum. Diameter of the disk = 650 mm and thickness = 16 mm. If the mold constant = 2.2 sec/mm² in Chvorinov's rule, how long will it take the casting to solidify?

Solution: Volume $V = \pi D^2 t / 4 = \pi (650)^2 (16) / 4 = 5,309,292 \text{ mm}^3$
 Area $A = 2\pi D^2 / 4 + \pi D t = \pi (650)^2 / 2 + \pi (650)(16) = 663,661 + 32,673 = 696,334 \text{ mm}^2$
 Chvorinov's rule: $T_{TS} = C_m (V/A)^2 = 2.2 (5,309,292 / 696,334)^2 = \mathbf{127.9 \text{ s} = 2.13 \text{ min}}$

- 10.15 (SI units) In casting experiments performed using a certain alloy and type of sand mold, it took 170 sec for a cube-shaped casting to solidify. The cube was 50 mm on a side. (a) Determine the value of the mold constant in Chvorinov's rule. (b) If the same alloy and mold type were used, find the total solidification time for a cylindrical casting in which the diameter = 50 mm and length = 50 mm.

Solution: (a) Volume $V = (50)^3 = 125,000 \text{ mm}^3$
 Area $A = 6 \times (50)^2 = 15,000 \text{ mm}^2$
 $(V/A) = 125,000 / 15,000 = 8.333 \text{ mm}$
 $C_m = T_{TS} / (V/A)^2 = 170 / (8.333)^2 = \mathbf{2.448 \text{ s/mm}^2}$

(b) Cylindrical casting with $D = 50 \text{ mm}$ and $L = 50 \text{ mm}$
 Volume $V = \pi D^2 L / 4 = \pi (50)^2 (50) / 4 = 98,175 \text{ mm}^3$
 Area $A = 2\pi D^2 / 4 + \pi D L = \pi (50)^2 / 2 + \pi (50)(50) = 11,781 \text{ mm}^2$
 $V/A = 98,175 / 11,781 = 8.333$
 $T_{TS} = 2.232 (8.333)^2 = \mathbf{170 \text{ s} = 2.833 \text{ min}}$

- 10.16 (SI units) Total solidification times of three casting geometries are to be compared: (1) a sphere with diameter = 10 cm, (2) a cylinder with diameter and length both = 10 cm, and (3) a cube with each side = 10 cm. The same casting alloy is used in all three cases. (a) Determine the relative solidification times for each geometry. (b) Based on the results of part (a), which geometric element would make the best riser? (c) If the mold constant = 3.5 min/cm² in Chvorinov's rule, compute the total solidification time for each casting.

Solution: For ease of computation, make the substitution 10 cm = 1 decimeter (1 dm)

(a) Chvorinov's rule: $T_{TS} = C_m (V/A)^2$
 (1) Sphere volume $V = \pi D^3 / 6 = \pi (1)^3 / 6 = \pi / 6 \text{ dm}^3$
 Sphere surface area $A = \pi D^2 = \pi (1)^2 = \pi \text{ dm}^2$
 $V/A = (\pi / 6) / \pi = 1/6 = 0.1667 \text{ dm}$
 Chvorinov's rule $T_{TS} = (0.1667)^2 C_m = \mathbf{0.02778 C_m}$

(2) Cylinder volume $V = \pi D^2 H / 4 = \pi (1)^2 (1) / 4 = \pi / 4 = 0.25 \pi \text{ dm}^3$
 Cylinder area $A = 2\pi D^2 / 4 + \pi D L = 2\pi (1)^2 / 4 + \pi (1)(1) = \pi / 2 + \pi = 1.5 \pi \text{ dm}^2$
 $V/A = 0.25 \pi / 1.5 \pi = 0.1667 \text{ dm}$
 Chvorinov's rule $T_{TS} = (0.1667)^2 C_m = \mathbf{0.02778 C_m}$

(3) Cube: $V = L^3 = (1)^3 = 1.0 \text{ dm}^3$
 Cube area $A = 6L^2 = 6(1)^2 = 6.0 \text{ dm}^2$
 $V/A = 1.0 / 6.0 = 0.1667 \text{ dm}$
 Chvorinov's rule $T_{TS} = (0.1667)^2 C_m = \mathbf{0.02778 C_m}$

(b) All three shapes are equivalent as risers.

