A Practitioner`s View on (Normalized) Mutual Information

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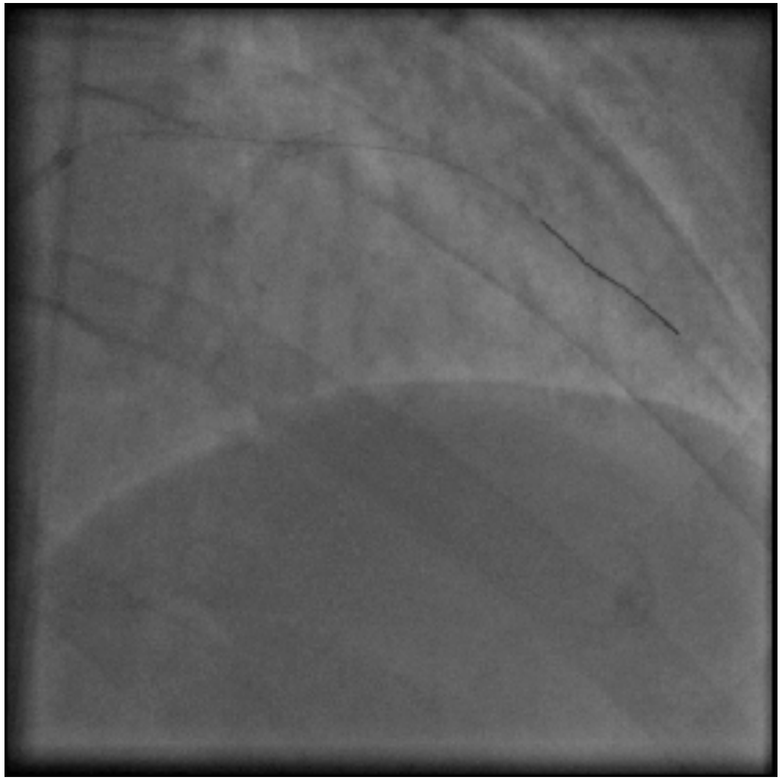
Overview

