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SOME STUDIES OF VARIABILITY OF RETURNS ON INVESTMENTS IN COMMON STOCKS*

LAWRENCE FISHER AND JAMES H. LORIE!

INTRODUCTION

We report here the findings of three studies we have conducted on the variability of returns on investments in common stocks listed on the New York Stock Exchange. One study examines the frequency distributions of returns on individual stocks for fifty-five specific periods ranging from one to forty years in length during the period 1926-65. A second examines the aggregated distributions of returns from investments in individual common stocks for nonoverlapping periods of equal length from one to twenty years. Aggregating frequency distributions of all such one-, five-, ten-, or twenty-year periods permits broader generalization about the behavior of the market, since these aggregated distribu-

* We are indebted to Harry Roberts for aid in understanding Gini's mean difference. Our exposition has benefitted from reactions to presentations at seminars at the University of Chicago and several other universities. Most of the many computer programs required for this study were prepared by Marvin Lipson. Some additional programming was done by Mark Case and Owen M. Hewett.

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tions are not dominated by the behavior of the market in any single period.

The third study deals with returns from investment in portfolios containing different numbers of common stocks on the New York Stock Exchange. Distributions were found for portfolios of six size ranges from one through 128 and for portfolios containing all such common stocks. The tables dealing with aggregated frequency distributions, paralleling the second study, are of greater general interest, we think, and are discussed in the text. The tables from which they were derived and which deal with specific periods, paralleling the first study, are of less direct interest and are presented in Appendix A.

Before discussing our results, we would like to indicate why we undertook these studies and the ways in which they are related to our earlier studies on average rates of return¹ and on outcomes for random investments.²

¹ Lawrence Fisher and James H. Lorie, "Rates of Return on Investments in Common Stocks," *Journal of Business* 37 (January 1964):1-21; Lawrence Fisher and James H. Lorie, "Rates of Return on Investments in Common Stocks: The Year-by-Year Record, 1926-65," *Journal of Business* 41 (July 1968):291-316.

² Lawrence Fisher, "Outcomes for 'Random' Investments in Common Stocks Listed on the New York Stock Exchange," *Journal of Business* 38 (April 1965):149-61.

For several reasons, studies of variability may be interesting. One of the most controversial and important subjects in the field of finance is risk. There is controversy about both methods of estimation and the nature of the relationship between risk and rates of return. The studies reported here do not deal directly with either of those controversial aspects of risk, but they do bear upon the general subject by providing the first comprehensive and well-based estimates of the effect of increasing the size of portfolios on the variability of returns—one of the most widely used estimates of risk. The earlier studies of average rates of return provided bench marks which have been widely used in evaluating the performance of average rates of return from portfolios; the studies reported here can be thought of as providing bench marks for evaluating the effectiveness of diversification in reducing variability of returns.

Another way of looking at the present studies would be to say that the earlier studies on average rates of return indicate only the average experience from investing in common stocks listed on the New York Stock Exchange without any indication of the inherent riskiness. The studies reported here indicate something about riskiness by providing detailed information on frequency distributions of returns.

These studies should prove more useful than the first author's earlier study of outcomes for random investments in common stocks, which also deals with variability of returns, because in these studies we are able to look at the variability of returns on portfolios as well as return on individual stocks. We can now look at portfolios because the current studies hold constant the holding period of the investments whose frequency distributions are reported. Looking at port-

folios is obviously desirable, since almost all investors with significant investments hold portfolios of more than one common stock. Moreover, there is much interest in the effect of changing the size of portfolios on variability in return.

The current studies are also superior in that they, unlike the earlier study on outcomes for random investments, take into account the value of investments even after they consist of assets other than the common stock in which the investment was originally made. This change in assets can occur where there are mergers, spin-offs, or delistings.

A section on general methodology follows these introductory remarks. It includes some comments on statistics that we have computed in the course of all three studies. Next are sections on the three studies, and finally appendixes containing the basic data for the last study. The results are presented primarily in tables which, we hope, will provide reference material for specialists in the field. Since we have spent considerable time examining the material in the tables, we will make a few comments. However, most analysis will be left to the reader.

GENERAL METHODS OF ANALYSIS

The distributions which will be described are in all cases the distributions of "wealth ratios." The wealth ratio is the ratio of the value of the investment at the end of the period to the amount invested. Much of the work in this field has been in terms of rates of return, since such rates are necessary in comparing investment results for periods of different lengths. We are free to use wealth ratios because we compare only periods of equal length.

We have used wealth ratios for two reasons. First, introspection and observation have persuaded us that it is extremely difficult to understand the significance of differences among annual rates of return for long periods of time. For example, few persons easily see that a difference between 5 percent per annum, compounded annually, and 10 percent per annum over a forty-year period produces wealth ratios which are strikingly different—approximately 7 and 45, respectively. The wealth ratio produced by the 10 percent annual return is 543 percent greater than the ratio produced by

period in question exceeds the compounding interval, and to an overestimate if the period is less than the compounding interval. This is exemplified in table 1.

Harry Markowitz⁴ uses returns, but they are not necessarily annual rates. They are simply one less than the corresponding wealth ratios, and they are typically expressed as percentages. In table 1, the Markowitz returns for stocks A, B, and C would be 0, 300 percent, and

TABLE 1

ILLUSTRATION OF RELATIONSHIP BETWEEN WEALTH RATIOS AND RATES OF RETURN

Stock	Wealth Ratio After 10 Years	Annual Rate of Return Compounded Annually (Percent)
A	1	0
В	4	14.9
С	7	21.5
Mean	4	12.1

The wealth ratio implied by an investment returning 12.1 percent annually and held for ten years is 3.14, not 4.0. Thus, using the mean rate of return to deduce the mean wealth ratio would lead to a significant underestimate.

the 5 percent return. The corresponding wealth ratios for annual rates of return of 9 percent and 10 percent are 31 and 45, respectively. The wealth ratio for the 10 percent rate of return exceeds that for the 9 percent rate of return by 44 percent.

The second reason for presenting data on wealth ratios rather than on rates of return is that data on rates of return are frequently misinterpreted. The most common mistake is to assume that one can deduce the mean wealth ratio from knowledge of the mean rate of return.³ Such an attempt leads to an underestimate of the mean wealth ratio if the

600 percent, respectively. We have used wealth ratios rather than Markowitz's returns, since the latter are easily confused with *annual* rates of return.

We recognize, however, that some readers do think about returns from investments as annual rates. In order to

³ See, for example, Marc Nerlove, "Factors Affecting Differences among Rates of Return on Investments in Individual Common Stocks," Review of Economics and Statistics 50 (August 1968): 312-31; and Eugene F. Brigham and James L. Pappas, "Rates of Return on Common Stock," Journal of Business 42 (July 1969): 302-20.

⁴ Harry Markowitz, Portfolio Selection: Efficient Diversification of Investments (New York: John Wiley & Sons, 1959).

facilitate translation from wealth ratios to annual rates of return, we present table 2, which simply indicates for periods of various lengths the rates of return corresponding to various wealth ratios.

In computing wealth ratios, commissions were charged when investments were originally made and when each dividend was reinvested, but the value of the investment at the end of each period was calculated on the basis of the market price on that date without taking into account any contingent transaction costs or taxes.⁵

For each frequency distribution of wealth ratios the following statistics are reported:

- 1. a) 5th centile
 - b) 10th centile
 - c) 20th centile
 - \vec{d}) 30th centile
 - e) 40th centile
 - f) 50th centile (median)
 - g) 60th centile
 - h) 70th centile
 - i) 80th centile
 - j) 90th centile
- k) 95th centile
- 2. The maximum
- 3. The minimum
- 4. The arithmetic mean
- 5. Measures of absolute dispersion
 - a) The standard deviation
 - b) The mean deviation
 - c) Gini's mean difference
- 6. Measures of relative dispersion
 - a) Coefficient of variation
 - b) Relative mean deviation
 - c) Gini's coefficient of concentration
- 7. Momental skewness
- 8. Kurtosis

⁵ The wealth ratios used were, in fact, the wealth ratios used to construct the table of annual rates of return with reinvestment of dividends for the tax-exempt investor in the cash-to-portfolio computations (part A of table 1 of the Fisher and Lorie 1968 article). For the methods of treating investments in stocks which were merged into or spun off other issues or which were delisted, see the 1968 article, p. 295, and the 1964 article, pp. 15–17.

All of the foregoing statistics should be familiar, with the possible exception of Gini's mean difference and Gini's coefficient of concentration. These statistics are discussed, among other places, in Gini's own work⁶ and in a text of Kendall and Stuart.⁷ Even so, it may be helpful for us to say something here about Gini's statistics.

In principle, to compute Gini's mean difference, one merely finds the absolute value of the difference between the elements of each possible pair of observations and divides by the number of such pairs. For example, consider the following three observations: 2, 4, 7. The following pairs are considered: 2 and 4, 2 and 7, and 4 and 7. The absolute values of the differences between the elements of these pairs are 2, 5, and 3, respectively. Thus Gini's mean difference is 10 divided by 3, or $3\frac{1}{3}$. If there are N observations, the number of possible pairs is equal to N(N-1)/2. When N is very large—as in our third study, for example—the volume of computations necessary for exact calculation is unbearable and estimation must be used.

The relationship between Gini's mean difference and Gini's coefficient of concentration is nearly analogous to that between the standard deviation and the coefficient of variation. To compute the coefficient of variation, one divides the standard deviation by the mean. To compute Gini's coefficient of concentration, one divides Gini's mean difference by twice the mean.

The discourteous reader might ask at this point why, instead of using statistics

⁶ Corrado Gini, *Memorie di metodologia statistica*, 2d ed. rev. Ernesto Pizzetti and T. Salvemini (Rome: Libreria Eredi Virgilio Veschi, 1955).

⁷ Maurice G. Kendall and Alan Stuart, Advanced Theory of Statistics in Three Volumes, 2d ed. (New York: Hafner Publishing Co., 1963), vol. 1.

