18 BULK DEFORMATION PROCESSES IN METALWORKING

Review Questions

18.1 What are the reasons why the bulk deformation processes are important commercially and technologically?

Answer. Reasons why the bulk deformation processes are important include: (1) they are capable of significant shape change when hot working is used, (2) they have a positive effect on part strength when cold working is used, and (3) most of the processes produce little material waste; some are net shape processes.

18.2 Name the four basic bulk deformation processes.

Answer. The four basic bulk deformation processes are (a) rolling, (2) forging, (3) extrusion, and (4) wire and bar drawing.

18.3 What is rolling in the context of the bulk deformation processes?

Answer. Rolling is a deformation process in which the thickness of the workpiece is reduced by compressive forces exerted by two opposing rolls. The rolls rotate, thus pulling and simultaneously squeezing the workpiece between them.

18.4 In rolling of steel, what are the differences between a bloom, a slab, and a billet?

Answer. A bloom is a rolled steel workpiece with a square cross section of about 150 mm by 150 mm. The starting work unit for a bloom is an ingot heated in a soaking pit. A slab is rolled from an ingot or a bloom and has a rectangular cross section of about 250 mm by 40 mm. A billet is rolled from a bloom and has a square cross section of about 40 mm by 40 mm.

18.5 List some of the products produced on a rolling mill.

Answer. Rolled products include flat sheet and plate stock, round bar and rod stock, rails, structural shapes such as I-beams and channels.

18.6 What is *draft* in a rolling operation?

Answer. Draft is the difference between the starting thickness and the final thickness as the workpiece passes between the two opposing rolls.

18.7 What is *sticking* in a hot-rolling operation?

Answer. Sticking is a condition in hot rolling in which the surface of the workpiece adheres to the rolls as the piece passes between the rolls, causing severe deformation of the metal below the surface in order to allow passage through the roll gap.

18.8 Identify some ways in which force in flat rolling can be reduced.

Answer. Ways to reduce force in flat rolling include (1) use hot rolling, (2) reduce draft in each pass, and (3) use smaller diameter rolls.

18.9 What is a two-high rolling mill?

Answer. A two-high rolling mill consists of two opposing rolls between which the work is compressed.

18.10 What is a reversing mill in rolling?

Answer. A reversing mill is a two-high rolling mill in which the direction of rotation of the rolls can be reversed to allow the work to pass through from either side.

18.11 Besides flat rolling and shape rolling, identify some additional bulk forming processes that are related to rolling.

Answer. Some other processes that use rolls are ring rolling, thread rolling, gear rolling, roll piercing, and roll forging.

18.12 What is forging?

Answer. Forging is a deformation process in which the workpiece is compressed between two dies, using impact or gradual pressure to form the part.

18.13 One way to classify forging operations is by the degree to which the work is constrained in the die. By this classification, name the three basic types.

Answer. The three basic types are (1) open die forging, (2) impression die forging, and (3) flashless forging.

18.14 Why is flash desirable in impression die forging?

Answer. Because its presence constrains the metal in the die so that it fills the details of the die cavity.

18.15 What is a trimming operation in the context of impression die forging?

Answer. Trimming is a shearing operation used to remove the flash on the workpiece after impression die forging.

18.16 What are the two basic types of forging equipment?

Answer. The two types of forging machines are hammers, which impact the work part, and presses, which apply a gradual pressure to the work.

18.17 What is isothermal forging?

Answer. Isothermal forging is a hot forging operation in which the die surfaces are heated to reduce heat transfer from the work into the tooling.

18.18 What is extrusion?

Answer. Extrusion is a compression forming operation in which a workpiece is forced to flow through a die opening, thus taking the cross-sectional shape of the die opening.

18.19 Distinguish between direct and indirect extrusion.

Answer. In direct extrusion, also known as forward extrusion, a metal billet is loaded into a container, and a ram compresses the material, forcing it to flow through a die opening at the opposite end of the container. In indirect extrusion, also known as backward extrusion, the die is incorporated into the ram, and as the ram compresses into the metal billet, the metal is forced to flow through the die opening in a direction that is opposite (backwards) of the ram motion.

18.20 Name some products that are produced by extrusion.

Answer. Products produced by continuous extrusion include structural shapes (window frames, shower stalls, channels), tubes and pipes, and rods of various cross sections. Products made by discrete extrusion include toothpaste tubes, aluminum beverage cans, and battery cases.

18.21 Why is friction a factor in determining the ram force in direct extrusion but not a factor in indirect extrusion?

Answer. Friction is a factor in direct extrusion because the work billet is squeezed against the walls of the container so that friction resists the movement of the billet toward the die opening. In indirect extrusion, the billet does not move relative to the container walls, and thus there is no friction between the billet and the container walls.

18.22 What does the centerburst defect in extrusion have in common with the roll piercing process?

Answer. They are both examples of how compressive stresses applied to the outside surface of a solid cylindrical cross section can create high tensile stresses in the interior of the cylinder.

18.23 Define wire drawing and bar drawing.

Answer. Wire and bar drawing are bulk deformation processes in which the cross section of a wire or bar is reduced by pulling (drawing) it through a die opening.

18.24 Although the workpiece in a wire drawing operation is obviously subjected to tensile stresses, how do compressive stresses also play a role in the process?

Answer. Compressive stresses are present in wire drawing because the starting metal is compressed as it is forced through the approach of the die opening.

18.25 In a wire drawing operation, why must the drawing stress never exceed the yield strength of the work metal?

Answer. Because if the drawing stress exceeded the yield strength, the metal on the exit side of the draw die would stretch rather than force metal to be pulled through the die opening.

Problems

Answers to problems labeled **(A)** are listed in an Appendix at the back of the book.

Rolling

18.1 **(A)** (SI units) A 30-mm-thick plate made of low carbon steel is reduced to 25 mm in one pass in a rolling operation. As the thickness is reduced, the plate widens by 4%. Yield strength of the steel = 174 MPa, and tensile strength = 290 MPa. The entrance speed of the plate = 77 m/min. Roll radius = 300 mm, and rotational speed = 45 rev/min. Determine (a) the minimum required coefficient of friction that would make this rolling operation possible, (b) exit velocity of the plate, and (c) forward slip.

Solution: (a) Maximum draft $d_{max} = \mu^2 R$
Given that $d = t_o - t_f = 30 - 25 = 5$ mm
$\mu^2 = 5/300 = 0.0167$

$\mu = (0.0167)^{0.5} = \mathbf{0.129}$

(b) Plate widens by 4%
$t_o w_o v_o = t_f w_f v_f$
$w_f = 1.04\ w_o$
$30(w_o)(12) = 25(1.04 w_o) v_f$
$v_f = 30(w_o)(77)/25(1.04 w_o) = 2310/26 = \mathbf{88.85\ m/min}$

(c) $v_r = 2\pi r N = 2\pi(300 \times 10^{-3})(45) = 84.82$ m/min

$s = (v_f - v_r)/v_r = (88.85 - 84.82)/84.82 = \mathbf{0.0475}$

18.2 (USCS units) A 3-in-thick slab is 10 in wide and 15 ft long. The thickness of the slab is reduced by 20% and width increases by 3% in a hot-rolling operation. If the entry speed of the slab is 40 ft/min, determine the (a) length and (b) exit velocity of the slab.

Solution: (a) After three passes, $t_f = (0.80)(3.0) = 2.40$ in
$w_f = (1.03)(10) = 10.3$ in
$t_o w_o L_o = t_f w_f L_f$
$(3)(10)(15 \times 12) = (2.4)(10.3) L_f$
$L_f = (3)(10)(15 \times 12)/(2.4)(10.3 \times 12) = \mathbf{18.2\ ft}$

(b) Given that $t_o w_o v_o = t_f w_f v_f$,
$v_f = (3)(10)(40)/(2.48 \times 10.3) = \mathbf{48.54\ ft/min}$

18.3 (SI units) A series of cold-rolling operations are used to reduce the thickness of a metal plate from 50 mm to 20 mm in a reversing two-high mill. Roll diameter = 600 mm, and coefficient of friction between rolls and work = 0.15. The specification is that the draft is equal on each pass. Determine the (a) minimum number of passes required and (b) draft for each pass?

Solution: (a) Maximum possible draft $d_{max} = \mu^2 R = (0.15)^2\,(300) = 6.75$ mm
Minimum number of passes = $(t_o - t_f)/d_{max} = (50 - 20)/6.75 = 4.44$ rounded up to **5 passes**

(b) Draft per pass $d = (50 - 20)/5 = \mathbf{6.0\ mm}$

18.4 (SI units) In the previous problem, suppose that the percent reduction rather than the draft were specified to be equal for each pass. (a) What is the minimum number of passes required? (b) What are the draft and exiting stock thickness for each pass?

Solution: (a) Maximum draft occurs on first pass: $d_{max} = \mu^2 R = (0.15)^2\,(300) = 6.75$ mm
This converts into a maximum possible reduction $x = 6.75/50 = 0.135$
Let x = fraction reduction per pass, and n = number of passes. The number of passes must be an integer. To reduce from $t_o = 50$ mm to $t_o = 20$ mm in n passes, the following relationship must be satisfied:
$50(1 - x)^n = 20$
$(1 - x)^n = 20/50 = 0.4$
$(1 - x) = 0.4^{1/n}$
Try $n = 5$: $(1 - x) = (0.4)^{1/5} = 0.8325$
$x = 1 - 0.8325 = 0.1674$, which exceeds the maximum possible reduction of 0.135.
Try $n = 6$: $(1 - x) = (0.4)^{1/6} = 0.8584$

$x = 1 - 0.8584 = 0.1416$, which exceeds the maximum possible reduction of 0.135.
Try $n = 7$: $(1 - x) = (0.4)^{1/7} = 0.8773$
$x = 1 - 0.8773 = 0.1227$, which is within the maximum possible reduction of 0.135.

