# Part VI Material Removal Processes

# 20 Theory of Metal Machining

## Chapter Contents



The ***material removal processes*** are a family of shaping operations (Figure 1.5) in which excess material is removed from a starting work part so that what remains is the desired final geometry. The "family tree" is shown in Figure 20.1. The most important branch of the family is ***conventional machining***, in which a sharp cutting tool is used to mechanically cut the material to achieve the desired geometry. The three most common machining processes are turning, drilling, and milling. The "other machining operations" in Figure 20.1 include shaping, planing, broaching, and sawing. This chapter begins the coverage of machining, which runs through Chapter 23.

Another group of material removal processes is the ***abrasive processes***, which mechanically remove material by the action of hard, abrasive particles. This process group, which includes grinding, is covered in Chapter 24. The "other abrasive processes" in Figure 20.1 include honing, lapping, and superfinishing. Finally, there are the ***nontraditional processes***, which use various energy forms other than a sharp cutting tool or abrasive particles to remove material. The energy forms include mechanical, electrochemical, thermal, and chemical.[1] The nontraditional processes are discussed in Chapter 25.

***Machining*** is a manufacturing process in which a sharp cutting tool is used to cut away material to leave

[1]Some of the mechanical energy forms in the nontraditional processes use abrasive particles, and so they overlap with the abrasive processes in Chapter 24.

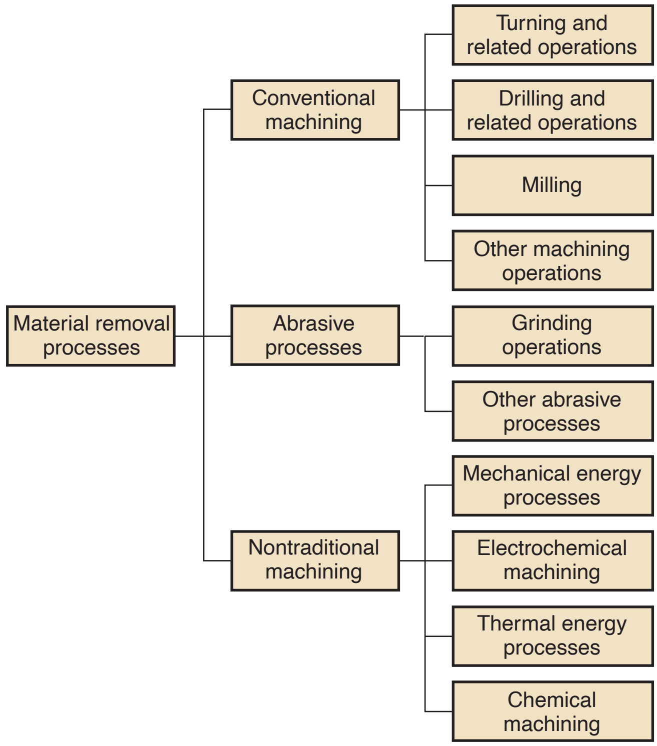


**FIGURE 20.1** Classification of material removal processes.

the desired part shape. The predominant cutting action in machining involves shear deformation of the work material to form a chip; as the chip is removed, a new surface is exposed. Machining is most frequently applied to shape metals. The process is illustrated in the diagram of Figure 20.2.

Machining is one of the most important manufacturing processes. The Industrial Revolution and the growth of the manufacturing-based economies of the world can be traced largely to the development of the various machining operations

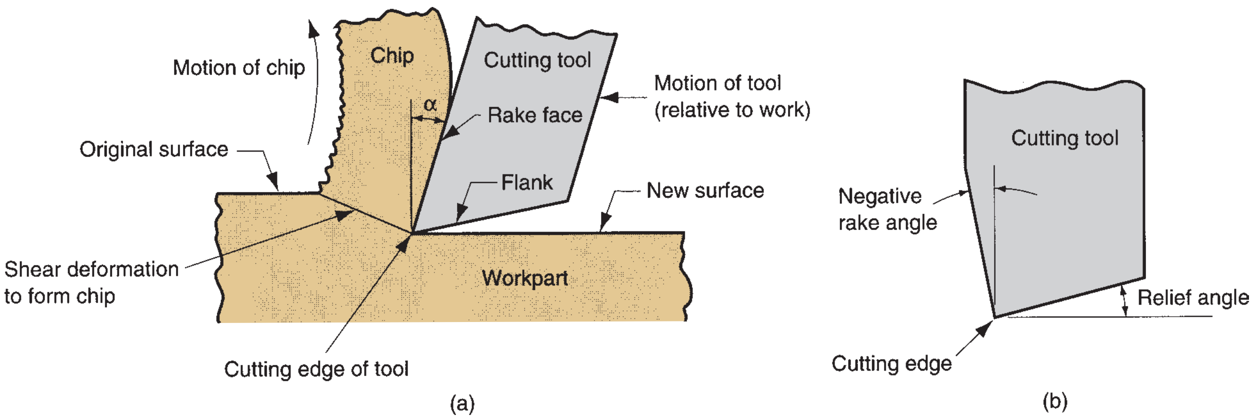


**FIGURE 20.2** (a) A cross-sectional view of the machining process. (b) Tool with negative rake angle; compare with positive rake angle in (a).

(Historical Note 21.1). Machining is important commercially and technologically for several reasons:

- ***Variety of work materials***. Machining can be applied to a wide variety of work materials. Virtually all solid metals can be machined. Plastics and plastic composites can also be cut by machining. Ceramics pose difficulties because of their high hardness and brittleness; however, most ceramics can be successfully cut by the abrasive machining processes discussed in Chapter 24.
- ***Variety of part shapes and geometric features***. Machining can be used to create any regular geometries, such as flat planes, round holes, and cylinders. By introducing variations in tool shapes and tool paths, irregular geometries can be created, such as screw threads and T-slots. By combining several machining operations in sequence, shapes of almost unlimited complexity and variety can be produced.
- ***Dimensional accuracy***. Machining can produce dimensions to very close tolerances. Some machining processes can achieve tolerances of $\pm 0.025$ mm ($\pm 0.001$ in), much more accurate than most other processes.
- ***Good surface finishes***. Machining is capable of creating very smooth surface finishes. Roughness values less than 0.4 microns (16 $\mu$in) can be achieved in conventional machining operations. Some abrasive processes can achieve even better finishes.

On the other hand, certain disadvantages are associated with machining and other material removal processes:

- ***Wasteful of material***. Machining is inherently wasteful of material. The chips generated in a machining operation are wasted material. Although these chips can usually be recycled, they represent waste in terms of the unit operation.
- ***Time consuming***. A machining operation generally takes more time to shape a given part than alternative shaping processes such as casting or forging.

Machining is generally performed after other manufacturing processes such as casting or bulk deformation (e.g., forging, bar drawing). The other processes create the general shape of the starting work part, and machining provides the final geometry, dimensions, and finish.

## 20.1 Overview of Machining Technology

Machining is not just one process; it is a group of processes. The common feature is the use of a cutting tool to form a chip that is removed from the work part. To perform the operation, relative motion is required between the tool and work. This relative motion is achieved in most machining operations by means of a primary motion, called the ***cutting speed***, and a secondary motion, called the ***feed***. The shape of the tool and its penetration into the work surface, combined with these motions, produces the desired geometry of the resulting work surface.

**Types of Machining Operations** There are many kinds of machining operations, each of which is capable of generating a certain part geometry and surface texture. These operations are discussed in considerable detail in Chapter 21, but for now it is appropriate to identify and define the three most common types: turning, drilling, and milling, illustrated in Figure 20.3.

