Sensors
Introduction to polarimetric imaging

Christophe Stolz
Master VIBOT
2012

Summary

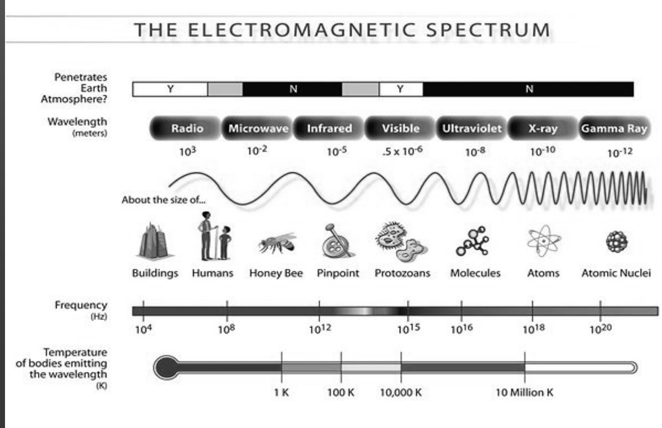
I. Introduction
II. Polarization of light
    polarisation states and parameters
III. Stokes Model
IV. O.Morel :
    Polarimetric imaging
    example and applications

I Introduction

- Non Conventional optical Imaging:
  Imaging techniques such as
  microscopy
  radar imaging
  polarimetric imaging
  holography
- Applications:
  ground data,
  metrology,
  health

I Introduction

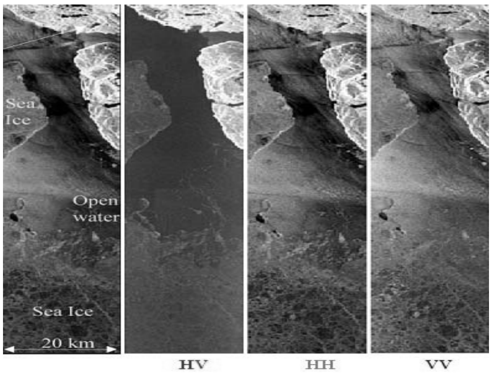
- Recall of EM spectrum



I Introduction

- Radar imaging : examples of images

C-band false colour composite and single channel intensity images from SIR-C for an ice infested region of the Labrador Sea off the coast of newfoundland.

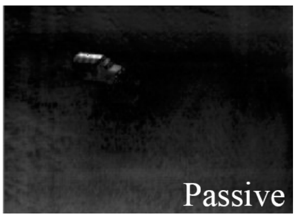


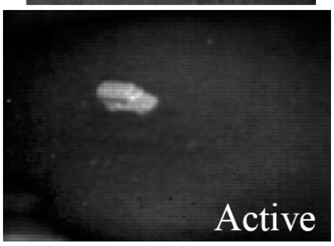
"Remote Sensing Glossary" of the Canada Centre for Remote Sensing

I Introduction

- Remote sensing : active imaging
  middle Infrared

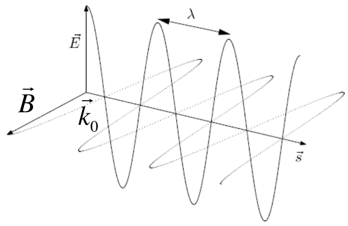
Truck with sand color on a the sand





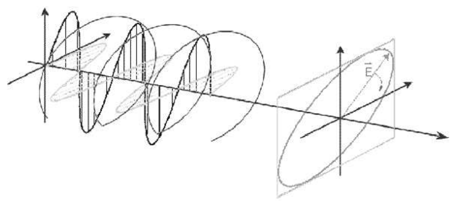
II 1. Polarization of light

- monochromatic plane wave
  vectors $\vec{E}, \vec{B}, \vec{k}_0$ forming a direct trihedral



$$\vec{E} = \vec{E}_0 e^{i\left(\omega t - \vec{k}_0 . \vec{r}\right)}$$

$$\left\|\vec{k}_0\right\| = k_0 = \frac{2\pi}{\lambda}$$

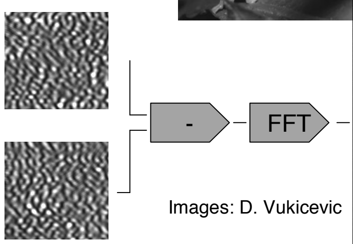


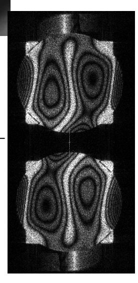
II Polarization of light

- Wave parameters
- Intensity $\Rightarrow$ $i = |a|^2$ $\Rightarrow$

- Frequency $c_n = \frac{1}{n} \cdot (\lambda_0 \nu); e_{ph} = h\nu$

- Phase $\varphi = 2\pi \frac{r}{\lambda_n}$



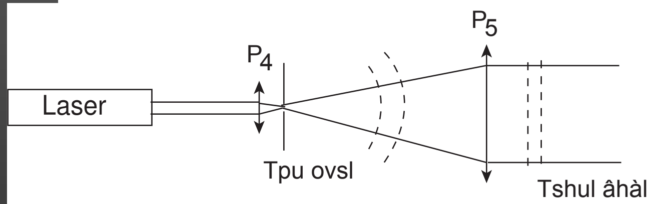


Images: D. Vukicevic

- Polarization

II Polarization of light

- How to realize a plane wave ?
- Laser beam expander



- Example at 633 nm, pinhole size of 15 μm

$$D_{pinhole} = \frac{F\lambda}{a}$$

II Polarization of light : Polarization states

- Polarization states
  in the preceding coordinate system

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = E_1\vec{e}_x + E_2\vec{e}_y \quad \text{where} \quad \begin{cases} E_x = E_{0x}\cos\left(\omega t - \vec{k}_0.\vec{r} - \varphi_1\right) \\ E_y = E_{0y}\cos\left(\omega t - \vec{k}_0.\vec{r} - \varphi_2\right) \end{cases}$$