(b) Pass 1: $d = 50(0.1227) =$ **6.135 mm**, $t_f = 50 - 6.135 =$ **43.86 mm**
Pass 2: $d = 43.86(0.1227) =$ **5.38 mm**, $t_f = 43.86 - 5.38 =$ **38.48 mm**
Pass 3: $d = 38.48(0.1227) =$ **4.72 mm**, $t_f = 38.48 - 4.72 =$ **33.76 mm**
Pass 4: $d = 33.76(0.1227) =$ **4.14 mm**, $t_f = 33.76 - 4.14 =$ **29.62 mm**
Pass 5: $d = 29.62(0.1227) =$ **3.63 mm**, $t_f = 29.62 - 3.63 =$ **25.99 mm**
Pass 6: $d = 25.99(0.1227) =$ **3.19 mm**, $t_f = 25.99 - 3.19 =$ **22.80 mm**
Pass 7: $d = 22.80(0.1227) =$ **2.80 mm**, $t_f = 22.80 - 2.80 =$ **20.00 mm**

18.5 (USCS units) A continuous hot-rolling mill has eight stands. The thickness of the starting slab = 3.0 in, its width = 15.0 in, and its length = 10 ft. The final thickness = 0.3 in. Roll diameter at each stand = 36 in, and rotational speed at stand number 1 = 30 rev/min. The speed of the slab entering stand 1 = 240 ft/min. Assume that no widening of the slab occurs during the rolling sequence. Percent reduction in thickness is equal at all stands, and it is assumed that the forward slip will be equal at each stand. Determine the (a) percent reduction at each stand, (b) rotational speed of the rolls at stands 2 through 8, and (c) forward slip. (d) What is the draft at stands 1 and 8? (e) What are the length and exit speed of the final strip exiting stand 8?

Solution: (a) To reduce from $t_o = 3.0$ in to $t_f = 0.3$ in over 8 stands, $3.0(1 - x)^8 = 0.3$
$(1 - x)^8 = 0.3/3.0 = 0.10$
$(1 - x) = (0.10)^{1/8} = 0.74989$
$x = 1 - 0.74989 =$ ***r* = 0.2501 = 25.01%** at each stand

(b) Forward slip s $= (v_f - v_r)/v_r$
$sv_r = v_f - v_r$
$(1 + s)v_r = v_f$
At stand 1: $(1 + s)v_{r1} = v_1$, where v_{r1} = roll speed, v_1 = exit speed of slab.
At stand 2: $(1 + s)v_{r2} = v_2$, where v_{r2} = roll speed, v_2 = exit speed of slab.
Etc.
At stand 8: $(1 + s)v_{r8} = v_8$, where v_{r8} = roll speed, v_8 = exit speed of slab.
By constant volume, $t_o w_o v_o = t_1 w_1 v_1 = t_2 w_2 v_2 = \ldots = t_8 w_8 v_8$
Since there is no change in width, $w_o = w_1 = w_2 = \ldots w_8$
Therefore, $t_o v_o = t_1 v_1 = t_2 v_2 = \ldots = t_8 v_8$
$t_o = 3.0$ in
$3v_o = 3(1 - r)v_1 = 3(1 - r)^2 v_2 = \ldots 3(1 - r)^8 v_8$, where $r = 0.2501$ as determined in part (a).
Since s is a constant, $v_{r1} : v_{r2} : \ldots : v_{r8} = v_1 : v_2 : \ldots : v_8$
Given that $N_{r1} = 30$ rev/min, $v_{r1} = \pi D N_{r1} = (2\pi \times 18/12)(30) = 282.78$ ft/min
In general $N_r = (30/282.78) = 0.10609 v_r$
$N_{r2} = 0.10609 \times 282.78/(1 - r) = 0.10609 \text{ x } 282.78/(1\text{-}0.2501) =$ **40 rev/min**
$N_{r3} = 0.10609 \times 282.78/(1 - r)^2 =$ **53.3 rev/min**
$N_{r4} = 0.10609 \times 282.78/(1 - r)^3 =$ **71.1 rev/min**
$N_{r5} = 0.10609 \times 282.78/(1 - r)^4 =$ **94.9 rev/min**

$N_{r6} = 0.10609 \times 282.78/(1 - r)^5 =$ **126.9.3 rev/min**
$N_{r7} = 0.10609 \times 282.78/(1 - r)^6 =$ **168.5 rev/min**
$N_{r8} = 0.10609 \times 282.78/(1 - r)^7 =$ **224.9 rev/min**

(c) Given $v_o = 240$ ft/min
$v_1 = 240/(1 - r) = 240/0.74989 = 320$ ft/min
$v_2 = 320/0.74989 = 426.8$ ft/min
From equations for forward slip, $(1 + s)v_{r1} = v_1$
$(1 + s)(282.78) = 320$
$(1 + s) = 320/282.78 = 1.132$ **s = 0.132**
Check with stand 2: given $v_2 = 426.8$ ft/min from above
$N_{r2} = 0.10609 v_{r2}$
Rearranging, $v_{r2} = N_{r2}/0.10609 = 9.426 N_{r2} = 0.426(40) = 377.04$ ft/min
$(1 + s)(377.04) = 426.8$
$(1 + s) = 426.8/377.14 = 1.132$ $s = 0.132$, as before

(d) Draft at stand 1 $d_1 = 3.0(0.2501) =$ **0.7503 in**
Draft at stand 8 $d_8 = 3.0(1 - 0.2501)^7(0.2501) =$ **0.10006 in**

(e) Length of final strip $L_f = L_8$
$t_o w_o L_o = t_8 w_8 L_8$
Given that $w_o = w_8$, $t_o L_o = t_8 L_8$
$3.0(10 \text{ ft}) = 0.3L_8$ $\mathbf{L_8 = 100}$ **ft**
$t_o w_o v_o = t_8 w_8 v_8$
$t_o v_o = t_8 v_8$
$v_8 = 240(3/0.3) =$ **2400 ft/min**

18.6 **(A)** (SI units) A low-carbon steel plate is 270 mm wide and 25 mm thick. It is reduced in one pass in a two-high rolling mill to a thickness of 20 mm. Roll radius = 600 mm, and roll speed = 30 m/min. Strength coefficient = 500 MPa, and strain hardening exponent = 0.25. Determine the (a) roll force, (b) roll torque, and (c) power required to perform the operation.

Solution: (a) Draft $d = 25 - 20 = 5$ mm
Contact length $L = (600 \times 5)^{.5} = 54.77$ mm
True strain $\varepsilon = \ln(25/20) = \ln 1.25 = 0.223$
$\bar{Y}_f = 500(0.223)^{0.25}/1.25 = 274.9$ MPa
Rolling force $F = 274.9(270)(54.77) =$ **4,065,194 N**

(b) Torque $T = 0.5(4{,}065{,}194)(54.77 \times 10^{-3}) =$ **111,325 N-m**

(c) $N = (30 \text{ m/min})/(2\pi \times 0.600) = 7.96$ rev/min $= 0.133$ rev/s
Power $P = 2\pi(0.133)(4{,}065{,}194)(54.77 \times 10^{-3}) = 186{,}061$ N-m/s $= 186{,}061$ W $=$ **186.1 kW**

18.7 (SI units) Solve the previous problem using a roll radius = 300 mm.

Solution: (a) Draft $d = 25 - 20 = 5$ mm
Contact length $L = (300 \times 5)^{.5} = 38.73$ mm
True strain $\varepsilon = \ln(25/20) = \ln 1.25 = 0.223$
$\bar{Y}_f = 500(0.223)^{0.25}/1.25 = 274.9$ MPa

Rolling force $F = 274.9(270)(38.73) =$ **2,874,657 N**

(b) Torque $T = 0.5(2{,}874{,}657)(38.73 \times 10^{-3}) =$ **55,668 N-m**

(c) $N = (30 \text{ m/min})/(2\pi \times 0.300) = 15.9$ rev/min $= 0.265$ rev/s
Power $P = 2\pi(0.265)(2{,}874{,}657)(38.73 \times 10^{-3}) = 185{,}378$ N-m/s $= 185{,}378$ W $=$ **185.4 kW**

Note that the force and torque are reduced as roll radius is reduced, but that the power remains the same (within calculation error) as in the previous problem.

18.8 (SI units) Solve the previous problem except that a cluster mill with working rolls of radius = 60 mm is used.

Solution: (a) Draft $d = 25 - 20 = 5$ mm,
Contact length $L = (60 \times 5)^{.5} = 17.32$ mm
True strain $\varepsilon = \ln(25/20) = \ln 1.25 = 0.223$
$\bar{Y}_f = 500(0.223)^{0.25}/1.25 = 274.9$ MPa
Rolling force $F = 274.9(270)(17.32) =$ **1,285,542 N**

(b) Torque $T = 0.5(1{,}285{,}542)(17.32 \times 10^{-3}) =$ **11,133 N-m**

(c) $N = (30 \text{ m/min})/(2\pi \times 0.060) = 79.58$ rev/min $= 1.326$ rev/s
Power $P = 2\pi(1.326)(1{,}285{,}542)(17.32 \times 10^{-3}) = 185{,}506$ N-m/s $= 185{,}506$ W $=$ **185.5 kW**

Note that this is the same power value (within calculation error) as in the two previous problems. In fact, power would probably increase because of lower mechanical efficiency in the cluster type rolling mill.

