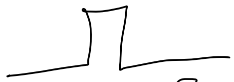
Math Basis

define



$$\ell(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Define

$$x_{sn}(t) = \frac{x_c(nT)}{\tau}\left[\ell(t-nT) - \ell(t-nT-\tau)\right]$$

Define

$$x_s(t) = \sum_{n=-\infty}^{\infty} x_{sn}(t)$$

Then

$$X_{sn}(s) = \frac{1}{\tau}\left(\frac{1-e^{-s\tau}}{s}\right) x_c(nT)e^{-snT}$$

and

$$X_s(s) = \frac{1}{\tau}\left(\frac{1-e^{-s\tau}}{s}\right)\sum_{n=-\infty}^{\infty} x_c(nT)e^{-snT}$$

Thus

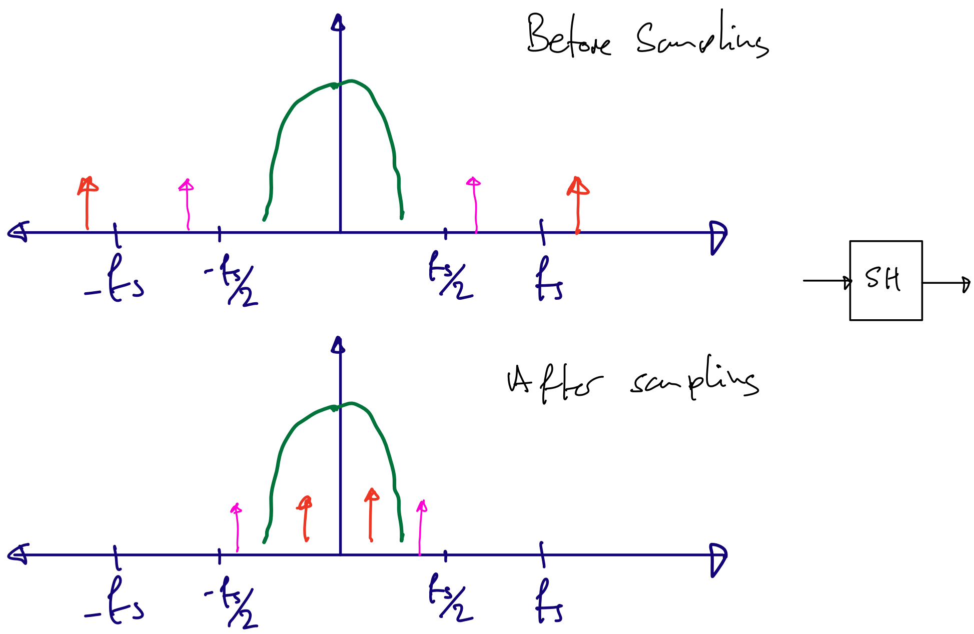
$$\xrightarrow{\tau \to 0} \quad X(s) = \sum_{n=-\infty}^{\infty} x_c(nT)e^{-snT}$$