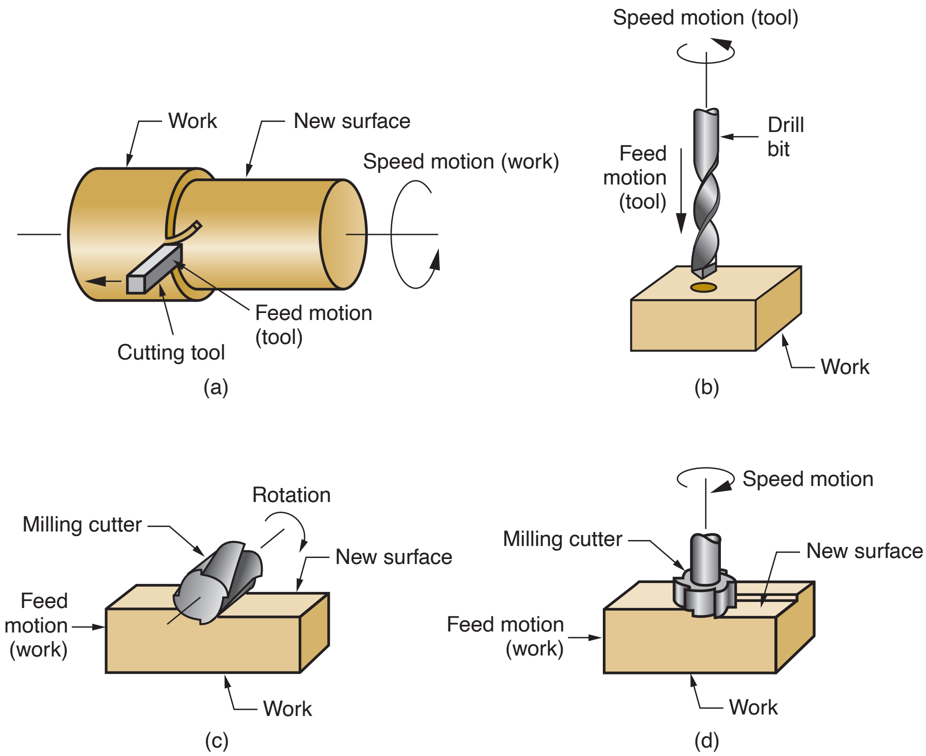


**FIGURE 20.3** The three most common types of machining processes: (a) turning, (b) drilling, and two forms of milling: (c) peripheral milling, and (d) face milling.

In ***turning***, a cutting tool with a single cutting edge is used to remove material from a rotating workpiece to generate a cylindrical shape, as in Figure 20.3(a). The speed motion in turning is provided by the rotating work part, and the feed motion is achieved by the cutting tool moving slowly in a direction parallel to the axis of rotation of the workpiece. ***Drilling*** is used to create a round hole. It is accomplished by a rotating tool that typically has two cutting edges. The tool is fed in a direction parallel to its axis of rotation into the work part to form the round hole, as in Figure 20.3(b). In ***milling***, a rotating tool with multiple cutting edges is fed slowly across the work material to generate a plane or straight surface. The direction of the feed motion is perpendicular to the tool's axis of rotation. The speed motion is provided by the rotating milling cutter. The two basic forms of milling are peripheral milling and face milling, as in Figure 20.3(c) and (d).

Other conventional machining operations include shaping, planing, broaching, and sawing (Section 21.6). Also, grinding and similar abrasive operations are often included within the category of machining. These processes commonly follow the conventional machining operations and are used to achieve a superior surface finish on the work part.

**The Cutting Tool** A cutting tool has one or more sharp cutting edges and is made of a material that is harder than the work material. The cutting edge serves to separate a chip from the parent work material, as in Figure 20.2. Connected to the cutting edge are two surfaces of the tool: the rake face and the flank. The rake face, which directs the flow of the newly formed chip, is oriented at a certain angle called the ***rake angle*** $\alpha$. It is measured relative to a plane perpendicular to the work surface.

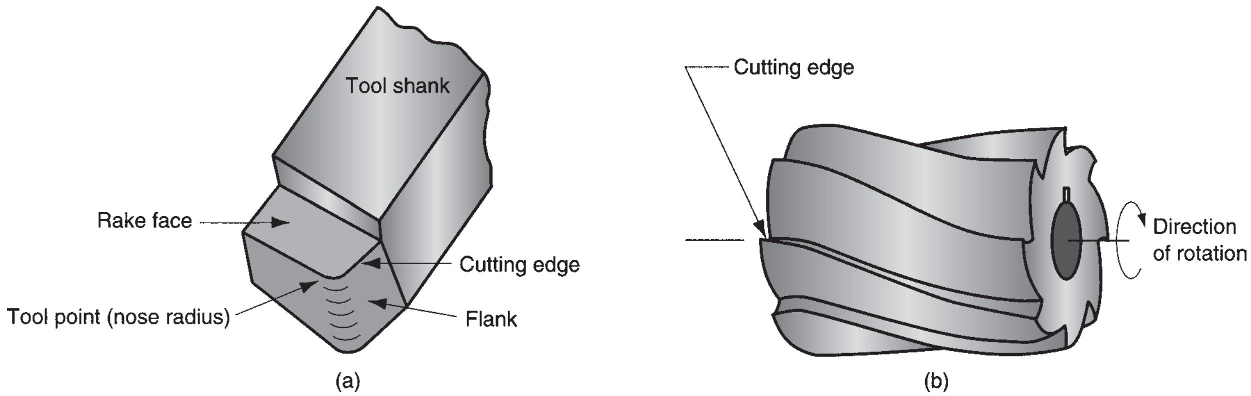


**FIGURE 20.4** (a) A single-point tool showing rake face, flank, and tool point; and (b) a helical milling cutter, representative of tools with multiple cutting edges.

The rake angle can be positive, as in Figure 20.2(a), or negative as in (b). The flank of the tool provides a clearance between the tool and the newly generated work surface, thus protecting the surface from abrasion, which would degrade the finish. This flank surface is oriented at an angle called the ***relief angle***.

Most cutting tools in practice have more complex geometries than those in Figure 20.2. There are two basic types, examples of which are illustrated in Figure 20.4: (a) single-point tools and (b) multiple-cutting-edge tools. A ***single-point tool*** has one cutting edge and is used for operations such as turning. In addition to the tool features shown in Figure 20.2, there is one tool point from which the name of this cutting tool is derived. During machining, the point of the tool penetrates below the original work surface of the part. The point is usually rounded to a certain radius, called the nose radius. ***Multiple-cutting-edge tools*** have more than one cutting edge and usually achieve their motion relative to the work part by rotating. Drilling and milling use rotating multiple-cutting-edge tools. Figure 20.4(b) shows a helical milling cutter used in peripheral milling. Although the shape is quite different from a single-point tool, many elements of tool geometry are similar. Single-point and multiple-cutting-edge tools and the materials used in them are discussed in more detail in Chapter 22.

**Cutting Conditions** Relative motion is required between the tool and work to perform a machining operation. The primary motion is accomplished at a certain ***cutting speed*** $v$. In addition, the tool must be moved laterally across the work. This is a much slower motion, called the ***feed*** $f$. The remaining dimension of the cut is the penetration of the cutting tool below the original work surface, called the ***depth of cut*** $d$. Collectively, speed, feed, and depth of cut are called the ***cutting conditions***. They form the three dimensions of the machining process, and for certain operations (e.g., most single-point tool operations) they can be used to calculate the material removal rate for the process:

$$R_{MR} = vfd \tag{20.1}$$

where $R_{MR}$ = material removal rate, mm$^3$/s (in$^3$/min); $v$ = cutting speed, m/s (ft/min), which must be converted to mm/s (in/min); $f$ = feed, mm (in); and $d$ = depth of cut, mm (in).

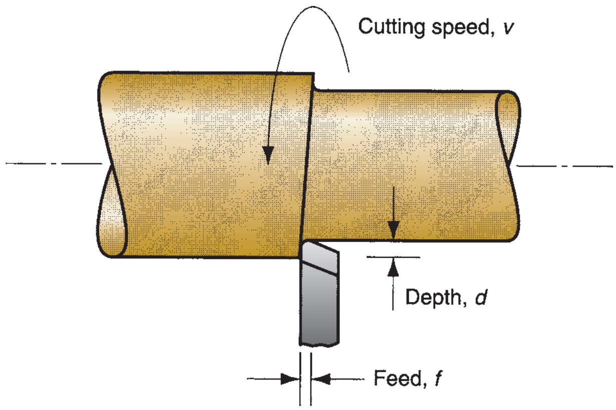


**FIGURE 20.5** Cutting speed, feed, and depth of cut for a turning operation.

The cutting conditions for a turning operation are depicted in Figure 20.5. Typical units used for cutting speed are m/s (ft/min). Feed in turning is expressed in mm/rev (in/rev), and depth of cut is expressed in mm (in). In other machining operations, interpretations of the cutting conditions may differ. For example, in a drilling operation, depth is interpreted as the depth of the drilled hole.

