Journal of Statistical Mechanics:
Theory and Experiment

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Journal of Statistical Mechanics: Theory and Experiment
An IOP and SISSA journal

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Received 8 July 2009
Accepted 22 October 2009
Published 18 November 2009

Online at stacks.iop.org/JSTAT/2009/P11005
doi:10.1088/1742-5468/2009/11/P11005

Abstract. We investigate the problem of controlling a group of mobile agents in a plane in order to move them towards a desired orbit via pinning control, in which each agent is associated with a chaotic oscillator coupled with those of neighboring agents, and the pinning strategy is to have the common linear feedback acting on a small fraction of agents by random selection. We explore the effects of the pinning probability, feedback gains and agent density in the pinning synchronization of a mobile agent network under a fast-switching constraint, and perform numerical simulations for validation. In particular, we show that there exists a critical pinning density for network synchronization with an unbounded region: above the threshold, the dynamical network can be controlled by pinning; below it, anarchy prevails. And for the network with a single bounded synchronization region, pinning control has little effect as regards enhancing network synchronizability.

Keywords: network dynamics, random graphs, networks, interacting agent models, nonlinear dynamics

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 1742-5468/09/P11005+13$30.00

Contents

1. Introduction

A significant problem under extensive study in recent years is that of how to regulate the collective dynamics of a complex network to obtain a synchronous behavior. The method of pinning control which involves applying localized feedback to a small fraction of network nodes to achieve the control goal has particularly attracted attention [1]–[8], partly because of the distributed nature of complex networks.

The study of the pinning strategy provides us with insights into the regulatory mechanisms in controlling complex networks of coupled dynamical systems, and endows us with heuristics for designing large artificial networks. Examples of such control can be commonly found in social networks, such as the emergence of collective behaviors and mass opinion formations [9, 10], and in biological systems, such as the mechanisms through which cell cycles are controlled and synchronized [11]–[13], as well as in communication systems, e.g., cluster-head nodes are able to manage and coordinate the information gathered by local nodes etc [14, 15].

From the perspective of control theory, the question that one comes across first is that of the controllability of the pinning control. Sorrentino suggested that the controllability of coupled dynamical networks via pinning can be assessed by means of a master stability function (MSF) approach [6, 16] which has been extensively used to handle synchronization of coupled systems. The MSF approach allows us to determine the stability of a linearly coupled dynamical network with a constant coupling matrix (or Laplacian) [17]–[19]. However, many real-world networks in biological, social, financial and communication systems exhibit time-varying topological structures: edges are deleted, added or rewired according to some evolving rules. Many researchers have devoted great attention to the study of this topic, including evolving network models that address dynamical linking according to predetermined rules [20]–[25], fast-switching synchronization with a characteristic timescale much shorter than that of the individual node's dynamics [26]–[28] and so on. All these investigations have made the regulation of complex networks with time-varying topologies possible.

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In this paper, we investigate the pinning control problem for a specific time-varying network model. The model arises from the interaction of mobile agents proposed by Frasca *et al* [28], and can be used for simplified representation, like clock synchronization in mobile robots [15], or task coordination of swarming animals [28]. How one controls the appearance of synchronized states of the dynamical network is of great significance in theory and potential applications. For the mobile agent network model, we introduce pinning control to regulate its synchronization, as an attempt to explain the control of complex time-varying systems. In this paper, we adopt the constraint of fast switching [26, 27] to derive synchronization conditions which relate synchronization to the pinning probability, the values of feedback gains and to the density of agents. Of course, the basic idea of pinning control is to reduce the number of controllers. Then how to determine the number of controlled agents is a challenging fundamental question in pinning control for a complex dynamical network. To the best of our knowledge, few results concerning the number of pinned nodes have been reported (a recent result gives an approximation via adaptive pinning control in [29]). This paper will give a positive answer for the number of pinned nodes in a pinning mobile agent network.

The rest of this paper is organized as follows. A mobile agent network model and the formulation of pinning control are introduced in section 2. In section 3, we investigate pinning synchronization for a controlled mobile agent network with two typical couplings, and we present theoretical results as well as illustrative examples for validation. Conclusions are finally drawn in section 4.

2. The problem formulation

Consider N mobile agents distributed in a two-dimensional space of size q with periodic boundary conditions. Each agent moves with velocity $\mathbf{v}_i(t)$ and direction of motion $\theta_i(t)$. The velocity $\mathbf{v}_i(t)$ is constant in modulus (denoted by v) and is updated in direction by the angle $\theta_i(t)$ for each time unit. The agents are considered as random walkers whose position and orientation are updated according to

$$\mathbf{y}_i(t+\Delta t) = \mathbf{y}_i(t) + \mathbf{v}_i(t)\Delta t, \tag{1}$$

$$\theta_i(t+\Delta t) = \eta_i(t+\Delta t), \tag{2}$$

where $\mathbf{y}_i(t)$ is the position of agent i in the plane at time t, $\eta_i(t)$, $i = 1, 2, \ldots, N$, are N independent random variables chosen at each time unit with uniform probability from the interval $[-\pi, \pi]$, and Δt is the time unit. In the following, we assume that the time unit is sufficiently small so that fast-switching synchronization is guaranteed.

