Diagnostic Medical Image Processing

Acquisition Specific Pre-Processing

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Diagnostic Medical Image Processing

Image Pre-Processing in X-Ray Imaging

X-ray

Image artifacts in X-ray can have many sources. We will consider in the following section artifacts that are due to the used detector technology.

Two typical devices are used to convert X-ray in intensity images:

- Image intensifiers (II, introduced ˜ 1940))
- Flat panel detectors (FD, introduced ˜ 2000)

Note:

- Both technologies are still used in hospitals.
- New equipment is mostly shipped with flat panels.
- Research systems use flat panels, image intensifiers are history.

X-ray Tube

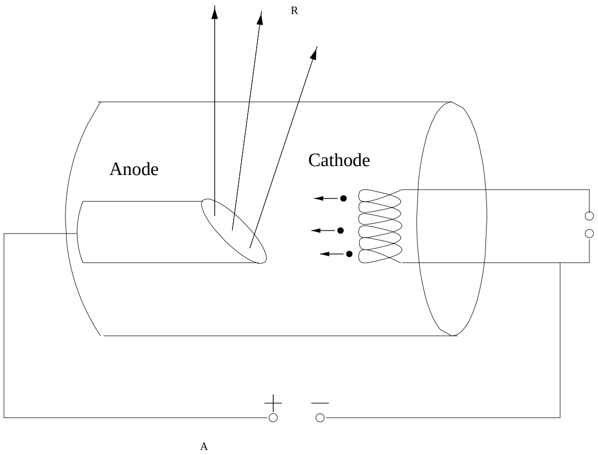

Figure: Concept of traditional X-ray Tube

X-ray Tube

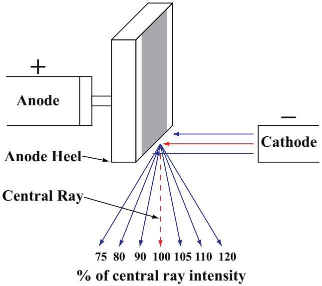

Figure: Heel effect (from http://www.nova.edu/ocean/coralxds)

The heel effects lead to images that show a gray level ramp. The elimination of these inhomgeneities will be discussed in upcoming lectures.

X-ray Tube

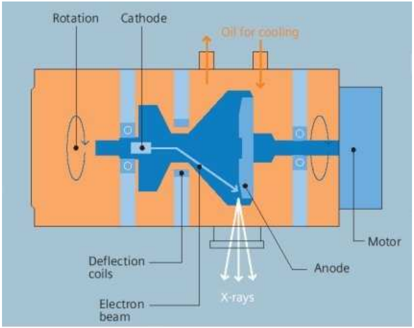

Figure: Concept of the Straton X-ray Tube (image: Siemens AG)

The **engineers from Erlangen** who developed this new X-ray tube were under the final four for the German Zukunftspreis 2005 (http://www.deutscher-zukunftspreis.de/newsite/aktuelles/07.shtml).

X-ray Tube

Figure: X-ray Tubes: Traditional X-Ray Tube (left), Straton X-Ray Tube (images: Siemens Healthcare)

X-Ray Detectors

Figure: Image intensifier (left) and flat panel detector (right)

X-Ray Detectors in Cardiology

Figure: C–arm device with image intensifier (left) and flat panel detector (right)

