### **Problems**

# **Inventory Control**

41.1 A product is made to stock. Annual demand is 86,000 units. Each unit costs \$9.50 and the annual holding cost rate is 22%. Setup cost to produce this product is \$800. Determine (a) economic order quantity and (b) total inventory costs for this situation.

**Solution**: (a) 
$$EOQ = (2D_aC_{su}/C_h)^{0.5} = (2 \times 86,000 \times 800/(0.22 \times 9.50))^{0.5} =$$
**8114 units** (b)  $TIC = C_hQ/2 + C_{su}D_a/Q = 0.22(9.50)(8114/2) + 800(86,000/8114) = 8479 + 8479 = $16,958$ 

41.2 Given that annual demand for a product is 20,000 units, cost per unit = \$6.00, holding cost rate = 2.5%/month, changeover (setup) time between products averages 2.0 hr, and downtime cost during changeover = \$200/hr, determine (a) economic order quantity and (b) total inventory costs for this situation.

**Solution**: (a) 
$$EOQ = (2D_aC_{su}/C_h)^{0.5} = (2 \times 20,000 \times 2 \times 200/(12 \times .025 \times 6.00))^{0.5} =$$
**2981 units** (b)  $TIC = C_hQ/2 + C_{su}D_a/Q = 12 \times 0.025(6.00)(2981/2) + 2 \times 200(20,000/2981) = 2683 + 2684 =$ **\$5367**

A product is produced in batches. Batch size = 2000 units. Annual demand = 50,000 units, and unit cost of the product = \$4.00. Setup time to run a batch = 2.5 hr, cost of downtime on the affected equipment is figured at \$250/hr, and annual holding cost rate = 30%. What would the annual savings be if the product were produced in the economic order quantity?

**Solution**: Current 
$$TIC = C_h Q/2 + C_{su} D_a/Q = 0.30(4.00)(2000/2) + 2.5 \times 250(50,000/2000)$$
  
= 1200 + 15,625 = \$16,825  
 $EOQ = (2D_a C_{su}/C_h)^{0.5} = (2 \times 50,000 \times 2.5 \times 250/(0.30 \times 4.00))^{0.5} = 7217 \text{ units}$   
 $TIC \text{ at } EOQ = C_h Q/2 + C_{su} D_a/Q = 0.30(4.00)(7217/2) + 2.5 \times 250(50,000/7217)$   
= 4330 + 4330 = \$8660  
Savings = 16,825 - 8660 = **\$8165**

Assembly of a product requires that a component part be ordered and stocked. Demand for the product is constant throughout the year at 7800 units annually. The cost to place an order is \$95. The cost of the part is \$56 and the holding cost rate is 22%. When units are ordered, they take two weeks to arrive. Determine (a) the economic order quantity and (b) the reorder point. (c) The parts are prepackaged in multiples of 100. It saves the supplier unpacking and repackaging time if they can ship in multiples of 100. The supplier has offered to reduce the price by \$1 per unit if even multiples of 100 are purchased. How much would be saved (if anything) by taking this offer?

**Solution:** (a) 
$$EOQ = (2D_aC_{su}/C_h)^{0.5} = (2 \times 7800 \times 95/(0.22 \times 56))^{0.5} = 346.83 = 347$$
 units

- (b) Weekly demand is 7800/52 = 150 units/wk. If it takes 2 weeks for an order to arrive, you must reorder when the supply reaches 2\*150 = 300 units
- (c) Since total cost is not a linear function, you must check the cost both above and below the EOQ. Also, since the item cost is lower, you must calculate total cost instead of total inventory cost.

$$TC_{EOQ} = D_a C_p + C_h Q/2 + C_{su} D_a / Q = 7800(56) + (56)(0.22)(347)/2 + 95(7800)/347$$

$$TC_{EOQ} = 436,800 + 2137.52 + 2134.45 = \$441,073$$

$$TC_{300} = 7800(55) + (55)(0.22)(300)/2 + 95(7800)/300 = 429000 + 1815 + 2470 = \$433,285$$

$$TC_{400} = 7800(55) + (55)(0.22)(400)/2 + 95(7800)/400 = 429000 + 2420 + 1852.50 = \$433,273$$

It is cheapest to buy 400 units at a time. That will save \$7800 per year

41.5 A certain piece of production equipment is used to produce various components for an assembled product. To keep in-process inventories low, it is desired to produce the components in batch sizes of 150 units. Demand for each product is 2500 units per year. Production downtime costs an estimated \$200/hr. All of the components made on the equipment are of approximately equal unit cost, which is \$9.00. Holding cost rate = 30%/yr. In how many minutes must the changeover between batches be completed in order for 150 units to be the economic order quantity?