18.9 (USCS units) A 4-in-thick aluminum slab is 10 in wide and 100 in long. It is reduced in one pass in a two-high rolling mill to a thickness = 3.5 in. The roll rotates at a speed = 12 rev/min and has a radius = 20 in. The aluminum has a strength coefficient = 25,000 lb/in^2 and a strain hardening exponent = 0.20. Determine the (a) roll force, (b) roll torque, and (c) power required to accomplish this operation.

Solution: (a) Draft $d = 4.00 - 3.5 = 0.5$ in,
Contact length $L = (20 \times 0.5)^{0.5} = 3.16$ in
True strain $\varepsilon = \ln(4.0/3.5) = \ln 1.143 = 0.1335$
$\bar{Y}_f = 25{,}000(0.1335)^{0.20}/1.20 = 13{,}928$ lb/in^2
Rolling force $F = \bar{Y}_f wL = 13{,}928(10)(3.16) =$ **440,125 lb**

(b) Torque $T = 0.5FL = 0.5(440{,}125)(3.16) =$ **789,700 in-lb**

(c) $N = 12$ rev/min
Power $P = 2\pi(12)(440{,}125)(3.16) = 104{,}863{,}473$ in-lb/min
Given that 1 hp = 396,000 in-lb/min,
$HP = (104{,}863{,}473)/(396{,}000) =$ **265 hp**

18.10 (SI units) A single-pass rolling operation reduces a 20-mm-thick plate to 18 mm. The starting plate is 200 mm wide. Roll radius = 250 mm and rotational speed = 12 rev/min. The work metal has a strength coefficient = 600 MPa and a strain hardening exponent = 0.22. Determine the (a) roll force, (b) roll torque, and (c) power required for this operation.

Solution: (a) Draft $d = 20 - 18 = 2.0$ mm
Contact length $L = (250 \times 2)^{.5} = 22.36$ mm = 0.02236 m
True strain $\varepsilon = \ln(20/18) = \ln 1.111 = 0.1054$
$\overline{Y}_f = 600(0.1054)^{0.22}/1.22 = 300$ MPa
Rolling force $F = 300(200)(22.36) =$ **1,341,600 N**

(b) Torque $T = 0.5(1{,}341{,}600)(0.02236)$ **= 14,999 N-m**

(c) Given that $N = 12$ rev/min = 0.2 rev/s
Power $P = 2\pi(0.2)(1{,}341{,}600)(0.02236) =$ **37,697 W = 37.7 kW**

18.11 **(A)** (USCS units) A hot-rolling operation is performed using rolls of diameter = 24 in. The rolling stand can exert a maximum force of 400,000 lb, and the mill has a maximum horsepower of 100 hp. It is desired to reduce a 1.5-in-thick plate by the maximum possible draft in one pass. The starting plate is 10 in wide. In the heated condition, the work material has a strength coefficient = 20,000 lb/in^2 and a strain hardening exponent = 0. Determine the (a) maximum possible draft, (b) associated true strain, and (c) maximum speed of the rolls for the operation.

Solution: (a) Assumption: maximum possible draft is determined by the force capability of the rolling mill and not by coefficient of friction between the rolls and the work.
Draft $d = 1.5 - t_f$
Contact length $L = (12d)^{0.5}$
$\overline{Y}_f = 20{,}000(\varepsilon)^0/1.0 = 20{,}000$ lb/in^2
Force $F = 20{,}000(10)(12d)^{0.5} = 400{,}000$ (the limiting force of the rolling mill)
$(12d)^{0.5} = 400{,}000/200{,}000 = 2.0$
$12\ d = 2.0^2 = 4$
$\boldsymbol{d = 4/12 = 0.333}$ **in**

(b) True strain $\varepsilon = \ln(1.5/t_f)$
$t_f = t_o - d = 1.5 - 0.333 = 1.167$ in
$\varepsilon = \ln(1.5/1.167) = \ln 1.285 =$ **0.251**

(c) Given maximum possible power HP = 100 hp = 100 x 396000 (in-lb/min)/hp = 39,600,000 in-lb/min
Contact length $L = (12 \times 0.333)^{0.5} = 2.0$ in
$P = 2\pi N(400{,}000)(2.0) = 5{,}026{,}548N$ in-lb/min
$5{,}026{,}548N = 39{,}600{,}000$
$N = 7.88$ rev/min
$v_r = 2\pi RN = 2\pi(12/12)(7.88) =$ **49.5 ft/min**

18.12 (USCS units) Solve the previous problem except that the operation is warm rolling and the strain-hardening exponent is 0.18. Assume the strength coefficient remains at 20,000 lb/in^2.

Solution: (a) Assumption (same as in previous problem): maximum possible draft is determined by the force capability of the rolling mill and not by coefficient of friction between the rolls and the work.
Draft $d = 1.5 - t_f$
Contact length $L = (12d)^{0.5}$

$\varepsilon = \ln(1.5/t_f)$
$\bar{Y}_f = 20{,}000(\varepsilon)^{0.18}/1.18 = 16{,}949\varepsilon^{.18}$
$F = \bar{Y}_f(10)(12d)^{0.5} = 34.641\bar{Y}_f\ (d)^{0.5} = 400{,}000$ (as given)
$\bar{Y}_f\ (d)^{0.5} = 400{,}000/34.641 = 11{,}547$
Now use trial-and-error to values of $\bar{Y}_f$ and d that fit this equation.
Try $d = 0.3$ in, $t_f = 1.5 - 0.3 = 1.2$ in
$\varepsilon = \ln(1.5/1.2) = \ln 1.25 = 0.223$
$\bar{Y}_f = 16{,}949(0.223)^{.18} = 13{,}134$ lb/in^2.
$(d)^{0.5} = 11{,}547/13{,}134 = 0.8791$
$d = 0.773$, which does not equal the initial trial value of $d = 0.3$

Try $d = 0.5$ in, $t_f = 1.5 - 0.5 = 1.0$ in
$\varepsilon = \ln(1.5/1.0) = \ln 1.50 = 0.4055$
$\bar{Y}_f = 16{,}949(0.4055)^{.18} = 14{,}538$ lb/in^2.
$(d)^{0.5} = 11{,}547/14{,}538 = 0.7942$
$d = 0.631$, which does not equal the trial value of $d = 0.5$

Try $d = 0.6$ in, $t_f = 1.5 - 0.6 = 0.9$ in
$\varepsilon = \ln(1.5/0.9) = 0.5108$
$\bar{Y}_f = 16{,}949(0.5108)^{.18} = 15{,}120$ lb/in^2.
$(d)^{0.5} = 11{,}547/15{,}120 = 0.7637$
$d = 0.583$, which is too much compared to $d = 0.6$

Try $d = 0.58$ in, $t_f = 1.5 - 0.58 = 0.92$ in
$\varepsilon = \ln(1.5/0.92) = \ln 1.579 = 0.489$
$\bar{Y}_f = 16{,}949(0.489)^{.18} = 15{,}007$ lb/in^2.
$(d)^{0.5} = 11{,}547/15{,}007 = 0.769$
$d = 0.592$, which is close but still above the trial value of $d = 0.55$

Try $d = 0.585$ in, $t_f = 1.50 - 0.585 = 0.915$ in
$\varepsilon = \ln(1.5/0.915) = 0.494$
$\bar{Y}_f = 16{,}949(0.494)^{.18} = 15{,}036$ lb/in^2.
$(d)^{0.5} = 11{,}547/15{,}036 = 0.768$
$d = 0.590$, which is close but still above the trial value of $d = 0.585$.

Try $d = 0.588$ in, $t_f = 1.50 - 0.588 = 0.912$ in
$\varepsilon = \ln(1.5/0.912) = 0.498$
$\bar{Y}_f = 16{,}949(0.498)^{.18} = 15{,}053$ lb/in^2.
$(d)^{0.5} = 11{,}547/15{,}053 = 0.767$
$d = 0.588$, which is almost the same as the trial value of ***d* = 0.588.**

(b) True strain $\varepsilon = \ln(1.5/0.912) =$ **0.498**

(c) Given maximum possible power HP = 100 hp = 100 × 396000 (in-lb/min)/hp
= 39,600,000 in-lb/min

Contact length $L = (12 \times 0.588)^{0.5} = 2.66$ in
$P = 2\pi N(400{,}000)(2.66) = 6{,}685{,}000N$ in-lb/min
$6{,}486{,}000N = 39{,}600{,}000$
$N = 5.92$ rev/min
$v_r = 2\pi RN = 2\pi(12/12)(5.92) =$ **37.2 ft/min**

Forging

18.13 **(A)** (SI units) A cylindrical part is upset-forged in an open die. The starting diameter = 50 mm and height = 40 mm. Height after forging = 30 mm. Coefficient of friction at the die-work interface = 0.20. Yield strength of the low-carbon steel = 105 MPa; its strength coefficient = 500 MPa and strain-hardening exponent = 0.25. Determine the force in the operation (a) just as the yield point is reached (yield at strain = 0.002), (b) at a height of 35 mm, and (c) at a height of 30 mm. Use of a spreadsheet calculator is recommended.