$\delta = \varphi_2 - \varphi_1$ The phase delay between E_1 and E_2

When: $\frac{d\delta}{dt} = 0$ The wave is polarized => three polarization states

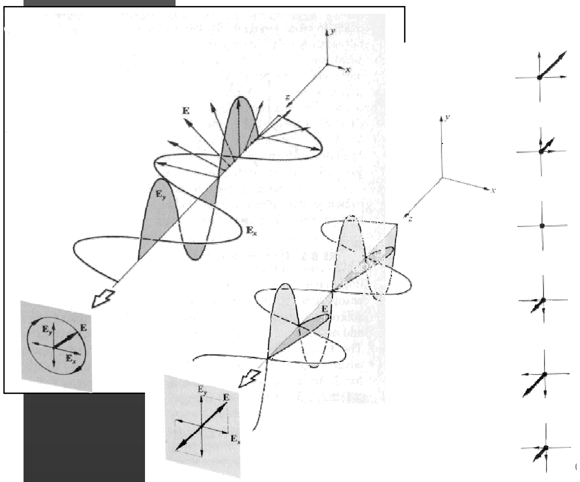
Linear polarization $\delta = m\pi$ with $m \in \mathbb{Z}$

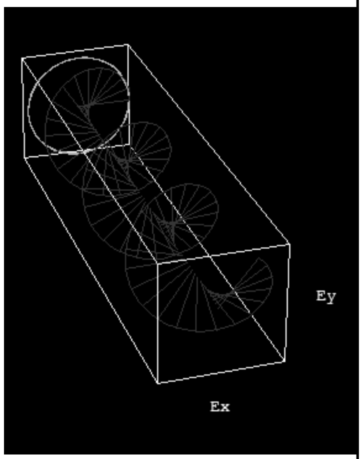
Elliptical polarization

Circular polarisation $\begin{cases} a_1 = a_2 \\ \delta = (2m+1)\frac{\pi}{2} \text{ avec } m \in \mathbb{Z} \end{cases}$

II Polarization of light : Polarization states

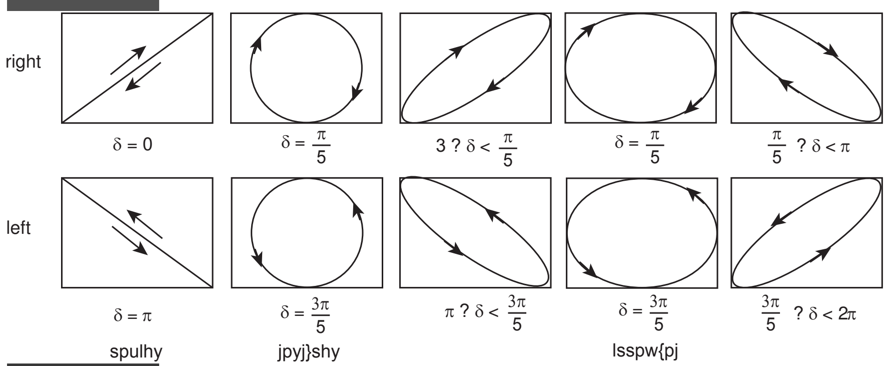
- 3D representations





II Polarization of light : Polarization states

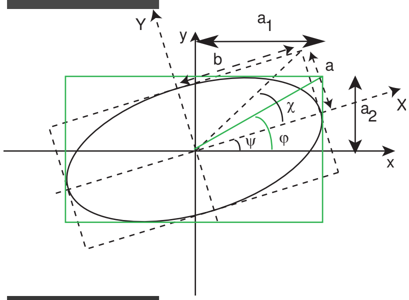
- Polarization states: polarization plane



II Polarization of light : Polarization states

- Other representation: $\begin{cases} E_x = E_0 \cos\varphi \cos\left(\omega t - \vec{k}_0 \vec{r} - \varphi_1\right) \\ E_y = E_0 \sin\varphi \cos\left(\omega t - \vec{k}_0 \vec{r} - \varphi_2\right) \end{cases}$

  with $E_0^{\ 2}$ the light intensity and $\tan\varphi = \dfrac{a_2}{a_1} = \dfrac{E_{0y}}{E_{0x}}$



Ellipse equation

$$\left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 - 2\frac{E_x}{E_{0x}}\frac{E_y}{E_{0y}}\cos\delta = \sin^2\delta$$

ψ azimuth $\in \left[0,\pi\right]$; $\tan 2\psi = \tan(2\varphi)\cos\delta$

χ ellipticity $\in \left[-\dfrac{\pi}{4},\dfrac{\pi}{4}\right]$; $\sin 2\chi = -\sin 2\varphi \sin\delta$

φ diagonal angle

II Polarization of light : Polarization states

- Ellipse equation in the X and Y basis
- Canonical form of the ellipse $\dfrac{E_X^{\ 2}}{a^2} + \dfrac{E_Y^{\ 2}}{b^2} = 1$
- Component of the electrical field:

$$E_X = E_x \cos\psi + E_y \sin\psi$$
$$E_Y = -E_x \sin\psi + E_y \cos\psi$$

$$I = E_{0x}^{\ 2} + E_{0y}^{\ 2} = a^2 + b^2$$

- Development and identification :

