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Fabien Guilbaud & Huyền Pham

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Optimal high-frequency trading with limit and market orders

FABIEN GUILBAUD[†] and HUYÊN PHAM^{*‡}

[†]EXQIM and Laboratoire de Probabilités et Modèles Aléatoires,
Université Paris 7 Diderot, CNRS, UMR 7599, France

[‡]Laboratoire de Probabilités et Modèles Aléatoires, Université Paris 7 Diderot,
CREST-ENSAE, and Institut Universitaire de France, CNRS, UMR 7599, France

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We propose a framework for studying optimal market-making policies in a limit order book (LOB). The bid–ask spread of the LOB is modeled by a tick-valued continuous-time Markov chain. We consider a small agent who continuously submits limit buy/sell orders at best bid/ask quotes, and may also set limit orders at best bid (resp. ask) plus (resp. minus) a tick for obtaining execution order priority, which is a crucial issue in high-frequency trading. The agent faces an execution risk since her limit orders are executed only when they meet counterpart market orders. She is also subject to inventory risk due to price volatility when holding the risky asset. The agent can then also choose to trade with market orders, and therefore obtain immediate execution, but at a less favorable price. The objective of the market maker is to maximize her expected utility from revenue over a short-term horizon by a trade-off between limit and market orders, while controlling her inventory position. This is formulated as a mixed regime switching regular/impulse control problem that we characterize in terms of a quasi-variational system by dynamic programming methods. Calibration procedures are derived for estimating the transition matrix and intensity parameters for the spread and for Cox processes modelling the execution of limit orders. We provide an explicit backward splitting scheme for solving the problem and show how it can be reduced to a system of simple equations involving only the inventory and spread variables. Several computational tests are performed both on simulated and real data, and illustrate the impact and profit when considering execution priority in limit orders and market orders.

Keywords: Applied mathematical finance; Trading strategies; Stochastic control; Quantitative finance techniques; Portfolio optimization; Market microstructure

JEL Classification: G1, G10, G11

1. Introduction

Most modern equity exchanges are organized as *order-driven* markets. In such a market, the price formation exclusively results from operating a *limit order book* (LOB), an order-crossing mechanism where *limit orders* are accumulated while waiting to be matched with incoming *market orders*. Any market participant is able

to interact with the LOB by posting either market orders or limit orders.[§]

In this context, *market making* is a class of strategies that consists of simultaneously posting limit orders to buy and sell during a continuous trading session. By doing so, market makers provide a counterpart to any incoming market orders: suppose that investor *A* wants to sell one share of a given security at time *t* and that investor *B*

*Corresponding author. Email: pham@math.jussieu.fr

[§]A market order of size *m* is an order to buy (sell) *m* units of the asset being traded at the lowest (highest) available price in the market; its execution is immediate. A limit order of size *ℓ* at price *q* is an order to buy (sell) *ℓ* units of the asset being traded at the specified price *q*; its execution is uncertain and achieved only when it meets a counterpart market order. Given a security, the *best bid* (resp. *ask*) price is the highest (resp. lowest) price among limit orders to buy (resp. to sell) that are active in the LOB. The *spread* is the difference, expressed in numéraire per share, of the best ask price and the best bid price, positive during the continuous trading session (Gould *et al.* 2010).