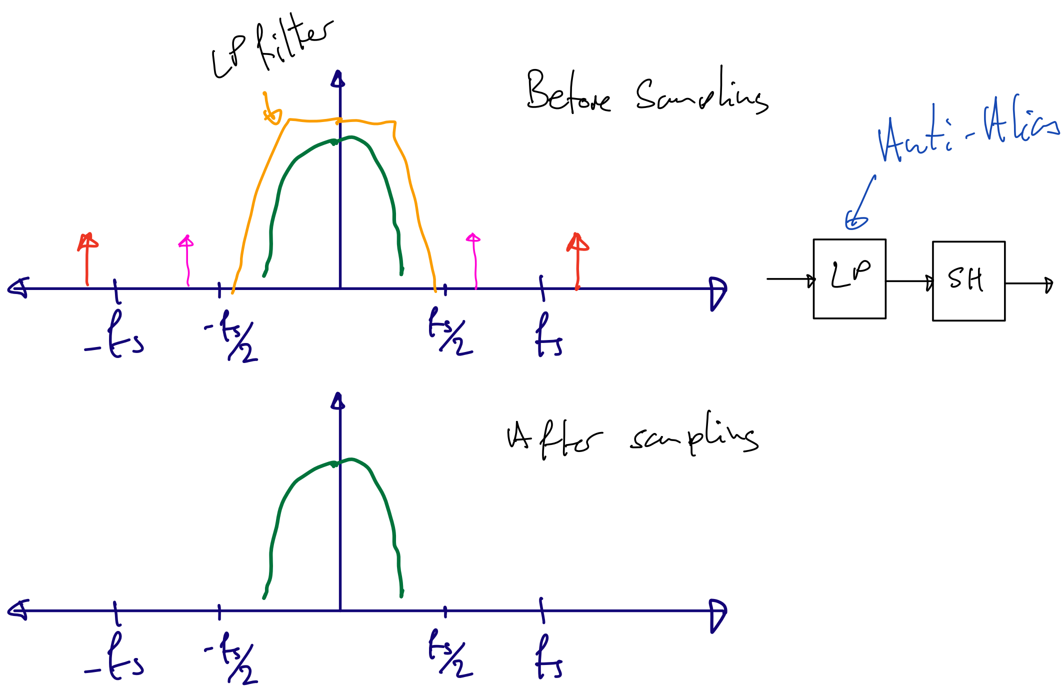
⋮ Bla Bla, rigorous math

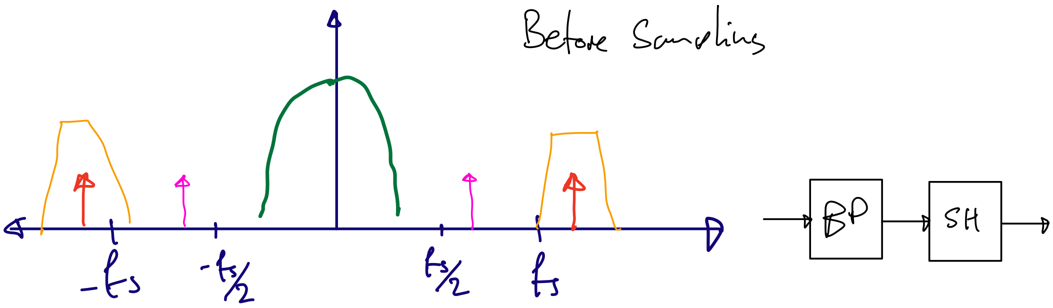
$$X_s(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\omega - \frac{jk2\pi}{T}\right)$$

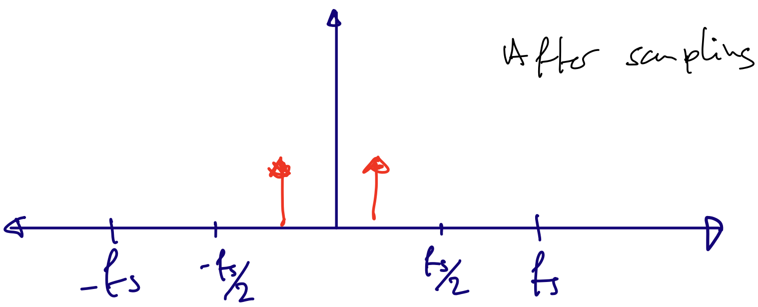
When you sample a signal, then there will be copies at every nfs

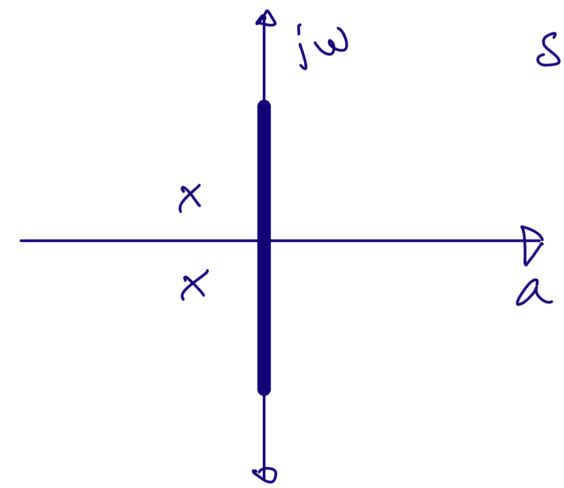
As such, $x(t)$ should be band limited before sampling






