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T2 ________		F2 ________
T3 ________	Problem Chosen	F3 ________
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2017
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Summary Sheet

Summary

This paper tries to optimize the passenger throughput at an Airport Security Checkpoint. First, we estimate the parameters and have the hypothesis testing by chi-square test. Then we make assumptions based on queuing theory.

We build a Multiplex Queueing System Model to identify bottlenecks for task A. Having considered about the complexity of security process, three-stage tandem system is developed to describe the efficiency at each check point. Passengers is divided into two types, Regular and Precheck. Moreover, two cases are discussed and the index of every check point in each stage has been studied. After a comprehensive analysis, the bottlenecks are clearly identified.

We have discussed the definition of throughput and time variance of the system. Then an evaluation function Q is developed to model the improvement of throughput and reduce of time variance. According to the bottlenecks identified in task A, we have developed two modifications and demonstrate how these modifications impact the process based on the objective function, as task B required.

Based on the Multiplex Queueing System Model and evaluation function developed in the previous two tasks, two types of cultural differences are discussed by sensitivity analysis for task C. We discussed what specific change these cultural norms are resulted. Then the sensitivity analysis is been use to demonstrate how can the security system accommodate these differences.

Finally, our recommendations are not only based the bottlenecks but also considering about culture norms. Moreover, we validate our model by Monte Carlo simulation, which shows our model is reliable. We have considered each check point and their mutual effects in the whole system. Conclusions are made based on the comprehensive analysis.

Keywords: Aviation Security Check; Tandem Queuing System; Evaluation Function

Contents

1 Introduction

1.1 Background

Aviation security checkpoints have been playing an important role in avoiding the terrorist attacks and protecting passengers. However, these checkpoints always lead to inconvenience since passengers have to spend more unpredicted time on queuing. Therefore, how to improve checkpoint efficiency and decrease variance of wait time while maximizing security is an urgent problem.

1.2 Problem Restatement and Analysis

Firstly, the TSA requires writers to set up a model to explore the passengers flow and identify the bottlenecks in the process of security check. After referring to literatures on queuing theory and some discussion, we decide to develop the multiplex queuing system to simulate the process of security check. The process is divided into three stages: the first stage is used to check passengers' documents; the second stage is designed to inspect passengers' body and baggage; and the third stage is where passengers collect their belongings and then exit.

Secondly, some modifications in the process should be given to optimize these bottlenecks and we are obliged to elaborate how these changes influence the model. Based on tandem queuing system, we design evaluation function to depict the passenger throughput and the wait time variance. Then we come up with some modification suggestions and explore how these modifications impact our queuing model.

Thirdly, the TSA asks participants to analyse sensitivity of security system under the influence of different cultures. We decide to discuss two particular culture norms in detail.

Finally, we are asked to propose some policy and recommendations based on our model.

2 Model Preparation

First, we analyse the existing data preliminary and make hypothesis testing. Based on these analysis, we can assume that arrival time follows Poisson distribution. Then we make some other rational and important assumptions.

2.1 Hypothesis Testing and Data Analysis

Data has been collected about how passengers proceed through each step of the security system, including arrival time, ID check time, body and baggage screening time. The approximate distribution of these data is obtained and the distribution pattern is verified by hypothesis testing. We take 30 seconds as the unit of observation time. Figure 1 is the frequency histograms of Arrival Times of TSA-precheck entrance.

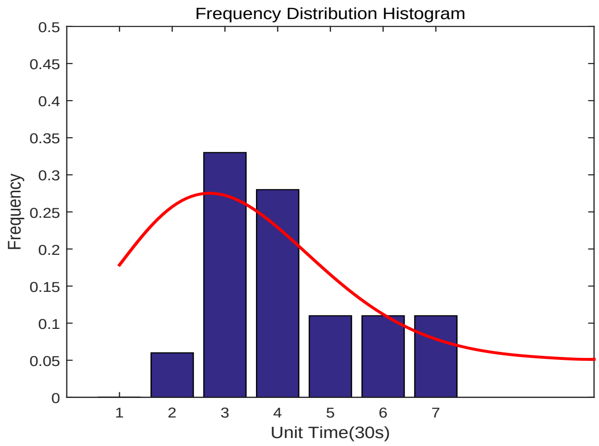


Figure 1: Frequency Distribution Histogram

We assume that the arrival time follows the Poisson distribution. Then, based on the exiting data, the hypothesis testing is performed by χ^2-square test. Based on basic queuing theory[3], the throughput of passenger is assumed to be stationary, non-after effected and normal in the process of input. Therefore, passengers' arrival can be assumed as a set of Poisson flow whose particle appears alone and source is infinite. Note $N(t)$ as the number of passengers whose arrival time is in the interval$(0, t]$. According to collected data from the airport, we tested whether $N(t)$ subjects to Poisson distribution with χ^2-square test.

The distribution law of $H_0 : N(t)$ is a Poisson distribution

$$P_k(t) = P\{N(t) = k\} = \frac{(\lambda t)^k e^{-\lambda t}}{k!}, k = 0, 1, 2 \cdots$$

Suppose that the distribution of passengers' arrival time follows the Poisson distribution:

$$P(x = n) = \frac{\lambda^n}{n!} e^{-\lambda}$$

The maximum likelihood estimate is used to find the estimated value:

$$\lambda \approx \widetilde{\lambda} = \frac{\sum_{k=0}^{n} k f_k}{n} = 3.222$$

which k is frequence, f_k is frequency.

Using χ^2 goodness-of-fit test:

Table 1: **Arrival time intensity(per 30s)**

	x=0	x=1	x=2	x=3	x=4	x=5	x=6
f_i	0	1	6	5	2	2	2

$$\chi^2 = \sum_{k=0}^{6} \frac{\left(f_k - \widetilde{f}_k\right)^2}{\widetilde{f}_k} \, (k = 0, \ldots, 6)$$

where $\widetilde{f}_k = nP_k$, The value of n is related to the amount of data samples, which $n = 18$ in our testing.

When $\chi^2 > \chi^2_{0.05}$ does not strictly follow the poisson distribution so we reject the null hypothesis. The condition that $\chi^2 < \chi^2_{0.05}$ follow the negative exponential distribution so we accept the null hypothesis.

The χ^2 goodness-of-fit process is shown in the Table 2.

