# Package 'timeFA'

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dynamic\_A

dynamic\_A

## **Description**

Get the adjacency matrix for plotting

#### Usage

```
dynamic_A(x, factor_count, simple.flag, threshold)
```

## Arguments

x input the original estimated loading matrix

simple.flag if True, only eliminate the entries below threshold and make all row sums to be

1; if False, the approach further eliminates the entries of the rows that are very

close to threshold value and only leaves the maximum entry of each row

threshold A parameter to eliminate very small entries of the loading matrix

# Value

The new loading matrix with all rows sum to be 1

em

Permutation matrix em

# Description

Permutation matrix em.

#### Usage

```
em(m, n, i, j)
```

generate 3

#### Value

Permutation matrix em #'@seealso trearrange

# **Examples**

```
em(m,n,i,j)
```

generate

Generate an AR(1) tensor time series

#### **Description**

For test only, randomly generate matrices A1,A2,..Ak by iid standard gaussians with given dimensions, and then generate an AR(1) tensor time series.

## Usage

```
generate(dim, t)
```

## Arguments

```
dim an array of dimentions of matrices A_1, A_2, \cdots, A_k
```

t length of time

#### Value

```
a list containing the following:
```

```
A matrices A_1,A_2,\cdots,A_k X AR(1) tensor time series #'@seealso run.test
```

```
dim <- c(1,2,3)
T <- 100
xx <- generate(c(m1,m2,m3),T)</pre>
```

4 MAR.SE

| grouping.loading | grouping.loading |
|------------------|------------------|
|------------------|------------------|

#### **Description**

Get the group of loadings

## Usage

```
grouping.loading(loading, ncluster, rowname, plot = T)
```

## Arguments

loading The estimated loading matrix

ncluster The number of clusters to use, usually the dimension of the factor matrix

rowname The name of the rows

plot plot the clustering graph, defacult True

#### Value

Loading matrix after grouping

| MAR.SE  | Asymptotic Covariano | ee Matrix of MAR1.otimes   |
|---------|----------------------|----------------------------|
| MAR. SE | Asymptotic Covarianc | e marrix of MAKT. Ottilles |

## Description

Asymptotic covariance Matrix of MAR1.otimes for given A, B and matrix-valued time series xx, see Theory 3 in paper.

# Usage

```
MAR.SE(xx, B, A, Sigma)
```

# Arguments

| XX  | T * p * q matrix-valued time series             |
|-----|---|
| В   | q by q matrix in MAR(1) model                   |
| Α   | p by p matrix in MAR(1) model                   |
| Sig | covariance matrix cov(vec(E_t)) in MAR(1) model |

## Value

asmptotic covariance matrix

MAR1.LS 5

#### **Examples**

```
# given T * p * q time series xx
out2=MAR1.LS(xx)
FnormLL=sqrt(sum(out2$LL))
xdim=p;ydim=q
out2Xi=MAR.SE(xx.nm,out2$RR*FnormLL,out2$LL/FnormLL,out2$Sig)
out2SE=sqrt(diag(out2Xi))
SE.A=matrix(out2SE[1:xdim^2],nrow=xdim)
SE.B=t(matrix(out2SE[-(1:xdim^2)],nrow=ydim))
```

MAR1.LS

Least Squares Iterative Estimation

## **Description**

Iterated least squares estimation in the model  $X_t = LL * X_{t-1} * RR + E_t$ .

#### Usage

```
MAR1.LS(xx, niter = 50, tol = 1e-06, print.true = FALSE)
```

#### **Arguments**

T \* p \* q matrix-valued time series

niter maximum number of iterations if error stays above tol

tol relative Frobenius norm error tolerance

print.true printe LL and RR

## Value

```
a list containing the following:
```

LL estimator of LL, a p by p matrix

RR estimator of RR, a q by q matrix

res residual of the MAR(1)

Sig covariance matrix  $cov(vec(E_t))$ 

dis Frobenius norm difference of last update

niter number of iterations

6 MAR1.otimes

MAR1.otimes

MLE under a structured covariance tensor

#### **Description**

MAR(1) iterative estimation with Kronecker covariance structure:  $X_t = LL * X_{t-1} * RR + E_t$  such that  $\Sigma = cov(vec(E_t)) = \Sigma_r \otimes \Sigma_l$ .