- Introduction
- Parzen Windowing
- Histogram Binning
- Jittering
- Conclusions

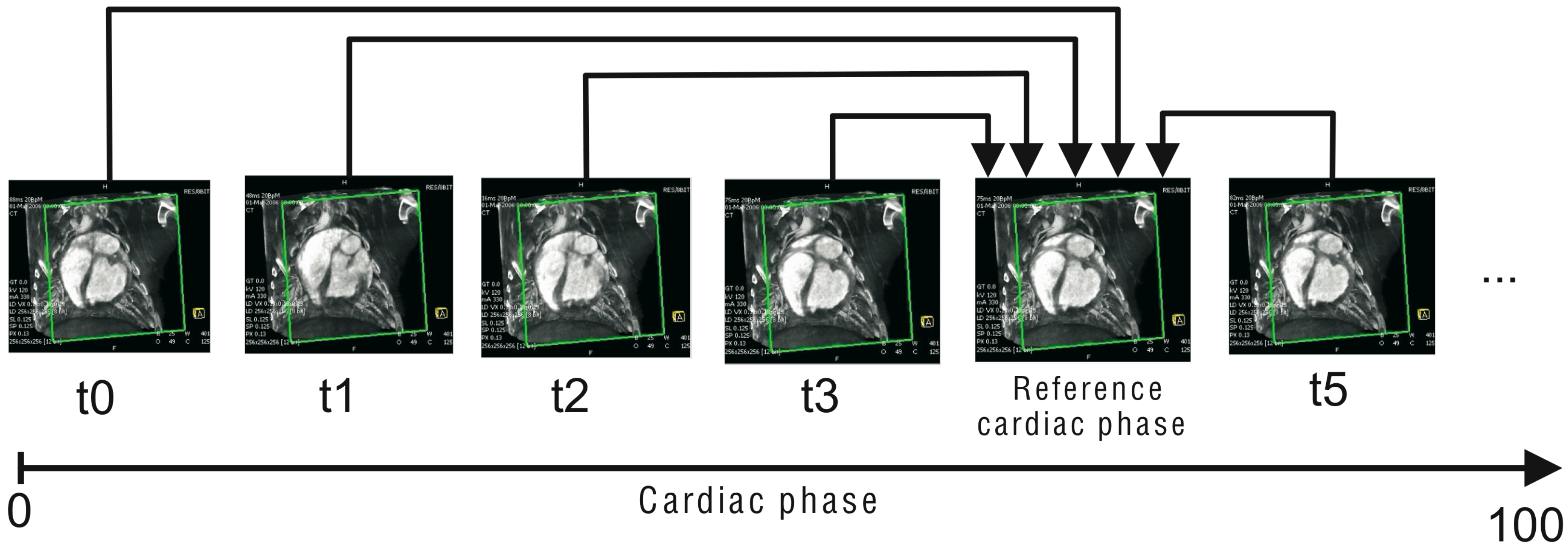
Cardiac Imaging



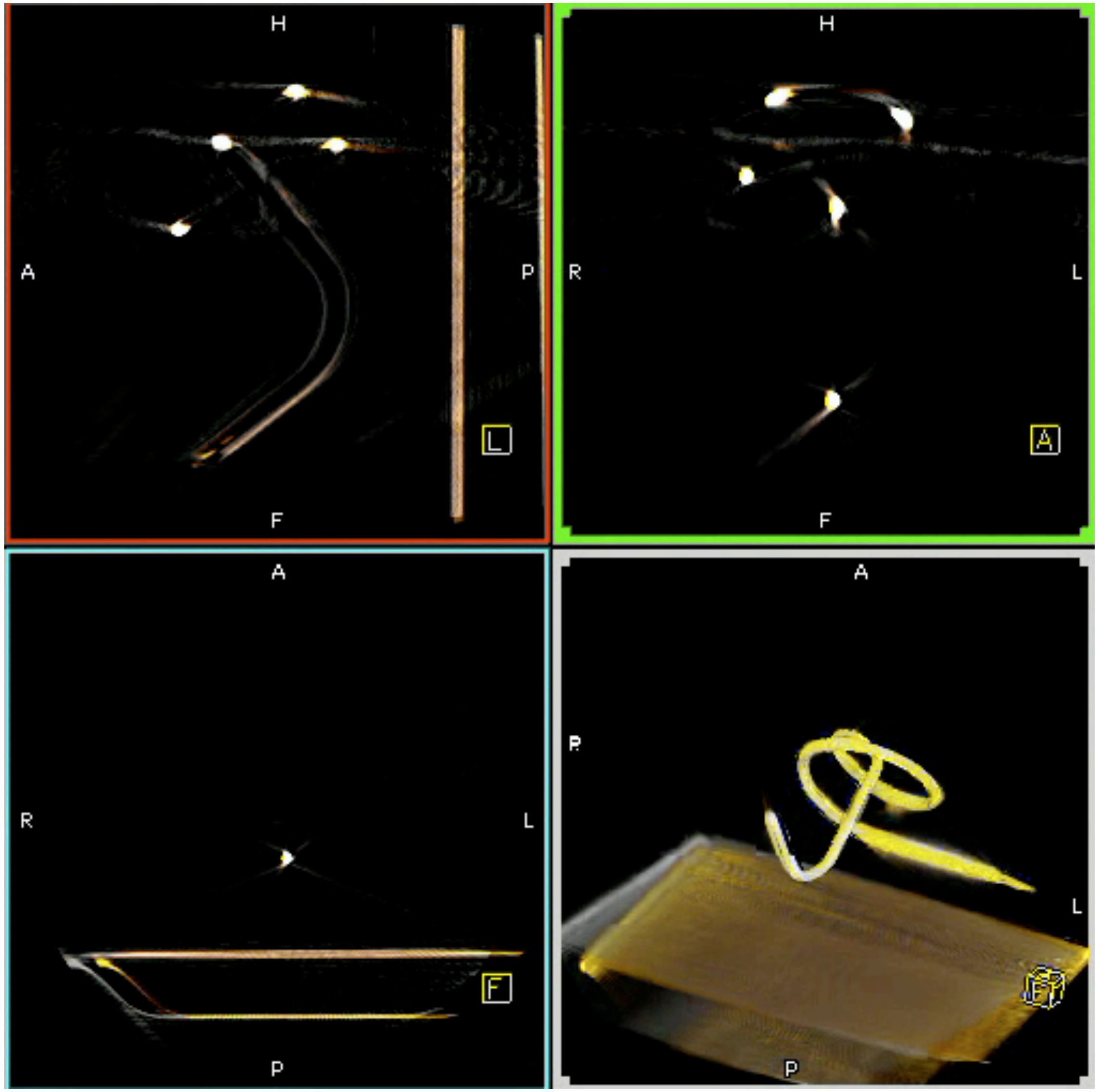
Registration and Reconstruction

- **3-D/3-D registration:**

  Reconstruct certain heart states, register 3-D volumes, interpolate deformation fields



Interventional Imaging



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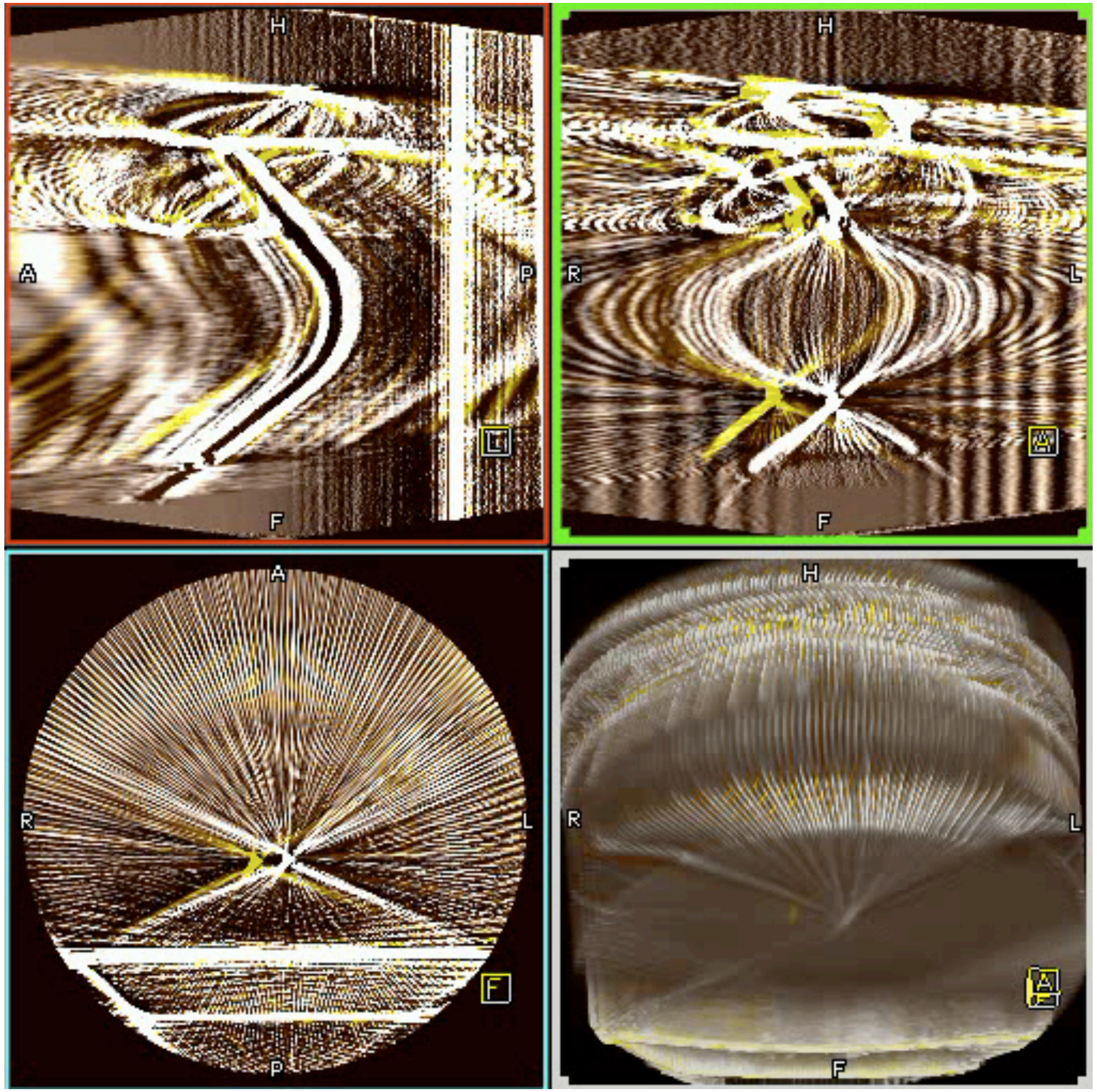
Interventional Imaging



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Goals of our work

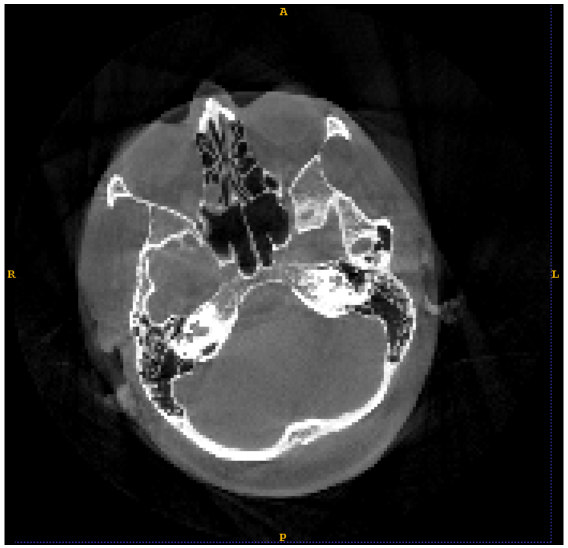
- Multi-modality registration
- Highly efficient registration (< 5 sec., CUDA)
- Parameter free and of clinical use
- Integration in the ITK/VTK framework
- Experimental evaluation using benchmark data sets (RIRE version 2.0: CT-MR, PET-MR, MR-MR)

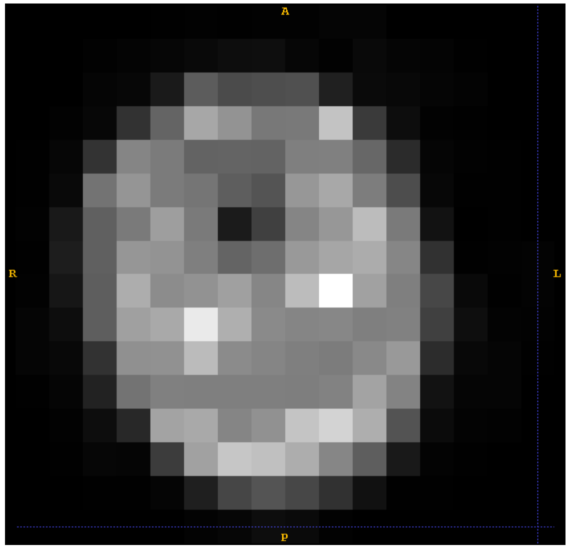
Registration

- Why another standard registration implementation?
  - Correct implementation of mathematical formulation
  - Numerically accurate multileveling (no drastical SNR reduction)
  - Statistical similarity measures

- Speed is achieved by subvoxel accurate registration on low resolution images
- High expectations on sublevel registrations

256x256





16x16

(Normalized) Mutual Information

- Universally usable registration has to be multimodal
  - Mutual Information is currently state-of-the-art
- Statistical information degrades with lower resolution
- Numerical problems of MI in low resolution images known from literature

Image Registration

- Normalized Mutual Information (NMI)

$$\mathcal{D}_{\text{NMI}}[R, T_\Phi] = -\frac{\mathcal{H}(R) + \mathcal{H}(T_\Phi)}{\mathcal{H}(R, T_\Phi)}$$

- Question:

  How to always achieve good PDF estimates?