Table 2: χ^2**-Square Test**

N	f_i	$\widehat{p_i}$	$n\widehat{p_i}$
0	0	0.03986	0.717595
1	1	0.128458	2.312249
2	6	0.206961	3.725291
3	5	0.222291	4.001238
4	2	0.179068	3.22322
5	2	0.115399	2.077186
6	2	0.061974	1.115526

According to the results in Table 2. We can get that,

$$\chi^2_{1-\alpha}(k - r - 1) = \chi^2_{1-0.05}(7 - 1 - 1) = \chi^2_{0.95}(5) = 11.07$$

$$\chi^2 = \sum_{k=1}^{6} \frac{(f_k - n\widehat{p_k})^2}{n\widehat{p_k}} = \sum_{k=1}^{6} \frac{{f_k}^2}{n\widehat{p_k}} - n = 4.268$$

As $4.268 < 11.07$,we accept the H_0 at the level of 0.95 and reckon that the passengers' arrival follows the Poisson distribution, that is $\lambda = 3.222$.

Similarly, the service time is presumed to have no memory, which means the service time for each passenger is independent and follows a negative exponential distribution. The service time of passengers can be tested through negative exponential distribution of goodness of fit χ^2test.

2.2 Assumptions

- passengers' arrival follows the Poisson distribution of the parameter λ.
- the service time of each passenger is independent and follows the negative exponential distribution of the same parameter.
- passengers are sufficient.
- the arrival and service are independent processes.

- it takes same time either passenger choose milimeter wave scanner or X-ray detector under the process of body scan.
- in zone C, people don't need to wait in queue to collect their luggage.
- apart from the main process of the security check, the rest of the process have no impact on our model.

2.3 Symbol Description

Table 3: **Symbol Description**

Symbol	Description
λ_1	intensity of Poisson distribution arrival time for regular passengers
λ_2	intensity of Poisson distribution arrival time for Pre-Check passengers
μ_{11}	service rate of ID Check Process(officer 1)
μ_{12}	service rate of ID Check Process(officer 2)
μ_{21}	service rate of X-Ray Scan Process(officer A)
μ_{22}	service rate of X-Ray Scan Process(officer B)
μ_2	service rate of Body Scan Process
ρ	utilization $\rho = \lambda/\mu$
W_a	mean waiting time in zone A
W_i	mean waiting time of luggage in second stage
W_p	mean waiting time of passengers in second stage
W_b	waiting time in zone B ($\mathrm{W}_b = \max\{\mathrm{W}_i, \mathrm{W}_p\}$)
W_c	available collecting time in zone C
L	average length of queue

Note that λ_2 is calculated in Hypothesis Testing and other values are estimated by mean level of existing data. For example, λ_1 is estimated by the mean intensity of arrival time for regular passengers. Similarly, $\lambda_1 = 4.63unit/min$, $\lambda_2 = 6.53unit/min$, $\mu_{11} = 4.79unit/min$, $\mu_{12} = 5.89unit/min$, $\mu_{21} = 7.96unit/min$, $\mu_{22} = 16.35unit/min$, $\mu_2 = 5.15unit/min$.

3 Multiplex Queueing System: Bottlenecks Identification

In this section, firstly we summarize and analyse the current process of security system. According to the hypothesis testing of the data and assumptions mentioned above, the current process is abstracted into a queuing network with a two-stage tandem structure. Then the average waiting time and utilization index of each stage are calculated. Finally we identify the bottlenecks under different cases.

3.1 Description of Current Process

Current process of security check covers three parts: document checking in zone A, baggage and body screening in zone B, collecting items and exiting in zone C. Figure 2 shows how it works. According to this process, we examine each part of this process and then make it as a whole queueing system.

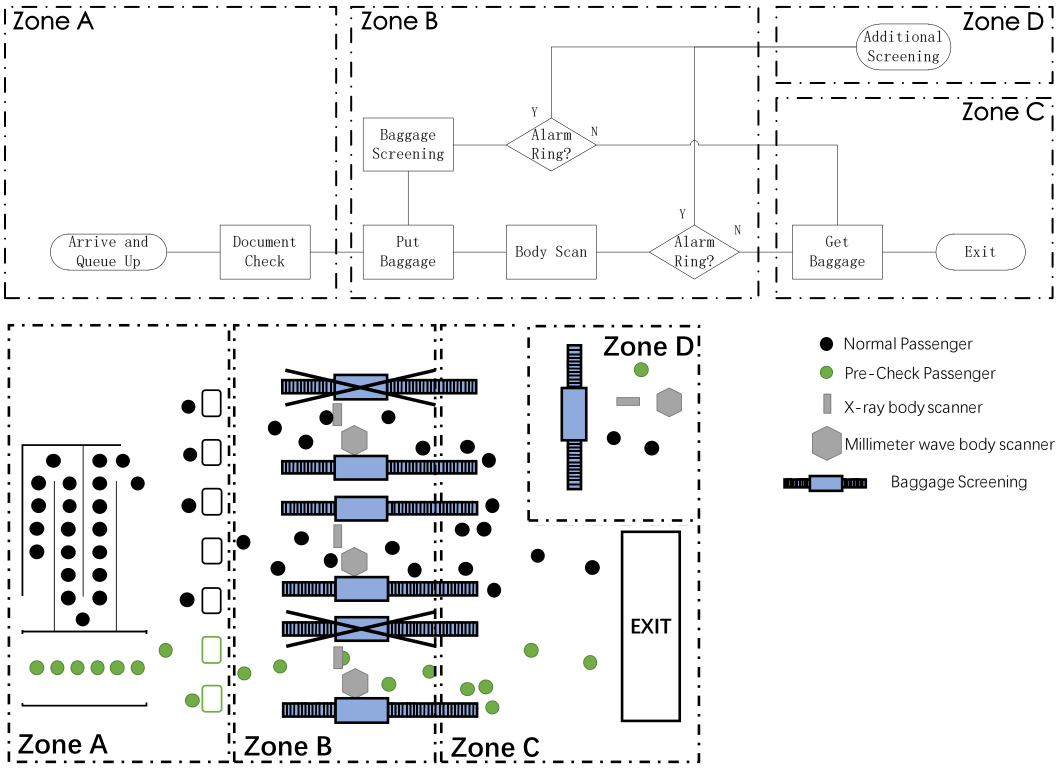


Figure 2: Workflow of The Current Process

3.2 First Stage: Queuing in Zone A

In Zone A, the passengers randomly arrive at the checkpoint and wait in a queue until a security officer can inspect their identifications and boarding documents. Specifically, waiting queues are divided into the TSA pre-check entrance and the regular entrance. The queue of TSA Pre-Check entrance in zone A can be seen as a single-channel to a single-service desk. The queue of the regular entrance in zone A can be assumed as a single-channel to multi-service desks. The schematic can be seen in Figure 3.