Moreover, we use N identical chaotic oscillators as the network nodes with each oscillator associated with an agent. Here each agent is characterized by a state variable $\mathbf{x}_i(t) \in \mathbb{R}^n$, and evolves according to a given chaotic orbit. Note that agents i and j are assigned to be adjacent (neighbors, connected by an edge) if $|\mathbf{y}_i(t) - \mathbf{y}_j(t)| < r$ at time t, where r is the interaction radius of the neighborhood, and $|\cdot|$ refers to an induced norm. Hence, we construct a time-varying network G by combining agents, oscillators and their dynamical laws. If not otherwise specified, symbols for graphs, topologies and their corresponding Laplacian matrices are not differentiated.

Suppose that we want to synchronize such a network with a chaotic state defined by

$$\lim_{t\to\infty} |x_i(t) - s(t)| = 0, \qquad \forall i, \tag{3}$$

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where $\mathbf{s}(t)$ is a given orbit such that $\dot{\mathbf{s}} = \mathbf{f}(\mathbf{s})$. We impose local feedback control on some of the mobile agents to achieve control goal (3). The controlled network can be generally formulated as

$$\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i) - \sigma \sum_{j=1}^{N} g_{ij}(t)\mathbf{h}(\mathbf{x}_j) - \sigma k_i B_i(\mathbf{h}(\mathbf{x}_i) - \mathbf{h}(\mathbf{s})), \tag{4}$$

where $i = 1, 2, \ldots, N$, $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^n$ governs the local dynamics of the oscillator, $\mathbf{h} : \mathbb{R}^n \to \mathbb{R}^n$ is a vectorial output function, σ is the overall coupling strength, and the Laplacian matrix $G(t) = [g_{ij}(t)] \in \mathbb{R}^{N\times N}$ is a zero-row-sum one which defines the neighborhood of each agent at time t. In detail, $g_{ij}(t) = g_{ji}(t) = -1$ if agents i and j are adjacent at time t; otherwise, $g_{ij}(t) = g_{ji}(t) = 0$. The diagonal entries of G satisfy

$$g_{ii}(t) = -\sum_{j=1, j\neq i}^{N} g_{ij}(t) = m(t), \tag{5}$$

where $m(t)$ is the number of neighbors of agent i at time t. The binary vector B—$B_i = 1$ ($B_i = 0$) if node i is controlled (not controlled)—is used to index the controlled nodes. Constant k_i is the feedback gain acting on node i. For simplicity, $\forall i = 1, \ldots, N$, let $k_i = k$.

For common complex networks that exhibit fixed topological structures and coupling strengths, the pinned nodes indexed with B are called local leaders since they are selected according to some property such as degree, weight, or centrality. Then the leaders guide their neighbors to the desired evolution through interactions between them. In static networks[3], especially in those that obey a scale-free degree distribution, selective pinning wherein the pinned nodes are chosen according to degree is proven to be more effective than random pinning [5]. However, it is very difficult to determine leaders for the controlled mobile agent network (4) mainly because of the fast-switching operation and random walks of moving nodes. Recalling that every agent is equivalent to every other, we then apply random pinning to the mobile agent network (4) instead of selective pinning.

3. Controllability analysis

In this section, we are interested in the pinning controllability, the number of controlled agents l, and the value of the feedback gain k. To assess this, we consider an extended network of $N + 1$ dynamical systems with node state variable $\mathbf{z}_i$ [6], where $\mathbf{z}_i(t) = \mathbf{x}_i(t)$ for $i = 1, 2, \ldots, N$ and $\mathbf{z}_{N+1}(t) = \mathbf{s}(t)$. Following this, the desired evolution $\mathbf{s}(t)$ is added to the original network (4) through an extra virtual node $\mathbf{z}_{N+1}$.

Let $\delta\mathbf{z}_i(t) = \mathbf{z}_i(t) - \mathbf{s}(t)$ be the deviation of the ith vector state from the synchronous solution. Define the following vectors:

$$\mathbf{Z} = (\mathbf{z}_1^{\mathrm{T}}, \mathbf{z}_2^{\mathrm{T}}, \ldots, \mathbf{z}_N^{\mathrm{T}}, \mathbf{z}_{N+1}^{\mathrm{T}})^{\mathrm{T}} \in \mathbb{R}^{(N+1)\times n},$$
$$\delta\mathbf{Z} = (\delta\mathbf{z}_1^{\mathrm{T}}, \delta\mathbf{z}_2^{\mathrm{T}}, \ldots, \delta\mathbf{z}_N^{\mathrm{T}}, \delta\mathbf{z}_{N+1}^{\mathrm{T}})^{\mathrm{T}} \in \mathbb{R}^{(N+1)\times n}.$$

Linearizing system (4) at the synchronous solution yields

$$\delta\dot{\mathbf{Z}} = [I_{N+1} \otimes \mathbf{Jf}(\mathbf{s}) - \sigma M(t) \otimes \mathbf{Jh}(\mathbf{s})]\delta\mathbf{Z}, \tag{6}$$

[3] Here, a static network means a network with a constant coupling matrix (Laplacian).