Machining operations usually divide into two categories, distinguished by purpose and cutting conditions: roughing cuts and finishing cuts. ***Roughing*** cuts are used to remove large amounts of material from the starting work part as rapidly as possible, in order to produce a shape close to the desired form, but leaving some material on the piece for a subsequent finishing operation. ***Finishing*** cuts are used to complete the part and achieve the final dimensions, tolerances, and surface finish. In production machining jobs, one or more roughing cuts are usually performed on the work, followed by one or two finishing cuts. Roughing operations are performed at high feeds and depths—feeds of 0.4–1.25 mm/rev (0.015–0.050 in/rev) and depths of 2.5–20 mm (0.100–0.750 in) are typical. Finishing operations are carried out at low feeds and depths—feeds of 0.125–0.4 mm (0.005–0.015 in/rev) and depths of 0.75–2.0 mm (0.030–0.075 in) are typical. Cutting speeds are lower in roughing than in finishing.

A ***cutting fluid*** is often applied to the machining operation to cool and lubricate the cutting tool. Cutting fluids are discussed in Section 22.4. Determining whether a cutting fluid should be used, and, if so, choosing the proper cutting fluid, is usually included within the scope of cutting conditions. Given the work material and tooling, the selection of these conditions is very influential in determining the success of a machining operation.

**Machine Tools** A machine tool is used to hold the work part, position the tool relative to the work, and provide power for the machining process at the speed, feed, and depth that have been set. By controlling the tool, work, and cutting conditions, machine tools permit parts to be made with great accuracy and repeatability, to tolerances of 0.025 mm (0.001 in) and better. The term ***machine tool*** applies to any power-driven machine that performs a machining operation, including grinding. The term is also applied to machines that perform metal forming and pressworking operations (Chapters 18 and 19).

The traditional machine tools used to perform turning, drilling, and milling are lathes, drill presses, and milling machines, respectively. Conventional machine tools are usually tended by a human operator, who loads and unloads the work parts, changes cutting tools, and sets the cutting conditions. Many modern machine tools are designed to accomplish their operations with a form of automation called computer numerical control (Section 37.3).

# 20.2 Theory of Chip Formation in Metal Machining

The geometry of most practical machining operations is somewhat complex. A simplified model of machining is available that neglects many of the geometric complexities, yet describes the mechanics of the process quite well. It is called the ***orthogonal*** cutting model, Figure 20.6. Although an actual machining process is three-dimensional, the orthogonal model has only two dimensions that play active roles in the analysis.

## 20.2.1 THE ORTHOGONAL CUTTING MODEL

By definition, orthogonal cutting uses a wedge-shaped tool in which the cutting edge is perpendicular to the direction of cutting speed. As the tool is forced into the material, the chip is formed by shear deformation along a plane called the ***shear plane***, which is oriented at an angle $\phi$ with the surface of the work. Only at the sharp cutting edge of the tool does failure of the material occur, resulting in separation of the chip from the parent material. Along the shear plane, where the bulk of the mechanical energy is consumed in machining, the material is plastically deformed.

The tool in orthogonal cutting has only two elements of geometry: (1) rake angle and (2) clearance angle. As indicated previously, the rake angle $\alpha$ determines the direction that the chip flows as it is formed from the work part; and the clearance angle provides a small clearance between the tool flank and the newly generated work surface.

During cutting, the cutting edge of the tool is positioned a certain distance below the original work surface. This corresponds to the thickness of the chip prior to chip

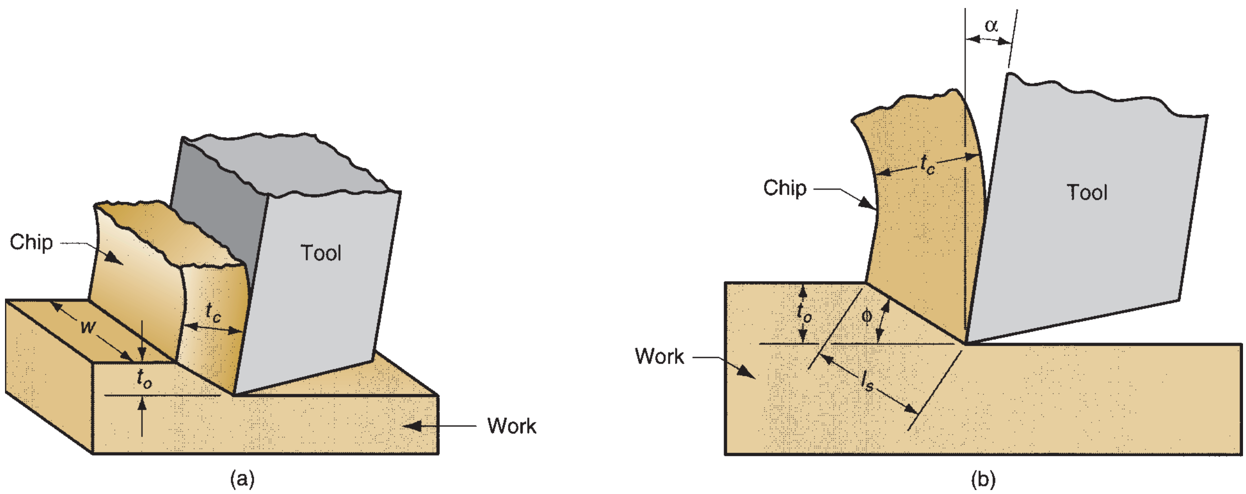


**FIGURE 20.6** Orthogonal cutting: (a) as a three-dimensional process, and (b) how it reduces to two dimensions in the side view.

formation, $t_o$. As the chip is formed along the shear plane, its thickness increases to $t_c$. The ratio of $t_o$ to $t_c$ is called the ***chip thickness ratio*** (or simply the ***chip ratio***) $r$:

$$r = \frac{t_o}{t_c} \tag{20.2}$$

Since the chip thickness after cutting is always greater than the corresponding thickness before cutting, the chip ratio will always be less than 1.0.

In addition to $t_o$, the orthogonal cut has a width dimension $w$, as shown in Figure 20.6(a), even though this dimension does not contribute much to the analysis in orthogonal cutting.

The geometry of the orthogonal cutting model allows us to establish an important relationship between the chip thickness ratio, the rake angle, and the shear plane angle. Let $l_s$ be the length of the shear plane. The following substitutions can be made: $t_o = l_s \sin \phi$, and $t_c = l_s \cos(\phi - \alpha)$. Thus,

$$r = \frac{l_s \sin \phi}{l_s \cos(\phi - \alpha)} = \frac{\sin \phi}{\cos(\phi - \alpha)}$$

This can be rearranged to determine $\phi$ as follows:

$$\tan \phi = \frac{r \cos \alpha}{1 - r \sin \alpha} \tag{20.3}$$

The shear strain that occurs along the shear plane can be estimated by examining Figure 20.7. Part (a) shows shear deformation approximated by a series of parallel

