Long Rod Penetrators (Tungsten, DU, Steel)

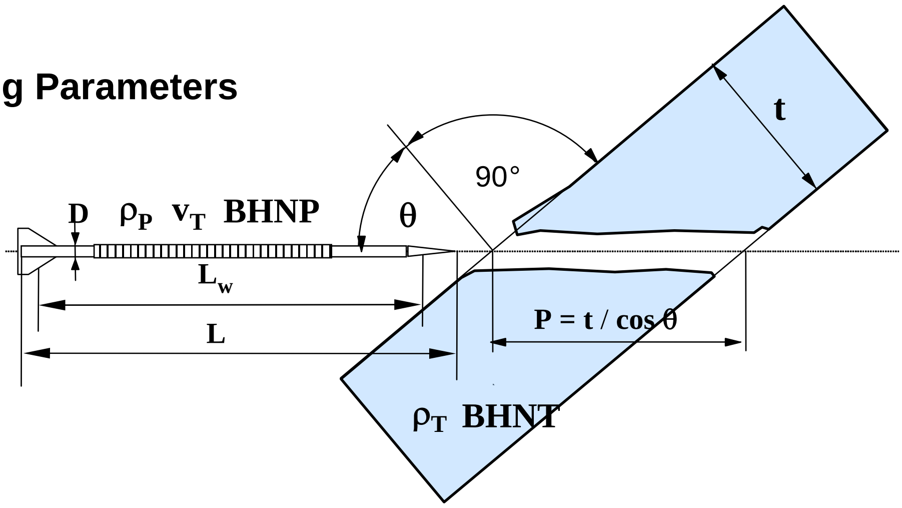
Perforation Limit in Oblique Single Targets

1. Perforation Limit Definition

A finite target perforation limit means that the projectile reaches the rear face of the target and spalling opens the penetration channel. The penetrator residue will have a length of one to one and a half diameters. In experiments the residue sticks in the exit crater or drops to the floor in the immediate vicinity of the target. The result is observable at the target site, whereas a semi-infinite target first has to be cut up laboriously.

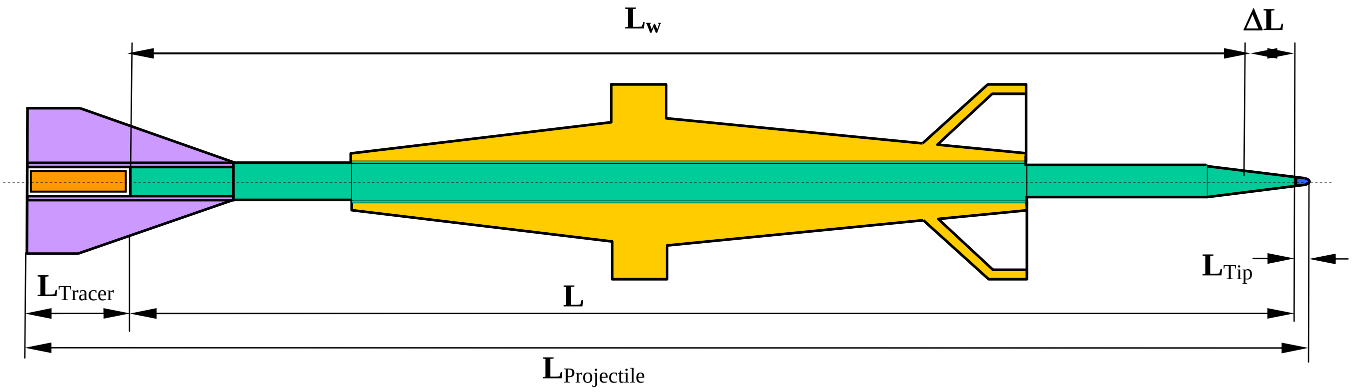
The described equation allows a quick estimation of the perforation limit of tungsten, DU or steel long rod penetrators at oblique RHA-targets.

2. Governing Parameters

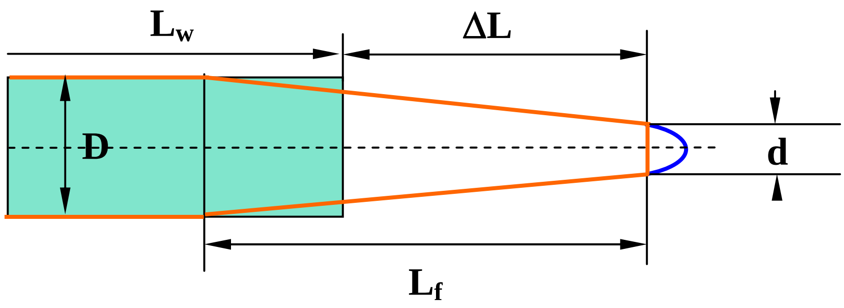


D	penetrator diameter in the grooves [mm]
L	overall length of tungsten [mm]
$\mathbf{L_w}$	working length [mm]
$\mathbf{v_T}$	impact velocity [km/s]
θ	angle of obliquity [°]
ρ_P	penetrator density [kg/m^3]
ρ_T	target density [kg/m^3]
t	target plate thickness [mm]
BHNP	Brinell Hardness Number of penetrator
BHNT	Brinell Hardness Number of target
P	perforation channel length in line of sight [mm]

3. Definition of the Working Length



Working length	$L_w = L - \Delta L$
with	$\Delta L = L_f \cdot [1\text{-}1/3(1+d/D+(d/D)^2)]$
	(Frustum replaced by a cylinder with equal volume)
in case of cylindric penetrator	$L_w = L$



4. Perforation Limit Equation

The equation for the normalized perforation is based on 5 multiplicative, dimensionless terms:

$$\frac{P}{L_w} = a \cdot \frac{1}{\tanh\left(b_0 + b_1 \cdot \frac{L_w}{D}\right)} \cdot \cos^m \theta \cdot \sqrt{\frac{\rho_P}{\rho_T}} \cdot e^{\frac{-s^2}{v_T^2}}$$

Term 1 The constant a is material dependant:

Material	**Tungsten**	**DU**	**Steel**
Constant a	0.994	0.825	1.104

Term 2 **Influence of L / D ratio**

$$f\left(\frac{L_w}{D}\right)=\frac{1}{\tanh\left(b_0+b_1\cdot\frac{L_w}{D}\right)}$$

The coefficients are assumed as identical for all penetrator materials with:

b0 = 0.283
b1 = 0.0656

Term 3 **Influence of target obliquity**

For all penetrator materials the best fit for the exponent is:

m = -0.224

Term 4 **Influence of penetrator to target density ratio (square root density-law)**

This term can be neglected in case of steel vs. steel (identical densities).

Term 5 **Influence of material and dynamic properties**

The numerator s^2 is a function of the penetrator density and of the Brinell Hardness Number of the target - and of the penetrator hardness in case of steel penetrators.

Tungsten and **DU** penetrators:

$$s^2=\frac{(c_0+c_1\cdot BHNT)\cdot BHNT}{\rho_P}$$

Material	**Tungsten**	**DU**
c_0	134.5	90.0
c_1	-0.148	-0.0849

Steel penetrators:

$$s^2=\frac{c\cdot BHNT^k\cdot BHNP^n}{\rho_P}$$

with: c = 9874

k = 0.3598
n = -0.2342

Usually the density of steel penetrator and target are identical. The equation is simplified as follow:

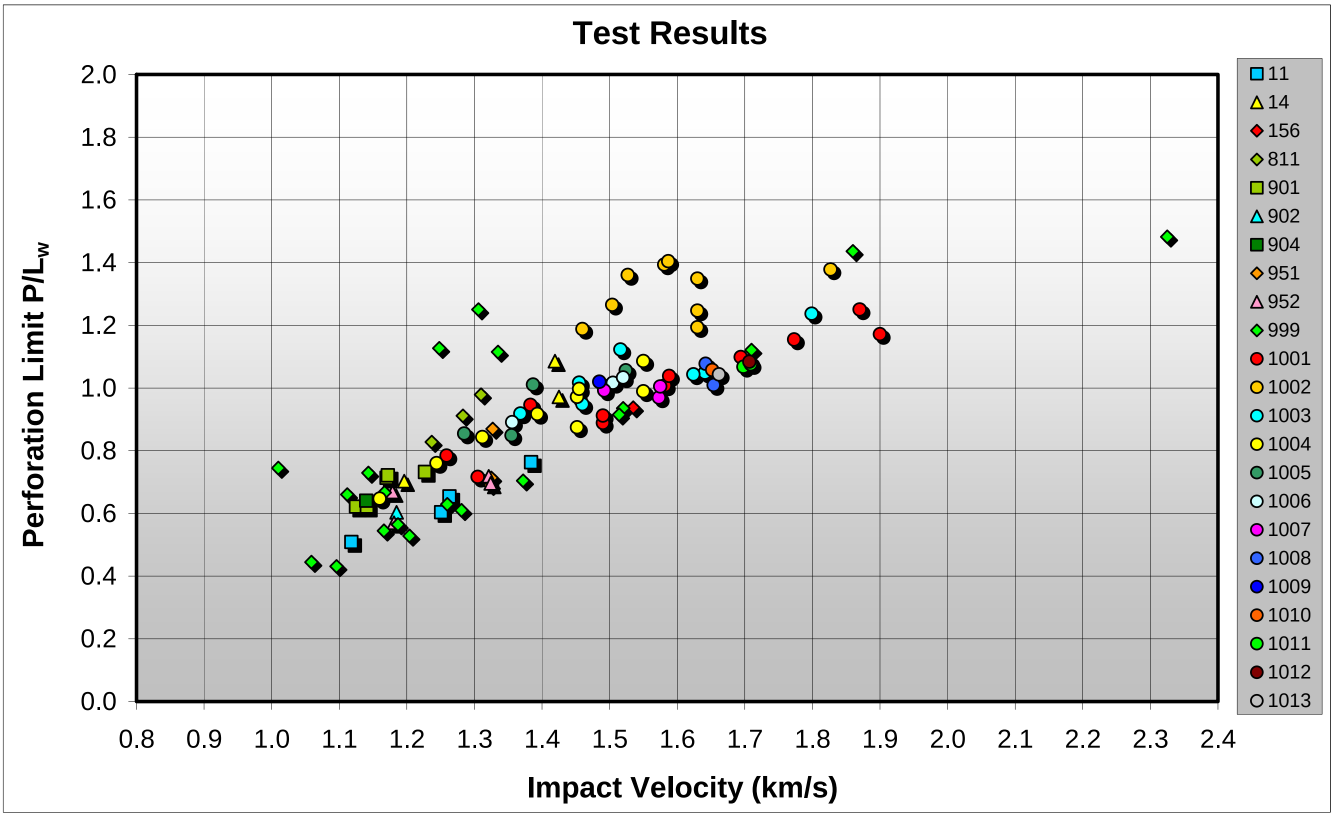
$$\frac{P}{L_w}=a\cdot\frac{1}{\tanh\left(b_0+b_1\cdot\frac{L_w}{D}\right)}\cdot\cos^{\mathrm{m}}\theta\cdot \mathrm{e}^{-\frac{c_S\cdot BHNT^k\cdot BHNP^n}{v_T^2}}$$

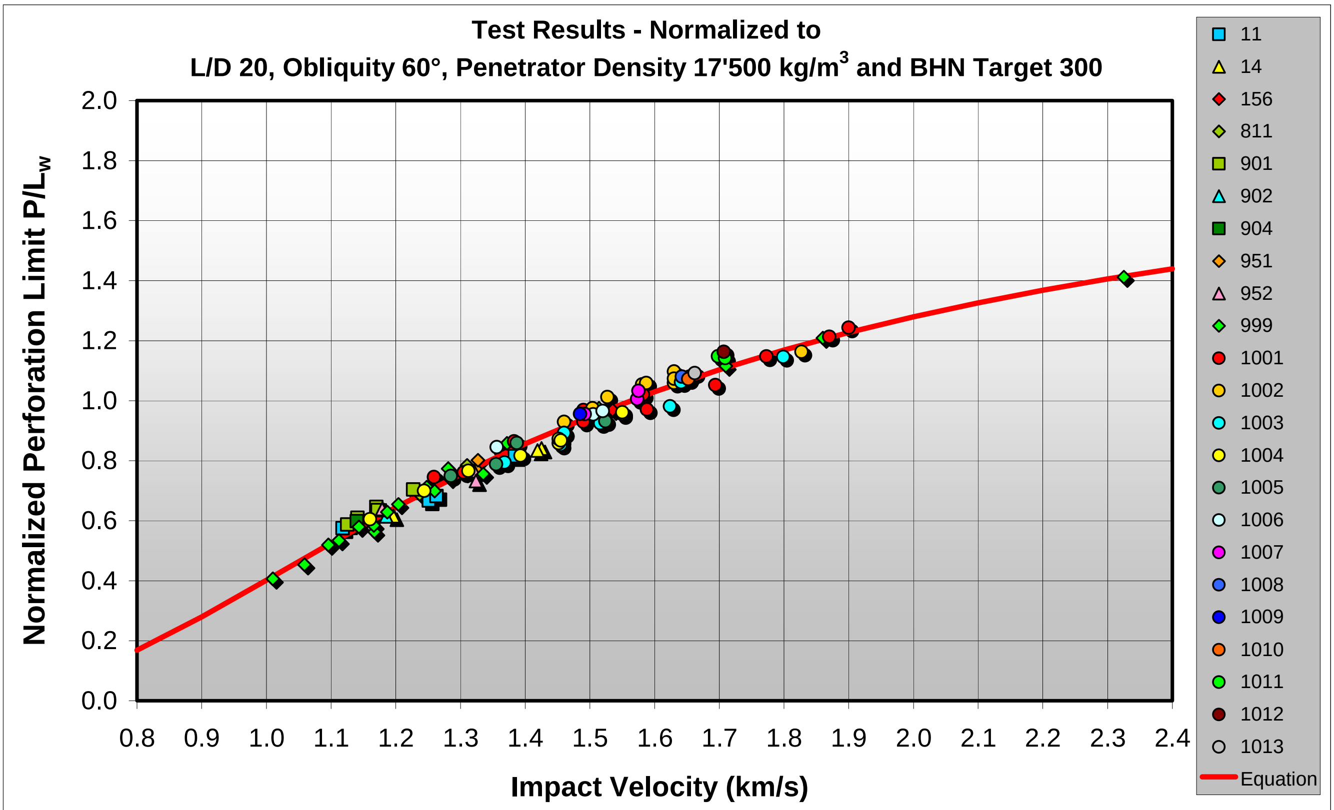
with: c_S = 9874 / 7850 = 1.258

5. Applied Datas

5.1 Tungsten Penetrators

Test results of 99 different penetrators: 79 own test results and 20 limit velocities (number 999 in the legend) selected from the SwRI data base [1].