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Parzen-Window Estimation

- **Efficient discretization:**
  - Discretization of pdf using histogram and Parzen estimator
  - Kernel width estimation
  - Number of bins and bin size
  - Apply methods from signal theory

Parzen-Window Estimation

- Discrete Parzen-window estimator:

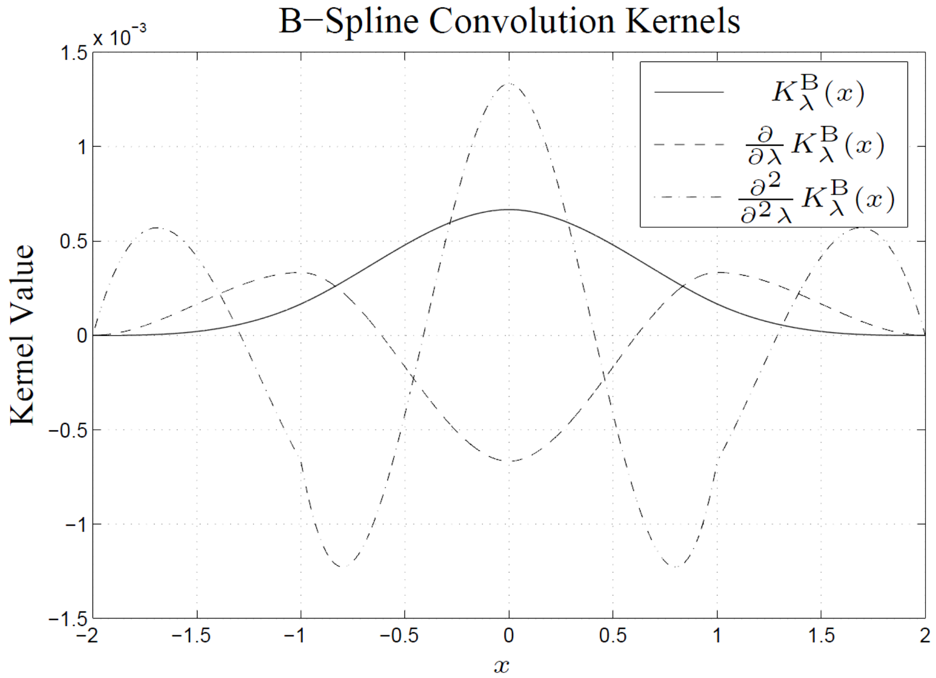
$$
\begin{aligned}
p_{\lambda,n}(x) &= \frac{1}{n}\sum_{i=1}^{n} K_\lambda(x - x_i) \\
\widehat{p}_{\lambda,n}(c_j) &= \sum_{i=1}^{b} h_n(c_i) K_\lambda(c_j - c_i) = (h_n \star K_\lambda)(c_j) \approx p_{\lambda,n}(c_j)
\end{aligned}
$$

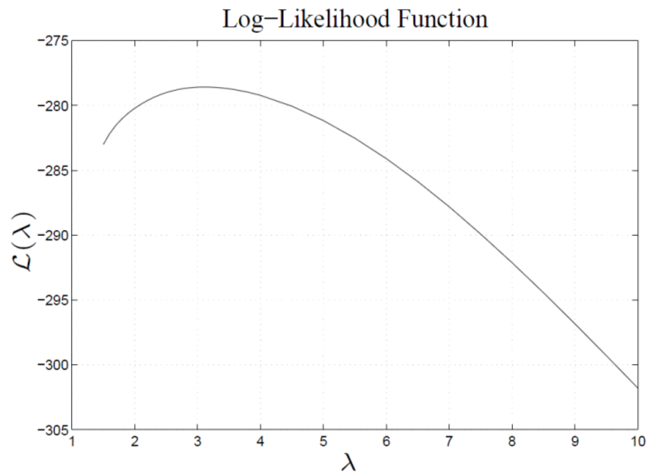
- Kernel width estimation (LOO-CV):

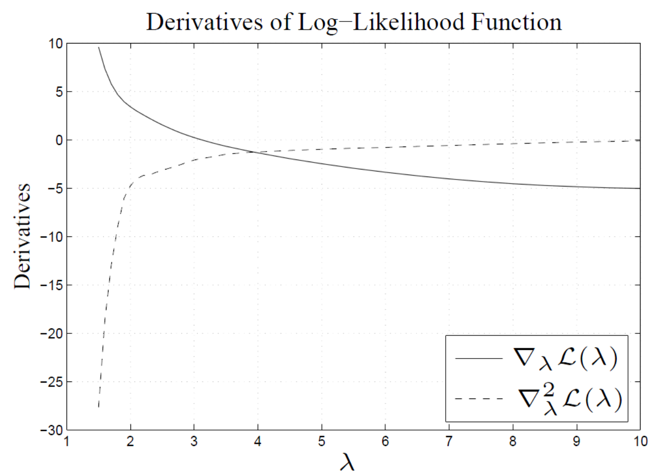
$$
\begin{aligned}
\mathcal{L}(\lambda) &= \sum_{j=1}^{n} \log \widehat{p}^{\,j}_{\lambda,n-1}(x_j) \\
\widehat{\lambda} &= \operatorname*{argmax}_{\lambda} \mathcal{L}(\lambda)
\end{aligned}
$$

Density Estimation

- Optimal kernel width determined by log-likelihood estimation based on a leave-one-out cross-validation
- Numerical optimization







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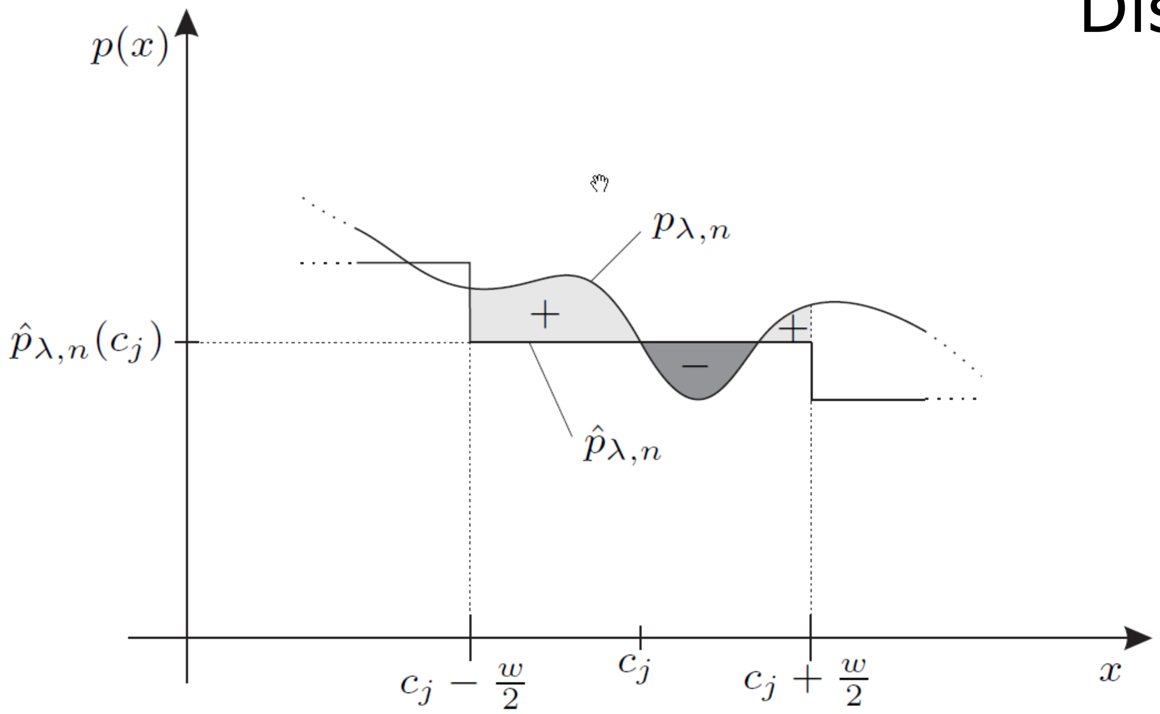
Discrete PDF Representation