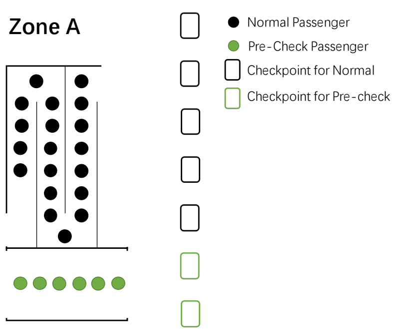


Figure 3: Schematic Diagram of Zone A

Only one passenger is allowed to be in front of a service desk at a time. When all service desks are busy, the remaining passengers have to wait until there is a service desk available. Service stations and a corresponding waiting queue constitute a queuing system. Based on the assumptions mentioned above, in this section, we consider the system with $c\,(c \geq 1, c \in \mathbb{N})$ independent service desks. When a passenger arrives, he can accept the service immediately as long as the service desk is available. Otherwise, if the desk is busy, the passenger has to queue up and wait until it is not busy.

In queuing theory[2], $M/M/1$ model and $M/M/c$ model are two common queuing models, which means arrival time follows the Poisson distribution of the parameter λ, the service time of each passenger is independent and follows the negative exponential distribution, the arrival and service are regarded as independent of each other. These two models both have only one queuing line in front of service desks. The $M/M/c$ model differs from $M/M/1$ in that, in $M/M/c$ model passengers are queued in front of C desks while passengers are queued in front of 1 desk in $M/M/1$ model.

According to the above analysis, in zone A, for the Pre-Check entrance line can be considered as $M/M/1$ queuing model, while the regular entrance can be considered as $M/M/5$ model.

Under the the statistical equilibrium, the passenger's waiting time distribution is determined by the following theorem[9],

Theorem 3.1. *In $M/M/1$ model, under the statistical equilibrium ($\rho < 1$), the distribution function of customer's waiting time is*

$$W_q(t) = P(W_q \leq t) = 1 - \rho e^{-\mu(1-\rho)t}, t \geq 0$$

The average waiting time is

$$W_q = E\left[W_q\right] = \frac{\rho}{\mu(1-\rho)}, \rho < 1$$

Lemma 3.2. *In $M/M/c$ model, let $\rho = \frac{\lambda}{\mu}, \rho_c = \frac{\lambda}{c\mu}, p_j = \lim\limits_{i\to\infty} P\left\{N(t) = j\right\}, j = 0, 1, 2, \cdots,$ When $\rho_c < 1$, there exists $\{p_j, j \geq 1\}$, and*

$$p_j = \begin{cases} \frac{1}{j!}\rho^j p_0, & 1 \leq j \leq c-1 \\ \\ \frac{1}{c^{j-c} \cdot c!}\rho^j p_0, & j \geq c \end{cases}$$

where $p_0 = \left[\sum\limits_{j=0}^{c-1} \frac{\rho^j}{j!} + \frac{c\rho^c}{c!(c-\rho)}\right]^{-1}$.

Theorem 3.3. *In $M/M/c$ model, under the statistical equilibrium $\left(\rho = \frac{\lambda}{c\mu} < 1\right)$, the distribution function of passenger's waiting time in $M/M/c$ is*

$$W_q(t) = 1 - \frac{p_c}{1-\rho_c} e^{-\mu(c-\rho)t}, t \geq 0$$

The average waiting time is

$$W_q = \frac{\rho_c}{\lambda(1-\rho_c)^2} \cdot p_c$$

3.3 The Other Two Stages: Security Check and Collect Items

In zone B and C. The procedure of both regular and pre-check passengers can be shown as Figure 4.

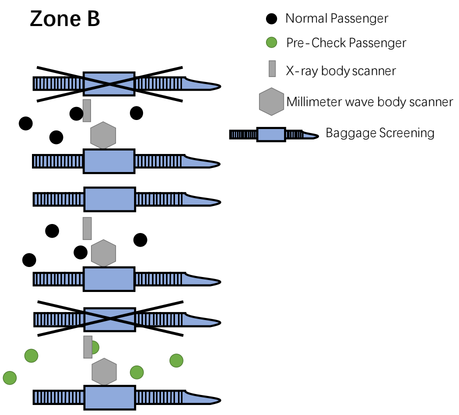


Figure 4: Schematic Diagram of Zone B

In this particular case, shown as Figure 4, the regular passengers can be considered as the $M/M/3$ queuing model. The Pre-check passengers can be considered as the $M/M/1$ queuing model.

Because baggage screening and passenger security are carried out in parallel, the process of baggage scanning and passenger security can be considered respectively. So the queuing process of item scanning and body scanning are been studied separately and we define the waiting time of scanning in zone B as

$$W_b = \max\{W_i, W_p\}$$

where W_i is the average waiting time of item scanning, and W_p is the average waiting time of people scanning, which can be both calculated according to Theorem 3.1 to 3.3.

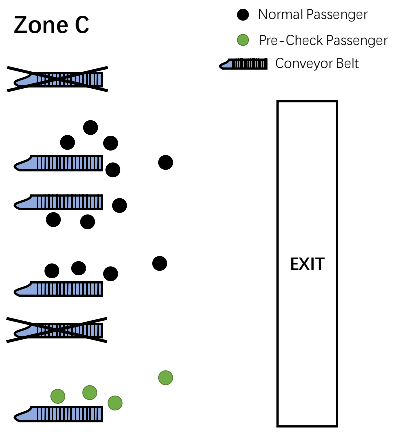


Figure 5: Schematic Diagram of Zone C

However, after passengers pass through the scanner or metal detector, they go straight to zone C, which is the terminal of the conveyor belt on the other side of the

X-ray scanner, to collect their belongings and depart the checkpoint area. Shown as Figure 5, it's not a complete queuing system since passengers may not keep in line in zone C when waiting for their baggage. So we consider the area between zone A and the body scanner is a queuing system, while the process after the scanner is not a queuing system. After they pass through the body scanner, they have to wait if their baggage has not been scanned yet, otherwise, they can just depart.

