PART I

1. Dataset and Preliminary Analysis

In this section we:

- · Read and concatenate the CSV files.
- Drop unnecessary columns.
- Compute basic statistics and perform sanity checks.
- Convert timestamps and check for missing values.

```
In [1]: import numpy as np
                  import pandas as pd
                   import matplotlib.pyplot as plt
                   from scipy import stats
                  from scipy.stats import expon
                   from statsmodels.tsa.stattools import acf
                   from statsmodels.graphics.tsaplots import plot_acf
In [2]: file_list = ['Data/SG/SG_20170117.csv.gz','Data/SG/SG_20170118.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170118.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG/SG_20170119.csv.gz','Data/SG/SG_20170119.csv.gz','Data/SG/SG/SG_20170119.csv.gz','Data/SG/SG/S
                                               'Data/SG/SG_20170125.csv.gz','Data/SG/SG_20170126.csv.gz','Data/SG/SG_20170127.csv.gz','Dat
                  Data_sg = {}
                   for i,file in enumerate(file_list):
                           Data_sg[file[8:-7]] = pd.read_csv(file, compression='gzip').drop(columns=['Unnamed: 0'])
                           Data_sg[file[8:-7]]['ets'] = pd.to_datetime(Data_sg[file[8:-7]]['ets'], format='%Y%m%d:%H:%M:%S.%f')
                   print(Data sg.keys())
                  Data_sg["SG_20170117"]
                dict_keys(['SG_20170117', 'SG_20170118', 'SG_20170119', 'SG_20170120', 'SG_20170123', 'SG_20170124', 'SG_
                20170125', 'SG_20170126', 'SG_20170127', 'SG_20170130', 'SG_20170131', 'SG_20170201'])
Out[2]:
                                                                                                                                                      bp0 bq0
                                                                                 ets etype eprice eqty eside
                                                                                                                                                                                  ap0
                              0 2017-01-17 09:01:00.270164
                                                                                                  A 45610
                                                                                                                        1400
                                                                                                                                            B 46010 1066 46085 1445
                              1 2017-01-17 09:01:00.312121
                                                                                                                           700
                                                                                                                                             S 46010 1066 46085 1445
                                                                                                         46485
                              2 2017-01-17 09:01:00.358162
                                                                                                  A 46000
                                                                                                                                             B 46010 1066 46085 1445
                                                                                                                           124
                              3 2017-01-17 09:01:00.359972
                                                                                                  A 45950
                                                                                                                           182
                                                                                                                                             B 46010 1066 46085 1445
                               4 2017-01-17 09:01:00.360001
                                                                                                  A 45970
                                                                                                                             22
                                                                                                                                             B 46010 1066 46085 1445
                   841144 2017-01-17 17:29:59.997678
                                                                                                                                             S 45820
                                                                                                                                                                   377 45845
                                                                                                  A 45845
                                                                                                                           187
                                                                                                                                                                                               187
                   841145 2017-01-17 17:29:59.997691
                                                                                                  C 45720
                                                                                                                        1250
                                                                                                                                            B 45820
                                                                                                                                                                    377 45845
                                                                                                                                                                                               187
                   841146 2017-01-17 17:29:59.997884
                                                                                                  C 45765
                                                                                                                                             B 45820
                                                                                                                                                                   377 45845
                                                                                                                                                                                               187
                                                                                                                           110
                   841147 2017-01-17 17:29:59.998327
                                                                                                  A 45740
                                                                                                                                             B 45820
                                                                                                                                                                    377 45845
                                                                                                                                                                                               187
                                                                                                                           110
                   841148 2017-01-17 17:29:59.998861
                                                                                                  C 45785
                                                                                                                                            B 45820
                                                                                                                                                                                               187
                                                                                                                          324
                                                                                                                                                                   377 45845
                 841149 rows × 9 columns
In [ ]:
```

```
In []:

In [3]: df_AllData = pd.concat(Data_sg.values(), ignore_index=True)
    print(f"Shape of concatenated DataFrame: {df_AllData.shape}")
    print("\nFirst 5 rows:")
    df_AllData.head()

Shape of concatenated DataFrame: (7827010, 9)
```

