Image Enhancement

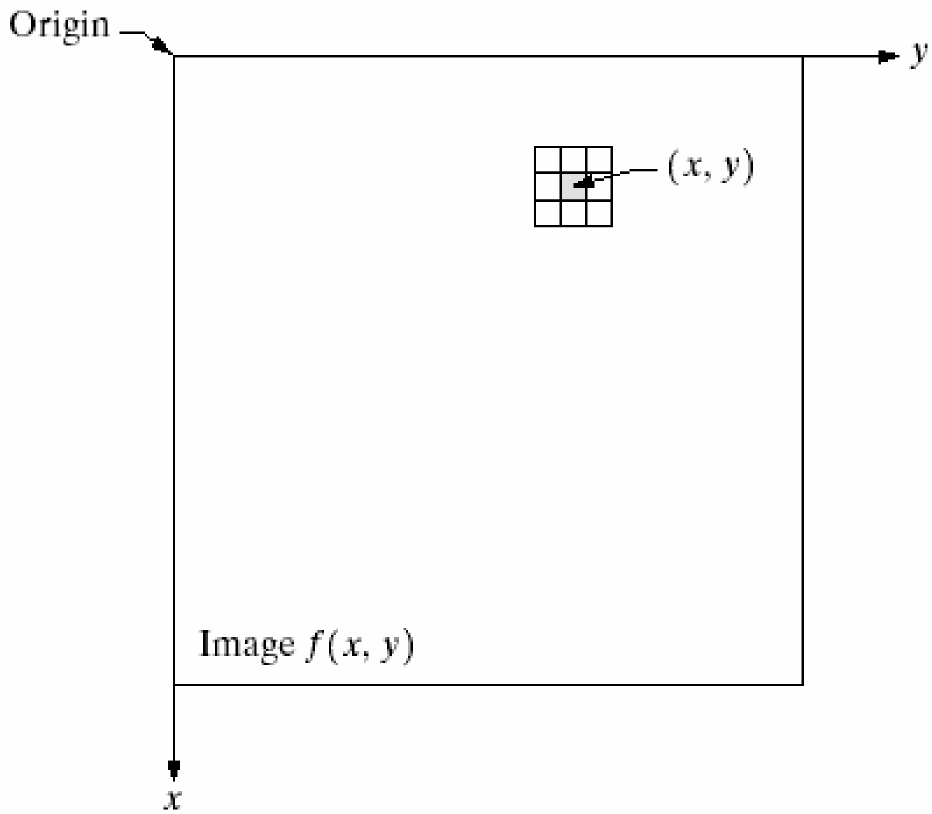
- To process an image so that the result is more suitable than the original image for a *specific* application.
- Spatial domain methods and frequency domain methods.

Spatial Domain Methods

- Procedures that operate directly on the aggregate of pixels composing an image
- $g(x, y) = T[f(x, y)]$
- A neighborhood about (x,y) is defined by using a square (or rectangular) subimage area centered at (x,y).

Image Enhancement in the Spatial Domain

FIGURE 3.1 A 3×3 neighborhood about a point (x, y) in an image.



Spatial Domain Methods

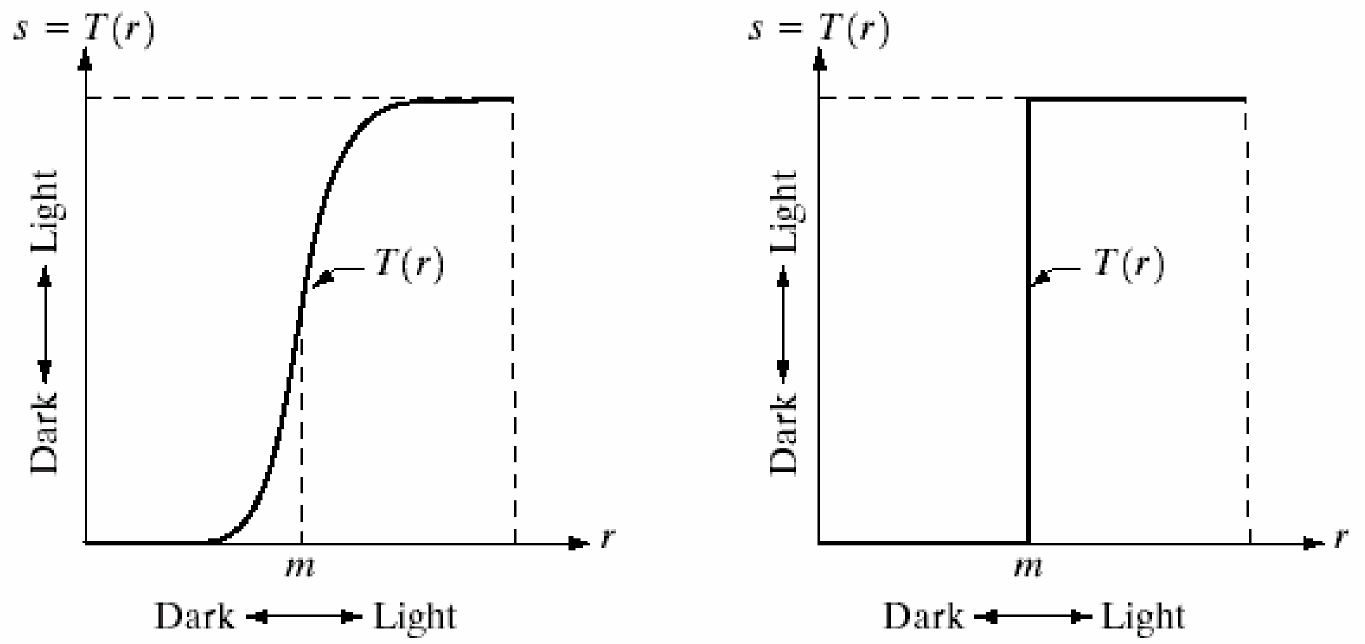
- When the neighborhood is 1 x 1 then g depends only on the value of f at (x,y) and T becomes a gray-level transformation (or mapping) function:

  $$s=T(r)$$

  r,s: gray levels of f(x,y) and g(x,y) at (x,y)

  - Point processing techniques (e.g. contrast stretching, thresholding)

Image Enhancement in the Spatial Domain



a b

FIGURE 3.2 Gray-level transformation functions for contrast enhancement.

Contrast Stretching **Thresholding**

Spatial Domain Methods

- **Mask processing or filtering**: when the values of f in a predefined neighborhood of (x,y) determine the value of g at (x,y).
  - Through the use of masks (or kernels, templates, or windows, or filters).

Enhancement by Point Processing

- These are methods based only on the intensity of single pixels.
  - r denotes the pixel intensity before processing.
  - s denotes the pixel intensity after processing.

Some Simple Intensity Transformations

- Image negatives
- Piecewise-Linear Transformation Functions:
  - Contrast stretching
  - Gray-level slicing
  - Bit-plane slicing

➢ Implemented via Look-Up Tables (LUT) where values of T are stored in a 1-D array (for 8-bit, LUT will have 256 values)

Image Enhancement in the Spatial Domain

FIGURE 3.3 Some basic gray-level transformation functions used for image enhancement.

Linear: Negative, Identity

Logarithmic: Log, Inverse Log

Power-Law: *n*th power, *n*th root

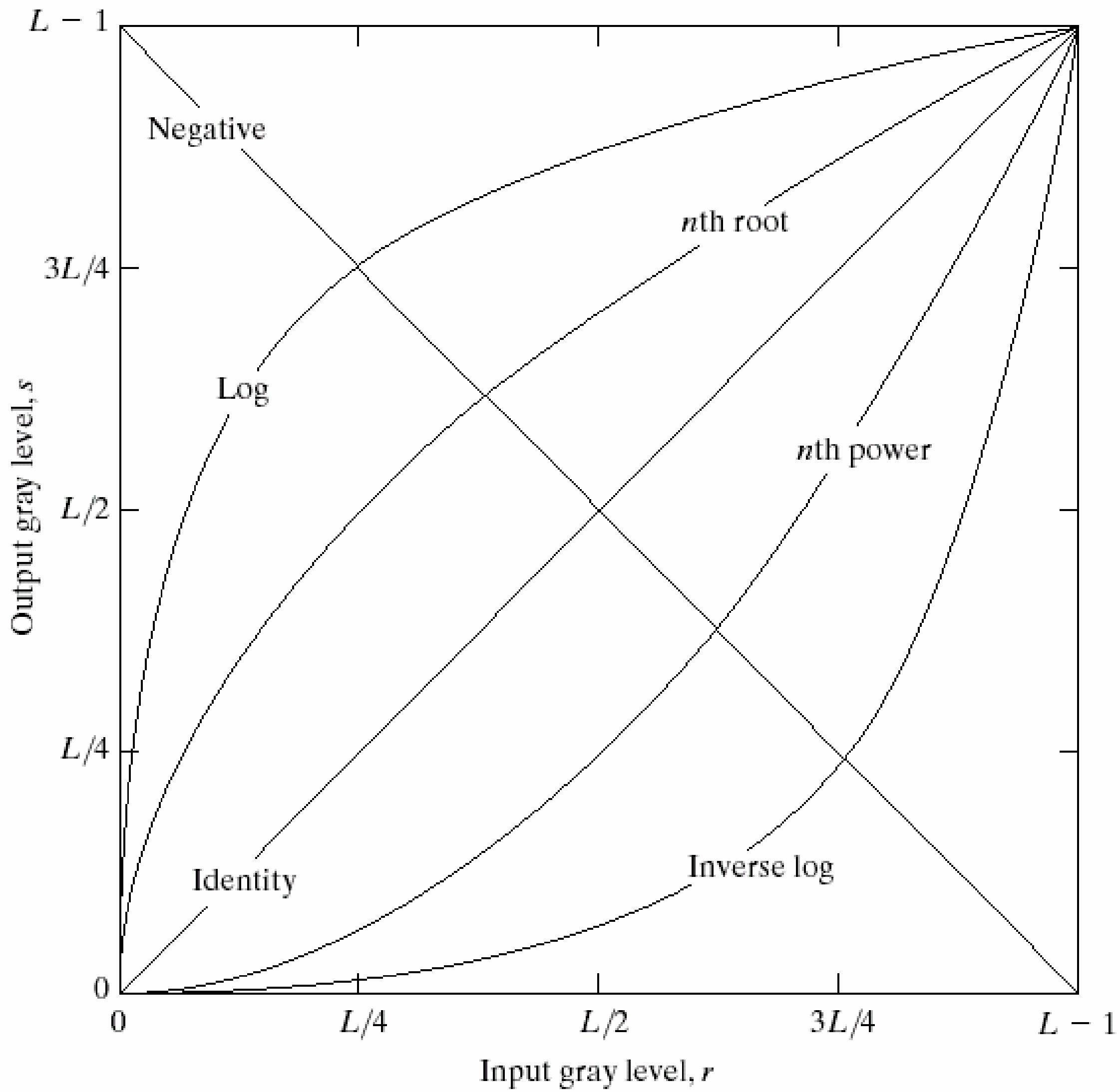
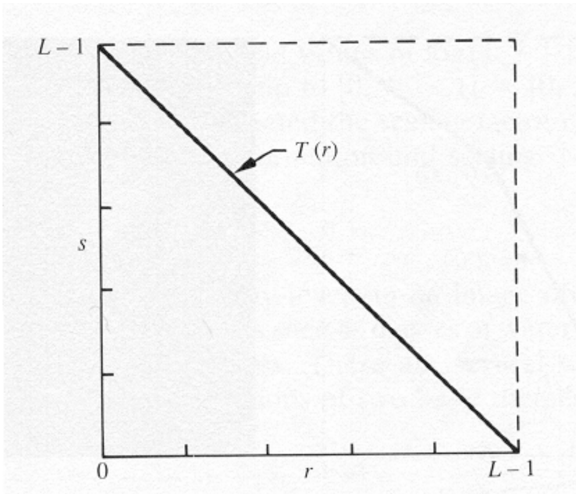


Image Negatives

- Are obtained by using the transformation function s=T(r).



[0,L-1] the range of gray levels

$s = L-1-r$

Image Enhancement in the Spatial Domain

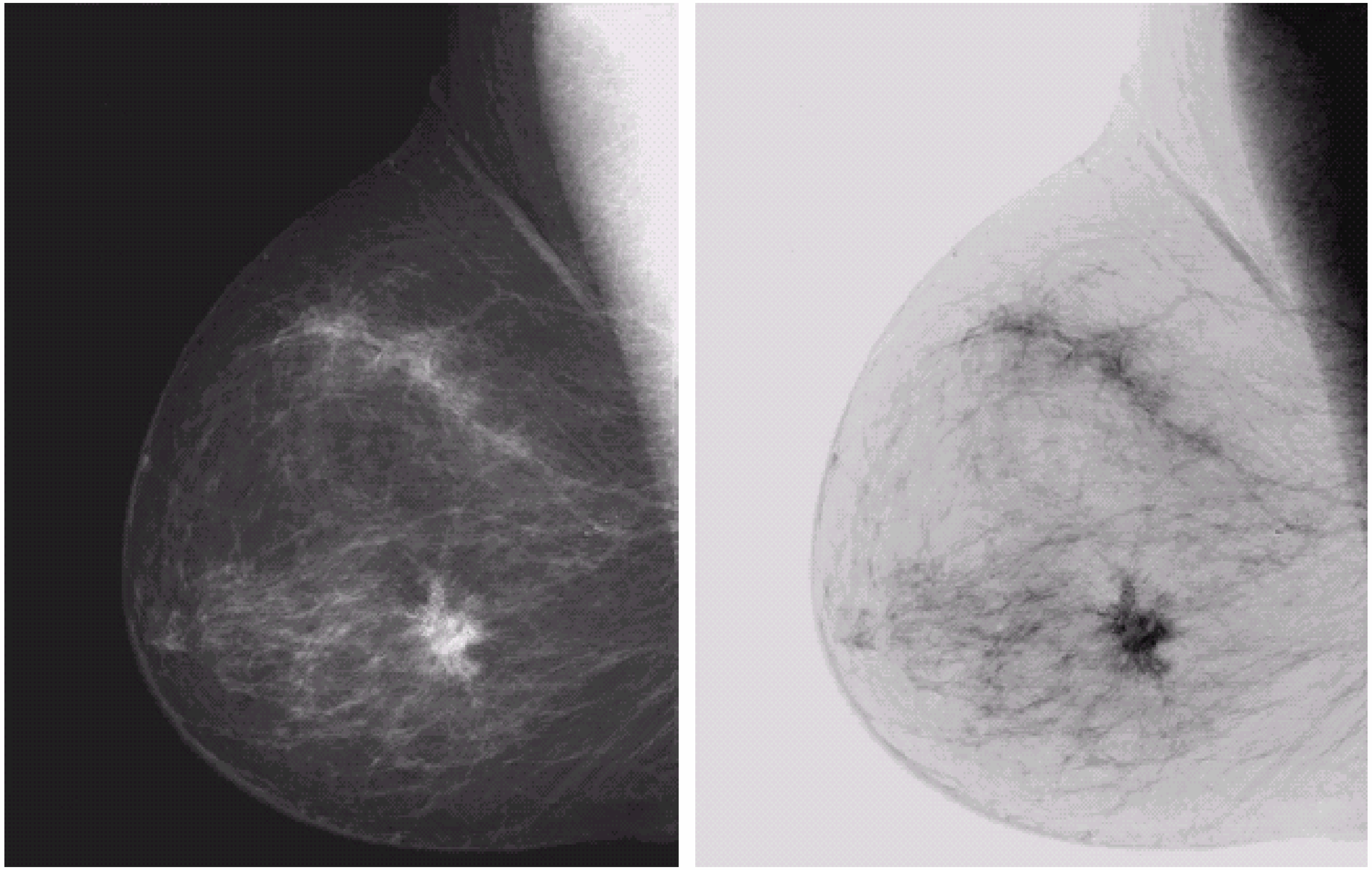


FIGURE 3.4 (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

Image Negatives

- Function reverses the order from black to white so that the intensity of the output image decreases as the intensity of the input increases.

- Used mainly in medical images and to produce slides of the screen.