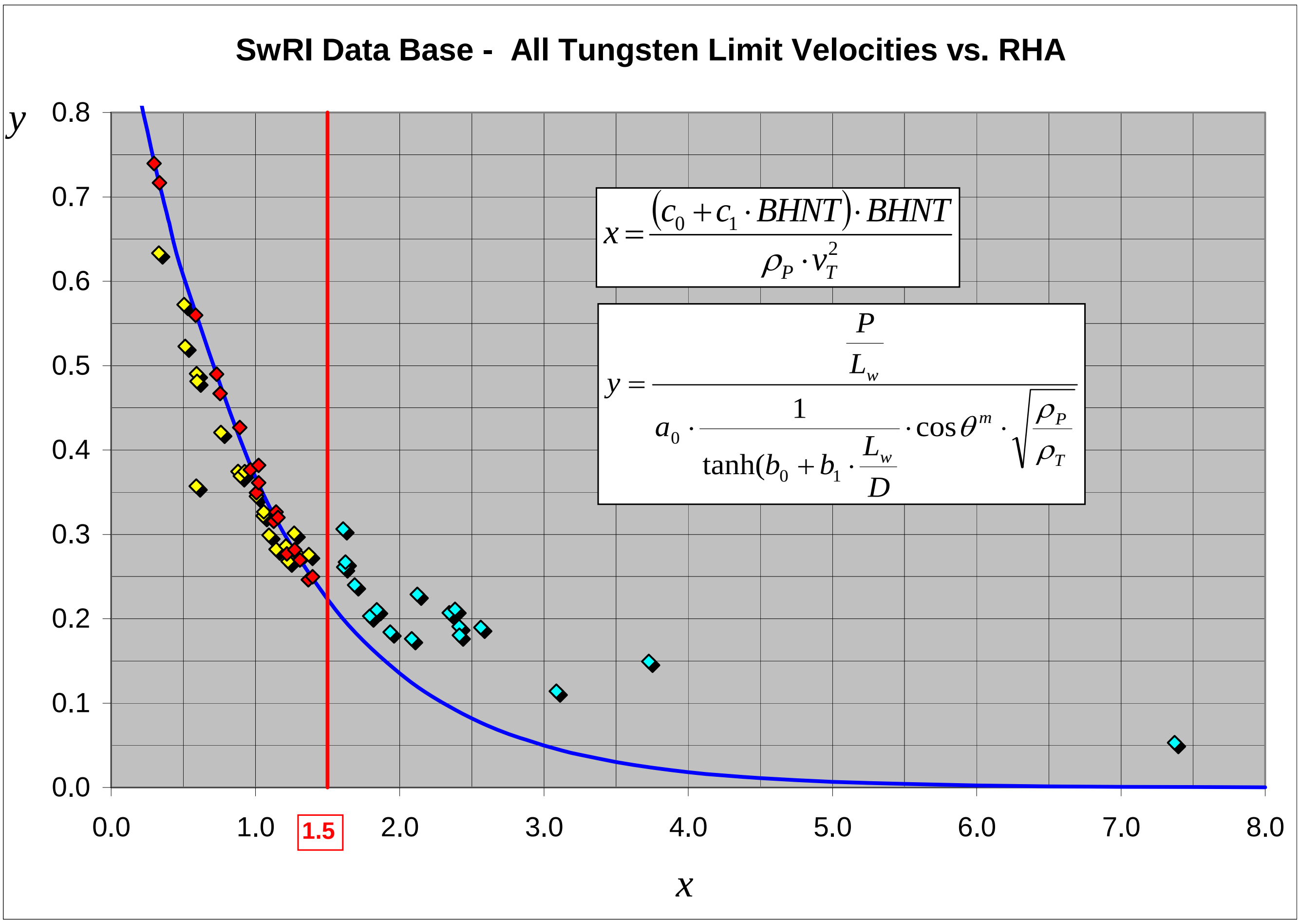
Range of results:

Penetrator

Aspect ratio L_w/D	4 - 36	
density	17’000 – 18’600	kg/m^3
impact velocty	1.0 – 2.3	km/s
Brinell Hardness Number	250 – 470	(no influence on perforation)

Target

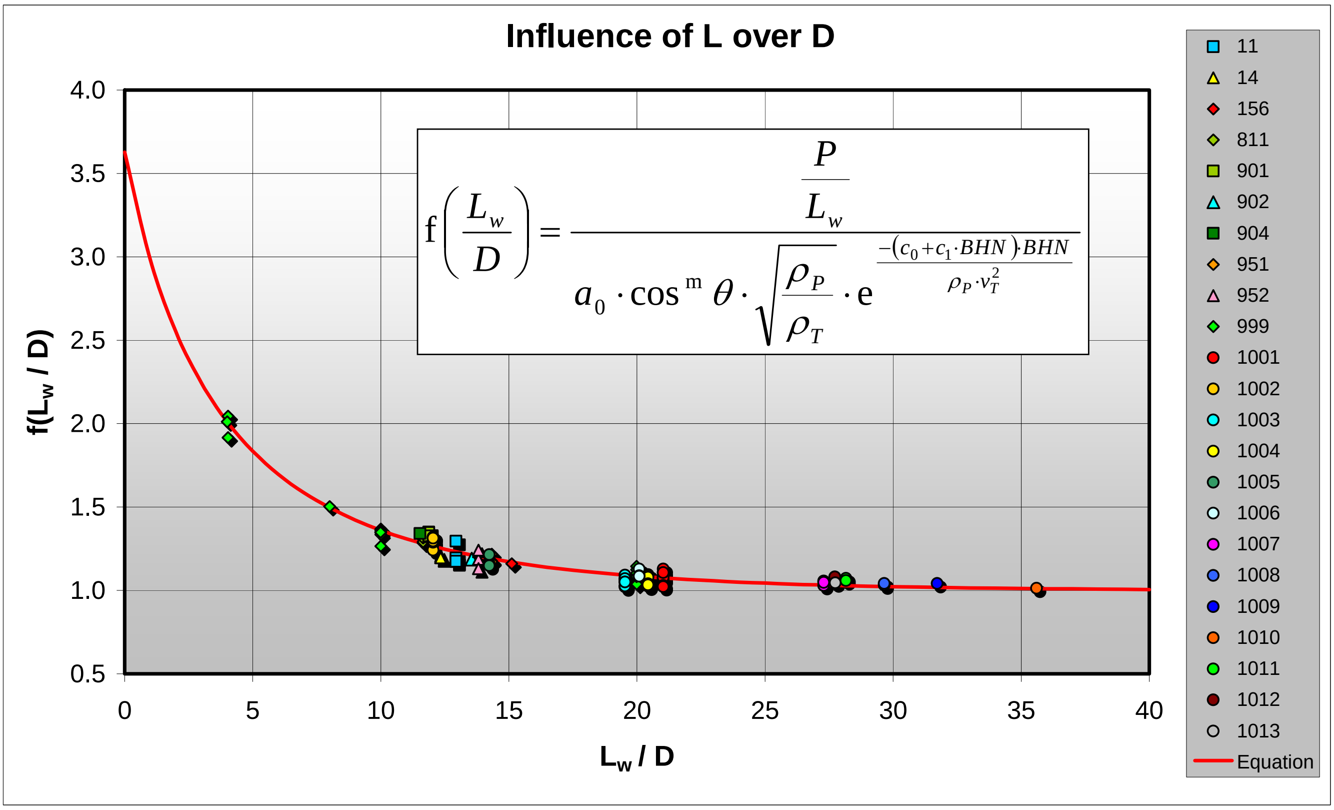
obliquity NATO	0 – 74	°
Brinell Hardess Number	180 – 460	