Before Sampling
$-f_s$
$-f_s/2$
$f_s/2$
f_s
BP
SH


After sampling
$-f_s$
$-f_s/2$
$f_s/2$
f_s



$s = a + jw$

$\times \Rightarrow$ poles. $|H(s)| \to \infty$

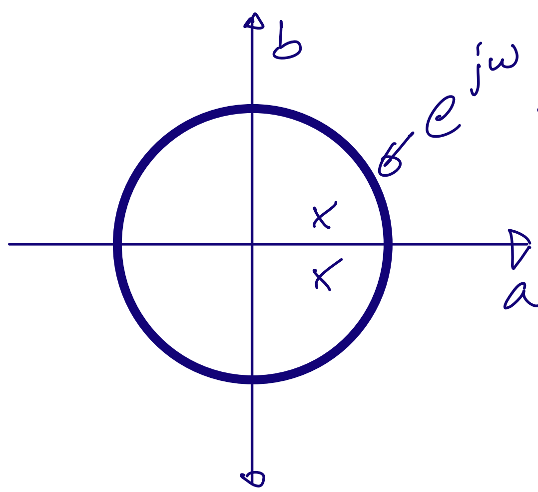
$0 \Rightarrow$ zeros. $|H(s)| \to 0$

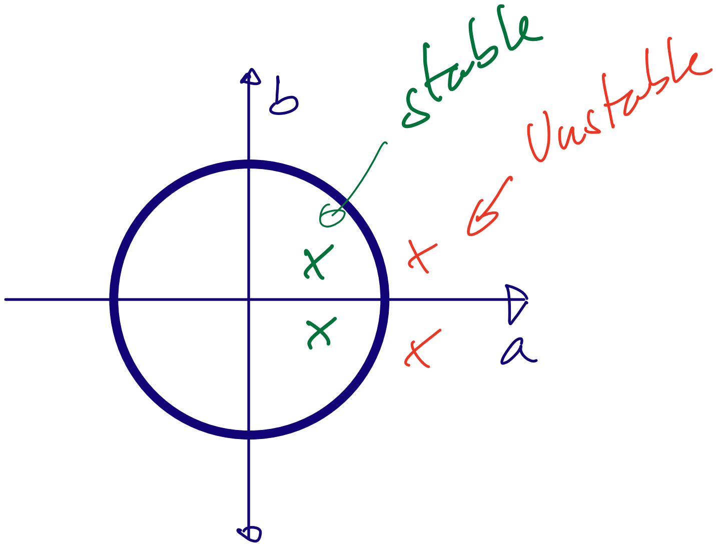
Discrete time

Spectra repeats every 2π

z-plane

$z = a + jb$





First order

$H(z) = \frac{b}{z-a}$

$$h(n) = \begin{cases} k & n<1 \\ a^{n-1}b + a^n k & n \geq 1 \end{cases}$$

$$y = bx + az^{-1}y$$

$$y - az^{-1}y = bx$$

$$H(z) = \frac{b}{1-az^{-1}}$$

FIR

IIR

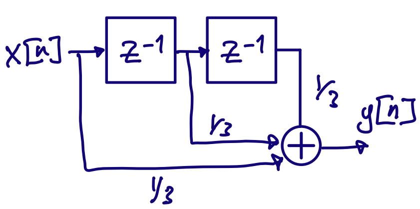
Finite impulse response

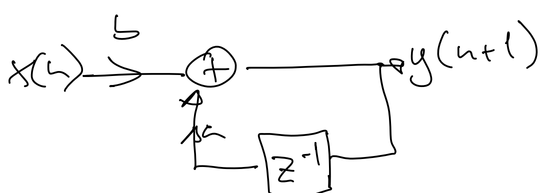
Infinite impulse response

