Homework 1

Convex Optimization

Due February 5

1. Consider convex functions $f_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, \cdots, k$. Prove that the set

$$\{x \mid f_i(x) \leq 0, i = 1, \cdots, k\}$$

is convex.

2. Consider a convex function $f : \mathbb{R}^n \to \mathbb{R}$. Prove that the set

$$\{(x, t) \mid f(x) \leq t\}$$

is convex.

3. (a) If $\boldsymbol{M}_1, \boldsymbol{M}_2 \in \boldsymbol{S}^2$ are positive definite, prove that $\boldsymbol{M}_1 + \boldsymbol{M}_2$ is positive definite.

(b) Prove that the set of all $n \times n$ positive definite symmetric matrices is convex.

4. Prove that the following set is convex:

$$\left\{ (x_1, x_2, x_3) \in \mathbb{R}^3 \mid \begin{bmatrix} x_1 + x_2 & x_1 - 2x_3 \\ x_1 - 2x_3 & x_2 + 3x_3 \end{bmatrix} \succeq 0 \right\}$$

5. Find a necessary and sufficient condition under which the following quadratic function is convex:

$$f(x) = \begin{bmatrix} \alpha_1 x_1 & \alpha_2 x_2 & \cdots & \alpha_n x_n \end{bmatrix} \boldsymbol{P} \begin{bmatrix} \alpha_1 x_1 \\ \alpha_2 x_2 \\ \vdots \\ \alpha_n x_n \end{bmatrix}$$

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