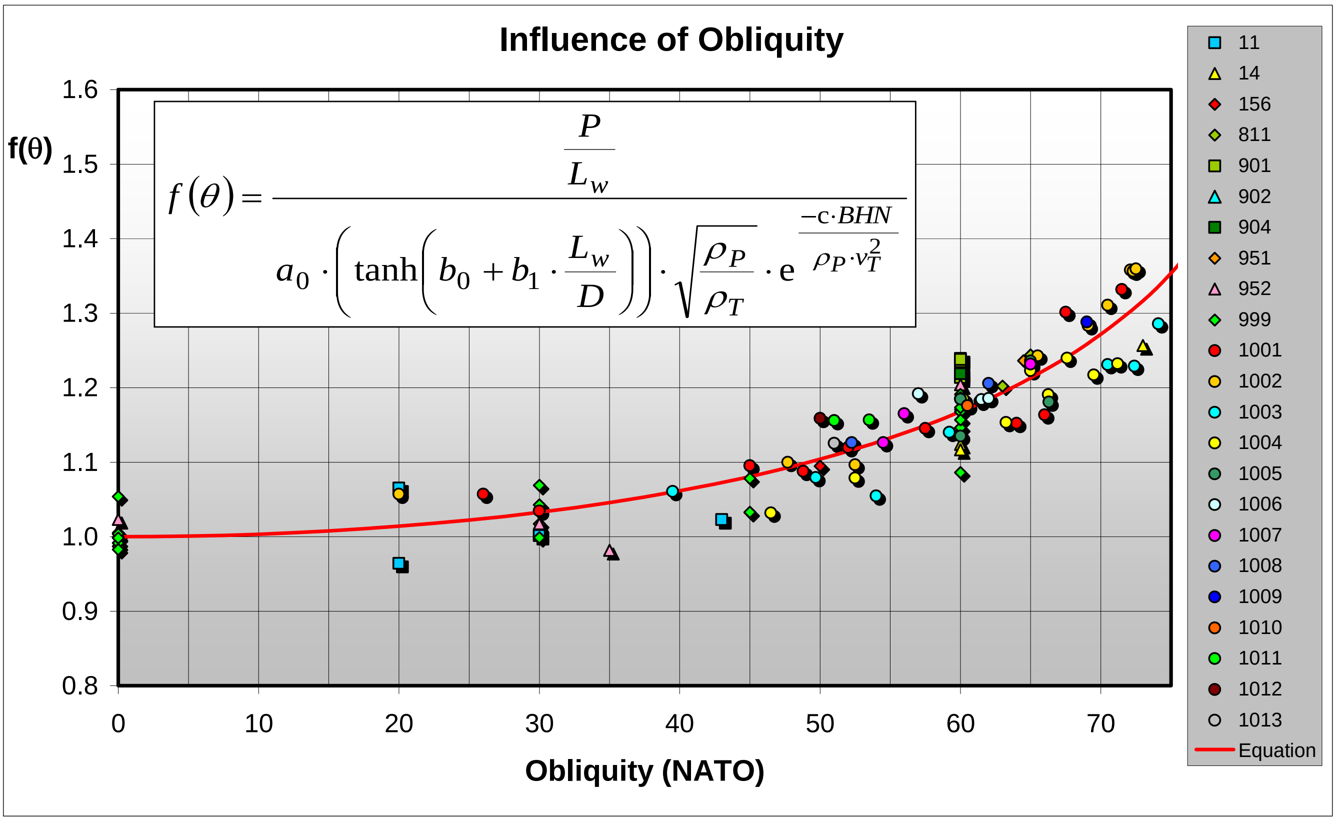


From the SwRI database all v-limits of tungsten rods versus RHA were choosed and putted into the chart above in dimensionless form. With a value of the abscissa bigger than 1.5, the results follow no longer to the equation. The penetration behaviour deviates from the hydrodynamic penetration too strongly. These results are outside of the validity range of the perforation equation. Some values of the lower velocity limit (x > 1.5) in function of the RHA Brinell Hardness at a penetrator density of 17’500 kg/m^3 are shown in the list below:

BHNT	Limit v_T (km/s)
200	0.894
300	1.015
400	1.071
500	1.073

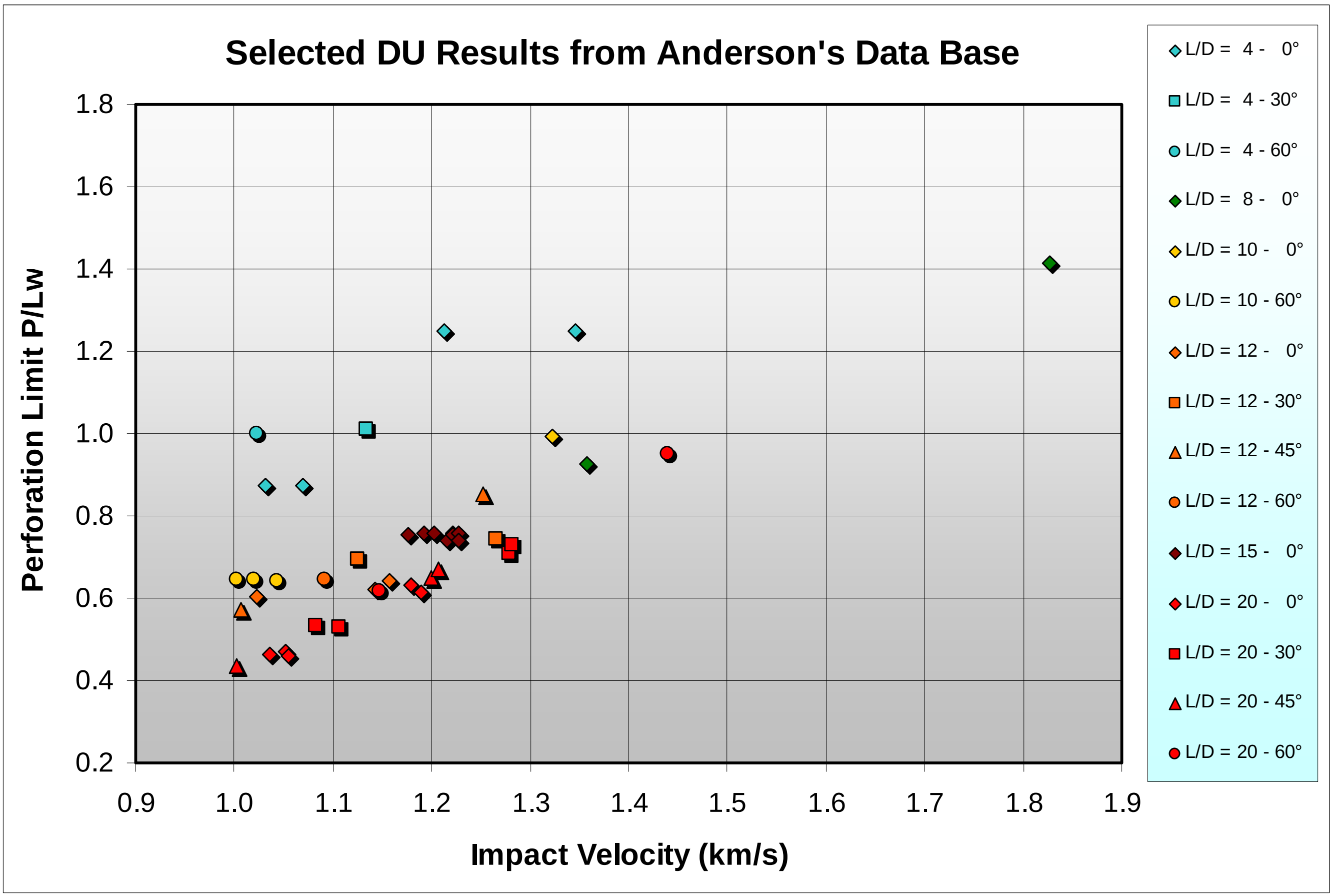
Many values from the database follow the perforation equation, while some of the results are strongly below the expected values. Disturbances (jaw etc) have a strong influence on the test results with scaled penetrators. The experience shows that many testing people often publish everyone result without questioning the plausibleness. For the derivation of the formula I didn't use obviously bad test-results.