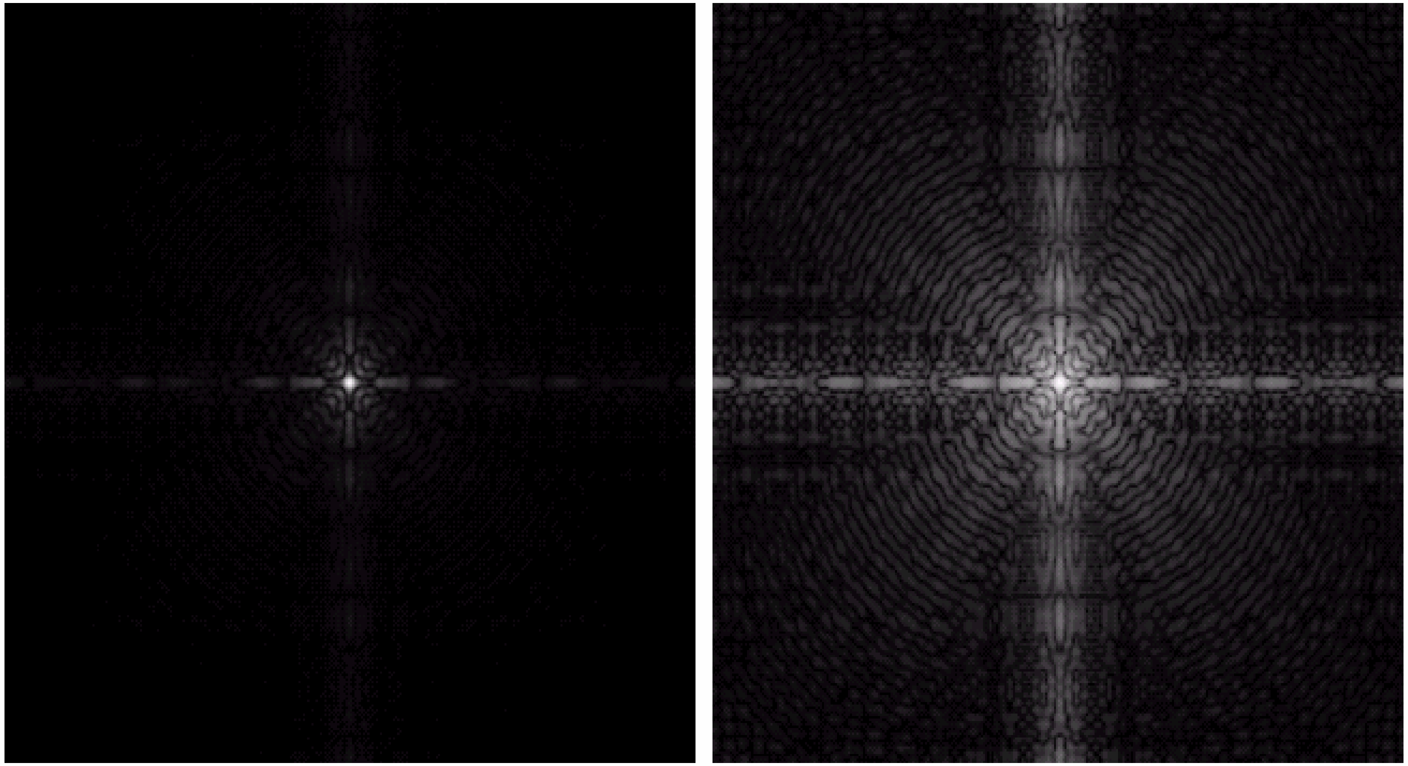
Log Transformations

$$s = c \log(1+r)$$

c: constant

- Compresses the dynamic range of images with large variations in pixel values

Image Enhancement in the Spatial Domain



a b

FIGURE 3.5 (a) Fourier spectrum. (b) Result of applying the log transformation given in Eq. (3.2-2) with $c = 1$.

Power-Law Transformations

$$s = cr^{\gamma}$$

c, γ : positive constants

- Gamma correction

Image Enhancement in the Spatial Domain

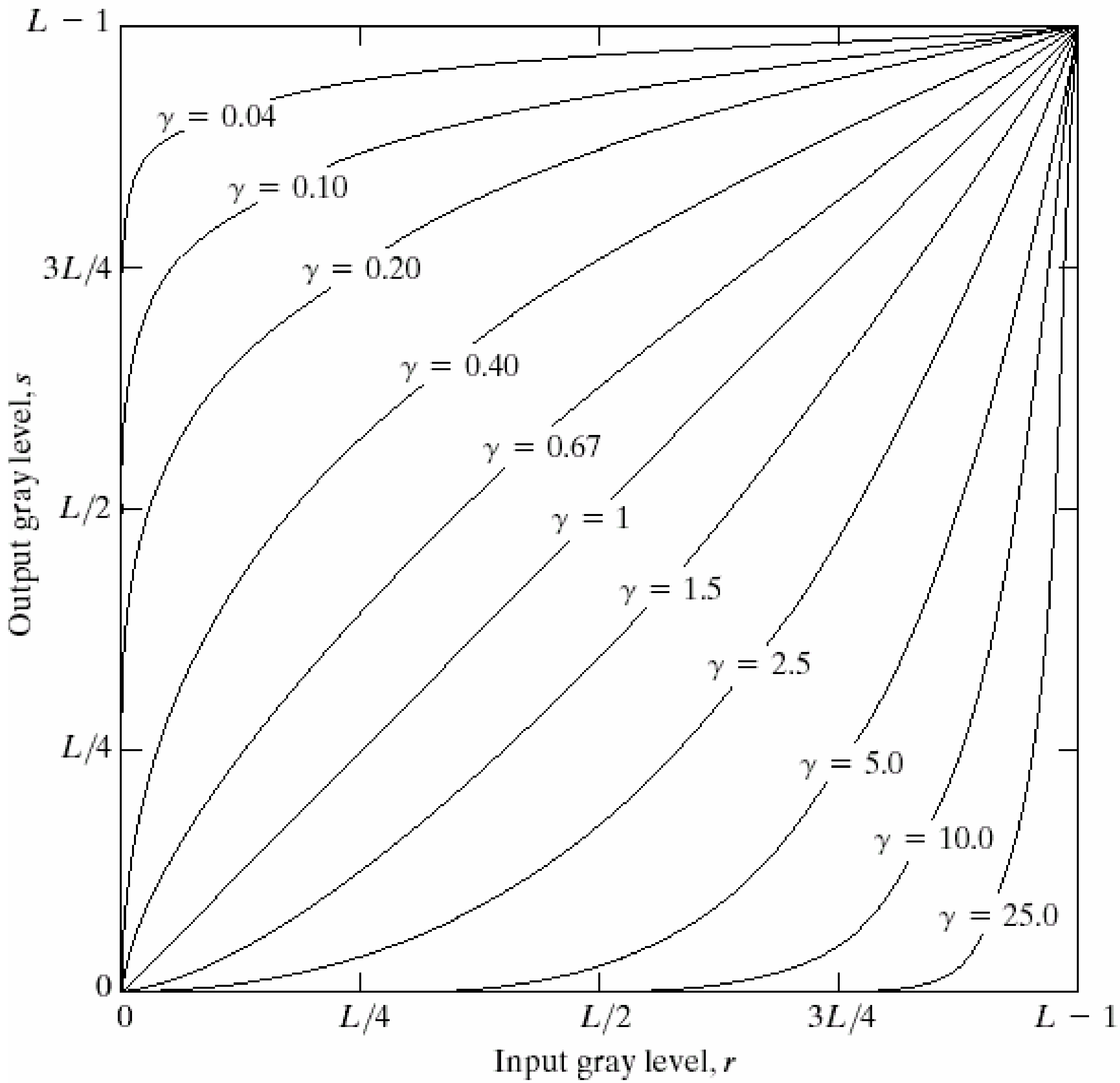
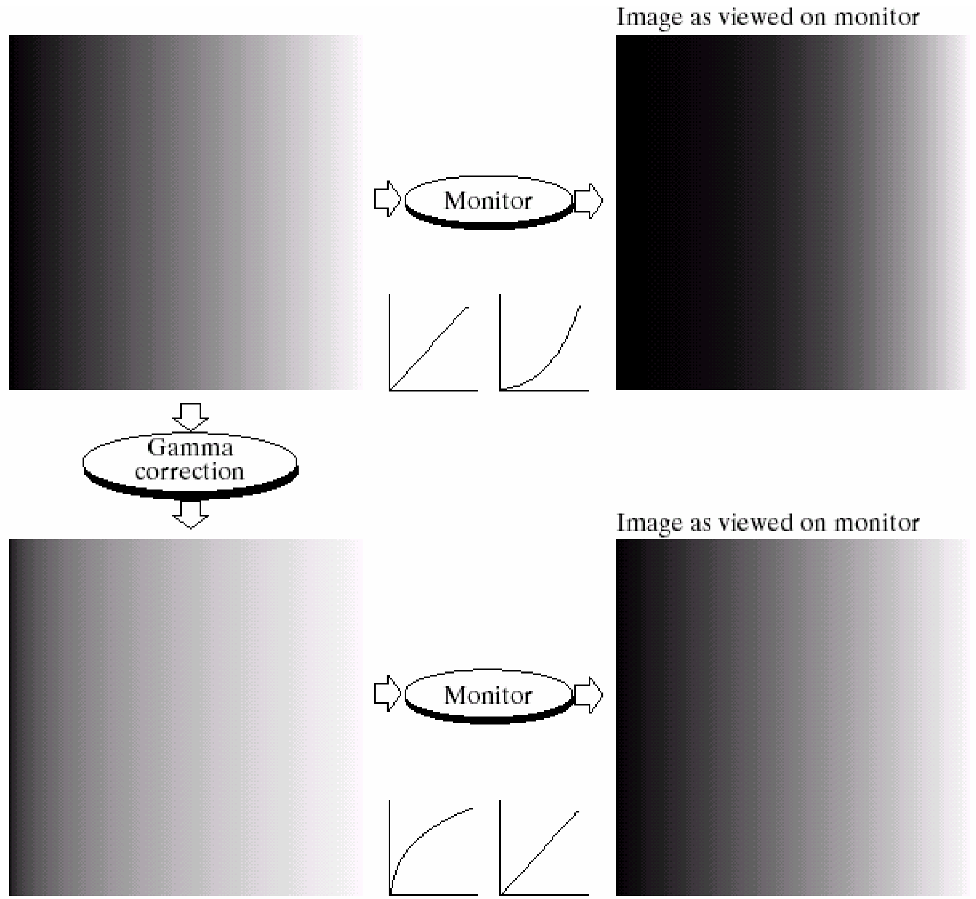


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases).

γ=c=1: identity

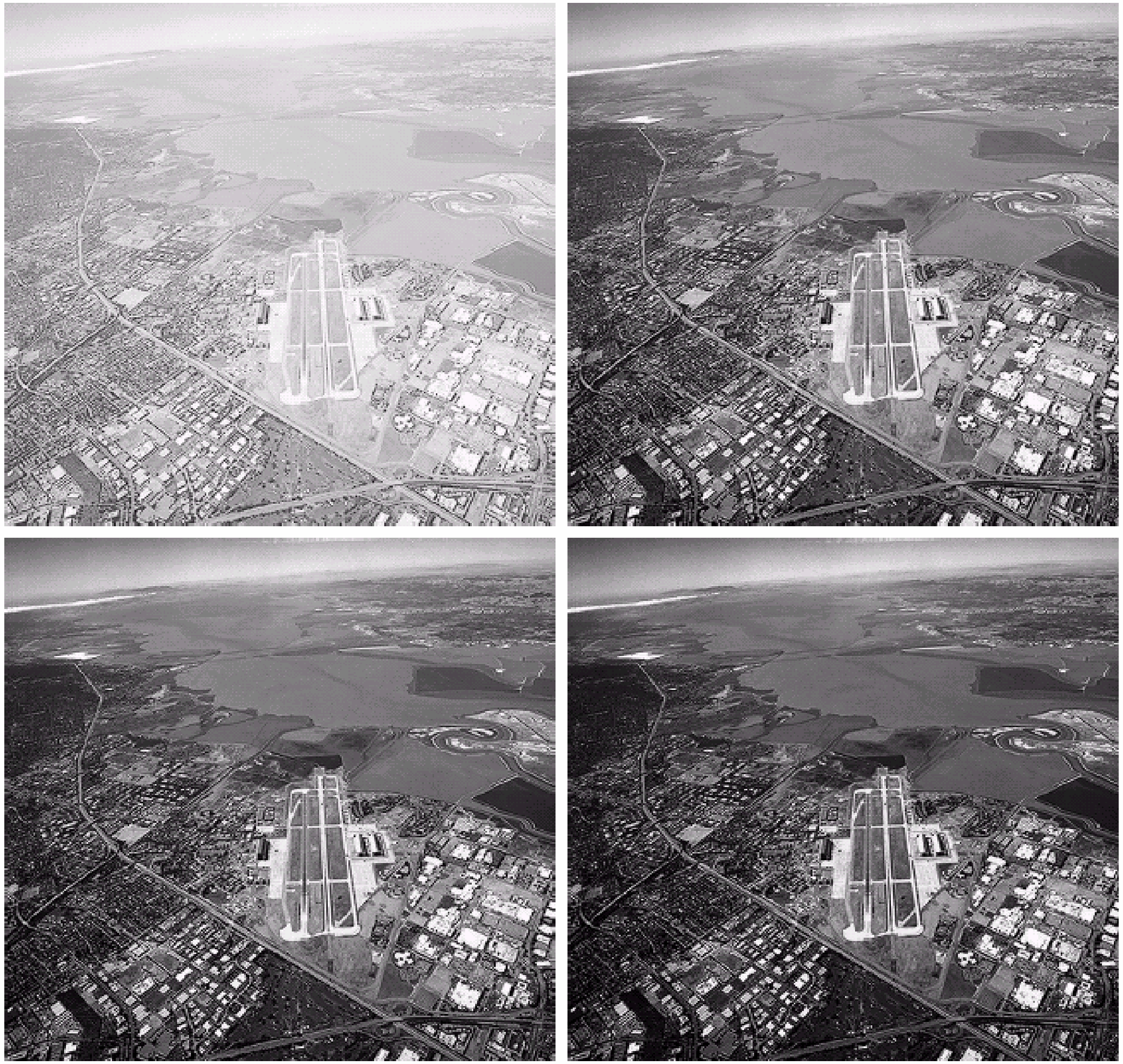
Image Enhancement in the Spatial Domain



a b
c d

FIGURE 3.7
(a) Linear-wedge gray-scale image.
(b) Response of monitor to linear wedge.
(c) Gamma-corrected wedge.
(d) Output of monitor.

Image Enhancement in the Spatial Domain



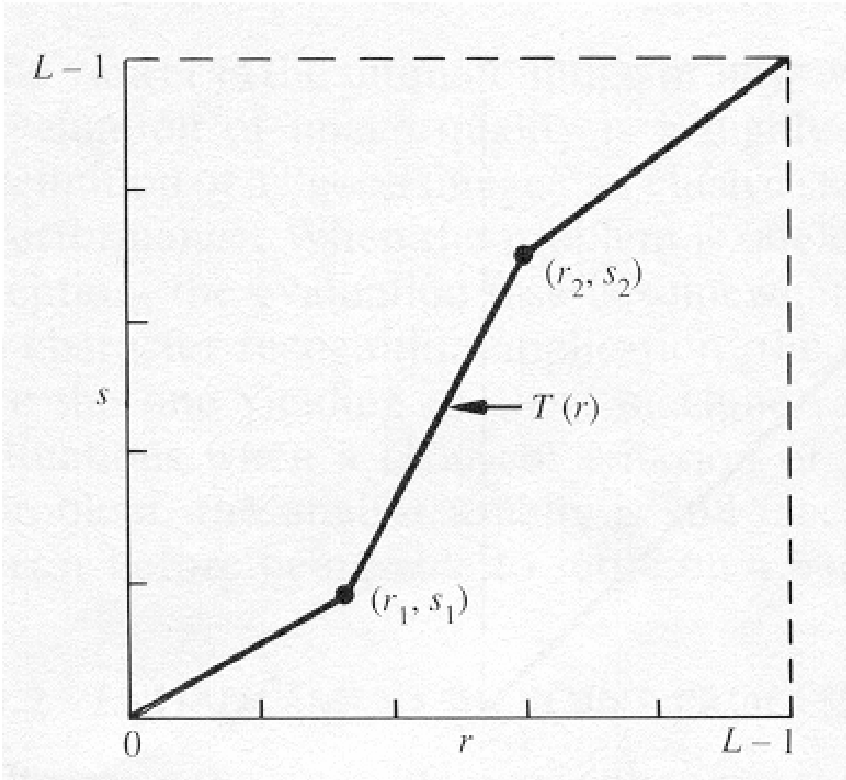
a b
c d

FIGURE 3.9 (a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and 5.0, respectively. (Original image for this example courtesy of NASA.)

