
Signal
Fourier
Problems
MATLAB
systems
z-Transform
data
signal
book
Transform
discrete-time
continuous-time
Control
analysis
FFT
Haar
wavelet
Systems
course
Discrete-Time
edition
system
Analysis
Solutions
students
DTFT
coefficients
text
time
speech
representation
transform
Representation
demos
used
use
function
covered
State
Web
Sampling
website
material
digital
version
third
Laplace
Design
previous
Section
shown
various
electrical
Filters
devices
Models
University
Prentice
properties
Version
begins
DFT
downloaded
transfer
Professor
Continuous-Time
one-semester
one-quarter
model
waveforms
aspects
variable
linear
parts
Toolbox
Simulink
Application
treatment
problems
theory
new
considered
study
engineering
Properties
illustrations
control
online
System
filtering

Contents

Chapter 1

Fourier Series

The frequency spectrum is a complex-valued function of the frequency variable, and thus it is usually specified in terms of an amplitude spectrum and a phase spectrum [1]. The complex exponential form is given by:

$$
\begin{aligned}
x(t) &= \sum_{-\infty}^{\infty} X_n e^{jn\omega_0 t} \\
X_n &= \int_{-T/2}^{T/2} x(t) e^{-jn\omega_0 t}\, dt
\end{aligned}
\tag{1.1}
$$

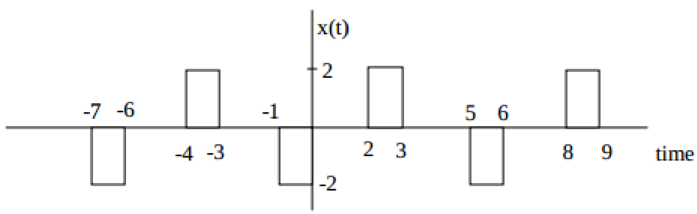
In the next exercises we will also be using the first and/or the second derivative of the previous expressions (**??**), and therefore we write them here explicitly:

$$
\begin{aligned}
\dot{x}(t) &= \sum_{-\infty}^{\infty} jn\omega_0 X_n e^{jn\omega_0 t} \\
jn\omega_0 X_n &= \int_{-T/2}^{T/2} \dot{x}(t) e^{-jn\omega_0 t}\, dt
\end{aligned}
\tag{1.2}
$$

$$\ddot{x}(t) = \sum_{-\infty}^{\infty} -n^2\omega_0^2 X_n e^{jn\omega_0 t}$$
$$-n^2\omega_0^2 X_n = \int_{-T/2}^{T/2} \dot{x}(t) e^{-jn\omega_0 t}\, dt \quad (1.3)$$

Problem 1

For the following signal:



a) Find the Fourier series.

b) Plot the spectra versus frequency, $\omega = n\omega_0$.

Solution

The period of the shown signal is $T = 6$ and therefore $\omega_0 = \frac{2\pi}{T} = \frac{\pi}{3}$.

Taking the derivative of $x(t)$ we get:

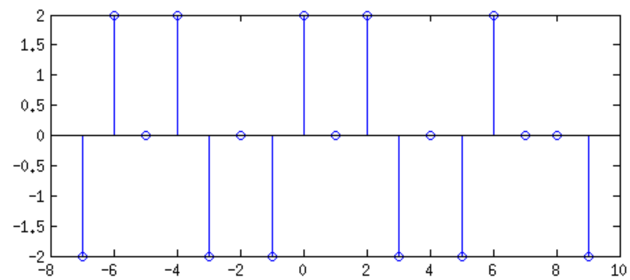


Figure 1.1: Derivative $\dot{x}$

The range t = [-3,3] contains one complete period of the signal. Usign (1.2) we have:

$$\dot{x}(t) = 2(-\delta(t+1) + \delta(t) + \delta(t-2) - \delta(t-3))$$

The Fourier coefficients are obtained with:

$$\begin{aligned}
jn\omega_0 X_n &= \frac{2}{6}\int_{-3}^{3}(-\delta(t+1) + \delta(t) + \delta(t-2) - \delta(t-3))e^{-jn\omega_0 t}\,dt \\
&= \frac{1}{3}(-e^{jn\omega_0} + 1 + e^{-2jn\omega_0} - e^{-3jn\omega_0}) \\
&= \frac{1}{3}[e^{\frac{jn\omega_0}{2}}(e^{\frac{-jn\omega_0}{2}} - e^{\frac{jn\omega_0}{2}}) - e^{\frac{-5jn\omega_0}{2}}(e^{\frac{-jn\omega_0}{2}} - e^{\frac{jn\omega_0}{2}})] \\
&= \frac{1}{3}[(e^{\frac{-jn\omega_0}{2}} - e^{\frac{jn\omega_0}{2}})(e^{-jn\omega_0}(e^{\frac{3jn\omega_0}{2}} - e^{\frac{-3jn\omega_0}{2}}))] \\
&= \frac{4j}{3}[\sin\frac{n\omega_0}{2}\sin\frac{3n\omega_0}{2}e^{-jn\omega_0}] \\
X_n &= \frac{-4j}{n\pi}[\sin(n\frac{\pi}{6})\sin(n\frac{\pi}{2})e^{-jn\frac{\pi}{3}}]
\end{aligned}$$