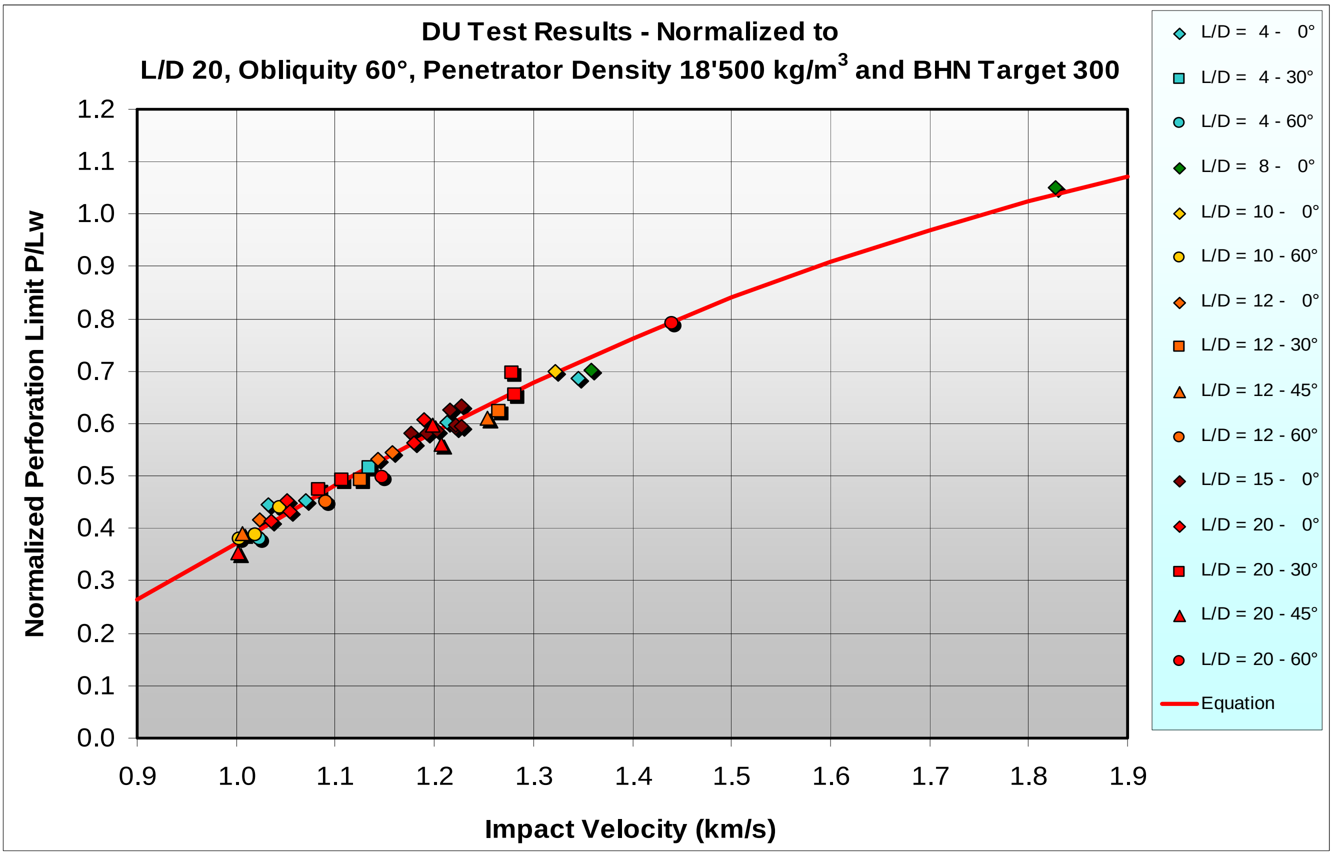

Influence of L over D
$$f\left(\frac{L_w}{D}\right) = \frac{\frac{P}{L_w}}{a_0 \cdot \cos^m \theta \cdot \sqrt{\frac{\rho_P}{\rho_T}} \cdot e^{\frac{-(c_0 + c_1 \cdot BHN) \cdot BHN}{\rho_P \cdot v_T^2}}}$$
$f(L_w / D)$
L_w / D
11
14
156
811
901
902
904
951
952
999
1001
1002
1003
1004
1005
1006
1007
1008
1009
1010
1011
1012
1013
Equation


Influence of Obliquity
$$f(\theta) = \frac{\frac{P}{L_w}}{a_0 \cdot \left(\tanh\left(b_0 + b_1 \cdot \frac{L_w}{D}\right)\right) \cdot \sqrt{\frac{\rho_P}{\rho_T}} \cdot e^{\frac{-c \cdot BHN}{\rho_P \cdot v_T^2}}}$$
f(θ)
Obliquity (NATO)
11
14
156
811
901
902
904
951
952
999
1001
1002
1003
1004
1005
1006
1007
1008
1009
1010
1011
1012
1013
Equation

5.2 Depleted Uranium (DU) Penetrators

42 limit velocities selected from the SwRI database [1].



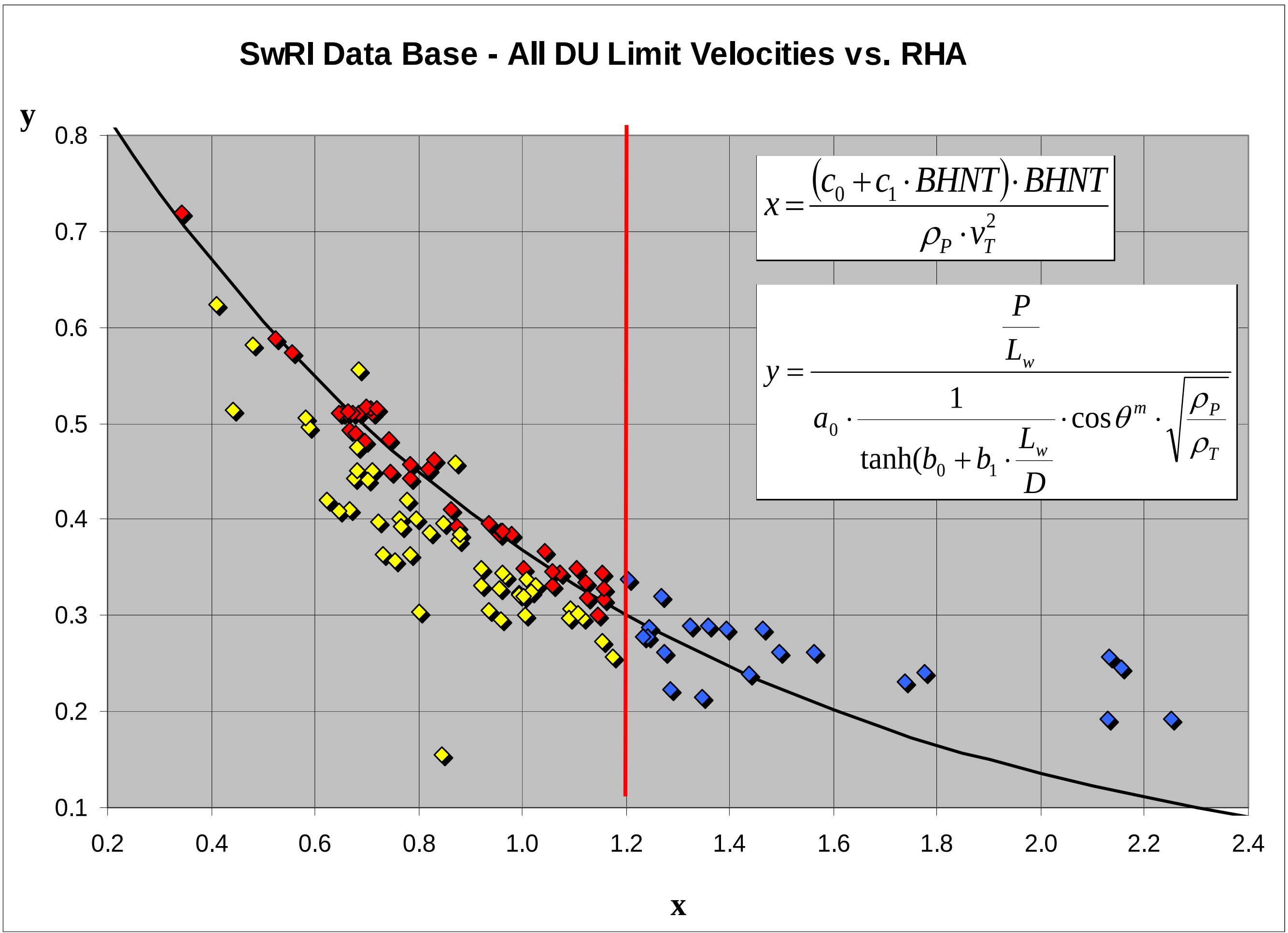


Range of results:
Penetrator

Aspect ratio L_w/D	4 - 20	
density	17'300 – 19'150	kg/m^3
impact velocty	1.0 – 1.8	km/s
Brinell Hardness Number	250 – 470	(no influence on perforation)