Piecewise-Linear Transformation Functions

Contrast Stretching

- To increase the dynamic range of the gray levels in the image being processed.



Contrast Stretching

- The locations of (r_1,s_1) and (r_2,s_2) control the shape of the transformation function.

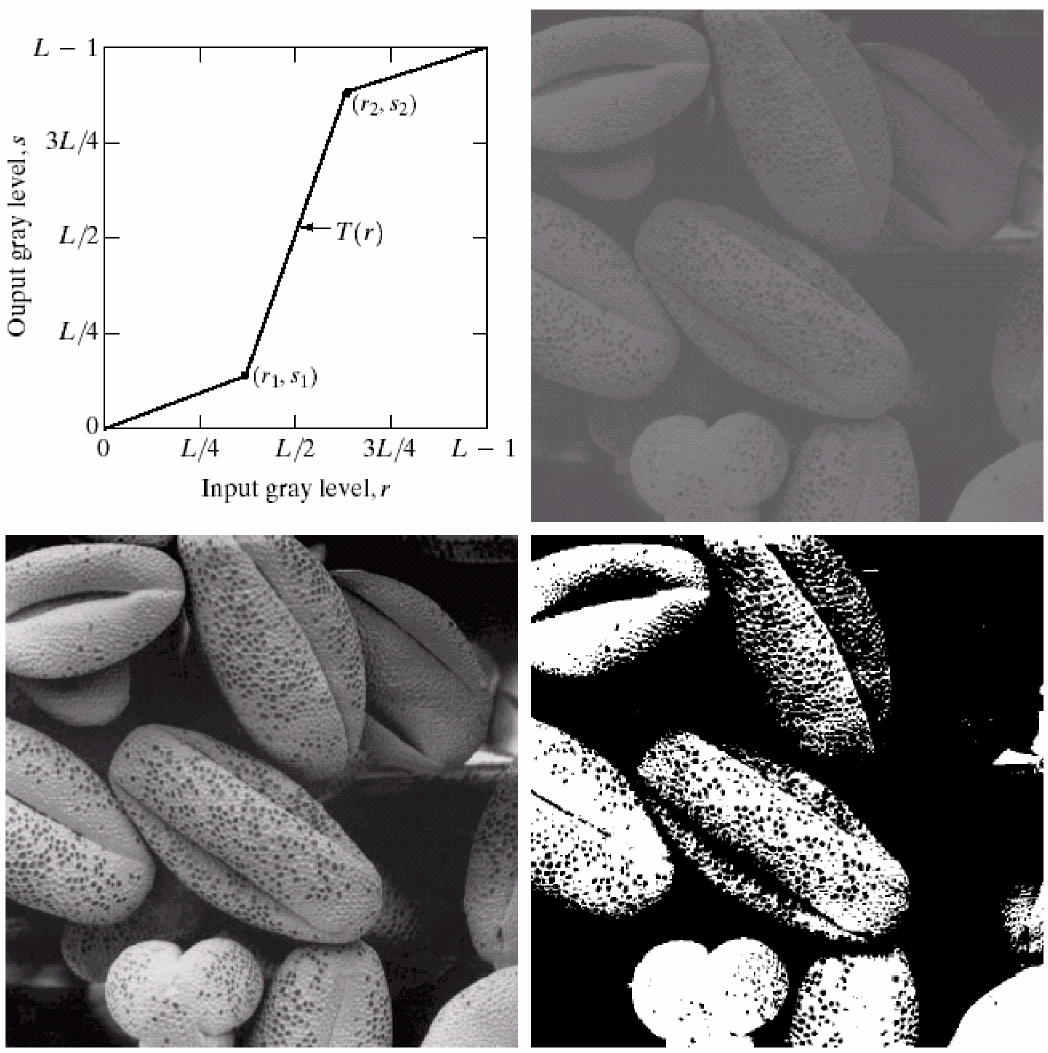
  – If $r_1= s_1$ and $r_2= s_2$ the transformation is a linear function and produces no changes.

  – If $r_1=r_2$, $s_1=0$ and $s_2=L-1$, the transformation becomes a thresholding function that creates a binary image.

Contrast Stretching

- More on function shapes:
  - Intermediate values of (r_1,s_1) and (r_2,s_2) produce various degrees of spread in the gray levels of the output image, thus affecting its contrast.
  - Generally, $r_1 \leq r_2$ and $s_1 \leq s_2$ is assumed.

Image Enhancement in the Spatial Domain

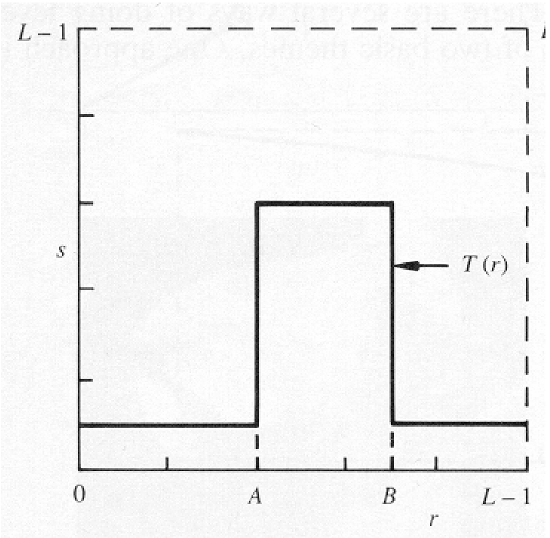


a b
c d

FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

Gray-Level Slicing

- To highlight a specific range of gray levels in an image (e.g. to enhance certain features).



One way is to display a high value for all gray levels in the range of interest and a low value for all other gray levels (binary image).

Gray-Level Slicing

– The second approach is to brighten the desired range of gray levels but preserve the background and gray-level tonalities in the image:

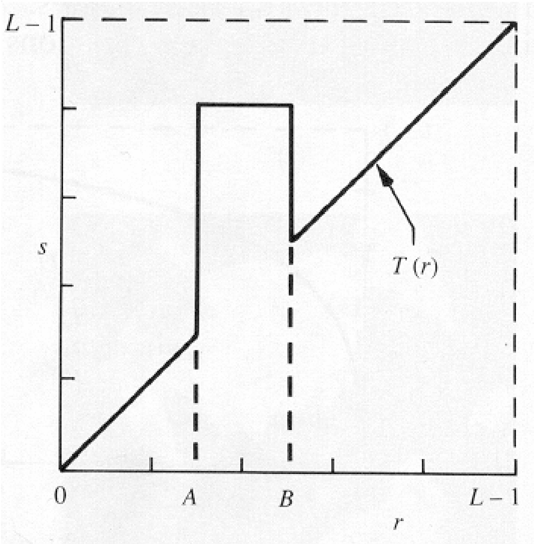


Image Enhancement in the Spatial Domain

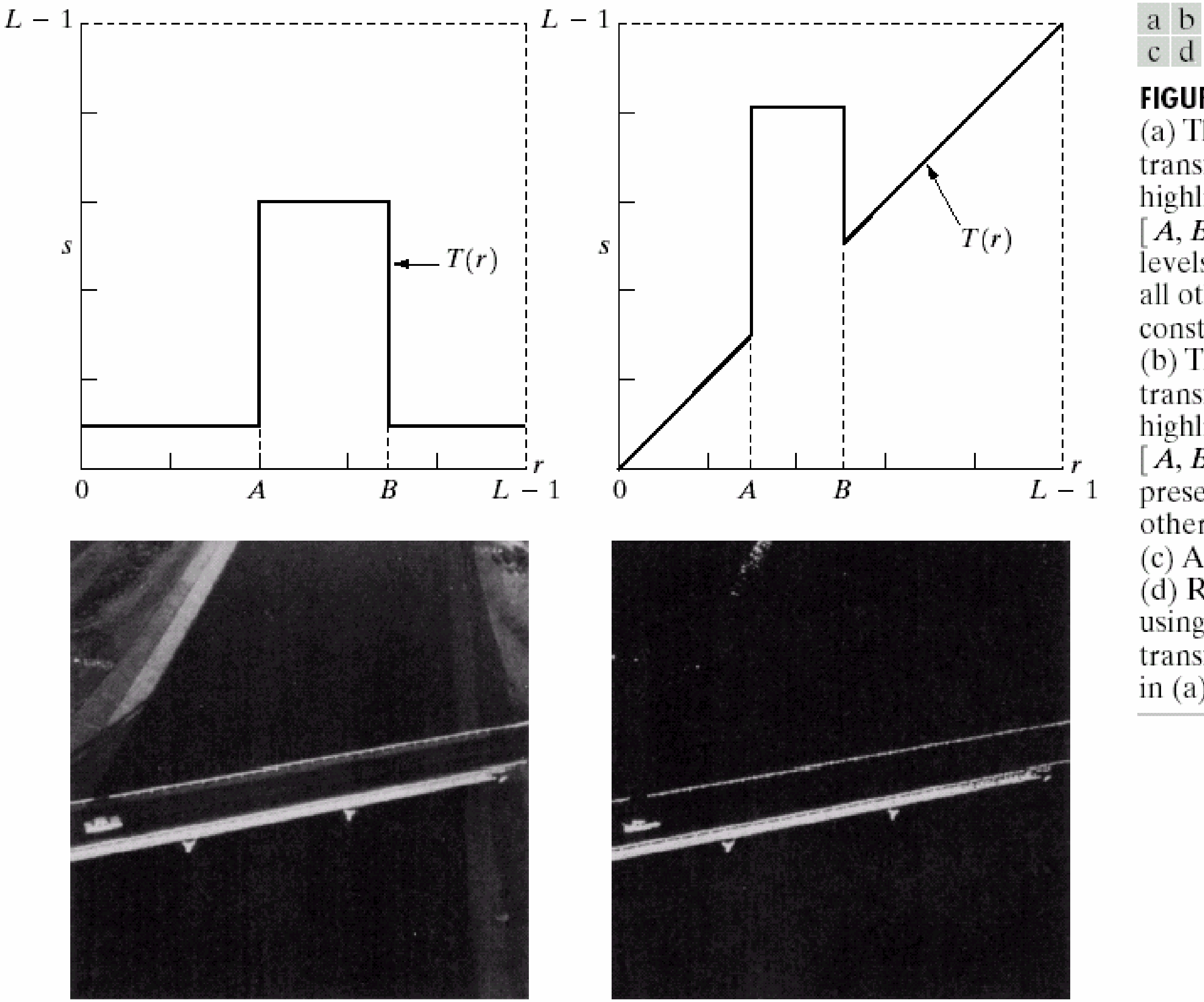


FIGURE 3.11 (a) This transformation highlights range $[A, B]$ of gray levels and reduces all others to a constant level. (b) This transformation highlights range $[A, B]$ but preserves all other levels. (c) An image. (d) Result of using the transformation in (a).

Bit-Plane Slicing

- To highlight the contribution made to the total image appearance by specific bits.
  - i.e. Assuming that each pixel is represented by 8 bits, the image is composed of 8 1-bit planes.
  - Plane 0 contains the least significant bit and plane 7 contains the most significant bit.

Bit-Plane Slicing

- More on bit planes:
  - Only the higher order bits (top four) contain visually significant data. The other bit planes contribute the more subtle details.
  - Plane 7 corresponds exactly with an image thresholded at gray level 128.

Image Enhancement in the Spatial Domain

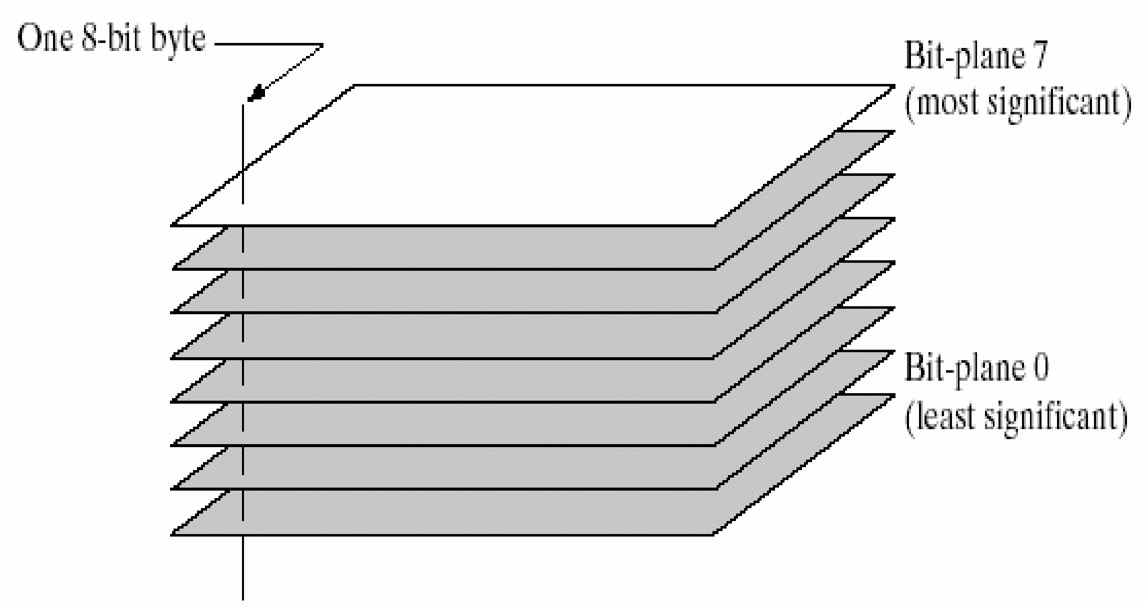


FIGURE 3.12 Bit-plane representation of an 8-bit image.

Image Enhancement in the Spatial Domain

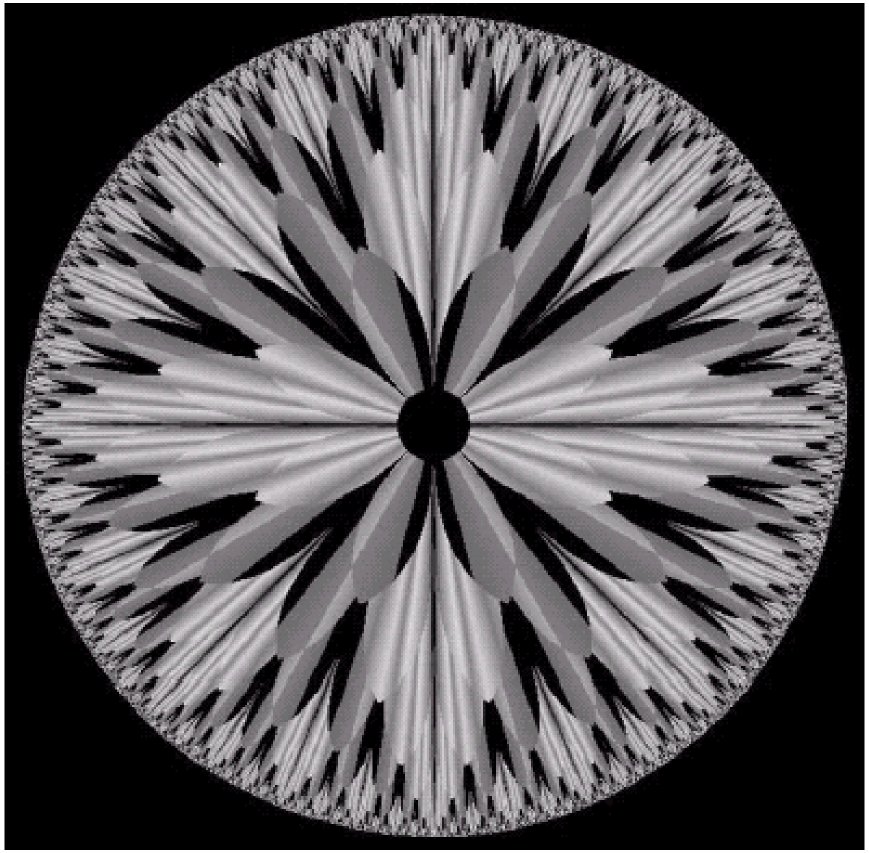


FIGURE 3.13 An 8-bit fractal image. (A fractal is an image generated from mathematical expressions). (Courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA.)