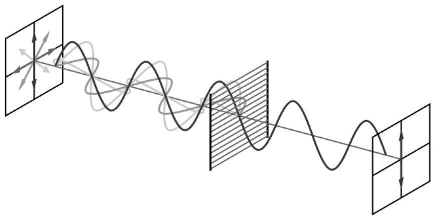
$$\sin^2\phi \left(\frac{\cos^2\psi}{a^2} + \frac{\sin^2\psi}{b^2}\right) = \frac{1}{A_1^2} \qquad (1)$$

$$\sin^2\phi \left(\frac{\sin^2\psi}{a^2} + \frac{\cos^2\psi}{b^2}\right) = \frac{1}{A_2^2} \qquad (2)$$

$$\sin^2\phi \cos\psi \sin\psi \left(\frac{1}{a^2} - \frac{1}{b^2}\right) = -\frac{\cos\phi}{A_1 A_2} \qquad (3)$$

II Polarization of light: producing

- Polarizer filter:
  shematized
  as a grid



- Dichroic polarizer :
  absorb one of the component
  of the incident light (anisotrpic)

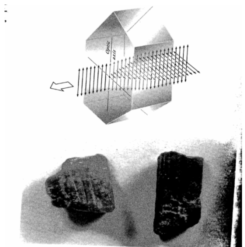


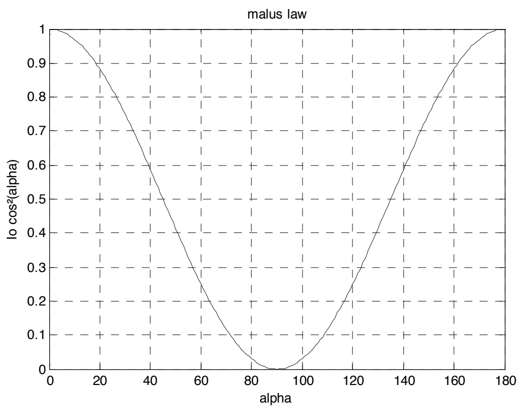
Figure 8.13 A dichroic crystal. The *E*-field parallel to the optic axis is transmitted without any diminution.The naturally occurring ridges evide in the photograph of the tourmaline crystals correspond to the optic axis. (Photo by E.H.)

- Malus law

$$I = I_0 \cos^2 \alpha$$

II Polarization of light: producing

- Malus law



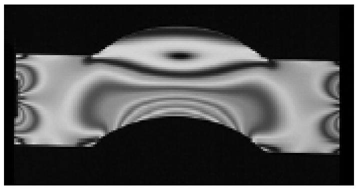
- Intensity after two crossed polarizer ?

II Polarization of light: producing

- Pictures seen throught polarizer(s)

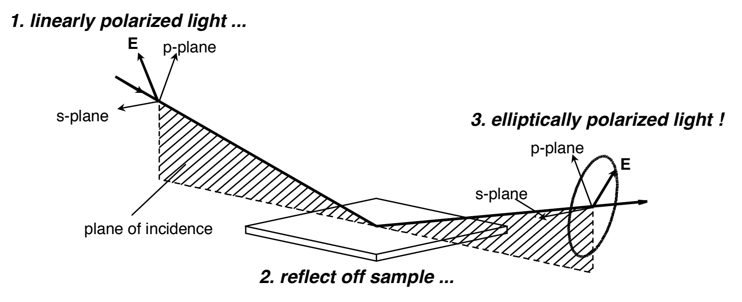
Reflection attenuation

Analysis of mechanic tension of plexiglass through crossed polarizers



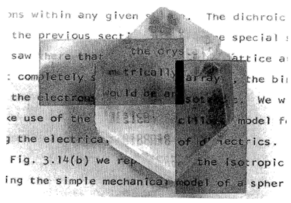
II Polarization of light: producing

- Reflection of a linearly polarized light



II Polarization of light: producing

- Anisotropic media: retarder plate
- Birefringent media: characterized by 2 orthogonal axis with 2 different index
  each index introduce a different phase speed
- ΔL : slow or extraordinary axis
- ΔR : fast or ordinary axis

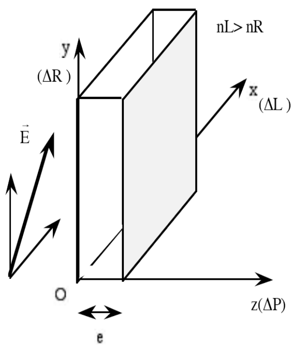


A calcite crystal (blunt corner on the bottom). The transmission axes of the two polarizers are parallel to their short edges. Where the image is doubled the lower, undeflected one is the ordinary image. Take a long look: there's a lot in this one. (Photo by E.H.)

II Polarization of light: producing

- Particular plates:
  - quarter wave plate
    output polarized elliptically,
    circular if input in an axis direction
  - half wave plate
    rotate linear polarization

$$\phi = \frac{(n_L - n_R)2\pi e}{\lambda}$$



II Polarization of light: modulating

- Pockels effect : eletro-optic effect in some crystals involving polarization modulation
- Ex : LiNbO3 crystals, birefringence depending of electrical field applied to it
- In that case $n_e - n_o = \alpha E$ with $E = \frac{V}{d}$ so $\phi = \frac{2\pi}{\lambda} \alpha L \frac{V}{d}$

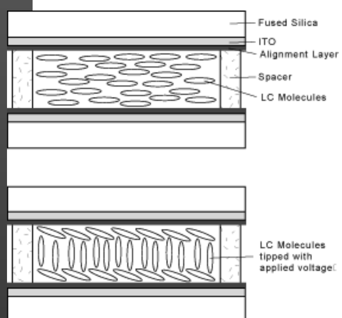
  d distance between electrodes
- Particular voltage Vπ => act like a half wave plate
- Applications: fast optical shutter (0, 90°), amplitude modulation