(c) If $C_m = 3.5 \text{ min/cm}^2 = 350 \text{ min/dm}^2$, then $T_{TS} = 350(0.02778) = \mathbf{9.723 \text{ min}}$. However, the volumes of the three geometries are different: (1) sphere $V = 0.524 \text{ dm}^3 = 524 \text{ cm}^3$, cylinder $V = 0.25\pi = 0.7854 \text{ dm}^3 = 785.4 \text{ cm}^3$, and (3) cube $V = 1.0 \text{ dm}^3 = 1000 \text{ cm}^3$. Accordingly, the answer to part (b) might be revised so that the sphere is chosen because it wastes less metal than the other shapes.

- 10.17 (USCS units) A steel casting has a cylindrical geometry with diameter = 4.0 in and weight = 20 lb. It takes 6.0 min to solidify when cast. Another cylindrical-shaped casting with the same diameter-to-length ratio weighs 12 lb. It is made of the same steel, and the same conditions of mold and pouring are used. Determine (a) the dimensions and (b) total solidification time of the lighter casting. The density of steel is given in Table 4.1.

Solution: (a) From Table 4.1, the density of steel $\rho = 0.284 \text{ lb/in}^3$

Weight $W = \rho V$, volume of the larger casting $V = W/\rho = 20/0.284 = 70.42 \text{ in}^3$

Volume $V = \pi D^2 L/4 = \pi(4)^2 L/4 = 4\pi L = 70.42 \text{ in}^3$

Length of the larger casting $L = 70.42/4\pi = 5.60 \text{ in}$

Now find dimensions of the smaller cylindrical casting with same D/L ratio and $W = 12 \text{ lb}$

Weight is proportional to volume. Volume of smaller casting $V = (12/20)(70.42) = 42.25 \text{ in}^3$

D/L ratio $= 4.0/5.60 = 0.714$; thus $L = (0.714^{-1})D = 1.40D$

Volume $V = \pi D^2 L/4 = \pi(D)^2(1.40D)/4 = 1.09956 D^3$

$D^3 = (42.25 \text{ in}^3)/1.09956 = 38.42 \text{ in}^3$

$D = (38.42)^{0.333} = \mathbf{3.374 \text{ in}}$

$L = 1.40(3.374) = \mathbf{4.724 \text{ in}}$

(b) To find the solidification time, first determine the mold constant in Chvorinov's rule.

Area of larger casting $A = 2\pi D^2/4 + \pi DL = 2\pi(4)^2/4 + \pi(4)(5.60) = 95.50 \text{ in}^2$

$(V/A) = 70.42/95.50 = 0.7374$

$C_m = 6.0/(0.7374)^2 = 11.035 \text{ min/in}^2$

Volume of smaller casting $V = \pi D^2 L/4 = \pi(3.374)^2(4.724)/4 = 42.24 \text{ in}^3$

$A = 2\pi D^2/4 + \pi DL = 0.5\pi(3.374)^2 + \pi(3.374)(4.724) = 67.95 \text{ in}^2$

$V/A = 42.24/67.95 = 0.622 \text{ in}$.

$T_{TS} = 11.035(0.622)^2 = \mathbf{4.265 \text{ min}}$

- 10.18 A cylindrical riser is to be used for a sand-casting mold. For a given cylinder volume, determine the diameter-to-length ratio that will maximize the time to solidify.

Solution: To maximize T_{TS} , the V/A ratio must be maximized.

Cylinder volume $V = \pi D^2 L/4$. $L = 4V/\pi D^2$

Cylinder area $A = 2\pi D^2/4 + \pi DL$

Substitute the expression for L from the volume equation in the area equation:

$A = \pi D^2/2 + \pi DL = \pi D^2/2 + \pi D(4V/\pi D^2) = \pi D^2/2 + 4 V/D$

Differentiate the area equation with respect to D :

$dA/dD = \pi D - 4 V/D^2 = 0$ Rearranging, $\pi D = 4V/D^2$

$D^3 = 4 V/\pi$

$D = (4 V/\pi)^{0.333}$

From the previous expression for L , substituting in the equation for D that we have developed,

$$L = 4V/\pi D^2 = 4V/\pi(4V/\pi)^{0.667} = (4V/\pi)^{0.333}$$

Thus, optimal values are $D = L = (4V/\pi)^{0.333}$, and therefore the **optimal D/L ratio = 1.0**

- 10.19 (A) (SI units) A riser in the shape of a sphere is to be designed for a sand-casting mold. The casting is a rectangular plate, with length = 200 mm, width = 100 mm, and thickness = 18 mm. If the total solidification time of the casting itself is known to be 3.5 min, determine the diameter of the riser so that it will take 25% longer for the riser to solidify.