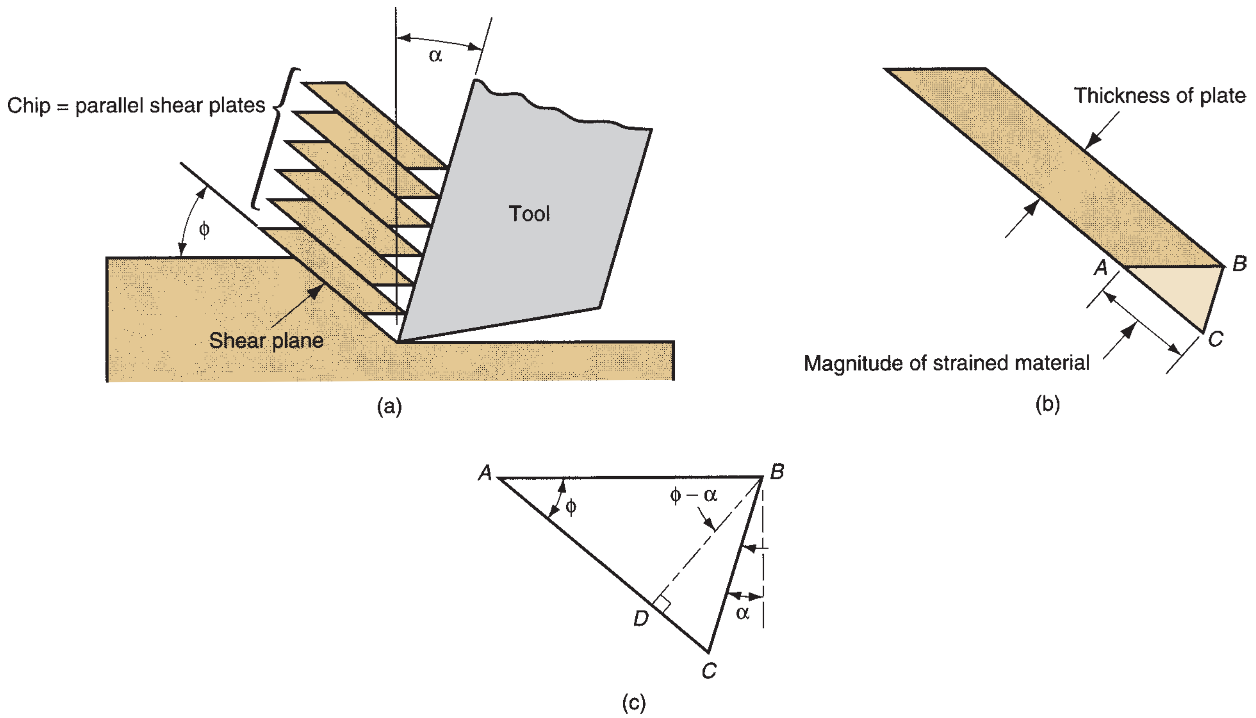


**FIGURE 20.7** Shear strain during chip formation: (a) chip formation depicted as a series of parallel plates sliding relative to each other; (b) one of the plates isolated to illustrate the definition of shear strain based on this parallel plate model; and (c) shear strain triangle used to derive Equation (20.4).

plates sliding against one another to form the chip. Consistent with the definition of shear strain (Section 3.1.4), each plate experiences the shear strain shown in Figure 20.7(b). Referring to part (c), this can be expressed as

$$\gamma = \frac{AC}{BD} = \frac{AD + DC}{BD}$$

which can be reduced to the following definition of shear strain in metal cutting:

$$\gamma = \tan(\phi - \alpha) + \cot \phi \tag{20.4}$$

**Example 20.1 Orthogonal cutting**

In a machining operation that approximates orthogonal cutting, the cutting tool has a rake angle = 10°. The chip thickness before the cut $t_o$ = 0.50 mm and the chip thickness after the cut $t_c$ = 1.125 mm. Calculate the shear plane angle and the shear strain in the operation.

**Solution:** The chip thickness ratio can be determined from Equation (20.2):

$$r = \frac{0.50}{1.125} = 0.444$$

The shear plane angle is given by Equation (20.3):

$$\tan \phi = \frac{0.444 \cos 10}{1 - 0.444 \sin 10} = 0.4738$$

$$\phi = \mathbf{25.4°}$$

Finally, the shear strain is calculated from Equation (20.4):

$$\gamma = \tan(25.4 - 10) + \cot 25.4$$

$$\gamma = 0.275 + 2.111 = \mathbf{2.386}$$

## 20.2.2 ACTUAL CHIP FORMATION

It should be noted that there are differences between the orthogonal model and an actual machining process. First, the shear deformation process does not occur along a plane, but within a zone. If shearing were to take place across a plane of zero thickness, it would imply that the shearing action must occur instantaneously as it passes through the plane, rather than over some finite (although brief) time period. For the material to behave in a realistic way, the shear deformation must occur within a thin shear zone. This more realistic model of the shear deformation process in machining is illustrated in Figure 20.8. Metal-cutting experiments have indicated that the thickness of the shear zone is only a few thousandths of an inch. Since the shear zone is so thin, there is not a great loss of accuracy in most cases by referring to it as a plane.

Second, in addition to shear deformation that occurs in the shear zone, another shearing action occurs in the chip after it has been formed. This additional shear is referred to as secondary shear to distinguish it from primary shear. Secondary shear results from friction between the chip and the tool as the chip slides along the rake

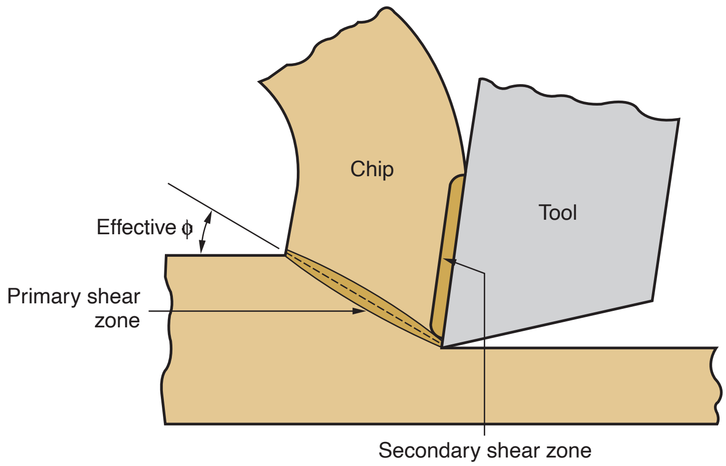


**FIGURE 20.8** More realistic view of chip formation, showing shear zone rather than shear plane. Also shown is the secondary shear zone resulting from tool–chip friction.

face of the tool. Its effect increases with increased friction between the tool and chip. The primary and secondary shear zones can be seen in Figure 20.8.

Third, formation of the chip depends on the type of material being machined and the cutting conditions of the operation. Four basic types of chip can be distinguished, illustrated in Figure 20.9:

➢ ***Discontinuous chip***. When relatively brittle materials (e.g., cast irons) are machined at low cutting speeds, the chips often form into separate segments (sometimes the segments are loosely attached). This tends to impart an irregular texture to the machined surface. High tool–chip friction and large feed and depth of cut promote the formation of this chip type.

➢ ***Continuous chip***. When ductile work materials are cut at high speeds and relatively small feeds and depths, long continuous chips are formed. A good surface finish typically results when this chip type is formed. A sharp cutting edge on the tool and low tool–chip friction encourage the formation of continuous chips. Long, continuous chips (as in turning) can cause problems with regard to chip disposal and/or tangling about the tool. To solve these problems, turning tools are often equipped with chip breakers (Section 22.3.1).

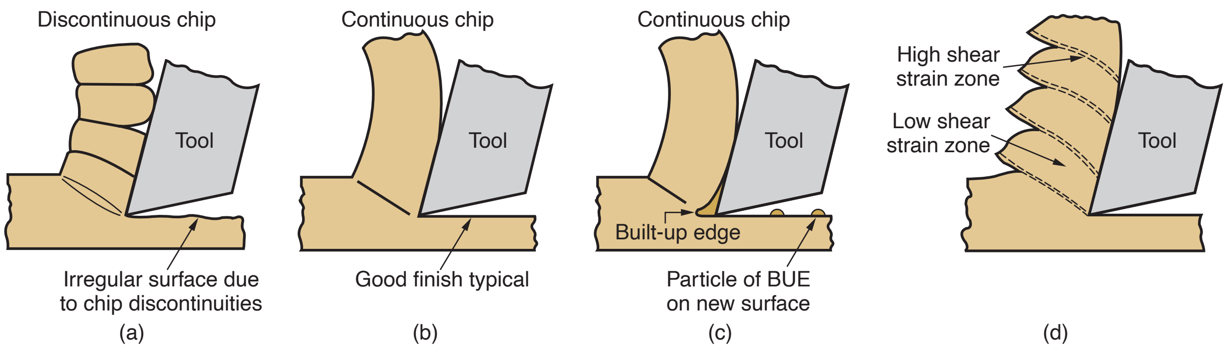


**FIGURE 20.9** Four types of chip formation in metal cutting: (a) discontinuous, (b) continuous, (c) continuous with built-up edge, (d) serrated.

