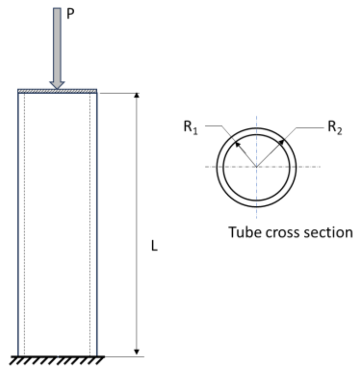
1. Consider the tube in the figure subjected to a compressive load.



$$\frac{P}{\pi\left(R_2^2 - R_1^2\right)} \le \sigma_a$$

$$P \le \frac{\pi^3 E\left(R_2^4 - R_1^4\right)}{16L^2}$$

$$R_1 = 0.080$$
$$0.088 \le R_2 \le 0.1$$
$$2 \le L \le 7$$
$$R_2 > R_1$$

All dimensions in meters

$\sigma_a = 250$ MPa

E=210 GPa

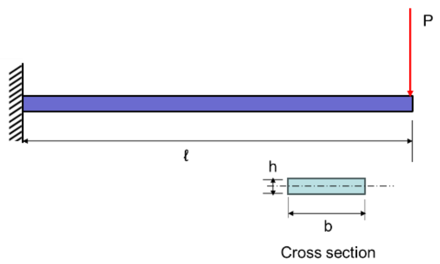
P=50000 N

Density=7800 kg/m^3

Determine the length and outer radius subject to the above conditions to minimize the tube's mass, which should be less than 10 kg.

2.

Consider a cantilever beam in the figure subjected to a load P at its tip.



ℓ=length, m

$I = \frac{bh^3}{12}$ I=moment of inertia of the cross-section, m^4

Maximum shearing stress, MPa, $\tau_{\max} = \frac{3P}{2bh}$

τ_a =Allowable shearing stress, 90 MPa

σ_a = Allowable bending stress, 250 MPa

$\sigma_{\max} = \frac{P\ell h}{2I}$ Maximum bending stress, MPa

P=load at the tip, 10 kN
E=Young’s modulus of elasticity: 210 GPa
ρ=density 7850 kg/m^3
Determine the dimensions of the beam to minimize its mass below 20 kg, and subject to the above conditions

$$0.002 \le b \le 0.007\, m$$
$$0.005 \le h \le 0.1\, m$$
$$1.5 \le \ell \le 7\, m$$
$$b > h$$

3.

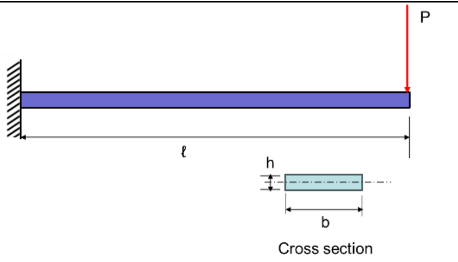
A steel cantilever beam of length ℓ has to be designed for minimum weight so it has its first natural frequency **above** 500 Hz. It is assumed to have a rectangular cross-section.
E=Young's modulus of elasticity of steel=210 GPa
ρ=density of steel= 7800 kg/m^3
$k_n \ell$=1.875 for the first natural frequency

$$k_n^4 = \frac{\omega_n^2 \rho_\ell}{EI}$$

$$I = \frac{bh^3}{12}$$ I=moment of inertia of the cross-section, m^4

$$f_n = \frac{1}{2\pi}\sqrt{\frac{EIk_n^4 \ell}{\rho_\ell}}$$ **Hz**

Determine the dimensions of the beam to minimize its mass below 5 kg, subject to the above conditions



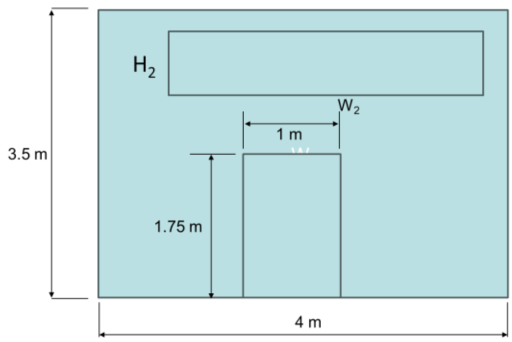
$$0.002 \le b \le 0.1\,m$$
$$0.005 \le h \le 0.2\,m$$
$$0.5 \le \ell \le 7\,m$$
$$b > h$$

4.