## Usage

```
MAR1.otimes(
    xx,
    LL.init = NULL,
    Sigl.init = NULL,
    Sigr.init = NULL,
    niter = 50,
    tol = 1e-06,
    print.true = FALSE
)
```

#### **Arguments**

#### Value

MAR1.projection 7

MAR1.projection

Projection Method

## **Description**

MAR(1) one step projection estimation in the model  $X_t = LL * X_{t-1} * RR + E_t$ .

## Usage

```
MAR1.projection(xx)
```

#### **Arguments**

XX

T \* p \* q matrix-valued time series

#### Value

```
a list containing the following:
```

```
LL estimator of LL, a p by p matrix
```

RR estimator of RR, a q by q matrix

res residual of the MAR(1)

Sig covariance matrix cov(vec(E\_t))

matrix\_factor

matrix\_factor

# Description

The main estimation function

#### Usage

```
matrix_factor(Yt, inputk1, inputk2, iscentering = 1, hzero = 1)
```

# Arguments

Yt Time Series data for a matrix

inputk1 The pre-determined row dimension of the factor matrix inputk2 The pre-determined column dimension of the factor matrix

iscentering The data is subtracted by its mean value

hzero Pre-determined parameter

8 mfmda

#### Value

```
a list containing the following:
eigval1 estimated row dimension of the factor matrix
eigval2 estimated column dimension of the factor matrix
loading1 estimated left loading matrix
loading2 estimated right loading matrix
Ft Estimated factor matrix with pre-determined number of dimensions
Ft.all Sum of Ft
Et The estimated residual, by subtracting estimated signal term from the data
```

## **Examples**

```
A <- 1:180
dim(A) <- c(3,3,20)
out = matrix_factor(A,3,3)
eig1 = out$eigval1
loading1 = out$loading1
Ft = out$Ft.all</pre>
```

mfmda

mfmda

## Description

This is a wrapper for all approaches

#### Usage

```
mfmda(Yt, approach = "3", hzero = 1, iscentering = 1)
```

## **Arguments**

approach Select estimation approaches, 1 for noniterative approach with no NaNs, 2 for

iterative approach with NaNs, 3 for iterative approach allowing NaNs.

hzero Pre-determined parameter

iscentering The data is subtracted by its mean value

Yc Time Series data for a matrix

# Value

The sample version of M matrix

```
A <- 1:180
dim(A) <- c(3,3,20)
M <- mfmda(A,"3",1,0)
```

mfmda.estqk 9

mfmda.estqk mfmda.estqk

## Description

Compute the estimated number of factors and the corresponding eigen-space

#### Usage

```
mfmda.estqk(Mhat, inputk = 1)
```

#### **Arguments**

Mhat The estimated value for matrix M

inputk The pre-determined number of dimension of factor matrix

#### Value

The estimated number of factors to use, the corresponding estimated Q matrix, the eigenvalue, the estimated Q matrix with requested number of factors

#### **Examples**

```
A <- 1:180

dim(A) <- c(3,3,20)

M <- mfmda(A,"3",1,0)

inputk <- 3

eig.ans <- mfmda.estqk(M,inputk)

khat <- eig.ans$estk

Qhat <- eig.ans$Qhatestk

eigval <- eig.ans$eigval

Qhatinputk <- eig.ans$Qhatinputk
```

mfmda.na.iter

mfmda.na.iter

## Description

The input data could have NaNs. The estimation approach is iterative.

## Usage

```
mfmda.na.iter(Yc, hzero)
```

## Arguments

Yc Time Series data for a matrix allowing NaNs

hzero Pre-determined parameter

#### Value

The sample version of M matrix

mfmda.nona.iter

mfmda.na.vec

mfmda.na.vec

## Description

This approach is for the vector-valued estimation with NaNs.