II Polarization of light: modulating

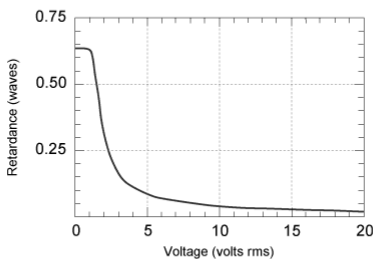
- Liquid crystals retarders

(a) Maximum Retardance (V=0)

(b) Minimum Retardance (V>>0)



Liquid Crystal Variable Retarder construction showing molecular alignment (a) without and (b) with applied voltage (drawing not to scale)



Retardance curve (with no compensator)

from Meadowlark

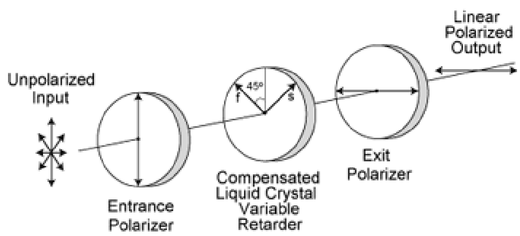
II Polarization of light: modulating

- Liquid Crystal retarder: retardance and Outout wave from voltage

Voltage (volts)	Retardance	Output
V ~ 2	$\delta = \lambda/2$	
2 < V < 4	$\lambda/4 < \delta < \lambda/2$	
V ~ 4	$\delta = \lambda/4$	
4 < V < 7	$0 < \delta < \lambda/4$	
V ~ 7	$\delta = 0$	

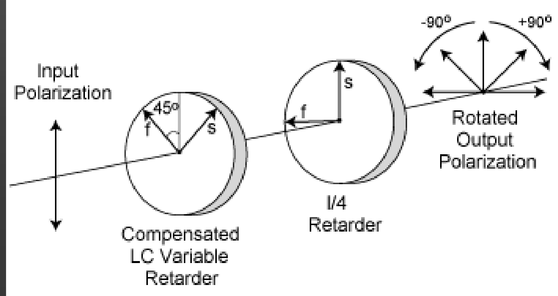
II Polarization of light: modulating

- Creating / modulating polarization:
- Polarizer + quarter wave plate: elliptic or circular polarization
- Polarization attenuator:



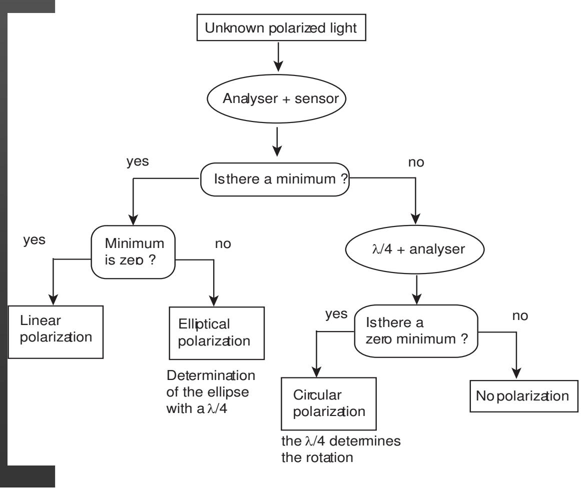
II Polarization of light: modulating

- Polarization rotation
- Half wave plate
- Or with liquid crystal cell



II Polarization of light: modulating

- How to determine the polarization of an unknown wave ?



III Stokes Model

- Stokes vector

$$s=\begin{pmatrix}s_0\\s_1\\s_2\\s_3\end{pmatrix}=\begin{pmatrix}I\\Q\\U\\V\end{pmatrix}=\begin{pmatrix}\langle E_x^*E_x\rangle+\langle E_y^*E_y\rangle\\ \langle E_x^*E_x\rangle-\langle E_y^*E_y\rangle\\ \langle E_xE_y^*\rangle+\langle E_x^*E_y\rangle\\ i\left[\langle E_xE_y^*\rangle-\langle E_x^*E_y\rangle\right]\end{pmatrix}=\begin{pmatrix}I_0+I_{90}\\ I_0-I_{90}\\ I_{45}-I_{-45}\\ I_d-I_g\end{pmatrix}$$

For measurement purpose

- Where :
  s_0 : intensity of light
  s_1, s_2 : difference between intensities respectively at 0° and 90° and at 45° and -45°
  s_3 : difference between right (d) and left (g) circular components

- Properties : for partial polarization $S_0 \geq \sqrt{S_1^2+S_2^2+S_3^2}$

III Stokes Model

- Examples of stokes vectors (normalized)

$\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$ Unpolarized

$\begin{pmatrix}1\\-1\\0\\0\end{pmatrix}$ Linearly polarized (vertical)

$\begin{pmatrix}1\\1\\0\\0\end{pmatrix}$ Linearly polarized (horizontal)

$\begin{pmatrix}1\\0\\1\\0\end{pmatrix}$ Linearly polarized (+45°)

$\begin{pmatrix}1\\0\\-1\\0\end{pmatrix}$ Linearly polarized (−45°)

$\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$ Right-hand circularly polarized

$\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}$ Left-hand circularly polarized

III Stokes Model

- Link with the polarization ellipse:

$$s = \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} E_{0x}{}^2 + E_{0y}{}^2 \\ E_{0x}{}^2 - E_{0y}{}^2 \\ 2E_{0x}E_{0y}\cos\delta \\ 2E_{0x}E_{0y}\sin\delta \end{pmatrix} = \begin{pmatrix} I \\ I_{pol}\cos 2\chi \cos 2\psi \\ I_{pol}\cos 2\chi \sin 2\psi \\ I_{pol}\sin 2\psi \end{pmatrix}$$