Solution: (a) $V = \pi D^2L/4 = \pi(50)^2(40)/4 = 78{,}540$ mm^3
Given $\varepsilon = 0.002$, $Y_f = 500(0.002)^{0.25} = 105.7$ MPa, and $h = 40 - 40(0.002) = 39.92$ mm
$A = V/h = 78{,}540/39.92 = 1967$ mm^2
$K_f = 1 + 0.4(0.2)(50)/39.92 = 1.10$
$F = 1.10(105.7)(1967) =$ **228,745 N**

(b) Given $h = 35$, $\varepsilon = \ln(40/35) = \ln 1.143 = 0.1335$
$Y_f = 500(0.1335)^{0.25} = 302.2$ MPa
$V = 78{,}540$ mm^3 from part (a) above.
At $h = 35$, $A = V/h = 78{,}540/35 = 2244$ mm^2
Corresponding $D = 53.5$ mm (from $A = \pi D^2/4$)
$K_f = 1 + 0.4(0.2)(53.5)/35 = 1.122$
$F = 1.122(302.2)(2244) =$ **760,989 N**

(c) Given $h = 30$, $\varepsilon = \ln(40/30) = \ln 1.333 = 0.2877$
$Y_f = 500(0.2877)^{0.25} = 366.2$ MPa
$V = 78{,}540$ mm^3 from part (a) above.
At $h = 30$, $A = V/h = 78{,}540/30 = 2618$ mm^2
Corresponding $D = 57.74$ mm (from $A = \pi D^2/4$)
$K_f = 1 + 0.4(0.2)(57.74)/30 = 1.154$
$F = 1.154(366.2)(2618) =$ **1,106,315 N**

18.14 (USCS units) A cylindrical work part with diameter = 2.5 in, and height = 2.5 in is upset forged in an open die to a height = 1.5 in. Coefficient of friction at the die-work interface = 0.10. The work metal has a strength coefficient = 40,000 lb/in^2 and strain-hardening exponent = 0.15. Yield strength = 15,750 lb/in^2. Determine the instantaneous force in the operation (a) just as the yield point is reached (yield at strain = 0.002), (b) at height $h = 2.3$ in, (c) $h = 2.1$ in, (d) $h = 1.9$ in, (e) $h = 1.7$ in, and (f) $h = 1.5$ in. Use of a spreadsheet calculator is recommended.

Solution: (a) $V = \pi D^2L/4 = \pi(2.5)^2(2.5)/4 = 12.273$ in^3
Given $\varepsilon = 0.002$, $Y_f = 40{,}000(0.002)^{0.15} = 15{,}748$ lb/in^2 and $h = 2.5 - 2.5(0.002) = 2.495$
$A = V/h = 12.273/2.495 = 4.92$ in^2
$K_f = 1 + 0.4(0.1)(2.5)/2.495 = 1.04$

F = 1.04(15,748)(4.92) = **80,579 lb**

(b) Given h = 2.3, ε = ln(2.5/2.3) = ln 1.087 = 0.0834
Y_f = 40,000(0.0834)$^{0.15}$ = 27,556 lb/in^2
V = 12.273 in^3 from part (a) above.
At h = 2.3, A = V/h = 12.273/2.3 = 5.34 in^2
Corresponding D = 2.61 (from $A = \pi D^2/4$)
K_f = 1 + 0.4(0.1)(2.61)/2.3 = 1.045
F = 1.045(27,556)(5.34) = **153,822 lb**

(c) Given h = 2.1, ε = ln(2.5/2.1) = ln 1.191 = 0.1744
Y_f = 40,000(0.1744)$^{0.15}$ = 30,780 lb/in^2
V = 12.273 in^3 from part (a) above.
At h = 2.1, A = V/h = 12.273/2.1 = 5.84 in^2
Corresponding D = 2.73 (from $A = \pi D^2/4$)
K_f = 1 + 0.4(0.1)(2.73)/2.1 = 1.052
F = 1.052(30,780)(5.84) = **189,236 lb**

(d) Given h = 1.9, ε = ln(2.5/1.9) = ln 1.316 = 0.274
Y_f = 40,000(0.274)$^{0.15}$ = 32,948 lb/in^2
V = 12.273 in^3 from part (a) above.
At h = 1.9, A = V/h = 12.273/1.9 = 6.46 in^2
Corresponding D = 2.87 (from $A = \pi D^2/4$)
K_f = 1 + 0.4(0.1)(2.87)/1.9 = 1.060
F = 1.060(32,948)(6.46) = **225,695 lb**

(e) Given h = 1.7, ε = ln(2.5/1.7) = ln 1.471 = 0.386
Y_f = 40,000(0.386)$^{0.15}$ = 34,673 lb/in^2
V = 12.273 in^3 from part (a) above.
At h = 1.7, A = V/h = 12.273/1.7 = 7.22 in^2
Corresponding D = 3.03 (from $A = \pi D^2/4$)
K_f = 1 + 0.4(0.1)(3.03)/1.7 = 1.071
F = 1.071(34,673)(7.22) = **268,176 lb**

(f) Given h = 1.5, ε = ln(2.5/1.5) = ln 1.667 = 0.511
Y_f = 40,000(0.511)$^{0.15}$ = 36,166 lb/in^2
V = 12.273 in^3 from part (a) above.
At h = 1.5, A = V/h = 12.273/1.5 = 8.182 in^2
Corresponding D = 3.23 (from $A = \pi D^2/4$)
K_f = 1 + 0.4(0.1)(3.23)/1.5 = 1.086
F = 1.086(36,166)(8.182) = **321,379 lb**

18.15 (USCS units) A cylindrical work part has a diameter = 2.5 in and a height = 4.0 in. It is upset forged to a height = 2.75 in. Coefficient of friction at the die-work interface = 0.10. The work material has a flow curve with strength coefficient = 25,000 lb/in^2 and strain-hardening exponent = 0.22. Determine the plot of force versus work height. Use of a spreadsheet calculator is recommended.

Solution: Volume of cylinder $V = \pi D^2L/4 = \pi(2.5)^2(4.0)/4$ = 19.635 in^3

Compute force F at selected values of height: h = (a) 4.0, (b) 3.75, (c) 3.5, (d) 3.25, (e) 3.0, (f) 2.75, and (g) 2.5. These values can be used to develop the plot. The shape of the plot will be similar to Figure 18.12 in the text.

At $\boldsymbol{h}$ **= 4.0**, assume yielding has just occurred and the height has not changed significantly. Use $\varepsilon = 0.002$ (the approximate yield point of metal).
At $\varepsilon = 0.002$, $Y_f = 25{,}000(0.002)^{0.22} = 6{,}370$ lb/in^2
Adjusting the height for this strain, $h = 4.0 - 4.0(0.002) = 3.992$
$A = V/h = 19.635/3.992 = 4.92$ in^2
$K_f = 1 + 0.4(0.1)(2.5)/3.992 = 1.025$
$F = 1.025(6{,}370)(4.92) =$ **32,125 lb**

At $\boldsymbol{h}$ **= 3.75**, $\varepsilon = \ln(4.0/3.75) = \ln 1.0667 = 0.0645$
$Y_f = 25{,}000(0.0645)^{0.22} = 13{,}680$ lb/in^2
$V = 19.635$ in^3 calculated above.
At $h = 3.75$, $A = V/h = 19.635/3.75 = 5.236$ in^2
Corresponding $D = 2.582$ (from $A = \pi D^2/4$)
$K_f = 1 + 0.4(0.1)(2.582)/3.75 = 1.028$
$F = 1.028(13{,}680)(5.236) =$ **73,601 lb**

At $\boldsymbol{h}$ **= 3.5**, $\varepsilon = \ln(4.0/3.5) = \ln 1.143 = 0.1335$
$Y_f = 25{,}000(0.1335)^{0.22} = 16{,}053$ lb/in^2
At $h = 3.5$, $A = V/h = 19.635/3.5 = 5.61$ in^2
Corresponding $D = 2.673$ (from $A = \pi D^2/4$)
$K_f = 1 + 0.4(0.1)(2.673)/3.5 = 1.031$
$F = 1.031(16{,}053)(5.61) =$ **92,808 lb**

At $\boldsymbol{h}$ **= 3.25**, $\varepsilon = \ln(4.0/3.25) = \ln 1.231 = 0.2076$
$Y_f = 25{,}000(0.2076)^{0.22} = 17{,}691$ lb/in^2
At $h = 3.25$, $A = V/h = 19.635/3.25 = 6.042$ in^2
Corresponding $D = 2.774$ (from $A = \pi D^2/4$)
$K_f = 1 + 0.4(0.1)(2.774)/3.25 = 1.034$
$F = 1.034(17{,}691)(6.042) =$ **110,538 lb**

At $\boldsymbol{h}$ **= 3.0**, $\varepsilon = \ln(4.0/3.0) = \ln 1.333 = 0.2874$
$Y_f = 25{,}000(0.2874)^{0.22} = 19{,}006$ lb/in^2
At $h = 3.0$, $A = V/h = 19.635/3.0 = 6.545$ in^2
Corresponding $D = 2.887$ (from $A = \pi D^2/4$)
$K_f = 1 + 0.4(0.1)(2.887)/3.0 = 1.038$
$F = 1.038(19{,}006)(6.545) =$ **129,182 lb**

At $\boldsymbol{h}$ **=2.75**, $\varepsilon = \ln(4.0/2.75) = \ln 1.4545 = 0.3747$
$Y_f = 25{,}000(0.3747)^{0.22} = 20{,}144$ lb/in^2
$V = 19.635$ in^3 calculated above.
At $h = 2.75$, $A = V/h = 19.635/2.75 = 7.140$ in^2
Corresponding $D = 3.015$ (from $A = \pi D^2/4$)
$K_f = 1 + 0.4(0.1)(3.015)/2.75 = 1.044$
$F = 1.044(20{,}144)(7.140) =$ **150,136 lb**

18.16 (SI units) A cold heading operation forms the head on a steel nail. The strength coefficient for the steel = 500 MPa, and the strain-hardening exponent = 0.22. Coefficient of friction at the die-work interface = 0.14. The diameter of the wire stock for the nail = 4.0 mm. The nail head diameter = 10 mm, and its thickness = 1.5 mm. The final length of the nail = 60 mm. (a) What length of stock must project out of the die in order to provide sufficient volume of work material for this upsetting operation? (b) Compute the maximum force that the punch must apply to form the head in this open-die operation.