Let $W_c = A_H - W_b$ be the mean waiting time in zone C, where A_H is the average of column H.

Especially, we noticed that some passengers and belongings may be flagged for additional inspection in zone D after being scanned in zone B, which means regardless of whether they will be sent to zone D or not, they have to be checked at Zone B first. So we can assume that zone D has little influence on the system of zone A to C.

3.4 Identify Bottlenecks:Based on Tandem Queueing System

In this section we study the tandem queuing system with two service stages.

The service doesn't end until passengers complete all service at each stage. In the airport security system, for example, passengers are supposed to complete the following tasks at each service stage: passengers' documents must be checked through zone A; the passengers and their baggage must be checked through zone B or zone D and items are collected in zone C.

When the passenger at the first stage completes his service and at the same time he finds that the second stage is empty, he would immediately go to the second stage to accept the service. However, if the second stage is busy, there are two different conditions as follows.

- He may leave the first stage and enter the queue of the second stage.
- He may stop at the first stage and would not leave until the second stage is ready for him. This would result in the first stage's stopping service for subsequent passengers.

As there are many inconspicuous details in this problem, this article will consider bottlenecks in the following two cases.

Case 1: No Queues Are Allowed at Second Stage

No queues are allowed, with the exception that the first stage may have an infinite queue.

The utilization $\rho = \frac{\lambda}{\mu}$, where λ is Poisson intensity of arrival time, μ is service intensity of service stage, the larger the ρ is, the more busy the system is.

For the two-stage problem in this case, Hunt[1] studied this case and figured out the maximum possible utilization is

$$\rho_{\max} = \mu_2(\mu_1 + \mu_2)/({\mu_1}^2 + \mu_2\mu_2 + {\mu_2}^2)$$

.

This parameter is used to compare different cases in the following discussion.

Case 2: Queues Are Allowed

Queues are allowed at the second stage, while the first stage may have an infinite queue. According to existing research, in this case it is difficult to work out any formal or analytic solution of the various indicators of the queuing system.

However, in this case, Hunt[1] gives a formula of utilization ratio

$$\rho_{\max} = \mu_2({\mu_1}^{q+1} - {\mu_2}^{q+1})/({\mu_1}^{q+2} - {\mu_2}^{q+2})$$

Compare $\rho_{\max}$ with results of other service desks and cases.

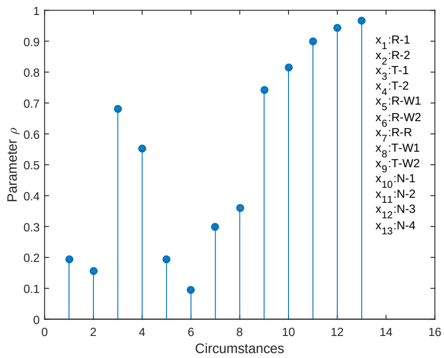


Figure 6: ρ in different check-points (labels are defined in Table 3 and 5

It is apparent the maximum possible utilization becomes larger when q gets bigger. So the capacity between zone A and zone B is important. It can be a bottleneck when capacity becomes too small.

The following analysis is based on the assumption that the capacity of the waiting area is sufficiently large, which means infinite queues are allowed in front of each service stage.

Based on the existing theory[7], when the first stage is under statistical equilibrium $(\rho < 1)$, the process of each stage in the tandem system is on the independent identical distribution. When each stage allows an infinite queue, the output rate of the first stage is the input of the second stage. So the number of incoming passengers in the latter service station is related to the parameter ρ of the previous service station in a tandem queuing system.

When $\rho_1 < 1$, it means that the first service stage is not fully loaded. The distribution of the first service stage's output is the same as its own input, that is, the distribution of the second service stage's input is the same as that of the first service stage. When $\rho_1 \geq 1$, the output is affected by the work efficiency of the first stage, because it is fully loaded.

According to Theorem 3.1, Theorem 3.3 and Definition 3.4, calculate parameters of efficiency at the first service stage as follows:

Table 4: **Symbol Description of Table 5**

Symbol	Description
$R-1$	ID Check Process of Regular Passengers(Officer 1)($M/M/5$)
$R-2$	ID Check Process of Regular Passengers(Officer 2)($M/M/5$)
$T-1$	ID Check Process of TSA Pre-Check Passengers(Officer 1)($M/M/2$)
$T-2$	ID Check Process of TSA Pre-Check Passengers(Officer 2)($M/M/2$)

Table 5: **Results of Zone A**

	R-1	R-2	T-1	T-2
parameter ρ	0.1934	0.1572	0.6812	0.5539
waiting time W_a	12.53	10.18	23.36	14.68
length of queue$L$	0.9676	0.7864	2.5417	1.5979

As the table above, the first service stage is under statistical equilibrium ($\rho < 1$), so the passengers' input of the the second stage is the same as that of the first stage, which follows the intensity λ_1, λ_2 Poisson distribution.

Based on the above conclusions, we can get the relevant parameters of the second stage as follows:

Table 6: **Symbol Description of Table 7**

Symbol	Description
$R-W1$	X-Ray Scan of Regular Passengers(Officer A)($M/M/3$)
$R-W2$	X-Ray Scan of Regular Passengers(Officer B)($M/M/3$)
$R-R$	Milimeter Wave Scan of Regular Passengers($M/M/2$)
$T-W1$	X-Ray Scan of TSA Pre-Check Passengers(Officer A)($M/M/1$)
$T-W2$	X-Ray Scan of TSA Pre-Check Passengers(Officer B)($M/M/1$)
$T-R$	Milimeter Wave Scan of TSA Pre-Check Passengers($M/M/1$)

Table 7: **Results of Zone B**

	R-W1	R-W2	R-R	T-W2	T-W1	T-R
parameter ρ	0.2007	0.0977	0.4649	0.3993	0.8205	≥ 1
waiting time(s) W_s	7.62	3.672	14.74	6.11	42.00	blockage
length of queue$L$	0.6085	0.2935	1.1860	0.6649	4.5697	blockage

Table 8: **Average Available collecting time in Zone C**

After the	R-W1	R-W2	R-R	T-W2	T-W1	T-R
available time(s) W_c	21	24.95	13.88	22.51	<0	-

Identify Bottlenecks

In the view of the utilization index, at T-R, $\rho \geq 1$, which means it cannot reach statistical equilibrium. Combing the queuing theory, as the time delays, the growth of the queue will be infinite, resulting in blockages.