Image Enhancement in the Spatial Domain

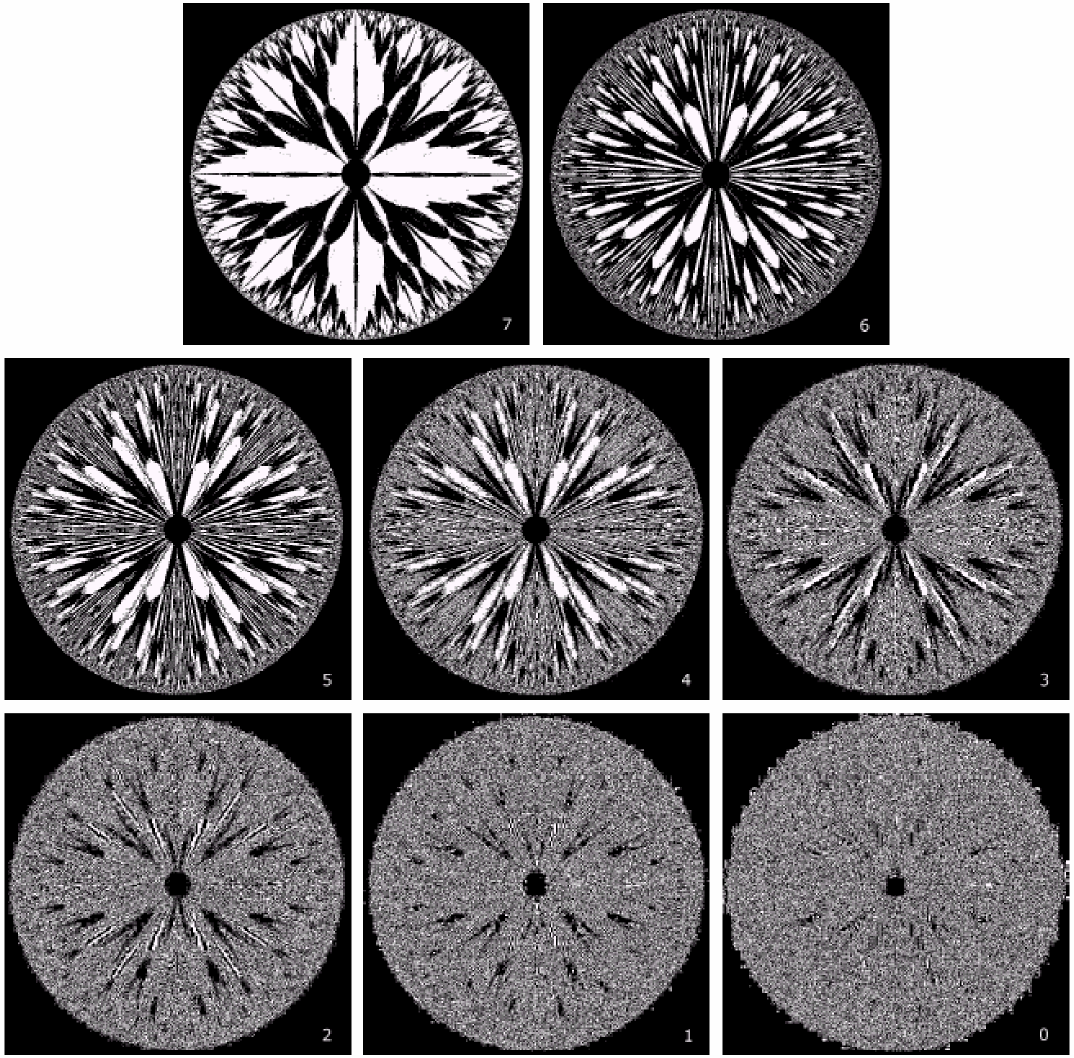
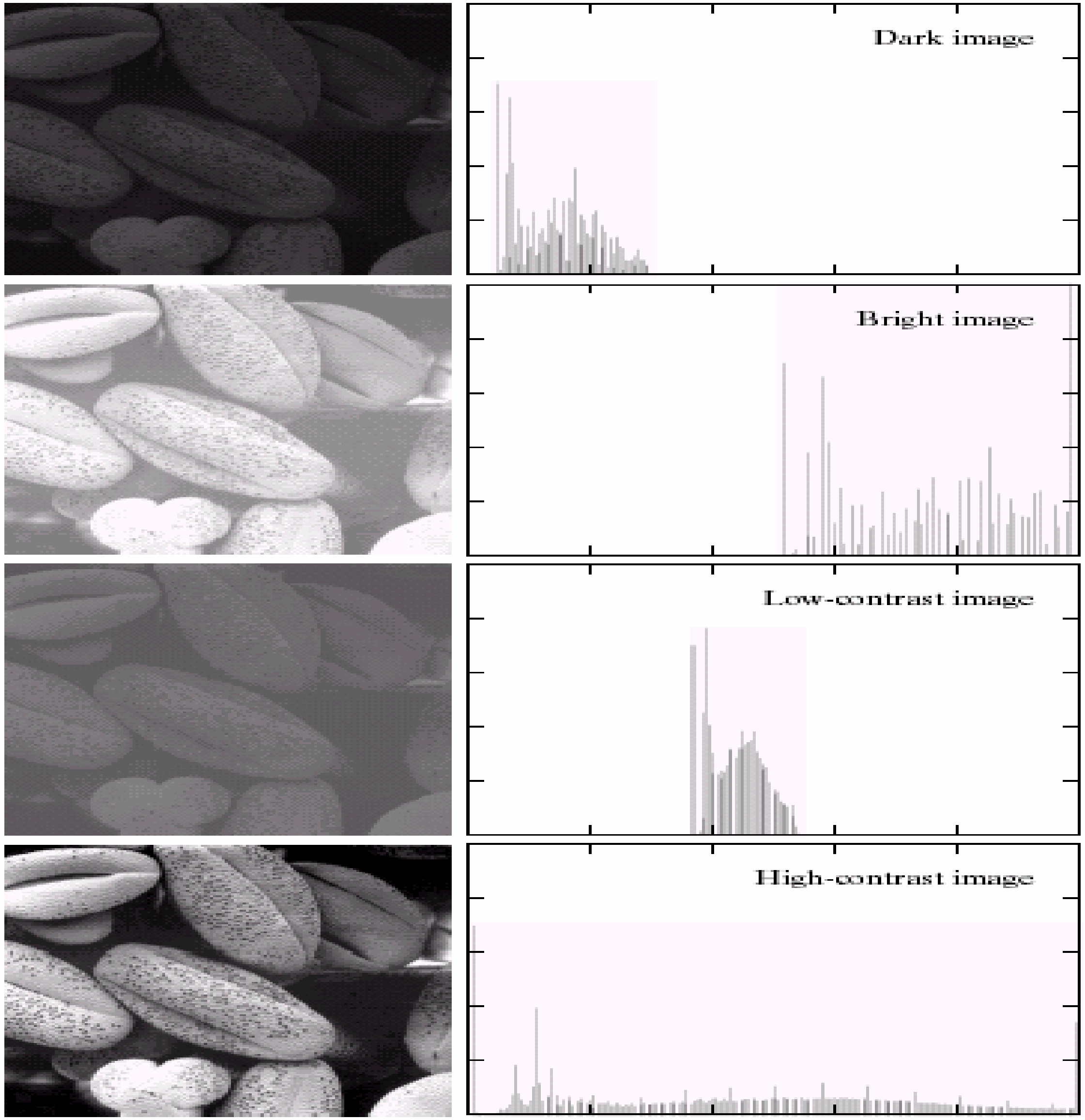


FIGURE 3.14 The eight bit planes of the image in Fig. 3.13. The number at the bottom, right of each image identifies the bit plane.

Histogram Processing

- The histogram of a digital image with gray levels from 0 to L-1 is a discrete function $h(r_k)=n_k$, where:
  - r_k is the kth gray level
  - n_k is the # pixels in the image with that gray level
  - n is the total number of pixels in the image
  - $k = 0, 1, 2, \ldots, L-1$
- Normalized histogram: $p(r_k)=n_k/n$
  - sum of all components = 1

Image Enhancement in the Spatial Domain



Histogram Processing

- The shape of the histogram of an image does provide useful info about the possibility for contrast enhancement.

- Types of processing:

  Histogram equalization

  Histogram matching (specification)

  Local enhancement

Histogram Equalization

- As mentioned above, for gray levels that take on discrete values, we deal with probabilities:

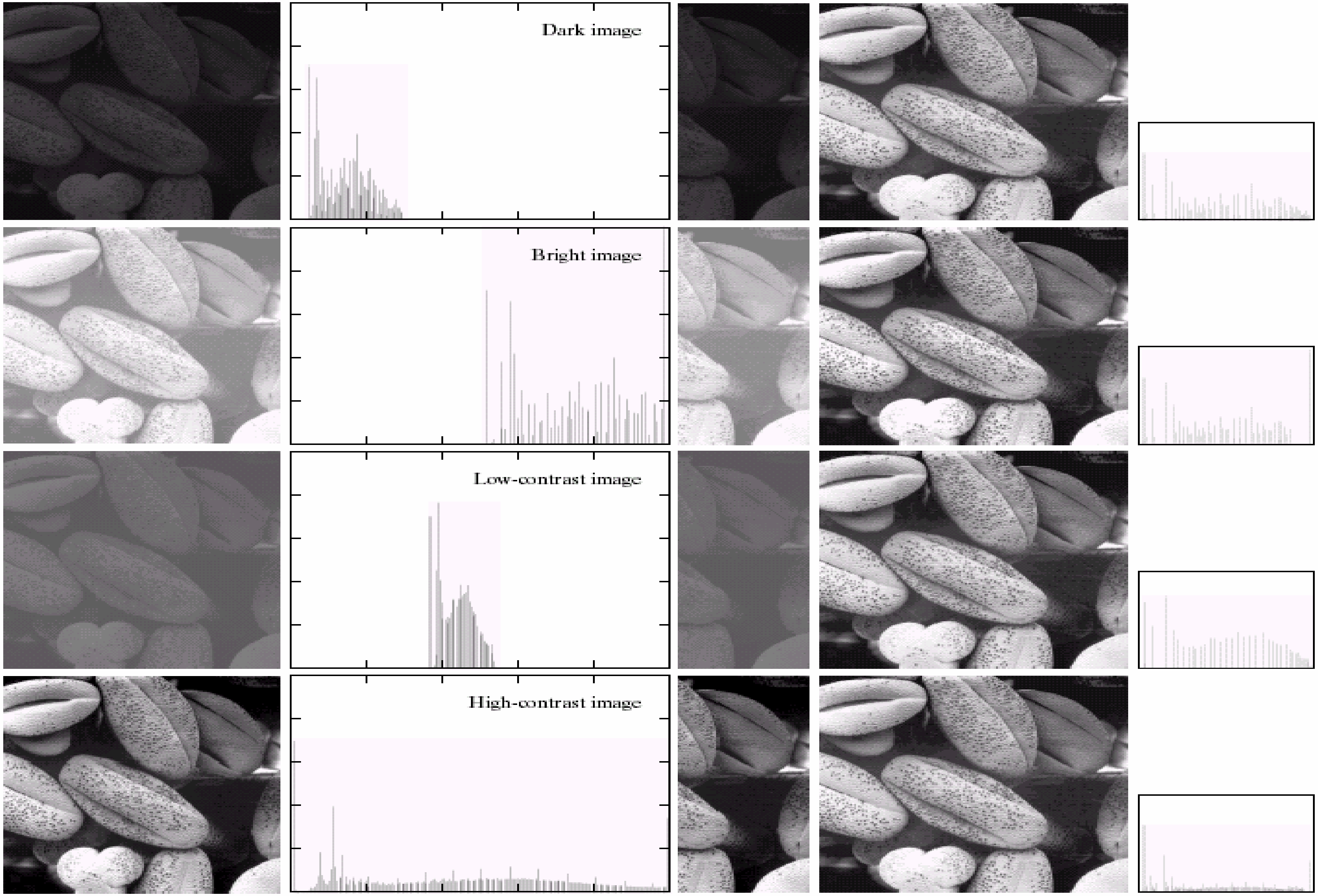
$$p_r(r_k)=n_k/n, \ k=0,1,.., L-1$$

  - The plot of $p_r(r_k)$ versus r_k is called a histogram and the technique used for obtaining a uniform histogram is known as histogram equalization (or histogram linearization).

Histogram Equalization

$$s_k = T(r_k) = \sum_{j=0}^{k} \frac{n_j}{n} = \sum_{j=0}^{k} p_r(r_j)$$

- Histogram equalization(HE) results are similar to contrast stretching but offer the advantage of full automation, since HE automatically determines a transformation function to produce a new image with a uniform histogram.


Dark image
Bright image
Low-contrast image
High-contrast image

Histogram Matching (or Specification)

- Histogram equalization does not allow interactive image enhancement and generates only one result: an approximation to a uniform histogram.
- Sometimes though, we need to be able to specify particular histogram shapes capable of highlighting certain gray-level ranges.

Histogram Specification

- The procedure for histogram-specification based enhancement is:
  - Equalize the levels of the original image using:

$$s = T(r_k) = \sum_{j=0}^{k} \frac{n_j}{n}$$

n: total number of pixels,

n_j: number of pixels with gray level r_j,

L: number of discrete gray levels

Histogram Specification

– Specify the desired density function and obtain the transformation function G(z):

$$v = G(z) = \sum_{0}^{z} p_z(w) \approx \sum_{i=0}^{z} \frac{n_i}{n}$$

p_z: specified desirable PDF for output

– Apply the inverse transformation function $z=G^{-1}(s)$ to the levels obtained in step 1.

Histogram Specification

- The new, processed version of the original image consists of gray levels characterized by the specified density $p_z(z)$.

In essence: $z = G^{-1}(s) \rightarrow \quad z = G^{-1}[T(r)]$

Histogram Specification

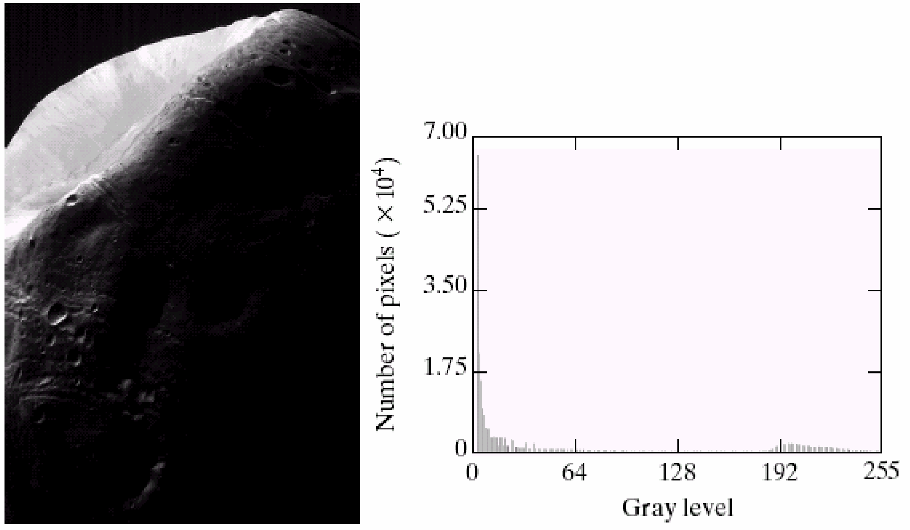
- The principal difficulty in applying the histogram specification method to image enhancement lies in being able to construct a meaningful histogram. So…

Histogram Specification

- Either a particular probability density function (such as a Gaussian density) is specified and then a histogram is formed by digitizing the given function,

- Or a histogram shape is specified on a graphic device and then is fed into the processor executing the histogram specification algorithm.

Image Enhancement in the Spatial Domain



a b

FIGURE 3.20 (a) Image of the Mars moon Photos taken by NASA's *Mars Global Surveyor*. (b) Histogram. (Original image courtesy of NASA.)