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where I_{N+1} is an identity matrix of order $N+1$, $\otimes$ stands for the Kronecker product, $\mathbf{Jf}$ and $\mathbf{Jh}$ are the Jacobians of the functions $\mathbf{f}$ and $\mathbf{h}$ evaluated at $\mathbf{s}(t)$, and $M(t) \in \mathbb{R}^{(N+1)\times(N+1)}$ is a square matrix written as

$$M(t) = \begin{bmatrix} g_{11} + B_1 k & g_{12} & \cdots & g_{1N} & -B_1 k \\ g_{21} & g_{22} + B_2 k & \cdots & g_{2N} & -B_2 k \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ g_{N1} & g_{N2} & \cdots & g_{NN} + B_N k & -B_N k \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}. \tag{7}$$

Without loss of generality, we apply local injections to the first l nodes with $1 \leq l \leq N$ (if another node is selected, we can rearrange the order of nodes in the network such that the pinned nodes are the first l nodes in the rearranged network). Then the feedback gain matrix is $K = \mathrm{diag}\{k, \ldots, k, 0, \ldots, 0\}$ with l nonzero entries. Let $\lambda_i(t)$ be the ith eigenvalue of matrix F defined as $F(t) = G(t) + K$. We easily derive that 0 and $\lambda_i(t)$, $i = 1, 2, \ldots, N$, are the $N+1$ eigenvalues of matrix $M(t)$.

As shown in [17]–[19], the MSF approach can be used to deal with almost all linearly coupled dynamical networks with static coupling configurations. Note that matrix $M(t)$ in equation (6) is regarded as the Laplacian of the network with node dynamics $\mathbf{z}_i(t)$, $i = 1, 2, \ldots, N, N+1$, like in network synchronization. However, it is very difficult to diagonalize $N+1$ independent linear systems from the mobile agent network (6) by means of the MSF approach. A significant result proved by Stilwell *et al* supplies a sufficient condition for fast-switching synchronization of time-varying networks [27]: if the time average of the Laplacian $L(t)$, defined as $\bar{L} = (1/T)\int_t^{t+T} L(\tau)\,\mathrm{d}\tau$, admits a stable synchronization manifold and if the switching between all the possible network configurations is fast enough, then the time-varying network with Laplacian $L(t)$ will synchronize, where T is a positive constant. This condition requires a constant average coupling matrix of the time-varying network. Fortunately, the coupling matrix of the mobile agent network can be averaged by means of a constant matrix written as

$$\bar{M} = \begin{bmatrix} \bar{L} + K & \mathbf{b} \\ 0 & 0 \end{bmatrix}, \tag{8}$$

where $\bar{L}$ is the average of the Laplacians $L(t)$ of network (4), and vector $\mathbf{b} = [k, \ldots, k, 0, \ldots, 0]^{\mathrm{T}}$. As has been shown in [28], a scaled all-to-all Laplacian can characterize the global behavior of the mobile agent network. In detail, for $N \geq 2$, the time average Laplacian $\bar{L} = pL_{\mathrm{A}}$, where p is the probability that two agents are neighbors, which is given by $p = \pi r^2/q^2$, and L_{A} is the all-to-all Laplacian matrix, i.e., the Laplacian of a completely connected network. Note that $L(t)$ represents the network Laplacian at time t, while the time average $\bar{L}$ is not the network Laplacian corresponding to a particular topology evolved from the mobile agent network, though it has the zero-row-sum property.

Therefore, we consider the local synchronization of the static average complex network instead of network (4):

$$\delta\dot{\mathbf{Z}} = [I_{N+1} \otimes \mathbf{Jf}(\mathbf{s}) - \sigma\bar{M} \otimes \mathbf{Jh}(\mathbf{s})]\delta\mathbf{Z}. \tag{9}$$

Since node $(N+1)$ is an extra virtual one, we omit it to obtain

$$\delta\dot{\mathbf{X}} = [I_N \otimes \mathbf{Jf}(\mathbf{s}) - \sigma\bar{F} \otimes \mathbf{Jh}(\mathbf{s})]\delta\mathbf{X}, \tag{10}$$

where $\delta\mathbf{X} = (\mathbf{x}_1^{\mathrm{T}}, \mathbf{x}_2^{\mathrm{T}}, \ldots, \mathbf{x}_N^{\mathrm{T}})^{\mathrm{T}} \in \mathbb{R}^{N\times N}$, $\bar{F} = \bar{L} + K$.

By matrix transformation, we obtain a block-diagonally decoupled form of equation (10):

$$\dot{\boldsymbol{\eta}}_i = [\mathbf{Jf}(\mathbf{s}) - \sigma\mu_i\mathbf{Jh}(\mathbf{s})]\boldsymbol{\eta}_i, \tag{11}$$

where $\mu_1 \le \mu_2 \le \cdots \le \mu_N$ are the eigenvalues of matrix $\bar{M}$. Since $\bar{L}$ is globally connected, any l nodes to be controlled will exhibit the same $\bar{M}$. Then we need to calculate the eigenvalues of matrix $\bar{M}$ according to the definition $\det(\mu I_N - \bar{M}) = 0$, where $\det(\cdot)$ represents the determinant. By row and column operations, we have

$$\mu_i = \left\{ \frac{(k+pN)}{2}\left(1 \pm \left(1 - \frac{4klp}{(k+pN)^2}\right)^{1/2}\right), \underbrace{pN, \ldots, pN}_{N-l-1}, \underbrace{pN+k, \ldots, pN+k}_{l-1} \right\}. \tag{12}$$

