Answer. (a).

19.14 Theoretically, the maximum reduction possible in a wire drawing operation, under the assumptions of a perfectly plastic metal, no friction, and no redundant work, is which of the following (one answer): (a) zero, (b) 0.63, (c) 1.0, or (d) 2.72?

Answer. (b).

19.15 Which of the following bulk deformation processes are involved in the production of nails for lumber construction (three best answers): (a) bar and wire drawing, (b) extrusion, (c) flashless forging, (d) impression die forging, (e) rolling, and (f) upsetting?

Answer. (a), (c), and (d). Bar stock is rolled, and then drawn into wire stock. Upset forged is used to form the nail head.

19.16 Johnson's formula is associated with which one of the four bulk deformation processes: (a) bar and wire drawing, (b) extrusion, (c) forging, and (d) rolling?

Answer. (b).

Problems

Rolling

19.1 A 42.0-mm-thick plate made of low carbon steel is to be reduced to 34.0 mm in one pass in a rolling operation. As the thickness is reduced, the plate widens by 4%. The yield strength of the steel plate is 174 MPa and the tensile strength is 290 MPa. The entrance speed of the plate is 15.0 m/min. The roll radius is 325 mm and the rotational speed is 49.0 rev/min. Determine (a) the minimum required coefficient of friction that would make this rolling operation possible, (b) exit velocity of the plate, and (c) forward slip.

Solution: (a) Maximum draft $d_{max} = \mu^2 R$
Given that $d = t_o - t_f = 42 - 34 = 8.0$ mm,
$\mu^2 = 8/325 = 0.0246$
$\mu = (0.0246)^{0.5} = \mathbf{0.157}$

(b) Plate widens by 4%.
$t_o w_o v_o = t_f w_f v_f$
$w_f = 1.04\, w_o$
$42(w_o)(15) = 34(1.04w_o)v_f$
$v_f = 42(w_o)(15)/ 34(1.04w_o) = 630/35.4 =$ **17.8 m/min**

(c) $v_r = \pi r^2 N = \pi(0.325)^2(49.0) = 16.26$ m/min

$s = (v_f - v_r)/v_r = (17.8 – 16.26)/16.26 = \mathbf{0.0947}$

19.2 A 2.0-in-thick slab is 10.0 in wide and 12.0 ft long. Thickness is to be reduced in three steps in a hot rolling operation. Each step will reduce the slab to 75% of its previous thickness. It is expected that for this metal and reduction, the slab will widen by 3% in each step. If the entry speed of the slab in the first step is 40 ft/min, and roll speed is the same for the three steps, determine: (a) length and (b) exit velocity of the slab after the final reduction.

Solution: (a) After three passes, $t_f = (0.75)(0.75)(0.75)(2.0) = 0.844$ in
$w_f = (1.03)(1.03)(1.03)(10.0) = 10.927$ in
$t_o w_o L_o = t_f w_f L_f$
$(2.0)(10.0)(12 \text{ x } 12) = (0.844)(10.927)L_f$
$L_f = (2.0)(10.0)(12 \text{ x } 12)/(0.844)(10.927) = 312.3$ in = **26.025 ft**

(b) Given that roll speed is the same at all three stands and that $t_o w_o v_o = t_f w_f v_f$,

Step 1: $v_f = (2.0)(10.0)(40)/(0.75 \text{ x } 2.0)(1.03 \text{ x } 10.0) = 51.78$ ft/min
Step 2: $v_f = (0.75 \text{ x } 2.0)(1.03 \text{ x } 10.0)(40)/(0.75^2 \text{ x } 2.0)(1.03^2 \text{ x } 10.0) = 51.78$ ft/min
Step 3: $v_f = (0.75^2 \text{ x } 2.0)(1.03^2 \text{ x } 10.0)(40)/(0.75^3 \text{ x } 2.0)(1.03^3 \text{ x } 10.0) =$ **51.78 ft/min**

19.3 A series of cold rolling operations are to be used to reduce the thickness of a plate from 50 mm down to 25 mm in a reversing two-high mill. Roll diameter = 700 mm and coefficient of friction between rolls and work = 0.15. The specification is that the draft is to be equal on each pass. Determine (a) minimum number of passes required, and (b) draft for each pass?

Solution: (a) Maximum draft $d_{max} = \mu^2 R = (0.15)^2 (350) = 7.875$ mm
Minimum number of passes = $(t_o - t_f)/d_{max} = (50 - 25)/7.875 =$ **3.17 → 4 passes**

(b) Draft per pass $d = (50 - 25)/4 =$ **6.25 mm**

19.4 In the previous problem, suppose that the percent reduction were specified to be equal for each pass, rather than the draft. (a) What is the minimum number of passes required? (b) What is the draft for each pass?

Solution: (a) Maximum possible draft occurs on first pass: $d_{max} = \mu^2 R = (0.15)^2 (350) = 7.875$ mm
This converts into a maximum possible reduction $x = 7.875/50 = 0.1575$
Let x = fraction reduction per pass, and n = number of passes. The number of passes must be an integer. To reduce from $t_o = 50$ mm to $t_o = 25$ mm in n passes, the following relationship must be satisfied:
$50(1 - x)^n = 25$
$(1 - x)^n = 25/50 = 0.5$
$(1 - x) = 0.5^{1/n}$
Try $n = 4$: $(1 - x) = (0.5)^{1/4} = 0.8409$
$x = 1 - 0.8409 = 0.1591$, which exceeds the maximum possible reduction of 0.1575.
Try $n = 5$: $(1 - x) = (0.5)^{1/5} = 0.87055$
$x = 1 - 0.87055 =$ **0.12945**, which is within the maximum possible reduction of 0.1575.

(b) Pass 1: $d = 50(0.12945) =$ **6.47 mm**, $t_f = 50 - 6.47 =$ **43.53 mm**
Pass 2: $d = 43.53(0.12945) =$ **5.63 mm**, $t_f = 43.53 - 5.63 =$ **37.89 mm**
Pass 3: $d = 37.89(0.12945) =$ **4.91 mm**, $t_f = 37.89 - 4.91 =$ **32.98 mm**
Pass 4: $d = 32.98(0.12945) =$ **4.27 mm**, $t_f = 32.98 - 4.27 =$ **28.71 mm**
Pass 5: $d = 28.71(0.12945) =$ **3.71 mm**, $t_f = 28.71 - 3.71 =$ **25.00 mm**

19.5 A continuous hot rolling mill has two stands. Thickness of the starting plate = 25 mm and width = 300 mm. Final thickness is to be 13 mm. Roll radius at each stand = 250 mm. Rotational speed at the first stand = 20 rev/min. Equal drafts of 6 mm are to be taken at each stand. The plate is wide enough relative to its thickness that no increase in width occurs. Under the assumption that the forward slip is equal at each stand, determine (a) speed v_r at each stand, and (b) forward slip s. (c) Also, determine the exiting speeds at each rolling stand, if the entering speed at the first stand = 26 m/min.

Solution: (a) Let t_o = entering plate thickness at stand 1. $t_o = 25$ mm. Let t_1 = exiting plate thickness at stand 1 and entering thickness at stand 2. $t_1 = 25 - 6 = 19$ mm.
Let t_2 = exiting plate thickness at stand 2. $t_2 = 19 - 6 = 13$ mm.
Let v_o = entering plate speed at stand 1.
Let v_1 = exiting plate speed at stand 1 and entering speed at stand 2.
Let v_2 = exiting plate speed at stand 2.
Let v_{r1} = roll speed at stand 1. $v_{r1} = \pi D N_r = \pi(2 \text{ x } 250)(10^{-3})(20) =$ **31.42 m/min**
Let v_{r2} = roll speed at stand 2. $v_{r2} = ?$

Forward slip $s = (v_f - v_r)/v_r$
$sv_r = v_f - v_r$
$(1 + s)v_r = v_f$

At stand 1, $(1 + s)v_{r1} = v_1$ (Eq. 1)
At stand 2, $(1 + s)v_{r2} = v_2$ (Eq. 2)
By constant volume, $t_o w_o v_o = t_1 w_1 v_1 = t_2 w_2 v_2$
Since there is no change in width, $w_o = w_1 = w_2$
Therefore, $t_o v_o = t_1 v_1 = t_2 v_2$
$1.0v_o = 0.75v_1 = 0.50v_2$
$v_2 = 1.5v_1$ (Eq. 3)
Combining (Eqs. 2 and 3), $(1 + s)v_{r2} = v_2 = 1.5v_1$
Substituting (Eq. 1), $(1 + s)v_{r2} = 1.5(1 + s)v_{r1}$, thus $v_{r2} = 1.5v_{r1}$
$v_{r2} = 1.5(31.42) =$ **47.1 m/min**

(b) $25v_o = 19v_1$
$v_1 = 25(26)/19 = 34.2$ m/min
(Eq. 1): $(1 + s)v_{r1} = v_1$
$(1 + s)(31.4) = 34.2$
$(1 + s) = 34.2/31.4 = 1.089$
$\boldsymbol{s = 0.089}$

(c) $v_1 =$ **34.2 m/min,** previously calculated in (b)
$v_2 = 1.5v_1 = 1.5(34.2) =$ **51.3 m/min**

19.6 A continuous hot rolling mill has eight stands. The dimensions of the starting slab are: thickness = 3.0 in, width = 15.0 in, and length = 10 ft. The final thickness is to be 0.3 in. Roll diameter at each stand = 36 in, and rotational speed at stand number 1 = 30 rev/min. It is observed that the speed of the slab entering stand 1 = 240 ft/min. Assume that no widening of the slab occurs during the rolling sequence. Percent reduction in thickness is to be equal at all stands, and it is assumed that the forward slip will be equal at each stand. Determine (a) percent reduction at each stand, (b) rotational speed of the rolls at stands 2 through 8, and (c) forward slip. (d) What is the draft at stands 1 and 8? (e) What is the length and exit speed of the final strip exiting stand 8?