First 5 rows:

```
Out[3]:
                                                                           ap0
                               ets etype eprice eqty eside
                                                               bp0 bq0
                                                                                 aq0
        0 2017-01-17 09:01:00.270164
                                       A 45610
                                                 1400
                                                          B 46010 1066 46085
                                                                                1445
        1 2017-01-17 09:01:00.312121
                                       A 46485
                                                  700
                                                          S 46010 1066 46085 1445
        2 2017-01-17 09:01:00.358162
                                       A 46000
                                                  124
                                                          B 46010 1066 46085 1445
        3 2017-01-17 09:01:00.359972
                                       A 45950
                                                  182
                                                          B 46010 1066 46085 1445
        4 2017-01-17 09:01:00.360001
                                                          B 46010 1066 46085 1445
                                       A 45970
                                                   22
In [4]: Data_sg_0117 = Data_sg["SG_20170117"]
        #df = df_AllData
        df = Data_sg_0117
In [5]: # Verify basic content
        print('_
        print("Event type counts:")
        print(df['etype'].value_counts())
        print("Side counts:")
        print(df['eside'].value_counts())
       Event type counts:
       etype
       Α
           388981
       C
            377310
       Μ
            61081
            13777
       Name: count, dtype: int64
       Side counts:
       eside
           432686
           408463
       Name: count, dtype: int64
In [6]: # Basic statistics for numerical columns
        print("\nSummary Statistics:")
        print(df[['eprice', 'eqty', 'bp0', 'bq0', 'ap0', 'aq0']].describe())
        # Convert timestamp to datetime
        df['ets'] = pd.to_datetime(df['ets'], format='%Y%m%d:%H:%M:%S.%f')
        # Time range of the dataset
        print(f"\nTime Range: {df['ets'].min()} to {df['ets'].max()}")
        # Events per second (liquidity measure)
        events_per_sec = df.resample('1s', on='ets').size().mean()
        print(f"\nAverage Events per Second: {events_per_sec:.2f}")
        # Transaction per second (liquidity measure)
        transaction_per_sec = df[df['etype'] == 'T'].resample('1s', on='ets').size().mean()
        print(f"\nAverage Transactions per Second: {transaction_per_sec:.2f}")
        # Check for missing values
        print("\nMissing values:")
        print(df.isnull().sum())
```

```
Summary Statistics:
            eprice
                                           bp0
                                                         ba0
                            eatv
count 841149.000000 841149.000000 841149.000000 841149.000000
      46151.758868 236.921342 46143.727497
mean
                                                623.455495
       272.820275
                     283.275695
                                   233.518584
                                                579.452229
std
       40000.000000 -14809.000000 45500.000000
                                                   1.000000
25%
       45990.000000 162.000000 45985.000000
                                                   250.000000
50%
       46135.000000
                       200.000000
                                  46130.000000
                                                  497.000000
75%
       46330.000000
                       250.000000
                                  46320.000000
                                                   830.000000
       60760.000000 25380.000000
                                  46720.000000 10932.000000
max
                ap0
count 841149.000000 841149.000000
       46160.476562
                     747.229150
mean
std
        233.056085
                       948.258226
       45520.000000
                       1.000000
min
25%
       46000.000000
                      326.000000
                    580.000000
50%
       46145.000000
75%
       46340.000000
                       944.000000
       46740.000000 26270.000000
Time Range: 2017-01-17 09:01:00.270164 to 2017-01-17 17:29:59.998861
Average Events per Second: 27.54
Average Transactions per Second: 0.45
Missing values:
ets
         0
etvpe
eprice
         0
eqty
        0
eside
bp0
ba0
         0
aq0
         0
dtvpe: int64
```

Data Sanity Checks: Trade Validity and Spread

Here we compute the bid-ask spread and validate trade prices. We:

- Compute the spread (ask bid) and its basic statistics.
- · Check for negative spreads.
- Validate that trade prices fall within the bid-ask range.
- Identify and print invalid trades.

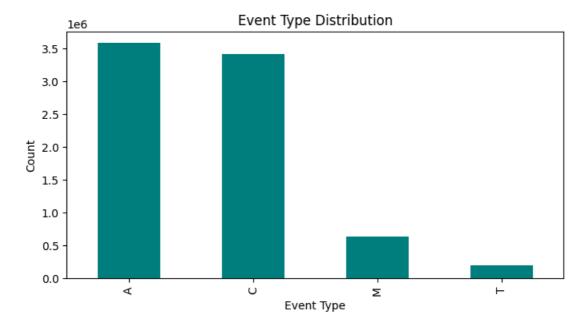
```
In [7]: df = df_AllData
        #df = Data_sg_0117
In [8]: # Compute spread and its statistics
        df['spread'] = df['ap0'] - df['bp0']
        print("\nSpread Statistics:")
        print(df['spread'].describe())
         # Check for negative spreads
        negative_spread = df[df['spread'] < 0]</pre>
        print(f"\nNegative spreads: {len(negative_spread)}")
        # Validate trade prices within bid/ask
        trades = df[df['etype'] == 'T']
        valid_trades = trades[(trades['eprice'] >= trades['bp0']) & (trades['eprice'] <= trades['ap0'])] ## Non,</pre>
        print(f"\nInvalid trades: {len(trades) - len(valid_trades)}")
         # Filter invalid trades and print details
        invalid_trades = trades[~trades.index.isin(valid_trades.index)]
         print("\nInvalid Trades:")
        print(invalid_trades[['ets', 'eprice', 'bp0', 'ap0']])
                                                                                                     ')
        print('
```

```
# For each invalid trade, find the closest preceding quote update
        for idx, row in invalid trades.iterrows():
            nearest_quote = df[df['ets'] <= row['ets']].iloc[-1]</pre>
            print(f"Trade time: {row['ets']}, Quote time: {nearest_quote['ets']}")
        print("\nData Sanity Check:")
        print("-3 invalid trades detected where eprice fell outside bp0/ap0.")
        print("-Inspection revealed these trades didnt have synchronization issue") #these 3 trades occur with b
                nor auction phase where trades could be non-continuous nor an unusual tick size")
        print("-These trades have been excluded from the continuous trading analysis to avoid bias.")
       Spread Statistics:
              7.827010e+06
       mean
               1.502254e+01
                7.337535e+00
       std
                5.000000e+00
       25%
                1.000000e+01
       50%
               1.500000e+01
                2.000000e+01
       75%
               1.500000e+02
       max
       Name: spread, dtype: float64
       Negative spreads: 0
       Invalid trades: 3
       Invalid Trades:
                                      ets eprice
                                                     bp0
       4655752 2017-01-26 12:52:32.761728 48400 48480 48505
       6465452 2017-01-31 10:24:05.806200 47150 47030 47070
       6672360 2017-01-31 12:48:30.842097 46250 46255
       Trade time: 2017-01-26 12:52:32.761728, Quote time: 2017-01-26 12:52:32.761728
       Trade time: 2017-01-31 10:24:05.806200, Quote time: 2017-01-31 10:24:05.806200
       Trade time: 2017-01-31 12:48:30.842097, Quote time: 2017-01-31 12:48:30.842097
       Data Sanity Check:
       -3 invalid trades detected where eprice fell outside bp0/ap0.
       -Inspection revealed these trades didnt have synchronization issue
       nor auction phase where trades could be non-continuous nor an unusual tick size
       -These trades have been excluded from the continuous trading analysis to avoid bias.
In [9]: df.iloc[6465452 -2: 6465452 +3, :]
Out[9]:
                                      ets etype eprice eqty eside
                                                                           bq0
                                                                                  ap0 aq0 spread
                                                                      bp0
        6465450 2017-01-31 10:24:05.806134
                                                 47030
                                                         250
                                                                 B 47030 1683 47180
                                                                                         50
                                                                                               150
        6465451 2017-01-31 10:24:05.806197
                                              A 47070
                                                         189
                                                                 S 47030
                                                                          1683 47070
                                                                                       189
                                                                                                40
        6465452 2017-01-31 10:24:05.806200
                                              T 47150
                                                                           1683
                                                                                47070
                                                                                                40
                                                                   47030
        6465453 2017-01-31 10:24:05 806204
                                                         124
                                                                          1683 47180
                                              T 47150
                                                                 B 47030
                                                                                         50
                                                                                               150
        6465454 2017-01-31 10:24:05.806251
                                              C 46985
                                                         100
                                                                 B 47030 1683 47180
                                                                                               150
```