As established in [17], network synchronization with the desired solution is achieved if the Lyapunov exponents for equation (11) are all negative, i.e., $\Gamma(\sigma\mu_i) < 0$ for $i = 1, 2, \ldots, N$, where Γ is the Lyapunov exponent. Once the MSF is assigned, we can derive an interval of values of $\sigma\mu_i$. Recent investigations show that two typical intervals of $\sigma\mu_i$ are bounded and unbounded (of course, there exist other intervals such as disconnected and empty ones, for different choices of MSF; see e.g., [30]–[32]) regions. For the Rössler oscillator that we used in the following simulations, the state dynamics of the oscillator is described by: $\dot{x}_1^i = -(x_2^i + x_3^i)$; $\dot{x}_2^i = x_1^i + ax_2^i$; $\dot{x}_3^i = b + x_3^i(x_1^i - c)$ with $\mathbf{x}^i = [x_1^i, x_2^i, x_3^i]^{\mathrm{T}}$; and parameters $a = b = 0.2$, $c = 7$. The MSF $\Gamma(\sigma\mu_i)$ is negative in the interval $[\gamma^a, \infty]$ if $\mathbf{Jh} = \mathrm{diag}\{0, 1, 0\}$ (type-1 coupling with an unbounded synchronization region) and in the interval (γ_1^b, γ_2^b) if $\mathbf{Jh} = \mathrm{diag}\{1, 0, 0\}$ (type-2 coupling with a single bounded synchronization region), where γ^a, γ_1^b and γ_2^b are all constants. By numerical calculation, we obtain $\gamma^a \approx 0.19$, $\gamma_1^b \approx 0.19$ and $\gamma_2^b \approx 4.62$. Thus, the controlled dynamical network (9) with the average Laplacian achieves synchronization in the corresponding intervals of $\sigma\mu_i$. Furthermore, under the fast-switching assumption, synchronization of the mobile agent network (4) about the desired evolution is guaranteed in the interval of $\sigma\mu_i$ obtained by the average network (9).

In the following, we investigate the pinning controllability of type-1 and type-2 coupled networks respectively.

3.1. The unbounded synchronization region

The average network with type-1 coupling achieves synchronization if the boundary condition reads

$$\mu_1 > \gamma^a/\sigma. \tag{13}$$

Equation (13) indicates that the smallest eigenvalue of $\bar{F}$ determines the network controllability. Substituting equation (12) into the above synchronization condition gives the relation between the controlled parameters and the synchronization threshold. If the value of the feedback gain k is sufficiently large, i.e., $k \gg pN$, it will result in a simple condition of synchronization:

$$\theta = \delta\rho > \frac{\gamma^a}{\pi r^2\sigma}, \tag{14}$$

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where $\delta = l/N$ is the pinning probability, ρ is the agent density, defined as $\rho = N/q^2$, and θ is the pinning density. Note that $pN = \pi r^2 \rho$ and the value of $\pi r^2 \rho$ is usually very small; thus the assumption $k \gg pN$ is reasonable.

From equation (14), the pinning density θ is the only factor regulating synchronization of the mobile agent network. For a given group of mobile agents, there exists a critical pinning density θ_{c}: synchronization of all agents takes place only when $\theta > \theta_{\mathrm{c}}$. Hence, we deduce for how many agents at least a dynamical network should be controlled by random pinning according to equation (14). Also notice that the controllability, characterized by μ_1, increases with the pinning density. Then there appears a question: can the upper bound of $\theta_{\mathrm{u}} = N/q^2$ ensure the threshold in equation (14) for a particular network of mobile agents? The answer is obviously positive since the smallest eigenvalue μ_1 is replaced by the constant k and thus equation (13) always holds. In other words, synchronization of the controlled network (4), no matter what the network size or agent density is, can be guaranteed under large injections for all mobile agents.

We carry on with simulations of the system at different control parameters including the pinning probability δ, network density ρ, and network size N. The other parameters are fixed so that fast switching is guaranteed: $\Delta t = 10^{-3}$, $v = 10^3$, and $r = 1$ for simplification. It is necessary to characterize network synchronization in a statistical method due to network evolution. We define the synchronization error $\delta x(t) = \sqrt{(\sum_{i=1}^{N} \delta \mathbf{x}_i^{\mathrm{T}} \delta \mathbf{x}_i)/(3N)}$, and the synchronization index $\langle \delta x \rangle = \langle \delta x(t) \rangle$ which is averaged over 100 realizations during a long enough time in the steady state from T to $T + \Delta T$. In simulations, let $T = 500$ and $\Delta T = 100$. Figure 1 reports the numerical results for pinning the agent network with type-1 coupling. As expected, the desired synchronization is lost when the pinning density is small, and for a large pinning density, the synchronization can always be obtained. The transition from a nonsynchronized behavior to a synchronized behavior appears when $\theta > 6.05 \times 10^{-3}$ in theory. All numerical results for different network sizes and pinning probabilities in figure 1 agree very well with the analytical value. We then say that the average network can be used to investigate collective behavior of the mobile agent network.