Solution: (a) To reduce from $t_o = 3.0$ in to $t_f = 0.3$ in over 8 stands, $3.0(1 - x)^8 = 0.3$
$(1 - x)^8 = 0.3/3.0 = 0.10$
$(1 - x) = (0.10)^{1/8} = 0.74989$
$x = 1 - 0.74989 = \boldsymbol{r = 0.2501 = 25.01\%}$ **at each stand.**

(b) Forward slip $s = (v_f - v_r)/v_r$
$sv_r = v_f - v_r$
$(1 + s)v_r = v_f$
At stand 1: $(1 + s)v_{r1} = v_1$, where v_{r1} = roll speed, v_1 = exit speed of slab.
At stand 2: $(1 + s)v_{r2} = v_2$, where v_{r2} = roll speed, v_2 = exit speed of slab.
Etc.
At stand 8: $(1 + s)v_{r8} = v_8$, where v_{r8} = roll speed, v_8 = exit speed of slab.
By constant volume, $t_o w_o v_o = t_1 w_1 v_1 = t_2 w_2 v_2 = \ldots = t_8 w_8 v_8$
Since there is no change in width, $w_o = w_1 = w_2 = \ldots w_8$
Therefore, $t_o v_o = t_1 v_1 = t_2 v_2 = \ldots = t_8 v_8$
$t_o = 3.0$,
$3v_o = 3(1 - r)v_1 = 3(1 - r)^2 v_2 = \ldots 3(1 - r)^8 v_8$, where $r = 0.2501$ as determined in part (a).
Since s is a constant, $v_{r1} : v_{r2} : \ldots : v_{r8} = v_1 : v_2 : \ldots : v_8$
Given that $N_{r1} = 30$ rev/min, $v_{r1} = \pi D N_{r1} = (2\pi \times 18/12)(30) = 282.78$ ft/min
In general $N_r = (30/282.78) = 0.10609v_r$
$N_{r2} = 0.10609 \times 282.78/(1-r) = 0.10609 \times 282.78/(1-0.2501) =$ **40 rev/min**
$N_{r3} = 0.10609 \times 282.78/(1-r)^2 =$ **53.3 rev/min**
$N_{r4} = 0.10609 \times 282.78/(1-r)^3 =$ **71.1 rev/min**
$N_{r5} = 0.10609 \times 282.78/(1-r)^4 =$ **94.9 rev/min**

$N_{r6} = 0.10609 \text{ x } 282.78/(1\text{-}r)^5 =$ **126.9.3 rev/min**
$N_{r7} = 0.10609 \text{ x } 282.78/(1\text{-}r)^6 =$ **168.5 rev/min**
$N_{r8} = 0.10609 \text{ x } 282.78/(1\text{-}r)^7 =$ **224.9 rev/min**

(c) Given $v_o = 240$ ft/min
$v_1 = 240/(1\text{-}r) = 240/0.74989 = 320$ ft/min
$v_2 = 320/0.74989 = 426.8$ ft/min
From equations for forward slip, $(1 + s)v_{r1} = v_1$
$(1 + s)(282.78) = 320$
$(1 + s) = 320/282.78 = 1.132$ **s = 0.132**
Check with stand 2: given $v_2 = 426.8$ ft/min from above
$N_{r2} = 0.10609v_{r2}$
Rearranging, $v_{r2} = N_{r2}/0.10609 = 9.426N_{r2} = 0.426(40) = 377.04$ ft/min
$(1 + s)(377.04) = 426.8$
$(1 + s) = 426.8/377.14 = 1.132$ $s = 0.132$, as before

(d) Draft at stand 1 $d_1 = 3.0(0.2501) =$ **0.7503 in**
Draft at stand 8 $d_8 = 3.0(1 - 0.2501)^7(0.2501) =$ **0.10006 in**

(e) Length of final strip $L_f = L_8$
$t_ow_oL_o = t_8w_8L_8$
Given that $w_o = w_8$, $t_oL_o = t_8L_8$
$3.0(10 \text{ ft}) = 0.3L_8$ $\mathbf{L_8 = 100}$ **ft**
$t_ow_ov_o = t_8w_8v_8$
$t_ov_o = t_8v_8$
$v_8 = 240(3/0.3) =$ **2400 ft/min**

19.7 A plate that is 250 mm wide and 25 mm thick is to be reduced in a single pass in a two-high rolling mill to a thickness of 20 mm. The roll has a radius = 500 mm, and its speed = 30 m/min. The work material has a strength coefficient = 240 MPa and a strain hardening exponent = 0.2. Determine (a) roll force, (b) roll torque, and (c) power required to accomplish this operation.

Solution: (a) Draft $d = 25 - 20 = 5$ mm,
Contact length $L = (500 \text{ x } 5)^{.5} = 50$ mm
True strain $\varepsilon = \ln(25/20) = \ln 1.25 = 0.223$
$\overline{Y}_f = 240(0.223)^{0.20}/1.20 = 148.1$ MPa
Rolling force $F = 148.1(250)(50) =$ **1,851,829 N**

(b) Torque $T = 0.5(1,851,829)(50 \text{ x } 10^{-3}) =$ **46,296 N-m**

(c) $N = (30 \text{ m/min})/(2\pi \text{ x } 0.500) = 9.55$ rev/min = 0.159 rev/s
Power $P = 2\pi(0.159)(1,851,829)(50 \text{ x } 10^{-3}) = 92,591$ N-m/s = **92,591 W**

19.8 Solve Problem 19.7 using a roll radius = 250 mm.

Solution: (a) Draft $d = 25 - 20 = 5$ mm,
Contact length $L = (250 \text{ x } 5)^{.5} = 35.35$ mm
True strain $\varepsilon = \ln(25/20) = \ln 1.25 = 0.223$
$\overline{Y}_f = 240(0.223)^{0.20}/1.20 = 148.1$ MPa
Rolling force $F = 148.1(250)(35.35) =$ **1,311,095 N**

(b) Torque $T = 0.5(1,311,095)(35.35 \text{ x } 10^{-3}) =$ **23,174 N-m**

(c) $N = (30 \text{ m/min})/(2\pi \text{ x } 0.250) = 19.1$ rev/min = 0.318 rev/s
Power $P = 2\pi(0.318)(1,311,095)(35.35 \text{ x } 10^{-3}) = 92,604$ N-m/s = **92,604 W**

Note that the force and torque are reduced as roll radius is reduced, but that the power remains the same (within calculation error) as in the previous problem.

19.9 Solve Problem 19.7, only assume a cluster mill with working rolls of radius = 50 mm. Compare the results with the previous two problems, and note the important effect of roll radius on force, torque and power.

Solution: (a) Draft d = 25 - 20 = 5 mm,
Contact length $L = (50 \times 5)^{.5} = 15.81$ mm
True strain $\varepsilon = \ln(25/20) = \ln 1.25 = 0.223$
$\overline{Y}_f = 240(0.223)^{0.20}/1.20 = 148.1$ MPa
Rolling force F = 148.1(250)(15.81) = **585,417 N**

(b) Torque $T = 0.5(585{,}417)(15.81 \times 10^{-3})$ = **4,628 N-m**

(c) N = (30 m/min)/(2π x 0.050) = 95.5 rev/min = 1.592 rev/s
Power $P = 2\pi(1.592)(585{,}417)(15.81 \times 10^{-3})$ = 92,554 N-m/s = **92,554 W**

Note that this is the same power value (within calculation error) as in Problems 19.7 and 19.8. In fact, power would probably increase because of lower mechanical efficiency in the cluster type rolling mill.

19.10 A 4.50-in-thick slab that is 9 in wide and 24 in long is to be reduced in a single pass in a two-high rolling mill to a thickness of 3.87 in. The roll rotates at a speed of 5.50 rev/min and has a radius of 17.0 in. The work material has a strength coefficient = 30,000 lb/in^2 and a strain hardening exponent = 0.15. Determine (a) roll force, (b) roll torque, and (c) power required to accomplish this operation.

Solution: (a) Draft d = 4.50 – 3.87 = 0.63 in,
Contact length $L = (17.0 \times 0.63)^{0.5} = 3.27$ in
True strain $\varepsilon = \ln(4.5/3.87) = \ln 1.16 = 0.1508$
$\overline{Y}_f = 30{,}000(0.1508)^{0.15}/1.15 = 19{,}642$ lb/in^2
Rolling force $F = \overline{Y}_f wL$ = 16,414(9.0)(3.27) = **483,000 lb**

(b) Torque $T = 0.5FL$ = 0.5(483000)(3.27) = **789,700 in-lb.**

(c) N = 5.50 rev/min
Power P = 2π(5.50)(483000)(3.27) = 54,580,500 in-lb/min
HP = (54,580,500 in-lb/min)/(396,000) = **138 hp**

19.11 A single-pass rolling operation reduces a 20 mm thick plate to 18 mm. The starting plate is 200 mm wide. Roll radius = 250 mm and rotational speed = 12 rev/min. The work material has a strength coefficient = 600 MPa and a strength coefficient = 0.22. Determine (a) roll force, (b) roll torque, and (c) power required for this operation.

Solution: (a) Draft d = 20 - 18 = 2.0 mm,
Contact length $L = (250 \times 2)^{.5} = 11.18$ mm = 0.0112 m
True strain $\varepsilon = \ln(20/18) = \ln 1.111 = 0.1054$
$\overline{Y}_f = 600(0.1054)^{0.22}/1.22 = 300$ MPa
Rolling force F = 300(0.0112)(0.2) = **0.672 MN = 672,000 N**

(b) Torque T = 0.5(672,000)(0.0112) **= 3,720 N-m**

(c) Given that N = 12 rev/min
Power P = 2π(12/60)(672,000)(0.0112) = **37,697 W**

19.12 A hot rolling mill has rolls of diameter = 24 in. It can exert a maximum force = 400,000 lb. The mill has a maximum horsepower = 100 hp. It is desired to reduce a 1.5 in thick plate by the maximum possible draft in one pass. The starting plate is 10 in wide. In the heated condition, the work material has a strength coefficient = 20,000 lb/in^2 and a strain hardening exponent = zero. Determine (a) maximum possible draft, (b) associated true strain, and (c) maximum speed of the rolls for the operation.

Solution: (a) Assumption: maximum possible draft is determined by the force capability of the rolling mill and not by coefficient of friction between the rolls and the work.
Draft $d = 1.5 - t_f$
Contact length $L = (12\text{d})^{0.5}$
$\overline{Y}_f = 20{,}000(\varepsilon)^0/1.0 = 20{,}000$ lb/in^2
Force $F = 20{,}000(10)\ (12d)^{0.5} = 400{,}000$ (the limiting force of the rolling mill)
$(12d)^{0.5} = 400{,}000/200{,}000 = 2.0$
$12\ d = 2.0^2 = 4$
$\boldsymbol{d = 4/12 = 0.333}$ **in**

(b) True strain $\varepsilon = \ln(1.5/t_f)$
$t_f = t_o - d = 1.5 - 0.333 = 1.167$ in
$\varepsilon = \ln(1.5/1.167) = \ln 1.285 = \mathbf{0.251}$

(c) Given maximum possible power $HP = 100$ hp = 100 x 396000 (in-lb/min)/hp = 39,600,000 in-lb/min
Contact length $L = (12 \times 0.333)^{0.5} = 2.0$ in
$P = 2\pi N(400{,}000)(2.0) = 5{,}026{,}548N$ in-lb/min
$5{,}026{,}548N = 39{,}600{,}000$
$N = 7.88$ rev/min
$v_r = 2\pi RN = 2\pi(12/12)(7.88) =$ **49.5 ft/min**

19.13 Solve Problem 19.12 except that the operation is warm rolling and the strain-hardening exponent is 0.18. Assume the strength coefficient remains at 20,000 lb/in^2.