$$H(z) = \frac{1}{2}\sum_{i=0}^{2} z^{-i}$$

$H(z) = \frac{b}{z-a}$

$$h(n) = \begin{cases} k & n<1 \\ a^{n-1}b + a^n k & n \geq 1 \end{cases}$$

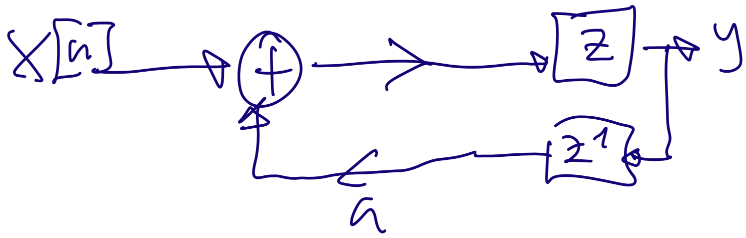




$y[n+1] = bx[n] + ay(n)$

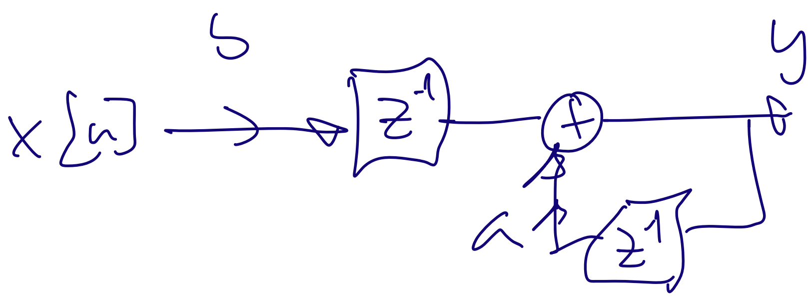
$$zy = bx + ay$$

$$y(z-a) = bx$$



$$y[n] = b\,x[n-1] + a\,y[n-1]$$

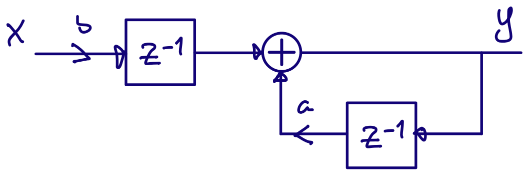
$$y = x b z^{-1} + a y z^{-1}$$



$$y(1 - a z^{-1}) = b x z^{-1}$$

$$H(z) = \frac{a z^{-1}}{1 - a z^{-1}}$$

$$= \frac{b}{z - a}$$



SC

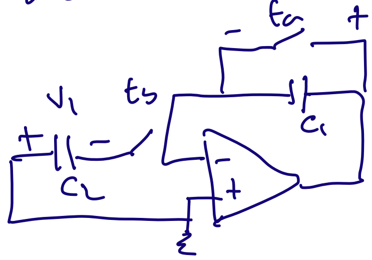
Why:

+ filter poles/zeros + gain determined by $\frac{C_x}{C_y}$ ratios. Accurate to 0.1%.

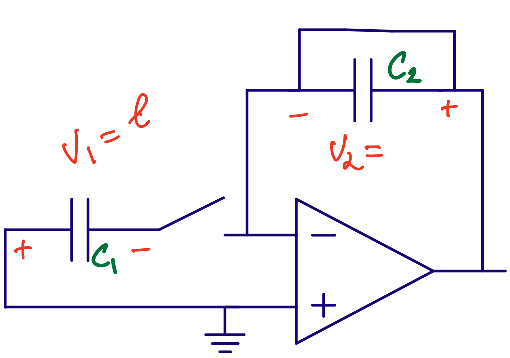
÷ Needs accurate freq ref.