It is also noted that the critical pinning density is quite independent of the number of controlled agents, the network size, and the pinning probability. This suggests that the controllability of the dynamical network (4) is determined not by l or δ, but by the pinning density θ. In particular, figure 2 illustrates the relation of the critical pinning probability δ_{c} and the critical agent density ρ_{c}. It is easy to see that ρ_{c} as a function of δ_{c} follows a power-law function in a log–log plot, where the linear slope is the critical pinning density θ_{c}.

3.2. The bounded synchronization region

For a type-2 coupled network, the boundary condition of the synchronization region is given by

$$\mu_1 > \gamma_1^b/\sigma, \tag{15}$$

$$\mu_N < \gamma_2^b/\sigma. \tag{16}$$

Sometimes, the eigenratio $R = \mu_N/\mu_1$ is defined for evaluating the bounded synchronization region. The lower the eigenratio R, the larger the synchronization region.

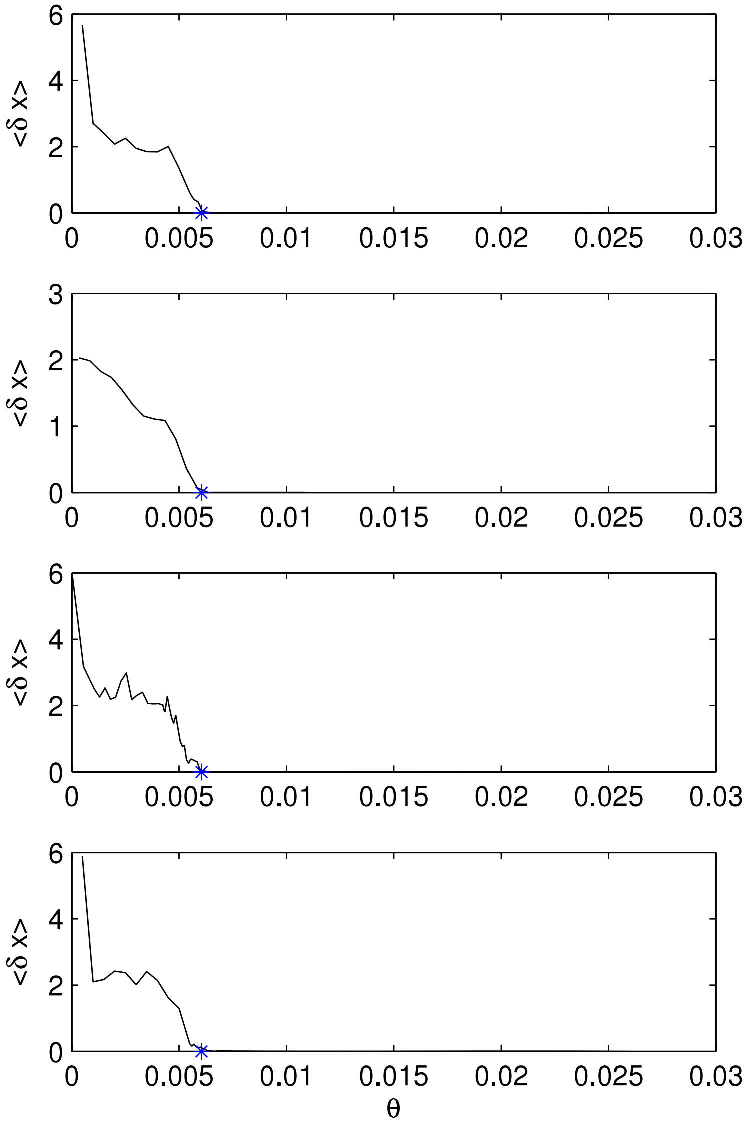

Figure 1. Synchronization index $\langle \delta x \rangle$ versus pinning density θ for identical systems at different parameters. The coupling strength is fixed to $\sigma = 10$; the feedback gain is selected as $k = 50$ to ensure that the assumption $k \gg pN$ is appropriate. We label the theoretical result for pinning density $\theta_c = 6.05 \times 10^{-3}$ with the mark ★ in this figure. The numerical value of the synchronization index is averaged over 100 realizations in the time interval $t \in [500, 600]$.

If the average Laplacian $\bar{L}$ is given beforehand, then the feedback gain matrix K directly affects the spectrum of $\bar{M}$. In other words, the eigenratio R is determined by the number of controlled nodes l and feedback gain k. We then seek the number of controlled nodes and an appropriate feedback gain by measuring R.

From equation (12), the value of the eigenratio R decreases with increase of the number of controlled nodes l. The minimum value of R is obtained if injections are applied to all agents. However, for the controlled dynamical network with type-2 coupling, one cannot always guarantee network synchronization even if all agents are selected to be

