Course: Applied Mathematics (D. Sidibé)

Final Exam: 2h

You must show all work and all reasoning - Full credit will be given only for clearly explained results!

■ PROBLEM 1 (45 Points)

Suppose A is a 2×2 matrix such that $Ax = x$ and $Ay = -y$, for $x \neq 0$ and $y \neq 0$.

1. What are the eigenvalues of A?
2. If you know the first column of A is $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, find the second column of A, i.e.

$$A = \begin{bmatrix} 2 & ? \\ 1 & ? \end{bmatrix}$$

3. For the matrix in question 2, find an invertible matrix S and a diagonal matrix Λ such that $A = S\Lambda S^{-1}$.
4. Find the matrix A^{101}.
5. If $Ax = x$ and $Ay = -y$, for $x \neq 0$ and $y \neq 0$, prove that x and y are independent vectors.

■ PROBLEM 2 (20 Points)

Find the line of equation $y = A + Bx$ which best fits the following data:

x	0	1	2
y	1	-1	-3

■ PROBLEM 3 (35 Points)

Let X and Y be independent random variables, each uniformly distributed on the interval $[0, 2]$.

1. What is the pdf of X? What are the mean and variance of X?
2. Find the mean and variance of XY.
3. Find the probability that $XY \geq 1$, i.e. $P(XY \geq 1)$.