TABLE 2
WEALTH RATIOS AND CORRESPONDING ANNUAL RATES OF RETURN
(COMPOUNDED ANNUALLY) FOR SPECIFIED PERIODS

Holding Period

Wealth Rate of Ratio Return Rate of Ratio Wealth Rate of Return Return Rate of Return Rate of Return Ret	5 Y	ears	10	Years	20	Years	39	11/12 Years
.01 -60.2	Wealth	Rate of	Wealth	Rate of	Wealth	Rate of	Weal	lth Rate of
.02 -54,3	Ratio	Return	Ratio	Return	Ratio	Return	Rati	lo Return
.02 -54,3	.01	-60.2	.01	-36.9	.01	-20.6		01 -10.9
.03 -50,4	.02		.02	-32.4	.02	-17.8		.02 -9.3
.04		-50.4						
.05	.04	-47.5	.04	-27.5	.05	-13.9		
.07 -41.2					-			2 -4.0
1.1 -36,9 .2 -14,9 .3 -5.8 .4 -2,3 .2 -27,5 .3 -11,3 .4 -4.5 .6 -1.3 .3 -21,4 .4 -8.8 .5 -3.4 1.0 0.0 .4 -16,7 .5 -6.7 .7 -1.8 1.5 1.0 .5 -12.9 .6 -5.0 .9 -0.5 2.2 2.0 .6 -9.7 .7 -3.5 1.1 0.5 3.2 3.0 .7 -6.9 .8 -2.2 1.4 1.7 4.6 3.9 .8 -4.4 .9 -1.0 1.8 3.0 6.4 4.8 .9 -2.1 1.1 1.0 2.3 4.3 8.6 5.5 1.0 0. 1.5 2.7 2.8 5.3 12. 6.4 1.1 1.9 1.5 4.1 3.4 6.3 15. <td< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></td<>								
.2 -27.5		-36.9			.3			
.3 -21.4		-27.5			.4			
.4 -16,7 .5 -6,7 .7 -1.8 1.5 1.0 .5 -12.9 .6 -5.0 .9 -0.5 2.2 2.0 .6 -9.7 .7 -3.5 1.1 0.5 3.2 3.0 .7 -6.9 .8 -2.2 1.4 1.7 4.6 3.9 .8 -4.4 .9 -1.0 1.8 3.0 6.4 4.8 .9 -2.1 1.1 1.0 2.3 4.3 8.6 5.5 1.0 0. 1.3 2.7 2.8 5.3 12. 6.4 1.1 1.9 1.5 4.1 3.4 6.3 15. 7.0 1.3 5.4 1.8 6.1 4.0 7.2 18. 7.5 1.4 7.0 2.0 7.2 4.7 8.0 22. 8.4 1.7 11.2 2.4 9.1 5.9 9.3 28. 8.4 1.7 11.2 2.4 9.1 5.9 9.3 28. <t< td=""><td></td><td></td><td></td><td></td><td>.5</td><td></td><td></td><td></td></t<>					.5			
.5 -12.9 .6 -5.0 .9 -0.5 2.2 2.0 .6 -9.7 .7 -3.5 1.1 0.5 3.2 3.0 .7 -6.9 .8 -2.2 1.4 1.7 4.6 3.9 .8 -4.4 .9 -1.0 1.8 3.0 6.4 4.8 .9 -2.1 1.1 1.0 2.3 4.3 8.6 5.5 1.0 0. 1.3 2.7 2.8 5.3 12. 6.4 1.1 1.9 1.5 4.1 3.4 6.3 15. 7.0 1.3 5.4 1.8 6.1 4.0 7.2 18. 7.5 1.4 7.0 0.2 0.7.2 4.7 8.0 22. 8.1 1.4 7.0 0.2 0.7.2 4.7 8.0 22. 8.1 1.7 11.2 2.4 9.1 5.9 9.3 28. 8		-16.7			.7			
.6 -9.7 .7 -3.5 1.1 0.5 3.2 3.0 .8 -4.4 .9 -1.0 1.8 3.0 6.4 4.8 .9 -2.1 1.1 1.0 2.3 4.3 8.6 5.5 1.0 0. 1.3 2.7 2.8 5.3 12. 6.4 1.1 1.9 1.5 4.1 3.4 6.3 15. 7.0 1.3 5.4 1.8 6.1 4.0 7.2 18. 7.5 1.4 7.0 2.0 7.2 4.7 8.0 22. 8.1 1.6 9.9 2.2 8.2 5.4 8.8 25. 8.4 1.7 11.2 2.4 9.1 5.9 9.3 28. 8.7 1.8 12.5 2.6 10.0 6.4 9.7 31. 9.0 2.0 14.9 2.8 10.8 7.0 10.2 35. 9					.9			
.8 -4.4 .9 -1.0 1.8 3.0 6.4 4.8 .9 -2.1 1.1 1.0 2.3 4.3 8.6 5.5 1.0 0. 1.3 2.7 2.8 5.3 12. 6.4 1.1 1.9 1.5 4.1 3.4 6.3 15. 7.0 1.3 5.4 1.8 6.1 4.0 7.2 18. 7.5 1.4 7.0 2.0 7.2 4.7 8.0 22. 8.1 1.6 9.9 2.2 8.2 5.4 8.8 25. 8.4 1.7 11.2 2.4 9.1 5.9 9.3 28. 8.7 1.8 12.5 2.6 10.0 6.4 9.7 31. 9.0 1.9 13.7 2.7 10.4 6.8 10.1 34. 9.2 2.0 14.9 2.8 10.8 7.0 10.2 35. <t< td=""><td>.6</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></t<>	.6							
.8 -4.4 .9 -1.0 1.8 3.0 6.4 4.8 .9 -2.1 1.1 1.0 2.3 4.3 8.6 5.5 1.0 0. 1.3 2.7 2.8 5.3 12. 6.4 1.1 1.9 1.5 4.1 3.4 6.3 15. 7.0 1.3 5.4 1.8 6.1 4.0 7.2 18. 7.5 1.4 7.0 2.0 7.2 4.7 8.0 22. 8.1 1.6 9.9 2.2 8.2 5.4 8.8 25. 8.4 1.7 11.2 2.4 9.1 5.9 9.3 28. 8.7 1.8 12.5 2.6 10.0 6.4 9.7 31. 9.0 1.9 13.7 2.7 10.4 6.8 10.1 34. 9.2 2.0 14.9 2.8 10.8 7.0 10.2 35. 9.3 2.1 16.0 2.9 11.2 7.1 10.3 36.		-6.9						
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1.1 1.9 1.5 4.1 3.4 6.3 15. 7.0 1.3 5.4 1.8 6.1 4.0 7.2 18. 7.5 1.4 7.0 2.0 7.2 4.7 8.0 22. 8.1 1.6 9.9 2.2 8.2 5.4 8.8 25. 8.4 1.7 11.2 2.4 9.1 5.9 9.3 28. 8.7 1.8 12.5 2.6 10.0 6.4 9.7 31. 9.0 1.9 13.7 2.7 10.4 6.8 10.1 34. 9.2 2.0 14.9 2.8 10.8 7.0 10.2 35. 9.3 2.1 16.0 2.9 11.2 7.1 10.3 36. 9.4 2.2 17.1 3.0 11.6 7.2 10.4 37. 9.5 2.4 19.1 3.2 12.3 7.4 10.5 39. 9.6 2.5 20.1 3.3 12.7 7.6 10.7 <td< td=""><td></td><td>0.</td><td></td><td>2.7</td><td>2.8</td><td>5.3</td><td>12</td><td>6.4</td></td<>		0.		2.7	2.8	5.3	12	6.4
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2.0 14.9 2.8 10.8 7.0 10.2 35. 9.3 2.1 16.0 2.9 11.2 7.1 10.3 36. 9.4 2.2 17.1 3.0 11.6 7.2 10.4 37. 9.5 2.3 18.1 3.1 12.0 7.3 10.5 38. 9.5 2.4 19.1 3.2 12.3 7.4 10.5 39. 9.6 2.5 20.1 3.3 12.7 7.6 10.7 40. 9.7 2.6 21.1 3.4 13.0 7.9 10.9 42. 9.8 2.7 22.0 3.5 13.3 8.3 11.2 44. 9.9 2.8 22.9 3.7 4.0 8.8 11.5 48. 10.2 2.9 23.7 4.0 14.9 9.4 11.9 53. 10.5 3.0 24.6 4.3 15.7 11. 12.7 60. 10.8 3.2 26.2 4.8 17.0 12. 13	1.8	12.5			6.4	9.7		
2.0 14.9 2.8 10.8 7.0 10.2 35. 9.3 2.1 16.0 2.9 11.2 7.1 10.3 36. 9.4 2.2 17.1 3.0 11.6 7.2 10.4 37. 9.5 2.3 18.1 3.1 12.0 7.3 10.5 38. 9.5 2.4 19.1 3.2 12.3 7.4 10.5 39. 9.6 2.5 20.1 3.3 12.7 7.6 10.7 40. 9.7 2.6 21.1 3.4 13.0 7.9 10.9 42. 9.8 2.7 22.0 3.5 13.3 8.3 11.2 44. 9.9 2.8 22.9 3.7 4.0 8.8 11.5 48. 10.2 2.9 23.7 4.0 14.9 9.4 11.9 53. 10.5 3.0 24.6 4.3 15.7 11. 12.7 60. 10.8 3.2 26.2 4.8 17.0 12. 13	1.9	13.7	2.7	10.4	6.8	10.1	34.	9.2
2.1 16.0 2.9 11.2 7.1 10.3 36. 9.4 2.2 17.1 3.0 11.6 7.2 10.4 37. 9.5 2.3 18.1 3.1 12.0 7.3 10.5 38. 9.5 2.4 19.1 3.2 12.3 7.4 10.5 39. 9.6 2.5 20.1 3.3 12.7 7.6 10.7 40. 9.7 2.6 21.1 3.4 13.0 7.9 10.9 42. 9.8 2.7 22.0 3.5 13.3 8.3 11.2 44. 9.9 2.8 22.9 3.7 14.0 8.8 11.5 48. 10.2 2.9 23.7 4.0 14.9 9.4 11.9 53. 10.5 3.0 24.6 4.3 15.7 11. 12.7 60. 10.8 3.2 26.2 4.8 17.0 12. 13.2 70. 11.2 3.6 29.2 4.8 17.0 12.	2.0		2.8	10.8	7.0	10.2	35	9.3
2.2 17.1 3.0 11.6 7.2 10.4 37. 9.5 2.3 18.1 3.1 12.0 7.3 10.5 38. 9.5 2.4 19.1 3.2 12.3 7.4 10.5 39. 9.6 2.5 20.1 3.3 12.7 7.6 10.7 40. 9.7 2.6 21.1 3.4 13.0 7.9 10.9 42. 9.8 2.7 22.0 3.5 13.3 8.3 11.2 44. 9.9 2.8 22.9 3.7 14.0 8.8 11.5 48. 10.2 2.9 23.7 4.0 14.9 9.4 11.9 53. 10.5 3.0 24.6 4.3 15.7 11. 12.7 60. 10.8 3.2 26.2 4.8 17.0 12. 13.2 70. 11.2 3.6 29.2 5.3 18.1 13. 13.7 80. 11.6 4.1 32.6 6.0 19.6 14. <td< td=""><td>2.1</td><td>16.0</td><td>2.9</td><td>11.2</td><td>7.1</td><td>10.3</td><td>36,</td><td>9.4</td></td<>	2.1	16.0	2.9	11.2	7.1	10.3	36,	9.4
2.3 18.1 3.1 12.0 7.3 10.5 38. 9.5 2.4 19.1 3.2 12.3 7.4 10.5 39. 9.6 2.5 20.1 3.3 12.7 7.6 10.7 40. 9.7 2.6 21.1 3.4 13.0 7.9 10.9 42. 9.8 2.7 22.0 3.5 13.3 8.3 11.2 44. 9.9 2.8 22.9 3.7 14.0 8.8 11.5 48. 10.2 2.9 23.7 4.0 14.9 9.4 11.9 53. 10.5 3.0 24.6 4.3 15.7 11. 12.7 60. 10.8 3.2 26.2 4.8 17.0 12. 13.2 70. 11.2 3.6 29.2 5.3 18.1 13. 13.7 80. 11.6 4.1 32.6 6.0 19.6 14. 14.1 90. 11.9 4.6 35.7 6.9 21.3 15. <t< td=""><td>2.2</td><td>17.1</td><td>3.0</td><td>11.6</td><td></td><td>10.4</td><td>37.</td><td>9.5</td></t<>	2.2	17.1	3.0	11.6		10.4	37.	9.5
2.5 20.1 3.3 12.7 7.6 10.7 40. 9.7 2.6 21.1 3.4 13.0 7.9 10.9 42. 9.8 2.7 22.0 3.5 13.3 8.3 11.2 44. 9.9 2.8 22.9 3.7 14.0 8.8 11.5 48. 10.2 2.9 23.7 4.0 14.9 9.4 11.9 53. 10.5 3.0 24.6 4.3 15.7 11. 12.7 60. 10.8 3.2 26.2 4.8 17.0 12. 13.2 70. 11.2 3.6 29.2 5.3 18.1 13. 13.7 80. 11.6 4.1 32.6 6.0 19.6 14. 14.1 90. 11.9 4.6 35.7 6.9 21.3 15. 14.5 100. 12.2 5.3 39.6 7.9 23.0 17. 15.2 120. 12.7 6.2 44.0 9.2 24.8 20.	2.3	18.1		12.0	7.3	10.5	38.	9.5
2.6 21.1 3.4 13.0 7.9 10.9 42. 9.8 2.7 22.0 3.5 13.3 8.3 11.2 44. 9.9 2.8 22.9 3.7 14.0 8.8 11.5 48. 10.2 2.9 23.7 4.0 14.9 9.4 11.9 53. 10.5 3.0 24.6 4.3 15.7 11. 12.7 60. 10.8 3.2 26.2 4.8 17.0 12. 13.2 70. 11.2 3.6 29.2 5.3 18.1 13. 13.7 80. 11.6 4.1 32.6 6.0 19.6 14. 14.1 90. 11.9 4.6 35.7 6.9 21.3 15. 14.5 100. 12.2 5.3 39.6 7.9 23.0 17. 15.2 120. 12.7 6.2 44.0 9.2 24.8 20. 16.7 200. 14.2 7.2 48.4 11. 27.1 22.	2.4	19.1	3.2	12.3	7.4	10.5	39,	9.6
2.7 22.0 3.5 13.3 8.3 11.2 44. 9.9 2.8 22.9 3.7 14.0 8.8 11.5 48. 10.2 2.9 23.7 4.0 14.9 9.4 11.9 53. 10.5 3.0 24.6 4.3 15.7 11. 12.7 60. 10.8 3.2 26.2 4.8 17.0 12. 13.2 70. 11.2 3.6 29.2 5.3 18.1 13. 13.7 80. 11.6 4.1 32.6 6.0 19.6 14. 14.1 90. 11.9 4.6 35.7 6.9 21.3 15. 14.5 100. 12.2 5.3 39.6 7.9 23.0 17. 15.2 120. 12.7 6.2 44.0 9.2 24.8 20. 16.2 150. 13.4 7.2 48.4 11. 27.1 22. 16.7 200. 14.2 8.6 53.8 13. 29.2 26.	2.5	20.1	3.3	12.7	7.6	10.7	40.	9.7
2.8 22.9 3.7 14.0 8.8 11.5 48. 10.2 2.9 23.7 4.0 14.9 9.4 11.9 53. 10.5 3.0 24.6 4.3 15.7 11. 12.7 60. 10.8 3.2 26.2 4.8 17.0 12. 13.2 70. 11.2 3.6 29.2 5.3 18.1 13. 13.7 80. 11.6 4.1 32.6 6.0 19.6 14. 14.1 90. 11.9 4.6 35.7 6.9 21.3 15. 14.5 100. 12.2 5.3 39.6 7.9 23.0 17. 15.2 120. 12.7 6.2 44.0 9.2 24.8 20. 16.2 150. 13.4 7.2 48.4 11. 27.1 22. 16.7 200. 14.2 8.6 53.8 13. 29.2 26. 17.7 250. 14.8 11. 61.5 16. 32.0 30. <td>2.6</td> <td>21.1</td> <td>3.4</td> <td>13.0</td> <td>7.9</td> <td>10.9</td> <td>42.</td> <td>9.8</td>	2.6	21.1	3.4	13.0	7.9	10.9	42.	9.8
2.9 23.7 4.0 14.9 9.4 11.9 53. 10.5 3.0 24.6 4.3 15.7 11. 12.7 60. 10.8 3.2 26.2 4.8 17.0 12. 13.2 70. 11.2 3.6 29.2 5.3 18.1 13. 13.7 80. 11.6 4.1 32.6 6.0 19.6 14. 14.1 90. 11.9 4.6 35.7 6.9 21.3 15. 14.5 100. 12.2 5.3 39.6 7.9 23.0 17. 15.2 120. 12.7 6.2 44.0 9.2 24.8 20. 16.2 150. 13.4 7.2 48.4 11. 27.1 22. 16.7 200. 14.2 8.6 53.8 13. 29.2 26. 17.7 250. 14.8 11. 61.5 16. 32.0 30. 18.5 300. 15.4 13. 67.0 19. 34.2 35. <td>2.7</td> <td>22.0</td> <td>3.5</td> <td>13.3</td> <td>8.3</td> <td>11.2</td> <td>44.</td> <td>9.9</td>	2.7	22.0	3.5	13.3	8.3	11.2	44.	9.9
3.0 24.6 4.3 15.7 11. 12.7 60. 10.8 3.2 26.2 4.8 17.0 12. 13.2 70. 11.2 3.6 29.2 5.3 18.1 13. 13.7 80. 11.6 4.1 32.6 6.0 19.6 14. 14.1 90. 11.9 4.6 35.7 6.9 21.3 15. 14.5 100. 12.2 5.3 39.6 7.9 23.0 17. 15.2 120. 12.7 6.2 44.0 9.2 24.8 20. 16.2 150. 13.4 7.2 48.4 11. 27.1 22. 16.7 200. 14.2 8.6 53.8 13. 29.2 26. 17.7 250. 14.8 11. 61.5 16. 32.0 30. 18.5 300. 15.4 13. 67.0 19. 34.2 35. 19.5 350. 15.8 16. 74.1 23. 36.8 42. </td <td>2.8</td> <td>22.9</td> <td>3.7</td> <td>14.0</td> <td>8.8</td> <td>11.5</td> <td>48.</td> <td>10.2</td>	2.8	22.9	3.7	14.0	8.8	11.5	48.	10.2
3.2 26.2 4.8 17.0 12. 13.2 70. 11.2 3.6 29.2 5.3 18.1 13. 13.7 80. 11.6 4.1 32.6 6.0 19.6 14. 14.1 90. 11.9 4.6 35.7 6.9 21.3 15. 14.5 100. 12.2 5.3 39.6 7.9 23.0 17. 15.2 120. 12.7 6.2 44.0 9.2 24.8 20. 16.2 150. 13.4 7.2 48.4 11. 27.1 22. 16.7 200. 14.2 8.6 53.8 13. 29.2 26. 17.7 250. 14.8 11. 61.5 16. 32.0 30. 18.5 300. 15.4 13. 67.0 19. 34.2 35. 19.5 350. 15.8 16. 74.1 23. 36.8 42. 20.5 400. 16.2 19. 80.2 29. 40.0 50.<	2.9	23.7	4.0	14.9	9.4	11.9	53,	10.5
3.6 29.2 5.3 18.1 13. 13.7 80. 11.6 4.1 32.6 6.0 19.6 14. 14.1 90. 11.9 4.6 35.7 6.9 21.3 15. 14.5 100. 12.2 5.3 39.6 7.9 23.0 17. 15.2 120. 12.7 6.2 44.0 9.2 24.8 20. 16.2 150. 13.4 7.2 48.4 11. 27.1 22. 16.7 200. 14.2 8.6 53.8 13. 29.2 26. 17.7 250. 14.8 11. 61.5 16. 32.0 30. 18.5 300. 15.4 13. 67.0 19. 34.2 35. 19.5 350. 15.8 16. 74.1 23. 36.8 42. 20.5 400. 16.2 19. 80.2 29. 40.0 50. 21.6 500. 16.8 24. 88.8 36. 43.1 60.	3.0	24.6	4.3	15.7	11.		60,	10.8
3.6 29.2 5.3 18.1 13. 13.7 80. 11.6 4.1 32.6 6.0 19.6 14. 14.1 90. 11.9 4.6 35.7 6.9 21.3 15. 14.5 100. 12.2 5.3 39.6 7.9 23.0 17. 15.2 120. 12.7 6.2 44.0 9.2 24.8 20. 16.2 150. 13.4 7.2 48.4 11. 27.1 22. 16.7 200. 14.2 8.6 53.8 13. 29.2 26. 17.7 250. 14.8 11. 61.5 16. 32.0 30. 18.5 300. 15.4 13. 67.0 19. 34.2 35. 19.5 350. 15.8 16. 74.1 23. 36.8 42. 20.5 400. 16.2 19. 80.2 29. 40.0 50. 21.6 500. 16.8 24. 88.8 36. 43.1 60.	3.2	26.2	4.8	17.0	12.	13.2	70.	11.2
4.6 35.7 6.9 21.3 15. 14.5 100. 12.2 5.3 39.6 7.9 23.0 17. 15.2 120. 12.7 6.2 44.0 9.2 24.8 20. 16.2 150. 13.4 7.2 48.4 11. 27.1 22. 16.7 200. 14.2 8.6 53.8 13. 29.2 26. 17.7 250. 14.8 11. 61.5 16. 32.0 30. 18.5 300. 15.4 13. 67.0 19. 34.2 35. 19.5 350. 15.8 16. 74.1 23. 36.8 42. 20.5 400. 16.2 19. 80.2 29. 40.0 50. 21.6 500. 16.8 24. 88.8 36. 43.1 60. 22.7 700. 17.8 30. 97.4 46. 46.6 73. 23.9 1,000. 18.9 38. 107.0 58. 50.1 <t< td=""><td>3.6</td><td>29.2</td><td>5.3</td><td>18.1</td><td>13.</td><td></td><td>80,</td><td>11.6</td></t<>	3.6	29.2	5.3	18.1	13.		80,	11.6
5.3 39.6 7.9 23.0 17. 15.2 120. 12.7 6.2 44.0 9.2 24.8 20. 16.2 150. 13.4 7.2 48.4 11. 27.1 22. 16.7 200. 14.2 8.6 53.8 13. 29.2 26. 17.7 250. 14.8 11. 61.5 16. 32.0 30. 18.5 300. 15.4 13. 67.0 19. 34.2 35. 19.5 350. 15.8 16. 74.1 23. 36.8 42. 20.5 400. 16.2 19. 80.2 29. 40.0 50. 21.6 500. 16.8 24. 88.8 36. 43.1 60. 22.7 700. 17.8 30. 97.4 46. 46.6 73. 23.9 1,000. 18.9 38. 107.0 58. 50.1 90. 25.2 1,300. 19.7	4.1	32.6	6.0	19.6	14.	14.1	90.	11.9
6.2 44.0 9.2 24.8 20. 16.2 150. 13.4 7.2 48.4 11. 27.1 22. 16.7 200. 14.2 8.6 53.8 13. 29.2 26. 17.7 250. 14.8 11. 61.5 16. 32.0 30. 18.5 300. 15.4 13. 67.0 19. 34.2 35. 19.5 350. 15.8 16. 74.1 23. 36.8 42. 20.5 400. 16.2 19. 80.2 29. 40.0 50. 21.6 500. 16.8 24. 88.8 36. 43.1 60. 22.7 700. 17.8 30. 97.4 46. 46.6 73. 23.9 1,000. 18.9 38. 107.0 58. 50.1 90. 25.2 1,300. 19.7	4.6	35.7	6.9	21.3	15.	14.5	100.	12.2
7.2 48.4 11. 27.1 22. 16.7 200. 14.2 8.6 53.8 13. 29.2 26. 17.7 250. 14.8 11. 61.5 16. 32.0 30. 18.5 300. 15.4 13. 67.0 19. 34.2 35. 19.5 350. 15.8 16. 74.1 23. 36.8 42. 20.5 400. 16.2 19. 80.2 29. 40.0 50. 21.6 500. 16.8 24. 88.8 36. 43.1 60. 22.7 700. 17.8 30. 97.4 46. 46.6 73. 23.9 1,000. 18.9 38. 107.0 58. 50.1 90. 25.2 1,300. 19.7	5.3	39.6	7.9	23.0	17.	15.2	120,	12.7
8.6 53.8 13. 29.2 26. 17.7 250. 14.8 11. 61.5 16. 32.0 30. 18.5 300. 15.4 13. 67.0 19. 34.2 35. 19.5 350. 15.8 16. 74.1 23. 36.8 42. 20.5 400. 16.2 19. 80.2 29. 40.0 50. 21.6 500. 16.8 24. 88.8 36. 43.1 60. 22.7 700. 17.8 30. 97.4 46. 46.6 73. 23.9 1,000. 18.9 38. 107.0 58. 50.1 90. 25.2 1,300. 19.7	6.2	44.0	9.2	24.8	20.	16.2	150,	13.4
11. 61.5 16. 32.0 30. 18.5 300. 15.4 13. 67.0 19. 34.2 35. 19.5 350. 15.8 16. 74.1 23. 36.8 42. 20.5 400. 16.2 19. 80.2 29. 40.0 50. 21.6 500. 16.8 24. 88.8 36. 43.1 60. 22.7 700. 17.8 30. 97.4 46. 46.6 73. 23.9 1,000. 18.9 38. 107.0 58. 50.1 90. 25.2 1,300. 19.7	7.2	48.4	11.	27.1	22.	16.7	200.	14.2
13. 67.0 19. 34.2 35. 19.5 350. 15.8 16. 74.1 23. 36.8 42. 20.5 400. 16.2 19. 80.2 29. 40.0 50. 21.6 500. 16.8 24. 88.8 36. 43.1 60. 22.7 700. 17.8 30. 97.4 46. 46.6 73. 23.9 1,000. 18.9 38. 107.0 58. 50.1 90. 25.2 1,300. 19.7	8.6	53.8	13.	29.2	26.	17.7	250,	14.8
16. 74.1 23. 36.8 42. 20.5 400. 16.2 19. 80.2 29. 40.0 50. 21.6 500. 16.8 24. 88.8 36. 43.1 60. 22.7 700. 17.8 30. 97.4 46. 46.6 73. 23.9 1,000. 18.9 38. 107.0 58. 50.1 90. 25.2 1,300. 19.7	11.	61.5	16.	32.0	30.		300.	15.4
19. 80.2 29. 40.0 50. 21.6 500. 16.8 24. 88.8 36. 43.1 60. 22.7 700. 17.8 30. 97.4 46. 46.6 73. 23.9 1,000. 18.9 38. 107.0 58. 50.1 90. 25.2 1,300. 19.7	13.	67.0	19.	34.2	35.	19.5	350	15.8
24. 88.8 36. 43.1 60. 22.7 700. 17.8 30. 97.4 46. 46.6 73. 23.9 1,000. 18.9 38. 107.0 58. 50.1 90. 25.2 1,300. 19.7	16.	74.1	23.	36.8	42.	20.5	400	16.2
30. 97.4 46. 46.6 73. 23.9 1,000. 18.9 38. 107.0 58. 50.1 90. 25.2 1,300. 19.7	19.	80.2	29.	40.0	50.	21.6	500.	16.8
38. 107.0 58. 50.1 90. 25.2 1,300. 19.7		88.8	36.	43.1	60.		700.	17.8
38. 107.0 58. 50.1 90. 25.2 1,300. 19.7	30.	97.4	46.	46.6	73.	23.9	1,000	18.9
49. 117.8 75. 54.0 111. 26.6 1,715. 20.5	38.	107.0	58.	50.1	90.			
	49.	117.8	75.	54.0	111.	26.6	1,715.	20.5