From the perspective of the average queuing length, at T-1 and T-W1, the average queuing length is significantly higher than those of other queues.

According to the average waiting time, it is longer at T-1 and T-W1. Therefore, we propose to optimize these places to reduce the waiting time of passengers.

In zone C, the average available time in T-W1 is minus, which means passengers don't have enough time for collecting items. However, this is due to the previous process T-W1. Further more, it's because the μ_{21} is small.

To sum up,

- the check-point that Pre-Check passengers go through either a millimeter wave scanner or metal detector.
- the check-point where the belongings of Pre-Check passengers get X-ray screened.

4 Modifications Based on Evaluation Function

In this section, first, we give the mathematical definition of passenger throughput and variance in wait time. An objective function is given based on these definitions and constraint conditions are given combining the actual situation in the current process. Then, based on the initial solution to this objective programming problem, we develop three potential modifications to the current process. At last, we model these changes to demonstrate the impact on the process.

4.1 Definition of Throughput and Variance in Wait Time

We give the mathematical definition of variance in wait time of the whole tandem system. According to the definition of $D(X)$ and $E(X)$,

$$D(X) = \int_0^{+\infty} (x-u)^2 f(x)dx = E(X^2) - [E(X)]^2$$

where

$$u = E(X) = \int_0^{+\infty} xf(x)dx$$

So the variance of waiting time $W_q(t) = 1 - \frac{p_c}{1-\rho_c} e^{-\mu(c-\rho)t}\ t \geq 0$ can be calculated.

Let $k_1 = \frac{p_c}{1-\rho_c}, k_2 = \mu(c-\rho)$.

$$
\begin{aligned}
E\left(X^2\right) &= \int_0^{+\infty} t^2 k_1 k_2 e^{-k_2 t} dt \\
&= k_1 k_2 \frac{t^2 e^{-k_2 t}}{-k_2}\Big|_0^{+\infty} - \int_0^{+\infty} k_1 k_2 \frac{t^2 e^{-k_2 t}}{-k_2} dt \\
&= \int_0^{+\infty} 2k_1 t e^{-k_2 t} dt \\
&= 2k_1 \frac{t e^{-k_2 t}}{-k_2}\Big|_0^{+\infty} - \int_0^{+\infty} 2k_1 \frac{e^{-k_2 t}}{-k_2} dt \\
&= \int_0^{+\infty} 2k_1 \frac{e^{-k_2 t}}{k_2} dt \\
&= 2k_1 \frac{e^{-k_2 t}}{-k_2^2}\Big|_0^{+\infty} \\
&= \frac{2k_1}{k_2^2}
\end{aligned}
$$

Then we have

$$
D(X) = E(X^2) - [E(X)]^2 = \frac{2p_c}{(1-\rho_c)\mu^2(c-\rho)^2} - \frac{{\rho_c}^2 {p_c}^2}{\rho^2 c^2 \mu^2 (1-\rho_c)^4} \tag{1}
$$

Similarly, in $M/M/c$ model, we have

$$
D[W_q] = \frac{\lambda(2\mu - \lambda)}{\mu^2(\mu-\lambda)^2} \tag{2}
$$

Now, for the whole tandem system, let X and Y be random variables of waiting time at the first stage and second stage. As we discussed in the previous section, X and Y are mutually independent when the capacity between the first and second stage is sufficient. According to knowledge of statistics, we know that

$$
D(X+Y) = D(X) + D(Y) + 2Cov(X,Y)
$$

where $Cov(X,Y)$ is covariance of X and Y, and $Cov(X,Y) = 0$ while X and Y are mutually independent.

$$
D(X+Y) = D(X) + D(Y)
$$

Let X be random variable of waiting time at the first stage (zone A), Y is random variable of waiting time at the second stage(zone B). Z is random variable of waiting time in zone C, p is the possibility that passengers been taken to zone D, which $pD(Z) = \epsilon$ can be seen as known data. X,Y,Z are mutually independent. So variance of waiting time of two-stage tandem system is,

$$
D(X+Y+pZ) = D(X) + D(Y) + \epsilon \tag{3}
$$

where D(X) and D(Y) depending on they are $M/M/c$ or $M/M/1$.

The passenger throughput is a indicator which can reflect the transport capacity of security system. Specifically, it is the number of passengers passed per unit of time. In the two-stage tandem queuing system, for the sufficient volume between first stage and second stage, there are four states of the system classified by ρ_1 and ρ_1. So the mathematical definition of passenger throughput of the whole tandem system is given by,

$$G = \chi_{(\rho_1<1)}\chi_{(\rho_2<1)}\lambda_1 + (1-\chi_{\rho_1<1})\chi_{(\rho_2<1)}u_1 + (1-\chi_{\rho_2<1})u_2 \tag{4}$$

where $\chi_{\rho_i<1}$ is characteristic function of statistical equilibrium $\rho_i < 1, i = 1, 2$. ρ_i refers to the ρ of the $i - th$ stage.

$$\chi_{\rho_i<1} = \begin{cases} 1 & \rho_i < 1, i = 1, 2 \\ 0 & \rho_i \geq 1, i = 1, 2 \end{cases}$$

To improve passenger throughput and reduce variance in wait time. We define an index Q to measure the performance of security systems.

$$Q(\lambda, \mu_1, \mu_2, c_1, c_2) = \frac{G}{\sqrt{D(X + Y + pZ)}}$$

where G and $D(X + Y + pZ)$ can be determined by (1)(2)(3)(4). Apparently, the larger Q it is, the more passenger throughout and less variance of waiting time the system has.

4.2 Modifications

Based on the analysis above, we give the following optimization:

1. For the bottlenecks of the security process from chapter 3.4, we recommend to increase the number of physical security equipments (like millimeter wave body scanner and X-ray body scanner) , item security equipment and service desk of the first service stage for Pre-Check passengers. Due to space constraints, we will discuss in detail about the increment of physical security devices as follows.