Phase angle

$$\delta = \arctan\left(\frac{S_3}{S_2}\right)$$

Diagonal angle

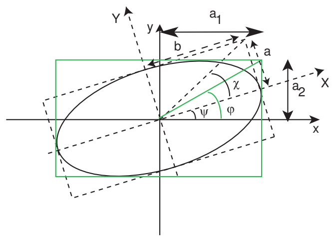
$$\varphi = \frac{1}{2}\arccos\left(\frac{S_1}{\sqrt{S_1{}^2 + S_2{}^2 + S_3{}^2}}\right)$$

ellipticity

$$\psi = \frac{1}{2}\tan^{-1}\left(S_2 / S_1\right)$$

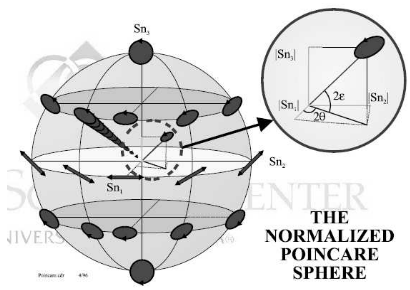
azimuth

$$\chi = \frac{1}{2}\arcsin\left(\frac{S_3}{\sqrt{S_1^2 + S_2^2 + S_3^2}}\right)$$



III Stokes Model

- Poincaré sphere
- Representation of polarization states:
  center is unpolarized light
  surface is polarized light
  equator plane represents
  linear waves
  poles represents
  circular waves
  north hemisphere is right
  elliptically polarized light
- Orthogonal polarization
  states are diametrically opposite



From R.A Chipman

III Stokes Model

- degree of polarization DOP

$$DOP = \frac{\sqrt{s_1^2 + s_2^2 + s_3^2}}{s_0} = \frac{I_{pol}}{I} \leq 1 \qquad \text{or} \qquad \frac{I_{\parallel} - I_{\perp}}{I_{\parallel} + I_{\perp}}$$

- degree of linear polarization DOLP

$$DOLP = \frac{\sqrt{s_1^2 + s_2^2}}{s_0}$$

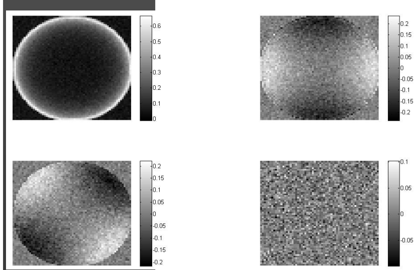
- Degree of circular polarization

$$DOCP = \frac{s_3}{s_0}$$

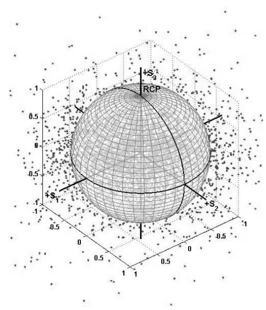
III Stokes Model

- Example of mapping

Stokes images



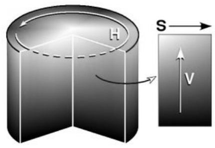
Representation on the sphere
Here 12% of the pixels are to be rejected



From J. Zallat

III Stokes Model

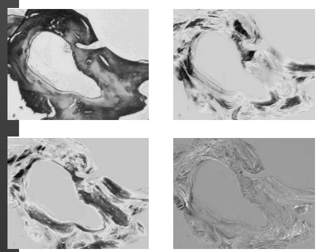
- Representation of Stokes vector in Color HSV coordinates



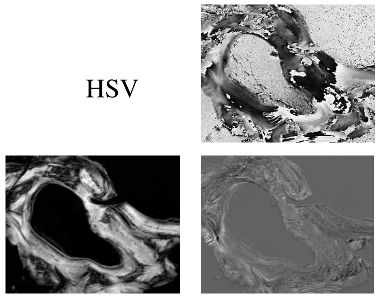
$$H = \tan^{-1}\left(\frac{\bar{S}_2}{\bar{S}_1}\right) \qquad S = \sqrt{\bar{S}_1^2 + \bar{S}_2^2} \qquad V = \frac{1}{2} - \frac{\bar{S}_3}{2}$$

- Example with medical images

Stokes



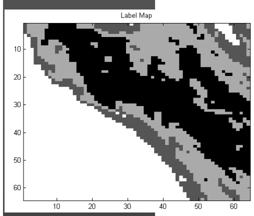
HSV

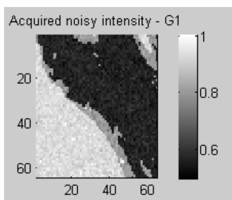


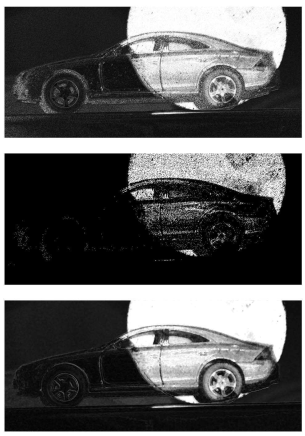
J. Zallat, C. Collet, A. De Martino and S. Ainouz, "Interpretation physique d’images codées en polarisation par prévisualisation couleurs", Journées Imagerie non conventionnelles, CNRS, 2006.

