MSc Computer Vision & Vibot

Applied Mathematics

Home work 2

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Eigenvalues and eigenvectors

NOTE

You can discuss the problems with each other, but your solutions must be independently written with your own words and formulations.

Problem 1

The first problem of this homework is the **problem 11** in the notes. Here we give some ideas to help you finding a solution.
This problem is an example of what is called *Markov processes*. To solve the problem we first need to find the transion matrix or Markov matrix.

- For a given day, say Day n, we divide the citizens of *Computer Vision Village* into two groups: group 1 contains all citizens who read the paper on Day n, and group 2 contains all citizens who do not buy the paper on Day n. We also define a vector

$$v_n = \begin{pmatrix} g_1 \\ g_2 \end{pmatrix} \in \mathbb{R}^2,$$

  where g_1 is the size of goup 1 and g_2 the size of group 2.

  1. Construct a 2×2 matrix $M = (m_{ij})$, where the entry m_{ij} gives the probability that a citizen in group j one day will be in group i the next day. M is the transition matrix of the Markov process (check that the columns of M add to 1).
  2. Show that we have $Mv_n = v_{n+1}$.
  3. Now, you are ready to answer questions 1 and 2.

- To answer question 3, think about eigenvalues of M. What does 'sales figures become stable' means in terms of eigenvalues/eigenvectors?

Problem 2

This second problem is about another application of eigenvalues/eigenvectors. It deals with the Google PageRank algorithm.

1. Read the paper *How Google ranks Web pages?* which is available from the Master's website and write a summary of the ranking method.

2. Expalin how the eigenvalue problem is solved.
3. Starting with a 5×5 Markov matrix, apply the algorithm described in Section 5 of the paper to find the eigenvector associated with eigenvalue 1.
4. **Additional question** (If you like and have time) Write this algorithm as a Matlab function.
5. Any other comment?