Diagnostic Medical Image Processing

Reconstruction

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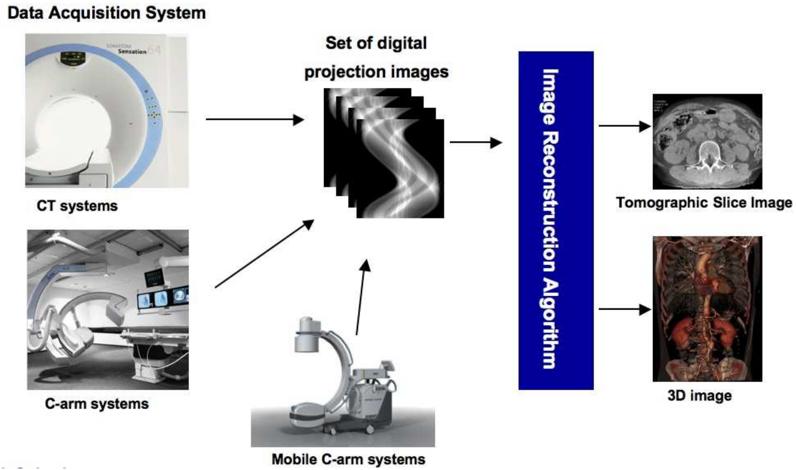
Historical notes

- C-arm CT was introduced as a commercial product in 1999.
- Reconstruction module was just an add on feature that time.
- Today C-arm CT became standard and a driving force of innovation in X-ray imaging.
- The major problem was the mathematical characterization and the calibration of the acquisition geometry.
- The breakthrough idea was the usage of homogeneous coordinates and of (3×4)–projection matrices that were standard in computer vision since the early 90-ies.
- In 2000 the first hardware accelerators (DSP's) for C-arm CT were shipped.
- Nowadays hardware accelerated C-arm CT is standard, and still in the major focus of research.

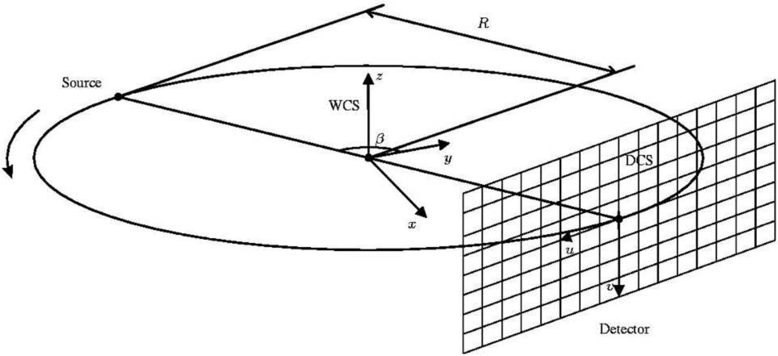
Diagnostic Medical Image Processing

Basic Idea



: Image courtesy of Holger Scherl, LME

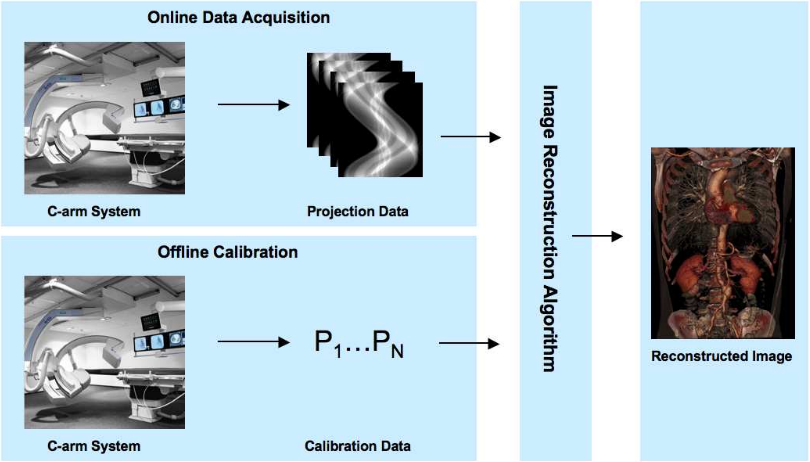
Basic Idea



WCS	World Coordinate System
DCS	Detector Coordinate System

: Image courtesy of Holger Scherl, LME

Basic Idea



: Image courtesy of Holger Scherl, LME

Principle of C-arm CT

$$P_i \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} \curvearrowright \begin{pmatrix} u/w \\ v/w \end{pmatrix}$$

Input 2-D X-ray projection images, projection matrices.

Step 1 Perform proper weighting scheme on images.

Step 2 Filter the projection images along horizontal lines.

Step 3 Perform a weighted back-projection along projection rays.