Chapter 3
Image Enhancement in the Spatial Domain

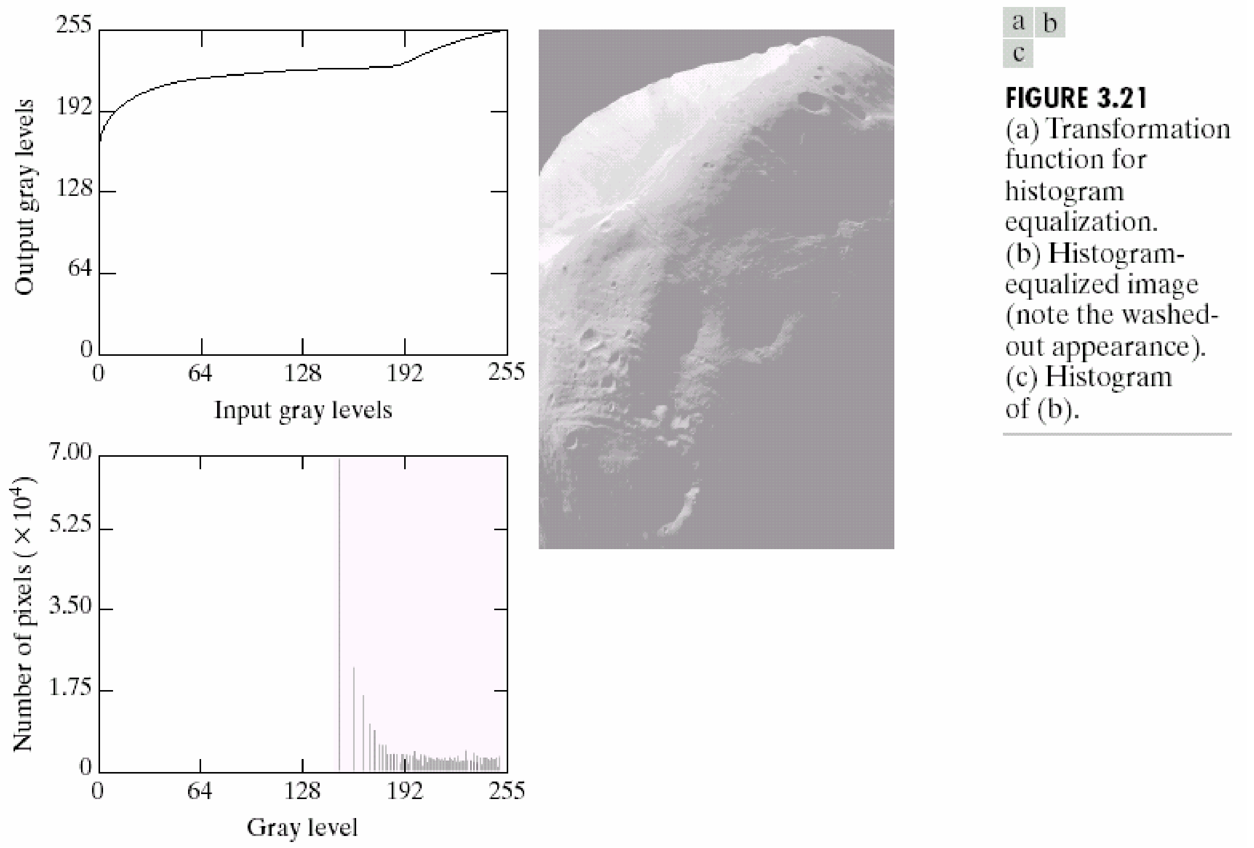
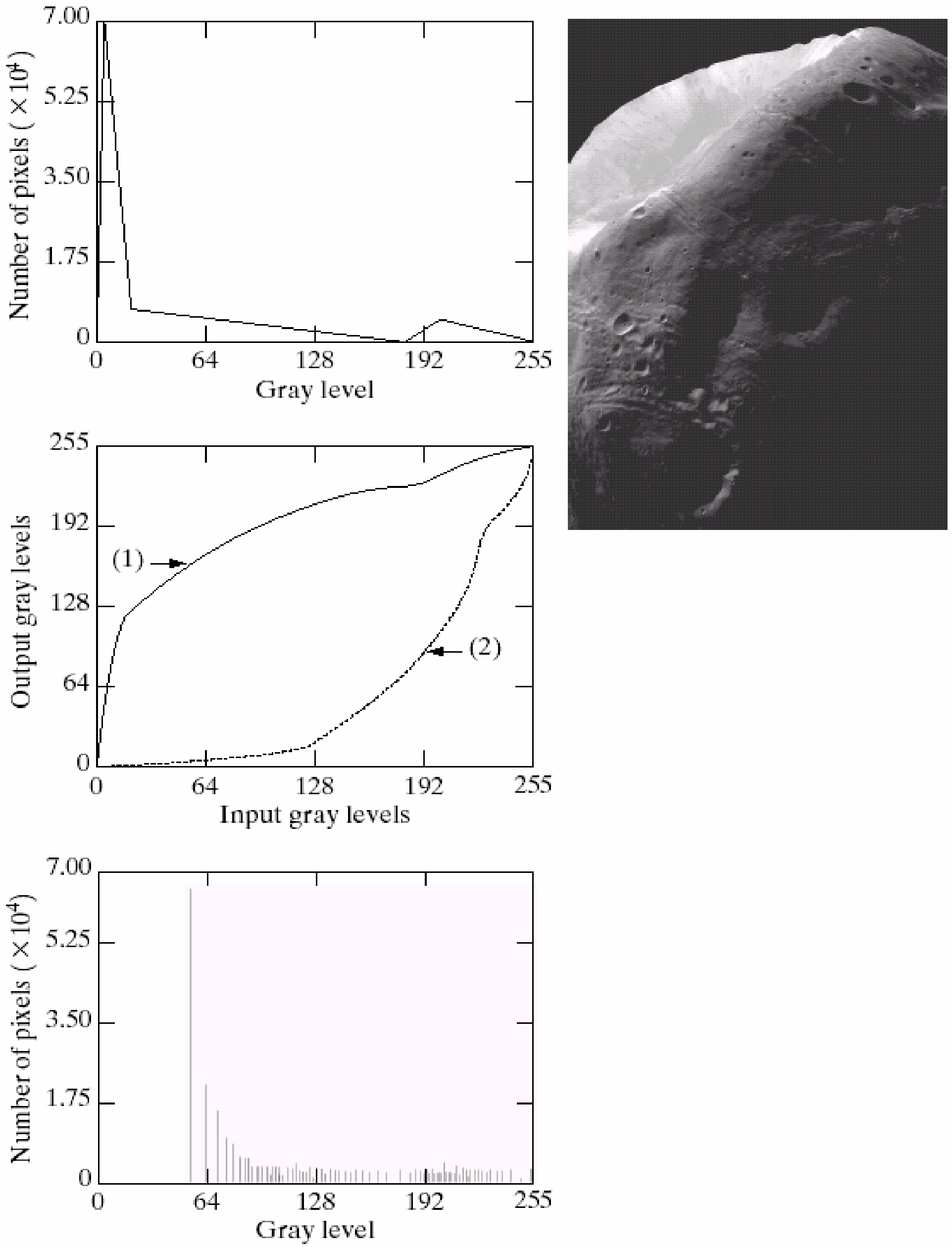


FIGURE 3.21 (a) Transformation function for histogram equalization. (b) Histogram-equalized image (note the washed-out appearance). (c) Histogram of (b).



a c
b
d

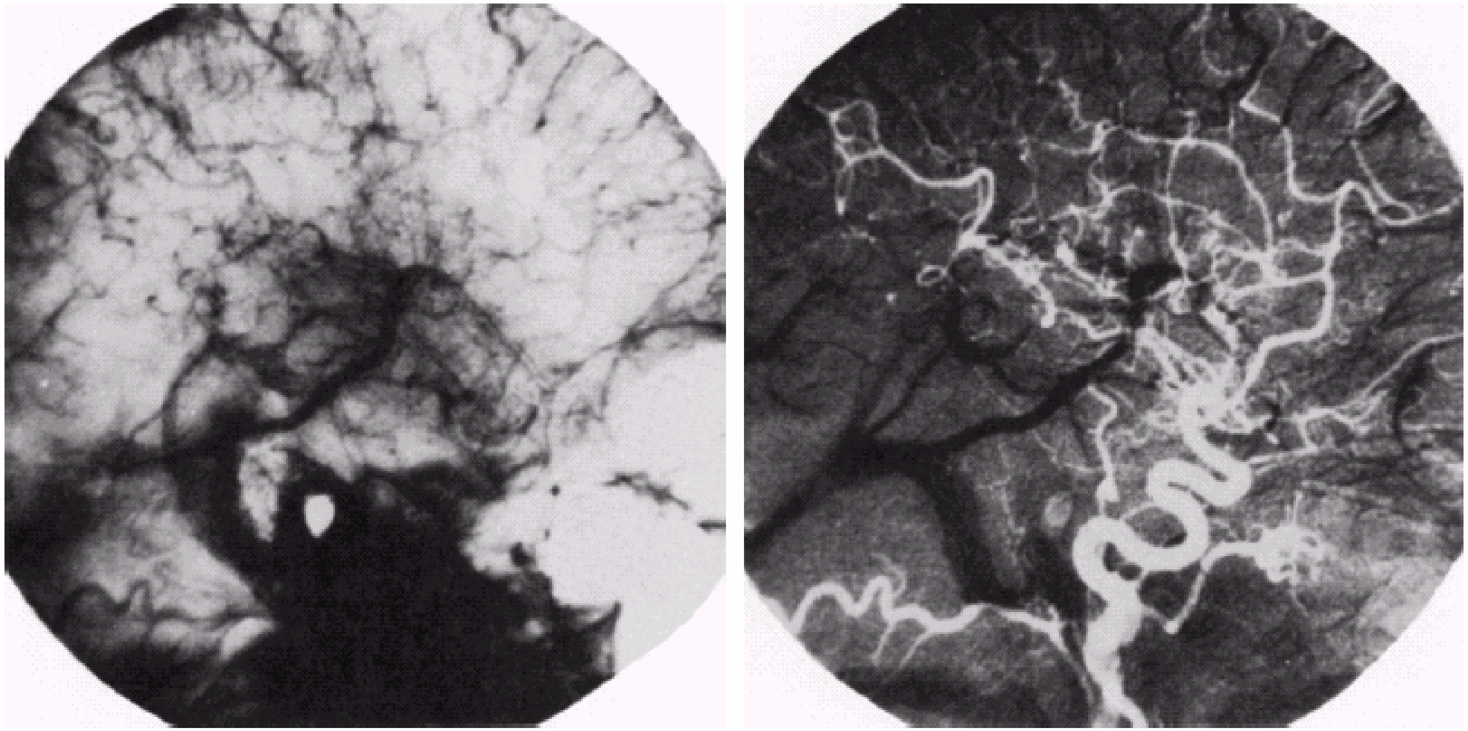
FIGURE 3.22
(a) Specified histogram.
(b) Curve (1) is from Eq. (3.3-14), using the histogram in (a); curve (2) was obtained using the iterative procedure in Eq. (3.3-17).
(c) Enhanced image using mappings from curve (2).
(d) Histogram of (c).

Image Subtraction

$$g(x, y) = f(x, y) - h(x, y)$$

- Mask mode radiography
  - h(x,y) is the mask

Image Enhancement in the Spatial Domain



a b

FIGURE 3.29 Enhancement by image subtraction. (a) Mask image. (b) An image (taken after injection of a contrast medium into the bloodstream) with mask subtracted out.

Image Averaging

- A noisy image:

$$g(x, y) = f(x, y) + n(x, y)$$

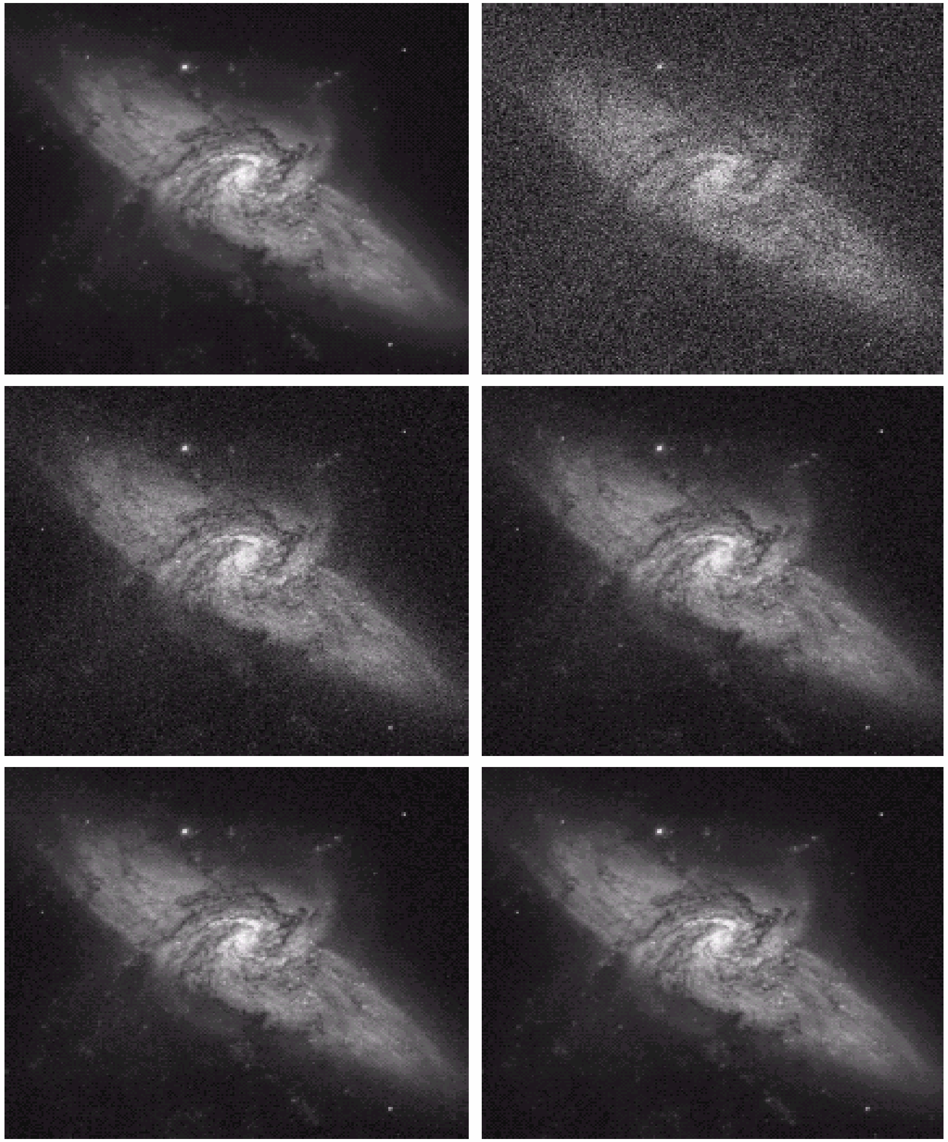
- Averaging M different noisy images:

$$\overline{g}(x, y) = \frac{1}{M}\sum_{i=1}^{M} g_i(x, y)$$

Image Averaging

- As M increases, the variability of the pixel values at each location decreases.
  - This means that $\bar{g}(x,y)$ approaches f(x,y) as the number of noisy images used in the averaging process increases.
- Registering of the images is necessary to avoid blurring in the output image.