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Image Intensifier

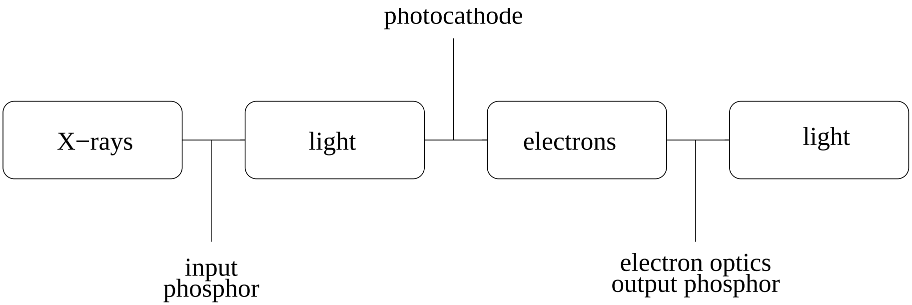
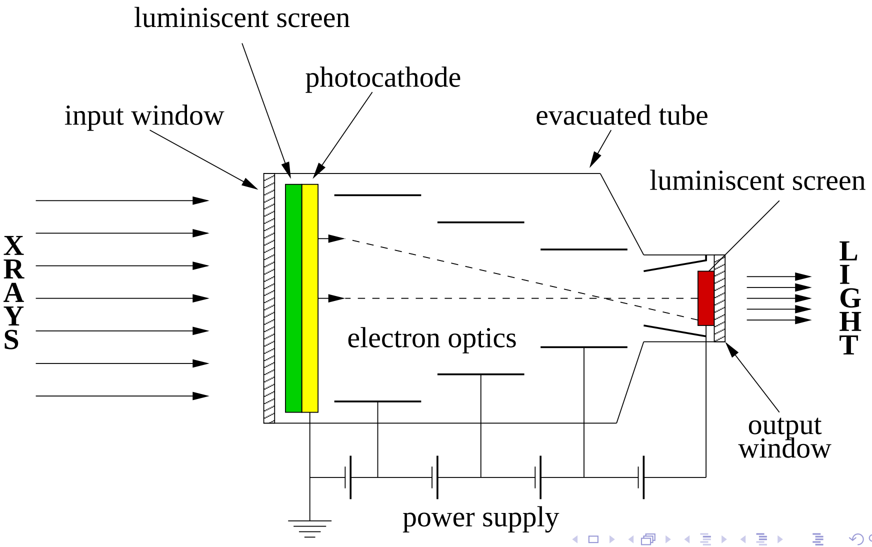


Figure: Basic principle of image intensifier

Image Intensifier



X-Ray to Intensity Conversion

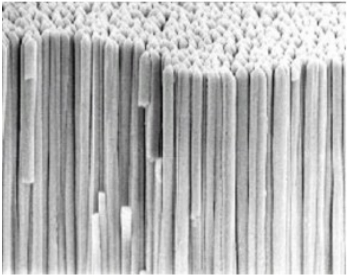


Figure: Due to its cristal structure, CsI minimizes lateral diffusion and scattering, i.e. it supports to preserve spatial resolution

Materials used in image intensifiers (II):

input luminescent screen:	CsI:Na
photocathode:	$SbCs_3$
output luminescent screen:	ZnCdS:Ag

Image Intensifier and Image Distortion

Image distortion using II technology is caused by several phenomena:

- magnetic field affects the accelerated electrons in vacuum tube
  - earth magnetic field
  - artificial magnetic field (for instance, MR scanner or Niobe system)
- scattering (veiling glare)
- convex entrance screen

Image Distortion

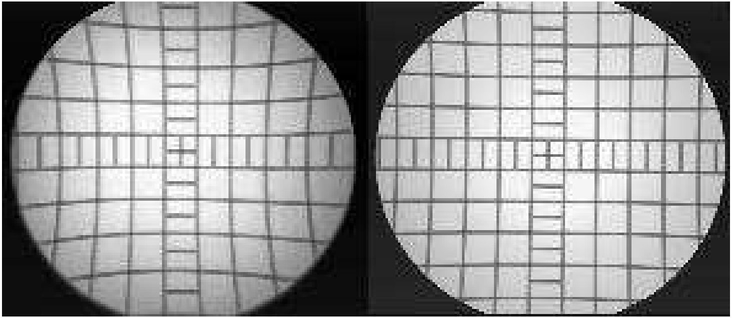


Figure: Example of an distorted (left) and an undistorted (right) image

Distortion Correction

Definition

Image undistortion (or **distortion correction**) is a image-to-image mapping that eliminates the distortions implied by the image acquisition device in the image plane.