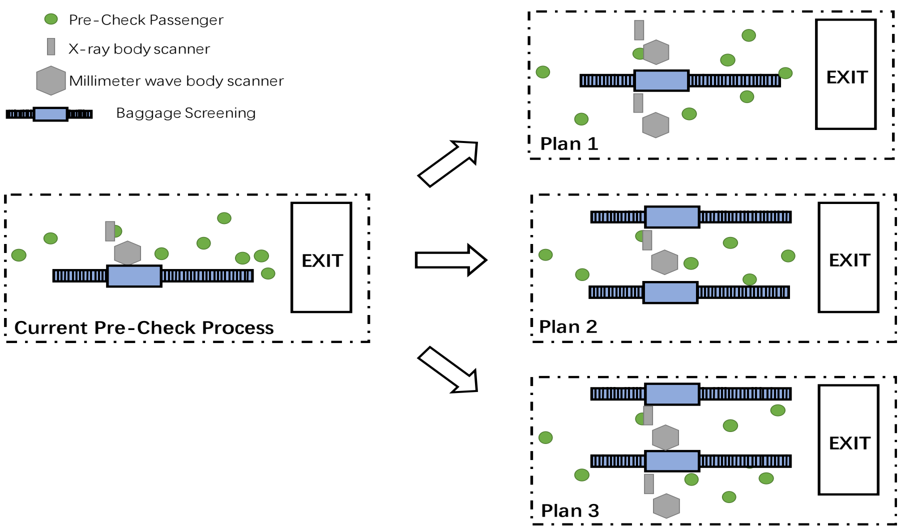


Figure 7: Modification 1

2. According to the bottleneck, we also develop a further classification for passengers. Based on the new types of the passengers, open corresponding channels. For example, people who are likely to encounter difficulties under the security process, such as the elderly and sick people and people who never take plane before, more staff should be arranged near the channel to assist and guide them; on the other hand, experienced people are familiar with the process. So they can complete the process with little help of the staff. Another example is the classification of passengers with or without luggage. There is no need for passengers without baggage to wait for items to be scanned, just taking the body scan process is enough. Correspondingly, with the help of additional staff's help, people with luggage move also faster than usual, leading to a better efficiency. Detailed discussion of the classification on baggage is given below.

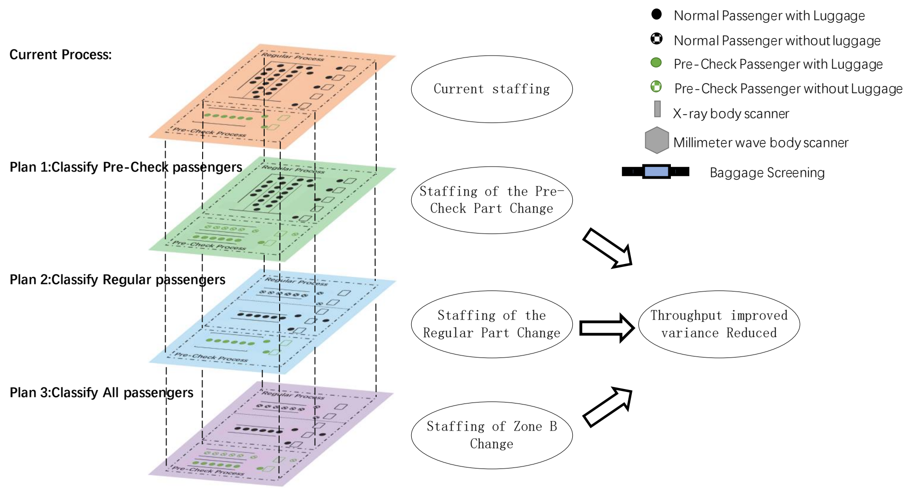


Figure 8: Modification 2

3. Increase queuing capacity of each stage. From chapter 3.4 we can see that because of the the lack of queuing space in second service stage, the service rate of the first phase of service is affected. So we can expand the queuing space of the second service stage. For example, the use of remote virtual queuing technology will help. Passenger only need to reach the security gate within the specified time period, during which the passenger has priority queuing. Rest area can be arranged between the first stage and second stage, where lies store and leisure facilities, reducing passenger waiting time while increasing the efficiency of security personnel.

4.3 Impact the Process of Modifications

Table 9: **The Title of Table**

the number of baggage scanner	1	2	3	4	5
waiting time W_i	41.99	9.07	7.73	7.57	7.54

we define the waiting time of scanning as $W_{bij} = \max\{W_i, W_j\}$ where W_i the average waiting time of item scanning, and W_p is the average waiting. For $i, j = 1, 2, 3, 4, 5$, the W_{bij} can be seen as Figure. Here we apply that $\lambda_1 = 6.5288\ unit/min$, $\mu_1 = 5.8939$

$unit/min$, $\mu_2 = \max\{\mu_{2i}, \mu_{2p}\}$ $unit/min$, $c_1 = 2$. So the first stage is $M/M/c$ model and the second is $M/M/c$ model.

To further study the impact of these modifications on the current process, let's see how they influence the performance index Q. Note that in our model, $W_{bij} = \max\{W_i, W_j\}$, which means if $W_i > W_j$, not only $W_{bij} = W_i$ but also $\mu_2 = \mu_{2i}$. Then the value of performance index Q under different cases is shown in Figure.

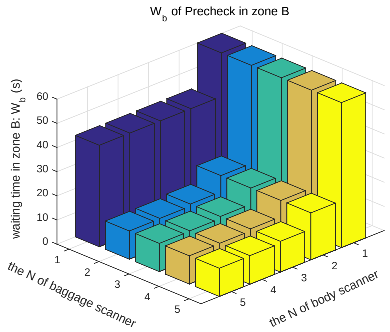


Figure 9: W_b of Precheck in zone B

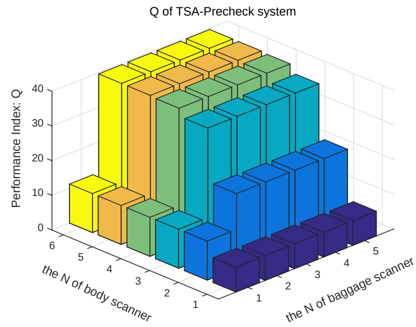


Figure 10: Q of TSA-Precheck system

We can see that, after the number of baggage scanner increased to two and the number of body scanner increased to three, the increase of the performance index Q is not so significant. Therefore, for the bottleneck found by Task a, we propose the following modifications, which is the most efficient choice. We suggest that in TSA-precheck system, body scanner should be increased to 3 and baggage scanner should be increased to two.