Solution: (a) Volume of nail head $V = \pi D_f^2 h_f/4 = \pi(10)^2(1.5)/4 = 117.8$ mm^3.
Cross-sectional area of wire stock $A_o = \pi D_o^2/4 = \pi(4.0)^2/4 = 12.57$ mm^2
$h_o = V/A_o = 117.8/12.57 =$ **9.37 mm**

(b) $\varepsilon = \ln(9.37/1.5) = \ln 6.25 = 1.83$
$Y_f = 500(1.83)^{0.22} = 571$ MPa
Area of head $A_f = \pi(10)^2/4 = 78.5$ mm^2
$K_f = 1 + 0.4(0.14)(10/1.5) = 1.37$
$F = 1.37(571)(78.5) =$ **61,558 N**

18.17 **(A)** (SI units) A hot upset-forging operation is performed in an open die. Initial diameter of the cylindrical work part = 30 mm, and its height = 50 mm. The part is upset to an average diameter = 45 mm. The work metal at this elevated temperature yields at 90 MPa. Coefficient of friction at the die-work interface = 0.40. Determine the (a) final height of the part and (b) maximum force in the operation.

Solution: (a) $V = \pi D_o^2 h_o/4 = \pi(30)^2(50)/4 = 35,343$ mm^3
$A_f = \pi D_f^2/4 = \pi(45)^2/4 = 1590$ mm^2
$h_f = V/A_f = 35,343/1590 =$ **22.2 mm**

(b) $\varepsilon = \ln(50/22.2) = \ln 2.25 = 0.811$
Given a hot forging operation, assume the strength coefficient is equal to the yield strength and the strain hardening exponent is zero.
$Y_f = 90(0.811)^0 = 90$ MPa
Force is maximum at largest area value, $A_f = 1590$ mm^2
$D = D_f = 45$ mm
$K_f = 1 + 0.4(0.4)(45/22.22) = 1.324$
$F = 1.324(90)(1590) =$ **189,469 N**

18.18 (SI units) A hydraulic forging press is capable of exerting a maximum force = 1,000,000 N. A cylindrical work part is to be cold-upset-forged. The starting part has diameter = 30 mm and height = 30 mm. The flow curve of the metal is defined by a strength coefficient = 400 MPa and strain-hardening exponent = 0.20. Determine the maximum reduction in height to which the part can be compressed with this forging press if the coefficient of friction = 0.1. A trial-and-error solution is required, so use of a spreadsheet calculator is recommended.

Solution: Volume of work $V = \pi D_o^2 h_o/4 = \pi(30)^2(30)/4 = 21,206$ mm^3.
Final area $A_f = 21,206/h_f$
$\varepsilon = \ln(30/h_f)$
$Y_f = 400\varepsilon^{0.2} = 400(\ln 30/h_f)^{0.2})$
$K_f = 1 + 0.4\mu(D_f/h_f) = 1 + 0.4(0.1)(D_f/h_f)$

Forging force $F = K_f Y_f A_f = (1 + 0.04D_f/h_f)(\ 400(\ln 30/h_f)^{0.2})(\ 21{,}206/h_f)$

Requires trial and error solution to find the value of h_f that matches a force = 1,000,000 N.
(1) Try h_f = 20 mm
A_f = 21,206/20 = 1060.3 mm^2
ε = ln(30/20) = ln 1.5 = 0.405
$Y_f = 400(0.405)^{0.2}$ = 333.9 MPa
$D_f = (4 \times 1060.3/\pi)^{0.5}$ = 36.7 mm
K_f = 1 + 0.04(36.7/20) = 1.073
F = 1.073(333.9)(1060.3) = **380,050 N**
Too low. Try a smaller value of h_f to increase F.

(2) Try h_f = 10 mm.
A_f = 21,206/10 = 2120.6 mm^2
ε = ln(30/10) = ln 3.0 = 1.099
$Y_f = 400(1.099)^{0.2}$ = 407.6 MPa
$D_f = (4 \times 2120.6/\pi)^{0.5}$ = 51.96 mm
K_f = 1 + 0.04(51.96/10) = 1.208
F = 1.208(407.6)(2120.6) = **1,043,998 N**
Slightly high. Need to try a value of h_f between 10 and 20, closer to 10.

(3) Try h_f = 11 mm
A_f = 21,206/11 = 1927.8 mm^2
ε = ln(30/11) = ln 2.7273 = 1.003
$Y_f = 400(1.003)^{0.2}$ = 400.3 MPa
$D_f = (4 \times 1927.8/\pi)^{0.5}$ = 49.54 mm
K_f = 1 + 0.04(51.12/11) = 1.18
F = 1.18(400.3)(1927.8) = **910,653 N**

(4) By linear interpolation, try h_f = 10 + (44/133) = 10.33 mm
A_f = 21,206/10.33 = 2052.8 mm^2
ε = ln(30/10.33) = ln 2.9042 = 1.066
$Y_f = 400(1.066)^{0.2}$ = 405.16 MPa
$D_f = (4 \times 2052.8/\pi)^{0.5}$ = 51.12 mm
K_f = 1 + 0.04(51.12/10.33) = 1.198
F = 1.198(405.16)(2052.8) = **996,364 N**

(5) By further linear interpolation, try h_f = 10 + (44/48)(0.33) = 10.30
A_f = 21,206/10.30 = 2058.8 mm^2
ε = ln(30/10.30) = ln 2.913 = 1.069
$Y_f = 400(1.069)^{0.2}$ = 405.38 MPa
$D_f = (4 \times 2058.8/\pi)^{0.5}$ = 51.2 mm
K_f = 1 + 0.04(51.2/10.3) = 1.199
F = 1.199(405.38)(2058.8) = **1,000,553 N**
Close enough! Maximum height reduction = 30.0 – 10.3 = **19.7 mm**

Using a spreadsheet calculator, the author's program (written in Excel) obtained a value of h = 19.69603 mm to achieve a force of 1,000,000 lb within one pound.

18.19 (SI units) A connecting rod is hot-forged in an impression die. The projected area of the part = 4150 mm^2. The design of the die causes flash to form during forging, so that the area, including flash, = 5820 mm^2. Part geometry is simple. At room temperature, the work metal has a strength coefficient = 800 MPa and a strain-hardening exponent = 0.15. As heated, the work metal yields at 200 MPa and has no tendency to strain-harden. Determine the maximum force required to perform the operation.

Solution: Since the work material has no tendency to work harden, $n = 0$
From Table 18.1, choose K_f =6.0
F = 6.0(200)(5820) = **6,984,000 N**

18.20 **(A)** (USCS units) A part is hot-forged in an impression die. The projected area of the part, including flash, = 14.1 in^2. After trimming, the part has a projected area of 10.3 in^2. Part geometry is complex. At room temperature, the work metal has a strength coefficient = 35,000 lb/in^2 and a strain-hardening exponent = 0.15. As heated, the work metal yields at 12,000 lb/in^2 and no strain hardening occurs. Determine the maximum force required to perform the forging operation.

Solution: Since the work material has no tendency to work harden, $n = 0$
From Table 18.1, choose K_f = 8.0
F = 8.0(12,000)(14.1) = **1,353,600 lb**

Extrusion

18.21 **(A)** (SI units) A cylindrical billet is 200 mm long and 100 mm in diameter. It is reduced by indirect extrusion to a 30-mm diameter. Die angle = 90°. In the Johnson equation, a = 0.8 and b = 1.5. In the flow curve for the work metal, strength coefficient = 250 MPa and strain-hardening exponent = 0.15. Determine the (a) extrusion ratio, (b) true strain (homogeneous deformation), (c) extrusion strain, (d) ram pressure, and (e) ram force.

Solution: (a) $r_x = A_o/A_f = D_o^2/D_f^2 = (100)^2/(30)^2 =$ **11.11**

(b) $\varepsilon = \ln r_x = \ln 11.11 =$ **2.41**

(c) $\varepsilon_x = a + b \ln r_x = 0.8 + 1.5(2.41) =$ **4.41**

(d) $\overline{Y}_f = 250(2.41)^{0.15}/1.15 = 248$ MPa
$p = 248(4.41) =$ **1094 MPa**

(e) $A_o = \pi D_o^2/4 = \pi(100)^2/4 = 7854$ mm^2
$F = 1094(7854) =$ **8,592,256 N**

18.22 (SI units) Solve the previous problem except direct extrusion is used instead of indirect extrusion. Determine the (a) extrusion ratio, (b) true strain (homogeneous deformation), (c) extrusion strain, (d) ram pressure, and (e) ram force. Solve parts (d) and (e) at the following two billet lengths during the process: L = 175 mm and L = 25 mm.