+ Linearity determined mostly by OTA gain (+ settling)

Comments OTAs

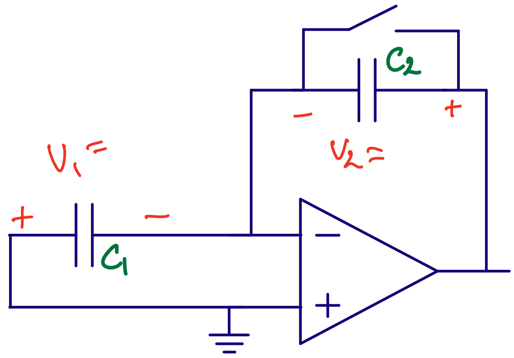


Time	Q_1	Q_2
t_a	0	$C_2 V_2$
t_s		0



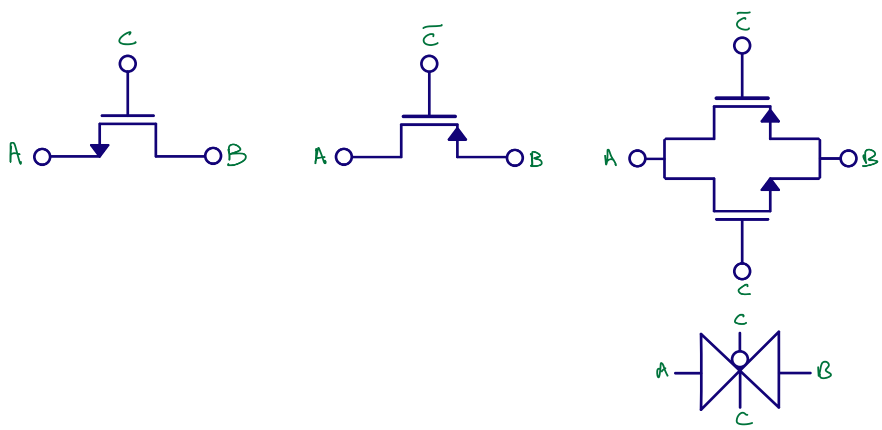
$Q_1 =$

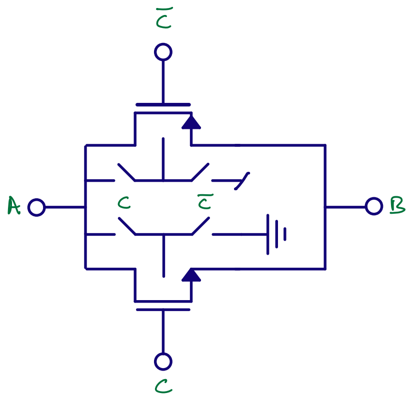
$Q_2 =$

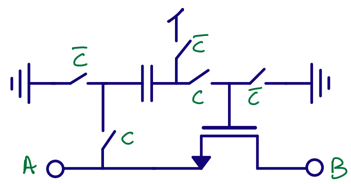


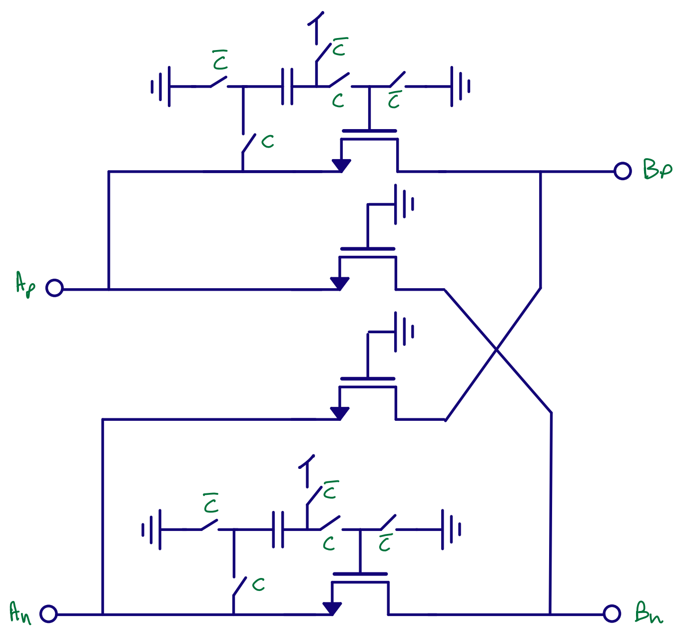
