Digital Signal Processing

Lab Tutorial-1*

1 Signal synthesis

Consider the following sine function

$$x(t) = sin(2\pi ft). \tag{1}$$

This signal for $f = 1Hz$ is generated as follows:

```
>> t = 0:0.01:2; % samples from 0-2 with 0.01 steps
>> x = sin(2*pi*t); % Evaluate sin(2 pi t)
>> plot(t,x,'b'); % Create plot with blue line
>> xlabel('t in sec'); ylabel('x(t)'); % Label axis
>> title('Plot of sin(2\pi t)'); % Title plot
```

$x(t)$, when sampled with the sampling frequency of f_s

$$x[n] = sin(2\pi \frac{f}{f_s} n). \tag{2}$$

For $f_s = 20$, the discrete-time sine function can be generated by:

```
>> n = 0:1:40; % sample index from 0 to 20
>> x = sin(0.1*pi*n); % Evaluate sin(0.2 pi n)
>> Hs = stem(n,x,'b','filled'); % Stem-plot
>> set(Hs,'markersize',4); % Change circle size
>> xlabel('n'); ylabel('x(n)'); % Label axis
>> title('Stem Plot of sin(0.2 pi n)'); % Title plot
```

1. Generate and display following digital signals:
   (a) Impulse function $\delta[n]$.
   (b) Unit step function $u[n]$.
   (c) Exponential function e^n.
2. How do you shift and fold (flip) these signals?

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2 Convolution

The output of any Linear Time Invariant (LTI) system is obtained by the convolution operation between input and the system response. Which is defined as

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau, \tag{3}$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]h[k] \tag{4}$$

for continuous and discreet time signals respectively. The convolution operation for a discreet causal LTI system is given by

$$y[n] = \sum_{k=0}^{n} x[k]h[n-k]. \tag{5}$$

The process of computing the convolution between x(k) and h(k) involves following four steps:

- Folding : Fold $h[k]$ about $k = 0$ to obtain $h[-k]$.
- Shifting: Shift $h[-k]$ by n_0 to the right (left if n_0 is negative), to obtain $h[n_0 - k]$.
- Multiplication: Multiply $x[k]$ by $h[n_0 - k]$ to obtain the product sequence $V_0[k] = x[k]h[n_0 - k]$.
- Summation: Sum all the values of the product sequence $V_0[k]$ to obtain the value of the output at times $n = n_0$.

1. Implement your own function for convolution that performs above four steps. What are the results that you have obtained for signals 1.1.(a)-(c) as input with $h[n] = \{\underline{1}, 2, 3, 4\}$. Comment your observations for (a) and (c).
2. How can you use your function to compute cross-correlation and auto-correlation?

3 2D-convolution and cross-correlation

The same idea of convolution in 1D can also be extended to 2D case, however, mostly designed as a non-causal system. Implement your own 2D-convolution function that takes an image and a convolution mask as inputs, and returns their convoluted output. Use your function as cross-correlation for the character recognition as follows:

- Download the file "Images.zip". Load images ”a.PNG" and "code.PNG" and binarize them.
- Take the cross-correlation between these two binarized 2D signals.
- Locate the best response position to recognize the character "a" in the image.

1. Report the center pixels of all the characters "a" that you are able to recognize.
2. How would you detect the same character if the image is rotated ("code_r.PNG")? Propose your own solution if possible.