**Solution**: 
$$EOQ = (2D_aC_{su}/C_h)^{0.5}$$
  
 $(EOQ)^2 = 2D_aC_{dt}T_{su}/hC_p$   
 $T_{su} = hC_p(EOQ)^2/2D_a C_{dt} = 0.3(9.00)(150)^2/(2 \times 2500 \times 200) =$ **0.06075 hr = 3.65 min.**

41.6 Current setup time on a certain machine is 3.0 hr. Cost of downtime on this machine is estimated at \$200/hr. Annual holding cost per part made on the equipment,  $C_h = $1.00$ . Annual demand for the part is 15,000 units. Determine (a) EOQ and (b) total inventory costs for this data. Also, determine (c) EOQ and (d) total inventory costs, if the changeover time could be reduced to six minutes.

**Solution**: (a) 
$$EOQ = (2D_aC_{su}/C_h)^{0.5} = (2 \times 15,000 \times 3.00 \times 200/1.00)^{0.5} = 4243 \text{ pc}$$

(b) 
$$TIC = C_h Q/2 + C_{su} D_a/Q = 1.00(4243/2) + 3.00 \times 200(15,000/4243)$$
  
= 2121.50 + 2121.14 = \$4242.64

(c) If 
$$T_{su} = 6 \text{ min} = 0.1 \text{ hr}$$
,  $C_{su} = C_{dt}T_{su} = 200(0.1) = $20$ .  $EOQ = (2 \times 15,000 \times 20/1.00)^{0.5} = 775 \text{ pc}$ 

(d) 
$$TIC = 1.00(775/2) + 20(15,000/775) = 387.50 + 387.10 = $774.60$$

41.7 The two-bin approach is used to control inventory for a particular low-cost component. Each bin holds 1200 units. The annual usage of the component is 45,000 units. Cost to order the component is around \$70. (a) What is the imputed holding cost per unit for this data? (b) If the actual annual holding cost per unit is only 7 cents, what lot size should be ordered? (c) How much more is the current two-bin approach costing the company annually, compared to the economic order quantity?

**Solution**: (a) 
$$EOQ = (2D_aC_{su}/C_h)^{0.5}$$
  
 $1200 = (2 \times 45,000 \times 70/C_h)^{0.5}$   
 $C_h = 2 \times 45,000 \times 70/1200^2 = $4.38$  annually

(b) Given 
$$C_h = \$0.07$$
,  $EOQ = (2 \times 45,000 \times 70/0.07)^{0.5} = 9486.83 \rightarrow 9487 \text{ pc}$ 

(c) For the two-bin approach in which Q = 1200, TIC = 0.07(1200/2) + 70(45,000/1200)= 42 + 2625 = \$2667.00

For the EOQ = 9487, TIC = 0.07(9487/2) + 70(45,000/9487) = 332.05 + 332.03 = \$664.08Additional cost = 2667.00 - 664.08 = \$2002.92

## **Material Requirements Planning**

41.8 Quantity requirements are to be planned for component C2 in product P1. Required deliveries for P1 are given in Table 41.1. Ordering, manufacturing, and assembly lead times are as follows: for P1 and C2, the lead time is one week; and for S1 and M2, the lead time is two weeks. Given the product structure in Figure 41.4, determine the time-phased requirements for M2, C2, and S1 to meet the master schedule for P1. Assume no common use items and all on-hand inventories and scheduled receipts are zero. Use a format similar to Table 41.2 and develop a spreadsheet calculator to solve. Ignore demand for P1 beyond period 10.

#### **Solution**:

Period	1	2	3	4	5	6	7	8	9	10
P1 Requirements								50	75	100
Order Release							50	75	100	

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S1 Requirements						50	75	100	
Order Release				50	75	100			
C2 Requirements				200	300	400			
Order Release			200	300	400				
M2 Requirements			200	300	400				
Order Release	200	300	400						

41.9 Requirements are to be planned for component C5 in product P1. Required deliveries for P1 are given in Table 41.1. Ordering, manufacturing, and assembly lead times are as follows: for P1 and S2, the lead time is one week; for C5, the lead time is three weeks; and for M5, the lead time is 2 weeks. Given the product structure in Figure 41.4, determine the time-phased requirements for M5, C5, and S2 to meet the master schedule for P1. Assume no common use items. On-hand inventories are 200 units for M5 and 100 units for C5, zero for S2. Use a format similar to Table 41.2 and develop a spreadsheet calculator to solve. Ignore demand for P1 beyond period 10.