Event Type Distribution

Plot the distribution of event types in the dataset.

```
In [10]: # Plot event type counts
    event_counts = df['etype'].value_counts()
    plt.figure(figsize=(8, 4))
    event_counts.plot(kind='bar', color='teal')
    plt.title('Event Type Distribution')
    plt.xlabel('Event Type')
    plt.ylabel('Count')
    plt.show()
```



The event type distribution shows significant class imbalance, with types A and C dominating. This is typical in limit order book data where order arrivals and cancellations are more frequent than trades or modifications. The dataset appears free of obvious issues, but I would verify:

- No missing values in critical columns
- Timestamps are chronological
- Price and quantity values are positive and reasonable

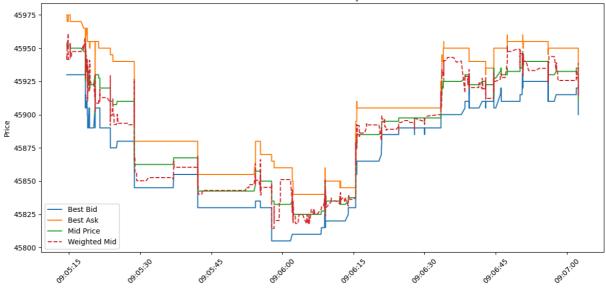
2. Prices and Tick Sizes

In this section we:

- Plot the best bid price, best ask price, mid price, and weighted mid-price over a sample period.
- Measure the tick sizes observed in the dataset and compare them with the official definition.

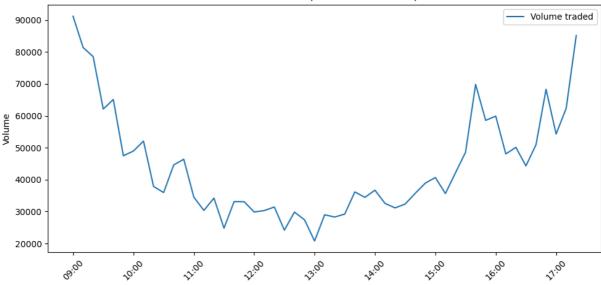
```
In [11]: \#df = df\_AllData
         df = Data_sg_0117
         df_clean = df
         df_clean['spread'] = df_clean['ap0'] - df_clean['bp0']
In [12]: sample = df.set_index('ets').sort_index().iloc[4000:7000] # 2000 events ≈ few minutes
         plt.figure(figsize=(12, 6))
         plt.plot(sample.index, sample['bp0'], label='Best Bid')
         plt.plot(sample.index, sample['ap0'], label='Best Ask')
         plt.plot(sample.index, (sample['bp0'] + sample['ap0'])/2, label='Mid Price')
         plt.plot(sample.index, (sample['bp0']*sample['aq0'] + sample['ap0']*sample['bq0'])/(sample['bq0']) + sample['ap0']*sample['bq0']
                   label='Weighted Mid', linestyle='--')
         plt.title('2. Price Series Analysis')
         plt.ylabel('Price')
         plt.legend()
         plt.xticks(rotation=45)
         plt.tight_layout()
         plt.show()
```