that are familiar to readers of English, we must refer to Gini's statistics. In the first place, Gini's mean difference gives us some information that is interesting in itself. It tells us the expected value of the difference in returns between two portfolios of any given size, including portfolios of one stock. In the second place, Gini's coefficient of concentration is useful in summarizing differences in returns

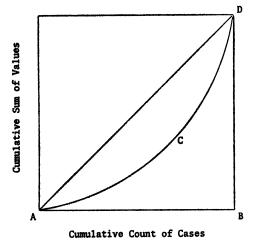


Fig. 1.—A Lorenz curve

to portfolios. The coefficient was originally developed to summarize differences in wealth or income and thus applies to our studies of differences among wealth ratios.

Many readers will be familiar with the Lorenz curve as exemplified in figure 1. Gini's coefficient of concentration measures the ratio of the area between the Lorenz curve ACD and the line AD to the total area of the triangle ABD. When applied to returns on portfolios, the interpretation is straightforward. If portfolios are ranked by the size of returns from the smallest to the largest, the locus of a Lorenz curve is readily drawn. The number of portfolios, M, is shown on the

horizontal axis; and the sum of the wealth ratios of these M (smallest) portfolios is shown on the vertical axis. If all portfolios had equal returns, the Lorenz curve would be identical with line AD. If all portfolios except one had zero wealth ratios and that one had a positive wealth ratio, the Lorenz curve would be virtually identical with curve ABD. In the former case, Gini's coefficient of concentration would be zero; in the latter case, one.

In the third place, we use Gini's statistics because many of the distributions we report here depart greatly from normality. For such distributions, the standard deviation of even a large sample may not give a very meaningful indication of the dispersion of the population. Gini's mean difference and coefficient of concentration are nonparametric measures and are invulnerable to this consequence of departure from normality. The mean deviation from the mean is also invulnerable to this adverse consequence of departure from normality. Gini's mean difference differs from the mean deviation by giving greater weight to extreme observations, thus taking care of a frequently made criticism of the mean deviation.

We use measures both of absolute and of relative dispersion. The absolute measures are interesting in themselves, but reliance on them exclusively would conceal some relationships which, as we will see, have been remarkably invariant for long periods of time.

THE STUDIES

STUDY 1: DISTRIBUTIONS OF WEALTH RATIOS FOR INVESTMENTS IN SINGLE STOCKS

The frequency distributions of wealth ratios for investments in single stocks for fifty-five time periods are summarized in table 3. Many of their general features should not surprise anyone. In general, the mean wealth ratios shown in column 16 increase with the length of the holding period, although there is substantial overlap. (For example, the greatest mean for a one-year period is greater than five of the eight means for five-year periods, and the worst mean for a five-year period exceeds only four of the 40 one-year means.)

The distributions for longer holding periods tend to have greater dispersion than the distributions for shorter periods, both absolutely and relatively. This tendency may be seen by looking at corresponding entries in columns (17–22) or by comparing ranges defined by columns (6) and (8), (5) and (9), (4) and (10), (3) and (11), (2) and (12), or (13) and (14).

For all periods studied, skewness of the distribution of wealth ratios was positive. This implies, as almost every investor knows, that the probability of deviating from the mean by very large amounts on the high side is greater than the probability of extremely large deviations on the low side. This skewness almost inevitably results from the simple arithmetic fact that it is impossible to lose more than 100 percent of one's investment, assuming that one does not buy on margin, while it is possible to make much more than 100 percent on one's money when one is lucky or wise. For all periods of five years or more, the maximum wealth ratio was at least 4.5 times as large as the mean wealth ratio. In fact, the maximum ratio was more than twice the mean in nine of the 40 one-year periods.

As would be expected with positive skewness, the mean is almost invariably greater than the median. There are a few exceptions for one-year periods when the positive skewness is slight. For almost all one-year periods and for all periods longer than one year the mean exceeds the median. The longer the period, the greater the difference.

For all periods except 1929, the kurtosis of the frequency distributions is greater than 3.0. That is, a greater proportion of the observations fall near the mean than is true for normal distributions. Nearness to the mean is measured in terms of standard deviations. In these particular distributions, the kurtosis is relatively small when skewness is slight. Thus the "peakedness" is caused largely by the presence of a few very large wealth ratios.

In table 3, it is interesting to note the lower levels of relative dispersion after 1943. For example, Gini's coefficient of concentration was always at least 0.16 from 1926 through 1943; thereafter, the coefficient was always less than 0.16.

While discussing dispersion, it is interesting to compare the six measures of dispersion: the standard deviation, the mean deviation, the mean difference, the coefficient of variation, the relative mean deviation, and Gini's coefficient of concentration. The important fact is that the standard deviation and measure of relative dispersion derived from it, the coefficient of variation, are more variable from period to period than are the other measures. We believe that the greater instability in the standard deviation and the coefficient of variation lends support to Mandelbrot's hypothesis that the dis-

⁸ As Kaplansky has stated (I. Kaplansky, "A Common Error concerning Kurtosis," *Journal of the American Statistical Association* 40 [June 1945]: 259), it is a vulgar error blandly or blindly to assume that high kurtosis necessarily implies great concentration around the rear. Since we have examined them in great detail, we know that the common interpretation is correct for these distributions.

tributions of returns on individual stocks over time have infinite variance.⁹

It is also mildly interesting to note that for our data the relationship between Gini's mean difference and the mean deviation was remarkably stable—always being near the ratio that would be

⁹ Benoit Mandelbrot, "Variation of Certain Speculative Prices," *Journal of Business* 36 (October 1963): 394–419.

expected if the distributions were normal. For normal distributions, the ratio of Gini's mean difference to the mean deviation is $\sqrt{2}$. Thus, it appears that

¹⁰ Derived from Kendall and Stuart, pp. 139, 241. For normal distributions the mean deviation is $\sqrt{(2/\pi)}$ (=0.80) times the standard deviation, and Gini's mean difference is $2/\sqrt{\pi}$ (=1.13) times the standard deviation. Note also that in table 3 the standard deviation is usually greater than Gini's mean difference.