Solution: (a) $r_x = A_o/A_f = D_o^2/D_f^2 = (100)^2/(30)^2 =$ **11.11**

(b) $\varepsilon = \ln r_x = \ln 11.11 =$ **2.41**

(c) $\varepsilon_x = a + b \ln r_x = 0.8 + 1.5(2.41) =$ **4.41**

(d) $\bar{Y}_f = 250(2.41)^{0.15}/1.15 = 248$ MPa
At L = 175 mm, pressure $p = 248(4.41 + 2 \times 175/100) =$ **1,961.7 MPa**
At L = 25 mm, pressure $p = 248(4.41 + 2 \times 25/100) =$ **1,217.7 MPa**

(e) $A_o = \pi D_o^2/4 = \pi(100)^2/4 = 7854$ mm^2
At L = 175 mm, Force $F = 1961.7(7854) =$ **15,407,192 N**
At L = 25 mm, Force $F = 1217.7(7854) =$ **9,563,816 N**

18.23 (USCS units) A 5.0-in-long cylindrical billet whose diameter = 2.0 in is reduced by indirect extrusion to a diameter = 0.50 in. Die angle = 90°. In the Johnson equation, $a = 0.8$ and $b = 1.5$. The work metal is low-carbon steel with a strength coefficient = 75,000 lb/in^2 and strain hardening exponent = 0.25. Determine the (a) extrusion ratio, (b) true strain (homogeneous deformation), (c) extrusion strain, (d) ram pressure, (e) ram force, and (f) power if the ram speed = 20 in/min.

Solution: (a) $r_x = A_o/A_f = D_o^2/D_f^2 = (2.0)^2/(0.50)^2 = 4^2 =$ **16.0**

(b) $\varepsilon = \ln r_x = \ln 16 =$ **2.773**

(c) $\varepsilon_x = a + b \ln r_x = 0.8 + 1.5(2.773) =$ **4.959**

(d) $\bar{Y}_f = 75{,}000(2.773)^{0.25}/1.25 = 77{,}423$ lb/in^2
$p = 77{,}423(4.959) =$ **383,934 lb/in²**

(e) $A_o = \pi D_o^2/4 = \pi(2.0)^2/4 = 3.1416$ in^2
$F = (383{,}934)(3.1416) =$ **1,206,164 lb**

(f) $P = 1{,}206{,}164(20) =$ **24,123,285 in-lb/min**
Given that 1 hp = 396,000 in-lb/min,
$HP = 24{,}123{,}285/396{,}000 =$ **60.92 hp**

18.24 (SI units) A low-carbon steel billet is 75 mm long with diameter = 35 mm. It is direct extruded to a diameter = 20 mm using an extrusion die angle = 75°. The steel's strength coefficient = 500 MPa, and its strain hardening exponent = 0.25. In the Johnson extrusion strain equation, $a = 0.8$ and $b = 1.4$. Determine the (a) extrusion ratio, (b) true strain (homogeneous deformation), (c) extrusion strain, and (d) ram pressure and force at L = 70, 50, 30, and 10 mm. Use of a spreadsheet calculator is recommended for part (d).

Solution: (a) $r_x = A_o/A_f = D_o^2/D_f^2 = (35)^2/(20)^2 =$ **3.0625**

(b) $\varepsilon = \ln r_x = \ln 3.0625 =$ **1.119**

(c) $\varepsilon_x = a + b \ln r_x = 0.8 + 1.4(1.119) =$ **2.367**

(d) $\bar{Y}_f = 500(1.119)^{0.25}/1.25 = 411.4$ MPa
$A_o = \pi(35)^2/4 = 962.1$ mm^2

It is appropriate to determine the volume of metal contained in the cone of the die at the start of the extrusion operation, to assess whether metal has been forced through the die opening by the time the billet has been reduced from L = 75 mm to L = 70 mm. For a cone-shaped die with angle = 75°, the height h of the frustum is formed by metal being compressed into

the die opening: The two radii are: $R_1 = 0.5D_o = 17.5$ mm and $R_2 = 0.5D_f = 10$ mm, and $h = (R_1 - R_2)/\tan 75 = 7.5/\tan 75 = 2.01$ mm.
Frustum volume $V = 0.333\pi h(R_1^2 + R_1R_2 + R_2^2) = 0.333\pi(2.01)(17.5^2 + 10 \times 17.5 + 10^2) = 1223.4$ mm^3. Compare this with the volume of the portion of the cylindrical billet between $L = 75$ mm and $L = 70$ mm. $V = \pi D_o^2\, h/4 = 0.25\pi(35)^2(75 - 70) = 4810.6$ mm^3. Since this volume is greater than the volume of the frustum, this means that the metal has extruded through the die opening by the time the ram has moved forward by 5 mm.

***L* = 70 mm**: pressure $p = 411.4(2.367 + 2 \times 70/35) =$ **2619.4 MPa**
Force $F = 2619.4(962.1) =$ **2,520,109 N**
***L* = 50 mm**: pressure $p = 411.4(2.367 + 2 \times 50/35) =$ **2149.2 MPa**
Force $F = 2149.2(962.1) =$ **2,067,757 N**
***L* = 30 mm**: pressure $p = 411.4(2.367 + 2 \times 30/35) =$ **1679.0 MPa**
Force $F = 1679.0(962.1) =$ **1,615,405 N**
***L* = 10 mm**: pressure $p = 411.4(2.367 + 2 \times 10/35) =$ **1208.9 MPa**
Force $F = 1208.9(962.1) =$ **1,163,053 N**

18.25 (USCS units) A direct extrusion operation is performed on a cylindrical billet with initial diameter = 2.0 in and length = 4.0 in. Die angle = 60°, and die orifice diameter is 0.50 in. In the Johnson extrusion strain equation, $a = 0.8$ and $b = 1.5$. The operation is carried out hot, and the hot metal yields at 13,000 lb/in^2 and does not strain-harden. (a) What is the extrusion ratio? (b) Determine the ram position at the point when the metal has been compressed into the cone of the die and starts to extrude through the die opening. (c) What is the ram pressure corresponding to this position? (d) Also determine the length of the final part if the ram stops its forward movement at the start of the die cone.

Solution: (a) $r_x = A_o/A_f = D_o^2/D_f^2 = (2.0)^2/(0.5)^2 =$ **16.0**

(b) The portion of the billet that is compressed into the die cone forms a frustum with $R_1 = 0.5D_o = 1.0$ in and $R_2 = 0.5D_f = 0.25$ in. The height of the frustum $h = (R_1 - R_2)/\tan 65 = (1.0 - 0.25)/\tan 60 = 0.433$ in. The volume of the frustum is
$V = 0.333\pi h(R_1^2 + R_1R_2 + R_2^2) = 0.333\pi(0.433)(1.0^2 + 1.0 \times 0.25 + 0.25^2) = 0.595$ in^3
The billet has advanced a certain distance by the time this frustum is completely filled and extrusion through the die opening is therefore initiated. The volume of billet compressed forward to fill the frustum is given by:
$V = \pi R_1^2(L_o - L_1) = \pi(1.0)^2(L_o - L_1)$
Setting this equal to the volume of the frustum,
$\pi(L_o - L_1) = 0.595$ in^3
$(L_o - L_1) = 0.595/\pi = 0.189$ in
$L_1 = 4.0 - 0.189 =$ **3.811 in**

(c) $\varepsilon = \ln r_x = \ln 16 = 2.7726$
$\varepsilon_x = a + b \ln r_x = 0.8 + 1.5(2.7726) = 4.959$
$\bar{Y}_f = 13{,}000(2.7726)^0/1.0 = 13{,}000$ lb/in^2
$p = 13{,}000(4.959 + 2 \times 3.811/2.0) =$ **114,000 lb/in^2**

(d) Length of extruded portion of billet = 3.811 in. With a reduction $r_x = 16$, the final part length, excluding the cone shaped butt remaining in the die is $L = 3.811(16) =$ **60.97 in.**

18.26 (USCS units) An indirect extrusion operation is performed on an aluminum billet with diameter = 2.0 in and length = 8.0 in. Final shape after extrusion is a rectangular cross section that is 1.0 in by 0.25 in. Die angle = 90°. The aluminum has a strength coefficient = 26,000 lb/in^2 and strain-hardening exponent = 0.20. In the Johnson extrusion strain equation, $a = 0.8$ and $b = 1.2$. (a) Compute the extrusion ratio, true strain, and extrusion strain. (b) What is the shape factor of the extrudate? (c) If the butt left in the container at the end of the stroke is 0.5 in long, what is the length of the extruded section? (d) Determine the ram pressure in the process.