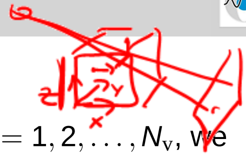
Output Reconstructed volume

Volume Elements: Voxels

The back-projection is done voxel-driven, i.e. each voxel of the $(N \times N \times N)$ volume is projected into the image plane for each projection image.

- The first element of the volume has the coordinates $\boldsymbol{v} = (v_1, v_2, v_3, 1)^\mathsf{T}$
- The increment in x-direction is denoted by $\Delta\boldsymbol{x} = (\Delta x, 0, 0, 1)^\mathsf{T}$.
- The increment in y-direction is denoted by $\Delta\boldsymbol{y} = (0, \Delta y, 0, 1)^\mathsf{T}$.
- The increment in z-direction is denoted by $\Delta\boldsymbol{z} = (0, 0, \Delta z, 1)^\mathsf{T}$.

Back-Projection Algorithm



For a given set of projection matrices $\boldsymbol{P}_n \in \mathbb{R}^{3\times 4}$, $n = 1, 2, \ldots, N_\mathrm{v}$, we get the following back-projection algorithm:

FOR $n = 1, 2, \ldots, N_\mathrm{v}$

- FOR $m = 1, 2, \ldots, N$
  - FOR $l = 1, 2, \ldots, N$
    - FOR $k = 1, 2, \ldots, N$
      - $\boldsymbol{p} = \boldsymbol{P}_n(\boldsymbol{v} + k\Delta\boldsymbol{x} + l\Delta\boldsymbol{y} + m\Delta\boldsymbol{z})$
      - $x = p_1/p_3$
      - $y = p_2/p_3$
      - Volume$[m][l][k]$ = Volume$[m][l][k] + \frac{1}{p_3^2}$filtered_image$(x, y)$

Efficiency Considerations

Due to the linearity properties we can write:

$$\begin{aligned} \boldsymbol{p} &= \boldsymbol{P}_n(\boldsymbol{v} + k \cdot \Delta\boldsymbol{x} + l \cdot \Delta\boldsymbol{y} + m \cdot \Delta\boldsymbol{z}) & (1) \\ &= \boldsymbol{P}_n\boldsymbol{v} + k\,\underbrace{\boldsymbol{P}_n \cdot \Delta\boldsymbol{x}}_{\boldsymbol{p}_x} + l\,\underbrace{\boldsymbol{P}_n \cdot \Delta\boldsymbol{y}}_{\boldsymbol{p}_y} + m\,\underbrace{\boldsymbol{P}_n \cdot \Delta\boldsymbol{z}}_{\boldsymbol{p}_z} & (2) \end{aligned}$$

We conclude:

- The vector $\boldsymbol{P}_n\boldsymbol{v}$ is constant for all iterations.
- The values of $\boldsymbol{p}_x$, $\boldsymbol{p}_y$, and $\boldsymbol{p}_z$ are constant for all iterations.
- The matrix vector product can be rewritten to get a more efficient back-projection.

Back-Projection Algorithm

compute: $\boldsymbol{v}_n$, $\boldsymbol{p}_x$, $\boldsymbol{p}_y$, and $\boldsymbol{p}_z$

FOR $m = 1, 2, \ldots, N$

- FOR $l = 1, 2, \ldots, N$
  - FOR $k = 1, 2, \ldots, N$
    - $\boldsymbol{p} = \boldsymbol{v}_n + k\boldsymbol{p}_x + l\boldsymbol{p}_y + m\boldsymbol{p}_z$
    - $x = p_1/p_3$
    - $y = p_2/p_3$
    - $\text{Volume}[m][l][k] = \text{Volume}[m][l][k] + \frac{1}{p_3^2}\text{filtered\_image}(x, y)$

Efficient Ratio Computation

- The most time consuming operation in the inner loop is the ratio computation.
- The projection geometry is computed in the calibration step. The values of p_3 and its range are know.
- Ratio $1/p_3$ can be efficiently computed by:
  - regression function (approx. errors known in advance)
  - look-up table

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Take Home Messages

- Acquisition Geometry in C-arm systems is characterized by projection matrices.
- Calibration is done offline and by technicians.
- Voxel-driven back-projection more or less determines the complexity of reconstruction.
- Hardware oriented implementation required.

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Further Readings

- Cone beam reconstruction was first introduced in 1984 by this paper:
  L.A. Feldkamp, L.C. Davis and J.W. Kress: Practical cone-beam algorithm, J Opt Soc Am, A6, 1984, p. 612-619
- The usage of homogeneous coordinates for C-arm reconstruction is described in:
  K. Wiesent, K. Barth, N. Navab, P. Durlak , T. Brunner, O. Schuetz, W. Seissler: Enhanced 3-D-reconstruction algorithm for C-arm systems suitable for interventional procedures, IEEE Transactions on Medical Imaging, 2000 May ; Vol. 19, No. 5:391-403