TABLE 3

FREQUENCY DISTRIBUTIONS OF WEALTH RATIOS FROM INVESTMENTS
IN INDIVIDUAL STOCKS LISTED ON THE NYSE, 1926-65

					entiles of	the Freque	ncy Distrib	utions			
						(Median					
Period of Investment	5th	10th	20th	30th	40th	50th	60th	70th	80th	90t)	95th
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
				ONE-YE	AR PERIOD	S					
1/30/26-12/31/26	•429	•560	•722	.845	.917	•991	1.060	1.115	1.183	1.316	1.485
12/31/26-12/31/27	.640	.752	.888	1.053	1.132	1.225	1.330	1.457	1.601	1.834	2.274
12/31/27-12/31/28	•725	.834	.976	1.064	1.153	1.251	1.376	1.512	1.711	2.136	2.795
12/31/28-12/31/29	•197	.279	• 405	.518	.604	.700	.774	.873	.977	1.123	1.218
12/31/29-12/31/30	-205	•265	•367	• 442	•521	•593	.674	•757	.868	•986	1.106
12/31/30-12/31/31	.158	.215	•292	•347	-401	•467	• 536	.615	•738	•909	1.044
12/31/31-12/31/32 12/31/32-12/30/33	•353 •657	•449 •856	•561 1•189	•654 1•402	.729 1.628	-828	•931	1.035	1.173	1.377	1.662
12/30/33-12/31/34	•566	.645	.759	.849	•938	1.849 1.029	2.075 1.134	2.394	2.742	3.331	4.338
12/31/34-12/31/35	.761	.864	1.029	1.134	1.232	1.353	1.492	1.262 1.656	1.428 1.927	1.728 2.342	2.017 2.747
12/31/35-12/31/36	.854	.943	1.052	1.154	1.235	1.334	1.446	1.583	1.773	2.130	2.461
12/31/36-12/31/37	.276	.315	.373	•420	.472	.517	.562	.624	.687	.798	.888
12/31/37-12/31/38	.693	.838	• 986	1.090	1.186	1.260	1.350	1.458	1.594	1.798	2.020
12/31/38-12/30/39	. 554	-626	•704	•790	.869	.947	1.021	1.089	1.183	1.319	1.478
12/30/39-12/31/40 12/31/40-12/31/41	•477	•584	.713	•795	• 852	.904	•952	1.005	1.069	1.194	1.331
12/31/40-12/31/41	•445 •844	•545	•666	.751	.820	-879	•947	1.016	1.096	1.234	1.412
12/31/42-12/31/43	1.032	.907 1.088	.986 1.195	1.054 1.249	1.120	1.190	1.273	1.380	1.527	1.766	2.106
12/31/43-12/30/44	1.029	1.088	1.151	1.202	1.331 1.245	1.403 1.304	1.499 1.372	1.624	1.812 1.556	2.130 1.750	2.560
12/30/44-12/31/45	1.179	1.238	1.316	1.372	1.427	1.500	1.578	1.672	1.805	2.074	2.025 2.329
12/31/45-12/31/46	• 529	• 592	•698	.785	.847	.898	.944	1.002	1.083	1.187	1.308
12/31/46-12/31/47	.617	.701	.795	.857	.910	.962	1.025	1.091	1.175	1.312	1.456
12/31/47-12/31/48	•600	.703	• 798	.858	.916	.967	1.007	1.050	1.107	1.212	1.321
12/31/48-12/31/49	-840	•906	• 997	1.061	1.122	1.178	1.233	1.302	1.374	1.481	1.617
12/31/49-12/30/50 12/30/50-12/31/51	.904 .832	•965	1.047	1.123	1.207	1.298	1.384	1.487	1.614	1.817	2.015
12/31/51-12/31/51	.771	.905 .841	•981 •920	1.039 .980	1.081 1.037	1.122	1.172	1.228	1.294	1.419	1.554
12/31/52-12/31/53	•602	.704	-805	.864	•918	1.088 .969	1.137 1.021	1.184	1.245	1.333	1.424
12/31/53-12/31/54	1.095	1.164	1.251	1.327	1.407	1.480	1.565	1.064 1.660	1.112 1.786	1.221	1.320 2.256
12/31/54-12/30/55	.829	.924	1.003	1.049	1.093	1.144	1.193	1.261	1.354	1.516	1.687
12/30/55-12/31/56	.711	•789	.880	.941	.989	1.026	1.082	1.141	1.236	1.378	1.518
12/31/56-12/31/57	.496	•567	•658	• 721	.791	•856	.923	.996	1.060	1.149	1.228
12/31/57-12/31/58	1.112	1.199	1.294	1.363	1.434	1.491	1.552	1.647	1.780	2.001	2.326
12/31/58-12/31/59 12/31/59-12/30/60	• 759	•845	.934	• 988	1.028	1.080	1.149	1.214	1.323	1.527	1.715
12/30/60-12/29/61	•591 •852	•652 •935	.749 1.037	.825 1.114	.887 1.176	•948	1.022	1.100	1.196	1.325	1.447
12/29/61-12/31/62	.544	.616	•696	.761	.813	1.235 .856	1.301 .909	1.373 .965	1.460 1.019	1.621	1.818
12/31/62-12/31/63	.808	.894	.985	1.041	1.089	1.137	1.186	1.247	1.344	1.114 1.488	1.207 1.657
12/31/63-12/31/64	.765	.875	.978	1.049	1.099	1.142	1.194	1.248	1.323	1.466	1.622
12/31/64-12/31/65	.856	.923	•995	1.056	1.116	1.196	1.275	1.387	1.510	1.732	1.963
				FIVE-YE	AR PERIO	s					
1.420.424 12.423.420											
1/30/26-12/31/30 12/31/30-12/31/35	.041 .098	.095 .227	•192 •417	•340 •660	.518 .897	•682 1•147	.876 1.412	1.116	1.443	1.884	2.476
12/31/35-12/31/40	.119	-218	•364	.539	.673	.832	.959	1.766 1.134	2.293 1.336	3.296 1.736	4.601 2.061
12/31/40-12/31/45	1.459	1.721	2.084	2.408	2.708	3.155	3.688	4.335	5.556	7.601	10.036
12/31/45-12/30/50	.477	•627	.865	1.007	1.148	1.302	1.481	1.702	1.960	2.409	2.838
12/30/50-12/30/55	.853	1.142	1.440	1.691	1.907	2.107	2.337	2.621	3.002	3.775	4.568
12/30/55-12/30/60	•532	•707	• 939	1.130	1.292	1.477	1.656	1.883	2.181	2.708	3.396
12/30/60-12/31/65	.761	.977	1.256	1.440	1.605	1.778	1.979	2.241	2.595	3.402	4.445
				TEN-YEA	R PERIODS	i					
1/30/26-12/31/35	.016	.048	.152	.256	.461	.688	1.007	1.368	1.850	2.730	4.297
12/31/35-12/31/45	.614	1.056	1.592	1.912	2.210	2.557	2.999	3.493	4.216	5.500	7.664
12/31/45-12/30/55 12/30/55-12/31/65	.621	.982	1.505	1.895	2.302	2.750	3.270	4.059	5.100	7.169	9.001
12/30/33-12/31/65	.836	1.157	1.654	2.035	2.427	2.814	3.208	3.681	4.289	5.473	7.075
					PERIODS						
1/30/26-12/31/45 12/31/45-12/31/65	.000 .912	.052 1.886	.324 3.357	•772 4•549	1.273	1.864 8.242	2.772 10.111	3.914 12.529	5.133	7.395	11.389
12. 32. 45-12. 32. 65	• /12	4.000	34331	40-YEAR		0.272	10.111	12.324	16.068	21.992	30.115
1400404 10401 ::=											
1/30/26-12/31/65	•000	.258	1.283	3.724	8.257	14.323	21.581	33.613	50.787	82.532	127.554

for the particular distribution we describe here, either measure provides a good estimate of the other. The mean deviation is usually easier to calculate.

STUDY 2: DISTRIBUTIONS OF WEALTH RATIOS AGGREGATED FOR NONOVERLAPPING PERIODS

In table 4 we present data on aggregated frequency distributions of wealth ratios from investments in individual

stocks on the New York Stock Exchange. When one considers individual periods separately, as in table 3, it is hard to make generalizations about the variability of experience in investing in stocks on the New York Stock Exchange because of the substantial changes from period to period.

We cannot, for example, tell the probability of gaining or losing a given

			Mean		e e		_	s	y g			
				E	Deviation	Mean	ent	. Mean	ent		_	A S
	Minimum		Arithmetic	darc	ē	's p	fici	tive	fici	Skewness	osis	anie
Period of Investment	Mînj	Maximum	Arit	Standard Deviation	Mean	Gini's Mean Difference	Coefficient of Variation	Relative M Deviation	Coefficient o	Skew	Kurtosis	Number of Companies
(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)
				ONE-YE	AR PERIOD	S						
1/30/26-12/31/26	.073	2.970	• 985	.343	.242	•355	•348	-245	.180	1.304	8.891	510
12/31/26-12/31/27 12/31/27-12/31/28	.000 .398	7.889 13.226	1.300 1.453	•577 •904	•374 •498	•546 •712	•444 •622	.288 .343	•210 •245	3.660 5.844	36.275 60.390	543 589
12/31/28-12/31/29	.000	1.851	.700	.318	•259	.361	.454	.370	• 258	.261	2.718	627
12/31/29-12/31/30 12/31/30-12/31/31	.046 .000	2.105 2.204	.620 .522	.286 .291	.229 .221	.318 .311	•461 •558	.369 .425	.257 .298	.685 1.323	4.047 6.365	717
12/31/31-12/31/32	.000	3.308	.891	.435	.319	•455	.488	.358	•298	1.481	7.462	737 732
12/31/32-12/30/33 12/30/33-12/31/34	.000	20.841	2.083	1.366	.873	1.253	•656	.419	.301	4.686	54.283	709
12/31/34-12/31/35	.000	9.481 6.077	1.139 1.507	•574 •675	•364 •485	.517 .687	•504 •448	.319 .322	•227 •228	5.358 1.830	67.343 9.327	707 706
12/31/35-12/31/36	-178	17.234	1.483	.814	.424	.605	.549	.286	.204	10.719	197.482	719
12/31/36-12/31/37 12/31/37-12/31/38	.109	1.372 7.187	.541 1.307	•195 •497	.153 .320	•215	•360 •380	•283 •245	.199	.815	4.073	744
12/31/38-12/30/39	.000	2.830	.967	.313	.234	•469 •334	•324	.242	.179 .173	3.998 1.140	44.628 6.735	780 775
12/30/39-12/31/40	.000	2.748	.901	.276	.195	.288	.306	.216	.160	.823	8.584	778
12/31/40-12/31/41 12/31/41-12/31/42	.000 .560	2.941 5.907	.898 1.311	.312 .501	•227 •320	.330 .452	•348 •382	.253 .244	.184 .172	1.101 3.358	7.515 21.015	788 797
12/31/42-12/31/43	.293	7.469	1.564	.644	.389	.550	.412	.249	.176	4.134	30.193	800
12/31/43-12/30/44	•417 •649	4.389 4.700	1.383 1.598	•353	.237	•339	•256	.171	.123	2.451	14.494	810
12/31/45-12/31/46	.254	2.230	•901	•422 •242	.283 .184	•400 •266	•264 •268	•177 •204	•125 •147	2.807 .609	16.262 4.739	826 853
12/31/46-12/31/47	• 348	2.577	.994	.260	.195	.280	.262	.196	.141	1.031	5.812	904
12/31/47-12/31/48 12/31/48-12/31/49	.337	4.544 2.885	.969 1.194	•259 •254	•169 •189	.250 .271	.268 .213	•175 •158	•129 •114	3.571 .967	44.575 7.704	939 963
12/31/49-12/30/50	.645	3.917	1.358	.378	.283	.397	.279	-208	.146	1.622	8.628	990
12/30/50-12/31/51	.135	4.047	1.149	-242	.168	•245	.211	.146	-107	2.249	24.699	1,010
12/31/51-12/31/52 12/31/52-12/31/53	.113	1.866 2.135	1.089 .968	.201 .215	•157 •162	.223 .234	•185 •222	.144 .168	.103 .121	.207 .389	3.915 4.939	1,029 1,044
12/31/53-12/31/54	.608	5.441	1.548	.392	.279	•397	.253	.180	.128	2.205	15.744	1,045
12/31/54-12/30/55 12/30/55-12/31/56	•163 •142	2.886 4.282	1.190 1.065	•270 •268	•194 •188	•280 •273	•227 •251	•163 •176	•118 •128	1.391 2.342	7.370 24.249	1,052 1,055
12/31/56-12/31/57	.268	2.266	.864	• 242	.191	.268	.280	.221	.155	.638	5.395	1,055
12/31/57-12/31/58	.803	5.077	1.579	• 440	. 285	.412	.279	.181	.131	2.873	16.763	1,077
12/31/58-12/31/59 12/31/59-12/30/60	•428 •253	3.372 2.380	1.144 .981	.310 .276	.219 .215	.314 .303	.271 .282	•191 •219	.137 .154	1.824 .886	9.521 5.001	1,067 1,088
12/30/60-12/29/61	.000	3.810	1.276	• 330	.229	•335	• 259	-180	•131	1.885	12.084	1,119
12/29/61-12/31/62 12/31/62-12/31/63	.146 .000	1.741 3.214	.865 1.176	.206 .287	.159	.228	.239	.184	.132	.364	4.102	1.142
12/31/63-12/31/64	•326	3.130	1.163	• 265	.198 .188	•291 •278	•244 •228	•168 •162	•124 •120	1.680	10.576 7.656	1,162 1,191
12/31/64-12/31/65	•289	5.426	1.282	-410	.283	•401	• 320	•221	.156	2.554	16.991	1,227
					EAR PERIO							
1/30/26-12/31/30 12/31/30-12/31/35	.000	4.487 11.841	.877 1.568	.778 1.585	.600 1.079	•822 1•506	.887 1.011	•684 •688	•468 •480	1.412 2.463	5.405 11.345	510 737
12/31/35-12/31/40	.000	10.457	.949	.822	.519	.741	.867	•547	.391	4.460	41.754	719
12/31/40-12/31/45	•C00	48.855	4.264	3.990	2.289	3.150	.936	•537	.369	5.010	41.665	788
12/31/45-12/30/50 12/30/50-12/30/55	.063 .113	6.514 10.794	1.455 2.335	.771 1.217	.576 .861	.811 1.240	•530 •521	•396 •369	•279 •266	1.525 1.836	7.489 9.107	853 1,010
12/30/55-12/30/60	.102	35.876	1.701	1.508	.737	1.067	•886	•433	.314	12.294	257.373	1,055
12/30/60-12/31/65	.159	18.598	2.086	1.382	.851	1.221	.663	.408	.293	4.087	34.270	1,119
1/30/26-12/31/35	•000	26 470	1.238		1.086	S 1.480	1.496	.877	.598	5.481	E4 100	510
12/31/35-12/31/45	.CC0	24.679 74.724	3.226	1.852 3.675	1.708	2.459	1.139	•529	.381	11.440	56.199 205.680	719
12/31/45-12/30/55	-047	21.753	3.526	2.766	2.012	2.779	•785	•571	.394	1.958	8.668	853
12/30/55-12/31/65	.084	22.340	3.241	2.350	1.506 R PERIODS	2.166	•725	•465	• 334	3.278	20.969	1.055
1/20/24-12/21/45	000	.0.7.0	2 2/-			4 01 0		074	FOC	2 712	22 205	510
1/30/26-12/31/45 12/31/45-12/31/65	.000 .116	40.763 110.916	3.361 10.766	4.759 10.593	2.943 7.083	4.018 9.866	1.416 .984	.876 .658	.598 .458	3.718 3.111	22.395 19.804	510 853
				40-YEAR	R PERIOD							
1/30/26-12/31/65	.000	1715.239	35.124	89.807	36.247	48.377	2.557	1.032	.689	13.439	242.255	510

amount by selecting a stock at random during a year selected at random. We know only the distribution of experience for the individual periods. To answer a variety of interesting questions (at least for the forty years 1926–65), we must combine the frequency distributions for each period, giving equal weight to each period's distribution.

Suppose one were interested in knowing the relative frequency with which one would have lost more than 20 percent of his money if he had bought a stock at random and held it for a year during the

forty-year period 1926-65. By reference to table 4, one can see that there was about a 20 percent chance of losing about 20 percent or more of one's money by investing in a stock for one year. Similarly, there was about a 37 percent chance of making 20 percent or more by investing in a stock for one year.

When one turns to the five-year periods, one can answer the same kinds of questions. For example, one lost about 20 percent or more of his money approximately 23 percent of the time. Conversely, one made at least 20 percent

TABLE 4

AGGREGATED FREQUENCY DISTRIBUTIONS OF WEALTH RATIOS FROM INVESTMENTS IN INDIVIDUAL STOCKS LISTED ON THE NYSE, 1926-65

			Per	iods		
Statistic	40 One- Year	20 One- Year (1926-45)	20 One- Year (1946-65)	8 Five- Year	4 Ten- Year	2 Twenty- Year
5th centil,	.466	.356	.663	.201	.130	.052
10th centile	.613	.480	.763	.391	.340	.288
20th centile	.796	.675	.879	.726	.894	1.006
30th centile	.911	.828	.961	.990	1,416	1.871
40th centile	1.003	.958	1.026	1,240	1.833	3.028
50th centile (median)	1.085	1.075	1.091	1.491	2.245	4.222
60th centile	1.173	1.192	1.161	1.762	2.709	5,626
70th centile	1.277	1.326	1.245	2.096	3,282	7.940
80th centile	1.423	1.500	1.359	2,564	4.099	11.194
90th centile	1.675	1.830	1.551	3,581	5,479	17,263
95th centile	1.975	2.230	1.743	4.875	7.451	22.878
Minimum	0.000	0.000	0.000	0.000	0.000	0.000
Maximum	20.841	20.841	5.441	48.855	74.724	110.916
Mean	1.148	1.158	1.138	1.904	2.808	7.064
Standard deviation	.554	.699	.355	2.064	2.892	9,008
Mean deviation	.351	.447	.255	1.145	1.761	5,956
Gini's mean difference	.518	.653	.367	1.640	2.505	8,052
Coefficient of variation	.483	.604	.312	1.084	1.030	1.275
Relative mean deviation	.306	. 386	.224	.601	.627	.843
Gini's coefficient of concentration	.226	.282	.161	.431	.446	.570
Skewness	5.339	5.062	1.791	7.197	7.315	3,485
Kurtosis	111.090	86.788	12,734	107.852	144.189	24.393
Number of cases	35,407	14,394	21,013	6,791	3,137	1,363

about half of the time. Naturally, the absolute variation in the wealth ratios increases as one moves from a one-year to a five-year holding period. Most of the increase is above the mean rather than below, as one would expect during periods when investors in common stocks generally received positive returns. It is important to note, however, that dispersion in the annual rates of return declines as the length of period increases. One can see this by interpreting the data in table 4 in connection with the conversion table presented earlier (table 2).