Solution: (a) Assumption (same as in previous problem): maximum possible draft is determined by the force capability of the rolling mill and not by coefficient of friction between the rolls and the work.
Draft $d = 1.5 - t_f$
Contact length $L = (12d)^{0.5}$
$\varepsilon = \ln(1.5/t_f)$
$\overline{Y}_f = 20{,}000(\varepsilon)^{0.18}/1.18 = 16{,}949\varepsilon^{.18}$
$F = \overline{Y}_f(10)(12d)^{0.5} = 34.641\,\overline{Y}_f\ (d)^{0.5} = 400{,}000$ (as given)
$\overline{Y}_f\ (d)^{0.5} = 400{,}000/34.641 = 11{,}547$

Now use trial-and-error to values of $\overline{Y}_f$ and d that fit this equation.
Try $d = 0.3$ in, $t_f = 1.5 - 0.3 = 1.2$ in
$\varepsilon = \ln(1.5/1.2) = \ln 1.25 = 0.223$
$\overline{Y}_f = 16{,}949(0.223)^{.18} = 13{,}134$ lb/in^2.
$(d)^{0.5} = 11{,}547/13{,}134 = 0.8791$
$d = 0.773$, which does not equal the initial trial value of $d = 0.3$

Try d = 0.5 in, $t_f = 1.5 - 0.5 = 1.0$ in
$\varepsilon = \ln(1.5/1.0) = \ln 1.50 = 0.4055$
$\overline{Y}_f = 16{,}949(0.4055)^{.18} = 14{,}538$ lb/in^2.
$(d)^{0.5} = 11{,}547/14{,}538 = 0.7942$
$d = 0.631$, which does not equal the trial value of $d = 0.5$

Try $d = 0.6$ in, $t_f = 1.5 - 0.6 = 0.9$ in
$\varepsilon = \ln(1.5/0.9) = 0.5108$
$\overline{Y}_f = 16{,}949(0.5108)^{.18} = 15{,}120$ lb/in^2.
$(d)^{0.5} = 11{,}547/15{,}120 = 0.7637$
$d = 0.583$, which is too much compared to $d = 0.6$

Try $d = 0.58$ in, $t_f = 1.5 - 0.58 = 0.92$ in
$\varepsilon = \ln(1.5/0.92) = \ln 1.579 = 0.489$
$\overline{Y}_f = 16{,}949(0.489)^{.18} = 15{,}007$ lb/in^2.
$(d)^{0.5} = 11{,}547/15{,}007 = 0.769$
$d = 0.592$, which is close but still above the trial value of $d = 0.55$

Try $d = 0.585$ in, $t_f = 1.50 - 0.585 = 0.915$ in
$\varepsilon = \ln(1.5/0.915) = 0.494$
$\overline{Y}_f = 16{,}949(0.494)^{.18} = 15{,}036$ lb/in^2.
$(d)^{0.5} = 11{,}547/15{,}036 = 0.768$
$d = 0.590$, which is close but still above the trial value of $d = 0.585$.

Try $d = 0.588$ in, $t_f = 1.50 - 0.588 = 0.912$ in
$\varepsilon = \ln(1.5/0.912) = 0.498$
$\overline{Y}_f = 16{,}949(0.498)^{.18} = 15{,}053$ lb/in^2.
$(d)^{0.5} = 11{,}547/15{,}053 = 0.767$
$d = 0.588$, which is almost the same as the trial value of $\boldsymbol{d = 0.588}$.

(b) True strain $\varepsilon = \ln(1.5/0.912) = \mathbf{0.498}$

(c) Given maximum possible power HP = 100 hp = 100 x 396000 (in-lb/min)/hp
= 39,600,000 in-lb/min
Contact length $L = (12 \times 0.588)^{0.5} = 2.66$ in
$P = 2\pi N(400{,}000)(2.66) = 6{,}685{,}000$N in-lb/min
$6{,}486{,}000N = 39{,}600{,}000$
$N = 5.92$ rev/min
$v_r = 2\pi RN = 2\pi(12/12)(5.92) = \mathbf{37.2\ ft/min}$

Forging

19.14 A cylindrical part is warm upset forged in an open die. The initial diameter is 45 mm and the initial height is 40 mm. The height after forging is 25 mm. The coefficient of friction at the die-work interface is 0.20. The yield strength of the work material is 285 MPa, and its flow curve is defined by a strength coefficient of 600 MPa and a strain-hardening exponent of 0.12. Determine the force in the operation (a) just as the yield point is reached (yield at strain = 0.002), (b) at a height of 35 mm, (c) at a height of 30 mm, and (d) at a height of 25 mm. Use of a spreadsheet calculator is recommended.

Solution: (a) $V = \pi D^2 L/4 = \pi(45)^2(40)/4 = 63{,}617$ mm^3
Given $\varepsilon = 0.002$, $Y_f = 600(0.002)^{0.12} = 284.6$ MPa, and $h = 40 - 40(0.002) = 39.92$
$A = V/h = 63{,}617/39.92 = 1594$ mm^2
$K_f = 1 + 0.4(0.2)(45)/39.92 = 1.09$
$F = 1.09(284.6)(1594) = \mathbf{494{,}400\ N}$

(b) Given $h = 35$, $\varepsilon = \ln(40/35) = \ln 1.143 = 0.1335$
$Y_f = 600(0.1335)^{0.12} = 471.2$ MPa
$V = 63{,}617$ mm^3 from part (a) above.
At $h = 35$, $A = V/h = 63617/35 = 1818$ mm^2

Corresponding $D = 48.1$ mm (from $A = \pi D^2/4$)
$K_f = 1 + 0.4(0.2)(48.1)/35 = 1.110$
$F = 1.110(471.2)(1818) =$ **950,700 N**

(c) Given $h = 30$, $\varepsilon = \ln(40/30) = \ln 1.333 = 0.2877$
$Y_f = 600(0.2877)^{0.12} = 516.7$ MPa
$V = 63{,}617$ mm^3 from part (a) above.
At $h = 30$, $A = V/h = 63{,}617/30 = 2120.6$ mm^2
Corresponding $D = 51.96$ mm (from $A = \pi D^2/4$)
$K_f = 1 + 0.4(0.2)(51.96)/30 = 1.138$
$F = 1.138(516.7)(2120.6) =$ **1,247,536 N**

(d) Given $h = 25$, $\varepsilon = \ln(40/25) = \ln 1.6 = 0.4700$
$Y_f = 600(0.470)^{0.12} = 548.0$ MPa
$V = 63{,}617$ mm^3 from part (a) above.
At $h = 25$, $A = V/h = 63{,}617/25 = 2545$ mm^2
Corresponding $D = 56.9$ mm (from $A = \pi D^2/4$)
$K_f = 1 + 0.4(0.2)(56.9)/25 = 1.182$
$F = 1.182(548.0)(2545) =$ **1,649,000 N**

19.15 A cylindrical workpart with $D = 2.5$ in and $h = 2.5$ in is upset forged in an open die to a height = 1.5 in. Coefficient of friction at the die-work interface = 0.10. The work material has a flow curve defined by: $K = 40{,}000$ lb/in^2 and $n = 0.15$. Yield strength = 15,750 lb/in^2. Determine the instantaneous force in the operation (a) just as the yield point is reached (yield at strain = 0.002), (b) at height $h = 2.3$ in, (c) $h = 2.1$ in, (d) $h = 1.9$ in, (e) $h = 1.7$ in, and (f) $h = 1.5$ in. Use of a spreadsheet calculator is recommended.

Solution: (a) $V = \pi D^2L/4 = \pi(2.5)^2(2.5)/4 = 12.273$ in^3
Given $\varepsilon = 0.002$, $Y_f = 40{,}000(0.002)^{0.15} = 15{,}748$ lb/in^2 and $h = 2.5 - 2.5(0.002) = 2.495$
$A = V/h = 12.273/2.495 = 4.92$ in^2
$K_f = 1 + 0.4(0.1)(2.5)/2.495 = 1.04$
$F = 1.04(15{,}748)(4.92) =$ **80,579 lb**

(b) Given $h = 2.3$, $\varepsilon = \ln(2.5/2.3) = \ln 1.087 = 0.0834$
$Y_f = 40{,}000(0.0834)^{0.15} = 27{,}556$ lb/in^2
$V = 12.273$ in^3 from part (a) above.
At $h = 2.3$, $A = V/h = 12.273/2.3 = 5.34$ in^2
Corresponding $D = 2.61$ (from $A = \pi D^2/4$)
$K_f = 1 + 0.4(0.1)(2.61)/2.3 = 1.045$
$F = 1.045(27{,}556)(5.34) =$ **153,822 lb**

(c) Given $h = 2.1$, $\varepsilon = \ln(2.5/2.1) = \ln 1.191 = 0.1744$
$Y_f = 40{,}000(0.1744)^{0.15} = 30{,}780$ lb/in^2
$V = 12.273$ in^3 from part (a) above.
At $h = 2.1$, $A = V/h = 12.273/2.1 = 5.84$ in^2
Corresponding $D = 2.73$ (from $A = \pi D^2/4$)
$K_f = 1 + 0.4(0.1)(2.73)/2.1 = 1.052$
$F = 1.052(30{,}780)(5.84) =$ **189,236 lb**

(d) Given $h = 1.9$, $\varepsilon = \ln(2.5/1.9) = \ln 1.316 = 0.274$
$Y_f = 40{,}000(0.274)^{0.15} = 32{,}948$ lb/in^2
$V = 12.273$ in^3 from part (a) above.
At $h = 1.9$, $A = V/h = 12.273/1.9 = 6.46$ in^2
Corresponding $D = 2.87$ (from $A = \pi D^2/4$)

$K_f = 1 + 0.4(0.1)(2.87)/1.9 = 1.060$
$F = 1.060(32{,}948)(6.46) =$ **225,695 lb**