$Q_1 =$

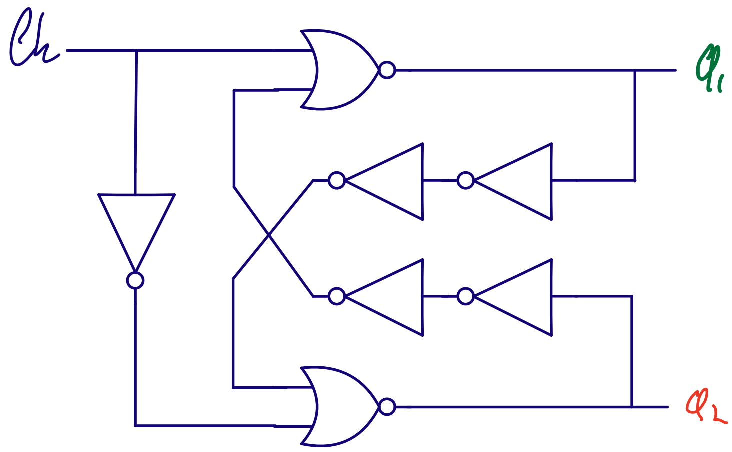
$Q_2 =$

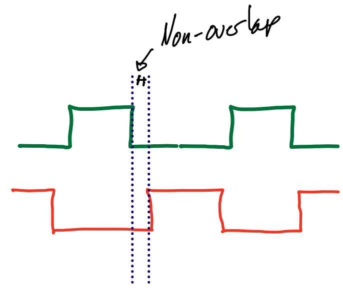



$\bar{C}$
A
c
$\bar{c}$
B
C

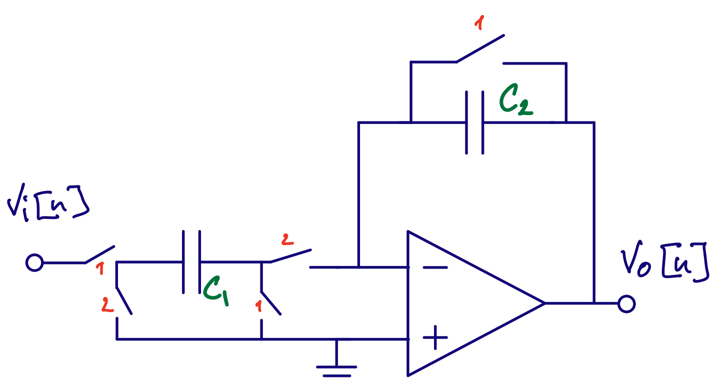

$\bar{c}$
$\bar{c}$
c
$\bar{c}$
c
A
B

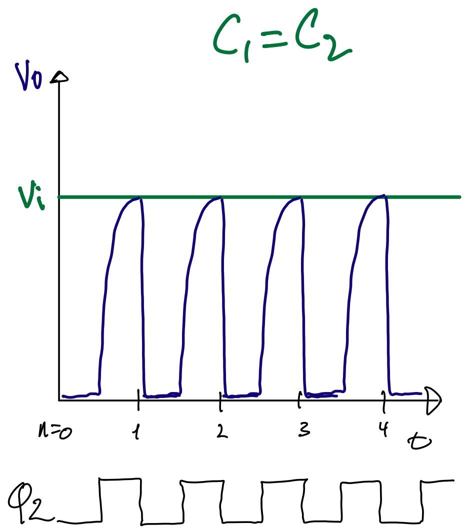

$\bar{c}$
$\bar{c}$
c
$\bar{c}$
c
B_p
A_p
$\bar{c}$
$\bar{c}$
c
$\bar{c}$
c
A_n
B_n