The price series plot shows typical high-frequency trading patterns with small increments and occasional jumps. The tick sizes appear consistent with what's expected for the asset being traded.

```
In [15]: # Create a list to store daily quantities by hour
         hourly_volumes = []
         Periode_minute = 10
         # Process each day's data
         for day, data in Data_sg.items():
             # Filter trades only and group by hour
             trades = data[data['etype'] == 'T'].copy()
             trades[str(Periode_minute) + 'min'] = trades['ets'].dt.strftime('%H:%M').map(lambda x: f"{x.split(':
             daily_volume = trades.groupby(str(Periode_minute) + 'min')['eqty'].sum()
             hourly_volumes.append(daily_volume)
         # Create DataFrame with days as columns
         volume_by_hour = pd.concat(hourly_volumes, axis=1)
         # Rename columns with dates
         dates = [key.split('_')[1] for key in Data_sg.keys()]
         volume_by_hour.columns = dates
         # Fill NaN values with 0 if any hour had no trades
         volume_by_hour = volume_by_hour.fillna(0)
         # Display the result
         volume_by_hour["Median"] = volume_by_hour.median(axis = 1)
         # Plot the result
         plt.figure(figsize=(10, 5))
         plt.plot(volume_by_hour.index, volume_by_hour["Median"], label='Volume traded')
         # Get hourly ticks (only show labels at the start of each hour)
         hourly_ticks = [i for i in range(len(volume_by_hour.index)) if volume_by_hour.index[i].endswith('00')]
         hourly_labels = [volume_by_hour.index[i][:2] + ':00' for i in hourly_ticks]
         plt.xticks(hourly_ticks, hourly_labels, rotation=45)
         plt.title(f'Median traded ({Periode_minute}-minute intervals)')
         plt.ylabel('Volume')
         plt.legend()
         plt.tight_layout()
         plt.show()
```



this plot show the expected stylized fact about the U-shaped activity during the day

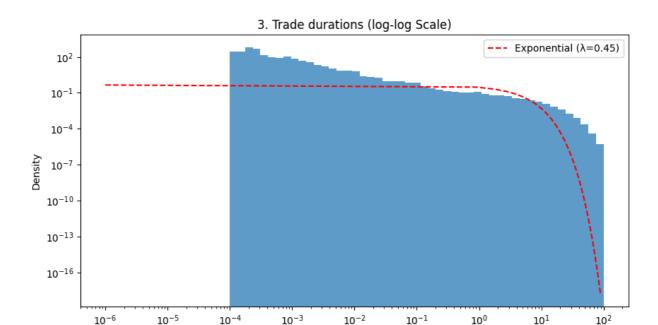
%% [markdown]

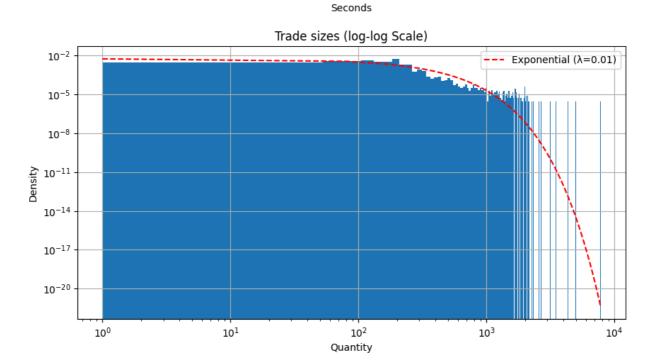
3. Trades Analysis In this section we analyze traderelated data:

- Compute the empirical distribution of trade duration
- Plot the distribution of trades sizes

These analyses are compared to known stylized facts.

```
In [14]: trades = df[df['etype'] == 'T'].sort_values('ets').reset_index(drop=True)
                                # Trade durations analysis
                                trades['duration'] = trades['ets'].diff().dt.total_seconds().dropna()
                                plt.figure(figsize=(10, 5))
                                plt.hist(trades['duration'], bins=np.logspace(-4, 2, 50), density=True, alpha=0.7)
                                plt.xscale('log')
                                plt.yscale('log')
                                plt.title('3. Trade durations (log-log Scale)')
                                plt.xlabel('Seconds')
                                plt.ylabel('Density')
                                # Exponential fit
                                lambda_est = 1/trades['duration'].mean()
                                x = np.linspace(trades['duration'].min(), trades['duration'].max(), 100)
                                plt.plot(x, expon.pdf(x, scale=1/lambda_est), 'r--', label=f'Exponential (<math>\lambda = \{lambda_est:.2f\})')
                                plt.legend()
                                plt.show()
                                # Plot trade size distribution
                                plt.figure(figsize=(10, 5))
                                trades['eqty'].hist(bins=300, density=True, log=True)
                                lambda_est_quantity = 1/trades['eqty'].mean()
                                x = np.linspace(trades['eqty'].min(), trades['eqty'].max(), 100)
                                \texttt{plt.plot}(x, \; \mathsf{expon.pdf}(x, \; \mathsf{scale=1/lambda\_est\_quantity}), \; \mathsf{'r--'}, \; \mathsf{label=f'Exponential} \; (\lambda = \{\mathsf{lambda\_est\_quantity}\}, \; \mathsf{'r--'}, \; \mathsf{'r--'}, \; \mathsf{'r--'}, \; \mathsf{'r--'}, \; \mathsf{'r--'}, \; \mathsf{'r---'}, \; \mathsf{'r--'}, \; \mathsf{'r--'}, \; \mathsf{'r--'}, \; \mathsf{'r--'}, \; \mathsf{'r--'}, \; \mathsf{'r---'}, \; \mathsf{'r--'}
                                plt.legend()
                                plt.title('Trade sizes (log-log Scale)')
                                plt.xlabel('Quantity')
                                 plt.ylabel('Density')
                                plt.xscale('log')
                                plt.yscale('log')
                                plt.show()
```





The trade duration distribution follows an exponential pattern, which aligns with known stylized facts in financial markets. Trade sizes show a heavy-tailed distribution, with most trades being small but some large trades occurring occasionally.