Solution: (a) $r_x = A_o/A_f$
$A_o = \pi D_o^2/4 = \pi(2)^2/4 = 3.142$ in^2
$A_f = 1.0 \times 0.25 = 0.25$ in^2
$r_x = 3.142/0.25 =$ **12.566**
$\varepsilon = \ln 12.566 =$ **2.531**
$\varepsilon_x = 0.8 + 1.3(2.531) =$ **4.09**

(b) To determine the die shape factor, determine the perimeter of a circle whose area is equal to that of the extruded cross section, $A = 0.25$ in^2. The radius of the circle is $R = (0.25/\pi)^{0.5} = 0.282$ in, $C_c = 2\pi(0.282) = 1.772$ in
The perimeter of the extruded cross section $C_x = 1.0 + 0.25 + 1.0 + 0.25 = 2.50$ in
$K_x = 0.98 + 0.02(2.50/1.772)^{2.25} =$ **1.023**

(c) Given that the butt length = 0.5 in
Original volume $V = (8.0)(\pi \text{ x } 2^2/4) = 25.133$ in^3. The final volume consists of two sections: (1) butt, and (2) extrudate. The butt volume $V_1 = (0.5)(\pi \times 2^2/4) = 1.571$ in^3. The extrudate has a cross-sectional area $A_f = 0.25$ in^2. Its volume $V_2 = LA_f = 25.133 - 1.571 = 23.562$ in^3. Thus, length $L = 23.562/0.25 =$ **94.247 in**

(d) $\overline{Y}_f = 26{,}000(1.145)^{0.2}/1.2 = 22{,}261$ lb/in^3
$p = 1.023(22{,}261)(4.09) =$ **93,142 lb/in^2**

18.27 **(A)** (SI units) An L-shaped structural section is direct extruded from an aluminum alloy billet whose length = 500 mm and diameter = 100 mm. Dimensions of the extruded cross section are given in Figure P18.27. The strength coefficient and strain-hardening exponent of the aluminum alloy are 240 MPa and 0.15, respectively (Table 3.4). The die angle = 90°, and the Johnson strain equation has constants $a = 0.8$ and $b = 1.5$. Determine the (a) extrusion ratio, (b) shape factor, and (c) length of the extruded section if the butt remaining in the container at the end of the ram stroke is 25 mm long. (d) Compute the maximum force required to drive the ram forward at the start of extrusion.

Solution: (a) $r_x = A_o/A_f$
$A_o = \pi(100)^2/4 = 7854$ mm^2
$A_f = 2(12 \times 50) = 1200$ mm^2
$r_x = 7854/1200 =$ **6.545**

(b) To determine the die shape factor, to determine the perimeter of a circle whose area is equal to that of the extruded cross section, $A = 1200$ mm^2. The radius of the circle is $R = (1200/\pi)^{0.5} = 19.54$ mm, $C_c = 2\pi(19.54) = 122.8$ mm.
The perimeter of the extruded cross section $C_x = 62 + 50 + 12 + 38 + 50 + 12 = 224$ mm
$K_x = 0.98 + 0.02(224/122.8)^{2.25} =$ **1.057**

(c) Total original volume $V = 0.25\pi(100)^2(500) = 3{,}926{,}991$ mm^3
The final volume consists of two sections: (1) butt, and (2) extrudate. The butt volume $V_1 = 0.25\pi(100)^2(25) = 196{,}350$ mm^3. The extrudate has a cross-sectional area $A_f = 1200$ mm^2. Its volume $V_2 = LA_f = 3{,}926{,}991 - 196{,}350 = 3{,}730{,}641$ mm^3.
Thus, length $L = 3{,}730{,}641/1200 =$ **3108.9 mm = 3.109 m**

(d) $\varepsilon = \ln 5.068 = 1.623$
$\varepsilon_x = 0.8 + 1.5(1.623) = 3.234$
$\overline{Y}_f = 240(1.623)^{0.15}/1.15 = 224.4$ MPa
Maximum ram force occurs at beginning of stroke when length is maximum at $L = 250$ mm
$p = K_x \overline{Y}_f\ (\varepsilon_x + 2L/D_o) = 1.057(224.4)(3.234 + 2(250)/100) = 1953$ MPa
$F = pA_o = 1953(7854) =$ **15,338,862 N = 15.34 MN**

18.28 (SI units) Determine the shape factor for each of the extrusion die orifice shapes in Figure P18.28.

Solution: (a) $A_x = 20 \times 60 = 1200$ mm, $C_x = 2(20 + 60) = 160$ mm
$A_o = \pi R^2 = 1200$
$R^2 = 1200/\pi = 381.97$, $R = 19.544$ mm, $C_c = 2\pi R = 2\pi(19.544) = 122.8$ mm
$K_x = 0.98 + 0.02(160/122.8)^{2.25} =$ **1.016**

(b) $A_x = \pi R_o^{\,2} - \pi R_i^{\,2} = \pi(25^2 - 22.5^2) = 373.06$ mm^2
$C_x = \pi D_o + \pi D_i = \pi(50 + 45) = 298.45$ mm
$R^2 = 373.06/\pi = 118.75$, $R = 10.897$ mm, $C_c = 2\pi R = 2\pi(10.897) = 68.47$ mm
$K_x = 0.98 + 0.02(298.45/68.47)^{2.25} =$ **1.53**

(c) $A_x = 2(5)(30) + 5(60 - 10) = 300 + 250 = 550$ mm^2
$C_x = 30 + 60 + 30 + 5 + 25 + 50 + 25 + 5 = 230$ mm
$A_o = \pi R^2 = 550$, $R^2 = 550/\pi = 175.07$, $R = 13.23$ mm
$C_c = 2\pi R = 2\pi(13.23) = 83.14$ mm
$K_x = 0.98 + 0.02(230/83.14)^{2.25} =$ **1.177**

(d) $A_x = 5(55)(5) + 5(85 - 5 \times 5) = 1675$ mm^2
$C_x = 2 \times 55 + 16 \times 25 + 8 \times 15 + 10 \times 5 = 680$ mm
$A_o = \pi R^2 = 1675$, $R^2 = 1675/\pi = 533.17$, $R = 23.09$ mm
$C_c = 2\pi R = 2\pi(23.09) = 145.08$ mm
$K_x = 0.98 + 0.02(680/145.08)^{2.25} =$ **1.626**

18.29 (SI units) In a direct extrusion operation, the cross section shown in Figure P18.28(b) is produced from a copper billet whose diameter = 100 mm and length = 400 mm. In the flow curve for copper, the strength coefficient = 300 MPa and strain-hardening exponent = 0.50 (Table 3.4). In the Johnson strain equation, $a = 0.8$ and $b = 1.5$. Determine the (a) extrusion ratio, (b) shape factor, and (c) force required to drive the ram forward during extrusion at the point in the process when the billet length remaining in the container = 450 mm. (d) Also determine the length of the extruded section at the end of the operation if the volume of the butt left in the container is 200,000 mm^3.

Solution: (a) $r_x = A_o/A_f$
$A_o = \pi(100)^2/4 = 7854$ mm^2
$A_f = A_x = \pi(50)^2/4 - \pi(45)^2/4 = 1963.5 - 1590.4 = 373.1$ mm^2
$r_x = 7854/373.1 =$ **21.05**

(b) To determine the die shape factor, determine the perimeter of a circle whose area is equal to that of the extruded cross section, $A_x = 373.1$ mm^2.
The radius of the circle is $R = (373.1/\pi)^{0.5} = 10.9$ mm, $C_c = 2\pi(10.9) = 68.5$ mm.
The perimeter of the extruded cross section $C_x = \pi(50) + \pi(45) = 298.5$ mm
$K_x = 0.98 + 0.02(298.5/68.5)^{2.25} =$ **1.53**

(c) $\varepsilon = \ln 21.05 = 3.047$
$\varepsilon_x = 0.8 + 1.5(3.047) = 5.37$
$\bar{Y}_f = 300(3.047)^{0.50}/1.50 = 349.1$ MPa
$p = K_x \bar{Y}_f \varepsilon_x = 1.53(349.1)(5.37 + 2(450)/100) = 7675.3$ MPa
$F = pA_o = 7675.3(7854) =$ **60,282,179 N = 60.3MN**

(d) Total original volume $V = \pi(100)^2(400)/4 = 3{,}141{,}593$ mm^3
The final volume consists of two sections: (1) butt, and (2) extrudate.
The butt volume as given $V_1 = 200{,}000$ mm^3.
The extrudate has a cross-sectional area $A_f = 373.1$ mm^2 from part (a)
Its volume $V_2 = LA_f = 3{,}141{,}593 - 200{,}000 = 2{,}941{,}593$ mm^3.
Thus, length $L = 2{,}941{,}593/373.1 =$ **7,884 mm = 7.884 m**

18.30 (SI units) A direct extrusion operation produces the cross section shown in Figure P18.28(d) from an aluminum alloy billet whose diameter = 150 mm and length = 900 mm. The flow curve parameters for the aluminum are $K = 240$ MPa and $n = 0.15$ (Table 3.4). In the Johnson strain equation, $a = 0.8$ and $b = 1.5$. Determine the (a) extrusion ratio, (b) shape factor, and (c) force required to drive the ram forward during extrusion at the point in the process when the billet length remaining in the container = 800 mm. (d) Also determine the length of the extrudate at the end of the operation if the volume of the butt left in the container is 500,000 mm^3.