Distortion Correction

We distinguish between two different types of image distortion:

- **Geometric distortion:** The acquisition device modifies the geometry of the mapped object. In simple terms, we expect that in undistorted images straight lines in 3-D end up as straight lines in the 2-D image plane.
- **Intensity distortion:** The acquistion device implies changes in intensities. In simple terms, we expect that in undistorted images identical tissue classes are mapped to identical intensities. Color normalization or homogenization of illumination

Distortion Correction

How can we correct geometric image distortion?

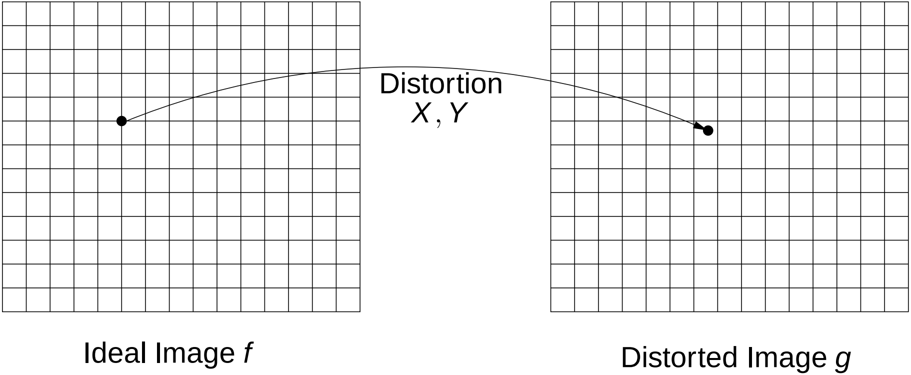


Figure: Image (un-)distortion: Mapping of pixels or image points

Geometric Image Undistortion: Core Problems

Problems to be solved for the implementation of a geometric image undistortion algorithm:

- Definition of a parametric or non-parametric **mapping** between undistorted and distorted image
- **Interpolation** of intensities of neighboring pixels, because lattice points of the undistorted image are not necessarily mapped to lattice points in the distorted image.
- Robust and reliable **estimation** of parameters or displacement vectors of the mapping
- Development of **efficient and robust algorithms** to run distortion correction (for instance, real-time image undistortion in cardiology with 30 fps (frames per second))

Image Undistortion: Model Design

Geometric image undistortion is a **three–stage process**:

- model design
- estimation of model parameters (calibration)
- inference

Image Undistortion: Model Design

Remarks on design issues:

- **Rule of thumb:** sample **always** in the space of your output.
- parametric vs. non–parametric model
- dimension of parameter space should be selected carefully ($\longrightarrow$ curse of dimensionality!)
- linear vs. non–linear estimators
- optimum use of available hardware (e.g., many-core architectures, graphics card (GPU), Cell processor, etc.)

Distortion Correction Mapping

Non-Parametric Mapping: look-up table or displacement vector field.

Parametric Mapping:

- image point of undistorted image: (x', y')
- image point of distorted image: (x, y)
- coefficients $u_{i,j}, v_{i,j} \in \mathbb{R}$
- we assume separable base functions and thus require univariate base functions: $b_k : \mathbb{R} \to \mathbb{R}$, where $k = 0, \ldots, d$.

$$x = X(x', y') = \sum_{i=0}^{d} \sum_{j=0}^{d-i} u_{i,j} b_j(y') b_i(x') \quad (1)$$

$$y = Y(x', y') = \sum_{i=0}^{d} \sum_{j=0}^{d-i} v_{i,j} b_j(y') b_i(x') \quad (2)$$

Image Undistortion

Example

Standard polynomials
We choose:

$$b_i(x) = x^i \tag{3}$$

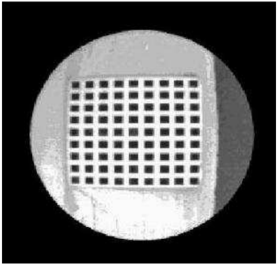
and get the following bi-variate polynomial for the x-coordinates:

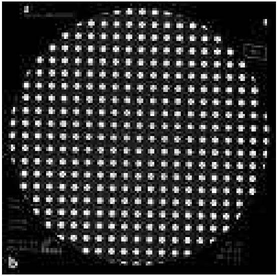
$$x = X(x', y') = \sum_{i=0}^{d} \sum_{j=0}^{d-i} u_{i,j} y'^j x'^i \tag{4}$$

Distortion Correction: Calibration of the Mapping

Definition

We call the estimation process of parameters that define the mapping of real-world objects into the camera image plane **calibration**.





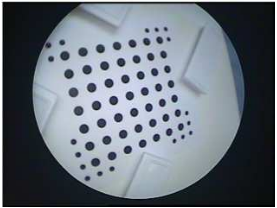


Figure: A few examples for a calibration patterns with squares, dots, and circles

Distortion Correction: Calibration of the Mapping

Calibration problem for distortion correction:

- N 2–D points on the planar calibration pattern are precisely known: $(x'_1, y'_1), (x'_2, y'_2), \dots (x'_N, y'_N)$
- N observed points in the distorted image: $(x_1, y_1), (x_2, y_2), \dots (x_N, y_N)$
- Problem: estimate the parameters of the distortion function.

Distortion Correction: Calibration of the Mapping

Definition

Least square estimation is a numerical procedure that fits a parametric or non-parametric curve to data points by the minimization of the sum of squared distances of data points from the curve.

Distortion Correction: Calibration of the Mapping

For our very concrete situation the least square estimation results in the following optimization problems:

$$\sum_{n=1}^{N} \left(X(x'_n, y'_n) - x_n\right)^2 \quad \rightarrow \quad \min \tag{5}$$

$$\sum_{n=1}^{N} \left(Y(x'_n, y'_n) - y_n\right)^2 \quad \rightarrow \quad \min \tag{6}$$

Distortion Correction

Set of N equations that are linear in $u_{i,j}$ and $v_{i,j}$:

$$x_n = \sum_{i=0}^{d} \sum_{j=0}^{d-i} u_{i,j} b_j(y'_n) b_i(x'_n) \tag{7}$$

$$y_n = \sum_{i=0}^{d} \sum_{j=0}^{d-i} v_{i,j} b_j(y'_n) b_i(x'_n) \tag{8}$$

This can be rewritten in matrix notation (here shown for x_i only):

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} u_{0,0} \\ u_{0,1} \\ \vdots \\ u_{d,0} \end{pmatrix} \tag{9}$$

Remarks on Measurement Matrix

The matrix $\boldsymbol{A}$ is the measurement matrix for the particular problem of image undistortion.

Exercises:

1. Compute the components of $\boldsymbol{A}$.
2. What is the rank of $\boldsymbol{A}$?
3. Suggest an algorithm to check the proper matching of (x_n, y_n) and (x'_n, y'_n).

Distortion Correction

Estimates:
Estimates of coefficients are:

$$\begin{pmatrix} u_{0,0} \\ u_{0,1} \\ \vdots \\ u_{d,d} \end{pmatrix} = \boldsymbol{A}^{\dagger} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} \tag{10}$$

where $\boldsymbol{A}^{\dagger}$ is the pseudo–inverse of $\boldsymbol{A}$. As we know, the pseudo–inverse can be computed by using the singular value decomposition.