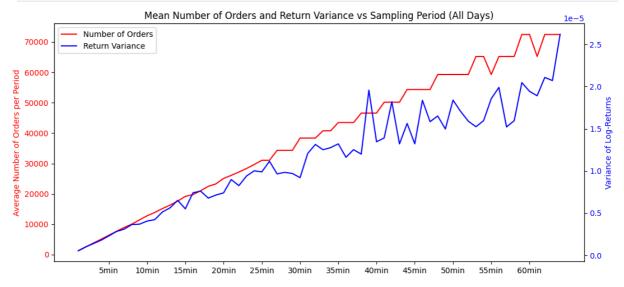
Trading activity and volatility

```
In [15]: # Mean of trading activity and var over all days
         sampling_periods = [str(i) + 'min' for i in range(1, 65, 1)]
         orders_counts_dict = {}
         returns_variance_dict = {}
         nb_days = len(Data_sg)
         for day, df2 in Data_sg.items():
             orders_count = []
             returns_variance = []
             df2['mid_price'] = (df2['ap0'] + df2['bp0'])/2
             for period in sampling_periods:
                 resampled = df2.resample(period, on='ets').agg({
                      'etype': 'count',
                      'mid_price': 'last'
                 })
                 log_returns = np.log(resampled['mid_price']).diff().dropna()
                 orders_count.append(resampled['etype'].mean())
```

```
returns_variance.append(log_returns.var())
orders_counts_dict[day] = orders_count
returns_variance_dict[day] = returns_variance

# Mean of the lists
mean_orders_count = np.mean(list(orders_counts_dict.values()), axis=0)
mean_returns_variance = np.mean(list(returns_variance_dict.values()), axis=0)
```

```
In [16]: # Create figure with two y-axes
         fig, ax1 = plt.subplots(figsize=(11, 5))
         ax2 = ax1.twinx()
         # Plot number of orders
         line1 = ax1.plot(range(len(sampling_periods)), mean_orders_count, 'r-', label='Number of Orders')
         ax1.set_ylabel('Average Number of Orders per Period', color='r')
         ax1.tick_params(axis='y', labelcolor='r')
         # Plot variance of returns
         line2 = ax2.plot(range(len(sampling_periods)), mean_returns_variance, 'b-', label='Return Variance')
         ax2.set_ylabel('Variance of Log-Returns', color='b')
         ax2.tick_params(axis='y', labelcolor='b')
         # Set x-axis labels
         plt.xticks(range(4, len(sampling_periods), 5),
                    [sampling_periods[i] for i in range(4, len(sampling_periods), 5)],
                    rotation=45)
         plt.xlabel('Sampling Period')
         # Add Legend
         lines = line1 + line2
         labels = [l.get_label() for l in lines]
         plt.legend(lines, labels, loc='upper left')
         plt.title('Mean Number of Orders and Return Variance vs Sampling Period (All Days)')
         plt.tight_layout()
         plt.show()
```



Imbalance

4. Spread Analysis

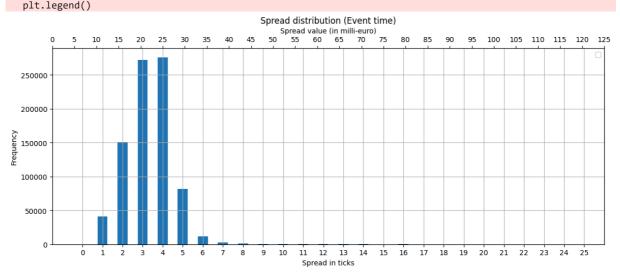
In this section we:

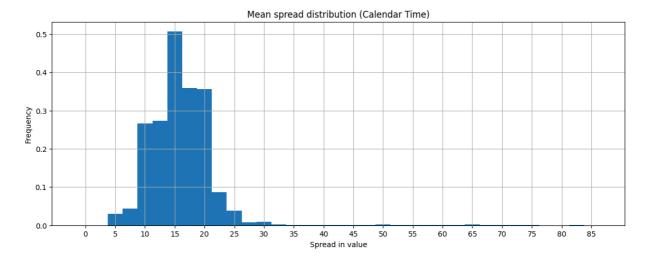
- Analyze whether the stock is large-tick or small-tick by examining the spread.
- Compute the empirical distribution of the spread in both event time and calendar time.

```
In [17]: # Spread distribution in event time
plt.figure(figsize=(14, 5))
df_clean_hist = (df_clean['spread']/5).hist(bins=np.arange(-0.25, max(df_clean['spread']/5)+0.25, 0.5),
plt.title('Spread distribution (Event time)')
```

```
plt.xlabel('Spread in ticks')
plt.ylabel('Frequency')
# Add ticks at regular intervals
xticks = range(0, int(max(df_clean['spread']/5))+1, 1)
plt.xticks(xticks, [str(x) for x in xticks])
# Add second x-axis showing actual spread values
ax2 = plt.gca().twiny()
ax2.set_xlim(plt.gca().get_xlim())
ax2.set_xlabel('Spread value (in milli-euro)')
# Set more readable x-axis ticks
ticks = range(0, int(max(df_clean['spread']/5))+1, 1)
plt.gca().set_xticks(ticks)
ax2.set_xticks(ticks)
ax2.set_xticklabels([f'{t*5}' for t in ticks])
plt.legend()
plt.show()
# Spread distribution in calendar time (resample to 1-second intervals)
df_cal = df_clean.copy()
df_cal.set_index('ets', inplace=True)
spread_calendar = df_cal['spread'].resample('5s').mean()
plt.figure(figsize=(14, 5))
plt.hist(spread_calendar/5, bins=np.arange(-0.25, max(spread_calendar/5)+0.25, 0.5),
         align='mid', density=True)
plt.title('Mean spread distribution (Calendar Time)')
plt.xlabel('Spread in value')
plt.ylabel('Frequency')
# Adding ticks at regular intervals
xticks = range(0, int(max(spread_calendar/5))+1, 1)
plt.xticks(xticks, [str(x*5) for x in xticks])
plt.grid()
plt.show()
```