(e) Given $h = 1.7$, $\varepsilon = \ln(2.5/1.7) = \ln 1.471 = 0.386$
$Y_f = 40{,}000(0.386)^{0.15} = 34{,}673$ lb/in^2
$V = 12.273$ in^3 from part (a) above.
At $h = 1.7$, $A = V/h = 12.273/1.7 = 7.22$ in^2
Corresponding $D = 3.03$ (from $A = \pi D^2/4$)
$K_f = 1 + 0.4(0.1)(3.03)/1.7 = 1.071$
$F = 1.071(34{,}673)(7.22) =$ **268,176 lb**

(f) Given $h = 1.5$, $\varepsilon = \ln(2.5/1.5) = \ln 1.667 = 0.511$
$Y_f = 40{,}000(0.511)^{0.15} = 36{,}166$ lb/in^2
$V = 12.273$ in^3 from part (a) above.
At $h = 1.5$, $A = V/h = 12.273/1.5 = 8.182$ in^2
Corresponding $D = 3.23$ (from $A = \pi D^2/4$)
$K_f = 1 + 0.4(0.1)(3.23)/1.5 = 1.086$
$F = 1.086(36{,}166)(8.182) =$ **321,379 lb**

19.16 A cylindrical workpart has a diameter = 2.5 in and a height = 4.0 in. It is upset forged to a height = 2.75 in. Coefficient of friction at the die-work interface = 0.10. The work material has a flow curve with strength coefficient = 25,000 lb/in^2 and strain hardening exponent = 0.22. Determine the plot of force vs. work height. Use of a spreadsheet calculator is recommended.

Solution: Volume of cylinder $V = \pi D^2 L/4 = \pi(2.5)^2(4.0)/4 = 19.635$ in^3
We will compute the force F at selected values of height h: h = (a) 4.0, (b) 3.75, (c) 3.5, (d) 3.25, (e) 3.0, (f) 2.75, and (g) 2.5. These values can be used to develop the plot. The shape of the plot will be similar to Figure 21.13 in the text.

At ***h* = 4.0**, we assume yielding has just occurred and the height has not changed significantly. Use $\varepsilon = 0.002$ (the approximate yield point of metal).
At $\varepsilon = 0.002$, $Y_f = 25{,}000(0.002)^{0.22} = 6{,}370$ lb/in^2
Adjusting the height for this strain, $h = 4.0 - 4.0(0.002) = 3.992$
$A = V/h = 19.635/3.992 = 4.92$ in^2
$K_f = 1 + 0.4(0.1)(2.5)/3.992 = 1.025$
$F = 1.025(6{,}370)(4.92) =$ **32,125 lb**

At ***h* = 3.75**, $\varepsilon = \ln(4.0/3.75) = \ln 1.0667 = 0.0645$
$Y_f = 25{,}000(0.0645)^{0.22} = 13{,}680$ lb/in^2
$V = 19.635$ in^3 calculated above.
At $h = 3.75$, $A = V/h = 19.635/3.75 = 5.236$ in^2
Corresponding $D = 2.582$ (from $A = \pi D^2/4$)
$K_f = 1 + 0.4(0.1)(2.582)/3.75 = 1.028$
$F = 1.028(13{,}680)(5.236) =$ **73,601 lb**

At ***h* = 3.5**, $\varepsilon = \ln(4.0/3.5) = \ln 1.143 = 0.1335$
$Y_f = 25{,}000(0.1335)^{0.22} = 16{,}053$ lb/in^2
At $h = 3.5$, $A = V/h = 19.635/3.5 = 5.61$ in^2
Corresponding $D = 2.673$ (from $A = \pi D^2/4$)
$K_f = 1 + 0.4(0.1)(2.673)/3.5 = 1.031$
$F = 1.031(16{,}053)(5.61) =$ **92,808 lb**

At ***h* = 3.25**, $\varepsilon = \ln(4.0/3.25) = \ln 1.231 = 0.2076$
$Y_f = 25{,}000(0.2076)^{0.22} = 17{,}691$ lb/in^2
At $h = 3.25$, $A = V/h = 19.635/3.25 = 6.042$ in^2

Corresponding $D = 2.774$ (from $A = \pi D^2/4$)
$K_f = 1 + 0.4(0.1)(2.774)/3.25 = 1.034$
$F = 1.034(17{,}691)(6.042) =$ **110,538 lb**

At $\boldsymbol{h}$ **= 3.0**, $\varepsilon = \ln(4.0/3.0) = \ln 1.333 = 0.2874$
$Y_f = 25{,}000(0.2874)^{0.22} = 19{,}006$ lb/in^2
At $h = 3.0$, $A = V/h = 19.635/3.0 = 6.545$ in^2
Corresponding $D = 2.887$ (from $A = \pi D^2/4$)
$K_f = 1 + 0.4(0.1)(2.887)/3.0 = 1.038$
$F = 1.038(19{,}006)(6.545) =$ **129,182 lb**

At $\boldsymbol{h}$ **=2.75**, $\varepsilon = \ln(4.0/2.75) = \ln 1.4545 = 0.3747$
$Y_f = 25{,}000(0.3747)^{0.22} = 20{,}144$ lb/in^2
$V = 19.635$ in^3 calculated above.
At $h = 2.75$, $A = V/h = 19.635/2.75 = 7.140$ in^2
Corresponding $D = 3.015$ (from $A = \pi D^2/4$)
$K_f = 1 + 0.4(0.1)(3.015)/2.75 = 1.044$
$F = 1.044(20{,}144)(7.140) =$ **150,136 lb**

19.17 A cold heading operation is performed to produce the head on a steel nail. The strength coefficient for this steel is 600 MPa, and the strain hardening exponent is 0.22. Coefficient of friction at the die-work interface is 0.14. The wire stock out of which the nail is made is 5.00 mm in diameter. The head is to have a diameter of 9.5 mm and a thickness of 1.6 mm. The final length of the nail is 120 mm. (a) What length of stock must project out of the die in order to provide sufficient volume of material for this upsetting operation? (b) Compute the maximum force that the punch must apply to form the head in this open-die operation.

Solution: (a) Volume of nail head $V = \pi D_f^2 h_f/4 = \pi(9.5)^2(1.6)/4 = 113.4$ mm^3.
$A_o = \pi D_o^2/4 = \pi(4.75)^2/4 = 19.6$ mm^2
$h_o = V/A_o = 113.4/19.6 =$ **5.78 mm**

(b) $\varepsilon = \ln(5.78/1.6) = \ln 3.61 = 1.2837$
$Y_f = 600(1.2837)^{0.22} = 634$ MPa
$A_f = \pi(9.5)^2/4 = 70.9$ mm^2
$K_f = 1 + 0.4(0.14)(9.5/1.6) = 1.33$
$F = 1.33(634)(70.9) =$ **59,886 N**

19.18 Obtain a large common nail (flat head). Measure the head diameter and thickness, as well as the diameter of the nail shank. (a) What stock length must project out of the die in order to provide sufficient material to produce the nail? (b) Using appropriate values for strength coefficient and strain hardening exponent for the metal out of which the nail is made (Table 3.4), compute the maximum force in the heading operation to form the head.

Solution: Student exercise. Calculations similar to those in the preceding problem for the data developed by the student.

19.19 A hot upset forging operation is performed in an open die. The initial size of the workpart is: $D_o = 25$ mm, and $h_o = 50$ mm. The part is upset to a diameter = 50 mm. The work metal at this elevated temperature yields at 85 MPa ($n = 0$). Coefficient of friction at the die-work interface = 0.40. Determine (a) final height of the part, and (b) maximum force in the operation.

Solution: (a) $V = \pi D_o^2 h_o/4 = \pi(25)^2(50)/4 = 24{,}544$ mm^3.
$A_f = \pi D_f^2/4 = \pi(50)^2/4 = 1963.5$ mm^2.
$h_f = V/A_f = 24{,}544/1963.5 =$ **12.5 mm.**

(b) $\varepsilon = \ln(50/12.5) = \ln 4 = 1.3863$

$Y_f = 85(1.3863)^0 = 85$ MPa
Force is maximum at largest area value, $A_f = 1963.5$ mm^2
$D = (4 \times 1963.5/\pi)^{0.5} = 50$ mm
$K_f = 1 + 0.4(0.4)(50/12.5) = 1.64$
$F = 1.64(85)(1963.5) =$ **273,712 N**

19.20 A hydraulic forging press is capable of exerting a maximum force = 1,000,000 N. A cylindrical workpart is to be cold upset forged. The starting part has diameter = 30 mm and height = 30 mm. The flow curve of the metal is defined by $K = 400$ MPa and $n = 0.2$. Determine the maximum reduction in height to which the part can be compressed with this forging press, if the coefficient of friction = 0.1. Use of a spreadsheet calculator is recommended.

Solution: Volume of work $V = \pi D_o^2 h_o/4 = \pi(30)^2(30)/4 = 21{,}206$ mm^3.
Final area $A_f = 21{,}206/h_f$
$\varepsilon = \ln(30/h_f)$
$Y_f = 400\varepsilon^{0.2} = 400(\ln 30/h_f)^{0.2})$
$K_f = 1 + 0.4\mu(D_f/h_f) = 1 + 0.4(0.1)(D_f/h_f)$
Forging force $F = K_f Y_f A_f = (1 + 0.04D_f/h_f)(\ 400(\ln 30/h_f)^{0.2})(\ 21{,}206/h_f)$

Requires trial and error solution to find the value of $\boldsymbol{h_f}$ that will match the force of **1,000,000 N**.
(1) Try $h_f = 20$ mm
$A_f = 21{,}206/20 = 1060.3$ mm^2
$\varepsilon = \ln(30/20) = \ln 1.5 = 0.405$
$Y_f = 400(0.405)^{0.2} = 333.9$ MPa
$D_f = (4 \times 1060.3/\pi)^{0.5} = 36.7$ mm
$K_f = 1 + 0.04(36.7/20) = 1.073$
$F = 1.073(333.9)(1060.3) =$ **380,050 N**
Too low. Try a smaller value of h_f to increase F.

(2) Try $h_f = 10$ mm.
$A_f = 21{,}206/10 = 2120.6$ mm^2
$\varepsilon = \ln(30/10) = \ln 3.0 = 1.099$
$Y_f = 400(1.099)^{0.2} = 407.6$ MPa
$D_f = (4 \times 2120.6/\pi)^{0.5} = 51.96$ mm
$K_f = 1 + 0.04(51.96/10) = 1.208$
$F = 1.208(407.6)(2120.6) =$ **1,043,998 N**
Slightly high. Need to try a value of h_f between 10 and 20, closer to 10.