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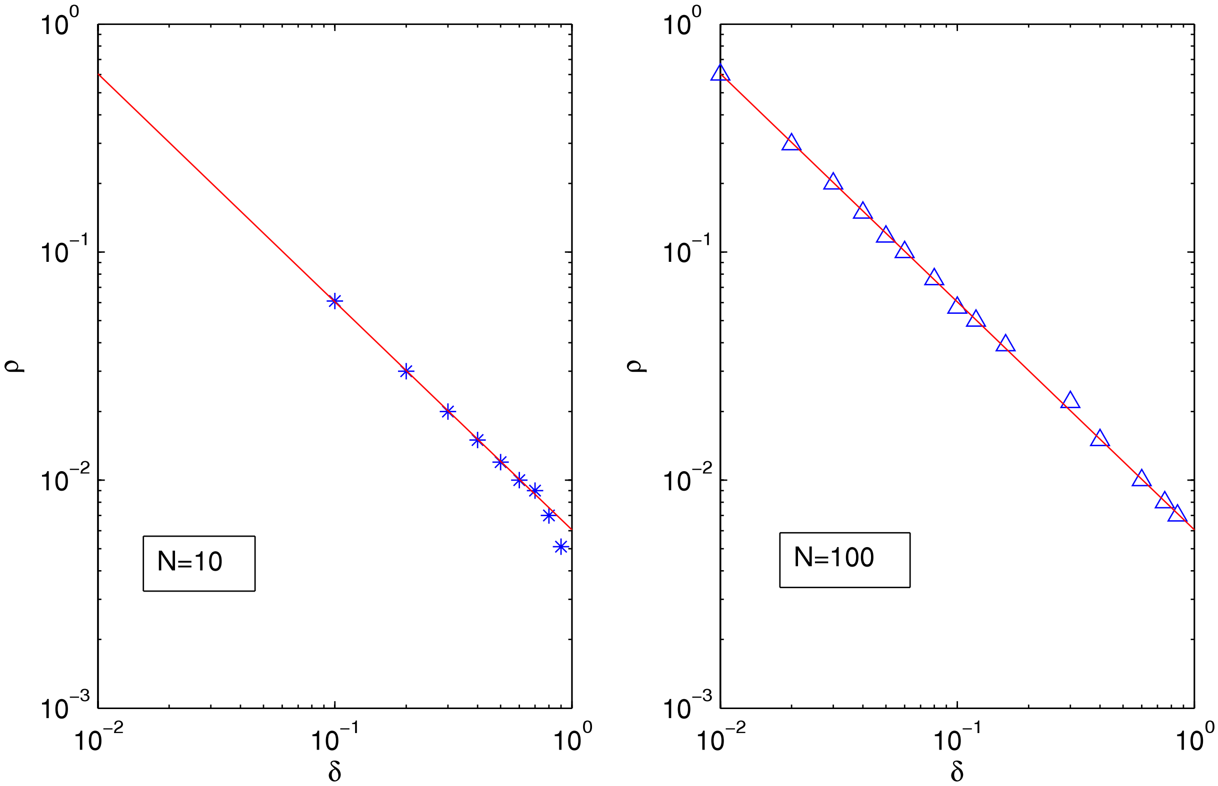

Figure 2. The relation of the critical pinning probability δ_c and critical agent density ρ_c at the synchronization threshold. The coupling strength is $\sigma = 10$, and the network sizes are respectively selected as 10 and 100. The numerical values of the synchronization index are averaged over 100 realizations in the time interval $t \in [500, 600]$.

pinned. This result is quite different from that for a type-1 coupled dynamical network. Figure 3 shows the numerical simulation of controlling all agents in a type-2 coupled network, where the desirable synchronization is achieved, as k belongs to the interval $(0.03, 0.25)$. The figure shows that a large k will lead to the maximum eigenvalue μ_N over the upper bound of the synchronization region, and a small k is likely to yield a small μ_1, so it cannot enter the region of synchronization. Also notice that in some of the literature, e.g., [4, 5], it is hypothesized that the feedback gains are sufficiently large. A recent simulation for heterogeneous complex networks with static topologies has shown that a too large or too small value of k can reduce the network controllability [6, 16]. For the mobile agent network with type-2 coupling, we also observe a similar result in figure 3. Therefore, the feedback gain plays a key role in synchronizing the dynamical network, and we have to choose the feedback gain carefully. An optimal solution for feedback gain k may be subject to $\min R$. To solve $\partial R/\partial k = 0$, we derive $k = pN$ for any fixed l. Substituting equation (12) and $k = pN$ into the boundary condition of synchronization gives the interval of ρ: the agents will achieve synchronization if the density satisfies

$$\frac{\gamma_1^b}{\pi r^2 \sigma(1-\sqrt{1-\delta})} = \rho_{c1} < \rho < \rho_{c2} = \frac{\gamma_2^b}{2\pi r^2 \sigma}. \tag{17}$$

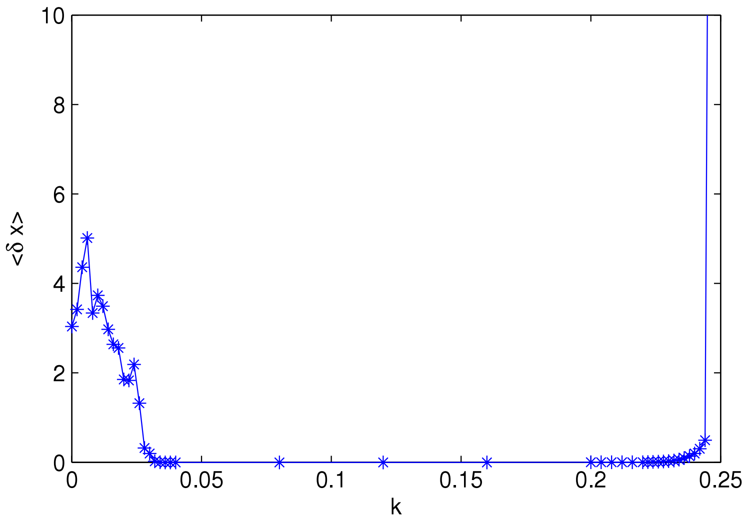

Figure 3. Synchronization index $\langle \delta x \rangle$ versus feedback gain k, where the network size is $N = 100$, the coupling strength is $\sigma = 10$, and the agent density $\rho = 0.07$.