Target

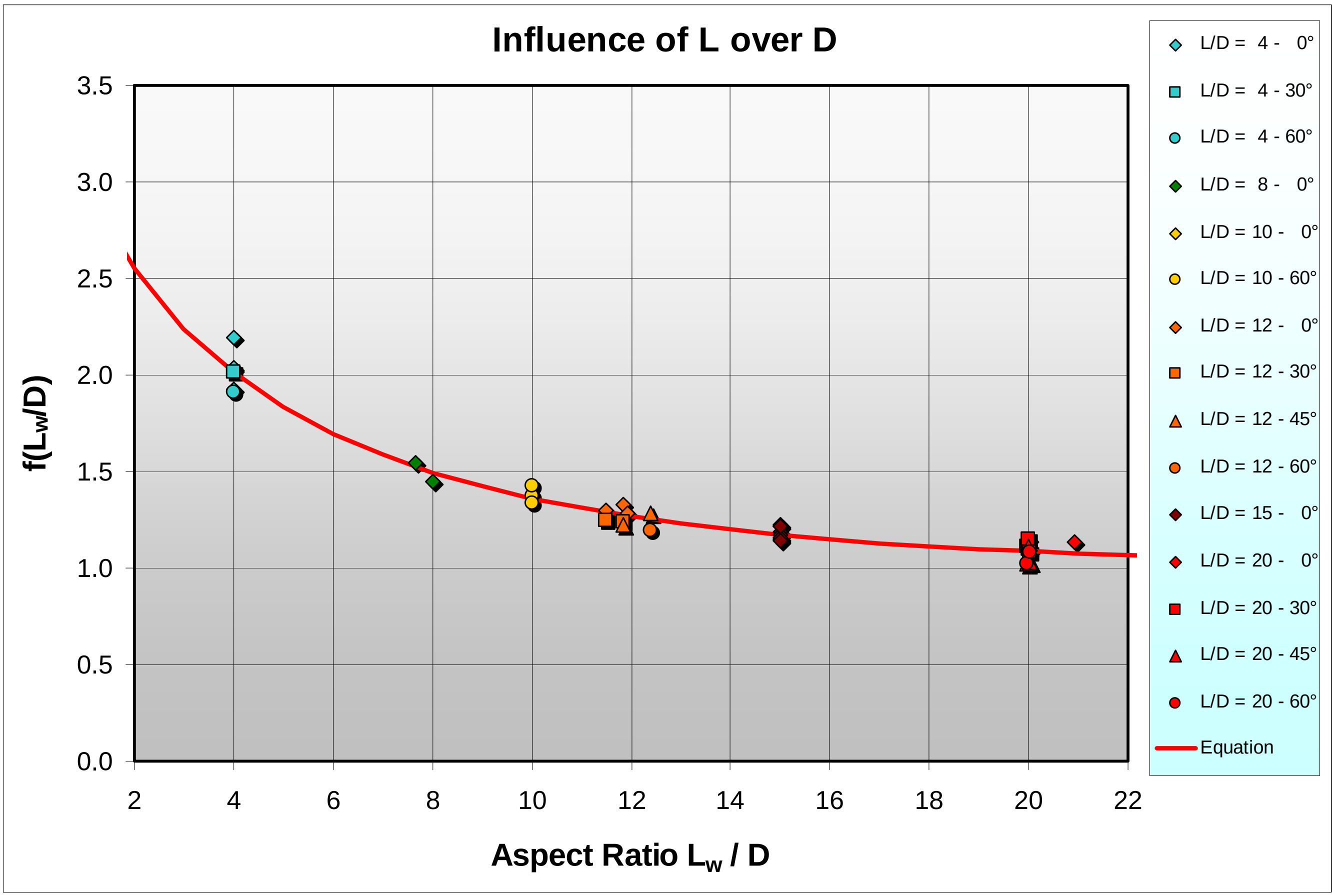
obliquity NATO	0 – 60	°
Brinell Hardness Number	270 – 430	



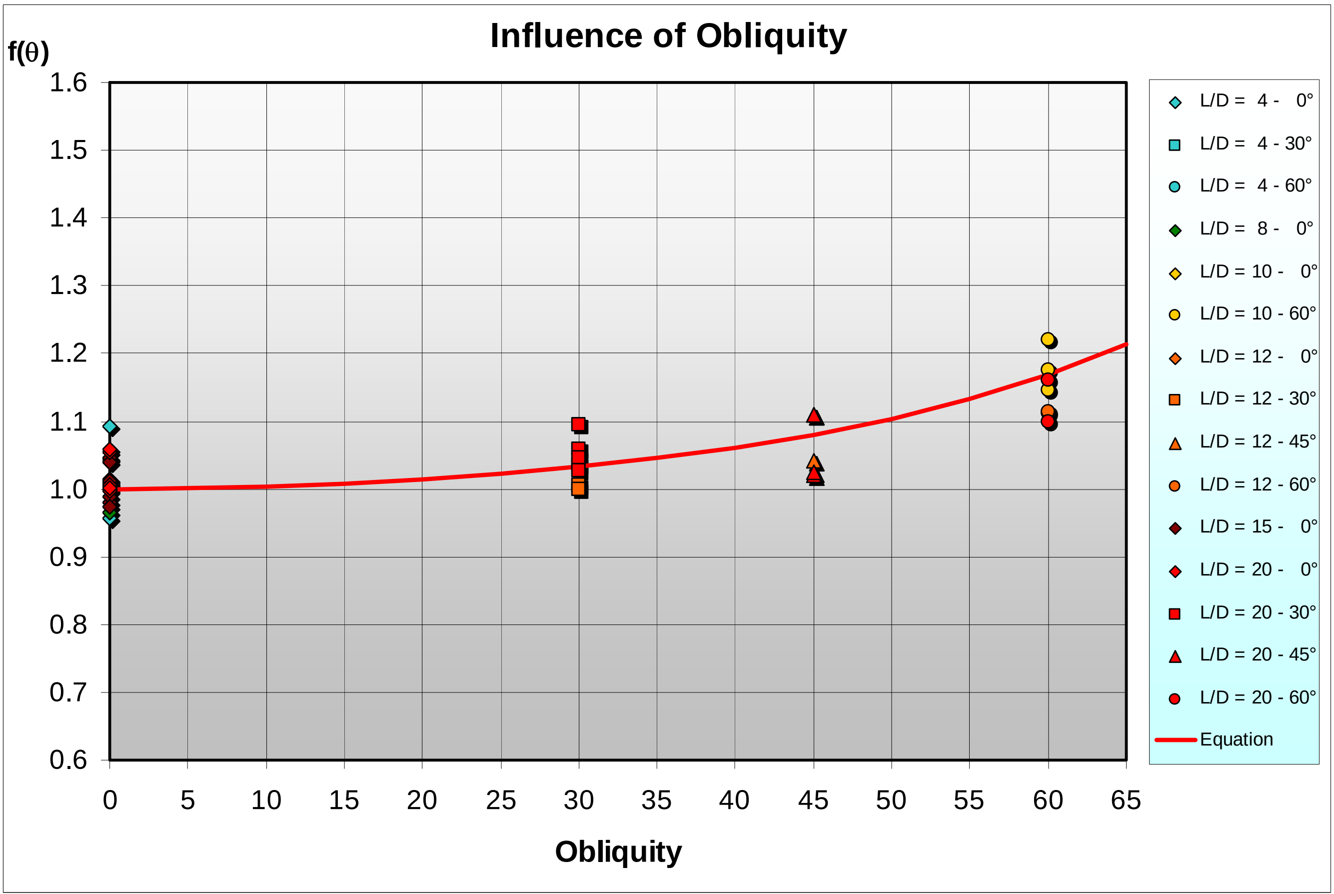
From the SwRI database all v-limits of DU rods versus RHA were choosed and putted into the above chart in dimensionless form. With a value of the abscissa bigger than 1.2, the results follow no longer to the equation. The penetration behaviour deviates from the hydrodynamic penetration too strongly. These results are outside of the validity range of the perforation equation. Some values of the lower velocity limit (x > 1.2) in function of the RHA Brinell Hardnes at a penetrator density of 18'600 kg/m^3 are shown in the list below :

BHNT	Limit v_T (km/s)
250	0.878
300	0.931
350	0.972
400	1.002
450	1.022

Here too many values from the database follow the perforation equation, while some of the results are strongly below the expected values. For the derivation of the formula I didn't use obviously bad test-results.