(3) Try $h_f = 11$ mm
$A_f = 21{,}206/11 = 1927.8$ mm^2
$\varepsilon = \ln(30/11) = \ln 2.7273 = 1.003$
$Y_f = 400(1.003)^{0.2} = 400.3$ MPa
$D_f = (4 \times 1927.8/\pi)^{0.5} = 49.54$ mm
$K_f = 1 + 0.04(51.12/11) = 1.18$
$F = 1.18(400.3)(1927.8) =$ **910,653 N**

(4) By linear interpolation, try $h_f = 10 + (44/133) = 10.33$ mm
$A_f = 21{,}206/10.33 = 2052.8$ mm^2
$\varepsilon = \ln(30/10.33) = \ln 2.9042 = 1.066$
$Y_f = 400(1.066)^{0.2} = 405.16$ MPa
$D_f = (4 \times 2052.8/\pi)^{0.5} = 51.12$ mm
$K_f = 1 + 0.04(51.12/10.33) = 1.198$
$F = 1.198(405.16)(2052.8) =$ **996,364 N**

(5) By further linear interpolation, try $h_f = 10 + (44/48)(0.33) = 10.30$
$A_f = 21{,}206/10.30 = 2058.8$ mm^2
$\varepsilon = \ln(30/10.30) = \ln 2.913 = 1.069$
$Y_f = 400(1.069)^{0.2} = 405.38$ MPa
$D_f = (4 \text{ x } 2058.8/\pi)^{0.5} = 51.2$ mm
$K_f = 1 + 0.04(51.2/10.3) = 1.199$
$F = 1.199(405.38)(2058.8) =$ **1,000,553 N**
Close enough! Maximum height reduction = 30.0 - 10.3 = **19.7 mm**

Using a spreadsheet calculator, the author's program (written in Excel) obtained a value of h = 19.69603 mm to achieve a force of 1,000,000 lb within one pound.

19.21 A part is designed to be hot forged in an impression die. The projected area of the part, including flash, is 16 in^2. After trimming, the part has a projected area of 10 in^2. Part geometry is complex. As heated the work material yields at 10,000 lb/in^2, and has no tendency to strain harden. At room temperature, the material yields at 25,000 lb/in^2 Determine the maximum force required to perform the forging operation.

Solution: Since the work material has no tendency to work harden, $n = 0$.
From Table 19.1, choose $K_f = 8.0$.
$F = 8.0(10{,}000)(16) =$ **1,280,000 lb.**

19.22 A connecting rod is designed to be hot forged in an impression die. The projected area of the part is 6,500 mm^2. The design of the die will cause flash to form during forging, so that the area, including flash, will be 9,000 mm^2. The part geometry is considered to be complex. As heated the work material yields at 75 MPa, and has no tendency to strain harden. Determine the maximum force required to perform the operation.

Solution: Since the work material has no tendency to work harden, $n = 0$.
From Table 19.1, choose K_f=8.0.
$F = 8.0(75)(9{,}000) =$ **5,400,000 N.**

Extrusion

19.23 A cylindrical billet that is 100 mm long and 50 mm in diameter is reduced by indirect (backward) extrusion to a 20 mm diameter. The die angle is 90°. The Johnson equation has $a = 0.8$ and $b = 1.4$, and the flow curve for the work metal has a strength coefficient of 800 MPa and strain hardening exponent of 0.13. Determine (a) extrusion ratio, (b) true strain (homogeneous deformation), (c) extrusion strain, (d) ram pressure, and (e) ram force.

Solution: (a) $r_x = A_o/A_f = D_o^2/D_f^2 = (50)^2/(20)^2 =$ **6.25**

(b) $\varepsilon = \ln r_x = \ln 6.25 =$ **1.833**

(c) $\varepsilon_x = a + b \ln r_x = 0.8 + 1.4(1.833) =$ **3.366**

(d) $\overline{Y}_f = 800(1.833)^{0.13}/1.13 = 766.0$ MPa
$p = 766.0(3.366) =$ **2578 MPa**

(e) $A_o = \pi D_o^2/4 = \pi(50)^2/4 = 1963.5$ mm^2
$F = 2578(1963.5) =$ **5,062,000 N**

19.24 A 3.0-in-long cylindrical billet whose diameter = 1.5 in is reduced by indirect extrusion to a diameter = 0.375 in. Die angle = 90°. In the Johnson equation, $a = 0.8$ and $b = 1.5$. In the flow curve for the work metal, K = 75,000 lb/in^2 and $n = 0.25$. Determine (a) extrusion ratio, (b) true strain (homogeneous deformation), (c) extrusion strain, (d) ram pressure, (e) ram force, and (f) power if the ram speed = 20 in/min.

Solution: (a) $r_x = A_o/A_f = D_o^2/D_f^2 = (1.5)^2/(0.375)^2 = 4^2 =$ **16.0**

(b) $\varepsilon = \ln r_x = \ln 16 =$ **2.773**

(c) $\varepsilon_x = a + b \ln r_x = 0.8 + 1.5(2.773) =$ **4.959**

(d) $\overline{Y}_f = 75{,}000(2.773)^{0.25}/1.25 = 77{,}423$ lb/in^2
$p = 77{,}423(4.959) =$ **383,934 lb/in²**

(e) $A_o = \pi D_o^2/4 = \pi(1.5)^2/4 = 1.767$ in^2
$F = (383{,}934)(1.767) =$ **678,411 lb.**

(f) $P = 678{,}411(20) =$ **13,568,228 in-lb/min**
$HP = 13{,}568{,}228/396{,}000 =$ **34.26 hp**

19.25 A billet that is 75 mm long with diameter = 35 mm is direct extruded to a diameter of 20 mm. The extrusion die has a die angle = 75°. For the work metal, $K = 600$ MPa and $n = 0.25$. In the Johnson extrusion strain equation, $a = 0.8$ and $b = 1.4$. Determine (a) extrusion ratio, (b) true strain (homogeneous deformation), (c) extrusion strain, and (d) ram pressure and force at $L = 70$, 60, 50, 40, 30, 20, and 10 mm. Use of a spreadsheet calculator is recommended for part (d).

Solution: (a) $r_x = A_o/A_f = D_o^2/D_f^2 = (35)^2/(20)^2 =$ **3.0625**

(b) $\varepsilon = \ln r_x = \ln 3.0625 =$ **1.119**

(c) $\varepsilon_x = a + b \ln r_x = 0.8 + 1.4(1.119) =$ **2.367**

(d) $\overline{Y}_f = 600(1.119)^{0.25}/1.25 = 493.7$ MPa
$A_o = \pi(35)^2/4 = 962.1$ mm^2
It is appropriate to determine the volume of metal contained in the cone of the die at the start of the extrusion operation, to assess whether metal has been forced through the die opening by the time the billet has been reduced from $L = 75$ mm to $L = 70$ mm. For a cone-shaped die with angle = 75°, the height h of the frustum is formed by metal being compressed into the die opening: The two radii are: $R_1 = 0.5D_o = 17.5$ mm and $R_2 = 0.5D_f = 10$ mm, and $h = (R_1 - R_2)/\tan 75 = 7.5/\tan 75 = 2.01$ mm
Frustum volume $V = 0.333\pi h(R_1^2 + R_1R_2 + R_2^2) = 0.333\pi(2.01)(17.5^2 + 10 \text{ x } 17.5 + 10^2) = 1223.4$ mm^3. Compare this with the volume of the portion of the cylindrical billet between $L = 75$ mm and $L = 70$ mm.
$V = \pi D_o^2\, h/4 = 0.25\pi(35)^2(75 - 70) = 4810.6$ mm^3
Since this volume is greater than the volume of the frustum, this means that the metal has extruded through the die opening by the time the ram has moved forward by 5 mm.

***L* = 70 mm**: pressure $p = 493.7(2.367 + 2 \text{ x } 70/35) =$ **3143.4 MPa**
Force $F = 3143.4(962.1) =$ **3,024, 321 N**
***L* = 60 mm**: pressure $p = 493.7(2.367 + 2 \text{ x } 60/35) =$ **2861.3 MPa**
Force $F = 2861.3(962.1) =$ **2,752,890 N**
***L* = 50 mm**: pressure $p = 493.7(2.367 + 2 \text{ x } 50/35) =$ **2579.2 MPa**
Force $F = 2579.2(962.1) =$ **2,481,458 N**
***L* = 40 mm**: pressure $p = 493.7(2.367 + 2 \text{ x } 40/35) =$ **2297.1 MPa**
Force $F = 2297.1(962.1) =$ **2,210,027 N**
***L* = 30 mm**: pressure $p = 493.7(2.367 + 2 \text{ x } 30/35) =$ **2014.9 MPa**
Force $F = 2014.9(962.1) =$ **1,938,595 N**
***L* = 20 mm**: pressure $p = 493.7(2.367 + 2 \text{ x } 20/35) =$ **1732.8 MPa**
Force $F = 1732.8(962.1) =$ **1,667,164 N**
***L* = 10 mm**: pressure $p = 493.7(2.367 + 2 \text{ x } 10/35) =$ **1450.7 MPa**
Force $F = 1450.7(962.1) =$ **1,395,732 N**

19.26 A 2.0-in-long billet with diameter = 1.25 in is direct extruded to a diameter of 0.50 in. The extrusion die angle = 90°. For the work metal, K = 45,000 lb/in^2, and n = 0.20. In the Johnson extrusion strain equation, a = 0.8 and b = 1.5. Determine (a) extrusion ratio, (b) true strain (homogeneous deformation), (c) extrusion strain, and (d) ram pressure at L = 2.0, 1.5, 1.0, 0.5 and zero in. Use of a spreadsheet calculator is recommended for part (d).

Solution: (a) $r_x = A_o/A_f = D_o^2/D_f^2 = (1.25)^2/(0.5)^2 =$ **6.25**

(b) $\varepsilon = \ln r_x = \ln 6.25 =$ **1.8326**

(c) $\varepsilon_x = a + b \ln r_x = 0.8 + 1.5(1.8326) =$ **3.549**

(d) $\overline{Y}_f = 45{,}000(1.8326)^{0.20}/1.20 = 42{,}330$ lb/in^2
$A_o = \pi(1.25)^2/4 = 1.227$ in^2
Unlike the previous problem, the die angle $\alpha = 90°$, so metal is forced through the die opening as soon as the billet starts to move forward in the chamber.