#### Usage

```
mfmda.na.vec(Yc, hzero)
```

## **Arguments**

Yc Time Series data for a matrix(dimensions n\*p\*q), allowing NA input

hzero Pre-scribed parameter h

#### Value

The sample version of M matrix

mfmda.nona.iter

mfmda.nona.iter

## **Description**

The input data do not have zeros. The estimation approach is iterative.

## Usage

```
mfmda.nona.iter(Yc, hzero)
```

## Arguments

Yc Time Series data for a matrix(dimensions n\*p\*q), no NA input allowed

hzero Pre-scribed parameter

## Value

The sample version of M matrix

mfmda.nona.noniter 11

mfmda.nona.noniter

mfmda.nona.noiter

## Description

The input data do not have zeros. The estimation approach is noniterative.

#### Usage

```
mfmda.nona.noniter(Yc, hzero)
```

## Arguments

Yc Time Series data for a matrix(dimensions n\*p\*q), no NA input allowed

hzero Pre-scribed parameter

#### Value

The sample version of M matrix

mfmda.nona.vec

mfmda.nona.vec

# Description

This approach is for the vector-valued estimation WITHOUT NaNs.

## Usage

```
mfmda.nona.vec(Yc, hzero)
```

## **Arguments**

Yc Time Series data for a matrix(dimensions n\*p\*q), no NA input allowed

hzero Pre-scribed parameter

# Value

The sample version of M matrix

```
A <- 1:180
dim(A) <- c(3,3,20)
M <- mfmda.nona.vec(A,2)
```

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PlotNetwork\_AB PlotNetwork\_AB

## Description

Plot the network graph

## Usage

```
PlotNetwork_AB(Ft, iterated_A, iterated_B = iterated_A, labels = use2)
```

# **Arguments**

Ft The estimated factor matrix iterated\_A The left loading matrix iterated\_B The right loading matrix

labels The row labels

#### Value

Plot the network graph

PM Permutation matrix PM

# Description

Permutation matrix PM.

# Usage

```
PM(m, n)
```

## **Arguments**

m an array of dimentions of matrices  $A_1, A_2, \dots, A_k$ 

n length of time

## Value

Permutation matrix PM #'@seealso trearrange

```
PM(m,n)
```

projection 13

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Kronecker Product Approximation

# Description

Kronecker product approximation used in Projection Method of matrix-value time series.

TAR(1) one step projection estimation in the model  $X_t = X_{t-1} \times A_1 \times \cdots \times A_K + E_t$ .

#### Usage

```
projection(xx, m1, m2, m3, n1, n2, n3)
projection(xx, m1, m2, m3, n1, n2, n3)
```

#### **Arguments**

```
xx T*m_1*\cdots*m_K tensor-valued time series m1 dim(A1) m2 dim(A2) m3 dim(A3) n1 nrow of A n2 nrow of B A mby n matrix such that <math>m=m1*n1 and n=m2*n2
```

## Value

```
a list containing two estimator (matrix) a list containing the estimation of matrices A_1,A_2,\cdots,A_K
```

#### See Also

```
MAR1.projection
```

```
A <- matrix(runif(6),ncol=2),
projection(A,3,3,2,2)
```

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rearrange

Rearrangement Operator

#### **Description**

Rearrangement Operator used for projection method.

## Usage

```
rearrange(A, m1, m2, n1, n2)
```

## Arguments

```
A m by n matrix such that m=m1*n1 and n=m2*n2 m1 ncol of A ncol of B nrow of A nrow of B
```

#### Value

rearengement matrix

## See Also

```
MAR1.projection
```

# **Examples**

```
A <- matrix(runif(6),ncol=2),
B <- matrix(runif(6),ncol=2),
M <- kronecker(B,A)
rearrange(M,3,3,2,2) == t(as.vector(A)) %*% as.vector(B)
'TRUE'</pre>
```

run.test

Test Function

## Description

For test only

# Usage

```
run.test(m1, m2, m3, n = 100, T)
```

TAR.SE

TAR.SE

Asymptotic Covariance Matrix of TAR1.LS

## Description

Asymptotic covariance Matrix of TAR1.LS for given A tensor-valued time series xx, see Theory 2 in paper.