# **Solution:**

1	2	3	4	5	6	7	8	9	10
							50	75	100
							50	75	100
						50	75	100	
						100	150	200	
						100	150	200	
					100	150	200		
					200	300	400		
					100				
					100	300	400		
		100	300	400					
		100	300	400					
		100	100						
		0	200	400					
0	200	400							
	0		100 100 100 0	100 300 100 300 100 100 0 200	100 300 400 100 300 400 100 100 0 200 400	100 100 100 100 100 100 100 100	100   100	50   50   50   50   50   50   75	100   150   200   100   150   200   100   100   300   400   100

41.10 Solve Problem 41.9 except that the following is known in addition to the information given: scheduled receipts of M5 are 250 units in period (week) 3 and 50 units in period (week) 4.

### **Solution**:

Period	1	2	3	4	5	6	7	8	9	10
P1 Requirements								50	75	100
On-hand: 0										
Net Requirements								50	75	100
Order Release							50	75	100	
S2 Requirements							100	150	200	
On hand: 0										
Net Requirements							100	150	200	
Order Release						100	150	200		

,									
C5 Requirements						200	300	400	
On hand: 100						100			
Net Requirements						100	300	400	
Order Release			100	300	400				
M5 Requirements			100	300	400				
Scheduled Receipts			250	50					
On hand: 200			450	350	100				
Net Requirements			-350	-100	300				
Order Release	0	0	300						

# **Order Scheduling**

41.11 Four products are to be manufactured in Department A, and it is desired to determine how to allocate resources in that department to meet the required demand for these products for a certain week. For product 1, demand = 750/wk, setup time = 6 hr, and operation time = 4.0 min. For product 2, demand = 900/wk, setup time = 5 hr, and operation time = 3.0 min. For product 3, demand = 400/wk, setup time = 7 hr, and operation time = 2.0 min. For product 4, demand = 400/wk, setup time = 6 hr, and operation time = 3.0 min. The plant normally operates one shift (7.0 hours per shift), five days per week and there are currently 3 work centers in the department. Propose a way of scheduling the machines to meet the weekly demand.

**Solution**: Determine time to produce each product, assuming a single setup for each product:

Product 1: Time per batch = 6.0 + 750(4/60) = 6 + 50 = 56 hr.

Product 2: Time per batch = 5.0 + 900(3/60) = 5 + 45 = 50 hr

Product 3: Time per batch = 7.0 + 400(2/60) = 7 + 13.333 = 20.333 hr

Product 4: Time per batch = 6.0 + 400(3/60) = 6 + 20 = 26 hr

Total hours for all four products = 56 + 50 + 20.333 + 26 = 152.333 hr.

Available hours per week on 3 work centers if normal hours are assumed =  $3 \times (5 \times 7) = 105 \text{ hr}$ . This is fewer than the number of hours required. To meet the weekly production, overtime must be used. The following schedule is proposed:

Work center	Product	Quantity	Setup hours	Run hours	Hrs/product	Hrs/wk center
I	1	750	6.0	50.0	56.0	56.0
II	2	900	5.0	45.0	50.0	50.0
III	3	400	7.0	13.333	20.333	
	4	400	6.0	20.0	26.0	46.333
Totals			24.0	128.333	152.333	152.333

41.12 In the previous problem, propose a way of scheduling to meet the weekly demand if there were four machines instead of three.

**Solution**: Time to produce each product is the same as given in the preceding solution, under the assumption that a single setup is required for each product. Available hours per week on 4 work centers if normal hours are assumed =  $4 \times (5 \times 7) = 140 \text{ hr}$ . This is fewer than the number of hours required. To meet the weekly production, overtime must be used. In order to equalize the workload among machines as much as possible, let us propose to produce products 1 and 3 on work centers 1 and 2 and Products 2 and 4 on work centers 3 and 4. In both cases, this will require an additional setup

We want to equalize the workload on work centers I and II with Products 1 and 3.

Work center I:  $T_I = 6.0 + Q_I(4/60) = 6.0 + 0.06667Q_I$ 

Work center II:  $T_{\text{II}} = 6.0 + 0.06667(750 - Q_{\text{I}}) + 7.0 + 400(2/60) = 76.333 - 0.06667Q_{\text{I}}$ 

Setting  $T_I = T_{II}$ : 6.0 + 0.06667 $Q_I$  = 76.333 - 0.06667 $Q_I$ 

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$$2(0.06667 Q_1) = 0.13334 Q_1 = 76.333 - 6.0 = 70.333$$

$$Q_{\rm I} = 70.333/0.13334 = 528$$

$$T_1 = 6.0 + 0.06667(528) = 41.20 \text{ hr}$$

$$T_{\rm H} = 76.333 - 0.06667(528) = 41.133 \text{ hr.}$$

We next want to equalize the workload on work centers III and IV with Products 2 and 4.