- Discretization error:

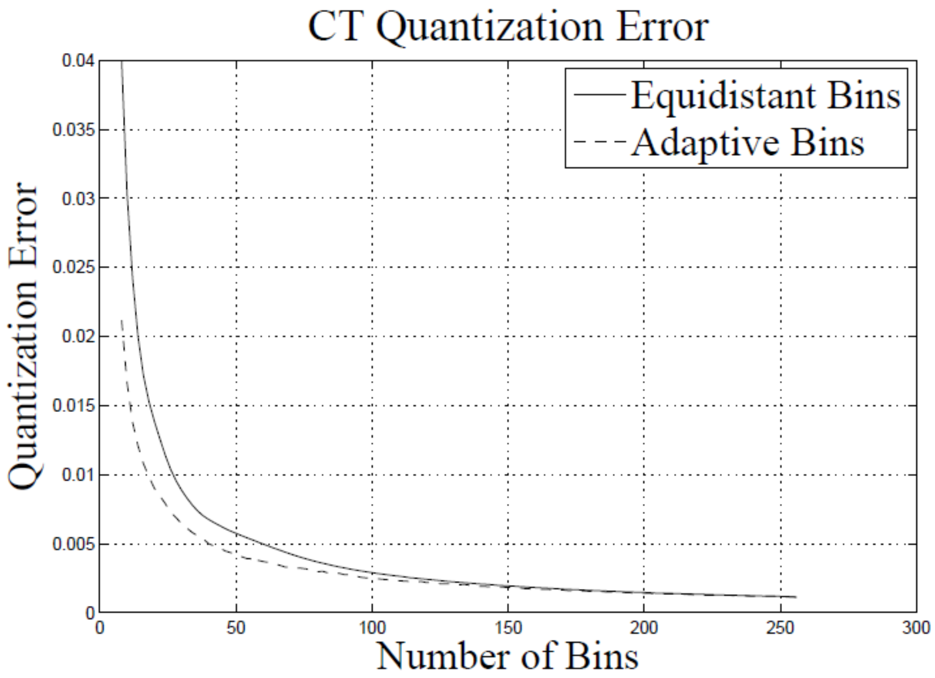
$$e(c_j) = \int_{c_j - \frac{w}{2}}^{c_j + \frac{w}{2}} p_{\lambda,n}(x)\ \mathrm{d}x - \underbrace{w\widehat{p}_{\lambda,n}(c_j)}_{\text{Discrete estimator}}$$

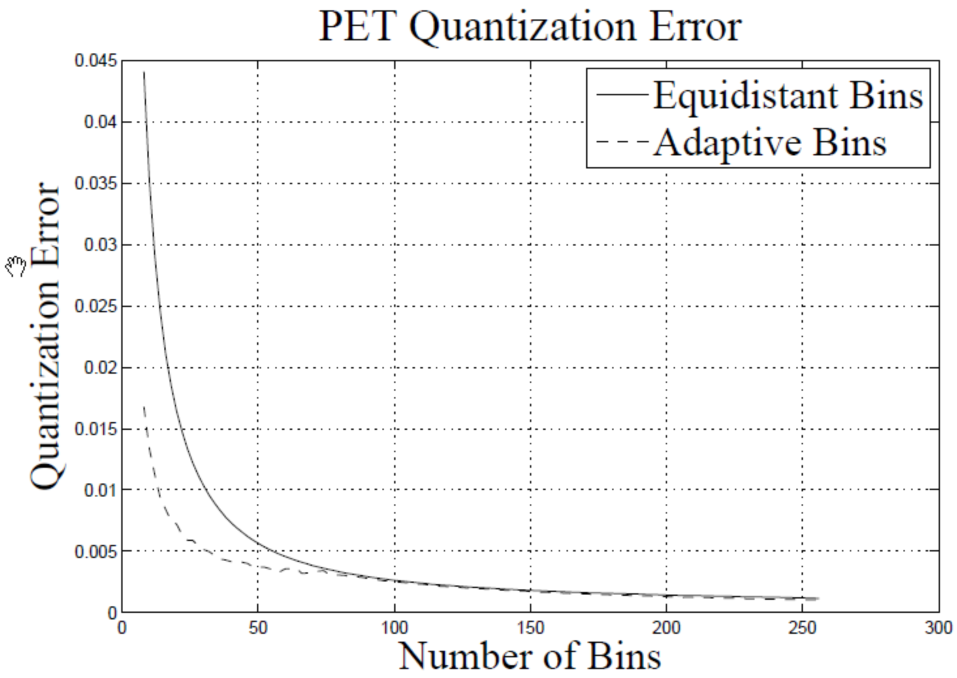


$$\boxed{e = \sum_{j=1}^{b} e(c_j)}$$

Discrete PDF Representation

- Discretization error is minimized w.r.t. number of bins
- Tradeoff between accuracy and efficiency
- Error threshold of 0.5% yields good results





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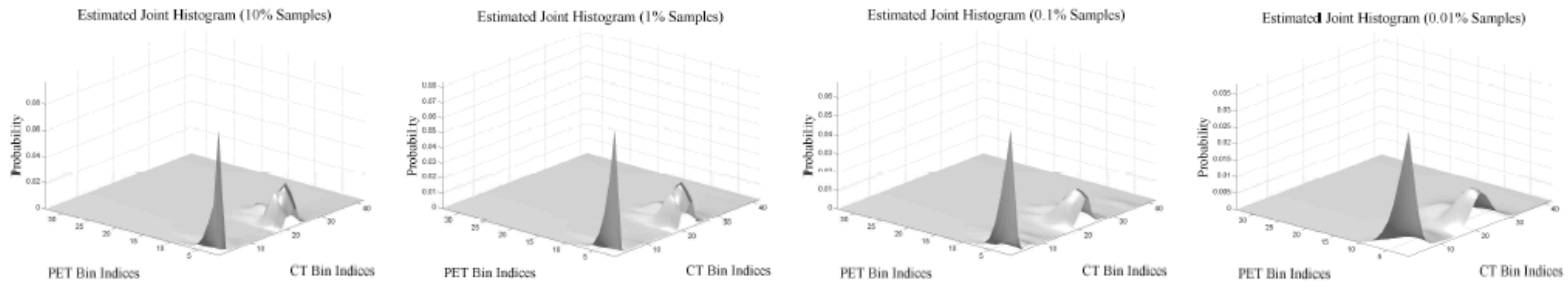
Estimated PDF



(a) (b) (c) (d)