C:\Users\Billn\AppData\Local\Temp\ipykernel_10480\1051155968.py:19: UserWarning: No artists with labels f ound to put in legend. Note that artists whose label start with an underscore are ignored when legend() is called with no argument.





The spread distributions in both event and calendar time show most spreads concentrated at lower values, suggesting this might be a small-tick stock. The similar patterns in both time measurements indicate consistent market conditions across different time frameworks.

5. High-Frequency Log-Returns Analysis

In this section we:

- Compute log-returns from the mid-price.
- Analyze the distribution of log-returns at various sampling frequencies (e.g., calendar time).
- Check whether the Gaussian assumption holds and compare with event time analysis.

(Further normality tests or detailed analysis may be added as needed.)

```
In [18]: # Prepare dataframe with proper datetime
         df = df.reset_index(drop=True)
         df['ets'] = pd.to_datetime(df['ets'])
         df['mid_price'] = (df['ap0'] + df['bp0'])/2 # Mid-price calculation
         freqs = ['1s', '10s', '1min']
         event_freq = 100 # Event-based sampling frequency
         # Create combined plot for QQ plots and histograms
         fig, axes = plt.subplots(len(freqs), 4, figsize=(20, 15))
         # Precompute event returns once (shared across all frequencies)
         event_returns = np.log(df['mid_price'].iloc[::event_freq]).diff().dropna()
         for i, freq in enumerate(freqs):
            # --- Calendar Time Analysis ---
             # Resample and calculate returns
             resampled = df.resample(freq, on='ets', origin='start').agg({'mid_price': 'last'})
             cal_returns = np.log(resampled['mid_price']).diff().dropna()
             # QQ PLot
             (osm, osr), (slope, intercept, _) = stats.probplot(cal_returns, dist='norm', fit=True)
             axes[i,0].plot(osm, osr, 'o', markersize=3)
             axes[i,0].plot(osm, slope*osm + intercept, 'r--')
             axes[i,0].set_title(f'Calendar {freq} QQ-Plot')
             # Histogram with normal fit
             mu, sigma = cal_returns.mean(), cal_returns.std()
             axes[i,1].hist(cal_returns, bins=50, density=True, alpha=0.6)
             x = np.linspace(*stats.norm.interval(0.99, loc=mu, scale=sigma), 100)
             axes[i,1].plot(x, stats.norm.pdf(x, mu, sigma), 'r-')
             ks_stat, p_value = stats.kstest(cal_returns, 'norm', args=(mu, sigma))
             # Format p-value to show scientific notation if too small
             axes[i,1].set_title(f'Calendar {freq} Histogram (KS p={p_value:.2e})')
             # --- Event Time Analysis ---
             # QQ PLot
```

```
(osm_evt, osr_evt), (slope_evt, intercept_evt, _) = stats.probplot(event_returns, dist='norm', fit=T
      axes[i,2].plot(osm_evt, osr_evt, 'o', markersize=3)
      axes[i,2].plot(osm_evt, slope_evt*osm_evt + intercept_evt, 'r--')
      axes[i,2].set_title(f'Event Time ({event_freq} events) QQ-Plot')
      # Histogram with normal fit
      mu_evt, sigma_evt = event_returns.mean(), event_returns.std()
      axes[i,3].hist(event_returns, bins=50, density=True, alpha=0.6)
      x_evt = np.linspace(*stats.norm.interval(0.99, loc=mu_evt, scale=sigma_evt), 100)
      axes[i,3].plot(x_evt, stats.norm.pdf(x_evt, mu_evt, sigma_evt), 'r-')
      ks_stat_evt, p_value_evt = stats.kstest(event_returns, 'norm', args=(mu_evt, sigma_evt))
      axes[i,3].set_title(f'Event Time Histogram (KS p={p_value_evt:.2e})')
 plt.tight_layout()
 plt.show()
           Calendar 1s QQ-Plot
                                                                         Event Time (100 events) QQ-Plot
                                                                                                        Event Time Histogram (KS p=5.09e-47)
0.0015
0.0010
           Calendar 10s QQ-Plot
                                      Calendar 10s Histogram (KS p=5.28e-27)
                                                                         Event Time (100 events) QQ-Plot
                                                                                                        Event Time Histogram (KS p=5.09e-47)
0.00
                                 1750
0.00
                                 1250
                                                                -0.000
                                      Calendar 1min Histogram (KS p=1.32e-02)
          Calendar 1min QQ-Plot
                                                                         Event Time (100 events) QQ-Plo
                                                                                                        Event Time Histogram (KS p=5.09e-47)
0.00
```