Solution: Casting volume $V = Lwt = 200(100)(18) = 360,000 \text{ mm}^3$

Casting area $A = 2(200 \times 100 + 200 \times 18 + 100 \times 18) = 50,800 \text{ mm}^2$

$$V/A = 360,000/50,800 = 7.0866$$

$$\text{Casting } T_{TS} = C_m(7.0866)^2 = 3.50 \text{ min}$$

$$C_m = 3.5/(7.0866)^2 = 0.0697 \text{ min/mm}^2$$

$$\text{Riser volume } V = \pi D^3/6 = 0.5236D^3$$

$$\text{Riser area } A = \pi D^2 = 3.1416D^2$$

$$V/A = 0.5236D^3/3.1416D^2 = 0.1667D$$

$$T_{TS} = 1.25(3.5) = 4.375 \text{ min} = 0.0697(0.1667D)^2 = 0.001936D^2$$

$$D^2 = 4.375/0.001936 = 2259.7 \text{ mm}^2$$

$$D = 47.5 \text{ mm}$$

- 10.20 (USCS units) A cylindrical riser with a diameter-to-length ratio = 1.0 is to be designed for a sand casting mold. The casting is shown in Figure P10.20; the units are inches. If the mold constant in Chvorinov's rule = 25.3 min/in², find the dimensions of the riser so that the riser will take 1.0 min longer to freeze than the casting itself.

Solution: Casting volume $V = V(5 \text{ in} \times 10 \text{ in rectangular plate}) + V(5 \text{ in half disk}) + V(\text{upright tube}) - V(3 \text{ in} \times 6 \text{ in rectangular cutout})$.

$$V(5 \text{ in} \times 10 \text{ in rectangular plate}) = 5 \times 12.5 \times 1.0 = 62.5 \text{ in}^3$$

$$V(5 \text{ in. half disk}) = 0.5\pi(5)^2(1)/4 = 9.817 \text{ in}^3$$

$$V(\text{upright tube}) = 3.0\pi(2.5)^2/4 - 4\pi(1.5)^2/4 = 7.657 \text{ in}^3$$

$$V(3 \text{ in} \times 6 \text{ in rectangular cutout}) = 3 \times 6 \times 1 = 18.0 \text{ in}^3$$

$$\text{Total } V = 62.5 + 9.817 + 7.657 - 18.0 = 61.974 \text{ in}^3$$

$$\text{Total } A = 1 \times 5 + 1(12.5 + 2.5\pi + 12.5) + 2(6+3) + 2(5 \times 12.5 - 3 \times 6) + 2(.5\pi(5)^2/4) - 2(1.5)^2\pi/4 + 2.5\pi(3) + 1.5\pi(3+1) = 203.36 \text{ in}^2$$

$$V/A = 61.974/203.36 = 0.305 \text{ in}$$

$$\text{Casting } T_{TS} = 25.3(0.305)^2 = 2.35 \text{ min}$$

$$\text{Riser design: specified } T_{TS} = 2.35 + 1.0 = 3.35 \text{ min}$$

$$\text{Riser volume } V = \pi D^2 L/4 = \pi D^3/4 = 0.25\pi D^3$$

$$\text{Riser area } A = \pi DL + 2\pi D^2/4 = \pi D^2 + 0.5\pi D^2 = 1.5\pi D^2$$

$$V/A = 0.25\pi D^3/1.5\pi D^2 = D/6$$

$$T_{TS} = C_m(V/A)^2$$

$$3.35 = 25.3(D/6)^2 = 0.703D^2$$

$$D^2 = 3.35/0.703 = 4.77 \text{ in}^2 \quad D = 2.18 \text{ in and } L = 2.18 \text{ in}$$