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Controlling the Motion of a Group of Mobile Agents

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Abstract—We propose a method of controlling an ensemble of mobile agents with variable coupling topology that is based on the principles of phase synchronization in a system of regular and chaotic oscillators. Results of modeling of the controlled motion of mobile agents in systems with serial, parallel, and strictly preset motion are presented.

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In recent years, increasing attention of researchers and engineers has been devoted to the problem of control over networks of autonomous mobile agents. This interest is related to the fact that a new approach to the description of static and dynamic structures in terms of a network of mobile agents has been successfully used in biology [1–5], medicine [6], social interactions [7, 8], and other fields. Investigation of the mechanisms of self-organization and collective motion of large groups in the nature (e.g., flying flocks of birds [4]) will simplify collective control over the operation of a number of robots [9–11] or transport vehicles [12] used by human beings. In most cases, the structure of such groups observed in living beings is random and variable with the time (e.g., neuronal ensembles [13], mobile communication networks [14]). For this reason, an increasing number of investigations have been devoted to networks with time-varying topology of coupling [15–18].

The present work is devoted to organization of control over an ensemble of mobile agents with a time-varying coupling structure.

By "mobile agent," we imply a material point moving on the (x, y) plane so that its trajectory coincides with the projection of a trajectory of some chaotic oscillator (assigned to this agent) onto the (x, y) plane. All results will be readily extended to the case of motions in a three-dimensional space. Without losing generality, let us consider a chaotic oscillator represented by the Rössler system:

$$\begin{cases} \dot{x}_i = -\omega_i y_i - z_i = f_i(y, z, \omega), \\ \dot{y}_i = \omega_i x_i + a y_i = g_i(x, y, \omega, a), \quad i = \overline{1, N}, \\ \dot{z}_i = b + z_i(x_i - c) = h_i(x, z, b, c), \end{cases} \tag{1}$$

where a, b, c, and ω are positive parameters. Below, we consider the case of $a = 0.22$, $b = 0.1$, $c = 8.5$, and ω_i values uniformly distributed on a certain segment.

The organization of control over the motion of agents in the space can be subdivided into two steps: (i) establishing a preset configuration of agents (in the case under consideration—serial or parallel) and (ii) driving agents to move along some trajectory.

In order to ensure a preset configuration of the ensemble of mobile agents, let us use the theory of chaotic phase synchronization. In addition to attaining a synchronous behavior of agents, this will make it possible to control the group as a whole rather than every individual agent. The use of chaotic oscillators determining the motion of agents allows us not to think of a synchronization domain on the (x, y) plane, since all agents will earlier or later accompany a sufficiently broad region of this plane.

The mode of coupling between agents in the ensemble is set differently, depending on a topology under consideration, but the coupling strength always obeys the following condition:

$$d = \begin{cases} \bar{d}, & \text{for} \quad (x_i - x_j)^2 + (y_i - y_j)^2 < r^2, \\ 0, & \text{otherwise}, \end{cases} \tag{2}$$

where $\tilde{d} = \text{const}$ and r determines the size of region G in which agents interact. This definition of the coupling strength corresponds to an assumption that the system topology varies with the time so that agents begin to interact only on approaching each other to a sufficiently short distance. Depending on the time of establishment of the coupling, we can speak of the formation of a single group or several groups of synchro-

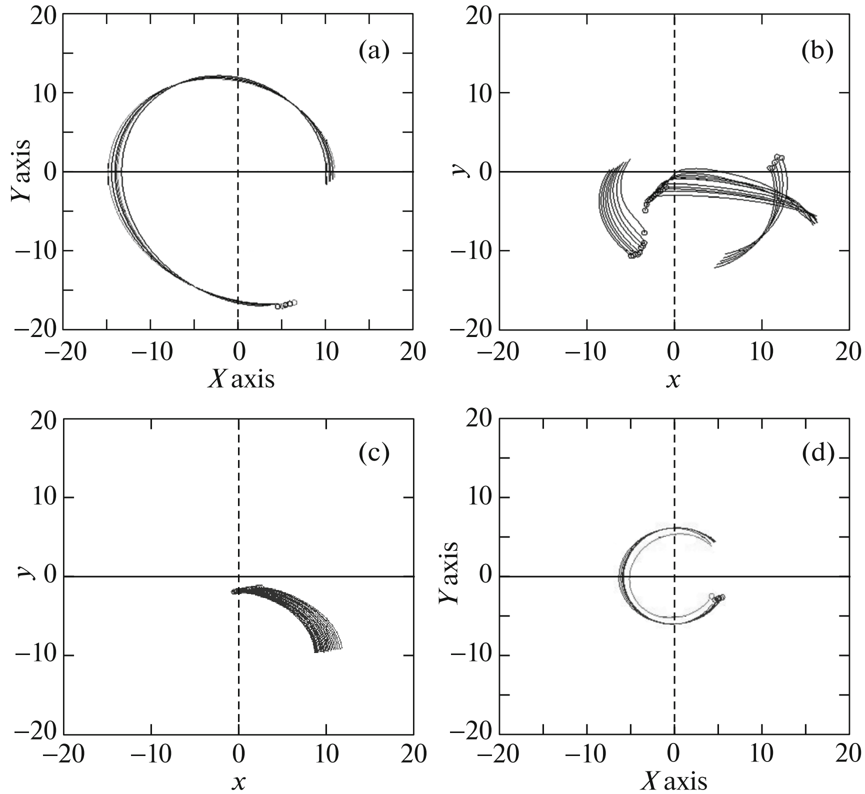

Fig. 1. Organized motion of mobile agents in system (1) with coupling strength $d' = 0.2$ at radius $r = 3$ ($a_i = 0.22$, $b_i = 0.1$, $c_i = 8.5$, $\omega_i \in [0.93; 1.07]$): (a) serial motion in a single group ($N = 8$), (b) parallel motion in groups ($N = 25$), (c) parallel motion in a single group ($N = 25$), and (d) driven motion along a limit cycle trajectory in the van der Pol system ($N = 8$).

nized agents that can be separately controlled. It is assumed that the coupling strength is sufficient for attaining phase synchronization in the system.