A concrete wall of 4mx3.5 area is facing traffic noise and a door of 1.75x1 m is already installed on it
The area of the window that has to be installed on this wall needs to be designed so the degradation of transmission loss is within limits.
The transmission loss of the wall at 1 kHz frequency is 40 dB; the transmission loss of the wooden door is 28 dB and that of the 3 mm glass window is 22 dB at the same 1 kHz frequency. Determine the dimensions of the windows so the average transmission loss is exactly equal to 30 dB.

$0.6 < W_2 < 3$ m

$0.4 < H_2 < 1.5$ m



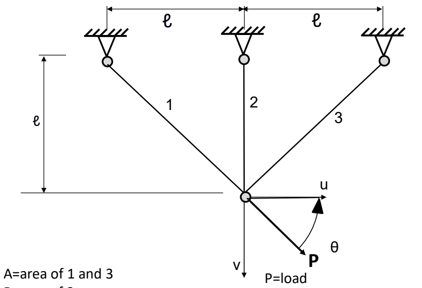
$$TL = 10\log\frac{1}{\tau}\ \text{dB}$$ TL=Transmission loss
τ=Transmission coefficient

$$\tau = \frac{1}{10^{\frac{TL}{10}}}$$

$$\bar{\tau} = \frac{\sum A_i \tau_i}{\sum A_i}$$ $\bar{\tau}$ average transmission coefficient

$$\overline{TL} = 10\log\frac{1}{\bar{\tau}}\ \text{dB}$$ Average transmission loss

Three-bar truss



A=area of 1 and 3
B=area of 2
ℓ_A =length of 1 and 3= $\ell\sqrt{2}$
ℓ_B =length of 2=ℓ

P=load
E=Young's modulus of elasticity
u: horizontal deflection
v:vertical deflection

D.N. Manik 4

Three-bar truss

A=area of 1 and 3
B=area of 2
ℓ_A =length of 1 and 3= $\ell\sqrt{2}$
ℓ_B =length of 2=ℓ

P=load
E=Young's modulus of elasticity
u: horizontal deflection
v:vertical deflection

Objective function
V=volume= $2A\,\ell_A + B\,\ell_B$
$= \ell\left(2\sqrt{2}A + B\right)$

Design variables:
A and B (areas of the truss members)

Statically indeterminate structure

$$u = \frac{\sqrt{2}\ell P\cos\theta}{EA}$$ Horizontal deflection $\leq \Delta u$

$$v = \frac{\sqrt{2}\ell P\sin\theta}{\left(A+\sqrt{2}B\right)E}$$ Vertical deflection $\leq \Delta v$

Three-bar truss

Stresses in each bar

$$\sigma_1 = \frac{1}{\sqrt{2}}\left(\frac{P\cos\theta}{A} + \frac{P\sin\theta}{A+\sqrt{2}B}\right)$$

$$\sigma_2 = \frac{\sqrt{2}P\sin\theta}{A+\sqrt{2}B}$$

$$\sigma_3 = \frac{1}{\sqrt{2}}\left(-\frac{P\cos\theta}{A} + \frac{P\sin\theta}{A+\sqrt{2}B}\right)$$

10/23/2024 D.N. Manik 6

Three-bar truss

$$\omega_n^2 = \frac{3EA}{\rho\ell^2\left(4A+\sqrt{2}B\right)} \geq \omega_a^2$$ Lowest natural frequency

$\sigma_1 \leq \sigma_a$

$\sigma_2 \leq \sigma_a$

σ_3 is significantly less and hence neglected

$I_i = \beta A_i^2$ Moment of inertia is proportional to the square of area cross section

10/23/2024 D.N. Manik 7

Three-bar truss

Compressive stress must be less than the buckling stress

$$-\sigma_1 \leq \frac{\pi^2 E\beta A}{2\ell^2}$$

$$-\sigma_2 \leq \frac{\pi^2 E\beta B}{\ell^2}$$

$$-\sigma_3 \leq \frac{\pi^2 E\beta A}{2\ell^2}$$

$A, B \geq A_{\min}$ The areas of the bars should be greater than a minimum area

10/23/2024 D.N. Manik 8

$\sigma_{1_a} = 35\,\text{MPa}$

$\sigma_{2_a} = 140\,\text{MPa}$

u=0.125mm

v=0.125 mm

E=70 GPa

ρ=2800 kg/m^3

$\beta = 1$

$\theta = 45^0$

$\ell = 1m$

Minimum area: 50 mm^2

Maximum area: 40000 mm^2

Design the areas subject to the above conditions