***L* = 2.0 in**: pressure p = 42,330(3.549 + 2 x 2.0/1.25) = **285,677 lb/in^2**
Force F = 285,677(1.227) = **350,579 lb**
***L* = 1.5 in**: pressure p = 42,330(3.549 + 2 x 1.5/1.25) = **251,813 lb/in^2**
Force F = 251,813(1.227) = **309,022 lb**
***L* = 1.0 in**: pressure p = 42,330(3.549 + 2 x 1.0/1.25) = **217,950 lb/in^2**
Force F = 217,950(1.227) = **267,465 lb**
***L* = 0.5 in**: pressure p = 42,330(3.549 + 2 x 0.5/1.25) = **184,086 lb/in^2**
Force F = 184,086(1.227) = **225,908 lb**
***L* = 0.0 in**: pressure p = 42,330(3.549 + 2 x 0.0/1.25) = **150,229 lb/in^2**
Force F = 150,229(1.227) = **184,351 lb**
These last values for L = 0 are not possible because of the increase in pressure and force due to the butt remaining in the extruder container at the end of the operation.

19.27 A direct extrusion operation is performed on a cylindrical billet with an initial diameter of 2.0 in and an initial length of 4.0 in. The die angle = 60° and orifice diameter is 0.50 in. In the Johnson extrusion strain equation, a = 0.8 and b = 1.5. The operation is carried out hot and the hot metal yields at 13,000 lb/in^2 and does not strain harden when hot. (a) What is the extrusion ratio? (b) Determine the ram position at the point when the metal has been compressed into the cone of the die and starts to extrude through the die opening. (c) What is the ram pressure corresponding to this position? (d) Also determine the length of the final part if the ram stops its forward movement at the start of the die cone.

Solution: (a) $r_x = A_o/A_f = D_o^2/D_f^2 = (2.0)^2/(0.5)^2 =$ **16.0**

(b) The portion of the billet that is compressed into the die cone forms a frustum with $R_1 = 0.5D_o$ = 1.0 in and $R_2 = 0.5D_f$ = 0.25 in. The height of the frustum h = (R_1 - R_2)/tan 65 = (1.0 - 0.25)/tan 60 = 0.433 in. The volume of the frustum is
$V = 0.333\pi h(R_1^2 + R_1R_2 + R_2^2) = 0.333\pi(0.433)(1.0^2 + 1.0 \text{ x } 0.25 + 0.25^2) = 0.595$ in^3
The billet has advanced a certain distance by the time this frustum is completely filled and extrusion through the die opening is therefore initiated. The volume of billet compressed forward to fill the frustum is given by:
$V = \pi R_1^2(L_o - L_1) = \pi(1.0)^2(L_o - L_1)$
Setting this equal to the volume of the frustum, we have
$\pi(L_o - L_1) = 0.595$ in^3
$(L_o - L_1) = 0.595/\pi = 0.189$ in
L_1 = 4.0 - 0.189 = **3.811 in**

(c) $\varepsilon = \ln r_x = \ln 16 = 2.7726$

$\varepsilon_x = a + b \ln r_x = 0.8 + 1.5(2.7726) = 4.959$
$\overline{Y}_f = 13{,}000(2.7726)^0/1.0 = 13{,}000$ lb/in^2
$p = 13{,}000(4.959 + 2 \times 3.811/2.0) =$ **114,000 lb/in^2**

(d) Length of extruded portion of billet = 3.811 in. With a reduction $r_x = 16$, the final part length, excluding the cone shaped butt remaining in the die is $L = 3.811(16) =$ **60.97 in.**

19.28 An indirect extrusion process starts with an aluminum billet with diameter = 2.0 in and length = 3.0 in. Final cross section after extrusion is a square with 1.0 in on a side. The die angle = 90°. The operation is performed cold and the strength coefficient of the metal $K = 26{,}000$ lb/in^2 and strain-hardening exponent $n = 0.20$. In the Johnson extrusion strain equation, $a = 0.8$ and $b = 1.2$. (a) Compute the extrusion ratio, true strain, and extrusion strain. (b) What is the shape factor of the product? (c) If the butt left in the container at the end of the stroke is 0.5 in thick, what is the length of the extruded section? (d) Determine the ram pressure in the process.

Solution: (a) $r_x = A_o/A_f$
$A_o = \pi D_o^2/4 = \pi(2)^2/4 = 3.142$ in^2
$A_f = 1.0 \times 1.0 = 1.0$ in^2
$r_x = 3.142/1.0 =$ **3.142**
$\varepsilon = \ln 3.142 =$ **1.145**
$\varepsilon_x = 0.8 + 1.3(1.145) =$ **2.174**

(b) To determine the die shape factor we need to determine the perimeter of a circle whose area is equal to that of the extruded cross section, $A = 1.0$ in^2. The radius of the circle is $R = (1.0/\pi)^{0.5} = 0.5642$ in, $C_c = 2\pi(0.5642) = 3.545$ in
The perimeter of the extruded cross section $C_x = 4(1.0) = 4.0$ in
$K_x = 0.98 + 0.02(4.0/3.545)^{2.25} =$ **1.006**

(c) Given that the butt thickness = 0.5 in
Original volume $V = (3.0)(\pi \times 2^2/4) = 9.426$ in^3
The final volume consists of two sections: (1) butt, and (2) extrudate. The butt volume $V_1 = (0.5)(\pi 2^2/4) = 1.571$ in^3. The extrudate has a cross-sectional area $A_f = 1.0$ in^2. Its volume $V_2 = LA_f = 9.426 - 1.571 = 7.855$ in^3. Thus, length $L = 7.855/1.0 =$ **7.855 in**

(d) $\overline{Y}_f = 26{,}000(1.145)^{0.2}/1.2 = 22{,}261$ lb/in^3
$p = 1.006(22{,}261)(2.174) =$ **48,698 lb/in^2**

19.29 An L-shaped structural section is direct extruded from an aluminum billet in which $L_o = 500$ mm and $D_o = 100$ mm. Dimensions of the cross section are given in Figure P19.29. Die angle = 90°. Determine (a) extrusion ratio, (b) shape factor, and (c) length of the extruded section if the butt remaining in the container at the end of the ram stroke is 25 mm.

Solution: (a) $r_x = A_o/A_f$
$A_o = \pi(100)^2/4 = 7854$ mm^2
$A_f = 2(12 \times 50) = 1200$ mm^2
$r_x = 7854/1200 =$ **6.545**

(b) To determine the die shape factor we need to determine the perimeter of a circle whose area is equal to that of the extruded cross section, $A = 1200$ mm^2. The radius of the circle is $R = (1200/\pi)^{0.5} = 19.54$ mm, $C_c = 2\pi(19.54) = 122.8$ mm.
The perimeter of the extruded cross section $C_x = 62 + 50 + 12 + 38 + 50 + 12 = 224$ mm
$K_x = 0.98 + 0.02(224/122.8)^{2.25} =$ **1.057**

(c) Total original volume $V = 0.25\pi(100)^2(500) = 3{,}926{,}991$ mm^3

The final volume consists of two sections: (1) butt, and (2) extrudate. The butt volume $V_1 = 0.25\pi(100)^2(25) = 196{,}350$ mm^3. The extrudate has a cross-sectional area $A_f = 1200$ mm^2. Its volume $V_2 = LA_f = 3{,}926{,}991 - 196{,}350 = 3{,}730{,}641$ mm^3.
Thus, length $L = 3{,}730{,}641/1200 =$ **3108.9 mm = 3.109 m**

19.30 The flow curve parameters for the aluminum alloy of Problem 19.29 are: $K = 240$ MPa and $n = 0.16$. If the die angle in this operation = 90°, and the corresponding Johnson strain equation has constants $a = 0.8$ and $b = 1.5$, compute the maximum force required to drive the ram forward at the start of extrusion.

Solution: From Problem 19.29, $r_x = 5.068$
$\varepsilon = \ln 5.068 = 1.623$
$\varepsilon_x = 0.8 + 1.5(1.623) = 3.234$
$\bar{Y}_f = 240(1.623)^{0.16}/1.16 = 223.6$ MPa
Maximum ram force occurs at beginning of stroke when length is maximum at $L = 250$ mm
$p = K_x\bar{Y}_f\ (\varepsilon_x + 2L/D_o) = 1.057(223.6)(\ 3.234 + 2(250)/88) = 2107.2$ MPa
$F = pA_o = 2107.2\ (6082.1) =$ **12,816,267 N**

19.31 A cup-shaped part is backward extruded from an aluminum slug that is 50 mm in diameter. The final dimensions of the cup are: OD = 50 mm, ID = 40 mm, height = 100 mm, and thickness of base = 5 mm. Determine (a) extrusion ratio, (b) shape factor, and (c) height of starting slug required to achieve the final dimensions. (d) If the metal has flow curve parameters $K = 400$ MPa and $n = 0.25$, and the constants in the Johnson extrusion strain equation are: $a = 0.8$ and $b = 1.5$, determine the extrusion force.