10% 1% 0.1% 0.01%

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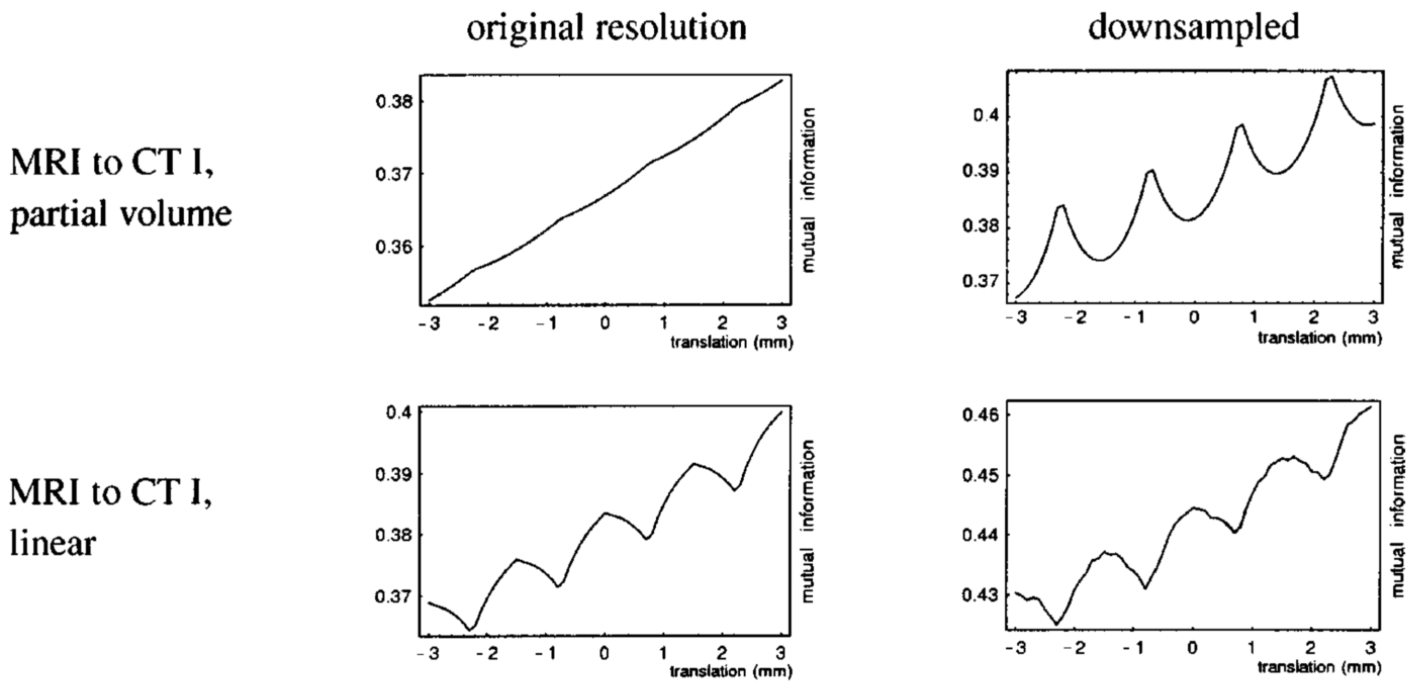
Artifacts in Mutual Information

- Problem:

  Interpolation has a high impact on local extrema due to grid aligning positions (common in medical images).

  Effect is increased at lower resolutions.