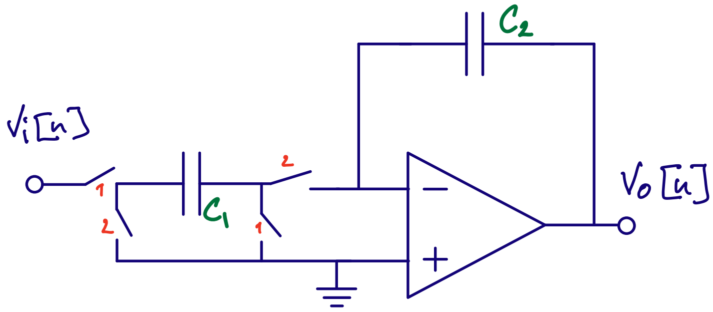




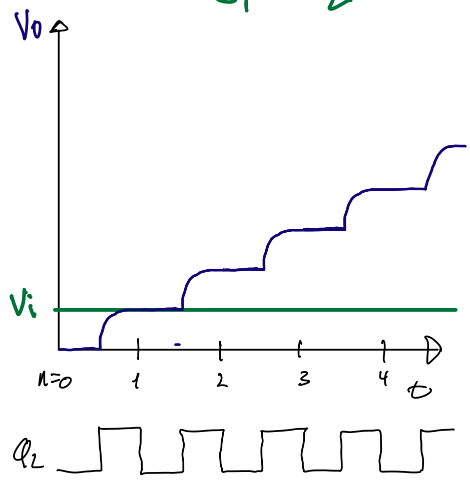
$$V_o[n+1] = \frac{C_1}{C_2} V_i[n]$$
$$\Downarrow$$
$$V_o z = \frac{C_1}{C_2} V_i \Rightarrow H(z) = \frac{C_1}{C_2} z^{-1}$$







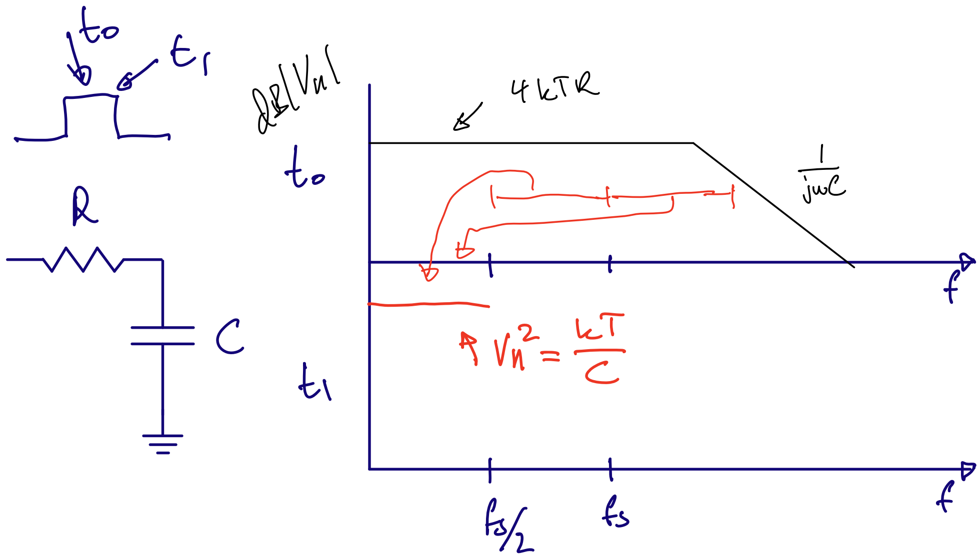
$C_1 = C_2$



$$V_o[n] = V_o[n-1] + \frac{C_1}{C_2} V_i[n-1]$$

$$V_o - z^{-1} V_o = \frac{C_1}{C_2} z^{-1} V_i$$

$$H(z) = \frac{C_1}{C_2} \frac{z^{-1}}{z^{-1}+1} = \frac{C_1}{C_2} \frac{1}{z-1}$$