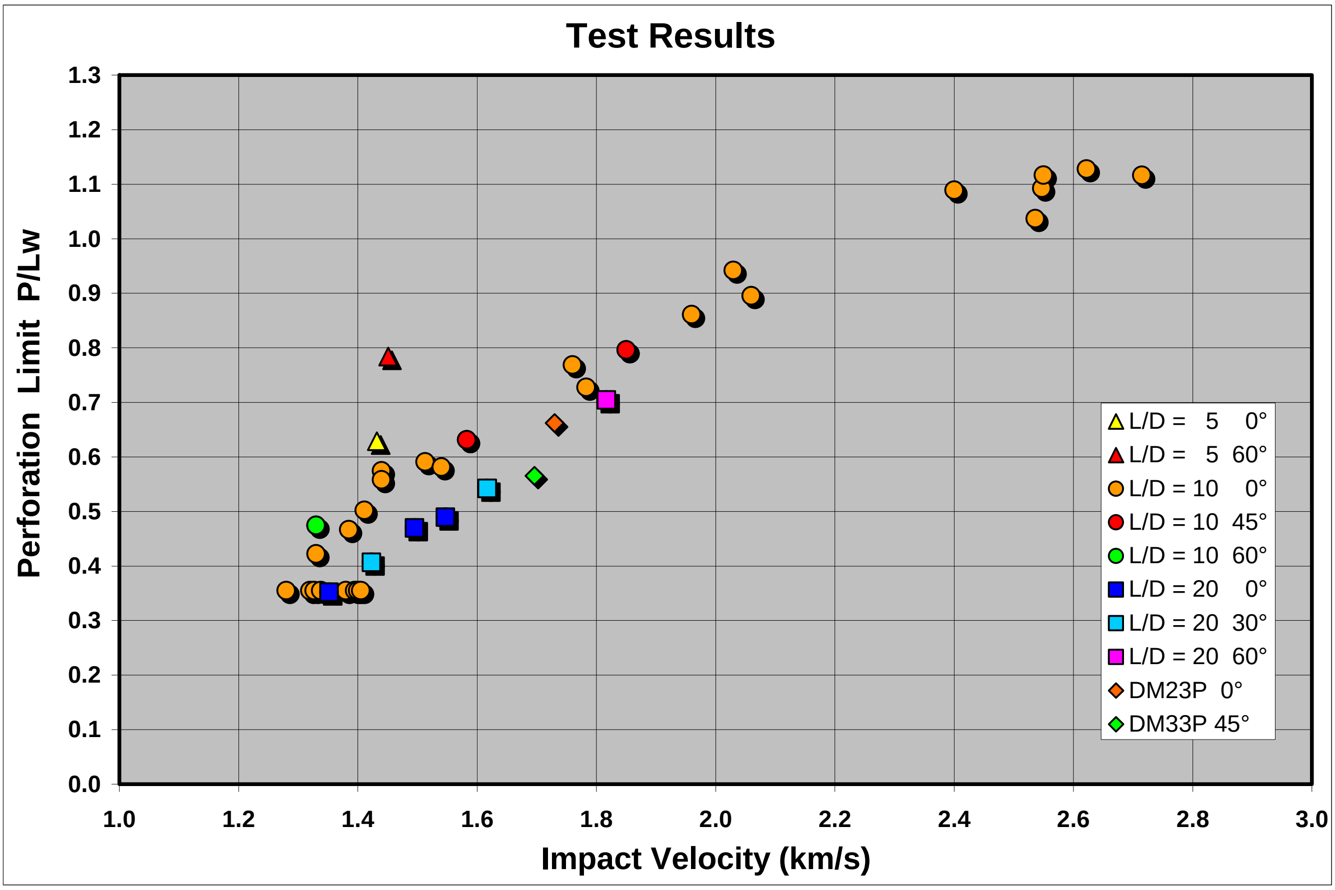


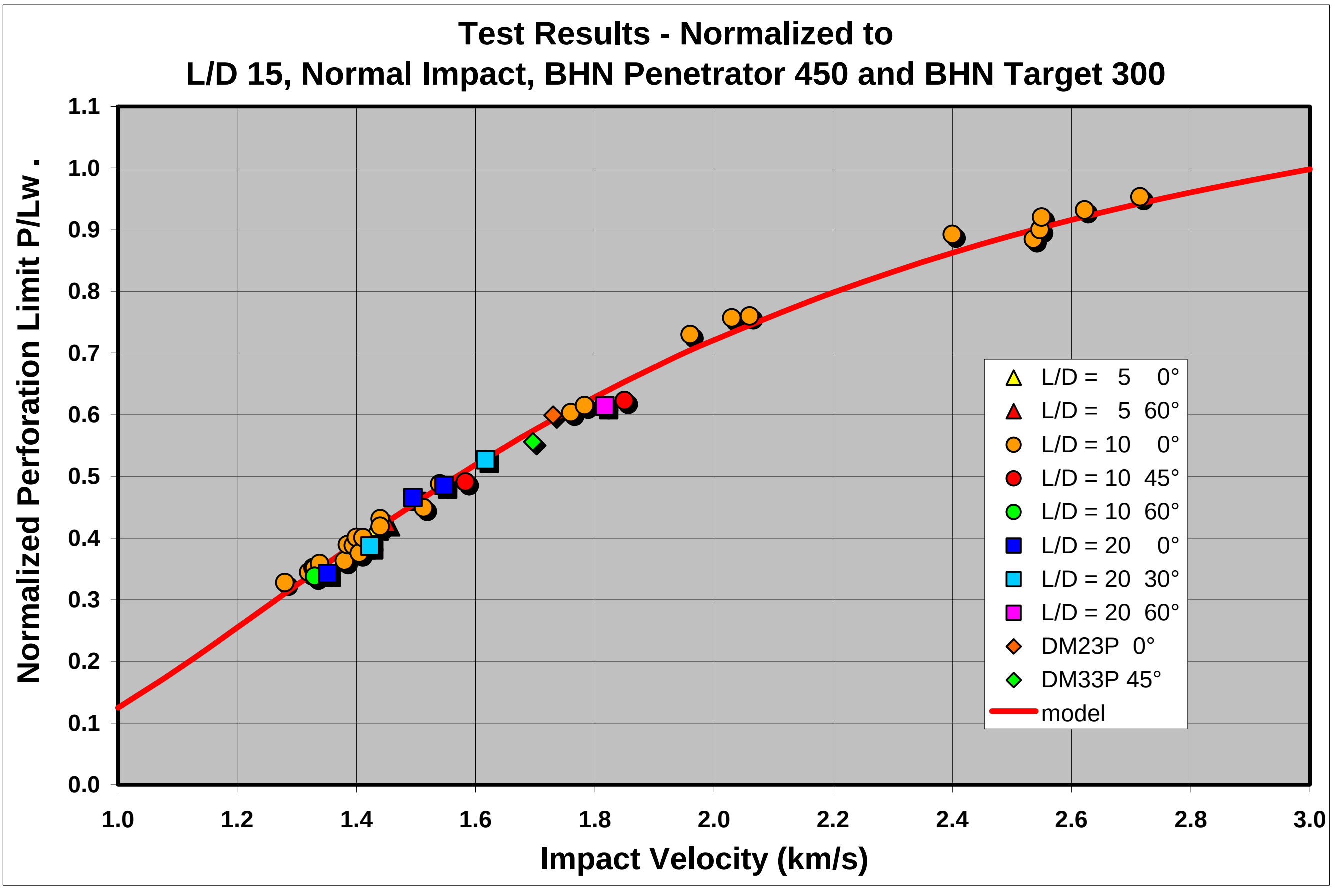
Influence of L over D
f(L_W/D)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
2
4
6
8
10
12
14
16
18
20
22
Aspect Ratio L_W / D
L/D = 4 - 0°
L/D = 4 - 30°
L/D = 4 - 60°
L/D = 8 - 0°
L/D = 10 - 0°
L/D = 10 - 60°
L/D = 12 - 0°
L/D = 12 - 30°
L/D = 12 - 45°
L/D = 12 - 60°
L/D = 15 - 0°
L/D = 20 - 0°
L/D = 20 - 30°
L/D = 20 - 45°
L/D = 20 - 60°
Equation



Influence of Obliquity
f(θ)
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0
5
10
15
20
25
30
35
40
45
50
55
60
65
Obliquity
L/D = 4 - 0°
L/D = 4 - 30°
L/D = 4 - 60°
L/D = 8 - 0°
L/D = 10 - 0°
L/D = 10 - 60°
L/D = 12 - 0°
L/D = 12 - 30°
L/D = 12 - 45°
L/D = 12 - 60°
L/D = 15 - 0°
L/D = 20 - 0°
L/D = 20 - 30°
L/D = 20 - 45°
L/D = 20 - 60°
Equation