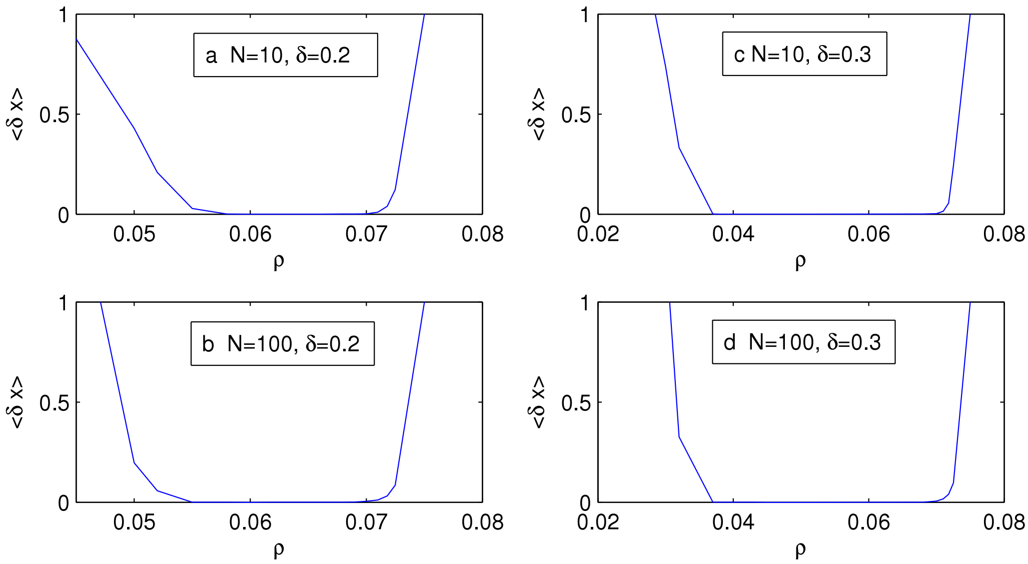

Figure 4. Synchronization index $\langle \delta x \rangle$ versus agent density ρ. The coupling strength is $\sigma = 10$, and the feedback gain is $k = pN$.

Figure 4 reports the numerical results for $N = 10$ and 100. As expected, for $\rho_{c1} < \rho < \rho_{c2}$, a synchronization motion will be established; otherwise the synchronization will be lost. This is related to the boundary condition of type-2 coupling. In particular, the upper bounds of ρ for different network sizes or pinning probabilities are almost the same; and the lower bounds of ρ for different network sizes remain the same when governed by the same pinning probability. That is to say, the critical density values are quite independent of network size N, which conforms with the analytical result. Note that equation (17)

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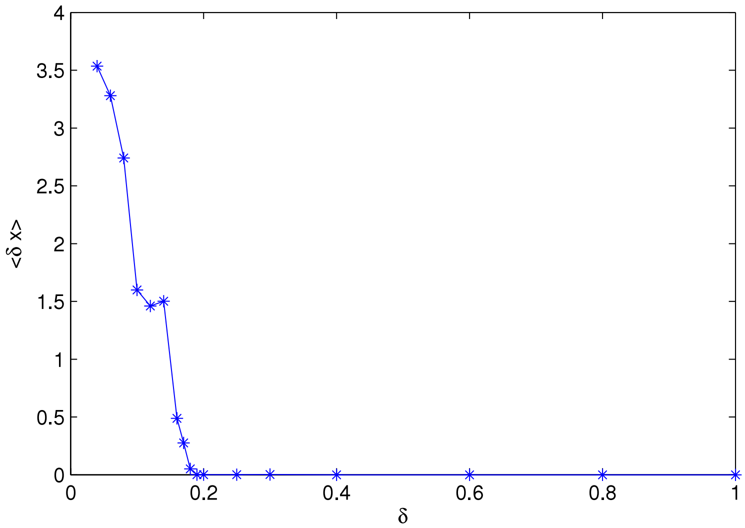

Figure 5. Synchronization index $\langle \delta x \rangle$ versus pinning probability δ. The coupling strength is $\sigma = 10$ and the feedback gain is $k = pN$.

implies an inequality for the pinning probability ρ, i.e.,

$$\frac{\gamma_1^b}{\pi r^2 \sigma (1 - \sqrt{1-\delta})} < \frac{\gamma_2^b}{2\pi r^2 \sigma}, \tag{18}$$

and then we have $\delta > 1 - (1 - 2\gamma_1^b/\gamma_2^b)^2 \approx 0.158$. Thus in the above simulations, the pinning probability must be larger than 15.8%. In other words, even when the feedback gain and agent density are both properly assigned, a synchronization motion cannot be established owing to the low pinning probability. A numerical result is given in figure 5 for the network size $N = 100$. The simulation shows that one needs to pin nearly a fifth of the agents to synchronize the entire network.