- ***Continuous chip with built-up edge***. When machining ductile materials at low-to-medium cutting speeds, friction between tool and chip tends to cause portions of the work material to adhere to the rake face of the tool near the cutting edge. This formation is called a built-up edge (BUE). The formation of a BUE is cyclical; it forms and grows, then becomes unstable and breaks off. Much of the detached BUE is carried away with the chip, sometimes taking portions of the tool rake face with it, which reduces the life of the cutting tool. Portions of the detached BUE that are not carried off with the chip become imbedded in the newly created work surface, causing the surface to become rough.

The preceding chip types were first classified by Ernst in the late 1930s [13]. Since then, the available metals used in machining, cutting tool materials, and cutting speeds have all increased, and a fourth chip type has been identified:

- ***Serrated chips*** (the term ***shear-localized*** is also used for this fourth chip type). These chips are semi-continuous in the sense that they possess a saw-tooth appearance that is produced by a cyclical chip formation of alternating high shear strain followed by low shear strain. This fourth type of chip is most closely associated with certain difficult-to-machine metals such as titanium alloys, nickel-base superalloys, and austenitic stainless steels when they are machined at higher cutting speeds. However, the phenomenon is also found with more common work metals (e.g., steels) when they are cut at high speeds [13].[2]

## 20.3 Force Relationships and the Merchant Equation

Several forces can be defined relative to the orthogonal cutting model. Based on these forces, shear stress, coefficient of friction, and certain other relationships can be defined.

### 20.3.1 FORCES IN METAL CUTTING

Consider the forces acting on the chip during orthogonal cutting in Figure 20.10(a). The forces applied against the chip by the tool can be separated into two mutually perpendicular components: friction force and normal force to friction. The ***friction force*** $F$ is the frictional force resisting the flow of the chip along the rake face of the tool. The ***normal force to friction*** $N$ is perpendicular to the friction force. These two components can be used to define the coefficient of friction between the tool and the chip:

$$\mu = \frac{F}{N} \tag{20.5}$$

The friction force and its normal force can be added vectorially to form a resultant force $R$, which is oriented at an angle $\beta$, called the friction angle. The friction angle is related to the coefficient of friction as

$$\mu = \tan \beta \tag{20.6}$$

[2]A more complete description of the serrated chip type can be found in Trent & Wright [13], pp. 348–367.

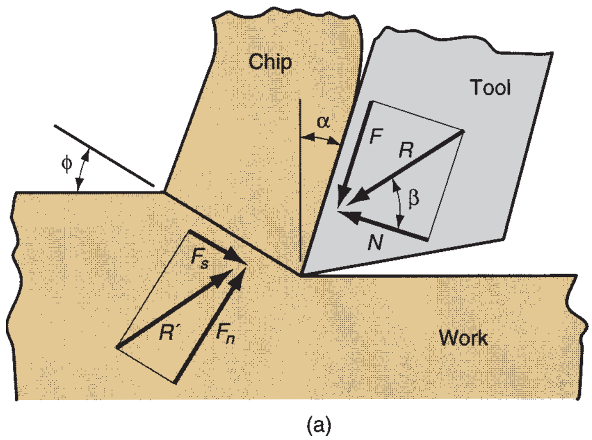


**FIGURE 20.10** Forces in metal cutting: (a) forces acting on the chip in orthogonal cutting, and (b) forces acting on the tool that can be measured.

In addition to the tool forces acting on the chip, there are two force components applied by the workpiece on the chip: shear force and normal force to shear. The ***shear force*** $F_s$ is the force that causes shear deformation to occur in the shear plane, and the ***normal force to shear*** $F_n$ is perpendicular to the shear force. Using the shear force, the shear stress that acts along the shear plane between the work and the chip can be defined:

$$\tau = \frac{F_s}{A_s} \tag{20.7}$$

where $A_s$ = area of the shear plane. This shear plane area can be calculated as

$$A_s = \frac{t_o w}{\sin\phi} \tag{20.8}$$

The shear stress in Equation (20.7) represents the level of stress required to perform the machining operation. Therefore, this stress is equal to the shear strength of the work material ($\tau = S$) under the conditions at which cutting occurs.

Vector addition of the two force components $F_s$ and $F_n$ yields the resultant force $R'$. In order for the forces acting on the chip to balance, this resultant $R'$ must be equal in magnitude, opposite in direction, and collinear with the resultant $R$.

None of the four force components $F$, $N$, $F_s$, and $F_n$ can be directly measured in a machining operation, because the directions in which they are applied vary with different tool geometries and cutting conditions. However, it is possible for the cutting tool to be instrumented using a force measuring device called a dynamometer, so that two additional force components acting against the tool can be directly measured: cutting force and thrust force. The ***cutting force*** $F_c$ is in the direction of cutting, the same direction as the cutting speed $v$, and the ***thrust force*** $F_t$ is perpendicular to the cutting force and is associated with the chip thickness before the cut $t_o$. The cutting force and thrust force are shown in Figure 20.10(b) together with their resultant force $R''$. The respective directions of these forces are known, so the force transducers in the dynamometer can be aligned accordingly.

Equations can be derived to relate the four force components that cannot be measured to the two forces that can be measured. Using the force diagram in Figure 20.11, the following trigonometric relationships can be derived:

$$F = F_c \sin\alpha + F_t \cos\alpha \tag{20.9}$$

$$N = F_c \cos\alpha - F_t \sin\alpha \tag{20.10}$$

$$F_s = F_c \cos\phi - F_t \sin\phi \tag{20.11}$$

$$F_n = F_c \sin\phi + F_t \cos\phi \tag{20.12}$$

If cutting force and thrust force are known, these four equations can be used to calculate estimates of shear force, friction force, and normal force to friction. Based on these force estimates, shear stress and coefficient of friction can be determined.

Note that in the special case of orthogonal cutting when the rake angle $\alpha = 0$, Equations (20.9) and (20.10) reduce to $F = F_t$ and $N = F_c$, respectively. Thus, in this special case, friction force and its normal force could be directly measured by the dynamometer.

**Example 20.2**
**Shear stress in machining**

Suppose in Example 20.1 that cutting force and thrust force are measured during an orthogonal cutting operation: $F_c = 1559$ N and $F_t = 1271$ N. The width of the orthogonal cutting operation $w = 3.0$ mm. Based on these data, determine the shear strength of the work material.

**Solution:** From Example 20.1, rake angle $\alpha = 10°$, and shear plane angle $\phi = 25.4°$. Shear force can be computed from Equation (20.11):

$$F_s = 1559 \cos 25.4 - 1271 \sin 25.4 = 863 \text{ N}$$

The shear plane area is given by Equation (20.8):

$$A_s = \frac{(0.5)(3.0)}{\sin 25.4} = 3.497 \text{ mm}^2$$

Thus the shear stress, which equals the shear strength of the work material, is

$$\tau = S = \frac{863}{3.497} = 247 \text{ N/mm}^2 = \mathbf{247\ MPa}$$

This example demonstrates that cutting force and thrust force are related to the shear strength of the work material. The relationships can be established in a more direct way. Recalling from Equation (20.7) that the shear force $F_s = S\,A_s$, the force diagram of Figure 20.11 can be used to derive the following equations:

$$F_c = \frac{St_o w\cos(\beta - \alpha)}{\sin\phi\cos(\phi + \beta - \alpha)} = \frac{F_s\cos(\beta - \alpha)}{\cos(\phi + \beta - \alpha)} \tag{20.13}$$

and

$$F_t = \frac{St_o w\sin(\beta - \alpha)}{\sin\phi\cos(\phi + \beta - \alpha)} = \frac{F_s\sin(\beta - \alpha)}{\cos(\phi + \beta - \alpha)} \tag{20.14}$$