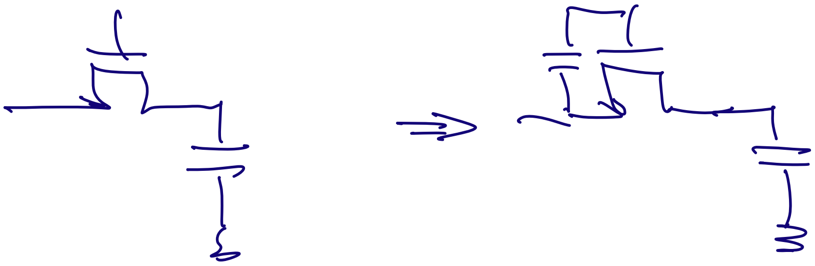


Both phases contribute noise!

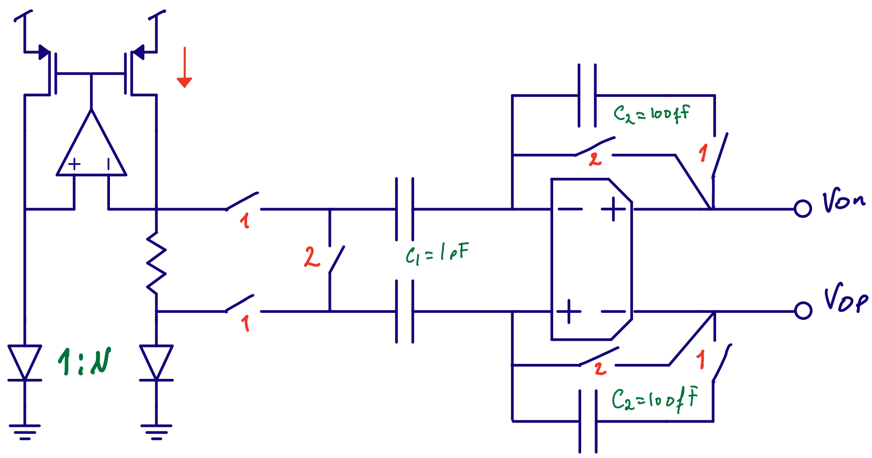
$$\sigma_t^2 = \sigma_{t_1}^2 + \sigma_{t_2}^2$$

Noise slide!

Simulation of charge injection?



$$Q_{CH} = -WLC_{ox}V_{eff}$$



$$\Delta V_{BE} = \frac{30\,mV}{64} \sim 0{,}5\,mV$$

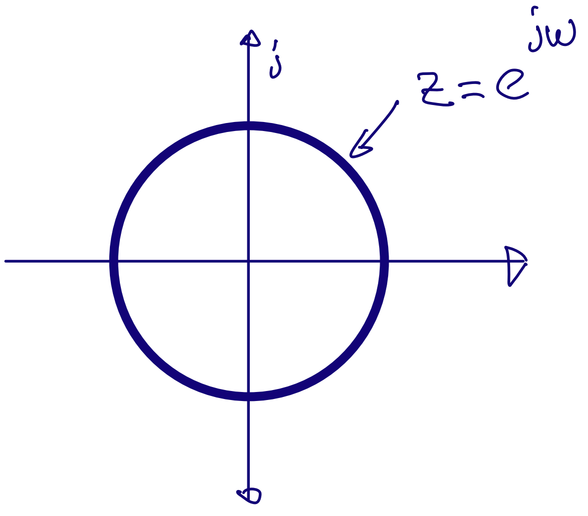
$$\Delta V_n \sim 100\,\mu V$$

$$1e^{-4}$$

$$1e^{-8} = \frac{kT}{C}$$

$$C = \frac{1{,}38e^{-23} \cdot (273 + 125)}{1e^{-8}}$$

$$= 2 \cdot 1{,}38e^{-15} \cdot 400 = 1{,}1\,pF$$



j
$z = e^{jw}$