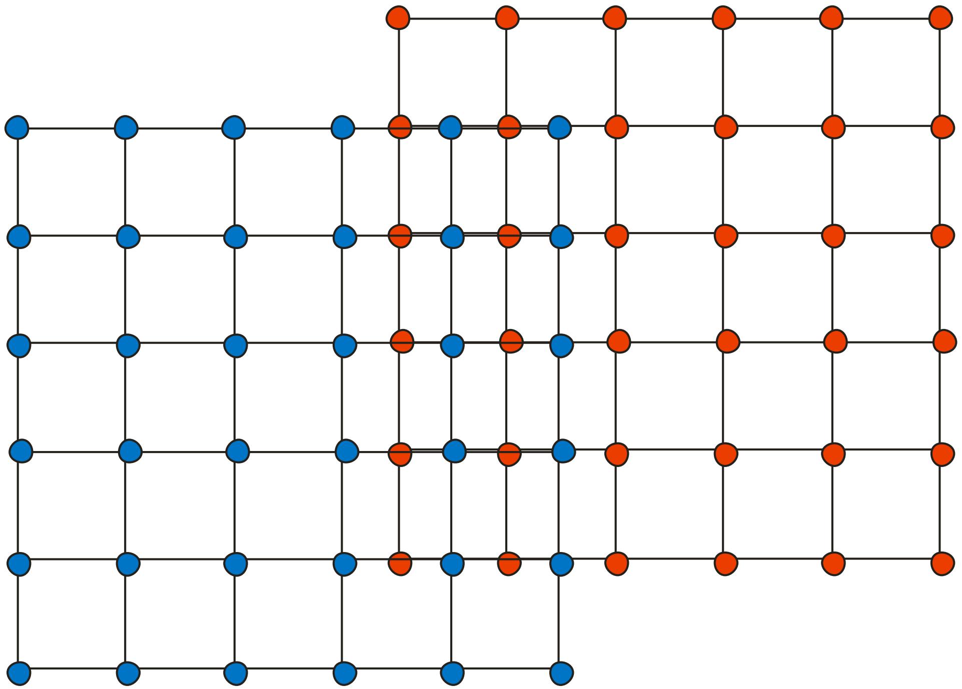


Taken from Pluim et.al. 2000

Artifacts in Mutual Information

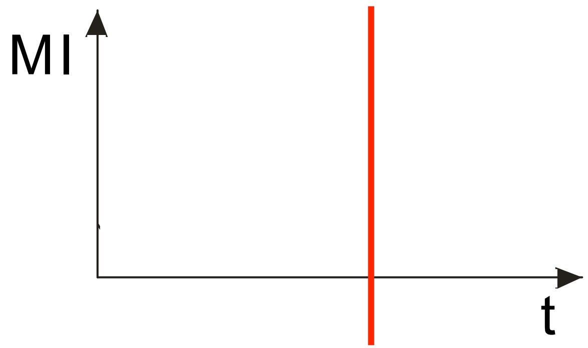
Image Translation

- Reference Image
- Template Image

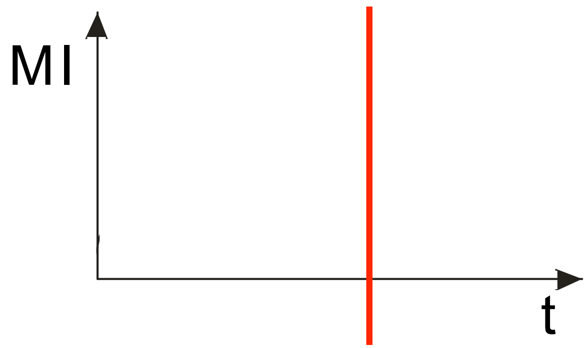


MI Energy

Partial Volume Interpolation



Linear Interpolation



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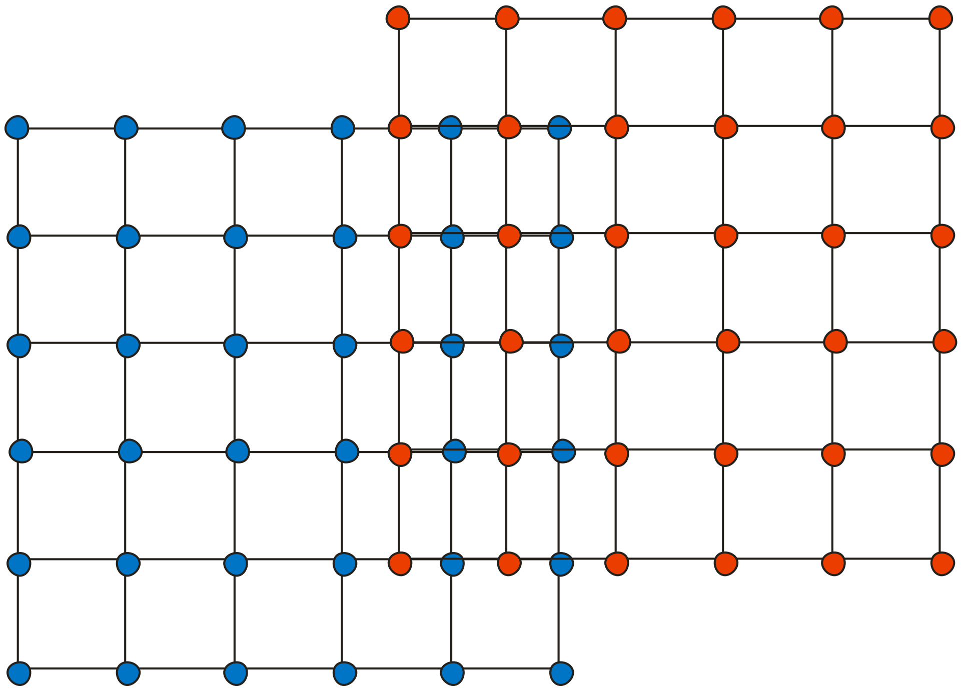
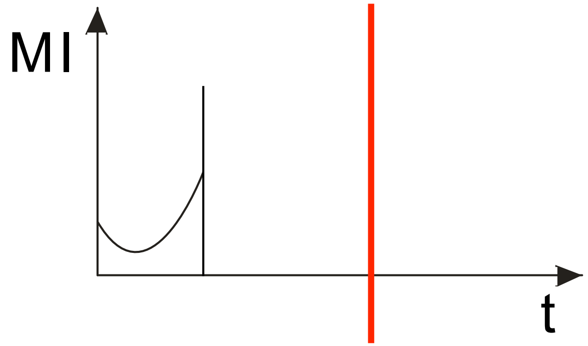
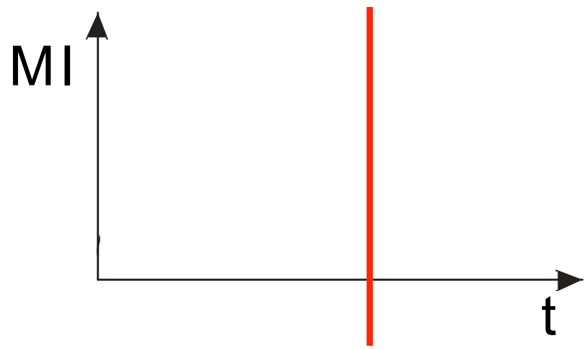


MI Energy

Partial Volume Interpolation



Linear Interpolation



Artifacts in Mutual Information

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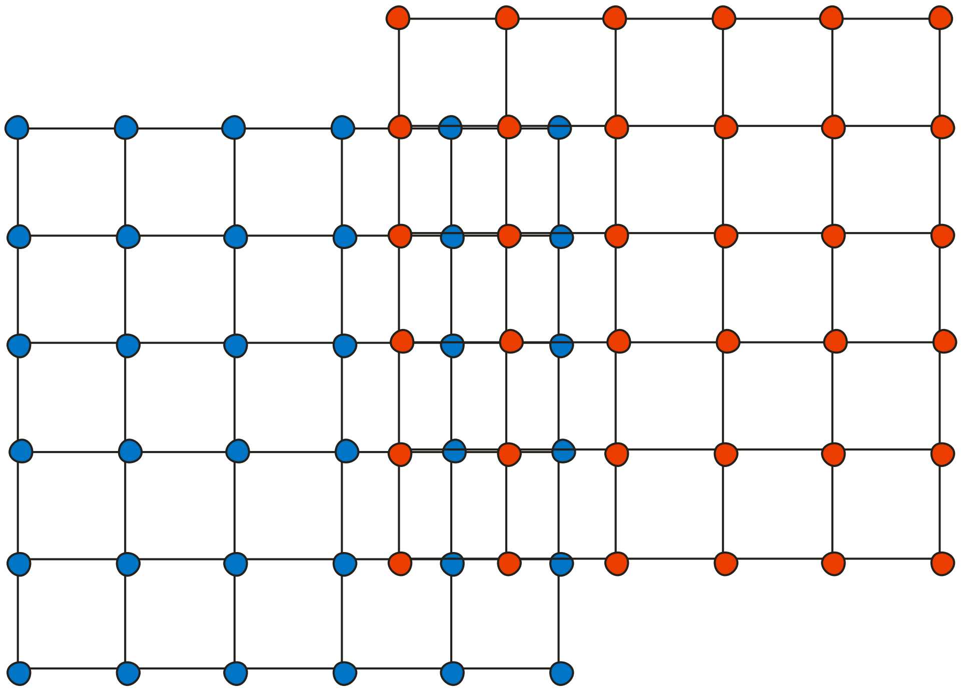
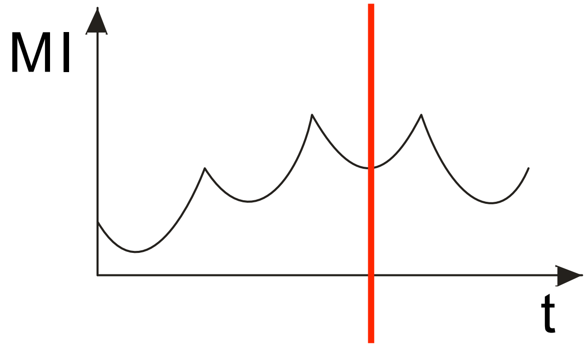
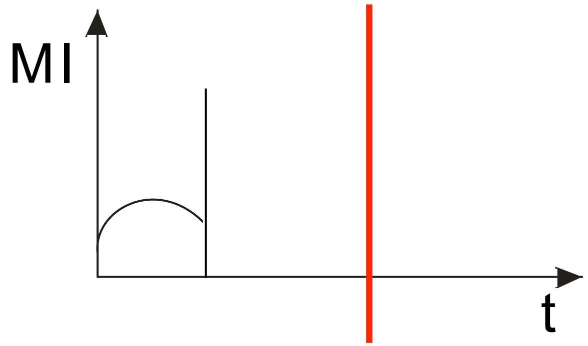


MI Energy

Partial Volume Interpolation

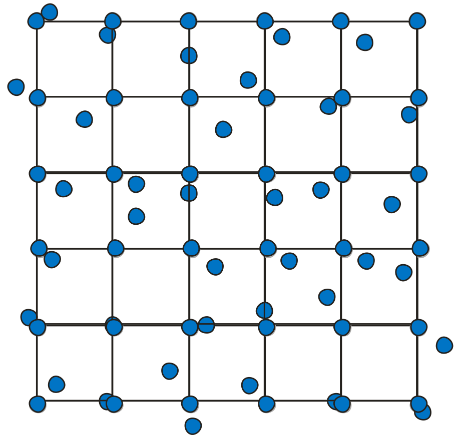


Linear Interpolation



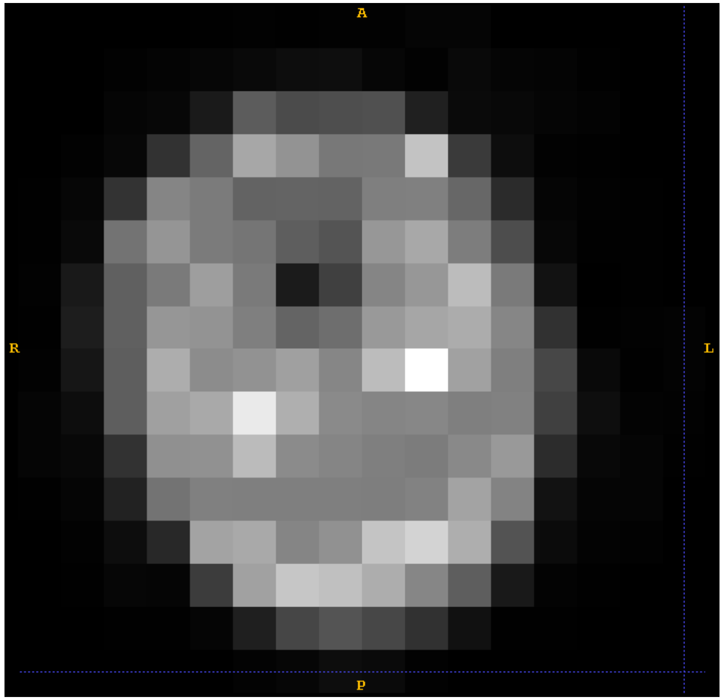
Improvements of Smoothness

- Jittering
  - Implementation in ITKx
  - Using random generator to disturb positions of voxels