Next we use Matlab to plot the magnitude and phase of the spectra using the script given in [2]

Listing 1.1: Calculate and plot magnitude and phase of Xn

```
n=1:15;
Xn=-4*j./n/pi.*sin(pi*n/6).*sin(n*pi/2).*exp(-j*n*pi/3);
n=-15:-1;
X_n=-4*j./n/pi.*sin(pi*n/6).*sin(n*pi/2).*exp(-j*n*pi/3);
Xn=[X_n 0 Xn];
n=-15:15;
```

```
subplot(211),stem(n,abs(Xn));
 title('|X_n|')
subplot(212),stem(n,angle(Xn))
 title('angle(X_n) in rad')
```

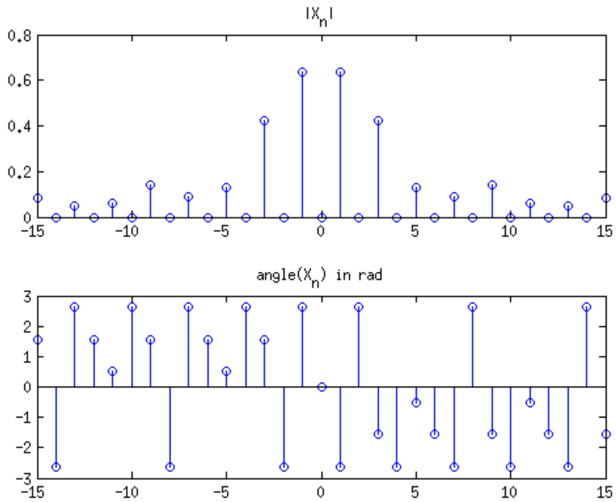


Figure 1.2: Magnitude and Angle X_n

We then plot the approximation of the function using its Fourier coefficients [2].

Listing 1.2: Approximation of x(t) with Fourier coefficients

```
function [x,t] = fapprox(N,T)
    t = -1.5*T:T/1000:1.5*T;
    w0 = 2*pi/T;
    X0 = 0;
    x = X0*ones(1,length(t)); % dc component
    for n=1:N,
     Xn = -4*j/n/pi*sin(pi*n/6)*sin(n*pi/2)*exp(-j*n*pi/3);
     X_n = conj(Xn);
     x = x + Xn*exp(j*n*w0*t) + X_n*exp(-j*n*w0*t);
    end
end
```

We do this for N=5 and N=50:

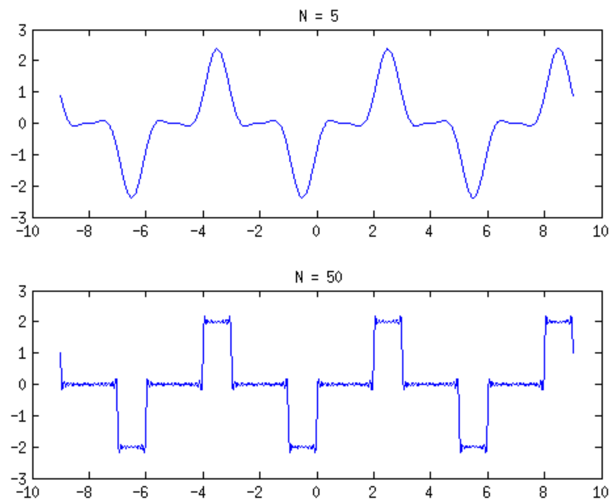
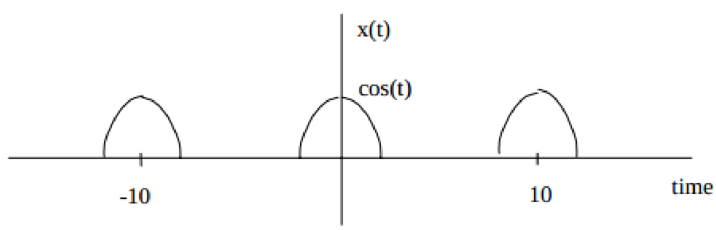


Figure 1.3: Approximation of x(t) by Xn

As we can see, the larger the N the more close to the original function we get. However, as a consequence of the Gibbs effect, we can't say that it'll be equal.

Problem 2

Repeat problem 1 for the following signal:



Solution

The period of the shown signal is $T = 10$ and therefore $\omega_0 = \frac{2\pi}{T} = \frac{\pi}{5}$.