Solution: (a) $r_x = A_o/A_f$
$A_o = 0.25\pi(50)^2 = 1963.75$ mm^2
$A_f = 0.25\pi(50^2 - 40^2) = 706.86$ mm^2
$r_x = 1963.75/706.86 =$ **2.778**

(b) To determine the die shape factor we need to determine the perimeter of a circle whose area is equal to that of the extruded cross section, $A = 706.86$ mm^2. The radius of the circle is $R = (706.86/\pi)^{0.5} = 15$ mm, $C_c = 2\pi(15) = 94.25$ mm.
The perimeter of the extruded cross section $C_x = \pi(50 + 40) = 90\pi = 282.74$ mm.
$K_x = 0.98 + 0.02(282.74/94.25)^{2.25} =$ **1.217**

(c) Volume of final cup consists of two geometric elements: (1) base and (2) ring.
(1) Base $t = 5$ mm and $D = 50$ mm. $V_1 = 0.25\pi(50)^2(5) = 9817.5$ mm^3
(2) Ring OD = 50 mm, ID = 40 mm, and $h = 95$ mm.
$V_2 = 0.25\pi(50^2 - 40^2)(95) = 0.25\pi(2500 - 1600)(95) = 67{,}151.5$ mm^3
Total $V = V_1 + V_2 = 9817.5 + 67{,}151.5 = 76{,}969$ mm^3
Volume of starting slug must be equal to this value $V = 76{,}969$ mm^3
$V = 0.25\pi(50)^2(h) = 1963.5h = 76{,}969$ mm^3
***h* = 39.2 mm**

(d) $\varepsilon = \ln 2.778 = 1.0218$
$\varepsilon_x = 0.8 + 1.5(1.0218) = 2.33$
$\bar{Y}_f = 400(1.0218)^{0.25}/1.25 = 321.73$ MPa
$p = K_x\ \bar{Y}_f\ \varepsilon_x = 1.217(321.73)(2.33) = 912.3$ MPa
$A_o = 0.25\pi(40)^2 = 1256.6$ mm^2
$F = 912.3(1256.6) =$ **1,146,430 N**

19.32 Determine the shape factor for each of the extrusion die orifice shapes in Figure P19.32.

Solution: (a) $A_x = 20 \text{ x } 60 = 1200$ mm, $C_x = 2(20 + 60) = 160$ mm
$A_o = \pi R^2 = 1200$
$R^2 = 1200/\pi = 381.97$, $R = 19.544$ mm, $C_c = 2\pi R = 2\pi(19.544) = 122.8$ mm
$K_x = 0.98 + 0.02(160/122.8)^{2.25} = \mathbf{1.016}$

(b) $A_x = \pi R_o^2 - \pi R_i^2 = \pi(25^2 - 22.5^2) = 373.06$ mm^2
$C_x = \pi D_o + \pi D_i = \pi(50 + 45) = 298.45$ mm
$R^2 = 373.06/\pi = 118.75$, $R = 10.897$ mm, $C_c = 2\pi R = 2\pi(10.897) = 68.47$ mm
$K_x = 0.98 + 0.02(298.45/68.47)^{2.25} = \mathbf{1.53}$

(c) $A_x = 2(5)(30) + 5(60 - 10) = 300 + 250 = 550$ mm^2
$C_x = 30 + 60 + 30 + 5 + 25 + 50 + 25 + 5 = 230$ mm
$A_o = \pi R^2 = 550$, $R^2 = 550/\pi = 175.07$, $R = 13.23$ mm
$C_c = 2\pi R = 2\pi(13.23) = 83.14$ mm
$K_x = 0.98 + 0.02(230/83.14)^{2.25} = \mathbf{1.177}$

(d) $A_x = 5(55)(5) + 5(85 - 5\text{x}5) = 1675$ mm^2
$C_x = 2 \text{ x } 55 + 16 \text{ x } 25 + 8 \text{ x } 15 + 10 \text{ x } 5 = 680$ mm
$A_o = \pi R^2 = 1675$, $R^2 = 1675/\pi = 533.17$, $R = 23.09$ mm
$C_c = 2\pi R = 2\pi(23.09) = 145.08$ mm
$K_x = 0.98 + 0.02(680/145.08)^{2.25} = \mathbf{1.626}$

19.33 A direct extrusion operation produces the cross section shown in Figure P19.32(a) from a brass billet whose diameter = 125 mm and length = 350 mm. The flow curve parameters of the brass are K = 700 MPa and $n = 0.35$. In the Johnson strain equation, $a = 0.7$ and $b = 1.4$. Determine (a) the extrusion ratio, (b) the shape factor, (c) the force required to drive the ram forward during extrusion at the point in the process when the billet length remaining in the container = 300 mm, and (d) the length of the extruded section at the end of the operation if the volume of the butt left in the container is 600,000 mm^3.

Solution: (a) $r_x = A_o/A_f$
$A_o = \pi(125)^2/4 = 12{,}272$ mm^2
$A_f = A_x = 20(60) = 1200$ mm^2
$r_x = 12272/1200 = \mathbf{10.23}$

(b) To determine the die shape factor we need to determine the perimeter of a circle whose area is equal to that of the extruded cross section, $A_f = 1200$ mm^2.
The radius of the circle is $R = (1200/\pi)^{0.5} = 19.544$ mm, $C_c = 2\pi(19.544) = 122.8$ mm.
The perimeter of the extruded cross section $C_x = 2(20 + 60) = 160$ mm
$K_x = 0.98 + 0.02(160/122.8)^{2.25} = \mathbf{1.016}$

(c) $\varepsilon = \ln 10.23 = 2.325$
$\varepsilon_x = 0.7 + 1.4(2.325) = 3.955$
$\overline{Y}_f = 700(2.325)^{0.35}/1.35 = 696.6$ MPa

$p = K_x \overline{Y}_f \varepsilon_x = 1.016(696.6)(3.955 + 2(300)/125) = 6196.3$ MPa
$F = pA_o = 6196.3(12{,}272) = \mathbf{76{,}295{,}200\ N}$

(d) Total original volume $V = \pi(125)^2(350)/4 = 4{,}295{,}200$ mm^3
The final volume consists of two sections: (1) butt, and (2) extrudate.
The butt volume as given $V_1 = 600{,}000$ mm^3.
The extrudate has a cross-sectional area $A_f = 1200$ mm^2.
Its volume $V_2 = LA_f = 4{,}295{,}200 – 600{,}000 = 3{,}695{,}200$ mm^3.
Thus, length $L = 3{,}695{,}200/1200 =$ **3079.3 mm = 3.079 m**

19.34 In a direct extrusion operation the cross section shown in Figure P19.32(b) is produced from a copper billet whose diameter = 100 mm and length = 500 mm. In the flow curve for copper, the strength coefficient = 300 MPa and strain hardening exponent = 0.50. In the Johnson strain equation, $a = 0.8$ and $b = 1.5$. Determine (a) the extrusion ratio, (b) the shape factor, (c) the force required to drive the ram forward during extrusion at the point in the process when the billet length remaining in the container = 450 mm, and (d) the length of the extruded section at the end of the operation if the volume of the butt left in the container is 350,000 mm^3.

Solution: (a) $r_x = A_o/A_f$
$A_o = \pi(100)^2/4 = 7854$ mm^2
$A_f = A_x = \pi(50)^2/4 - \pi(45)^2/4 = 1963.5 – 1590.4 = 373.1$ mm^2
$r_x = 7854/373.1 =$ **21.05**

(b) To determine the die shape factor we need to determine the perimeter of a circle whose area is equal to that of the extruded cross section, $A_x = 373.1$ mm^2.
The radius of the circle is $R = (373.1/\pi)^{0.5} = 10.9$ mm, $C_c = 2\pi(10.9) = 68.5$ mm.
The perimeter of the extruded cross section $C_x = \pi(50) + \pi(45) = 298.5$ mm
$K_x = 0.98 + 0.02(298.5/68.5)^{2.25} =$ **1.53**

(c) $\varepsilon = \ln 21.05 = 3.047$
$\varepsilon_x = 0.8 + 1.5(3.047) = 5.37$
$\overline{Y}_f = 300(3.047)^{0.50}/1.50 = 349.1$ MPa

$p = K_x \overline{Y}_f \varepsilon_x = 1.53(349.1)(5.37 + 2(450)/100) = 7675.3$ MPa
$F = pA_o = 7675.3(7854) =$ **60,282,179 N**

(d) Total original volume $V = \pi(100)^2(500)/4 = 3,926,991$ mm^3
The final volume consists of two sections: (1) butt, and (2) extrudate.
The butt volume as given $V_1 = 350,000$ mm^3.
The extrudate has a cross-sectional area $A_f = 373.1$ mm^2.
Its volume $V_2 = LA_f = 3,926,991 – 350,000 = 3,576,991$ mm^3.
Thus, length $L = 3,576,991/373.1 =$ **9,587.2 mm = 9.587 m**

19.35 A direct extrusion operation produces the cross section shown in Figure P19.32(c) from an aluminum billet whose diameter = 150 mm and length = 500 mm. The flow curve parameters for the aluminum are $K = 240$ MPa and $n = 0.16$. In the Johnson strain equation, $a = 0.8$ and $b = 1.2$. Determine (a) the extrusion ratio, (b) the shape factor, (c) the force required to drive the ram forward during extrusion at the point in the process when the billet length remaining in the container = 400 mm, and (d) the length of the extruded section at the end of the operation if the volume of the butt left in the container is 600,000 mm^3.

Solution: (a) $r_x = A_o/A_f$
$A_o = \pi(150)^2/4 = 17,671.5$ mm^2
$A_f = A_x = 60(5) + 2(25)(5) = 300 + 250 = 550$ mm^2
$r_x = 17,671.5/550 =$ **32.1**

(b) To determine the die shape factor we need to determine the perimeter of a circle whose area is equal to that of the extruded cross section, $A_x = 550$ mm^2.
$C_x = 30 + 60 + 30 + 5 + 25 + 50 + 25 + 5 = 230$ mm
$A_o = \pi R^2 = 550$, $R^2 = 550/\pi = 175.07$, $R = 13.23$ mm
$C_c = 2\pi R = 2\pi(13.23) = 83.14$ mm
$K_x = 0.98 + 0.02(230/83.14)^{2.25} =$ **1.177**

(c) $\varepsilon = \ln 32.1 = 3.47$
$\varepsilon_x = 0.8 + 1.2(3.47) = 4.96$

$\overline{Y}_f = 240(3.47)^{0.16}/1.16 = 252.5$ MPa

$p = K_x \overline{Y}_f \varepsilon_x = 1.177(252.5)(4.96 + 2(400)/150) = 3059.1$ MPa

$F = pA_o = 3059.1(17{,}671.5) =$ **54,058,912 N**

(d) Total original volume $V = \pi(150)^2(500)/4 = 8{,}835{,}750$ mm^3
The final volume consists of two sections: (1) butt, and (2) extrudate.
The butt volume as given $V_1 = 600{,}000$ mm^3.
The extrudate has a cross-sectional area $A_f = 550$ mm^2.
Its volume $V_2 = LA_f = 8{,}835{,}750 - 600{,}000 = 8{,}235{,}750$ mm^3.
Thus, length $L = 8{,}835{,}750/550 =$ **14,974 mm = 14.974 m**

19.36 A direct extrusion operation produces the cross section shown in Figure P19.32(d) from an aluminum billet whose diameter = 150 mm and length = 900 mm. The flow curve parameters for the aluminum are $K = 240$ MPa and $n = 0.16$. In the Johnson strain equation, $a = 0.8$ and $b = 1.5$. Determine (a) the extrusion ratio, (b) the shape factor, (c) the force required to drive the ram forward during extrusion at the point in the process when the billet length remaining in the container = 850 mm, and (d) the length of the extruded section at the end of the operation if the volume of the butt left in the container is 600,000 mm^3.