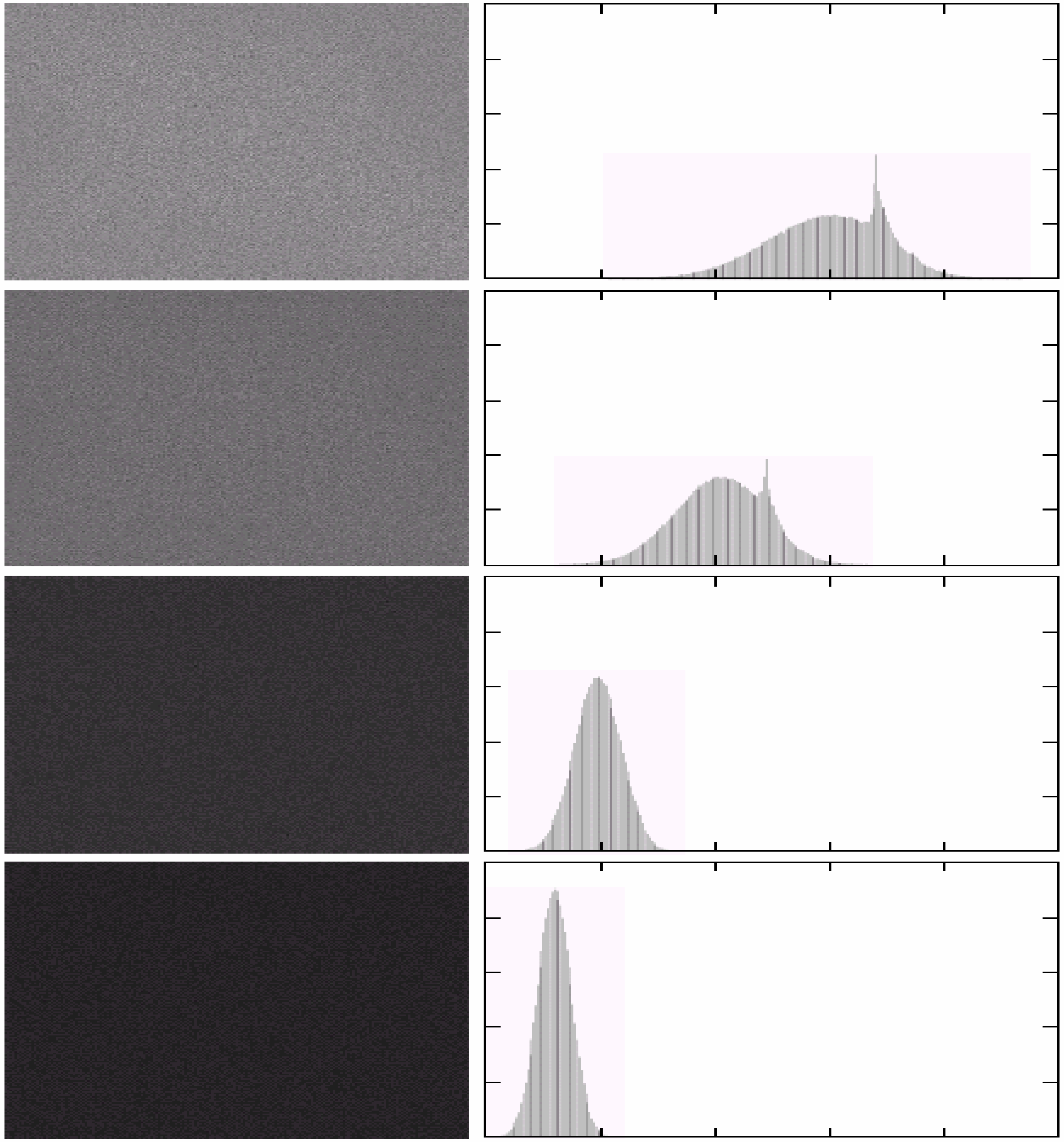
Image Enhancement in the Spatial Domain



a b
c d
e f

FIGURE 3.30 (a) Image of Galaxy Pair NGC 3314. (b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels. (c)–(f) Results of averaging $K = 8, 16, 64,$ and 128 noisy images. (Original image courtesy of NASA.)

Image Enhancement in the Spatial Domain



a b

FIGURE 3.31
(a) From top to bottom: Difference images between Fig. 3.30(a) and the four images in Figs. 3.30(c) through (f), respectively.
(b) Corresponding histograms.

Local Enhancement

- When it is necessary to enhance details over smaller areas

- To devise transformation functions based on the gray-level distribution in the neighborhood of every pixel

Local Enhancement

- The procedure is:
  - Define a square (or rectangular) neighborhood and move the center of this area from pixel to pixel.
  - At each location, the histogram of the points in the neighborhood is computed and either a histogram equalization or histogram specification transformation function is obtained.

Local Enhancement

- More procedure:
  - This function is finally used to map the gray level of the pixel centered in the neighborhood.
  - The center is then moved to an adjacent pixel location and the procedure is repeated.

Spatial Filtering

- Use of spatial masks for image processing (spatial filters)
- Linear and nonlinear filters
- Low-pass filters eliminate or attenuate high frequency components in the frequency domain (sharp image details), and result in image blurring.

Spatial Filtering

- High-pass filters attenuate or eliminate low-frequency components (resulting in sharpening edges and other sharp details).

- Band-pass filters remove selected frequency regions between low and high frequencies (for image restoration, not enhancement).

Spatial Filtering

$$g(x,y)=\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)\,f(x+s,y+t)$$

a=(m-1)/2 and *b=(n-1)/2*,
m x *n* (odd numbers)

- For *x*=0,1,…,M-1 and *y*=0,1,…,N-1
- Also called *convolution* (primarily in the frequency domain)

Spatial Filtering

- The basic approach is to sum products between the mask coefficients and the intensities of the pixels under the mask at a specific location in the image:

$$R = w_1 z_1 + w_2 z_2 + \ldots + w_9 z_9$$

(for a 3 x 3 filter)

Image Enhancement in the Spatial Domain

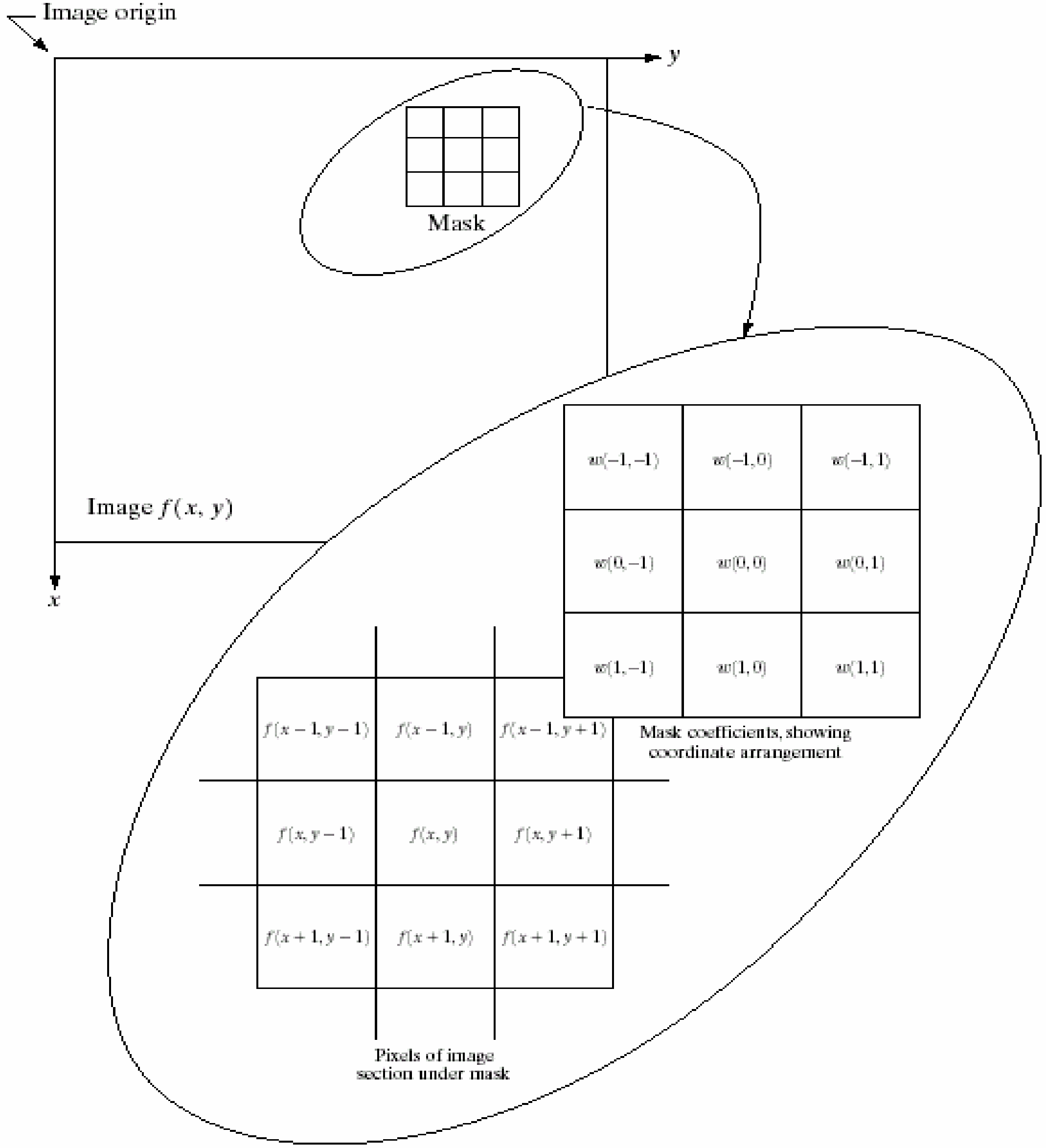


FIGURE 3.32 The mechanics of spatial filtering. The magnified drawing shows a 3×3 mask and the image section directly under it; the image section is shown displaced out from under the mask for ease of readability.

Spatial Filtering

- Non-linear filters also use pixel neighborhoods but do not explicitly use coefficients
  - e.g. noise reduction by median gray-level value computation in the neighborhood of the filter

Smoothing Filters

- Used for blurring (removal of small details prior to large object extraction, bridging small gaps in lines) and noise reduction.

- Low-pass (smoothing) spatial filtering
  – Neighborhood averaging

  \- Results in image blurring

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

Image Enhancement in the Spatial Domain

FIGURE 3.33 Another representation of a general 3 × 3 spatial filter mask.

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

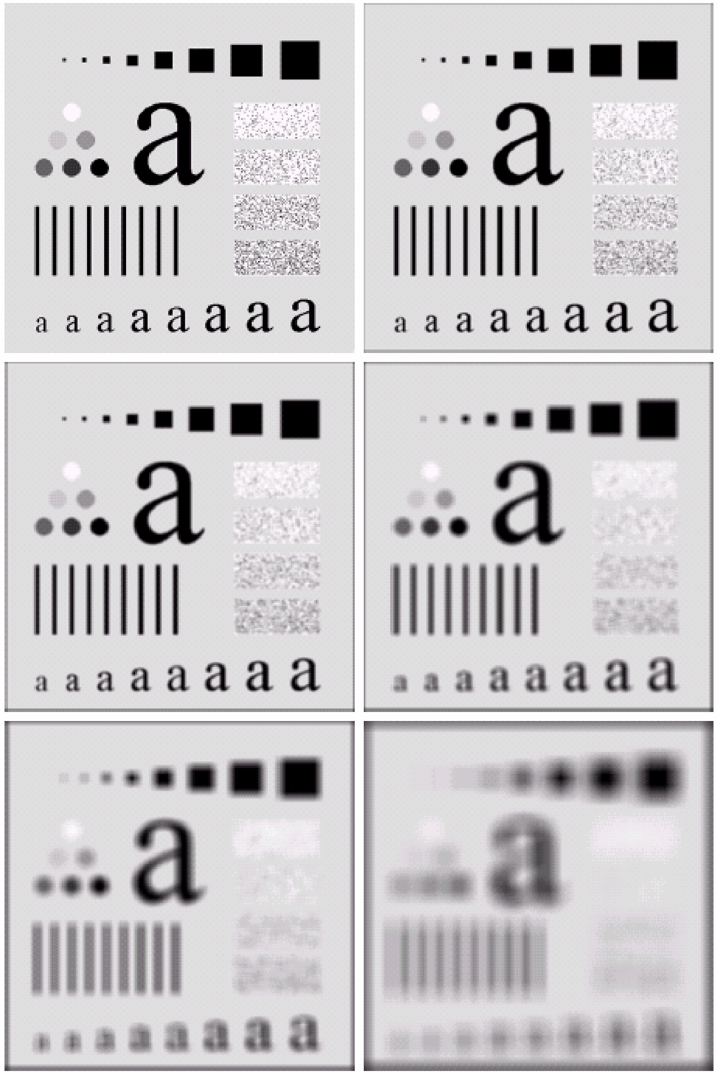
Image Enhancement in the Spatial Domain

$$\frac{1}{9}\times\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline\end{array}\qquad\frac{1}{16}\times\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline\end{array}$$

a b

FIGURE 3.34 Two 3×3 smoothing (averaging) filter masks. The constant multipli er in front of each mask is equal to the sum of the values of its coefficients, as is required to compute an average.

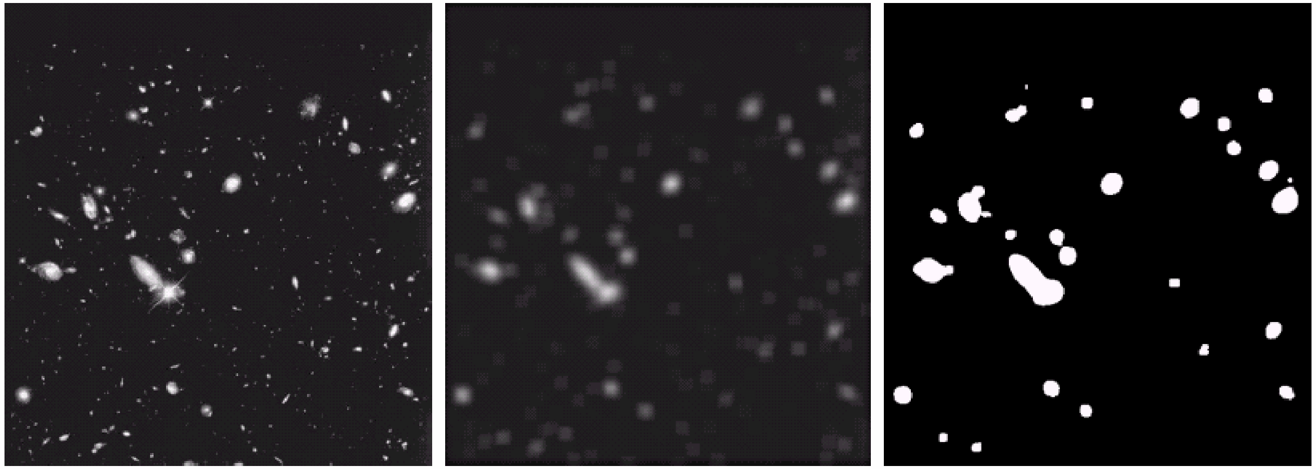
Image Enhancement in the Spatial Domain



a b
c d
e f

FIGURE 3.35 (a) Original image, of size 500×500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $n = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50×120 pixels.

Image Enhancement in the Spatial Domain



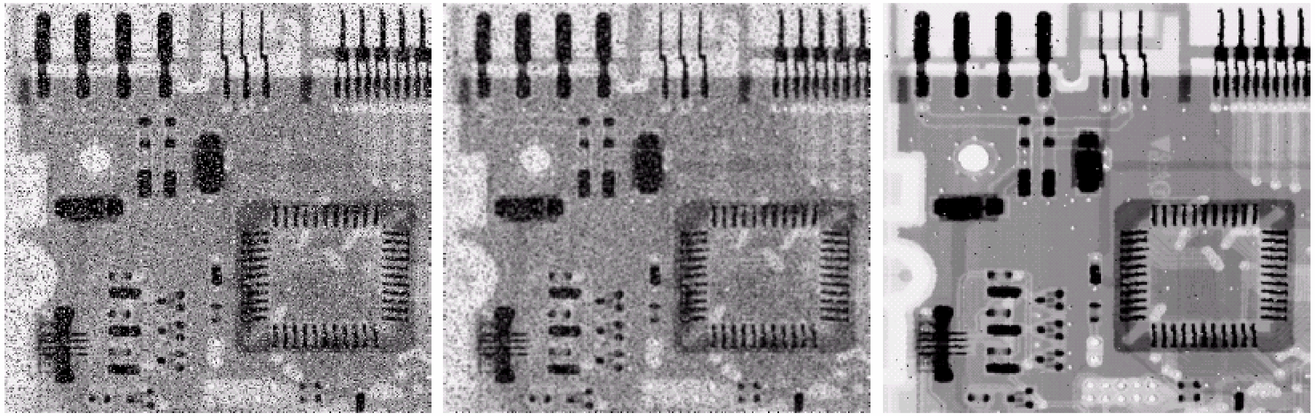
a b c

FIGURE 3.36 (a) Image from the Hubble Space Telescope. (b) Image processed by a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

Smoothing Filters

- Median filtering (nonlinear)
  - Used primarily for noise reduction (eliminates isolated spikes)
  - The gray level of each pixel is replaced by the median of the gray levels in the neighborhood of that pixel (instead of by the average as before).

Chapter 3
Image Enhancement in the Spatial Domain



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Sharpening Filters

- To highlight fine detail or to enhance blurred detail.
  - smoothing ~ integration
  - sharpening ~ differentiation

- Categories of sharpening filters:

  - Derivative operators
  - Basic highpass spatial filtering
  - High-boost filtering

Derivative Filters

- Averaging is analogous to integration and causes blurring, so differentiation is expected to have opposite results and sharpen an image.