#### Usage

```
TAR.SE(xx, C, B, A, Sigma)
```

#### **Arguments**

| xx  | $T * m_1 * \cdots * m_K$ tensor-valued time series |
|-----|--|
| С   | m3 by m3 matrix in MAR(1) model                    |
| В   | m2 by m2 matrix in MAR(1) model                    |
| Α   | m1 by m1 matrix in MAR(1) model                    |
| Sig | covariance matrix cov(vec(E_t)) in TAR(1) model    |

#### Value

asmptotic covariance matrix

TAR1.LS

Least Squares Iterative Estimation for Tensor-Valued Time Series

## Description

Iterated least squares estimation in the model  $X_t = X_{t-1} \times A_1 \times \cdots \times A_K + E_t$ .

#### Usage

```
TAR1.LS(xx, niter = 1000, tol = 1e-06, print.true = FALSE)
```

# Arguments

```
\begin{array}{ll} \text{xx} & T*m_1*\cdots*m_K \text{ tensor-valued time series} \\ \text{niter} & \text{maximum number of iterations if error stays above tol} \\ \text{tol} & \text{relative Frobenius norm error tolerance} \\ \text{print.true} & \text{printe } A_i \end{array}
```

#### Value

```
a list containing the following: 
 A estimator of coeficient matrices A_1, A_2, \cdots, A_K res residual of the MAR(1) 
 Sig covariance matrix cov(vec(E_t)) 
 niter number of iterations
```

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TAR1.VAR

Stacked vector AR(1) Model for Tensor-Valued Time Series

## Description

```
vector AR(1) Model.
```

## Usage

```
TAR1.VAR(xx)
```

#### **Arguments**

XX

$$T * m_1 * \cdots * m_K$$
 tensor-valued time series

#### Value

```
a list containing the following: coef coeficient of the fitted VAR(1) model res residual of the VAR(1) model
```

trearrange

trearrange

# Description

(alpha version) rearrangement operator for tensor.

## Usage

```
trearrange(A, m1, m2, m3, n1, n2, n3)
```

var1

 $Stacked\ vector\ AR(1)\ Model$ 

# Description

```
vector AR(1) Model.
```

## Usage

```
var1(xx)
```

## **Arguments**

хх

T \* p \* q matrix-valued time series

vector\_factor 17

#### Value

```
a list containing the following:  \\  \mbox{coef coeficient of the fitted VAR}(1) \mbox{ model} \\ \mbox{res residual of the VAR}(1) \mbox{ model} \\ \mbox{}
```

#### **Examples**

```
out.var1=var1(xx)
sum(out.var1$res**2)
```

vector\_factor

vector\_factor

#### **Description**

The main estimation function, vector version

#### Usage

```
vector_factor(Yt, inputk.vec, iscentering = 1, hzero = 1)
```

#### **Arguments**

Yt Time Series data for a matrix

inputk.vec The pre-determined dimensions of the factor matrix in vector

iscentering The data is subtracted by its mean value

hzero Pre-determined parameter

#### Value

a list containing the following:

eigval1 estimated dimensions of the factor matrix

loading estimated loading matrices

Ft Estimated factor matrix with pre-determined number of dimensions

 ${\tt Ft.all} \ \ Sum \ of \ Ft$ 

Et The estimated random term, by subtracting estimated signal term from the data

```
A <- 1:180

dim(A) <- c(3,3,20)

M <- mfmda(A,"3",1,0)

eig.ans <- vector_factor(M,3,0,1)

khat <- eig.ans$estk

Qhat <- eig.ans$Qhatestk

eigval <- eig.ans$eigval

Q1hatinputk <- eig.ans$Qhatinputk
```

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