Remarks on Parameterization

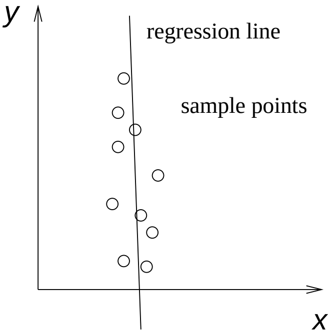


Figure: Regression line with infinite slope

Remarks on Parameterization

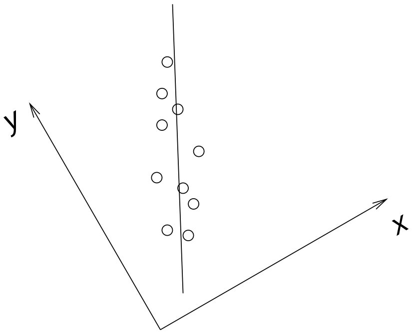


Figure: Regression line (rotated reference coordinate system)

Remarks on Parameterization

The parameterization of the straight line decides on the sensitivity of estimated parameters to variations in input data. A well-conditioned problem might appear ill-conditioned if the parameterization of the problem is not done properly.

For straight lines we observe:

- line representation $y = mx + t$ has singularities: The more parallel the regression line to y-axis, the larger m; for lines parallel to the y-axis, we observe the singularity $m = \infty$.
- a **fair** representation of straight lines is $x \ \sin\alpha + y \ \cos\alpha = d$.

Conclusion:

Select a parameterization that is independent from orthogonal transforms of the reference coordinate system.

Remarks on Parameterization

Definition

Interpolation defines the estimation of unknown data between observed data. In addition we require the interpolation curve to fit all the training data.

Definition

Like interpolation **regression** defines a technique to discover a mathematical relationship between multiple variables using a set of data points, i.e. training data. In regression it is not required that the regression curve fits the training data perfectly.

The regression function is usually estimated using a least square approach. The transition from interpolation to regression is smoothly, and some authors do not differentiate these two techniques explicitly.

Remarks on Parameterization

Definition

Overfitting is defined as training a model, for instance, parametric model, so that it well fits the training data, but fails to predict well in between and outside the data.

Overfitting can occur, if a complex model (for instance, a model with many parameters) is trained with a sparse set of data, i.e. too few training examples.

Remarks on Parameterization

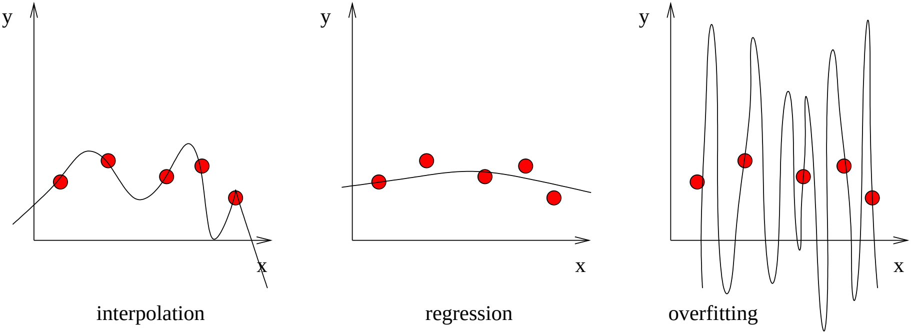


Figure: Interpolation vs. regression vs. overfitting

Remarks on Parameter Estimation

- **Problem**
  Compute the sensitivity of the estimated parameters from a set of N point correspondences.
- The parameters shall fit for all data that is processed by the used algorithm. How can we figure if the estimated parameters are sufficient for the observed data in practice? We might get different data as input for the algorithm
- To compute the sensitivity (robustness) of the estimated parameters we need many data samples
- But what if we do not have many samples?

Parameter Estimation: Bootstrapping

Solution *As an allusion to lifting oneself up by one's own bootstraps, the term* **bootstrapping** *means using a special process to perform a task that one would be unable to do in general. ('The Surprising Adventures of Baron Münchhausen')*

Parameter Estimation: Bootstrapping

- We discuss the following problem: given a random sample $\mathbf{X} = (X_1, X_2, \cdots, X_n)$ from an unknown probability distribution F, estimate the sampling distribution of some prespecified random variable $R(\mathbf{X}, F)$, on the basis of the observed data $\mathbf{x}$
- Standard jackknife theory gives an approximate mean and variance in the case $R(\mathbf{X}, F) = \theta(\hat{F}) - \theta(F), \theta$ some parameter of interest
- A general method, called the 'bootstrap,' is introduced from Bradley Efron (1979); it works satisfactorily on a variety of estimation problems
- The jackknife is shown to be a linear approximation method for the bootstrap.
- **The bootstrap is distinguished from the jackknife procedure, used to detect outliers, and cross-validation, used to make sure that results are repeatable**