For ten-year periods, one lost 20 percent or more of his money less than 20 percent of the time and made a profit of at least 20 percent about three-quarters of the time (table 4). It is possible to make other similar observations from table 4.

STUDY 3: THE EFFECT OF INCREASING THE NUMBER OF STOCKS IN A PORTFOLIO ON THE DISTRIBUTION OF RETURNS

Some preliminary comments.—Now we shall discuss the most interesting study in this article. The study concerns the wealth ratios resulting from investment in portfolios of specified numbers of stocks, ranging from one through 128 and in all stocks listed on the New York Stock Exchange. The ratios refer to all of the 40 one-year periods, the eight possible nonoverlapping five-year periods, the four possible ten-year periods, and the two 20-year periods. We also present data for the first twenty years and the last twenty years of the forty-year period so as to permit a comparison of the twenty years ending with the last year of World War II and the first twenty years of the postwar period.

Much of the previous work on the effect of portfolio size on the dispersion of wealth ratios is discussed and summarized in Brealey.¹¹ Other empirical work has been done by Evans and Archer.¹² This work has generally been concerned only with the effect of diversification on the standard deviation of returns or on the standard deviation of annual rates of return over time. These studies are subject to a serious bias in that they are based on investment only in stocks which were listed throughout the period of study. The elimination of stocks which merged into other stocks or were delisted is the source of the bias.

Additional empirical work is not required to find the effect of diversification on the variance or standard deviation of returns when the mean and variance for each period are known. This is true for the following reasons: (1) the variance for any period for portfolios (randomly selected) of more than one stock can be calculated from knowledge of the variance of returns from investment in portfolios of one stock, and (2) the variance for several periods considered together (that is, aggregated) can be calculated from knowledge of the means and variances for the individual periods. The variance among wealth ratios of stocks or portfolios is equal to the sum of their average variances for the periods under consideration and the variance of the means. Diversification by random selection reduces the average variance within each period but does not affect the variance of the means.

If we had been content to rely on the variance and its derivative statistics, we could have avoided much expense in

¹¹ Richard A. Brealey, An Introduction to Risk and Return from Common Stocks (Cambridge, Mass.: M.I.T. Press, 1969).

¹² John L. Evans and Stephen H. Archer, "Diversification and the Reduction of Dispersion: An Empirical Analysis," *Journal of Finance* 23 (December 1968): 761–67.

using the computer merely by algebraically calculating the statistics. We incurred the computer expense because the variance and its derivatives have been under suspicion since Mandelbrot's work seven years ago. 13

Table 5 shows the frequency distribution of returns for portfolios of different sizes. The frequency distributions for portfolios containing one stock were derived from complete enumeration of all possible such portfolios for the nonoverlapping periods selected. These distributions are also shown in table 4. We also

13 Mandelbrot (n. 9 above).

used complete enumeration to find the frequency distributions for portfolios containing two different stocks. We assumed equal initial investment in each stock and also assumed that dividends were reinvested in the stock which paid them.

For portfolios containing 8, 16, 32, and 128 stocks, we used simple random selection of individual stocks without replacement. It is possible, however, that this process produced two or more identical portfolios. We were unable to construct frequency distributions of portfolios of these sizes on the basis of complete

TABLE 5
AGGREGATED FREQUENCY DISTRIBUTIONS OF WEALTH RATIOS FROM INVESTMENTS
IN RANDOMLY SELECTED PORTPOLIOS CONTAINING SPECIFIED NUMBERS
OF STOCKS LISTED ON THE MYSE, 1926-65

Number and	Size of Portfolio	,			Centile	s of the Ag	gregated Fr	equency Di	stributions			
Length	Sampling	,					(Median)					
of Periods	Method	5th_	10th	20th	30th	40th	50th	60th	70th	80th	90th	95th
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
PER IODS						40 YEARS	1926-196	5				
40 ONE-YEAR	1E 2E	•466 •539	.613 .681	•796 •838	.911 .939	1.003 1.021	1.085	1.173 1.182	1.277	1.423	1.675 1.622	1.975 1.855
	8S 8R	•582	•745	.883	• 964	1.044	1.121	1.196	1.286	1.406	1.569	1.719
	165	.584 .583	.747 .763	.883 .893	.964 .965	1.045 1.046	1.122	1.198	1.288	1.407	1.570	1.718
	16R	.587	.763	.894	.965	1.046	1.129 1.130	1.202 1.205	1.290 1.293	1.408	1.561	1.680
	32\$.581	.775	. 899	964	1.043	1.136	1.205	1.295	1.408	1.555	1.684 1.648
	32R	• 588	.768	.902	.968	1.041	1.140	1.214	1.299	1.406	1.565	1.657
	1285	•576	.781	•900	-966	1.035	1.147	1.200	1.301	1.403	1.555	1.606
	A11	.580	.782	.900	.968	1.030	1.147	1.192	1.303	1,418	1.556	1.588
8 FIVE-YEAR	16	-201	.391	•726	•990	1.240	1.491	1.762	2.096	2.564	3.581	4.875
	2E	.418	.605	.879	1.109	1.328	1.553	1.803	2.110	2.555	3.444	4. 533
	88	.678	.804	1.021	1.239	1.435	1.627	1.842	2.101	2.473	3.355	4.278
	8R 16S	.680 .748	-805	1.023	1.242	1.437	1.631	1.848	2.107	2.480	3.423	4.366
	168	•746	•847 •845	1.036 1.036	1.287 1.292	1.470	1.641	1.847	2.093	2.405	3.508	4.308
	325	.794	.868	1.023	1.336	1.473 1.491	1.650 1.639	1.866 1.845	2.113 2.097	2.415 2.360	3.695 3.674	4.490 4.327
	32R	.769	.849	1.018	1.347	1.503	1.665	1.916	2.171	2.404	4.051	4.672
	1285	.851	.891	.976	1.416	1.517	1.633	1.818	2.109	2.316	3.987	4.335
	A11						1.635					
4 TEN-YEAR	16	.130	.340	.894	1.416	1.833	2.245	2.709	3.282	4.099	5.479	7.451
	26	.360	.683	1.250	1.727	2.129	2.496	2.885	3.340	3.959	5.086	6.235
	85	•736	.979	1.607	2.233	2.571	2.838	3.097	3.383	3.746	4.324	4.881
	8R 16S	•754 •870	.992	1.618	2.255	2.596	2.861	3.118	3.404	3.763	4.335	4.879
	16R	.888	1.065 1.080	1.545 1.551	2.452 2.504	2.749 2.804	2.968 3.016	3.173 3.221	3.387 3.433	3.661	4.062	4.436
	325	.972	1.123	1.471	2.636	2.889	3.061	3.226	3.389	3.690 3.603	4.090 3.904	4•447 4•172
	32R	-898	1.026	1.334	2.797	3.036	3.207	3.357	3.524	3.696	3.992	4.314
	1285	1.109	1.185	1.351	2.895	3.070	3.185	3.284	3.384	3.525	3.671	3.829
	A11						3,233					
2 TWENTY-YR	18	.052	-288	1.006	1.871	3.028	4.222	5.626	7.940	11.194	17.263	22.878
	2E	.517	.959	1.893	2.763	3.722	4.981	6.601	8.647	11.282	15.653	20.082
	85	1.608	2.010	2.661	3.390	4.465	6.201	7.895	9.424	11.129	13.516	15.839
	8R	1.660	2.060	2.717	3.464	4.531	6.242	7.888	9.408	11.100	13.468	15.770
	16S 16R	2.012 2.100	2.360 2.437	2.911 2.992	3.502 3.587	4.335 4.418	6.359 6.423	8.501	9.845	11.135	12.984	14.497
	325	2.350	2.665	3.070	3.514	4.078	6.467	8.523 9.058	9.856 10.174	11.105 11.110	12.924 12.523	14.378
	32R	2.293	2.569	2.980	3.393	3.848	6.376	9.058	10.174	11.110	12.523	13.417 13.352
	1285	2.791	2.962	3.272	3.506	3.739	6.377	9.924	10.447	10.971	11.494	12.574
	A11						7.064					

enumeration because of the enormous volume of necessary computation. For example, the number of possible portfolios containing eight different stocks that could be selected from a list of 1,000 stocks is more than 24 quintillion. ¹⁴ At current costs for computer time, complete enumeration of all such portfolios of eight stocks would have cost approximately \$150 trillion. Instead of complete enumeration, we used a sample of all possible portfolios. The sample numbers are indicated in the table. The smallest sample size was approximately 32,000

 14 2.4115 \times 10¹⁹.

portfolios in a given period.¹⁵ We believe that with random samples of this size there are no significant biases or errors in the portrayals in the frequency distributions for the specified periods.

As indicated earlier, there were two methods of random sampling. The first has already been described as simple random sampling without replacement. Samples of this type are designated in the

¹⁵ These sample sizes were selected so as to make the total number of stocks selected approximately the same regardless of the size of the portfolio. The actual numbers (32,768, etc.) are powers of two, which were convenient to use in the computer programming.

Number and Length of Periods	Size of Portfolio and Sampling Method	Sample Minimum	Sample Maximum	Arithmetic Mean	Standard Deviation	Mean Deviation	Gini's Mean Difference	Coefficient of Variation	Relative Mean Deviation	Coefficient of Concentration	Skewness	Kurtosis	Number of Portfolios Examined
Z10	SES	<u> </u>	<u> </u>		<u> </u>	Σ	<u> </u>	0.6	<u> 2 Z</u>	00	<u> </u>	<u> </u>	ZAH
(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
PERIODS						40 YEARS	1926-196	5					
40 ONE-YEAR	1E 2E 8S 8R 16S 16R 32S 32R 128S	.000 .000 .164 .185 .273 .284 .344 .373 .434	20.841 14.428 6.272 6.171 4.434 4.406 3.533 3.261 2.525	1.148 1.148 1.148 1.148 1.148 1.149 1.148 1.150 1.148	.554 .451 .354 .353 .335 .334 .325 .324 .318	. 351 . 307 . 265 . 264 . 257 . 253 . 253 . 249	.518 .449 .381 .381 .367 .367 .359 .359 .359	.483 .393 .308 .307 .292 .290 .283 .281 .277	.306 .268 .231 .230 .224 .223 .220 .220 .217	.226 .196 .166 .166 .160 .157 .156 .154	5.339 3.097 1.028 1.037 .680 .661 .502 .474 .380	7.606 7.837 5.069	35,407 16,357,749 5,242,880 2,621,440 1,310,720 1,310,720 655,360 655,360
8 FIVE-YEAR	1E 2E 8S 8R 16S 16R 32S 32R 128S	.000 .000 .103 .122 .268 .278 .427 .428 .668	48.855 45.698 16.196 15.239 11.391 10.258 8.355 7.618 5.713	1.904 1.904 1.904 1.918 1.905 1.934 1.903 1.966 1.906	2.064 1.623 1.190 1.209 1.103 1.145 1.051 1.145 1.019	1.145 .995 .827 .837 .787 .809 .762 .814 .745	1.640 1.417 1.171 1.189 1.113 1.154 1.075 1.165 1.043	1.084 .852 .625 .631 .579 .592 .552 .582	.601 .523 .434 .436 .413 .418 .401 .414 .391	.431 .372 .307 .310 .292 .298 .282 .296 .274	7.197 4.713 2.239 2.212 1.794 1.754 1.557 1.537 1.409	107.852 47.629 11.032 10.601 7.059 6.528 5.316 4.981 4.293	6,791 3,023,639 1,048,576 524,288 524,288 262,144 262,144 131,072 131,072
4 TEN-YEAR	1E 2E 8S 8R 16S 16R 32S 32R 128S	.000 .000 .049 .079 .192 .228 .475 .476 .797	74.724 51.189 16.967 16.527 10.241 9.974 6.930 7.813 4.637	2.808 2.808 2.804 2.824 2.807 2.849 2.806 2.896 2.804	2.892 2.144 1.325 1.342 1.137 1.164 1.030 1.150 .942	1.761 1.418 .991 .994 .881 .898 .822 .940 .786	2.505 2.039 1.431 1.436 1.254 1.270 1.131 1.239 .974	1.030 .763 .472 .475 .405 .409 .367 .397 .336	.627 .505 .354 .352 .314 .315 .293 .325 .280	.446 .363 .255 .254 .223 .201 .214 .174	7.315 4.564 1.037 1.205 .104 .236 -0.459 -0.371 -0.945	144.189 62.253 8.955 10.180 3.940 4.525 2.580 2.801 2.255	3,137 1,307,279 524,288 262,144 131,072 131,072 65,536
2 TWENTY-YR	1E 2E 8S 8R 16S 16R 32S 32R 128S	.000 .000 .172 .154 .545 .646 1.208 1.190 2.213	110.916 94.155 41.127 36.448 28.019 24.726 21.171 19.714 14.266	7.064 7.064 7.070 7.086 7.063 7.095 7.058 7.002 7.061	9.008 6.883 4.702 4.656 4.221 4.156 3.961 3.999 3.758	5.956 4.983 3.903 3.857 3.735 3.678 3.701 3.769 3.699	8.052 6.778 5.189 5.139 4.748 4.679 4.438 4.465 4.053	1.275 .974 .665 .657 .598 .586 .561 .571 .532	.843 .705 .552 .544 .529 .518 .524 .538	.570 .480 .367 .363 .336 .330 .314 .319 .287	3.485 2.327 .893 .893 .510 .498 .272 .243 .065	24.393 12.326 3.586 3.591 2.265 2.237 1.603 1.522 1.127	1,363 493,173 262,144 131,072 65,536 65,536 32,768 32,768

TABLE 5, CONTINUED

AGGREGATED FREQUENCY DISTRIBUTIONS OF MEALTH RATIOS FROM INVESTMENTS IN RANDOMLY SELECTED PORTFOLIOS CONTAINING SPECIFIED NUMBERS OF STOCKS LISTED ON THE NYSE, 1926-65