It has been found through the above discussions that synchronization of a mobile agent network with type-2 coupling is not easy to achieve through pinning control. The value of the feedback gain k, the agent density ρ and the pinning probability δ are all constrained by the boundary condition. As a result, we favor pinning control for those networks whose eigencoupling $\sigma\lambda_N$ is located in the negative region of the MSF.

4. Discussion and conclusions

From the above-mentioned theoretical results and numerical simulations, we see that pinning control is not always a good solution for enhancing synchronization of mobile agent networks: it has little effect on networks with type-2 coupling which, however, are quite popular in various investigations. In order to improve the pinning controllers, one can replace the localized 'output' feedback control by a 'state' feedback control, i.e.,

$$\mathbf{u}_i = -\sigma k_i B_i (\mathbf{x}_i - \mathbf{s}). \tag{19}$$

It is easy to verify that the controlled network (4) will synchronize about the desired state $\mathbf{s}(t)$ if all nodes are controlled by means of output feedback pinning. In other words,

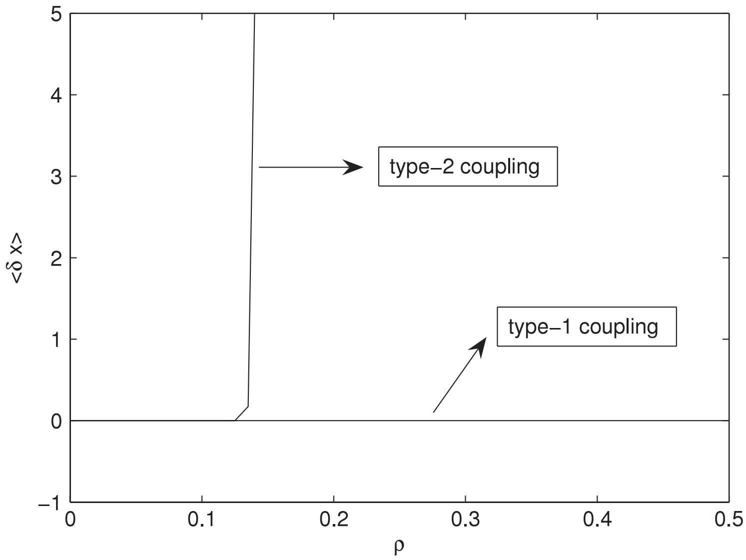

Figure 6. Synchronization index $\langle \delta x \rangle$ versus density ρ with $N = 100$ and $\delta = 0.1$. The coupling strength is $\sigma = 10$; the feedback gain is $k = 1$ and $m = 10$. The results are averaged over 100 realizations.

there must exist a certain number of nodes, l^*, such that the complex dynamical network achieves synchronization under the pinning strategy (19). The detailed proof is omitted here since it can be easily deduced by construction of the Lyapunov function. Another method is to control the output function (or inner-coupling matrix) $\mathbf{Jh}$. The analytical result shows that the synchronizability of the controlled network will be remarkably enhanced by controlling the output function; for more details, one can see [32, 33].

Here we introduce a fast-switching pinning to control the mobile agent network. Assume that the pinned nodes are randomly selected at each time unit $\Delta t_{\mathrm{p}} = m\Delta t$, where m is an integer. If Δt_{p} is small enough, then the pinning strategy allows us to control almost all agents during a short period of time. And the controllability of the mobile agent network can be treated as in section 3. We then calculate the eigenvalues of the average matrix $\bar{M}$ to obtain $\mu_i = pN + \delta k$. For the network with type-1 coupling, pinning only one agent with sufficiently large feedback can guarantee network synchronization. For the network with type-2 coupling, we write down the boundary condition for synchronization as $\gamma_1^b/\sigma < \pi r^2 \rho + \delta k < \gamma_2^b/\sigma$. It is easy to see that network synchronization takes place when

$$\rho < \rho_{\mathrm{c}} = \gamma_2^b / \pi r^2 \sigma. \tag{20}$$

Then we say that the fast-switching pinning can enhance the pinning controllability of a network with type-2 coupling. Figure 6 shows the corresponding simulations, which agree well with the theoretical results. Actually, equation (20) suggests that the real size of the system is measured not by the network size N but by the density ρ.

Regulating the complex dynamics that takes place on networks with a great number of interconnected individuals is of primary importance. In this paper, we investigate the problem of controlling a group of mobile agents in order to move them towards a desired synchronization evolution. We attempt to provide an insight into the regulatory

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mechanisms of a time-varying complex network via random pinning. We theoretically and numerically explore the effects of the pinning probability, feedback gains and agent density in pinning synchronization of mobile agent networks under the fast-switching constraint. In particular, for a given network with an unbounded synchronization region, the pinning density θ determines the controllability; however for a MSF that has a single bounded interval, pinning control has little impact: synchronization of the controlled network is easily lost, as a very small or a very large network density or feedback gain is selected in pinning. Besides, we introduce a fast-switching pinning to control the mobile agent network. These findings could be useful for controlling complex dynamical networks.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (NSFC) No. 60736021, NSFC-Guangdong No. U0735003, 863 Program No. 2006AA04Z184, No. 2007AA041406, the Specialized Research Fund for the Doctoral Program of Higher Education No. 20050335020, and the Key Technologies R&D Program of Zhejiang Province No. 2006C11066.

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