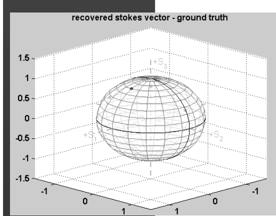
III Stokes Model

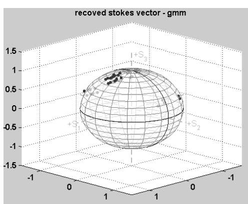
- Noise recovery











Sfikas JOSA A 2011

III Stokes Model

- Mueller matrix :
  represent the action of an optical system on an incident wave represented by its stokes vector

$$\vec{S}_o = \left[M \right] \cdot \vec{S}_i$$

$$[M] = \begin{bmatrix} M_{00} & M_{01} & M_{02} & M_{03} \\ M_{10} & M_{11} & M_{12} & M_{13} \\ M_{20} & M_{21} & M_{22} & M_{23} \\ M_{30} & M_{31} & M_{32} & M_{33} \end{bmatrix} = M_{00} \begin{pmatrix} 1 & \vec{D}^T \\ \vec{P} & \mathbf{m} \end{pmatrix}$$

III Stokes Model

- Examples of Mueller matrices

Polarizer (perfect) oriented with angle α (rotator):

$$M_{pol} = \frac{1}{2} \begin{pmatrix} 1 & \cos 2\alpha & \sin 2\alpha & 0 \\ \cos 2\alpha & \cos^2 2\alpha & \cos 2\alpha \sin 2\alpha & 0 \\ \sin 2\alpha & \cos 2\alpha \sin 2\alpha & \sin^2 2\alpha & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Retarder with a delay δ oriented with angle α:

$$M_{ret} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c^2 + s^2 \cos\delta & sc(1-\cos\delta) & -s\sin\delta \\ 0 & sc(1-\cos\delta) & s^2 + c^2\cos\delta & c\sin\delta \\ 0 & s\sin\delta & -c\sin\delta & \cos\delta \end{pmatrix} \qquad c = \cos 2\alpha, \text{ s=sin}2\alpha$$

Depolarizer

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & c \end{pmatrix} \qquad |a|, |b|, |c| \le 1$$

III Stokes Model

- Mueller matrix :
- Polarisance : DOP of the emerging light when incident light is unpolarized

$$\vec{P}^T = \frac{1}{m_{00}}\left(m_{10} \quad m_{20} \quad m_{30}\right) \qquad P = \frac{1}{m_{00}}\sqrt{m_{10}^2 + m_{20}^2 + m_{30}^2} \le 1$$

- Diattenuation: DOP of the emerging light when incident light is polarized

$$\vec{D}^T = \frac{1}{m_{00}}\left(m_{01} \quad m_{02} \quad m_{03}\right) \qquad D = \frac{T_{\max} - T_{\min}}{T_{\max} + T_{\min}} = \frac{1}{m_{00}}\sqrt{m_{01}^2 + m_{02}^2 + m_{03}^2} \le 1$$

III Stokes Model

- Link with Jones vector : From coherence matrix terms

$$S = A\Gamma \text{ with } \Gamma = \begin{bmatrix} \langle E_x E_x^* \rangle \\ \langle E_y E_y^* \rangle \\ \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -i \\ 0 & 0 & -1 & i \end{bmatrix}$$

- From S'=MS we can derive the Mueller matrix:

$$M = A\left(J \otimes J^*\right)A^{-1}$$

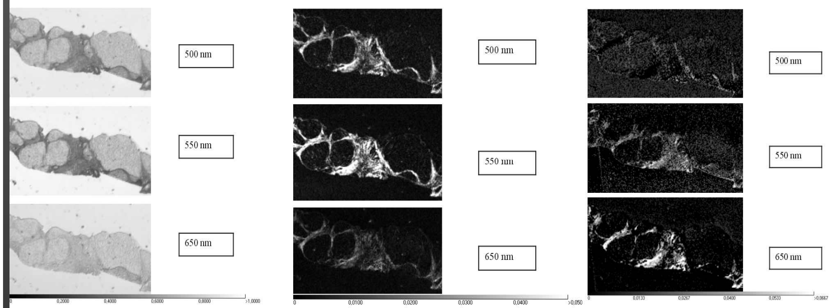
- Define H as the physical admissibility of matrix M

$$H = \frac{1}{4}\sum_{k,l=0}^{3} m_{kl} D_{kl}$$

M is physically valid if the eigenvalues of H are real positive

III Stokes Model

- Other decomposition: $\mathbf{M} = \mathbf{M}_{dep}\,\mathbf{M}_{ret}\,\mathbf{M}_{diatt}$
- Images of bones cuts



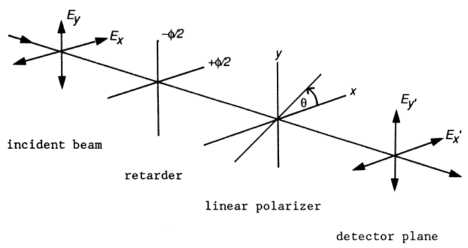
Depolarised transmission (0 à 1) | *Scalar Diattenuation* (0 à 0.05) | *Scalar delay* (0 à 0.07)

$$R = \cos^{-1}\left[\frac{1}{2}\mathrm{Tr}(\mathbf{M}_{ret}) - 1\right]$$

B. Laude-Boulesteix, A. De Martino, B. Drévillon, and L. Schwartz, "Mueller Polarimetric Imaging System with Liquid Crystals," Appl. Opt. 43, 2824-2832 (2004).