Number and	Size of Portfolio/				Centiles	of the Agg	regated Fre	quency Dist	ributions			
Length of Periods	Sampling Method	5th	10th	20th	30th	40th	(Median) 50th	60th	70th	80th	90th	95th
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
PERIODS						20 YEARS	1926-194	5				
20 ONE-YEAR	1E 2E 8S 8R 16S 16R 32S 32R 128S	.356 .438 .510 .512 .524 .527 .530 .532 .533	.480 .545 .582 .584 .583 .587 .581 .588	.675 .718 .760 .760 .771 .769 .779 .769 .781	.828 .859 .892 .893 .902 .903 .908 .913 .909	.958 .979 1.001 1.002 .995 .998 .986 .994 .976	1.075 1.098 1.145 1.149 1.167 1.173 1.189 1.196 1.221	1.192 1.218 1.272 1.275 1.295 1.311 1.318 1.322	1.326 1.344 1.378 1.380 1.389 1.392 1.396 1.400 1.400	1.500 1.497 1.497 1.499 1.504 1.504 1.505 1.505	1.830 1.761 1.675 1.677 1.656 1.656 1.622 1.640 1.593	2.230 2.070 1.891 1.887 1.828 1.816 1.796
4 FIVE-YEAR	1E 2E 8S 8R 16S 16R 32S 32R 128S	.100 .286 .586 .591 .677 .740 .712	.207 .421 .678 .680 .748 .746 .794 .769	.435 .622 .804 .806 .847 .845 .868 .849	.673 .798 .914 .917 .935 .934 .937 .924	.920 .983 1.036 1.039 1.038 1.039 1.023 1.018	1.200 1.206 1.195 1.199 1.194 1.201 1.194 1.222 1.191	1.561 1.535 1.454 1.460 1.461 1.480 1.482 1.527	2.022 2.091 2.028 2.035 1.895 1.909 1.795 1.844 1.680	2.722 2.878 3.222 3.303 3.472 3.658 3.674 4.051 3.987	4.123 4.217 4.243 4.327 4.307 4.489 4.327 4.672 4.335	6.168 5.517 5.082 5.177 4.950 5.113 4.783 5.080 4.549
2 TEN-YEAR	1E 2E 8S 8R 16S 16R 32S 32R 128S	.036 .195 .590 .607 .749 .763 .871 .813	.133 .362 .736 .754 .870 .888 .972 .898	.386 .700 .979 .992 1.065 1.080 1.123 1.026	.772 1.043 1.246 1.258 1.268 1.276 1.271 1.157	1.255 1.427 1.621 1.631 1.545 1.551 1.471 1.334 1.351	1.702 1.859 2.108 2.141 2.217 2.307 2.242 2.305 1.967	2.084 2.277 2.498 2.537 2.640 2.740 2.756 3.073 2.952	2.596 2.725 2.842 2.891 2.925 3.037 2.994 3.326 3.109	3.367 3.302 3.236 3.293 3.221 3.344 3.233 3.590 3.270	4.578 4.273 3.812 3.870 3.639 3.788 3.568 3.991	6.213 5.435 4.350 4.400 4.055 4.268 3.911 4.961 3.672

TABLE 5, CONTINUED

AGGREGATED FREQUENCY DISTRIBUTIONS OF WEALTH RATIOS FROM INVESTMENTS IN RANDOMLY SELECTED PORTFOLIOS CONTAINING SPECIFIED NUMBERS OF STOCKS LISTED ON THE NYSE, 1926-65

Number and	Size of Portfolio/				Centiles	of the Agg	regated Fre	quency Dist	ributions			
Length of Periods	Sampling Method	5th	10th	20th	30th	40th	(Median) 50th	60th	70th	80th	90th	95th
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
PER IODS						20 YEARS	1945-196	5				
20 ONE-YEAR	1E 2E 8S 8R 16S 16R 32S 32R 128S	.663 .746 .829 .831 .847 .849 .857 .860	.763 .820 .876 .876 .887 .888 .891 .894 .887	.879 .913 .944 .944 .948 .951 .954 .962	.961 .982 1.005 1.005 1.005 1.005 1.004 1.003 .994	1.026 1.042 1.063 1.062 1.067 1.067 1.074 1.065 1.078	1.091 1.100 1.114 1.113 1.120 1.127 1.128 1.140	1.161 1.162 1.163 1.165 1.166 1.167 1.174 1.169	1.245 1.236 1.218 1.219 1.214 1.216 1.210 1.219 1.199	1.359 1.335 1.299 1.300 1.285 1.287 1.277 1.282 1.277	1.551 1.497 1.454 1.454 1.450 1.448 1.453 1.443 1.456	1.743 1.650 1.567 1.566 1.557 1.559 1.558 1.560 1.561
4 FIVE-YEAR	1E 2E 8S 8R 16S 16R 32S 32R 128S	.579 .861 1.175 1.177 1.267 1.267 1.331 1.329 1.400	.797 1.015 1.280 1.281 1.351 1.350 1.397 1.393	1.057 1.227 1.427 1.428 1.473 1.471 1.495 1.494 1.507	1.264 1.399 1.554 1.556 1.583 1.586 1.590 1.600	1.460 1.560 1.678 1.682 1.702 1.712 1.710 1.743 1.712	1.656 1.724 1.810 1.816 1.837 1.855 1.863 1.942 1.923	1.875 1.903 1.953 1.960 1.982 2.004 2.016 2.099 2.054	2.131 2.116 2.115 2.121 2.130 2.148 2.151 2.219 2.165	2.474 2.407 2.312 2.316 2.291 2.305 2.284 2.336 2.267	3.149 2.924 2.600 2.602 2.506 2.516 2.448 2.483 2.364	3.916 3.481 2.865 2.868 2.702 2.573 2.610 2.431
2 TEN-YEAR	1E 2E 8S 8R 16S 16R 32S 32R 128S	.749 1.290 2.144 2.160 2.439 2.451 2.661 2.721 2.924	1.073 1.590 2.350 2.364 2.614 2.626 2.800 2.867 3.021	1.576 2.003 2.620 2.630 2.831 2.845 2.962 3.020	1.960 2.340 2.836 2.845 2.993 3.005 3.105 3.153 3.219	2.358 2.661 3.034 3.040 3.151 3.161 3.224 3.262 3.291	2.787 2.992 3.241 3.244 3.308 3.313 3.342 3.371 3.363	3.233 3.361 3.463 3.465 3.479 3.479 3.472 3.493 3.445	3.843 3.824 3.719 3.718 3.669 3.663 3.613 3.619 3.535	4.661 4.483 4.068 4.059 3.922 3.906 3.785 3.773 3.625	6.361 5.610 4.643 4.628 4.290 4.256 4.020 3.992 3.738	8.260 6.839 5.146 5.118 4.620 4.559 4.268 4.201 3.904

Number and Length of Periods	Size of Portfolio and Sampling Method	Sample Minimum	Sample Maximum	Arithmetic Mean	Standard Deviation	Mean Deviation	Gini's Mean Difference	Coefficient of Variation	Relative Mean Deviation	Coefficient of Concentration	Skewness	Kurtosis	Number of Portfolios Examined
(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)		(25)		
	(13)	(10)	(177	(10)	(197				(23)	(24)	(25)	(26)	(27)
PERIODS	15	000	20.041	1 150	400		1926-194		201	202	5 0/0		
20 ONE-YEAR	1E 2E 8S 8R 16S 16R 32S 32R 128S	.000 .000 .164 .185 .273 .284 .344 .373 .434	20.841 14.428 6.272 6.171 4.434 4.406 3.533 3.261 2.525	1.158 1.158 1.159 1.159 1.159 1.157 1.157 1.162 1.158	.699 .569 .448 .447 .425 .422 .410 .403	. 447 . 399 . 355 . 354 . 348 . 347 . 345 . 343	.653 .574 .493 .492 .476 .474 .465 .464	.604 .492 .387 .386 .367 .364 .356 .353 .348	•386 •344 •306 •306 •301 •300 •298 •297 •296	.282 .248 .213 .212 .205 .204 .201 .200 .196	5.062 2.880 .869 .874 .529 .507 .358 .322 .237	86.788 31.712 5.628 5.805 3.691 3.709 2.993 2.946 2.606	14,394 5,246,994 2,621,440 1,310,720 1,310,720 655,360 655,360 327,680 327,680
4 FIVE-YEAR	1E 2E 8S 8R 16S 16R 32S 32R 128S	.000 .000 .103 .122 .268 .278 .427 .428 .668	48.855 45.698 16.196 15.239 11.391 10.258 8.355 7.618 5.713	1.914 1.914 1.914 1.937 1.917 1.962 1.912 1.999 1.917	2.615 2.091 1.588 1.615 1.490 1.549 1.431 1.562 1.397	1.485 1.355 1.226 1.244 1.196 1.235 1.177 1.258 1.170	2.050 1.825 1.561 1.591 1.494 1.557 1.442 1.577 1.394	1.366 1.092 .830 .834 .778 .789 .748 .781	.776 .708 .640 .642 .624 .629 .615 .629 .611	.535 .477 .408 .411 .390 .397 .377 .395 .363	6.257 3.996 1.805 1.762 1.413 1.346 1.212 1.147 1.070	75.783 32.107 6.781 6.447 4.158 3.765 2.761 2.391	2,754 969,210 524,288 262,144 262,144 131,072 131,072 65,536
2 TEN-YEAR	1E 2E 8S 8R 16S 16R 32S 32R 128S	.000 .000 .049 .079 .192 .228 .475 .476 .797	74.724 51.189 16.967 16.527 10.241 9.974 6.930 7.813 4.637	2.232 2.232 2.227 2.269 2.229 2.316 2.230 2.390 2.228	3.075 2.284 1.415 1.473 1.222 1.322 1.113 1.398 1.023	1.845 1.550 1.189 1.202 1.091 1.134 1.039 1.268 1.000	2.302 1.935 1.459 1.497 1.327 1.420 1.239 1.540 1.118	1.378 1.023 .636 .649 .548 .571 .499 .585 .459	.826 .695 .534 .530 .489 .490 .466 .530 .449	.516 .433 .328 .330 .298 .307 .278 .322 .251	10.879 6.927 2.162 2.326 1.079 1.191 .511 .562 .103	225.901 98.839 14.642 15.430 5.646 5.836 2.630 2.448 1.233	1,229 387,916 262,144 131,072 131,072 65,536 65,536 32,768 32,768
Number and Length of Periods	Size of Portfolio and Sampling Method	Sample Minimum	Sample Maximum	Arithmetic Mean	Standard Deviation	Mean Deviation	Gini's Mean Difference	Coefficient of Variation	Relative Mean Deviation	Coefficient of Concentration	Skewness	Kurtosis	Number of Portfolios Examined
(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
PERIODS						20 YEARS	1945-196						
20 ONE-YEAR	1E 2E 8S 8R 16S 16R 32S 32R 128S	.000 .174 .534 .535 .637 .658 .697 .711 .777	5.441 4.951 2.644 2.580 2.214 2.141 1.989 1.930 1.765	1.138 1.138 1.138 1.138 1.138 1.138 1.138 1.139 1.138	.355 .287 .223 .222 .210 .210 .203 .202 .198	.255 .216 .174 .174 .165 .166 .160 .161	.367 .307 .246 .246 .233 .233 .227 .226 .221	.312 .252 .196 .195 .185 .184 .179 .178	.224 .189 .153 .153 .145 .146 .141 .142 .137	.161 .135 .108 .108 .103 .103 .100 .099 .097	1.791 1.233 .769 .772 .708 .710 .681 .670 .664	12.734 7.080 3.673 3.654 3.259 3.228 3.059 3.007 2.917	21,013 11,110,755 2,621,440 1,310,720 1,310,720 655,360 655,360 327,680 327,680
4 FIVE-YEAR	1E 2E 8S 8R 16S 16R 32S 32R 128S	.063 .107 .553 .639 .846 .910 1.022 1.197	35.876 24.524 7.417 7.554 5.052 4.934 3.507 3.471 2.746	1.894 1.894 1.894 1.899 1.894 1.906 1.895 1.932 1.894	1.296 .947 .557 .562 .460 .470 .404 .422 .355	.806 .635 .428 .430 .377 .383 .348 .370 .320	1.152 .903 .597 .600 .514 .524 .462 .484 .408	.684 .500 .294 .296 .243 .247 .213 .218 .187	.425 .335 .226 .226 .199 .201 .184 .191 .169	.304 .238 .158 .158 .136 .138 .122 .125 .108	6.646 4.318 1.458 1.528 .699 .732 .309 .167 .052	129.423 57.353 8.981 9.500 3.666 3.866 2.168 1.972 1.596	4,037 2,054,429 524,288 262,144 262,144 131,072 131,072 65,536 65,536
2 TEN-YEAR	1E 2E 8R 16S 16R 32S 32R 128S	.047 .064 .837 1.035 1.451 1.523 1.999 1.912 2.600	22.340 21.851 9.564 9.523 7.562 6.935 5.787 5.420 4.409	3.383 3.383 3.381 3.380 3.385 3.381 3.382 3.401 3.379	2.571 1.820 .917 .902 .653 .630 .469 .425 .256	1.677 1.286 .793 .787 .671 .662 .605 .613 .572	2.485 1.865 1.004 .990 .726 .703 .527 .479 .290	.760 .538 .271 .267 .193 .186 .139 .125 .076	.496 .380 .235 .233 .198 .196 .179 .180 .169	.367 .276 .148 .146 .107 .104 .078 .070 .043	2.499 1.775 .886 .668 .611 .494 .358 .289	13.148 8.049 4.242 4.139 3.613 3.488 3.230 3.029 2.821	1,908 919,363 262,144 131,072 65,536 65,536 32,768 32,768

table with the letter S. A second method of random sampling was also used, and the results of this method are indicated in the table with the letter R. In the second method, we took steps to insure that the portfolios were well diversified by industry. All the common stocks on the New York Stock Exchange were assigned to thirty-four industry groups. ¹⁶ Our

16 See Appendix, table A1.

method of random selection insured that no more than one stock fell in any single industry group. The greater the number of stocks in an industry, the greater the probability of including that industry in the portfolio. But the greater the number in the industry, the smaller the probability of including any particular stock.

We will not distinguish between these two different random methods of sampling in discussing the results, since the

TABLE 6

DISPERSION OF RETURNS ON N-STOCK PORTFOLIOS AS PERCENTAGE
OF DISPERSION OF ONE-STOCK PORTFOLIOS
(Based on portfolios of stocks from NYSE for 1926-65 or as specified)

		Num	ber of	Stocks	in Port	folio	
MEASURE OF RELATIVE DISPERSION For holding period(s)	1	_2	_8	<u>16</u>	32	128	A11 (Market)
Coefficient of Variation							
40 one-year	100	81	64	60	59	57	57
20 one-year (1926-45)	100	81	64	61	59	58	57
20 one-year (1946-65)	100	81	63	59	57	56	55
8 five-year	100	79	58	53	51	49	49
4 ten-year	100	74	46	39	36	33	32
2 twenty-year	100	76	52	47	44	42	41
Relative Mean Deviation							
40 one-year	100	88	75	73	72	71	71
20 one-year (1926-45)	100	89	79	78	77	77	76
20 one-year (1946-65)	100	85	68	65	63	61	61
8 five-year	100	87	72	69	67	65	65
4 ten-year	100	81	56	50	47	45	45
2 twenty-year	100	84	65	63	62	62	62
Gini's Coefficient of Concentration							
40 one-year	100	87	74	71	69	68	68
20 one-year (1926-45)	100	88	76	73	71	70	69
20 one-year (1946-65)	100	84	67	64	62	60	59
8 five-year	100	84	67	64	62	60	59
4 ten-year	100	81	57	50	45	39	34
2 twenty-year	100	84	64	59	55	50	46

two methods of selecting the sample did not produce significantly different results. Although there was a slight reduction in dispersion within individual periods as a result of the constrained random sampling, this reduction was almost exactly offset by the increased dispersion of the means among periods. Thus, when periods were aggregated, the distributions from the two methods of sampling became almost the same. The findings.—In considering the findings discussed here, it is important to remember that initial equal investments were made in each stock included in any portfolio and that there was no subsequent reallocation of resources to preserve the equality of investment. This is not an investment strategy we advocate; again, it was chosen to make certain that the distributions were affected only by the number of stocks in the portfolio.