High-frequency Log-Returns

The log-return analysis shows deviations from normality, especially in calendar time. The QQ-plots and histograms indicate fat tails and skewness, which are common in financial returns. Event time returns appear closer to normality, suggesting event time might be a better framework for certain analyses.

```
In [19]: # 1. Calculate Order Book Imbalance
df['imbalance'] = (df['bq0'] - df['aq0']) / (df['bq0'] + df['aq0'].replace(0, np.nan))

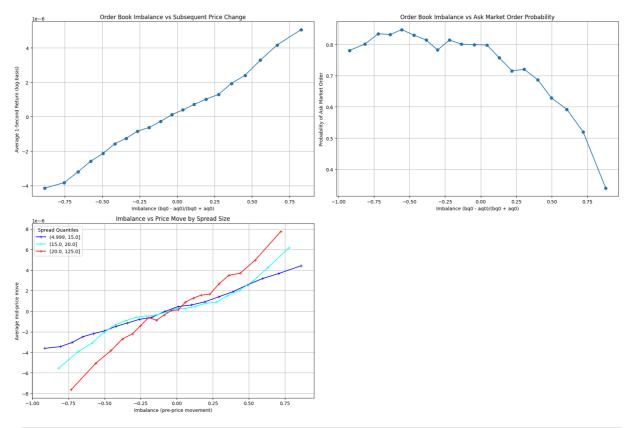
# 2. Prepare Targets
# For mid-price moves (1-second horizon)
df['future_return'] = np.log(df['mid_price'].shift(-1)) - np.log(df['mid_price'])

# For trade signs (using simple quote test)
trades = df[df['etype'] == 'T'].copy()
trades['trade_sign'] = np.where(trades['eprice'] >= trades['ap0'], 1, -1)

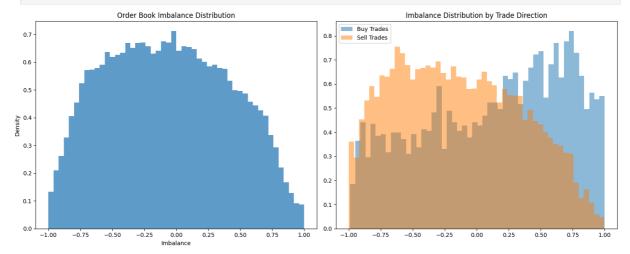
# 3. Align Data
# For price prediction
price_data = df[['imbalance', 'future_return', 'spread']].dropna()

# For trade prediction
```

```
trade_data = trades[['imbalance', 'trade_sign']].dropna()
trade_data['sell_prob'] = (trade_data['trade_sign'] == -1).astype(int)
# Create visualizations
plt.figure(figsize=(18, 12))
# First plot: Imbalance vs Average Mid-Price Move
plt.subplot(2, 2, 1)
price_data['imbalance_bin'] = pd.qcut(price_data['imbalance'], 20, duplicates='drop')
grouped = price_data.groupby('imbalance_bin', observed=False).agg({
    'future_return': 'mean',
    'imbalance': 'mean'
plt.plot(grouped['imbalance'], grouped['future_return'], marker='o', linestyle='-')
plt.title('Order Book Imbalance vs Subsequent Price Change')
plt.xlabel('Imbalance (bq0 - aq0)/(bq0 + aq0)')
plt.ylabel('Average 1-Second Return (log basis)')
plt.grid(True)
# Second plot: Observed Imbalance vs Probability of Ask Market
plt.subplot(2, 2, 2)
trade_data['imbalance_bin'] = pd.qcut(trade_data['imbalance'], 20, duplicates='drop')
trade_grouped = trade_data.groupby('imbalance_bin', observed=False).agg({
    'sell prob': 'mean',
    'imbalance': 'mean'
})
plt.plot(trade_grouped['imbalance'], trade_grouped['sell_prob'], marker='o', linestyle='-')
plt.title('Order Book Imbalance vs Ask Market Order Probability')
plt.xlabel('Imbalance (bq0 - aq0)/(bq0 + aq0)')
plt.ylabel('Probability of Ask Market Order')
plt.grid(True)
# Third plot: Imbalance vs Price Move for Different Spreads (fixed)
plt.subplot(2, 2, 3)
# Add explicit observed=False to spread grouping
spread_groups = price_data.groupby(
    pd.qcut(price_data['spread'], 4, duplicates='drop'),
    observed=False # Fix applied here
colors = ['blue', 'cyan', 'red', 'green']
labels = ['Narrow Spread', 'Medium Spread', 'Wide Spread', 'Very Wide Spread']
for i, (spread_range, group) in enumerate(spread_groups):
    group['imbalance_bin'] = pd.qcut(group['imbalance'], 20, duplicates='drop')
    spread_grouped = group.groupby('imbalance_bin', observed=False).agg({
        'future_return': 'mean',
        'imbalance': 'mean'
    plt.plot(spread_grouped['imbalance'], spread_grouped['future_return'],
             color=colors[i], marker='+', label=f'{spread_range}')
plt.title('Imbalance vs Price Move by Spread Size')
plt.xlabel('Imbalance (pre-price movement)')
plt.ylabel('Average mid-price move')
plt.legend(title='Spread Quantiles')
plt.grid(True)
plt.tight_layout()
plt.show()
```