5.3 Steel Penetrators





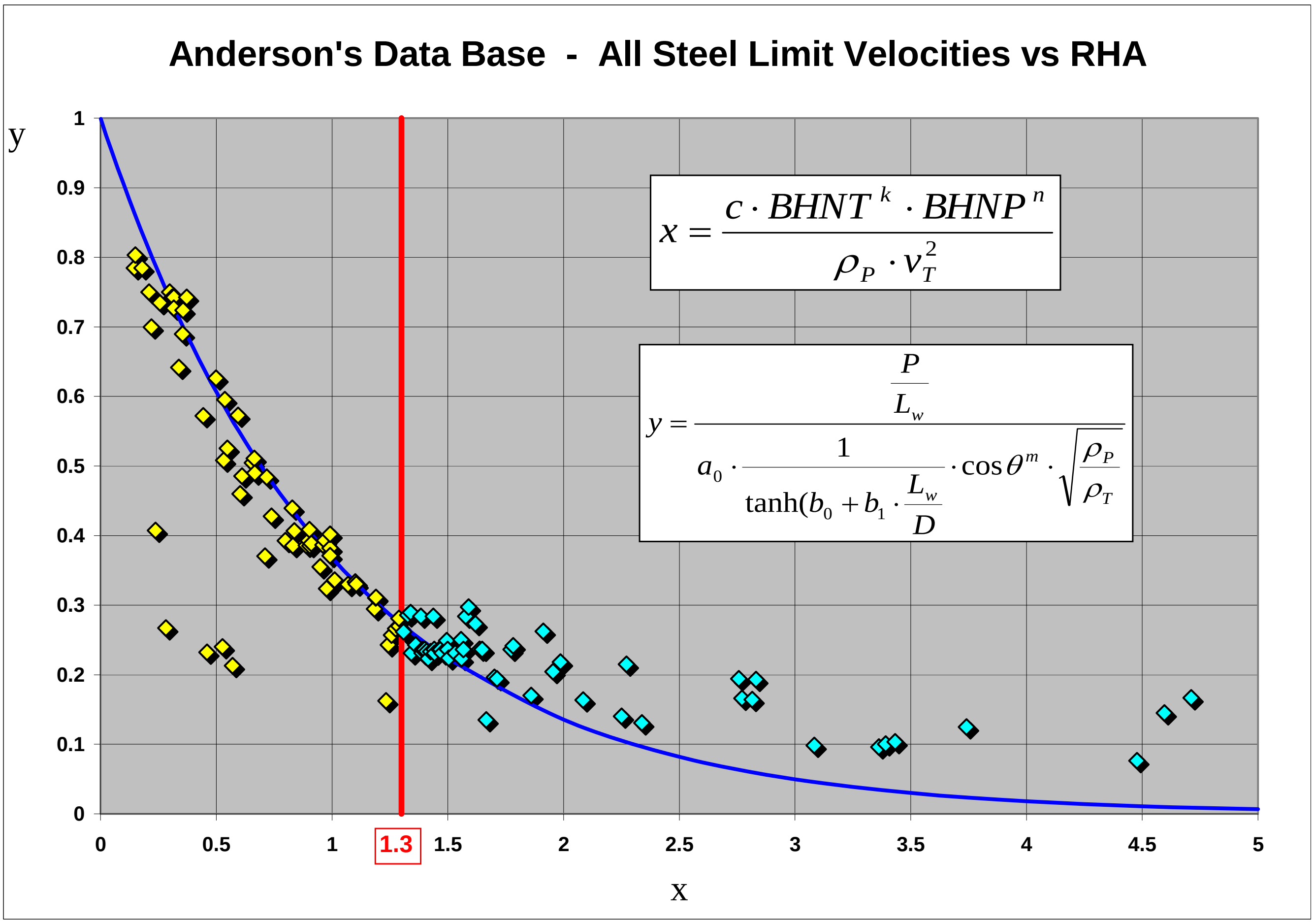
Range of results:

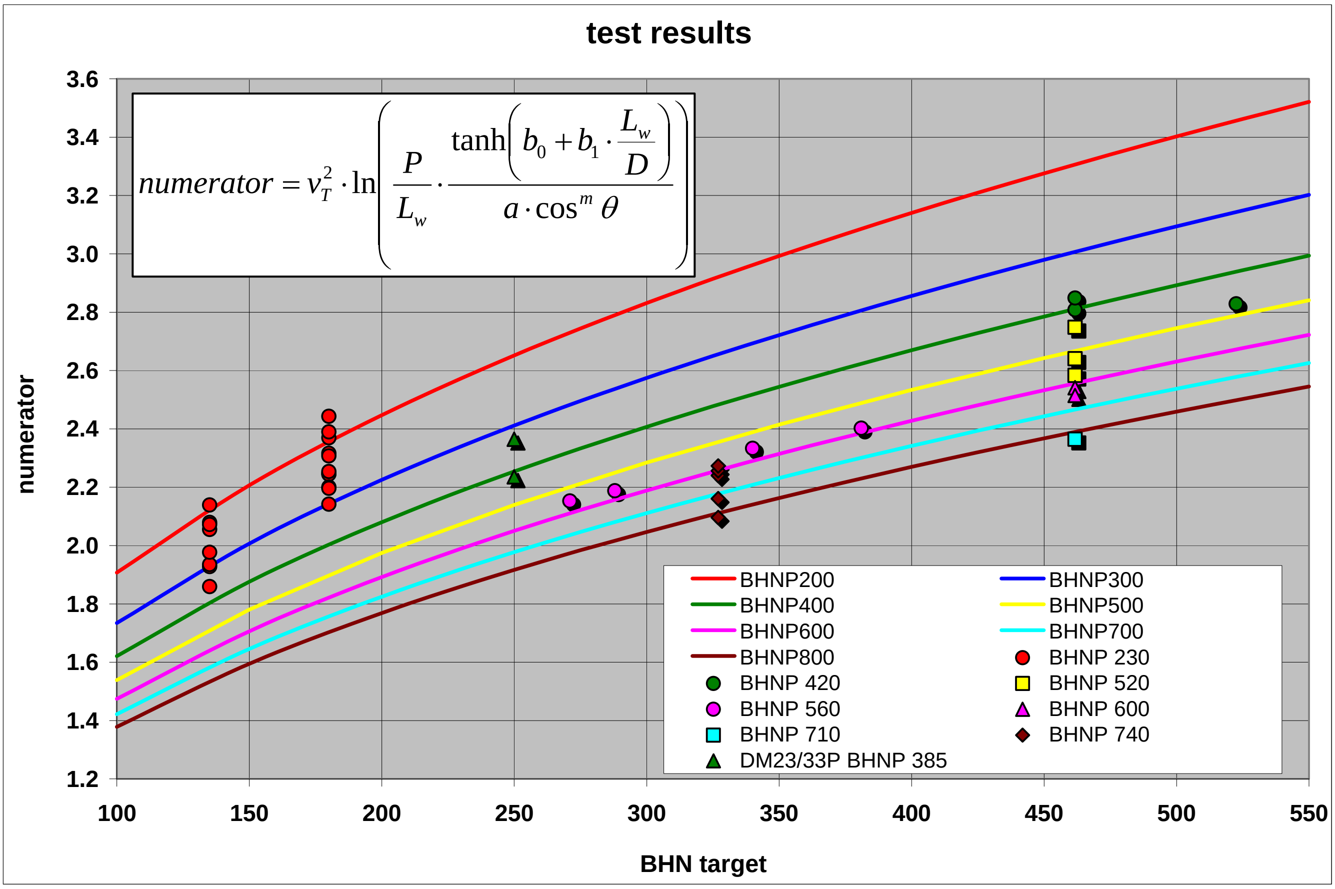
Penetrator

Aspect ratio L_w/D	5 - 20	
density	7'850	kg/m^3
impact velocty	1.3 – 2.7	km/s
Brinell Hardness Number	230 – 740	(has influence on perforation)

Target

obliquity NATO	0 – 60	°
Brinell Hardness Number	135 – 520	





References

[1] ***A Penetration Mechanics Database***
C.E. Anderson Jr. , B.L. Morris, D.L. Littlefield
Southwest Research Institute, San Antonio, Texas
January 1992

Steffisburg, June 2009 — Willi Odermatt