Classification for passengers results in the increase on both μ_1 and μ_2. So we increase the μ_1 and μ_2 for 5% to 50% level and see the impact on Q. We can see that when $\mu_1 + \mu_2$

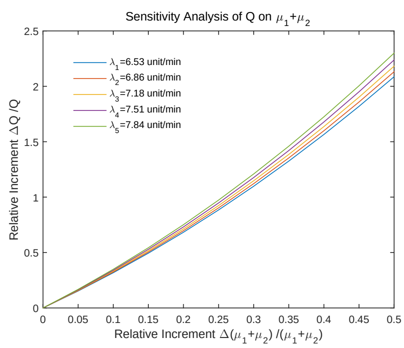


Figure 11: Sensitivity Analysis of Q on $\mu_1 + \mu_2$

has increased 50%, the $\Delta Q/Q$ has increased almost 220%, which means this modification is especially efficient to the improve throughput and reduce time variance.

Table 10: **The Title of Table**

the number of body scanner	1	2	3	4	5
waiting time W_j	Blockage	19.43	12.71	11.83	11.67

5 Sensitive Analysis: On Different Cultural Norms

Let us consider how the following behavior characteristics impact our model under different cultural norms.

Definition 5.1. *(The sensitivity of variable t to parameter r)[1] Let the relative change of r be $\frac{\Delta r}{r}$, if the change of r is Δr. Similarly, the relative change of t is $\frac{\Delta t}{t}$ if the change of r is Δr. Let $\Delta r \to 0$. If the limit exists, this limit is called sensitivity and we denote it by $S(t, r)$ i.e,*

$$\frac{\Delta t}{\Delta r}\frac{r}{t} \to \frac{dt}{dr}\frac{r}{t} = S(t, r)$$

5.1 Slow Traveler

We define slower travelers as people who are enjoying their holiday leisurely. In this harmonious and quiet preset, people put emphasis on enjoying their vacation, so it's obvious that these people have no sense of urgency at all, leading to passengers' slower behaviors in the process of a security check, then the service rates $\mu_i(i = 1, 2)$ have different degrees of reduction. What's more, people deeply respect and prioritize the personal space of others, which means passengers tend to keep a longer distance with each other in a queue. This would essentially affect the intensity of Poisson particles λ.

In order to exclude the impact of other variables on the study, we use the control variable method, passenger classifications are not discussed, all parameters are the same as the parameters of regular process, in which $\mu_1 = 5.89unit/min$, $\mu_2 = 7.96unit/min$,$\lambda = 4.6unit/min$. The following is our sensitivity analysis of the objective function with λ for different $\mu_i(i = 1, 2)$ cases:

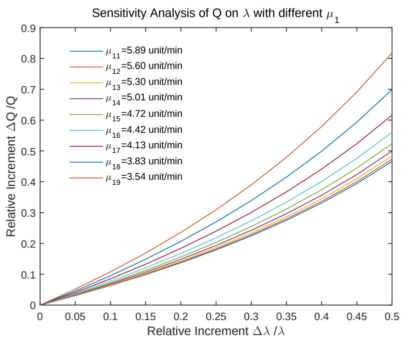


Figure 12: Sensitivity Analysis of Q on μ_1

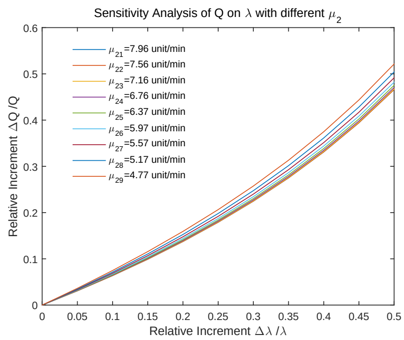


Figure 13: Sensitivity Analysis of Q on μ_2

Sensitivity of Q to λ denoted by Q_λ, $Q_\lambda = \frac{\Delta Q}{\Delta \lambda}\frac{\lambda}{Q}$. From the figures above we can draw the following conclusions:

1.When λ changes by 5%, the objective function Q changes by about 3.2%, its practical significance is that when the passengers' Poisson distribution intensity decreased by 5%, Q increased by 3.2%. Generally speaking, the security system's throughput and variance in wait time is improved.

2.The smaller the $\mu_i(i = 1, 2)$, the bigger Q_λ will be, that is, under the condition that λ changes on the same degree, the smaller the $\mu_i(i = 1, 2)$, the bigger the Q will change affected by λ. This means that when the service rate is lower, the condition of the airport security is more easily affected by the situation of incoming passengers.

3.Under the same change degree, the effect of μ_1's change on Q_λ is greater than that of μ_2's change.

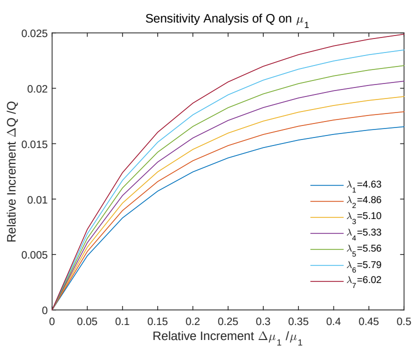
5.2 Collective Efficiency

Cultural norms of some countries, such as Switzerland, tend to focus on the collective interest, that is, emphasis on collective efficiency. We define people who emphasize collective efficiency are those who tend to choose to sacrifice their personal interests and comforts for the collective interest.

Specific to this article, passengers who have emphasis on collective efficiency will take measures, like preparing their luggage for screening in advance and collecting their luggage quickly, to improve queue efficiency, which mainly reflected in model is the increment of service rate $\mu_i(i = 1, 2)$.

We believe that collective priority and personal priority are opposite. People who prioritizing individual efficiency will take actions for their own convenience that will be obstacle to the progress of the whole system, such as cutting in the queue or failing to observe security rules. In the model, this is reflected in the reduction $\mu_i(i = 1, 2)$.