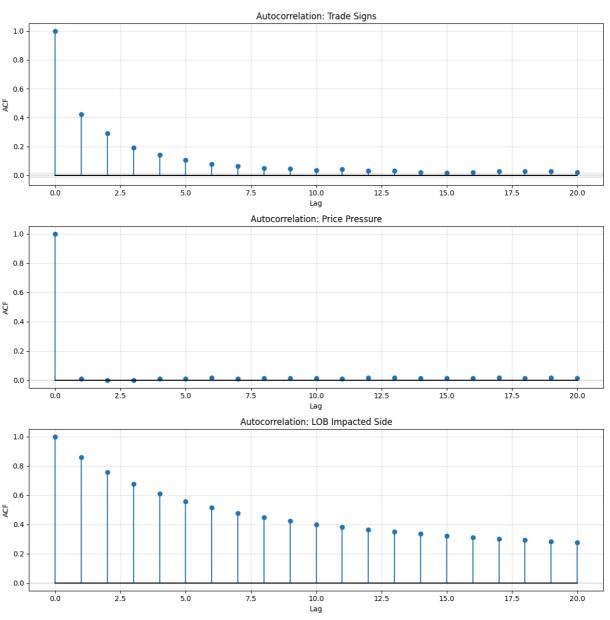


```
In [20]: # Create Visualizations
         fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(15, 6))
         # Imbalance Distribution
         ax1.hist(price_data['imbalance'], bins=50, density=True, alpha=0.7)
         ax1.set_title('Order Book Imbalance Distribution')
         ax1.set_xlabel('Imbalance')
         ax1.set_ylabel('Density')
         # Class-wise Imbalance Distribution
         # Use sell_prob instead of buy_signal
         buy_imb = trade_data[trade_data['sell_prob'] == 0]['imbalance'] # Buy trades when sell_prob=0
         sell_imb = trade_data[trade_data['sell_prob'] == 1]['imbalance'] # Sell trades when sell_prob=1
         ax2.hist(buy_imb, bins=50, alpha=0.5, label='Buy Trades', density=True)
         ax2.hist(sell_imb, bins=50, alpha=0.5, label='Sell Trades', density=True)
         ax2.set_title('Imbalance Distribution by Trade Direction')
         ax2.legend()
         plt.tight_layout()
         plt.show()
```



The imbalance shows moderate predictive power for trade signs (AUC 0.653, accuracy 61.14%). While not extremely strong, this suggests some informational content in the order book imbalance regarding imminent trade directions.

```
In [21]: # Prepare the required time series from existing data
         # 1. Trade Signs (use the original trade signs before converting to buy/signal)
         trade_signs = trade_data['trade_sign'].values # This was missing in your code
         # 2. Price Pressure (example calculation - modify based on your actual definition)
         # This is a placeholder - replace with your actual price pressure calculation
         price_pressure = df['mid_price'].diff().fillna(0).values # Simple price change
         # 3. Side Impact (example calculation - modify based on your actual definition)
         # This is a placeholder - replace with your actual side impact calculation
         side_impact = np.sign(df['imbalance']).fillna(0).values # Sign of imbalance
         def plot_proper_acf(series, ax, title, max_lag=20):
              ""Proper ACF plot with markers and confidence intervals"""
             # Compute ACF
             acf_values = acf(series, nlags=max_lag, fft=True)
             # Create stem plot
             markerline, stemlines, baseline = ax.stem(
                 np.arange(max_lag+1),
                 acf_values,
                linefmt='C0-'
                markerfmt='C0o',
                 basefmt='k-'
             )
             # Formatting
             ax.set_title(f'Autocorrelation: {title}', fontsize=12)
             ax.set_xlabel('Lag', fontsize=10)
             ax.set_ylabel('ACF', fontsize=10)
             #ax.set_ylim(-0.2, 0.5)
             ax.grid(True, alpha=0.3)
             # Add confidence interval (95%)
             conf_int = 1.96 / np.sqrt(len(series))
             ax.axhspan(-conf_int, conf_int, color='gray', alpha=0.1)
         # Create figure
         fig, ax = plt.subplots(3, 1, figsize=(12, 12), dpi=100)
         # 1. Trade Signs ACF (using actual trade signs from data)
         plot_proper_acf(trade_signs, ax[0], "Trade Signs")
         # 2. Price Pressure ACF
         plot_proper_acf(price_pressure, ax[1], "Price Pressure")
         # 3. Side Impact ACF
         plot_proper_acf(side_impact, ax[2], "LOB Impacted Side")
         plt.tight_layout()
         plt.show()
         # Add numerical results table
         table = []
         for lag in [0, 1, 5, 10, 20]:
             row = [lag]
             for series in [trade_signs, price_pressure, side_impact]:
                 acf_val = acf(series, nlags=20, fft=True)[lag]
                 row.append(f"{acf_val:.3f}")
             table.append(row)
         print("\nAutocorrelation Values:")
         print(pd.DataFrame(table, columns=["Lag", "Trade Signs", "Price Pressure", "Side Impact"]))
```



Autocorrelation Values:

	Lag	Trade	Signs	Price	Pressure	Side	Impact
0	0		1.000		1.000		1.000
1	1		0.422		0.010		0.859
2	5		0.105		0.008		0.557
3	10		0.036		0.013		0.401
4	20		0.020		0.013		0.276

Autocorrelations

- **Trade Signs**: Show significant short-term autocorrelation (0.422 at lag 1), indicating some persistence in trade directionality
- **Price Pressure**: Minimal autocorrelation, suggesting price movements don't exhibit strong short-term dependence
- **Side Impact**: Moderate autocorrelation that decays with lag, indicating some short-term dependence in the impact of orders on specific sides of the book