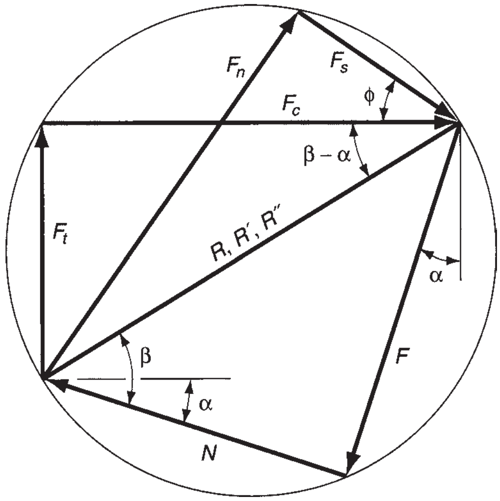


**FIGURE 20.11** Force diagram showing geometric relationships between $F$, $N$, $F_s$, $F_n$, $F_c$, and $F_t$.

These equations allow one to estimate cutting force and thrust force in an orthogonal cutting operation if the shear strength of the work material is known.

### 20.3.2 THE MERCHANT EQUATION

One of the important relationships in metal cutting was derived by Eugene Merchant [10]. Its derivation was based on the assumption of orthogonal cutting, but its general validity extends to three-dimensional machining operations. Merchant started with the definition of shear stress expressed in the form of the following relationship derived by combining Equations (20.7), (20.8), and (20.11):

$$\tau = \frac{F_c \cos\phi - F_t \sin\phi}{(t_o\, w/\sin\phi)} \tag{20.15}$$

Merchant reasoned that, out of all the possible angles emanating from the cutting edge of the tool at which shear deformation could occur, there is one angle $\phi$ that predominates. This is the angle at which shear stress is just equal to the shear strength of the work material, and so shear deformation occurs at this angle. For all other possible shear angles, the shear stress is less than the shear strength, so chip formation cannot occur at these other angles. In effect, the work material will select a shear plane angle that minimizes energy. This angle can be determined by taking the derivative of the shear stress $\tau$ in Equation (20.15) with respect to $\phi$ and setting the derivative to zero. Solving for $\phi$, the relationship named after Merchant is obtained:

$$\phi = 45 + \frac{\alpha}{2} - \frac{\beta}{2} \tag{20.16}$$

Among the assumptions in the Merchant equation is that shear strength of the work material is a constant, unaffected by strain rate, temperature, and other factors. Because this assumption is violated in practical machining operations, Equation (20.16) must be considered an approximate relationship rather than an accurate mathematical equation. Nevertheless, consider its application in the following example.

**Example 20.3 Estimating friction angle**

Using the data and results from the previous examples, determine (a) the friction angle and (b) the coefficient of friction.

**Solution:** (a) From Example 20.1, $\alpha = 10°$, and $\phi = 25.4°$. Rearranging Equation (20.16), the friction angle can be estimated:

$$\beta = 2\,(45) + 10 - 2\,(25.4) = \mathbf{49.2°}$$

(b) The coefficient of friction is given by Equation (20.6):

$$\mu = \tan 49.2 = \mathbf{1.16}$$

**Lessons Based on the Merchant Equation** The real value of the Merchant equation is that it defines the general relationship between rake angle, tool–chip friction, and shear plane angle. The shear plane angle can be increased by (1) increasing the rake angle and (2) decreasing the friction angle (and coefficient of friction) between the tool and the chip. Rake angle can be increased by proper tool design, and friction angle can be reduced by using a lubricant cutting fluid.

The importance of increasing the shear plane angle can be seen in Figure 20.12. If all other factors remain the same, a higher shear plane angle results in a smaller shear plane area. Since the shear strength is applied across this area, the shear force required to form the chip will decrease when the shear plane area is reduced. A greater shear plane angle results in lower cutting energy, lower power requirements, and lower cutting temperature. These are good reasons to try to make the shear plane angle as large as possible during machining.

**Approximation of Turning by Orthogonal Cutting** The orthogonal model can be used to approximate turning and certain other single-point machining operations so long as the feed in these operations is small relative to depth of cut. Thus, most

faiz big force small

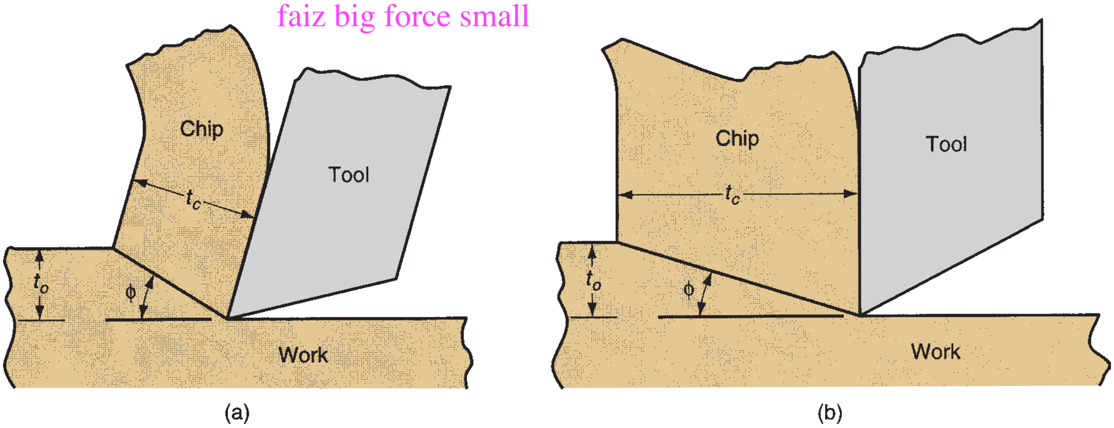


**FIGURE 20.12** Effect of shear plane angle $\phi$: (a) higher $\phi$ with a resulting lower shear plane area; (b) smaller $\phi$ with a corresponding larger shear plane area. Note that the rake angle is larger in (a), which tends to increase shear angle according to the Merchant equation.

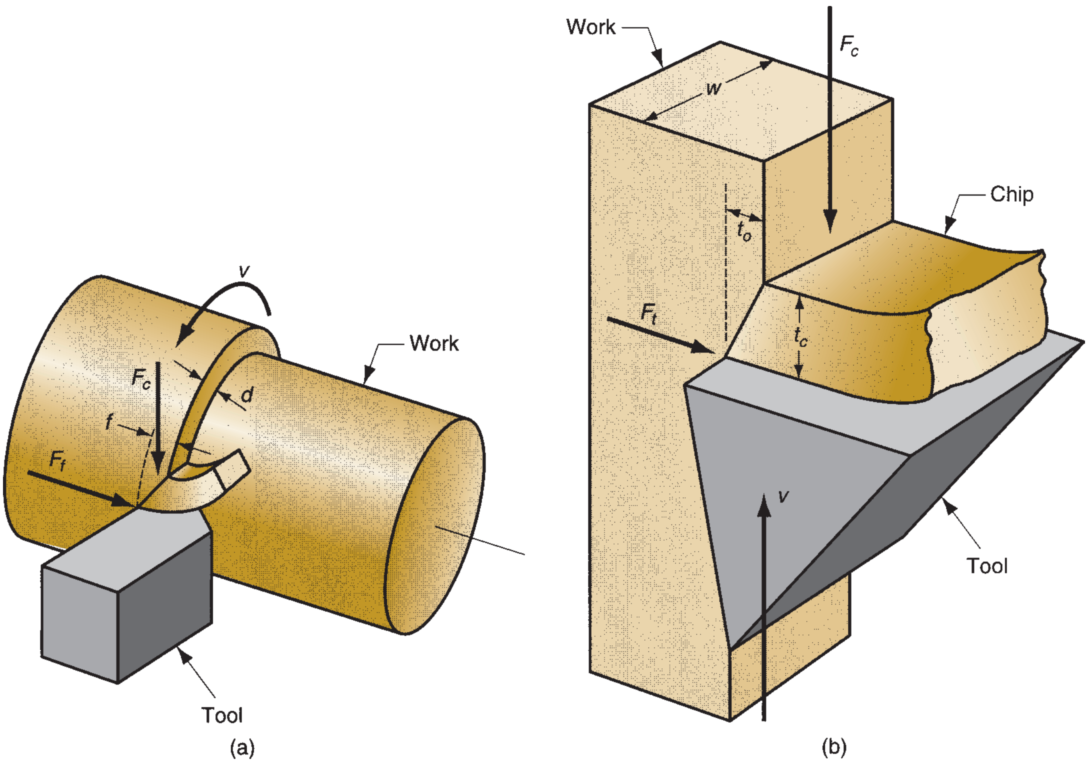


**FIGURE 20.13** Approximation of turning by the orthogonal model: (a) turning; and (b) the corresponding orthogonal cutting.

of the cutting will take place in the direction of the feed, and cutting on the point of the tool will be negligible. Figure 20.13 indicates the conversion from one cutting situation to the other.