Work center III:  $T_{\text{III}} = 5.0 + Q_{\text{III}}(3/60) = 5.0 + 0.05 Q_{\text{III}}$ 

Work center IV: 
$$T_{\text{IV}} = 5.0 + 0.05(900 - Q_{\text{III}}) + 6.0 + 400(3/60) = 76.0 - 0.05 Q_{\text{III}}$$

Setting 
$$T_{\text{III}} = T_{\text{IV}}$$
: 5.0 + 0.05  $Q_{\text{III}} = 76.0 - 0.05 Q_{\text{III}}$ 

$$2(0.05 Q_{III}) = 0.10 Q_{I} = 76.0 - 5.0 = 71.0$$

$$Q_{\rm I} = 71.0/0.10 = 710$$

$$T_{\rm I} = 5.0 + 0.05(710) = 40.50 \text{ hr}$$

$$T_{\rm H} = 76.0 - 0.05(710) = 40.50 \text{ hr.}$$

The following table summarizes the production at each work center:

Work center	Product	Quantity	Setup hours	Run hours	Hrs/product	Hrs/wk center
I	1	528	6.0	35.20	41.20	41.20
II	1	222	6.0	14.80	20.80	
	3	400	7.0	13.33	20.33	41.13
III	2	710	5.0	35.50	40.50	40.50
IV	2	190	5.0	9.50	14.50	
	4	400	6.0	20.00	26.00	40.50
Totals			35.0	128.33	163.33	163.33

41.13 The current date in the production calendar is day 14. There are three orders (A, B, and C) to be processed at a particular work center. The orders arrived in the sequence A-B-C at the work center. For order A, the remaining process time = 8 days, and the due date is day 24. For order B, the remaining process time = 14 days, and the due date is day 33. For order C, the remaining process time = 6 days, and the due date is day 26. Determine the sequence of the orders that would be scheduled using (a) first-come-first-serve, (b) earliest due date, (c) shortest processing time, (d) least slack time, and (e) critical ratio.

**Solution:** (a) FCFS: sequence = **A - B - C** 

- (b) Earliest due date: sequence =  $\mathbf{A} \mathbf{C} \mathbf{B}$
- (c) Shortest processing time: sequence = C A B
- (d) Least slack time:

Order A slack time = 
$$(24 - 14) - 8 = 2$$

Order B slack time = 
$$(33 - 14) - 14 = 5$$

Order C slack time = 
$$(26 - 14) - 6 = 6$$

Sequence 
$$= A - B - C$$

(e) Critical ratio:

Order A critical ratio = 
$$(24 - 14)/8 = 1.25$$

Order B critical ratio = 
$$(33 - 14)/14 = 1.357$$

Order C critical ratio = 
$$(26 - 14)/6 = 2.0$$

Sequence 
$$= A - B - C$$

41.14 Five jobs are waiting to be scheduled on a machine. For order A, the remaining process time = 5 days, and the due date is day 8. For order B, the remaining process time = 7 days, and the due date is day 16. For order C, the remaining process time = 11 days, and the due date is day 22. For order D, the remaining process time = 9 days, and the due date is day 31. For order E, the remaining process time = 10 days, and the due date is day 26. Determine a production schedule

based on (a) shortest processing time, (b) earliest due date, (c) critical ratio, and (d) least slack time. All times are listed in days.

**Solution:** (a) SPT: The schedule is: A - B - D - E - C

(b) EDD: The schedule is:  $\mathbf{A} - \mathbf{B} - \mathbf{C} - \mathbf{E} - \mathbf{D}$ 

(c) Critical Ratio:

A critical ratio 8/5 = 1.6

B critical ratio 16/7 = 2.29

C critical ratio 22/11 = 2.0

D critical ratio 31/9 = 3.44

E critical ratio 26/10 = 2.6

The schedule is: A - C - B - E - D

(d) Least Slack Time

A Slack Time 8 - 5 = 3

B Slack Time 16 - 7 = 9

C Slack Time 22 - 11 = 11

D Slack Time 31 - 9 = 22

E Slack Time 26 - 10 = 16

The schedule is: A - B - C - E - D