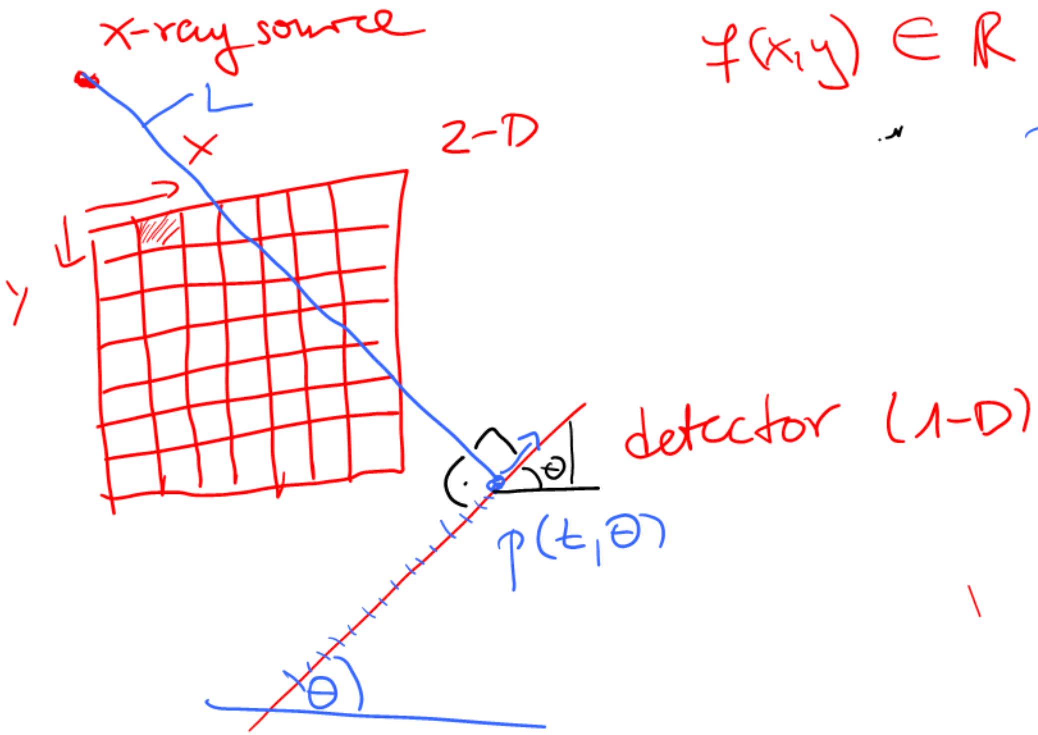
Computed tomography for dummies

$f(x,y) \in \mathbb{R}$



Beer's Law:

$$p(t,\theta) = I = I_0 \, e^{-\int_{(x,y)\in L} f(x,y)\,dx\,dy}$$

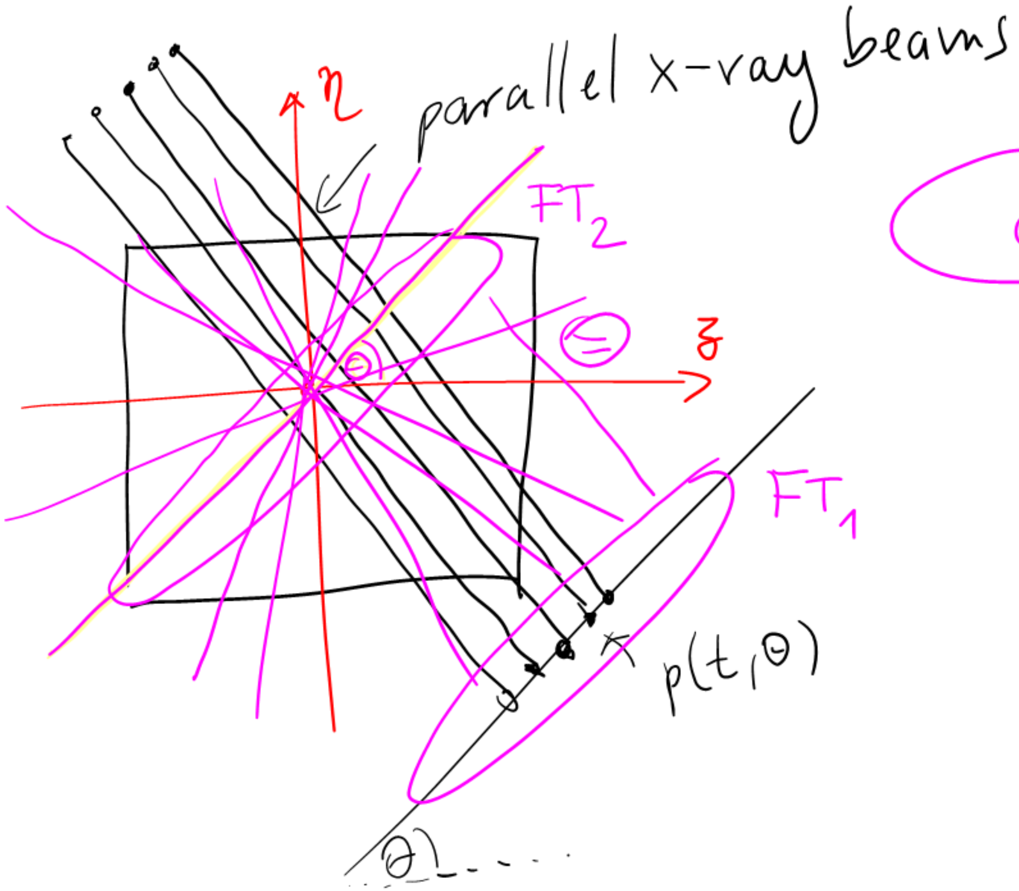
$$-\ln \frac{I}{I_0} = \int_{(x,y)\in L} f(x,y)\,dx\,dy$$

Characterization of the projection line: $\vec{n}^T \vec{x} = d$ where $\vec{n} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$

Using Dirac's delta function: $\delta(\cos\theta \cdot x + \sin\theta \cdot y - d) \cdot f(x,y)$

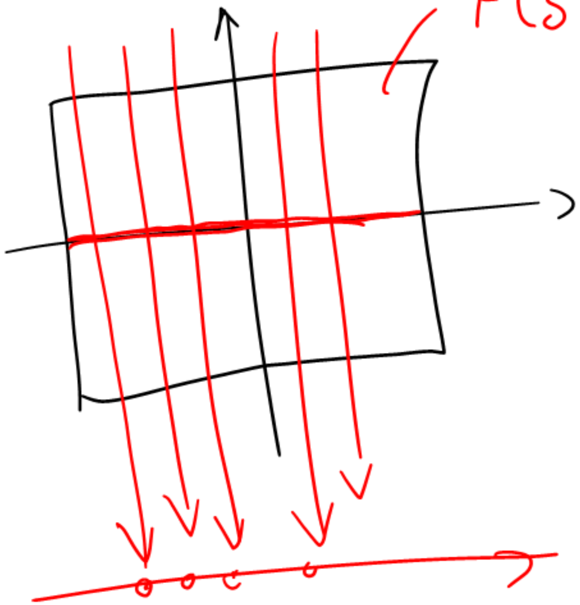
where: $\delta(x) = \begin{cases} 1, & x = 0 \\ 0, & \text{otherwise} \end{cases}$

Fourier Slice Theorem (FST)



COOL

Proof: FST



$$F(\xi,\eta) = c\cdot\iint_{-\infty}^{+\infty} f(x,y)\, e^{-i\cdot 2\pi(\xi x+\underset{\uparrow\atop 0}{\eta} y)}\, dx\, dy$$

$$F(\xi,0) = \int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty} f(x,y)\, e^{-i2\pi\xi x}\, dx\right] dy$$

$$-\log\frac{I}{I_0} = \int_{(x,y)\in L_i} f(x,y)\, dx\, dy = p(t,\theta) = p(t,\theta\equiv 0)$$

$$FT_\eta\big(p(t,0)\big) = \int_{-\infty}^{+\infty} p(t,0)\cdot e^{-i2\pi t\xi}\, dt$$

$$= \int_{-\infty}^{+\infty}\left[\int_{-\infty\atop (x,y)\in L_i}^{+\infty} f(x,y)\, dx\, dy\right]\cdot e^{-i2\pi t\xi}\, dt$$