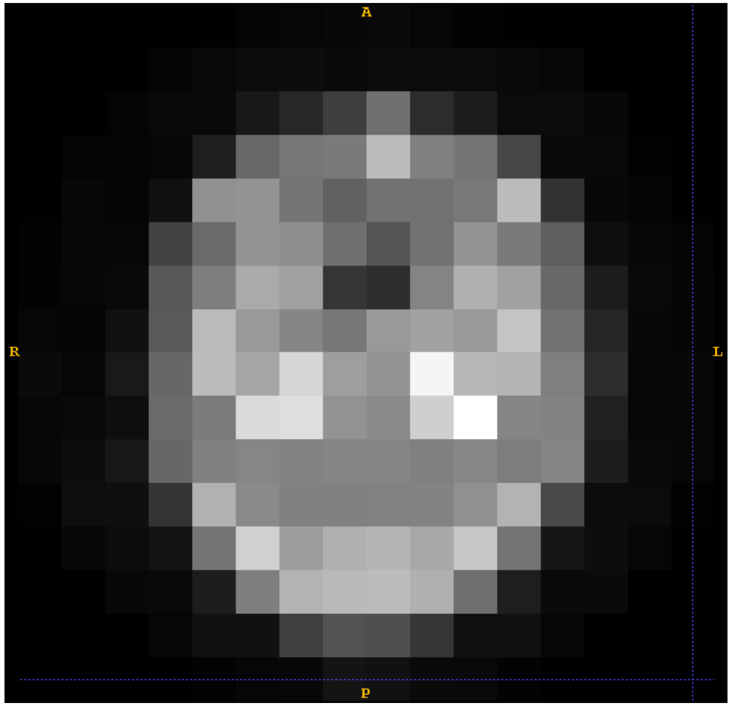


Artifacts in Mutual Information

- The numerical results on the following slides are computed from a CT with a DynaCT registration
- Level 16 x 16 x 16
- Resolution: 15.56 x 15.56 x 12.16