Parameter Estimation: Bootstrapping

- **Bootstrapping I:**
  - estimate parameters from k different subsets of given point correspondences
  - compute the covariance matrix of parameter vector
- **Bootstrapping II:**
  - estimate parameters from k different $N - 1$ point correspondences
  - map the unconsidered point into the distorted image using the estimate
  - compute the mean error of mapped points

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Scaling of Input Data

- proper scaling of data is **crucial** for the quality of the output (usually overseen by engineers!).
- limited numerical accuracy requires certain ranges
- "data normalization must not be considered optional" (Richard Hartley)
- select the optimum scaling by minimization of the condition number to minimize sensibility **and** to find a proper data range:

$$\kappa(\boldsymbol{A}^{\mathsf{T}}\boldsymbol{A}) \quad \rightarrow \quad \min \quad . \tag{11}$$

Scaling of Input Data

Example

- Use a polynomial of total degree 5 to undistort images.
- Input images are 1024×1024–image.
- The x and y coordinates are represented in pixels, i.e. $x, y \in 1, 2, \ldots 1024$
- The monomials range from 1 to $1024^5 = 1125899906842624$
- The result has to be between 0 and 1023!!!

Think about it! Do you have a good feeling in doing this?

Minimization of κ

The Gramian matrix can be used to test for linear independence of functions.
Any decrease of the condition number will be useful, even if it is not a global optimum!
Method to compute a proper scaling:

- select constants k and l
- scale all data points (x_i, y_i) to (kx_i, ly_i)
- rewrite (9) and compute new $\boldsymbol{A}$
- compute condition number $\kappa(\boldsymbol{A}^\mathrm{T}\boldsymbol{A})$
- minimize with respect to k and l, e.g. by gradient descent
- finally, recover the original coefficients $u_{i,j}, v_{i,j}$ and invert the scaling process

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Efficient Evaluation of Polynomials

Problem: each acquired image requires a distortion correction.
Observations:

- bivariate polynomials are univariate polynomials where coefficients are univariate polynomials:

$$x = \sum_{i=0}^{d} \left(\sum_{j=0}^{d-i} u_{i,j} y'^j \right) x'^i \tag{12}$$

- univariate polynomials are evaluated using Horner's scheme

$$p(x) = \sum_{i=0}^{d} a_i x^i = (\dots(a_d x + a_{d-1})x + \dots)x + a_0 \quad . \tag{13}$$

Efficient Evaluation of Polynomials

Definition

Horner scheme is named after William George Horner, is an algorithm for the efficient evaluation of polynomials in monomial form. Horner's method describes a manual process by which one may approximate the roots of a polynomial equation. The Horner scheme can also be viewed as a fast algorithm for dividing a polynomial by a linear polynomial (Wikipedia)

- for each line we get a univariate polynomial ($y' =$ const)
- row and column increments are constant
- arithmetic progression: reuse of former evaluations

Conclusion: After an initialization, for each pixel only sums have to be computed.

Arithmetic Progression

Definition

Arithmetic Progression is a sequence of numbers such that the difference of any two successive members of the sequence is a constant.

Problem:
Evaluate a polynomial $p(x)$ at $x_0, x_0 + h, x_0 + 2h, \ldots$ efficiently.
Solution:

$$p(x_0 + kh) = \binom{k}{0}\beta_0 + \binom{k}{1}\beta_1 + \ldots \binom{k}{d}\beta_d \quad , \tag{14}$$

where β_j is computed initially by the following algorithm.