Below, we consider three model problems of control over the motion of agents. The first two problems are related to the use of mutual chaotic synchronization for the organization of stable structures. The third problem demonstrates control over the motion of agents in these structures.

Problem 1: *serial motion in a chain of agents*. This motion is organized by introducing the following mode of coupling in the second equation of system (1):

$$\begin{cases} \dot{x}_i = f_i(y, z, \omega), \\ \dot{y}_i = g_i(x, y, \omega, a) + \sum\limits_j d(y_j - y_i), \quad i = \overline{1, N}, \\ \dot{z}_i = h_i(x, z, b, c). \end{cases} \quad (3)$$

In the case of phase synchronization [19], a certain (limited but not fixed) phase shift takes place that can be controlled by the coupling strength. The presence of this shift ensures serial (one by one) motion of agents without collisions. Coupling strength d' in this case is set large enough to attain phase synchronization. Since the agents do not reach region G simultaneously, they are sequentially synchronized with the and, hence, the power of the cluster (group) of agents in this organized motion sequentially increases. A number of such clusters of mutually synchronized agents can be formed. The variant with a single global cluster in which all agents move one by one in a chain is illustrated in Fig. 1a. The phase synchronization involves a certain (limited but not fixed) phase shift that can be controlled by changing the coupling strength.

Problem 2: parallel motion in a chain of agents. In this case, in addition to the coupling for mutual synchronization as in the preceding problem, we introduce a "repulsive" coupling with respect to coordinate x:

$$\begin{cases} \dot{x}_i = f_i(y, z, \omega) + \sum\limits_j d(x_j - x_i) + \sum\limits_j \dfrac{d}{(x_i - x_j)}, \\ \dot{y}_i = g_i(x, y, \omega, a) \qquad\qquad i = \overline{1, N}, \\ \dot{z}_i = h_i(x, z, b, c). \end{cases} \quad (4)$$

This coupling provides that, if two agents approach each other, opposite forces arise that drive these agents to repulse until reaching a certain finite spacing. This behavior is illustrated in Fig. 1b (cluster synchronization) and Fig. 1c (global synchronization).

Problem 3: motion of agents along a preset trajectory. For solving this task, agents constituting system (1) are supplemented with an additional agent that moves along a preset trajectory. This is the driving agent, and the other (driven) agents must follow its motion. In nature, this behavior corresponds to that of, e.g., a leader in a herd. Coupling between the driving and other agents is unilateral and leads to forced synchronization. Below, we consider two examples of driving an ensemble of mobile agents on preset trajectories.

Example 1. The preset trajectory represents a limit cycle in the van der Pol system. The van der Pol oscillator can be described as follows:

$$\begin{cases} \dot{X} = -Y, \\ \dot{Y} = X + \mu(1 - X^2)Y, \end{cases} \tag{5}$$

where μ is a small parameter. Dynamic equations for the other agents are written as

$$\begin{cases} \dot{x}_i = f_i(y, z, \omega), \\ \dot{y}_i = g_i(x, y, \omega, a) + \sum_j d(y_j - y_i) + D(Y - y_i), \quad i = \overline{1, N}, \\ \dot{z}_i = h_i(x, z, b, c), \end{cases} \tag{6}$$

where coupling D is included by analogy with d, but only when a mobile agent approaches the driving agent that moves along the preset trajectory.

Depending on a regime realized in the van der Pol oscillator, it is possible either to bring all agents to a static state where they cease to move (so that equilibrium is established in all dynamical systems) or drive them to a trajectory that is close to the limit cycle (see Fig. 1d).

Example 2. The target trajectory represents a chaotic trajectory in the Lorentz system

$$\begin{cases} \dot{X} = \sigma(Y - X), \\ \dot{Y} = X(r - Z) - Y, \\ \dot{Z} = XY - bZ, \end{cases} \tag{7}$$

where the parameters are selected so as to realize a chaotic regime. In order to obey control of the Lorentz system, it was found necessary to introduce coupling in both y and x coordinates:

$$\begin{cases} \dot{x}_i = f_i(y, z, \omega) + D(X - x_i), \\ \dot{y}_i = g_i(x, y, \omega, a) + \sum_j d(y_j - y_i) + D(Y - y_i), \quad i = \overline{1, N}, \\ \dot{z}_i = h_i(x, z, b, c). \end{cases} \tag{8}$$

In this case, a mobile agent corresponding to the Lorentz attractor gradually captures all other mobile agents while remaining on the own trajectory.

Thus, the results of investigation of the collective control of a group of mobile agents based on a chaotic Rössler oscillator show that the entire ensemble can be driven to move as a single group or several groups. By adding various kinds of dynamic coupling either between mobile agents in the ensemble or with an "external" agent, it is possible to obtain different configurations of agents moving over various trajectories. Using this approach, it is possible to organize adaptive control over mobile agents depending on the external conditions.

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