The interpretation of cutting conditions is different in the two cases. The chip thickness before the cut $t_o$ in orthogonal cutting corresponds to the feed $f$ in turning, and the width of cut $w$ in orthogonal cutting corresponds to the depth of cut $d$ in turning. In addition, the thrust force $F_t$ in the orthogonal model corresponds to the feed force $F_f$ in turning. Cutting speed and cutting force have the same meanings in the two cases. Table 20.1 summarizes the conversions.

**TABLE • 20.1** Conversion key: turning operation vs. orthogonal cutting.

| Turning Operation | Orthogonal Cutting Model |
|---|---|
| Feed $f$ = | Chip thickness before cut $t_o$ |
| Depth $d$ = | Width of cut $w$ |
| Cutting speed $v$ = | Cutting speed $v$ |
| Cutting force $F_c$ = | Cutting force $F_c$ |
| Feed force $F_f$ = | Thrust force $F_t$ |

## 20.4 Power and Energy Relationships in Machining

A machining operation requires power. The cutting force in a production machining operation might exceed 1000 N (several hundred pounds), as suggested by Example 20.2. Typical cutting speeds are several hundred m/min. The product of cutting force and speed gives the power (energy per unit time) required to perform a machining operation:

$$P_c = F_c v \tag{20.17}$$

where $P_c$ = cutting power, N-m/s or W (ft-lb/min); $F_c$ = cutting force, N (lb); and $v$ = cutting speed, m/s (ft/min). In U.S. customary units, power is traditionally expressed as horsepower by dividing ft-lb/min by 33,000. Hence,

$$HP_c = \frac{F_c v}{33{,}000} \tag{20.18}$$

where $HP_c$ = cutting horsepower, hp. The gross power required to operate the machine tool is greater than the power delivered to the cutting process because of mechanical losses in the motor and drive train in the machine. These losses can be accounted for by the mechanical efficiency of the machine tool:

$$P_g = \frac{P_c}{E} \quad \text{or} \quad HP_g = \frac{HP_c}{E} \tag{20.19}$$

where $P_g$ = gross power of the machine tool motor, W; $HP_g$ = gross horsepower; and $E$ = mechanical efficiency of the machine tool. Typical values of $E$ for machine tools are around 90%.

It is often useful to convert power into power per unit volume rate of metal cut. This is called the ***unit power***, $P_u$ (or ***unit horsepower***, $HP_u$), defined:

$$P_u = \frac{P_c}{R_{MR}} \quad \text{or} \quad HP_u = \frac{HP_c}{R_{MR}} \tag{20.20}$$

where $R_{MR}$ = material removal rate, mm³/s (in³/min). The material removal rate can be calculated as the product of $vt_o w$. This is Equation (20.1) using the conversions from Table 20.1. Unit power is also known as the ***specific energy*** $U$.

$$U = P_u = \frac{P_c}{R_{MR}} = \frac{F_c v}{v t_o w} = \frac{F_c}{t_o w} \tag{20.21}$$

The units for specific energy are typically N-m/mm³(in-lb/in³). However, the last expression in Equation (20.21) suggests that the units might be reduced to N/mm² (lb/in²). It is more meaningful to retain the units as N-m/mm³ or J/mm³ (in-lb/in³).

**Example 20.4 Power relationships in machining**

Continuing with the previous examples, determine cutting power and specific energy in the machining operation if the cutting speed = 100 m/min. Summarizing the data and results from previous examples, $t_o$ = 0.50 mm, $w$ = 3.0 mm, $F_c$ = 1557 N.

**Solution:** From Equation (20.18), power in the operation is

$$P_c = (1557\text{ N})(100\text{ m/min}) = 155{,}700\text{ N–m/min} = 155{,}700\text{ J/min} = 2595\text{ J/s} = 2595\text{ W}$$

Specific energy is calculated from Equation (20.21):

$$U = \frac{155{,}700}{100(10^3)(3.0)(0.5)} = \frac{155{,}700}{150{,}000} = \mathbf{1.038\ N\text{-}m/mm^3}$$

**TABLE • 20.2** Values of unit horsepower and specific energy for selected work materials using sharp cutting tools and chip thickness before the cut $t_o = 0.25$ mm (0.010 in).

| Material | Brinell Hardness | Specific Energy $U$ or Unit Power $P_u$: N-m/mm³ | Specific Energy $U$ or Unit Power $P_u$: in-lb/in³ | Unit Horsepower $HP_u$ hp/(in³/min) |
|---|---|---|---|---|
| Carbon steel | 150–200 | 1.6 | 240,000 | 0.6 |
| | 201–250 | 2.2 | 320,000 | 0.8 |
| | 251–300 | 2.8 | 400,000 | 1.0 |
| Alloy steels | 200–250 | 2.2 | 320,000 | 0.8 |
| | 251–300 | 2.8 | 400,000 | 1.0 |
| | 301–350 | 3.6 | 520,000 | 1.3 |
| | 351–400 | 4.4 | 640,000 | 1.6 |
| Cast irons | 125–175 | 1.1 | 160,000 | 0.4 |
| | 175–250 | 1.6 | 240,000 | 0.6 |
| Stainless steel | 150–250 | 2.8 | 400,000 | 1.0 |
| Aluminum | 50–100 | 0.7 | 100,000 | 0.25 |
| Aluminum alloys | 100–150 | 0.8 | 120,000 | 0.3 |
| Brass | 100–150 | 2.2 | 320,000 | 0.8 |
| Bronze | 100–150 | 2.2 | 320,000 | 0.8 |
| Magnesium alloys | 50–100 | 0.4 | 60,000 | 0.15 |

Data compiled from [6], [8], [11], and other sources.

Unit power and specific energy provide a useful measure of how much power (or energy) is required to remove a unit volume of metal during machining. Using this measure, different work materials can be compared in terms of their power and energy requirements. Table 20.2 presents a listing of unit horsepower and specific energy values for selected work materials.

The values in Table 20.2 are based on two assumptions: (1) the cutting tool is sharp, and (2) the chip thickness before the cut $t_o = 0.25$ mm (0.010 in). If these assumptions are not met, some adjustments must be made. For worn tools, the power required to perform the cut is greater, and this is reflected in higher specific energy and unit horsepower values. As an approximate guide, the values in the table should be multiplied by a factor between 1.00 and 1.25 depending on the degree of dullness of the tool. For sharp tools, the factor is 1.00. For tools in a finishing operation that are nearly worn out, the factor is around 1.10, and for tools in a roughing operation that are nearly worn out, the factor is 1.25.

Chip thickness before the cut $t_o$ also affects the specific energy and unit horsepower values. As $t_o$ is reduced, unit power requirements increase. This relationship is referred to as the ***size effect***. For example, grinding, in which the chips are extremely small by comparison to most other machining operations, requires very high specific energy values. The $U$ and $HP_u$ values in Table 20.2 can still be used to estimate horsepower and energy for situations in which $t_o$ is not equal to 0.25 mm (0.010 in) by applying a correction factor to account for any difference in chip thickness before the cut. Figure 20.14 provides values of this correction factor as a function of $t_o$. The unit horsepower and specific energy values in Table 20.2 should be multiplied by the appropriate correction factor when $t_o$ differs from 0.25 mm (0.010 in).