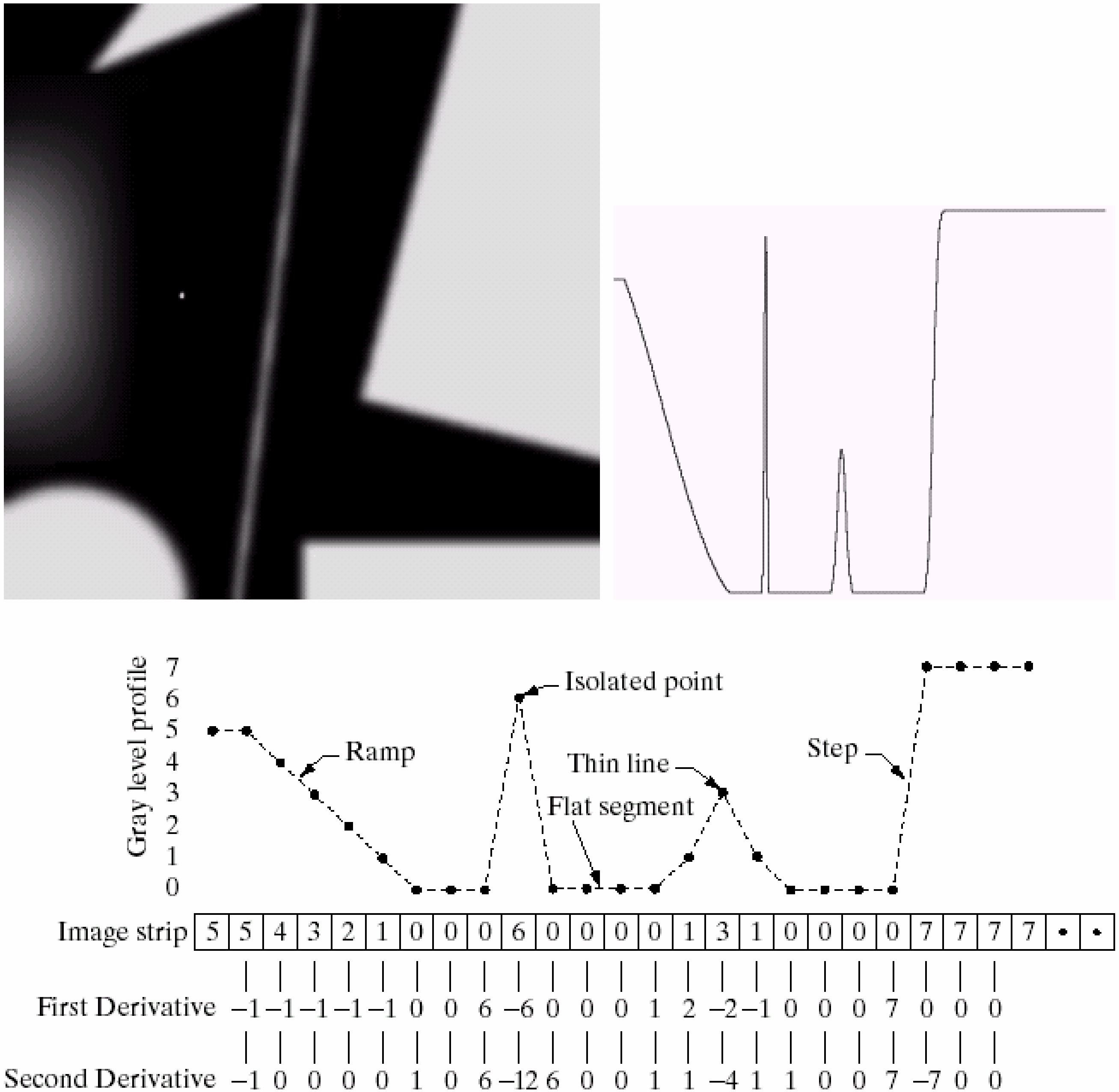
Derivatives

- First derivative

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

- Second derivative

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$



a b
c

FIGURE 3.38
(a) A simple image. (b) 1-D horizontal gray-level profile along the center of the image and including the isolated noise point.
(c) Simplified profile (the points are joined by dashed lines to simplify interpretation).

Digital Function Derivatives

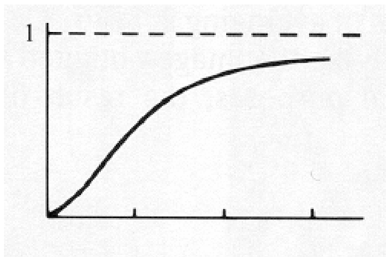
- First derivative:
  - 0 in constant gray segments
  - Non-zero at the onset of steps or ramps
  - Non-zero along ramps
- Second derivative:
  - 0 in constant gray segments
  - Non-zero at the onset and end of steps or ramps
  - 0 along ramps of constant slope.

Observations

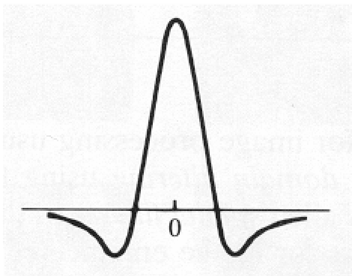
- 1st order derivatives produce thicker edges in an image
- 2nd order derivatives have stronger response to fine detail
- 1st order derivatives have stronger response to a gray lever step
- 2nd order derivatives produce a doble response at step cahnges in gray level
- 2nd order derivativeshave stronger response to a line than to a step and to a point than to a line

Basic Highpass Spatial Filtering

- Cross section of frequency domain filter:



- Cross section of spatial domain filter:



Basic Highpass Spatial Filtering

- The filter should have positive coefficients near the center and negative in the outer periphery:

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline -1 & -1 & -1 \\ \hline -1 & 8 & -1 \\ \hline -1 & -1 & -1 \\ \hline \end{array}$$

Basic Highpass Spatial Filtering

- The sum of the coefficients is 0, indicating that when the filter is passing over regions of almost stable gray levels, the output of the mask is 0 or very small.
- Some scaling and/or clipping is involved (to compensate for possible negative gray levels after filtering).

2-D, 2nd Order Derivatives for Image Enhancement

- Isotropic filters: rotation invariant
- Laplacian (linear operator):

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

- Discrete version:

$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

Laplacian

- Digital implementation:

$$\nabla^2 f = [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)] - 4f(x,y)$$

- Two definitions of Laplacian: one is the negative of the other
- Accordingly, to recover background features:

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & (I) \\ f(x,y) + \nabla^2 f(x,y) & (II) \end{cases}$$

I: if the center of the mask is negative

II: if the center of the mask is positive

Simplification

- Filter and recover original part in one step:

$$g(x,y)=f(x,y)-[f(x+1,y)+f(x-1,y)+f(x,y+1)+f(x,y-1)]+4f(x,y)$$

$$g(x,y)=5f(x,y)-[f(x+1,y)+f(x-1,y)+f(x,y+1)+f(x,y-1)]$$

Image Enhancement in the Spatial Domain

(a)

0	1	0
1	−4	1
0	1	0

(b)

1	1	1
1	−8	1
1	1	1

(c)

0	−1	0
−1	4	−1
0	−1	0

(d)

−1	−1	−1
−1	8	−1
−1	−1	−1

a	b
c	d

FIGURE 3.39 (a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4). (b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

a b
c d

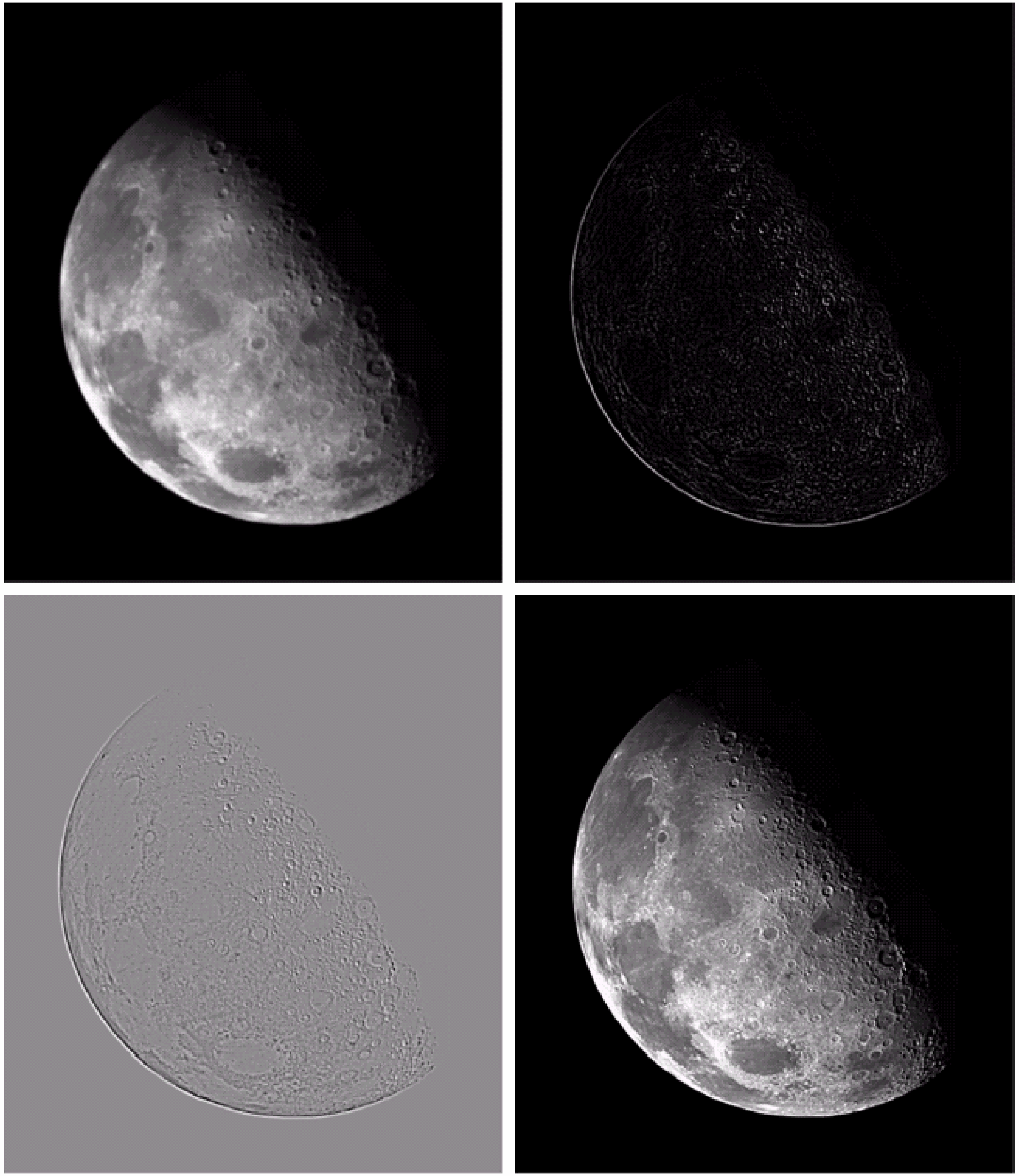
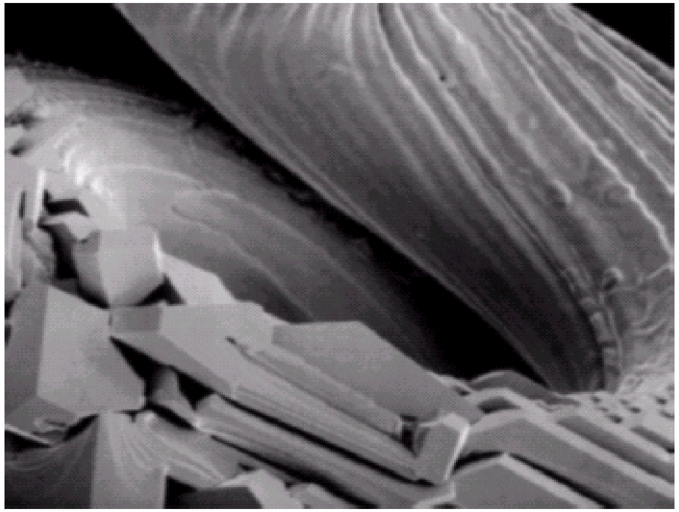


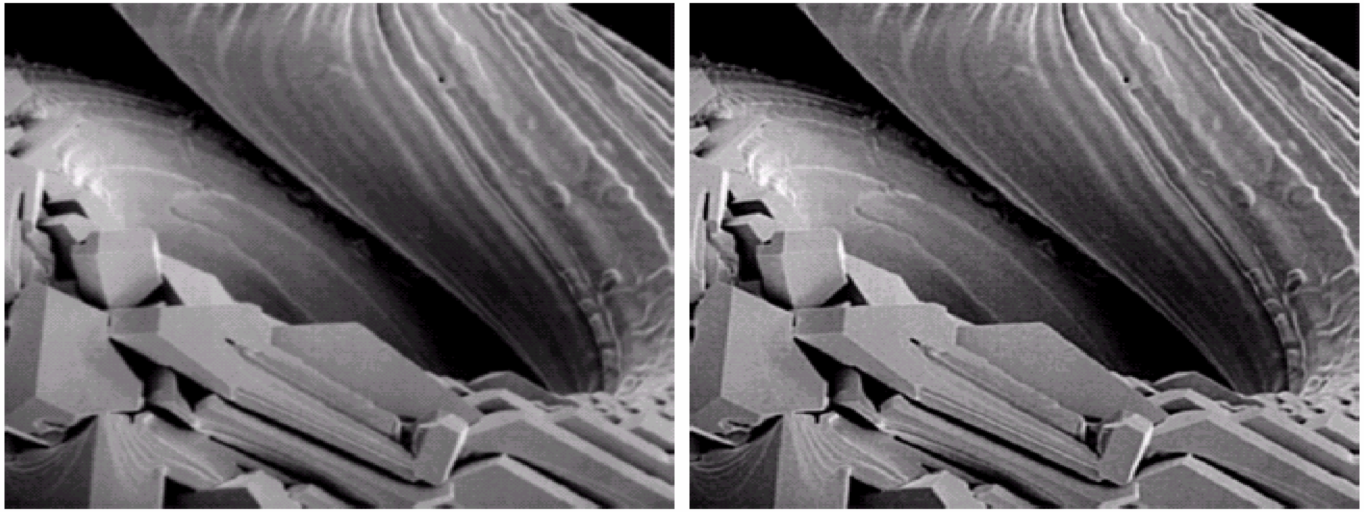
FIGURE 3.40 (a) Image of the North Pole of the moon. (b) Laplacian-filtered image. (c) Laplacian image scaled for display purposes. (d) Image enhanced by using Eq. (3.7-5). (Original image courtesy of NASA.)

Image Enhancement in the Spatial Domain

0	−1	0
−1	5	−1
0	−1	0

−1	−1	−1
−1	9	−1
−1	−1	−1





a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

High-boost Filtering

- Unsharp masking: $f_s(x,y) = f(x,y) - \overline{f}(x,y)$
- Highpass filtered image =

  Original – lowpass filtered image.