Arithmetic Progression Algorithm

FOR $i := 1 \dots d$
$\beta_i = p(x_0 + j \cdot h)$
FOR $i := 1 \dots d$
FOR $j := d, d-1, d-2, \dots i$
$\beta_j = \beta_j - \beta_{j-1}$

Figure: Auxiliary variables for computation of arithmetic progression

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Interpolation of Intensities

- **Nearest Neighbor Interpolation** is the simplest interpolation method. We assign to an image point in between pixels the intensity value of the closest pixel. Usually the Euclidean distance is applied.
  Mostly the results of nearest neighbor interpolation are not well appreciated. The images appear crispy and noisy, though the interpolation method is extremely fast.
- In **Bilinear Interpolation** we compute for the new image point a weighted mean of neigboring intensities. Here the intuition is applied that pixels closer to the new image point have an higher impact on the final intensity value.
  In most practical applications where interpolation is required, bilinear interpolation is applied.

Interpolation of Intensities

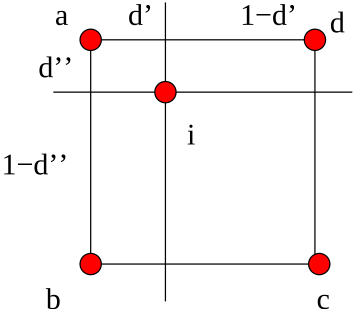

Figure: Interpolation

Computation of the intensity *i*:

- nearest neighbor interpolation

$$i = a$$

- bilinear interpolation

$$\begin{aligned} x &= a(1-d') + dd' \\ y &= b(1-d') + cd' \\ i &= x(1-d'') + yd'' \end{aligned}$$

Final Distortion Correction Algorithm

FOR $x :=$ first to last line		
	FOR $y :=$ first to last column	
		compute coordinate $x = X(x, y)$
		compute coordinate $y = Y(x, y)$
		compute intensity $f(x, y) = \text{interpol}(g, x, y)$

Figure: Image undistortion routine

Hardware Accelerated Image Warping

- image is decomposed into squares
- map the vertices of the square
- undistortion can be implemented using texture mapping unit
- bi–linear interpolation hardware supported in GPUs
- texture mapping hardware supported in GPUs

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Application of Image Undistortion: Endoscopy

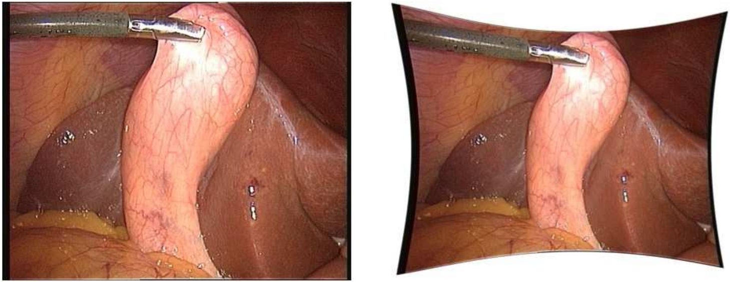

Figure: Original, distorted endoscope image and the result of distortion correction

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Take Home Messages

- In X-ray imaging different detector technologies are used that cause different artifacts.
- Image intensifiers yield images that are distorted.
- We are now familiar with the core concepts of calibration and least square estimators.
- We learnt about the efficient evaluation of polynomials; it is worth thinking through a problem in very detail that appears as simple as the evaluation of polynomials.
- Data normalization is mandatory.
- Bilinear interpolation is sufficient for nearly all practical problems.

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Further Readings

- An excellent overview of different detectors used in X–ray equipment can be found in the book edited by Heinz Morneburg: Bildgebende Systeme für die Diagnostik (in German)
- In case you need to learn more about polynomials and the efficient evaluation of polynomials you have to read Volume 2 of Prof. Knuth's classic work on
  The Art of Computer Programming.
- Literature on the bias-variance trade–off:
  - The Elements of Statistical Learning
  - Here you can find a simple and easy to read introduction
- Information on the Distortion Correction Products can be found on vendors homepages. Try, for instance, www.siemensmedical.com and search for Distortion Correction.