III Stokes Model

- Stokes vector measurement: classical configuration
- Polarizer and Retarder



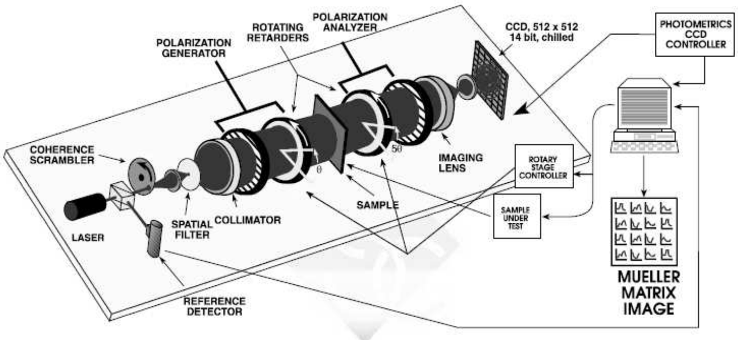
- Intensity at the output:

$$I(\alpha,\delta) = \frac{1}{2}\left[S_0 + S_1\cos 2\alpha + S_2\sin 2\alpha\cos\delta + S_3\sin 2\alpha\sin\delta\right]$$

III Stokes Model

- Stokes/Mueller polarimeter :

Polarization State Generator $\xrightarrow{\text{Si}}$ sample $\xrightarrow{\text{MSi}}$ Polarization State Analyser



III Stokes Model

- Stokes measurement: with a rotating quarter wave plate: $\delta = 90°$

$$S_0 = I(0°, 0°) + I(90°, 0°)$$
$$S_1 = I(0°, 0°) - I(90°, 0°)$$
$$S_2 = 2I(45°, 0°) - S_0$$
$$S_3 = 2I(45°, 90°) - S_0$$

- Take care of absorption loss in components p separately measured electric field

$$E_x^{'} = E_x e^{i\phi/2} p_x \quad \text{with } p_x = e^{-\alpha_x}$$
$$E_y^{'} = E_y e^{i\phi/2} p_y \quad \text{with } p_y = e^{-\alpha_y}$$

- Mueller matrix for anisotropic absorbing retarder

$$M = \frac{1}{2}\begin{pmatrix} p_x^2 + p_y^2 & p_x^2 - p_y^2 & 0 & 0 \\ p_x^2 - p_y^2 & p_x^2 + p_y^2 & 0 & 0 \\ 0 & 0 & 2p_x p_y \cos\phi & 2p_x p_y \sin\phi \\ 0 & 0 & -2p_x p_y \sin\phi & 2p_x p_y \cos\phi \end{pmatrix}$$

III Stokes Model

- Anisotropic retarder: behave like a polarizer and a retarder
- Then for a polarizer at angle γ

$$M = \frac{p}{2}\begin{pmatrix} 1 & \cos 2\gamma & 0 & 0 \\ \cos 2\gamma & 1 & 0 & 0 \\ 0 & 0 & \sin 2\gamma \cos\phi & \sin 2\gamma \sin\phi \\ 0 & 0 & -\sin 2\gamma \sin\phi & \sin 2\gamma \cos\phi \end{pmatrix}$$

$$p^2 = p_x^{\ 2} + p_y^{\ 2}$$

- Intensity at the output

$$I(\phi,\theta) = \frac{p^2}{2}\left[(1+\cos 2\theta \cos 2\gamma)S_0 + (\cos 2\gamma + \cos 2\theta)S_1 + (\sin 2\gamma \cos\phi \sin 2\theta)S_2 + (\sin 2\gamma \sin\phi \sin 2\theta)S_3\right]$$

- So:

$$S_0 = \frac{1}{p^2}\left[I(0°,0°) + I(90°,0)\right]$$

$$S_1 = \frac{1}{p^2}\left[I(0°,0°) - I(90°,0)\right]$$

$$S_2 = \frac{1}{p^2} I(45°,0°) - S_0$$

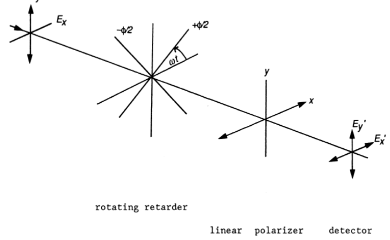
$$S_3 = \frac{1}{p^2} I(45°,90°) - S_0$$

Drawback, measuring p

III Stokes Model

- Stokes measurement : Fourier analysis with a quarter wave plate
- Stokes vector at the output

$$S' = \frac{1}{2}(S_0 + S_1 \cos^2 2\theta + S_2 \sin 2\theta \cos 2\theta - S_3 \sin 2\theta)\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$



- So $S'_0 = I(\theta)$

$$I(\theta) = \frac{1}{2}(S_0 + S_1 \cos^2 2\theta + S_2 \sin 2\theta \cos 2\theta - S_3 \sin 2\theta)$$

III Stokes Model

- Stokes measurement : Fourier analysis
- Recognize a Fourier series

$$I(\omega t) = \frac{1}{2}[A - B\sin 2\omega t + C\cos 4\omega t + D\sin 4\omega t] \quad A = S_0 + \frac{S_1}{2} \quad B = S_3 \quad C = \frac{S_1}{2} \quad D = \frac{S_2}{2}$$

- Then :

$$A = \frac{1}{\pi}\int_0^{2\pi} I(\theta)\,d\theta \qquad B = \frac{2}{\pi}\int_0^{2\pi} I(\theta)\sin 2\theta\,d\theta$$

$$C = \frac{2}{\pi}\int_0^{2\pi} I(\theta)\cos 4\theta\,d\theta \qquad D = \frac{2}{\pi}\int_0^{2\pi} I(\theta)\sin 4\theta\,d\theta$$

