Course: Applied Mathematics (D. Sidibé)

Mid-Term Exam: 1h

You must show all work and all reasoning - Full credit will be given only for clearly explained results!

■ PROBLEM 1 (50 Points)

Let $A = \begin{bmatrix} 0 & -1 \\ 4 & 0 \end{bmatrix}$.

1. Find the eigenvalues and eigenvectors of A.
2. Find the SVD decomposition of A, i.e. $A = U\Sigma V^T$.

■ PROBLEM 2 (50 Points)

For each of the following, you have to either

- find a matrix (or matrices) satisfying the given description
- or prove that such matrix (matrices) cannot exist.

1. The column space of A contains the vectors $\begin{bmatrix} 4 \\ 3 \\ 7 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 3 \\ 7 \\ 8 \end{bmatrix}$, and the nullspace of A contains the vectors $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 5 \\ -6 \end{bmatrix}$.
2. The rowspace of B contains the vectors $\begin{bmatrix} 1 \\ 2 \\ 4 \\ 6 \end{bmatrix}$, $\begin{bmatrix} -7 \\ 2 \\ 4 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ -2 \\ -3 \\ -4 \end{bmatrix}$ and the nullspace of B contains the vector $\begin{bmatrix} 4 \\ 2 \\ 3 \\ 7 \end{bmatrix}$.
3. C is a 4×2 matrix, D is a 2×4 matrix and the product CD has a one-dimensional nullspace.