Solution: (a) $r_x = A_o/A_f$
$A_o = \pi(150)^2/4 = 17{,}671.5$ mm^2
$A_f = A_x = 5(55)(5) + 5(85 - 5(5)) = 1675$ mm^2
$r_x = 17{,}671.5/1675 =$ **10.55**

(b) To determine the die shape factor we need to determine the perimeter of a circle whose area is equal to that of the extruded cross section, $A_x = 1675$ mm^2.
$C_x = 2 \times 55 + 16 \times 25 + 8 \times 15 + 10 \times 5 = 680$ mm
$A_o = \pi R^2 = 1675$, $R^2 = 1675/\pi = 533.17$, $R = 23.09$ mm
$C_c = 2\pi R = 2\pi(23.09) = 145.08$ mm
$K_x = 0.98 + 0.02(680/145.08)^{2.25} =$ **1.626**

(c) $\varepsilon = \ln 10.55 = 2.36$
$\varepsilon_x = 0.8 + 1.5(2.36) = 4.33$
$\overline{Y}_f = 240(2.36)^{0.16}/1.16 = 237.4$ MPa

$p = K_x \overline{Y}_f \varepsilon_x = 1.626(237.4)(4.33 + 2(850)/150) = 6046.2$ MPa

$F = pA_o = 6046.2(17{,}671.5) =$ **106,846,146 N**

(d) Total original volume $V = \pi(150)^2(900)/4 = 15{,}904{,}313$ mm^3
The final volume consists of two sections: (1) butt, and (2) extrudate.
The butt volume as given $V_1 = 600{,}000$ mm^3.
The extrudate has a cross-sectional area $A_f = 1675$ mm^2.
Its volume $V_2 = LA_f = 15{,}904{,}313 - 600{,}000 = 15{,}304{,}313$ mm^3.
Thus, length $L = 15{,}304{,}313/1675 =$ **9,137 mm = 9.137 m**

Drawing

19.37 A spool of wire has a starting diameter of 2.5 mm. It is drawn through a die with an opening that is to 2.1 mm. The entrance angel of the die is 18° degrees. Coefficient of friction at the work-die interface is 0.08. The work metal has a strength coefficient of 450 MPa and a strain hardening coefficient of 0.26. The drawing is performed at room temperature. Determine (a) area reduction, (b) draw stress, and (c) draw force required for the operation.

Solution: (a) $r = (A_o - A_f)/A_o$
$A_o = 0.25\pi(2.50)^2 = 4.91$ mm^2

$A_f = 0.25\pi(2.1)^2 = 3.46$ mm^2
$r = (4.91 - 3.46)/4.91 = \mathbf{0.294}$

(b) Draw stress σ_d:
$\varepsilon = \ln(4.91/3.46) = \ln 1.417 = 0.349$
$\overline{Y}_f = 450(0.349)^{0.26}/1.26 = 271.6$ MPa
$\phi = 0.88 + 0.12(D/L_c)$
$D = 0.5(2.5 + 2.1) = 2.30$
$L_c = 0.5(2.5 - 2.1)/\sin 18 = 0.647$
$\phi = 0.88 + 0.12(2.30/0.647) = 1.31$
$\sigma_d = \overline{Y}_f (1 + \mu/\tan \alpha)\phi(\ln A_o/A_f) = 271.6(1 + 0.08/\tan 18)(1.31)(0.349) =$ **154.2 MPa**

(c) Draw force F:
$F = A_f\, \sigma_d = 3.46(154.2) =$ **534.0 N**

19.38 Rod stock that has an initial diameter of 0.50 in is drawn through a draw die with an entrance angle of 13°. The final diameter of the rod is = 0.375 in. The metal has a strength coefficient of 40,000 lb/in^2 and a strain hardening exponent of 0.20. Coefficient of friction at the work-die interface = 0.1. Determine (a) area reduction, (b) draw force for the operation, and (c) horsepower to perform the operation if the exit velocity of the stock = 2 ft/sec.

Solution: (a) $r = (A_o - A_f)/A_o$
$A_o = 0.25\pi(0.50)^2 = 0.1964$ in^2
$A_f = 0.25\pi(0.35)^2 = 0.1104$ in^2
$r = (0.1964 - 0.1104)/0.1964 = \mathbf{0.4375}$

(b) Draw force F:
$\varepsilon = \ln(0.1964/0.1104) = \ln 1.778 = 0.5754$
$\overline{Y}_f = 40{,}000(0.5754)^{0.20}/1.20 = 29{,}845$ lb/in^2
$\phi = 0.88 + 0.12(D/L_c)$
$D = 0.5(.50 + 0.375) = 0.438$
$L_c = 0.5(0.50 - 0.375)/\sin 13 = 0.2778$
$\phi = 0.88 + 0.12(0.438/0.2778) = 1.069$
$F = A_f\, \overline{Y}_f (1 + \mu/\tan \alpha)\phi(\ln A_o/A_f)$
$F = 0.1104(29{,}845)(1 + 0.1/\tan 13)(1.069)(0.5754) =$ **2907 lb**

(c) $P = 2907(2 \text{ ft/sec} \times 60) = 348{,}800$ ft/lb/min
$HP = 348800/33{,}000 =$ **10.57 hp**

19.39 Bar stock of initial diameter = 90 mm is drawn with a draft = 15 mm. The draw die has an entrance angle = 18°, and the coefficient of friction at the work-die interface = 0.08. The metal behaves as a perfectly plastic material with yield stress = 105 MPa. Determine (a) area reduction, (b) draw stress, (c) draw force required for the operation, and (d) power to perform the operation if exit velocity = 1.0 m/min.

Solution: (a) $r = (A_o - A_f)/A_o$
$A_o = 0.25\pi(90)^2 = 6361.7$ mm^2
$D_f = D_o - d = 90 - 15 = 75$ mm,
$A_f = 0.25\pi(75)^2 = 4417.9$ mm^2
$r = (6361.7 - 4417.9)/6361.7 = \mathbf{0.3056}$

(b) Draw stress σ_d:
$\varepsilon = \ln(6361.7 /4417.9) = \ln 1.440 = 0.3646$

$\overline{Y}_f = k = 105$ MPa
$\phi = 0.88 + 0.12(D/L_c)$
$D = 0.5(90 + 75) = 82.5$ mm
$L_c = 0.5(90 - 75)/\sin 18 = 24.3$ mm
$\phi = 0.88 + 0.12(82.5/24.3) = 1.288$
$\sigma_d = \overline{Y}_f (1 + \mu/\tan \alpha)\phi(\ln A_o/A_f) = 105(1 + 0.08/\tan 18)(1.288)(0.3646) =$ **61.45 MPa**

(c) $F = A_f \sigma_d = 4417.9\ (61.45) =$ **271,475 N**

(d) $P = 271{,}475(1$ m/min$) = 271{,}475$ N-m/min $= 4524.6$ N-m/s $=$ **4524.6 W**

19.40 Wire stock of initial diameter = 0.125 in is drawn through two dies each providing a 0.20 area reduction. The starting metal has a strength coefficient = 40,000 lb/in^2 and a strain hardening exponent = 0.15. Each die has an entrance angle of 12°, and the coefficient of friction at the work-die interface is estimated to be 0.10. The motors driving the capstans at the die exits can each deliver 1.50 hp at 90% efficiency. Determine the maximum possible speed of the wire as it exits the second die.

Solution: First draw: $D_o = 0.125$ in, $A_o = 0.25\pi(0.125)^2 = 0.012273$ in^2 009819 in^2
$\varepsilon = \ln(0.012273/0.009819) = \ln 1.250 = 0.2231$
$r = (A_o - A_f)/A_o$, $A_f = A_o(1 - r) = 0.012773(1 - 0.2) = 0.$
$\overline{Y}_f = 40{,}000(0.2231)^{0.15}/1.15 = 27{,}775$ lb/in^2
$\phi = 0.88 + 0.12(D/L_c)$
$D_f = 0.125(1 - r)^{0.5} = 0.125(0.8)^{.5} = 0.1118$ in
$D = 0.5(.125 + 0.1118) = 0.1184$
$L_c = 0.5(0.125 - 0.1118)/\sin 12 = 0.03173$
$\phi = 0.88 + 0.12(0.1184/0.03173) = 1.33$
$F = A_f \overline{Y}_f (1 + \mu/\tan \alpha)\phi(\ln A_o/A_f)$
$F = 0.09819(27{,}775)(1 + 0.1/\tan 12)(1.33)(0.2231) = 119$ lb
1.5 hp at 90% efficiency = 1.5 x 0.90(33,000 ft-lb/min)/60 = 742.5 ft-lb/sec
$P = Fv = 119v = 742.5$
$v = 742.5/119 =$ **6.24 ft/sec**

Second draw: $D_o = 0.1118$ in, $A_o = 0.25\pi(0.1118)^2 = 0.009819$ in^2
$r = (A_o - A_f)/A_o$, $A_f = A_o(1 - r) = 0.009819(1 - 0.2) = 0.007855$ in^2
$\varepsilon = \ln(0.009819/0.007855) = \ln 1.250 = 0.2231$
Total strain experienced by the work metal is the sum of the strains from the first and second draws:
$\varepsilon = \varepsilon_1 + \varepsilon_2 = 0.2231 + 0.2231 = 0.4462$
$\overline{Y}_f = 40{,}000(0.4462)^{0.15}/1.15 = 30{,}818$ lb/in^2
$\phi = 0.88 + 0.12(D/L_c)$
$D_f = 0.1118(1 - r)^{0.5} = 0.1118(.8)^{.5} = 0.100$ in
$D = 0.5(0.1118 + 0.100) = 0.1059$
$L_c = 0.5(0.1118 - 0.100)/\sin 12 = 0.0269$
$\phi = 0.88 + 0.12(0.1059/0.0269) = 1.35$
$F = A_f \overline{Y}_f (1 + \mu/\tan \alpha)\phi(\ln A_o/A_f)$
$F = 0.007855(30{,}818)(1 + 0.1/\tan 12)(1.35)(0.4462) = 214$ lb.
1.5 hp at 90% efficiency = 742.5 ft-lb/sec as before in the first draw.
$P = Fv = 214v = 742.5$
$v = 742.5/214 =$ **3.47 ft/sec**

Note: The calculations indicate that the second draw die is the limiting step in the drawing sequence. The first operation would have to be operated at well below its maximum possible speed; or the second draw die could be powered by a higher horsepower motor; or the reductions to achieve the two stages could be reallocated to achieve a higher reduction in the first drawing operation.