Polarization exercises with Matlab

1 Simulation using Mueller computation

1.1 Optical devices functions

1. Create a function that represents a linear polarizer. The following prototype can be used:

   ```
   s_out = polarizer(s_in,a)
   ```

   where `s_in` and `a` represent respectively the Stokes vector of the incoming light $\mathbf{s}_{in}$ and the polarizer angle α. The Stokes vector of the output `s_out` is given according to:

   $$\mathbf{s}_{out} = \underbrace{\frac{1}{2}\begin{bmatrix} 1 & \cos 2\alpha & \sin 2\alpha & 0 \\ \cos 2\alpha & \cos^2 2\alpha & \cos 2\alpha \sin 2\alpha & 0 \\ \sin 2\alpha & \cos 2\alpha \sin 2\alpha & \sin^2 2\alpha & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{M_{pola}(\alpha)} \cdot \mathbf{s}_{in}$$

2. In the same way, create a function `retarder` that represents a variable retarder oriented at ψ with a phase equal to δ. The Mueller matrix is given by:

   $$M_{ret}(\delta,\psi) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c^2 + s^2\cos\delta & sc(1-\cos\delta) & -s\sin\delta \\ 0 & sc(1-\cos\delta) & s^2 + c^2\cos\delta & c\sin\delta \\ 0 & s\sin\delta & -c\sin\delta & \cos\delta \end{bmatrix}$$

   with $c = \cos 2\psi$ et $s = \sin 2\psi$.

1.2 Polarization parameters computation

The Stokes vector can be written as:

$$\mathbf{s} = \begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} I \\ I\rho\cos 2\varphi \\ I\rho\sin 2\varphi\cos\delta \\ I\rho\sin 2\varphi\sin\delta \end{bmatrix}$$

where $I = s_0$ and $\rho = \frac{\sqrt{s_1^2+s_2^2+s_3^3}}{s_0}$ are respectively the intensity light and the degree of polarization. The phase δ and the angle of linear polarization φ can be written:

$$\delta = \arctan\frac{s_3}{s_2}, \ \ \varphi = \frac{1}{2}\text{atan2}\frac{s_2}{s1\cos\delta}.$$

1. Create a function that converts a Stokes vector into a “physical” set of parameters:

   $$[s_0,\, s_1\,, s_2\,, s_3] \to \{I,\, \rho,\, \varphi,\, \delta\}$$

2. Create a function that converts a “physical” set of parameters into a Stokes vector:

   $$\{I,\, \rho,\, \varphi,\, \delta\} \to [s_0,\, s_1\,, s_2\,, s_3]$$

1.3 Applications

1. Show that after passing through a quarter-wave plate ($\delta = \pi/2$) oriented at 45°, a linearly polarized wave oriented at 0° becomes circularly polarized.

2. Show that after passing through a half-wave plate, a linearly polarized wave oriented at α° becomes linearly polarized oriented at $-\alpha$°.

3. Create a program that illustrates the Malus' Law.

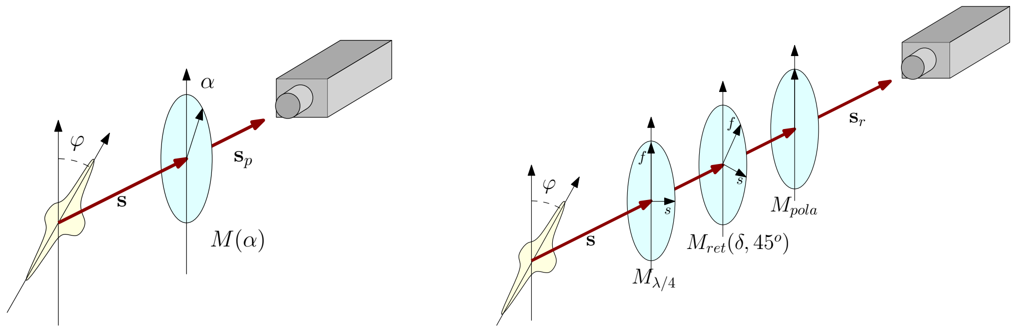
4. Create a new function that enables to draw the shape of the polarized component of the beam. The ellipse can be expressed as:

$$\begin{cases} x(t) = I\rho\cos(\omega t)\cos\varphi, \\ y(t) = I\rho\cos(\omega t + \delta)\sin\varphi, \end{cases}$$

   where ω should be chosen wisely. The function should take a Stokes vector as input and should plot the ellipse according to the input.

5. Let be a beam partially linearly polarized. Show that with a rotating polarizer placed in front of the sensor a sinusoidal relationship is obtained between the angle of the polarizer and the light intensity measured.

6. Show that by taking $\delta = 2\alpha$, the following two set-ups are equivalent:



2 Polarization imaging

The folder "imapola" contains two sets of images obtained from a simplified polarization imaging prototype (only valid to get the parameters of a partially linearly polarized light).

1. From the set n°1, compute the polarization parameters: I, ρ, and φ.

2. Compute the corresponding color image with the mapping of your choice: Wolff or Tyo.

3. Repeat question 1 and 2 with the set n°2. In addition, you can also compute the image that represents the approximation error.