Solution: (a) $r_x = A_o/A_f$
$A_o = \pi(150)^2/4 = 17{,}671$ mm^2
$A_f = A_x = 5(55)(5) + 5(85 - 5(5)) = 1675$ mm^2
$r_x = 17{,}671/1675 =$ **10.55**

(b) To determine the die shape factor, determine the perimeter of a circle whose area is equal to that of the extruded cross section, $A_x = 1675$ mm^2.
$C_x = 2 \times 55 + 16 \times 25 + 8 \times 15 + 10 \times 5 = 680$ mm
$A_o = \pi R^2 = 1675$, $R^2 = 1675/\pi = 533.17$, $R = 23.09$ mm
$C_c = 2\pi R = 2\pi(23.09) = 145.08$ mm

$K_x = 0.98 + 0.02(680/145.08)^{2.25} =$ **1.626**

(c) $\varepsilon = \ln 10.55 = 2.36$
$\varepsilon_x = 0.8 + 1.5(2.36) = 4.33$
$\bar{Y}_f = 240(2.36)^{0.15}/1.15 = 237.4$ MPa
$p = K_x \bar{Y}_f \varepsilon_x = 1.626(237.4)(4.33 + 2(800)/150) = 5789$ MPa
$F = pA_o = 5789(17{,}671) =$ **102,298,534 N = 102.3 MN**

(d) Total original volume $V = \pi(150)^2(900)/4 = 15{,}904{,}313$ mm^3
The final volume consists of two sections: (1) butt, and (2) extrudate.
The butt volume as given $V_1 = 500{,}000$ mm^3.
The extrudate has a cross-sectional area $A_f = 1675$ mm^2 from part (a)
Its volume $V_2 = LA_f = 15{,}904{,}313 - 500{,}000 = 15{,}404{,}313$ mm^3
Thus, length $L = 15{,}404{,}313/1675 =$ **9,197 mm = 9.197 m**

Drawing

18.31 **(A)** (SI units) A spool of copper wire has a starting diameter of 2.5 mm. It is drawn through a die with an opening that is 2.1 mm. The entrance angle of the die = 18°. Coefficient of friction at the work-die interface is 0.08. The pure copper has a strength coefficient = 300 MPa and a strain-hardening coefficient = 0.50. The operation is performed at room temperature. Determine the (a) area reduction, (b) draw stress, and (c) draw force required for the operation.

Solution: (a) $r = (A_o - A_f)/A_o$
$A_o = 0.25\pi(2.50)^2 = 4.91$ mm^2
$A_f = 0.25\pi(2.1)^2 = 3.46$ mm^2
$r = (4.91 - 3.46)/4.91 =$ **0.294**

(b) Draw stress σ_d:
$\varepsilon = \ln(4.91/3.46) = \ln 1.417 = 0.349$
$\bar{Y}_f = 300(0.349)^{0.5}/1.5 = 118.2$ MPa
$\phi = 0.88 + 0.12(D/L_c)$
$D = 0.5(2.5 + 2.1) = 2.30$
$L_c = 0.5(2.5 - 2.1)/\sin 18 = 0.647$
$\phi = 0.88 + 0.12(2.30/0.647) = 1.31$
$\sigma_d = \bar{Y}_f(1 + \mu/\tan\alpha)\phi(\ln A_o/A_f) = 118.2(1 + 0.08/\tan 18)(1.31)(0.349) =$ **67.3 MPa**

(c) Draw force $F = A_f\,\sigma_d = 3.46(67.3) =$ **233 N**

18.32 (USCS units) Aluminum rod stock with a starting diameter = 0.50 in is drawn through a draw die with an entrance angle = 13°. The final diameter of the rod is = 0.375 in. The metal has a strength coefficient = 25,000 lb/in^2 and a strain-hardening exponent = 0.20. Coefficient of friction at the work-die interface = 0.1. Determine the (a) area reduction, (b) draw force for the operation, and (c) horsepower to perform the operation if the exit velocity of the stock = 5 ft/sec.

Solution: (a) $r = (A_o - A_f)/A_o$

$A_o = 0.25\pi(0.50)^2 = 0.1964$ in^2
$A_f = 0.25\pi(0.35)^2 = 0.1104$ in^2
$r = (0.1964 - 0.1104)/0.1964 =$ **0.4375**

(b) Draw force F:
$\varepsilon = \ln(0.1964/0.1104) = \ln 1.778 = 0.5754$
$\bar{Y}_f = 25{,}000(0.5754)^{0.20}/1.20 = 18{,}653$ lb/in^2
$\phi = 0.88 + 0.12(D/L_c)$
$D = 0.5(.50 + 0.375) = 0.438$
$L_c = 0.5(0.50 - 0.375)/\sin 13 = 0.2778$
$\phi = 0.88 + 0.12(0.438/0.2778) = 1.069$
$F = A_f\,\bar{Y}_f\,(1 + \mu/\tan\alpha)\phi(\ln A_o/A_f)$
$F = 0.1104(18{,}653)(1 + 0.1/\tan 13)(1.069)(0.5754) =$ **1815 lb**

(c) $P = 1815(5 \text{ ft/sec} \times 60) = 544{,}500$ ft/lb/min
$HP = 544{,}500/33{,}000 =$ **16.5 hp**

18.33 (SI units) Bar stock of initial diameter = 90 mm is drawn with a draft = 15 mm. The draw die has an entrance angle = 18°, and the coefficient of friction at the work-die interface = 0.08. The metal behaves as a perfectly plastic material with yield stress = 105 MPa. Determine the (a) area reduction, (b) draw stress, (c) draw force required for the operation, and (d) power to perform the operation at an exit velocity = 1.0 m/min.

Solution: (a) $r = (A_o - A_f)/A_o$
$A_o = 0.25\pi(90)^2 = 6361.7$ mm^2
$D_f = D_o - d = 90 - 15 = 75$ mm,
$A_f = 0.25\pi(75)^2 = 4417.9$ mm^2
$r = (6361.7 - 4417.9)/6361.7 =$ **0.3056**

(b) Draw stress σ_d:
$\varepsilon = \ln(6361.7/4417.9) = \ln 1.440 = 0.3646$
$\bar{Y}_f = 105$ MPa
$\phi = 0.88 + 0.12(D/L_c)$
$D = (90 + 75)/2 = 82.5$ mm
$L_c = 0.5(90 - 75)/\sin 18 = 24.3$ mm
$\phi = 0.88 + 0.12(82.5/24.3) = 1.288$
$\sigma_d = \bar{Y}_f\,(1 + \mu/\tan\alpha)\phi(\ln A_o/A_f) = 105(1 + 0.08/\tan 18)(1.288)(0.3646) =$ **61.45 MPa**

(c) $F = A_f\,\sigma_d = 4417.9\,(61.45) =$ **271,475 N**

(d) $P = 271{,}475(1 \text{ m/min}) = 271{,}475$ N-m/min = 4524.6 N-m/s = **4524.6 W**

18.34 (USCS units) Wire stock of initial diameter = 0.125 in is drawn through two dies, each providing a 0.20 area reduction. The starting metal has a strength coefficient = 40,000 lb/in^2 and a strain-hardening exponent = 0.15. Each die has an entrance angle of 12°, and the coefficient of friction at the work-die interface is estimated to be 0.10. The motors driving the capstans at the die exits can each deliver 1.50 hp at 90% efficiency. Determine the maximum possible speed of the wire as it exits the second die.

Solution: First draw: D_o = 0.125 in, $A_o = 0.25\pi(0.125)^2 = 0.012273$ in^2
$r = (A_o - A_f)/A_o$, $A_f = A_o(1 - r) = 0.012273(1 - 0.2) = 0.0098175$
$\varepsilon = \ln(0.012273/0.0098175) = \ln 1.250 = 0.2232$
$\bar{Y}_f = 40{,}000(0.2232)^{0.15}/1.15 = 27{,}776$ lb/in^2
$\phi = 0.88 + 0.12(D/L_c)$
$D_f = 0.125(1 - r)^{0.5} = 0.125(0.8)^{.5} = 0.1118$ in
$D = 0.5(.125 + 0.1118) = 0.1184$
$L_c = 0.5(0.125 - 0.1118)/\sin 12 = 0.03173$
$\phi = 0.88 + 0.12(0.1184/0.03173) = 1.33$
$F = A_f\,\bar{Y}_f\,(1 + \mu/\tan\alpha)\phi(\ln A_o/A_f)$
$F = 0.\,0098175(27{,}776)(1 + 0.1/\tan 12)(1.33)(0.2231) = 119$ lb
1.5 hp at 90% efficiency = 1.5 × 0.90(33,000 ft-lb/min)/60 = 742.5 ft-lb/sec
$P = Fv = 119v = 742.5$
$v = 742.5/119 =$ **6.24 ft/sec**

Second draw: D_o = 0.1118 in, $A_o = 0.25\pi(0.1118)^2 = 0.009818$ in^2
$r = (A_o - A_f)/A_o$, $A_f = A_o(1 - r) = 0.009818(1 - 0.2) = 0.007855$ in^2
$\varepsilon = \ln(0.009818/0.007855) = \ln 1.250 = 0.2232$
Total strain experienced by the work metal is the sum of the strains from the first and second draws:
$\varepsilon = \varepsilon_1 + \varepsilon_2 = 0.2232 + 0.2232 = 0.4464$
$\bar{Y}_f = 40{,}000(0.4464)^{0.15}/1.15 = 30{,}818$ lb/in^2
$\phi = 0.88 + 0.12(D/L_c)$
$D_f = 0.1118(1 - r)^{0.5} = 0.1118(.8)^{.5} = 0.100$ in
$D = 0.5(0.1118 + 0.100) = 0.1059$
$L_c = 0.5(0.1118 - 0.100)/\sin 12 = 0.0269$
$\phi = 0.88 + 0.12(0.1059/0.0269) = 1.35$
$F = A_f\,\bar{Y}_f\,(1 + \mu/\tan\alpha)\phi(\ln A_o/A_f)$
$F = 0.007855(30{,}818)(1 + 0.1/\tan 12)(1.35)(0.4464) = 214$ lb
1.5 hp at 90% efficiency = 742.5 ft-lb/sec as before in the first draw
$P = Fv = 214v = 742.5$
$v = 742.5/214 =$ **3.47 ft/sec**

Note: The calculations indicate that the second draw die is the limiting step in the drawing sequence. The first operation would have to be operated at well below its maximum possible speed; or the second draw die could be powered by a higher horsepower motor; or the reductions to achieve the two stages could be reallocated to achieve a higher reduction in the first drawing operation.