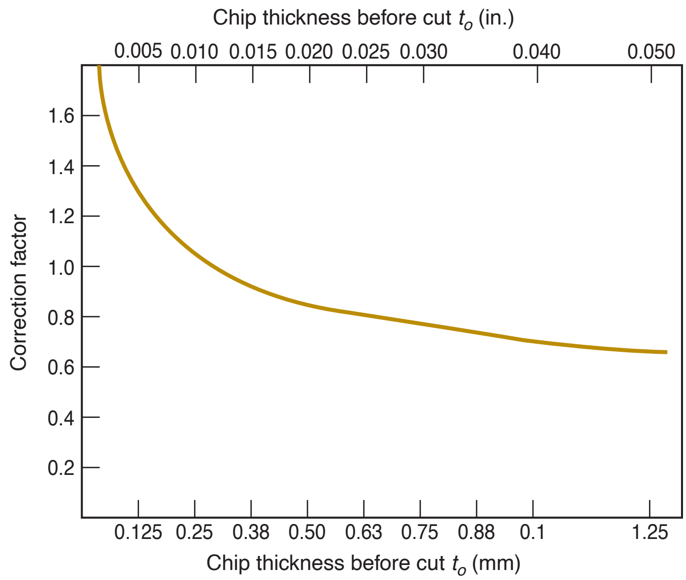


**FIGURE 20.14** Correction factor for unit horsepower and specific energy when values of chip thickness before the cut $t_o$ are different from 0.25 mm (0.010 in).

**TABLE • 20.3** Troubleshooting Guide for Power Problems

| Problem | Possible Solutions |
| --- | --- |
| Cutting power requirements too high for machine tool | Reduce cutting speed |
| | Reduce depth of cut and/or feed |
| | Use a more machinable work material |
| | Use a machine tool with more power |
| | Use a cutting fluid |
| | Use a cutting tool with higher rake angle |

In addition to tool sharpness and size effect, other factors also influence the values of specific energy and unit horsepower for a given operation. These other factors include rake angle, cutting speed, and cutting fluid. As rake angle or cutting speed are increased, or when cutting fluid is added, the $U$ and $HP_u$ values are reduced slightly.

Table 20.3 presents a troubleshooting guide that summarizes the actions that can be taken to mitigate problems in which the power requirements of the machining operation exceed the capacity of the machine tool.

## 20.5 Cutting Temperature

Of the total energy consumed in machining, nearly all of it (~ 98%) is converted into heat. This heat can cause temperatures to be very high at the tool–chip interface—over 600°C (1100°F) is not unusual. The remaining energy (~2%) is retained as elastic energy in the chip.

Cutting temperatures are important because high temperatures (1) reduce tool life, (2) produce hot chips that pose safety hazards to the machine operator, and

(3) can cause inaccuracies in work part dimensions due to thermal expansion of the work material. This section discusses the calculation and measurement of temperatures in machining.

## 20.5.1 ANALYTICAL METHODS TO COMPUTE CUTTING TEMPERATURES

There are several analytical methods to calculate estimates of cutting temperature. References [3], [5], [9], and [15] present some of these approaches. The method by Cook [5] was derived using experimental data for a variety of work materials to establish parameter values for the resulting equation. The equation can be used to predict the increase in temperature at the tool–chip interface during machining:

$$\Delta T = \frac{0.4U}{\rho C}\left(\frac{vt_o}{K}\right)^{0.333} \tag{20.22}$$

where $\Delta T$ = mean temperature rise at the tool–chip interface, C° (F°); $U$ = specific energy in the operation, N-m/mm$^3$ or J/mm$^3$ (in-lb/in$^3$); $v$ = cutting speed, m/s (in/sec); $t_o$ = chip thickness before the cut, m (in); $\rho C$ = volumetric specific heat of the work material, J/mm$^3$-C (in-lb/in$^3$-F); $K$ = thermal diffusivity of the work material, m$^2$/s (in$^2$/sec).

**Example 20.5 Cutting temperature**

For the specific energy obtained in Example 20.4, calculate the increase in temperature above ambient temperature of 20°C. Use the given data from the previous examples in this chapter: $v$ = 100 m/min, $t_o$ = 0.50 mm. In addition, the volumetric specific heat for the work material = 3.0 ($10^{-3}$) J/mm$^3$-C, and thermal diffusivity = 50 ($10^{-6}$) m$^2$/s (or 50 mm$^2$/s).

**Solution:** Cutting speed must be converted to mm/s: $v$ = (100 m/min)($10^3$ mm/m)/(60 s/min) = 1667 mm/s. Equation (20.22) can now be used to compute the mean temperature rise:

$$\Delta T = \frac{0.4(1.038)}{3.0(10^3)}\,°\text{C}\left(\frac{1667(0.5)}{50}\right)^{0.333} = (138.4)(2.552) = \mathbf{353°C}$$

## 20.5.2 MEASUREMENT OF CUTTING TEMPERATURE

Experimental methods have been developed to measure temperatures in machining. The most frequently used measuring technique is the ***tool–chip thermocouple***. This thermocouple consists of the tool and the chip as the two dissimilar metals forming the thermocouple junction. By properly connecting electrical leads to the tool and work part (which is connected to the chip), the voltage generated at the tool–chip interface during cutting can be monitored using a recording potentiometer or other appropriate data-collection device. The voltage output of the tool–chip thermocouple (measured in mV) can be converted into the corresponding temperature value by means of calibration equations for the particular tool–work combination.

The tool–chip thermocouple has been utilized by researchers to investigate the relationship between temperature and cutting conditions such as speed and feed.

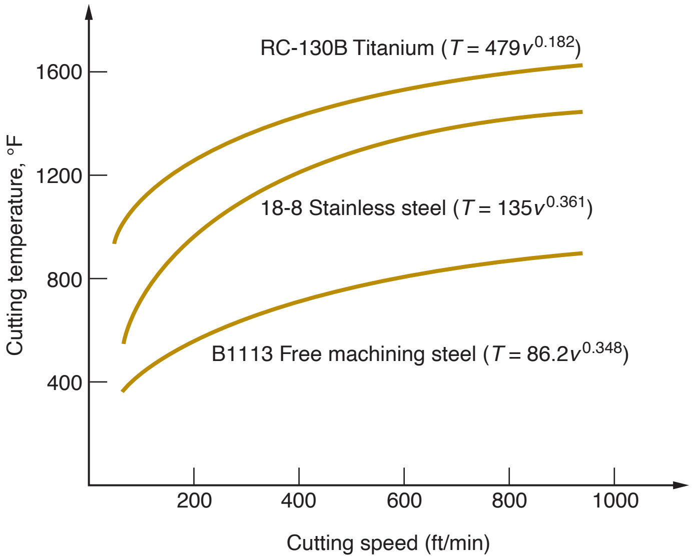


**FIGURE 20.15** Experimentally measured cutting temperatures plotted against speed for three work materials, indicating general agreement with Equation (20.23). Based on data in [9])[3]

Trigger [14] determined the speed–temperature relationship to be of the following general form:

$$T = K\,v^m \tag{20.23}$$

where $T$ = measured tool–chip interface temperature and $v$ = cutting speed. The parameters $K$ and $m$ depend on cutting conditions (other than $v$) and work material. Figure 20.15 plots temperature versus cutting speed for several work materials, with equations of the form of Equation (20.23) determined for each material. A similar relationship exists between cutting temperature and feed; however, the effect of feed on temperature is not as strong as cutting speed. These empirical results tend to support the general validity of the Cook equation: Equation (20.22).

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[7] Kalpakjian, S., and Schmid, S. ***Manufacturing Processes for Engineering Materials***, 5th ed.


[3]The units reported in the Loewen and Shaw ASME paper [9] were °F for cutting temperature and ft/min for cutting speed. Those units have been retained in the plots and equations of our figure.