- If A is an amplification factor then:
  - High-boost = A · original – lowpass (blurred)

    = (A-1) · original + original – lowpass

    = (A-1) · original + highpass

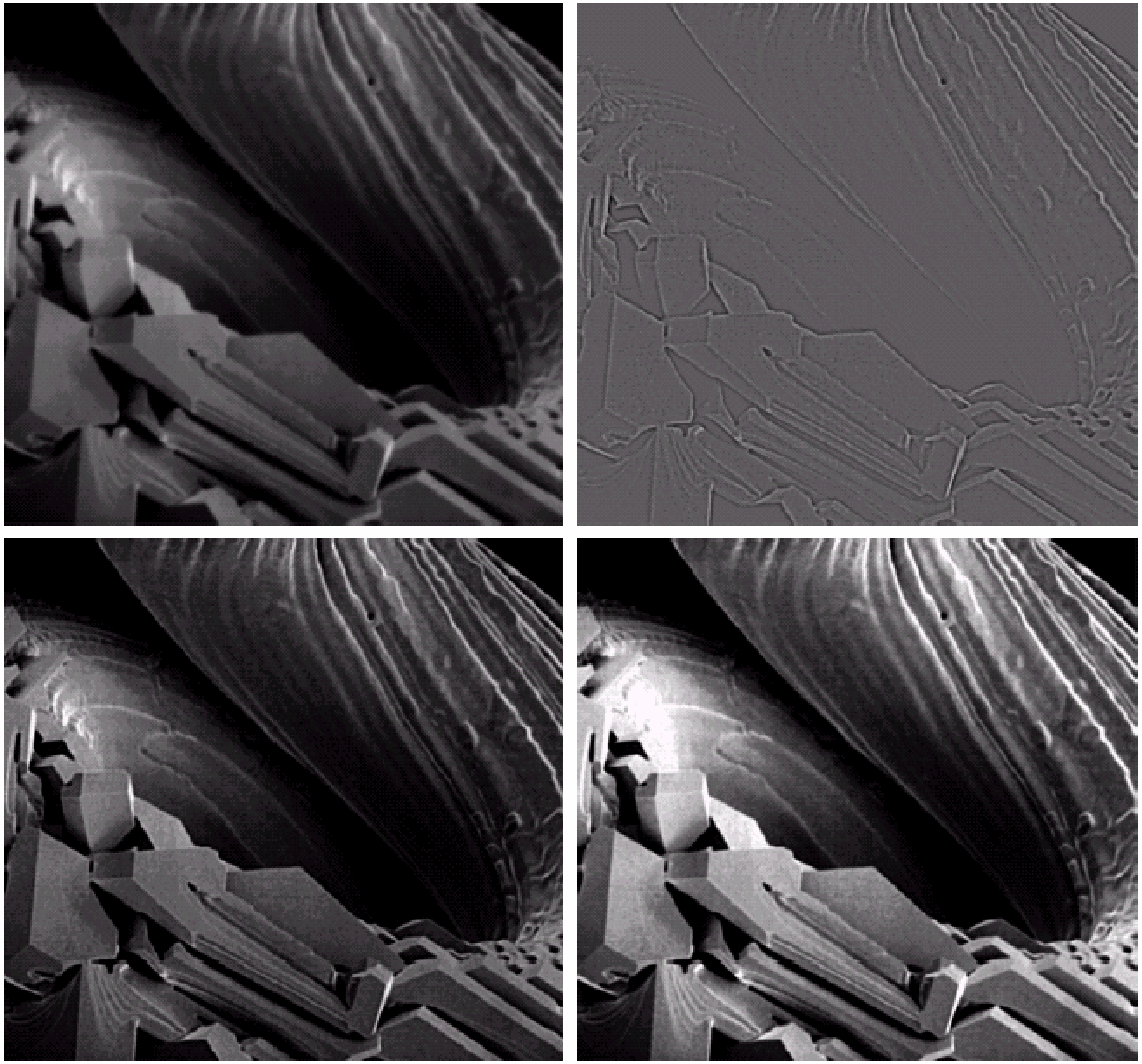
High-boost Filtering

- A=1 : standard highpass result
- A>1 : the high-boost image looks more like the original with a degree of edge enhancement, depending on the value of A.

$$\frac{1}{9} \times \begin{array}{|c|c|c|}\hline -1 & -1 & -1 \\ \hline -1 & w & -1 \\ \hline -1 & -1 & -1 \\ \hline \end{array}$$

$w = 9A - 1$, $A \geq 1$

Image Enhancement in the Spatial Domain



a b
c d

FIGURE 3.43 (a) Same as Fig. 3.41(c), but darker. (a) Laplacian of (a) computed with the mask in Fig. 3.42(b) using $A = 0$. (c) Laplacian enhanced image using the mask in Fig. 3.42(b) with $A = 1$. (d) Same as (c), but using $A = 1.7$.

1st Derivatives

- The most common method of differentiation in Image Processing is the *gradient*:

$$\nabla F = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix} \quad \text{at } (x,y)$$

- The magnitude of this vector is:

$$\nabla f = mag(\nabla f) = [G_x^2 + G_y^2]^{\frac{1}{2}} = \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

The Gradient

- Non-isotropic
- Its magnitude (often call the gradient) is rotation invariant
- Computations:

$$\nabla f \approx |G_x| + |G_y|$$

- Roberts uses:

$$G_x = (z_9 - z_5)$$

$$G_y = (z_8 - z_6)$$

- Approximation (Roberts Cross-Gradient Operators):

$$\nabla f \approx |z_9 - z_5| + |z_8 - z_6|$$

a
b c
d e

FIGURE 3.44 A 3×3 region of an image (the z's are gray-level values) and masks used to compute the gradient at point labeled z_5. All masks coefficients sum to zero, as expected of a derivative operator.

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Derivative Filters

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

At z_5, the magnitude can be approximated as:

$$\nabla f \approx [(z_5 - z_8)^2 + (z_5 - z_6)^2]^{1/2}$$

$$\nabla f \approx |z_5 - z_8| + |z_5 - z_6|$$

Derivative Filters

- Another approach is:

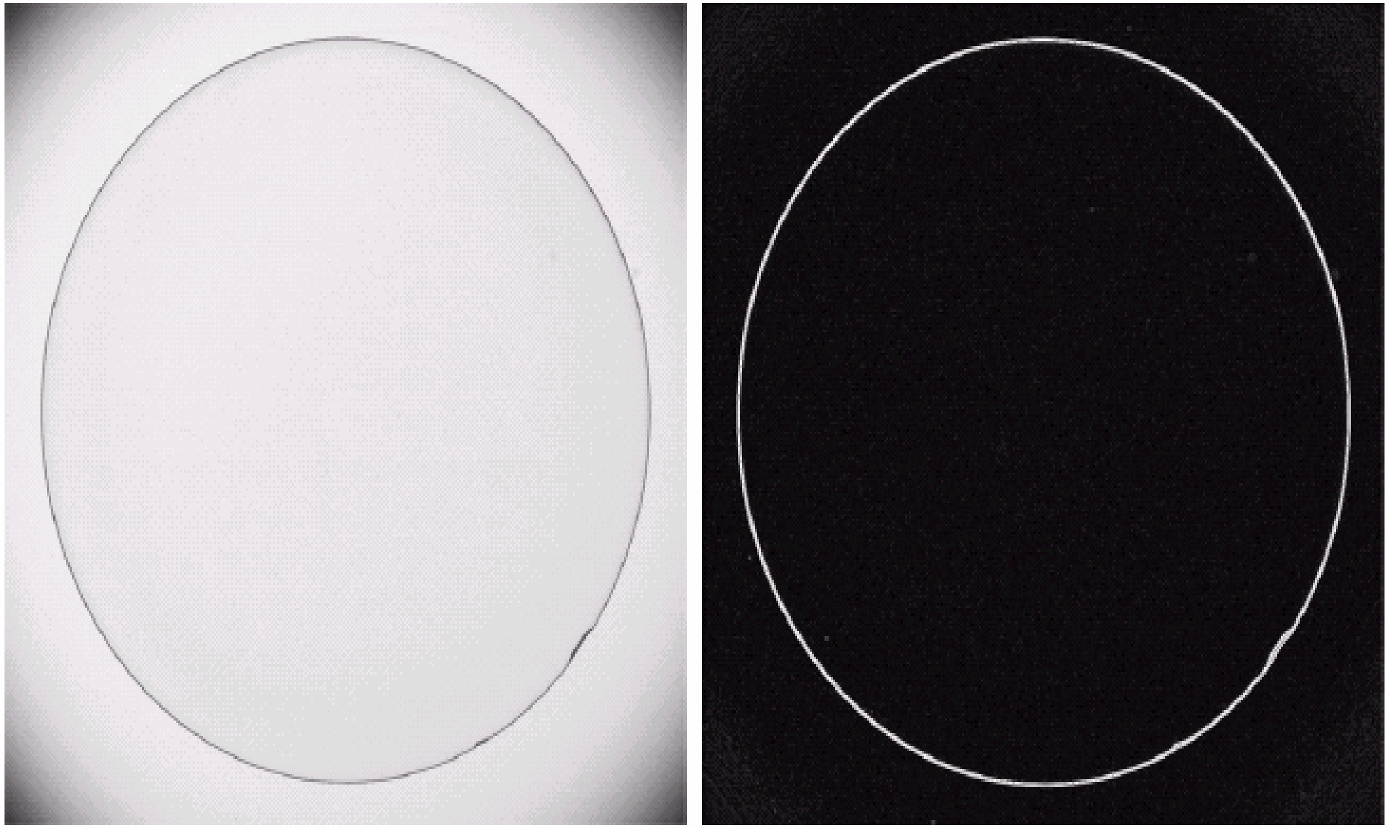
$$\nabla f \approx [(z_5 - z_9)^2 + (z_6 - z_8)^2]^{1/2}$$

$$\nabla f \approx | z_5 - z_9 | + | z_6 - z_8 |$$

- One last approach is (Sobel Operators):

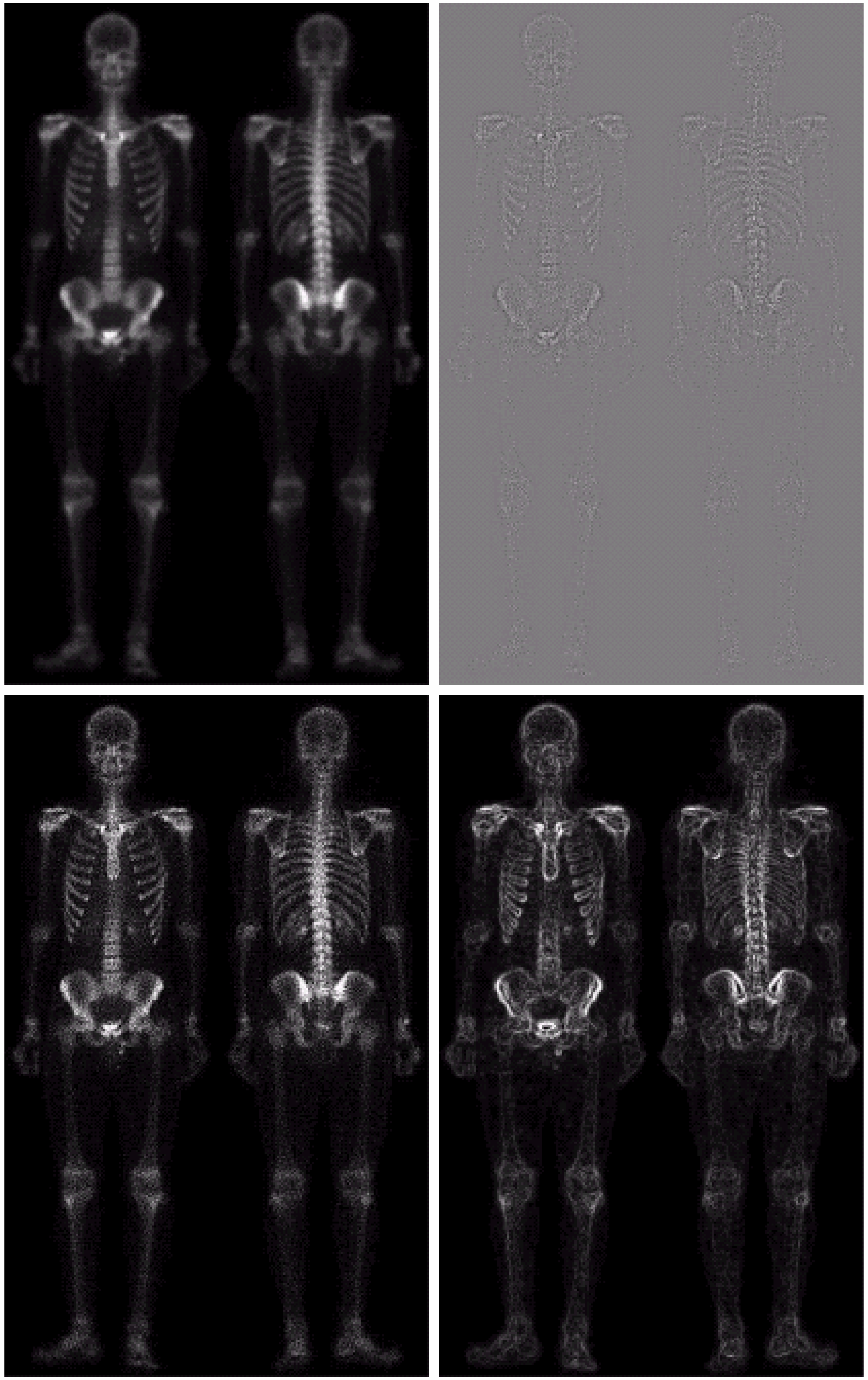
$$\nabla f = \left|(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)\right| + \left|(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)\right|$$

Image Enhancement in the Spatial Domain



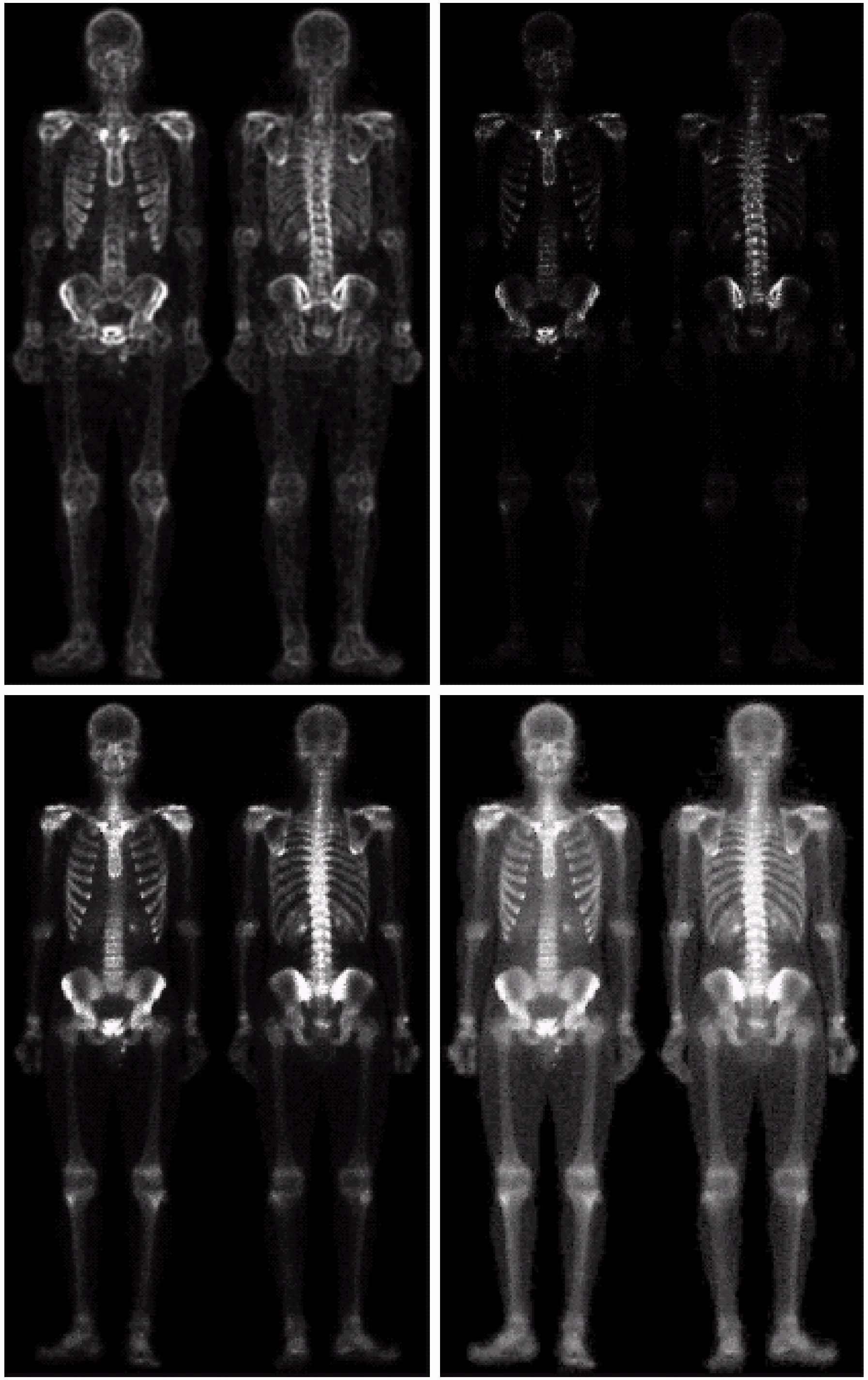
a b

FIGURE 3.45 Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)



a	b
c	d

FIGURE 3.46
(a) Image of whole body bone scan.
(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel of (a).



e f
g h

FIGURE 3.46 *(Continued)* (e) Sobel image smoothed with a 5×5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)