TABLE 7

DISPERSION OF RETURNS ON N-STOCK PORTFOLIOS AS PERCENTAGE
OF DISPERSION OF MARKET PORTFOLIOS
(Based on portfolios of stocks from NYSE for 1926-65 or as specified)

		Numbe	r of St	ocks in	Portfo	lio	
MEASURE OF RELATIVE DISPERSION For holding period(s)	1	2	8	16	32	128	All (Market)
••							
Coefficient of Variation	176	143	112	106	103	101	100
40 one-year			112	106	103	101	100
20 one-year (1926-45)	175	142					100
20 one-year (1946-65)	180	146	113	107	103	101	
8 five-year	205	161	118	109	104	101	100
4 ten-year	316	234	145	124	113	103	100
2 twenty-year	243	186	127	114	107	102	100
Relative Mean Deviation							
40 one-year	142	124	107	104	102	101	100
20 one-year (1926-45)	131	117	104	102	101	100	100
20 one-year (1946-65)	165	140	113	107	104	101	100
8 five-year	154	134	111	106	103	100	100
4 ten-year	224	181	126	112	105	100	100
2 twenty-year	161	135	105	101	100	100	100
Gini's Coefficient of Concentration							
40 one-year	148	128	109	105	103	101	100
20 one-year (1926-45)	145	127	110	106	103	101	100
20 one-year (1946-65)	169	141	113	107	104	101	100
8 five-year	161	139	115	109	105	102	100
4 ten-year	291	237	167	146	132	113	100
2 twenty-year	217	183	140	128	120	110	100

There is only one important generalization about table 5. It is that portfolios containing eight stocks have frequency distributions strikingly similar to those of portfolios containing larger numbers of stocks—including all listed stocks—except for the tails beyond the fifth and ninetieth centiles. The tails beyond those centiles get progressively shorter as the number of stocks in the portfolio in-

creases. This fact causes the measures of dispersion to get smaller, despite the nearly identical distributions between the fifth and ninetieth centiles.

Tables 6, 7, and 8 summarize the information in table 5 with respect to the effect on relative dispersion of changing the number of stocks in a portfolio. The tables are easily read. The market as a whole generally had 50–75 percent as

TABLE 8

PERCENT OF POSSIBLE REDUCTION IN RELATIVE DISPERSION ACHIEVED THROUGH INCREASING THE NUMBER OF STOCKS IN THE PORTFOLIO (Based on portfolios of stocks from NYSE for 1926-65 or as specified)

		Nui	mber of	Stocks	in Por	tfolio	
MEASURE OF RELATIVE DISPERSION For holding period(s)	1	_2	_8	<u>16</u>	32	128	All (Market)
Coefficient of Variation							
40 one-year	0	43	84	92	96	99	100
20 one-year (1926-45)	0	43	84	92	96	99	100
20 one-year (1946-65)	0	43	84	92	96	99	100
8 five-year	0	42	83	91	96	99	100
4 ten-year	0	38	79	89	94	99	100
2 twenty-year	0	40	81	90	95	99	100
Relative Mean Deviation							
40 one-year	0	42	84	91	95	98	100
20 one-year (1926-45)	0	45	87	94	96	99	100
20 one-year (1946-65)	0	39	80	89	94	99	100
8 five-year	0	37	79	89	95	99	100
4 ten-year	0	35	79	90	96	100	100
2 twenty-year	0	43	91	99	100	100	100
Gini's Coefficient of Concentration							
40 one-year	0	41	81	90	94	98	100
20 one-year (1926-45)	0	39	79	87	93	98	100
20 one-year (1946-65)	0	40	81	89	94	98	100
8 five-year	0	36	76	85	91	97	100
4 ten-year	0	28	65	76	84	93	100
2 twenty-year	0	29	66	76	83	92	100

much dispersion as did one-stock portfolios, depending on the periods and measure of dispersion (table 6). Conversely, one-stock portfolios have roughly one and one-third to twice as much dispersion as the market (table 7). The opportunity to reduce dispersion by increasing the number of stocks in the portfolio is rapidly exhausted (table 8). Roughly, 40 percent of achievable reduction is obtained by holding two stocks; 80 percent, by holding eight stocks; 90 percent, by holding sixteen stocks; 95 percent, by holding thirty-two stocks; and 99 percent, by holding 128 stocks (table 8).

APPENDIX A

AGGREGATED FREQUENCY DISTRIBUTIONS FOR PORTFOLIOS OF SPECIFIED SIZES

Table A1 shows the frequency distributions of wealth ratios for portfolios of specified sizes for the fifty-five periods. These distributions were aggregated to produce tables 4 and 5.

Since the statistics for portfolios having eight or more stocks were based on samples, it is unlikely that the minimum and maximum wealth ratios for any samples were the true minima and maxima. Table A2 shows the true minima and maxima for portfolios of eight or more stocks for each of the fifty-five periods.

TABLE A1

FREQUENCY DISTRIBUTIONS OF MEALTH RATIOS FROM INVESTMENTS IN RANDOMLY SELECTED PORTPOLIOS

CONTAINING SPECIFIED NUMBERS OF STOCKS LISTED ON THE NYSE, 1926-65

Number and	Size of Portfolio	,			Cen	tiles of th	e Frequency	Distributi	ons	_		
Length of Periods	Sampling Method	5th	10th	20th	30th	40th	(Median) 50th	60th	70th	80th	90th	95th
<u> </u>						- 10 00	<u> </u>		7000	<u> </u>	3001	3341
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
					ONE-YEA	R PERIODS						
1404 12424		4.00		700								
1/26-12/26	1 E 2 E	•429 •626	•560 •703	•722 •796	.845 .865	.917 .923	•991 •976	1.060 1.026	1.115 1.079	1.183 1.143	1.316 1.249	1.485 1.382
	85	-802	.839	.886	.919	.949	.977	1.005	1.038	1.077	1.139	1.196
	8R 16S	.801 .853	.838 .881	.884 .914	.918 .938	.947 .960	.976 .980	1.004 1.002	1.037 1.025	1.078 1.053	1.141 1.095	1.201 1.132
	16R	. 851	.879	.912	.935	. 957	.978	•999	1.023	1.052	1.095	1.132
	32S 32R	•892 •886	•912 •904	•936 •928	•953 •945	•969 •961	•983 •974	•998 •989	1.015	1.034 1.024	1.061	1.084
	1285	.942	.952	.963	.971	.978	.985	.992	.999	1.008	1.019	1.029
12/26-12/27	1 E 2 E	•640 •797	.752 .895	.888 1.011	1.053	1.132 1.175	1.225	1.330	1.457	1.601 1.532	1.834 1.739	2.274 1.957
	85	1.024	1.077	1.142	1.192	1.235	1.277	1.321	1.372	1.435	1.536	1.637
	8R 16S	1.026 1.097	1.077	1.140 1.185	1.189 1.221	1.231 1.253	1.273	1.317 1.318	1.367 1.354	1.430 1.400	1.529 1.474	1.631 1.557
	16R	1.095	1.134	1.182	1.216	1.247	1.278	1.309	1.345	1.389	1.459	1.530 1.477
	32S 32R	1.153	1.181	1.216	1.243 1.235	1.267 1.256	1.291 1.278	1.313	1.340 1.325	1.374 1.355	1.427 1.397	1.477 1.439
	1 285	1.230	1.244	1.262	1.275	1.287	1.299	1.310	1.323	1.338	1.359	1.377
12/27-12/28	16	725	.834	•976	1.064	1 162	1.251	1.376	1.512	1.711	2 124	3 705
12/21-12/20	ŽĒ	.725 .879	.960	1.064	1.148	1.153 1.227	1.310	1.402	1.517	1.689	2.136 2.049	2.795 2.459
	8S 8R	1.095	1.149	1.219	1.276	1.330	1.386	1.449	1.524	1.626	1.811	2.026
	16S	1.096 1.176	1.148 1.220	1.217 1.279	1.274 1.326	1.326 1.369	1.381 1.413	1.442 1.460	1.517 1.518	1.618 1.597	1.802 1.731	2.031 1.878
	16R	1.167	1.208	1.265	1.309	1.350	1.391	1.438	1.492	1.568	1.699	1.839
	32S 32R	1.244	1.281 1.237	1.325 1.275	1.362 1.306	1.395 1.335	1.430 1.363	1.468 1.393	1.510 1.431	1.566 1.479	1.662 1.558	1.754 1.630
	1285	1.344	1.365	1.392	1.412	1.430	1.448	1.466	1.486	1.512	1.547	1.579
12/28-12/29	16	.197	.279	•405	.518	.604	.700	.774	.873	.977	1.123	1.218
	2E	.339	.412	.505	.574	•636	•694	.752	.816	.890	•994	1.079
	8S 8R	•518 •520	•556 •558	-604 -604	.639 .637	•669 •667	•698 •696	.726 .723	.757 .753	•794 •789	.844 .838	.886 .879
	165	•572	.599	.633	.657	•679	• 699	.719	.740	.765	.801	.830
	16R 32S	.573 .609	.600 .629	.632 .653	•655 •670	.675 .685	•693 •699	.712 .713	.733 .728	.758 .746	.791 .770	.819 .790
	328	•612	.629	.651	•666	.679	.691	.703	.717	734	.756	•775
	1285	.659	•667	•679	.686	.693	.700	.707	.713	.721	•732	•742
12/29-12/30	16	-205	.265	.367	•442	.521	•593	.674	.757	.868	•986	1.106
	2E 8S	.313	-370	-446	•504	. 557	•607	.658	.715	.783	.882	.967
	85 8R	•#62 •463	.494 .496	•535 •537	•565 •568	.591 .594	.616 .620	•641 •645	•669 •673	•702 •705	•750 •753	•791 •793
	165	•506	.531	-560	.581	.600	-618	• 636	.655	.679	.711	•793 •739
	16R 32S	•514 •539	.537 .556	.566 .578	.588 .593	.606 .606	.624 .619	.642 .631	.662 .645	.684 .661	.716 .683	.743 .702
	32R	•550	.566 .590	.588	• 603	.615	.628	.640	•653	•669	.692	.711
	1285	.582	•590	.601	•607	.614	•620	•625	•632	.639	•649	•658
12/30-12/31	16	.158	.215	.292	.347	.401	•467	.536	-615	.738	•909	1.044
	2E 8S	.241	.288 .397	.350	.399 .463	.446 .489	•493 •514	•545 •540	.603 .568	.677 .603	.788 .656	.891 .702
	8R	.367 .371	.401	.434 .439	.469	• 494	.519	.545	.574	.610	•662	.708
	165	.410	-432	•460	-481	.500	•518	•536	•556	•580 •593	.61 6	.646
	16R 32S	.420 .442	.443 .458	.472 .478	.493 .494	.511 .507	.529 .520	.548 .533	•568 •547	•564	.628 .587	.658 .607
	32R 128S	.463 .483	.480 .491	.500 .502	.515 .509	.529 .515	•542 •521	.555 .527	.568 .534	.585 .541	.609 .552	.627 .560
	1200	*****	• . , •	• • • • • • • • • • • • • • • • • • • •	****	****	****		••••	****	****	***************************************
12/31-12/32	1E 2E	.353 .475	.449 .551	.561 .646	•654 •719	•729 •786	.828 .851	.931 .920	1.035	1.173 1.103	1.377 1.272	1.662
	85	.663	.707	.763	.805	.842	.878	.916	.959	1.012	1.093	1.163
	8R 16S	.665 .726	.709 .759	.763 .799	.805 .830	.842 .857	.877 .883	.915 .911	.957 .941	1.009 J978	1.088 1.032	1.160 1.078
	16R	.723	.756	.797	.827	.854	.879	.906	.935	.970	1.022	1.066
	325	.772	•796	.827	.849	.869	-887	.906	•927	•953	•989	1.020
	32R 128S	.763 .834	.785 3846	.813 .862	.834 .873	.852 .882	.871 .891	.900	.908 .909	.931 .921	.963 .937	•991 •950
12/32-12/33	16	.657	.856	1.189	1.402	1.628	1.849	2.075	2.394	2.742	3.331	4.338
-5, 25-15, 33	2E	• 986	1.165	1.401	1.587	1.755	1.925	2.106	2.319	2.608	3.128	3.691
	8S 8R	1.459	1.568 1.557	1.712	1.822 1.802	1.919	2.018 1.998	2.124 2.101	2.245 2.221	2.397 2.373	2.634 2.613	2.865
	165	1.619	1.704	1.814	1.897	1.971	2.045	2.121	2.208	2.312	2.480	2.839 2.662
	16R	1.586	1.669	1.777	1.856	1.929	2.000	2.075	2.160	2.273	2.448	2.658
	32S 32R	1.744	1.806 1.768	1.889	1.950	2.005 1.956	2.057 2.007	2.113 2.064	2.176 2.128	2.256 2.219	2.383	2.517 2.576
	1285	1.915	1.950	1.992	2.023	2.052	2.080	2.107	2.138	2.174	2.227	2.274