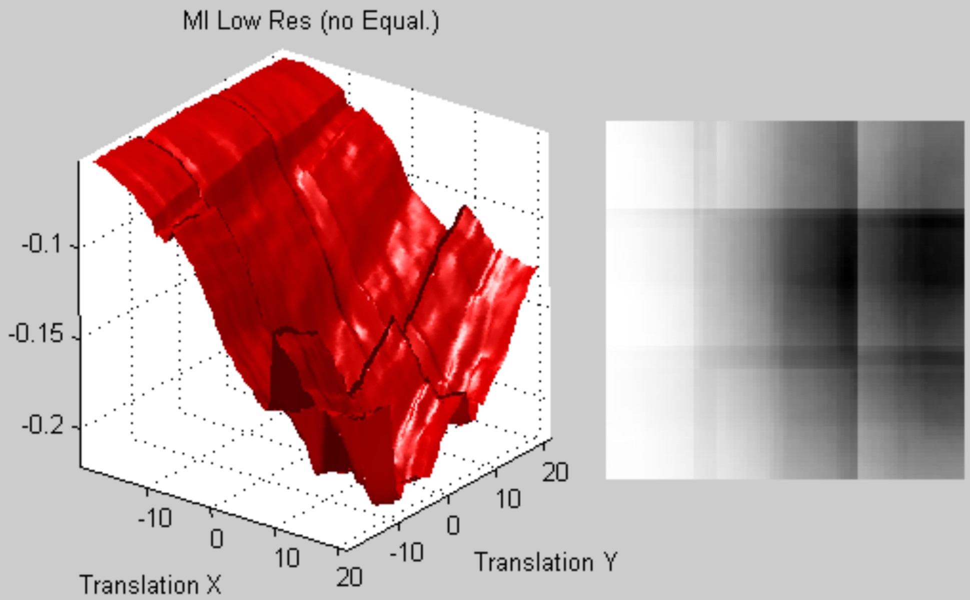
CT



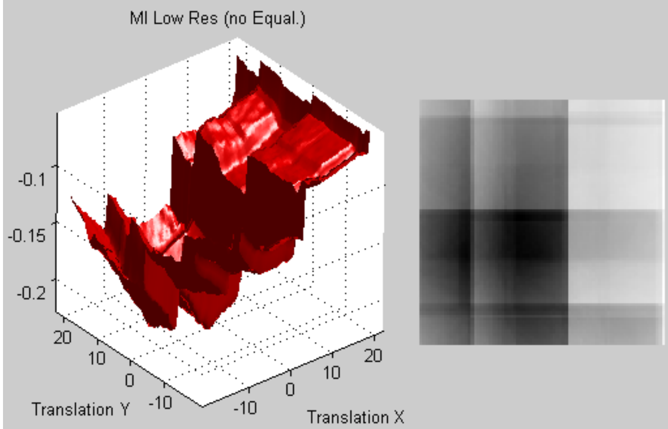
Dyna CT

Numerical Results I

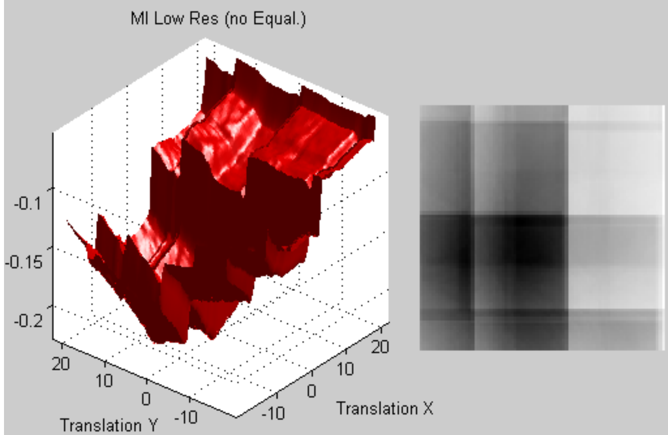
- MI



Initial position



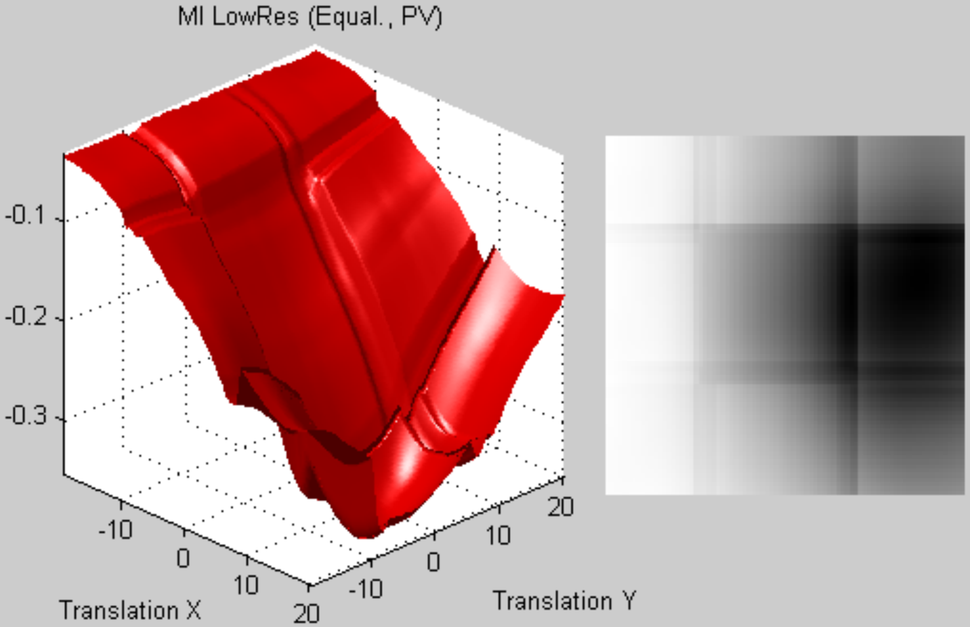
After translation



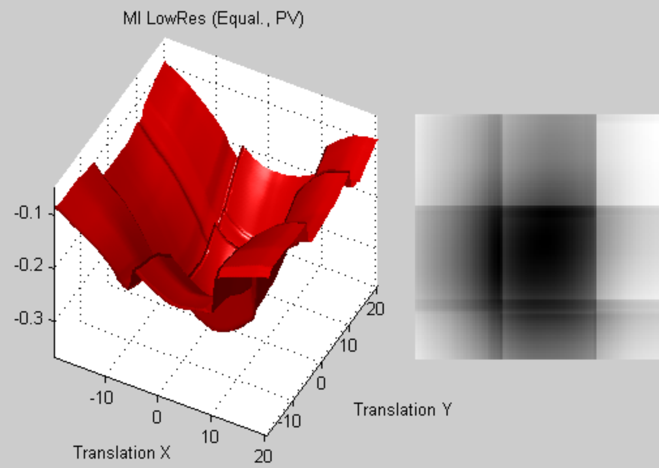
After rotation

Numerical Results I

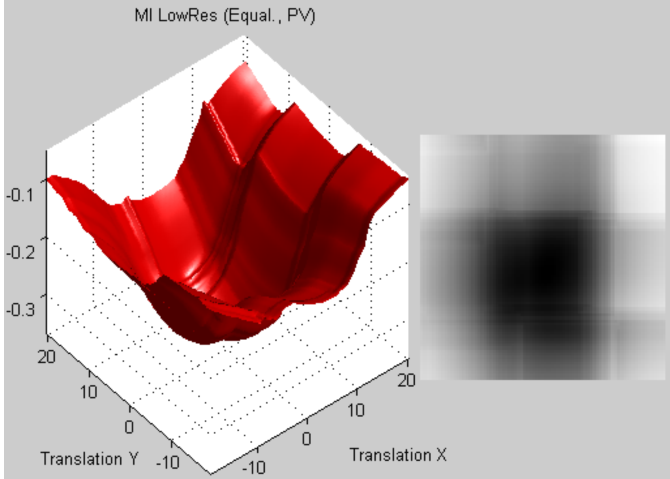
- MI with Histogram Equalization, Partial Volume interpolation



Initial position



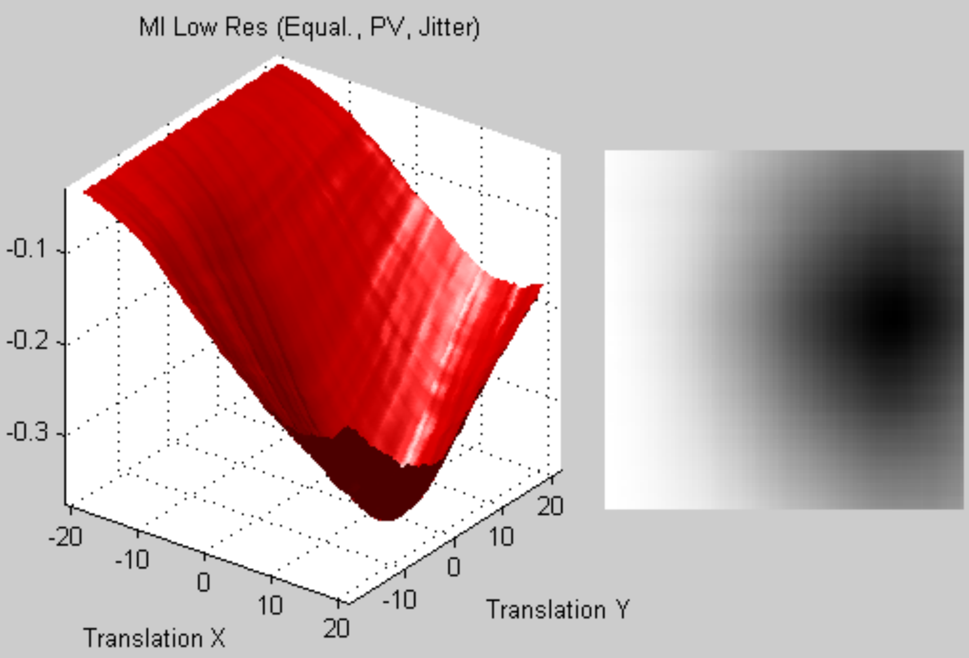
After translation



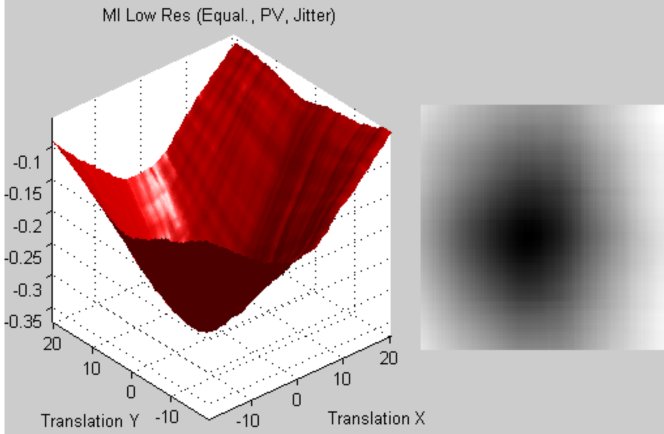
After rotation

Numerical Results I

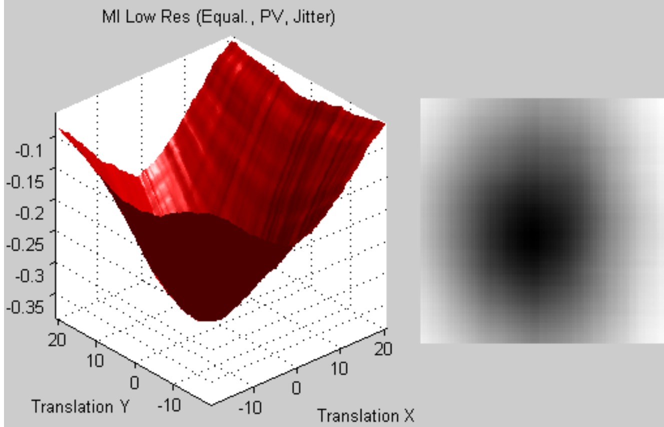
- MI with Histogram Equalization, Partial Volume Interpolation, Jittering



Initial position



After translation



After rotation

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Conclusions

- Non-parametric estimation of pdf‘s underestimated
- Empirical results on pdf‘s nice, but useless
- Resolve regularities by jittering
- Benchmark datasets