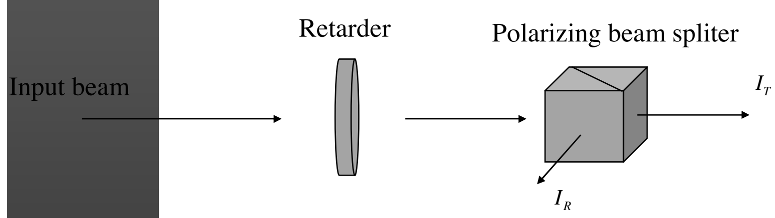
So

$$S_0 = A - C$$
$$S_1 = 2C$$
$$S_2 = 2D$$
$$S_3 = B$$

- In practice: plate in a static mount rotated by a stepper motor

III Stokes Model

- Stokes measurement principle: configuration with LC



The retarder element is composed with two liquid crystal wave plate

During the measurement of the Q (s1) and V (s3) components the first LC plate is held to zero retardance. It is then tuned to a quarter wave of retardance for the measurement of the U (s2) component

III Stokes Model

- Stokes measurement principle: configuration with LC
- First LCVR tuned to no retardance
- Measuring S1
  second LCVR tuned to zero and half wave retardance

$$S_1 = \frac{1}{2}\left[\frac{I_T(0) - I_T(\lambda/2)}{I_T(0) + I_T(\lambda/2)} - \frac{I_R(0) - I_R(\lambda/2)}{I_R(0) + I_R(\lambda/2)}\right]$$

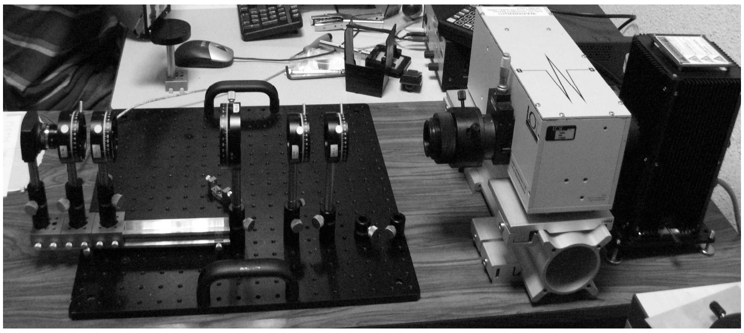
- Measuring S3
  second LCVR tuned to positive quarter and negative quarter wave

$$S_3 = \frac{1}{2}\left[\frac{I_T(\lambda/4) - I_T(-\lambda/4)}{I_T(\lambda/4) + I_T(-\lambda/4)} - \frac{I_R(\lambda/4) - I_R(-\lambda/4)}{I_R(\lambda/4) + I_R(-\lambda/4)}\right]$$

- Measuring S2 : first LCVR tuned as a quarter wave plate then same calculation

III Stokes Model

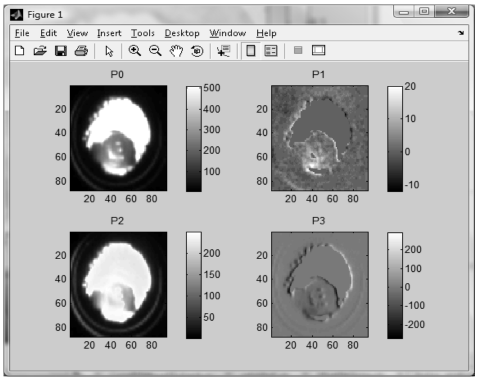
- Example of manual Stokes/Mueller setup
- Halogen light source and monochromator



PSA Object PSG

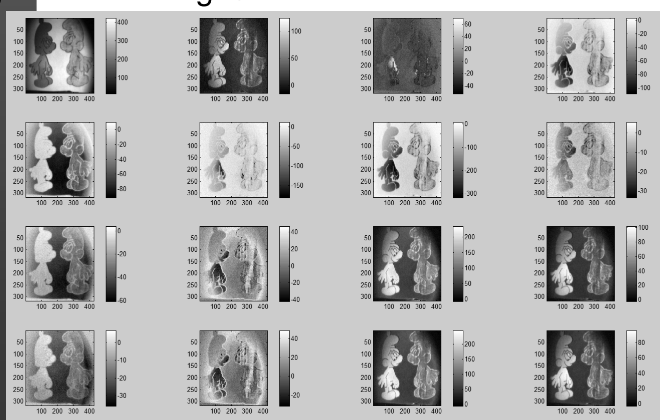
III Stokes Model

- Example of results ; cf labs
- Measured Stokes vector of a linear wave polarized at 45°



III Stokes Model

- Output of the polarimeter: S_0= AMW S_I
- Rough Mueller images

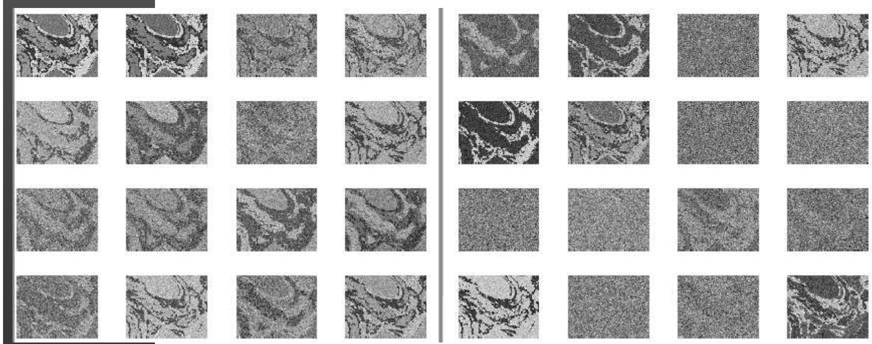


- Need precise calibration procedure

III Stokes Model

- Influence of noise

Noise is amplified



Intensities images SNR=10 dB　　Mueller

References

- D. L. Goldstein, *Polarized light second Edition (Optical Engineering)* (CRC, 2003).
- Polarization of light, Serge huard, John Wiley, 1997
- Meadowlark optics web site, LCD celles and applications