Same as chapter 5.1, we use the control variable method, passenger classifications are not discussed and $\mu_1 = 5.89unit/min$, $\mu_2 = 7.96unit/min$,$\lambda = 4.6unit/min$. The following is our sensitivity analysis of the performance index Q with $\mu_i(i = 1, 2)$ for different λ cases:



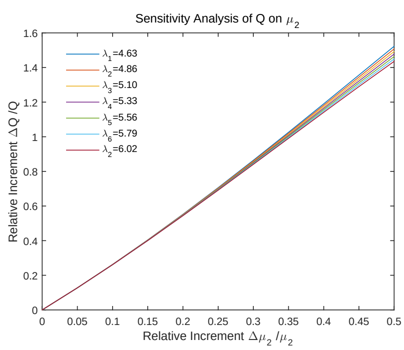


Figure 14: Sensitivity Analysis of Q on μ_1 Figure 15: Sensitivity Analysis of Q on μ_2

According to the definition 4.1, the sensitivity of Q on μ is the the slope of the line when it is close to zero. The sensitivity of Q on u_{11} is 0.09. The sensitivity of Q on u_{21} is 2.84. The sensitivity of Q on u_{21} is much bigger than u_{11}. From these results, we know that it is very important to be efficiently at the scanning check point. It tells us that it would be much better if passengers prepare their baggage checking in advance.

6 Model Validation

We validate our model by Monte Carlo simulation. A set of Poisson data streams are randomly generated. We set the $\lambda_1 = 6.53 unit/min$, $\mu_1 \in (4,6) unit/min$, $\mu_2 \in (4,6) unit/min$, and assume that there are 3 service desks at each stage. According to the current process, we can do the Monte Carlo simulation by increase the test unit number N to very big. Here we let $N = 10000$,

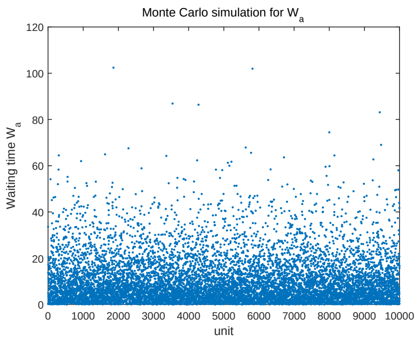


Figure 16: MC simulation in zone A

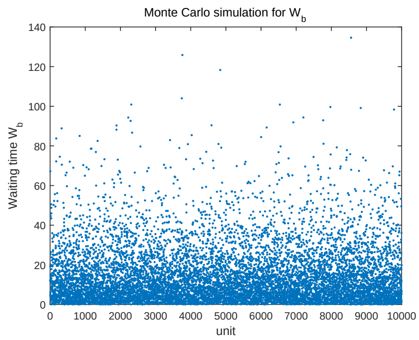


Figure 17: MC simulation in zone B

Then we can calculated these test units' mean waiting time $W = (W_{ai} + W_{bi})/N$. In our simulation we get $W = 23.25$, which is close to our model's result $W_{R-2} + W_{R-R} = 24.92$. It refers to that our model is reliable.

7 Strengths and Weaknesses

7.1 Strengths

- **Our model is reliable**
  Because we have made some assumptions and have the hypothesis testing. Besides, we validate our model by Monte Carlo method.

- **Our model is comprehensive**
  Based on queuing theory, we demonstrate each stage of the airport security system by considering it as a tandem system. The parameters in each check point are calculated. Besides, we give an evaluation function to reflect the throughput and variance of waiting time. Moreover, we give some effective suggestions, include culture norms, facilities, etc. The impact of different factors are fully discussed as sensitivity analysis in this paper.

- **Our model is a Multiplex Queueing System**
  Our model is not only a single model. We considered all mutually effects between each stage. Moreover, we have discussed many different cases and situations in our model, which make our results more reasonable.

7.2 Weaknesses

- **Lack of data in zone D**
  Because there is no relevant data of zone D, it is difficult to establish an effective model in zone D, resulting in a relatively simple dealing with zone D.

- **More flexible model is needed**
  Our assumptions may not be satisfied in the actual situation. For a more elaborate description of the security system, we are supposed to propose a more flexible model.

8 Future Work

1. With a more detailed and sufficient number of data on passenger security, we can consider the simulation based on cellular automata, analyze the security process from the individual level, and modify and improve the model's existing deficiencies based on the result of simulation, making the model more comprehensive.

2. Add the situation of zone D to the model and correct the model results to be realistic

3. Considering the cost of waiting time of passengers and the staff and equipment cost of the airport, the original objective function Q can be improved to make the model include more factors.

9 Recommendations

As we all know, the TSA is interested at how to improve checkpoint efficiency and reduce variance of waiting time while maximizing security. Here are some recommendations are given on a global scale and particular cultural norms.

Applicable on a global scale

1. Increase the number of body scanner to three and baggage scanner to two. Based on our multiplex queuing system, a bottleneck occurs when the performance index of service stage is too small. It means that the higher performance index, the shorter queue. Maintaining other factors unchanged, we simulate the queuing system and find the maximum performance index. The maximum can be reached when body scanner is increased to three and baggage scanner is increased to two.

2. Classify passengers into more types and offer more responding lanes and staffing. Based on the existing passenger classification (Pre-Check program), the passenger can be classified into more types according to their taking-flight records. Besides, more security staffing is needed to assist passengers in carrying their baggage. By simulating the queuing system, this modification is verified to be effective.

3. Increase the queuing capacity between the first and the second service stage, such as setting up a lounge area and using virtual queuing. Through our analysis and calculation, a bottleneck is most likely to occur between the first and the second service

stage. Therefore, to increase the queuing capacity between the first and the second service stage in a creative way is of great importance. Setting up a lounge area and using virtual queuing is a solution worth trying.

Based on particular cultural norms

1. While in the period when there are more passengers who emphasis on individual efficiency, for example, before and after the Chinese New Year holiday, based on our calculation, it is effective to increase personnel to maintain order and maximize the operation of equipment, in order to reduce the possibility of security congestion.

2. As for similar periods with many slower travelers, the airport can consider not taking measures of high cost to adjust the current condition. Because it shows that measures donâĂŹt make a difference in calculated results.

To summarize, we cling to the belief that our model is a powerful tool to help you optimize the security system of airport. And above suggestions based on our model and analysis are worth trying.

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