If we take the first and second derivative of $x(t)$ we get:

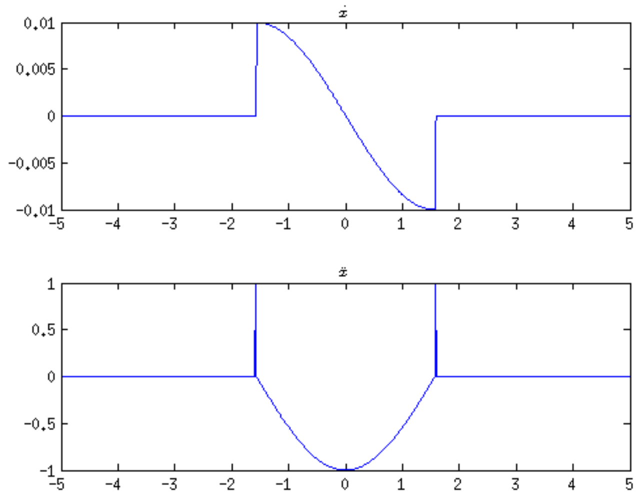


Figure 1.4: Derivative $\dot{x}$

The range t = [-5,5] contains one complete period of the signal. Applying (1.2) we have:

$$\ddot{x}(t) = -x(t) + \delta(t + \pi/2) + \delta(t - \pi/2)$$

$$\sum_{-\infty}^{\infty} -n^2\omega_0^2 X_n e^{jn\omega_0 t} = \sum_{-\infty}^{\infty} X_n e^{jn\omega_0 t} + \delta(t + \pi/2) + \delta(t - \pi/2)$$

$$\sum_{-\infty}^{\infty} (1 - n^2\omega_0^2) X_n e^{jn\omega_0 t} = \delta(t + \pi/2) + \delta(t - \pi/2)$$

Whe can now obtain X_n with:

$$
\begin{aligned}
(1-n^2\omega_0^2)X_n &= \frac{1}{T}\int_{-5}^{5} \delta(t+\pi/2)+\delta(t-\pi/2)e^{-jn\omega_0 t}\,dt \\
&= \frac{1}{T}(e^{jn\frac{\omega_0}{2}} + e^{-jn\frac{\omega_0}{2}}) \\
X_n &= \frac{1}{5(1-\frac{n^2\pi^2}{25})}\cos(\frac{n\pi^2}{10})
\end{aligned}
$$

Next we use Matlab to plot the magnitude and phase of the spectra using the script given in [2]

Listing 1.3: Calculate and plot magnitude and phase of Xn

```
n=-10:10;
Xn=cos(pi/2*n*w0)/5./(1-(n*w0).^2);
subplot(121),stem(n,abs(Xn))
 title('|X_n|')
subplot(122),stem(n,angle(Xn))
 title('angle(X_n) in rad')
```

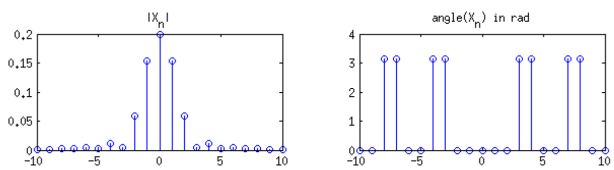


Figure 1.5: Magnitude and Angle X_n

We then plot the approximation of the function using its Fourier coefficients [2].

Listing 1.4: Approximation of x(t) with Fourier coefficients

```
function [x,t] = fapprox2(N,T)
    w0 = 2*pi/T;
    t = -1.5*T:T/1000:1.5*T;
    c0 = 1/5;
    x = c0*ones(1,length(t)); % dc component
    for n=1:N,
     cn = cos(pi/2*n*w0)/5/(1-(n*w0)^2);
     c_n = cn;
     x = x + cn*exp(j*n*w0*t) + c_n*exp(-j*n*w0*t);
    end
    plot(t,x)
    title([' N = ',num2str(N)])
end
```

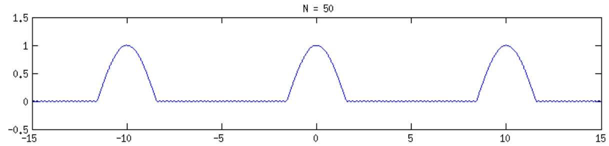


Figure 1.6: Approximation of x(t) by Xn

Chapter 2

Fourier Transform

Chapter 3

Haar Transform

Bibliography

[1] Edward Kamen and Bonnie Heck. *Fundamentals of Signals and Systems: With MATLAB Examples*. Prentice Hall PTR, 2000.

[2] Georgia Tech School of Electrical and Computer Engineering. Worked Problems, Chapter 4. `http://users.ece.gatech.edu/~bonnie/book/worked_problems/Chap4_Fseries_sol.pdf`. [Online; accessed Apr-2013].