Number and Length of Periods	Size of Portfolio and Sampling Method	Sample Minimum	Sample Maximum	Arithmetic Mean	Standard Deviation	Mean Deviation	Gini's Mean Difference	Coefficient of Variation	Relative Mean Deviation	Coefficient of Concentration	Skewness	Kurtosis	Number of Portfolios Examined
(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
					ONE-YEAR	RPERIODS							
1/26-12/26	1E 2E 8S 8R 16S 16R 32S 32R	.073 .174 .529 .554 .674 .692 .740 .789	2.970 2.950 1.700 1.843 1.365 1.410 1.257 1.198	.985 .985 .984 .984 .985 .983 .985 .977	.343 .242 .120 .122 .084 .085 .058 .057	.242 .179 .094 .095 .067 .068 .046 .046	.355 .259 .134 .135 .095 .096 .066	.348 .246 .122 .124 .086 .087 .059 .059	.245 .181 .095 .096 .068 .069 .047 .047	.180 .132 .068 .069 .048 .049 .033 .033	1.304 .920 .436 .499 .306 .346 .197 .228	8.891 5.910 3.637 3.744 3.246 3.283 3.088 3.007 2.939	510 129,795 131,072 65,536 65,536 32,768 32,768 16,384 16,384
12/26-12/27	1E 2E 8S 8R 16S 16R 32S 32R	.000 .000 .627 .632 .835 .834 .992 .977	7.889 6.338 2.805 2.881 2.238 2.227 1.805 1.745	1.300 1.300 1.300 1.296 1.300 1.291 1.298 1.284 1.300	.577 .408 .203 .199 .143 .136 .098 .089	.374 .278 .150 .148 .108 .105 .077 .072	.546 .404 .215 .212 .155 .148 .109 .099	.444 .314 .156 .154 .110 .105 .076 .069	.288 .214 .115 .114 .083 .081 .059 .056	.210 .155 .083 .082 .060 .057 .042 .039	3.660 2.581 1.254 1.261 .895 .812 .568 .449	36.275 19.478 6.823 6.922 4.825 4.718 3.653 3.595 2.994	543 147,153 131,072 65,536 65,536 32,768 32,768 16,384
12/27-12/28	1E 2E 8S 8R 16S 16R 32S 32R	.398 .407 .778 .803 .908 .961 1.053 1.061	13.226 10.450 4.001 4.080 3.069 2.802 2.288 2.208 1.752	1.453 1.453 1.451 1.454 1.454 1.431 1.454 1.383 1.453	.904 .639 .316 .313 .223 .213 .155 .132	.498 .388 .223 .222 .165 .163 .120 .125	.712 .554 .317 .314 .235 .225 .170 .144	.622 .440 .218 .216 .154 .149 .107 .096	.343 .267 .154 .153 .114 .114 .082 .090	.245 .191 .109 .108 .081 .079 .058 .052	5.844 4.122 2.021 2.033 1.404 1.398 .931 1.015	60.390 31.444 9.670 9.757 6.161 6.136 4.254 4.764 3.008	589 173,166 131,072 65,536 65,536 32,768 32,768 16,384
12/28 -1 2/29	1E 2E 8S 8R 16S 16R 32S 32R	.000 .026 .216 .283 .360 .355 .476 .509	1.851 1.754 1.214 1.169 1.024 1.003 .924 .883	.700 .700 .699 .697 .699 .695 .699 .692	.318 .225 .112 .109 .079 .075 .055 .049	.259 .181 .090 .087 .063 .060 .044 .040	.361 .254 .126 .123 .089 .084 .062 .055	.454 .321 .160 .156 .112 .108 .079 .071	.370 .258 .128 .125 .090 .086 .063 .057	.258 .182 .090 .088 .063 .061 .044 .040	.261 .184 .096 .083 .059 .C79 .029 .078	2.718 2.856 2.981 2.928 2.952 3.045 2.956 2.977 2.948	627 196,251 131,072 65,536 65,536 32,768 32,768 16,384 16,384
12/29-:2/30	1E 2E 8S 8R 16S 16R 32S 32R	.046 .051 .241 .260 .347 .383 .432 .452	2.105 1.961 1.121 1.123 .950 .927 .822 .825	.620 .620 .620 .623 .620 .626 .619 .629	.286 .202 .100 .100 .070 .070 .049 .049	.229 .160 .080 .080 .056 .056 .039 .039	.318 .226 .113 .113 .079 .079 .056	.461 .326 .162 .161 .114 .111 .079 .077	.369 .258 .128 .128 .091 .089 .063 .063	.257 .182 .091 .091 .064 .063 .045 .044	.685 .483 .237 .223 .153 .133 .102 .105	4.047 3.516 3.127 3.095 3.021 3.009 2.970 2.978 2.995	717 256,686 131,072 65,536 65,536 32,768 32,768 16,384
12/30-12/31	1E 2b 8S 8R 16S 16R 32S 32R	.000 .000 .164 .185 .273 .284 .344 .373	2.204 2.114 1.079 1.132 .910 .899 .770 .730	.522 .522 .521 .527 .521 .533 .522 .543	.291 .206 .102 .103 .072 .072 .050	.221 .160 .081 .081 .057 .058 .040 .043	.311 .225 .114 .115 .081 .081 .057 .056	.558 .394 .196 .196 .138 .135 .096 .092	.425 .306 .155 .154 .110 .108 .077 .079	.298 .216 .110 .109 .078 .076 .054 .052	1.323 .933 .452 .458 .328 .286 .190 .143	6.365 4.667 3.358 3.419 3.223 3.116 3.024 2.940 3.000	737 271,216 131,072 65,536 65,536 32,768 32,768 16,384
12/31-12/32	1E 2E 8S 8R 16S 16R 32S 32R 128S	.000 .000 .375 .356 .520 .513 .622 .569	3.308 3.188 1.787 1.758 1.521 1.367 1.217 1.151	.891 .891 .890 .890 .885 .890 .873	.435 .307 .153 .151 .108 .104 .075 .069	.319 .232 .120 .119 .085 .083 .060 .058	.455 .331 .170 .169 .121 .117 .085 .078	.488 .345 .171 .170 .121 .118 .084 .080	.358 .260 .135 .134 .096 .094 .067 .066	. 255 . 186 . 096 . 095 . 068 . 066 . 048 . 045 . 022	1.481 1.045 .506 .505 .379 .320 .249 .189	7.462 5.211 3.514 3.516 3.288 3.191 3.083 3.010 3.009	732 267,546 131,072 65,536 65,536 32,768 32,768 16,384
12/32-12/33	1E 2E 8S 8R 16S 16R 32S 32R	.000 .000 .832 .843 1.142 1.143 1.316 1.381	20.841 14.428 6.272 6.171 4.434 4.406 3.533 3.261 2.525	2.083 2.083 2.082 2.066 2.083 2.049 2.082 2.048 2.085	1.366 .965 .479 .486 .339 .353 .236 .258	.873 .650 .350 .352 .253 .264 .182 .203	1.253 .933 .500 .500 .363 .371 .260 .279	.656 .463 .230 .235 .163 .172 .114 .126	.419 .312 .168 .170 .122 .129 .087 .099	.301 .224 .120 .121 .087 .091 .062 .068	4.686 3.306 1.628 1.805 1.147 1.375 .762 .963	54.283 28.455 9.059 9.873 5.934 6.520 4.204 4.196 3.031	709 250,986 131,072 65,536 65,536 32,768 32,768 16,384 16,384

TABLE A1, CONTINUED FREQUENCY DISTRIBUTIONS OF MEALTH RATIOS FROM INVESTMENTS IN RANDOMLY SELECTED PORTFOLIOS CONTAINING SPECIFIED NUMBERS OF STOCKS LISTED ON THE NYSE, 1926-65

Size of Portfolio/ Sampling Centiles of the Frequency Distributions Number and Length (Median) 50th of Periods 5th 10th 20th 30th 40th 60th 70th 80th 90th 95 th (1) (2) (3) (4) (5) (6) (7) (8) (9) (10) (11)(12) (13) ONE-YEAR PERIODS (CONTINUED) 1.728 1.558 1.364 1.372 1.302 1.314 1.262 1.263 1.428 1.362 1.266 1.272 1.234 1.246 1.209 1.219 1.178 1E 2E 8S 8R 16S 16R 32S .566 .690 .883 .893 .947 .967 .998 .645 .760 .927 .937 .983 1.002 1.024 1.134 1.150 1.155 1.161 1.154 1.167 1.149 1.166 12/33-12/34 1.029 .849 .930 1.031 1.039 1.061 1.078 1.262 1.242 1.204 1.210 1.189 1.202 1.176 1.189 .759 .855 .986 .994 1.027 1.045 1.001 1.072 1.078 1.092 1.107 1.105 1.125 1.761 1.462 1.469 1.376 1.388 1.324 1.312 1.072 1.119 1.122 1.137 1.127 1.080 1.145 32R 1.104 1285 1.068 1.082 1-100 1.113 1.125 1.492 1.532 1.542 1.546 1.537 1.546 1.529 1.546 1.656 1.663 1.610 1.614 1.584 1.593 1.561 1.577 2.342 2.117 1.824 1.825 1.726 1.735 1.658 1.670 .761 .907 1.163 1.169 1.254 1.269 1.327 1.350 .864 1.007 1.226 1.232 1.303 1.316 1.361 1.384 2.747 2.394 1.939 1.943 1.801 1.232 1.323 1.426 1.432 1.453 1.465 1.471 1.490 1.492 1.353 1.421 1.483 1.488 1.494 1.927 1.829 1.694 1.697 1E 2E 8S 1.029 1.132 1.308 12/34-12/35 1.134 1.230 1.370 1.375 1.411 1.308 1.313 1.365 1.377 1.407 1.429 8R 16S 16R 32S 32R 128S 1.641 1.423 1.441 1.462 1.477 1.505 1.500 1.517 1.506 1.650 1.600 1.615 1.552 1.808 1.705 1.717 1.598 1.052 1.152 1.296 1.295 1.346 1.344 1.380 1.386 .943 1.054 1.231 1.231 1.296 1.293 1.341 1.351 1.334 1.385 1.442 1.439 1.455 1.450 1.583 1.580 1.550 1.545 1.533 1.528 1.517 1.773 1.722 1.624 1.619 1.586 1.580 1.555 1.552 .854 .979 1.181 1.182 1.257 1.446 1.473 1.491 1.489 1.491 1.486 1E 2E 8S 8R 1.154 1.231 1.348 1.346 1.385 12/35-12/36 2.461 1.235 1.307 1.395 1.392 1.420 1.417 2.130 1.975 1.740 1.735 1.670 1.662 1.619 1.605 2.252 1.854 1.849 1.759 1.748 1.723 1.665 165 1.382 16R 1.254 1.312 1.321 1.391 325 1.440 1.517 1285 1.613 .798 .721 .631 .628 .604 .599 .585 .570 .888 .786 .659 .656 .623 .618 .597 .582 .562 .563 .555 .554 .551 .549 .549 .537 .276 .338 .434 .434 .463 .464 .486 .478 .472 .495 .520 .519 .527 .525 .532 .521 .517 .528 .537 .536 .539 .536 .540 .528 .624 .602 .574 .572 .565 .562 .558 .545 .315 .374 .455 .455 .480 .480 .498 .489 .687 .650 .597 1E 2E 8S 8R 12/36-12/37 .373 .423 .482 .482 .500 .498 .512 .502 .420 .461 .502 .502 .514 .513 .522 .512 .597 .595 .581 .577 .569 .556 16S 16R 32S 32R 1285 .693 .849 1.067 1.068 1.134 1.128 1.181 1.184 .838 .950 1.115 1.115 1.168 1.162 1.206 1.209 1.090 1.144 1.218 1.215 1.242 1.234 1.259 1.262 1.284 1.260 1.277 1.291 1.287 1.295 1.286 1.299 1.302 1.594 1.517 1.419 1.413 1.389 1.383 1.370 1.391 1E 2E 8S 8R .986 1.065 1.174 1.172 1.210 1.186 1.212 1.255 1.252 1.269 12/37-12/38 1.350 1.458 1.798 2.020 1.350 1.345 1.328 1.323 1.322 1.314 1.319 1.325 1.458 1.422 1.368 1.364 1.352 1.344 1.341 1.352 1.798 1.661 1.496 1.490 1.449 1.452 1.416 1.459 2.020 1.802 1.576 1.575 1.523 1.557 1.466 16S 16R 32S 32R 128S 1.205 1.236 1.240 1.272 1.261 1.283 1.183 1.125 1.055 1.059 1.030 1.039 1.011 1.042 .626 .709 .832 .835 .871 .877 .869 .899 .934 .936 .944 .951 1.478 1.351 1.160 1.021 1.089 .947 12/38-12/39 16 - 554 -704 .790 1E 2E 8S 8R 16S 16R 32S .554 .648 .798 .801 .846 .852 .881 .905 .704 .789 .874 .877 .902 .907 .921 .790 .848 .906 .909 .924 .931 .937 .947 .948 .960 .963 .964 .970 .965 .993 .999 .988 .990 .983 .990 .979 1.008 1.234 1.111 1.114 1.056 1.018 1.022 1.005 1.013 .994 1.023 1.164 1.068 1.078 1.037 1.067 1.000 1.111 1.059 1.089 .924 .934 32R 1285 .946 . 954 .961 1.069 1.036 .977 .977 .956 .960 1.005 .982 .947 .945 .934 .937 .795 .811 .851 .850 .865 .866 .876 .889 .852 .858 .876 .875 .883 .884 .888 .902 1E 2E 8S 8R 16S 16R 32S 32R .477 .597 .748 .751 .792 .796 .825 .837 .584 .672 .782 .783 .816 .819 .841 .854 .952 .939 .922 .920 1.194 1.122 1.023 1.024 1.331 1.204 1.066 1.069 .904 .899 .899 .713 .755 .822 12/39-12/40 .822 .844 .846 .861 .874 .897 .899 .918 .912 .993 .963 .979 1.022 .982 901 .900 .927 .907 .940 .914 .957 .921 .999 1.016 .985 .948 .948 .935 .935 .545 .642 .764 .766 .802 .803 .829 .836 1.234 1.159 1.039 1.041 .998 .996 .968 .969 .445 .572 .729 .730 .777 .751 .785 .838 .839 .856 .855 .820 .836 .866 .875 .875 .879 .883 .891 .893 .894 .947 .931 .918 .919 .914 .913 1.412 1.277 1.089 .666 .726 .807 .808 .833 .833 1E 2E 8S 8R 12/40-12/41 1.096 1.051 .984 .985 .960 .959 .943 .945 1.088 1-029 16S 16R 1.027 •989 •989 •941 .811 .819 .856 32S 32R 128S .896 .887 .891 .858 .876 .900 .898 .873 .884 .914 .928

Number and Longth of Periods	Size of Portfolio and Sampling Wethod	Sample Minimum	Sample Maximum	Arithmetic Mean	Standard Deviation	Mean Deviation	Gini's Mean Difference	Coefficient of Variation	Relative Mean Deviation	Coefficient of Concentration	Skerness	Kurtosis	Number of Portfolios Examined
(14)	(15)	(16)	(17)	(18)	(19) ONE-YEAR	(20)	(21) (CONTINUE	(22) D)	(23)	(24)	(25)	(26)	(27)
12/33-12/34	1E 2E 8S 8R 16S 16R 32S 32R	.090 .154 .626 .627 .744 .721 .856 .899	9.481 6.789 2.975 2.609 2.046 2.093 1.689 1.642 1.326	1.139 1.139 1.139 1.147 1.139 1.154 1.138 1.153	.574 .406 .202 .201 .142 .139 .100 .089	.364 .269 .144 .143 .105 .102 .076 .068	•517 •384 •207 •206 •150 •148 •109 •098	.504 .356 .178 .176 .124 .121 .087 .078	.319 .236 .127 .125 .092 .088 .067 .059	.227 .169 .091 .090 .066 .064 .048 .042	5.358 3.780 1.874 1.866 1.258 1.293 .871 .783 .302	67.343 34.938 10.659 10.388 6.423 6.465 4.473 4.378 3.020	707 249,571 131,072 65,536 65,536 32,768 32,768 16,384
12/34-12/35	1E 2E 8S 8R 16S 16R 32S 32R	.000 .000 .768 .716 .946 .970 1.109 1.180	6.077 5.683 3.035 3.008 2.433 2.499 2.103 2.020 1.731	1.507 1.507 1.508 1.512 1.507 1.517 1.505 1.523 1.507	.675 .477 .238 .237 .167 .165 .116 .112	.485 .359 .187 .186 .132 .130 .092 .089	.687 .508 .265 .263 .187 .185 .130 .126	.448 .317 .158 .157 .111 .109 .077 .074	.322 .238 .124 .123 .088 .086 .061 .059	.228 .168 .088 .087 .062 .061 .043 .041	1.830 1.291 .630 .640 .452 .446 .310 .320	9.327 6.137 3.710 3.738 3.390 3.362 3.173 3.189 2.994	706 248,865 131,072 65,536 65,536 32,768 32,768 16,384
12/35-12/36	1E 2E 8S 8R 16S 16R 32S 32R	.178 .186 .853 .869 1.004 1.010 1.138 1.140	17.234 11.272 4.318 4.095 3.005 2.834 2.299 2.265 1.748	1.483 1.483 1.481 1.485 1.479 1.481 1.478	.814 .575 .285 .285 .204 .195 .140 .122	.424 .321 .178 .177 .133 .130 .098 .087	.605 .460 .257 .256 .194 .187 .141 .125	.549 .388 .192 .192 .137 .132 .095 .082	.286 .217 .120 .120 .089 .088 .066 .059	.204 .155 .087 .086 .065 .063 .048 .042	10.719 7.563 3.723 3.777 2.629 2.585 1.787 1.665	197.482 99.551 26.134 26.489 13.936 14.164 7.928 8.485 3.202	719 258,121 131,072 65,536 65,536 32,768 32,768 16,384
12/36-12/37	1E 2E 8S 8R 16S 16R 32S 32R	.109 .123 .276 .314 .350 .368 .414 .419	1.372 1.345 .867 .893 .762 .786 .690 .666	.541 .541 .540 .540 .538 .541 .529	.195 .138 .069 .068 .048 .047 .034 .031	.153 .109 .055 .054 .039 .037 .027 .027	.215 .154 .077 .076 .055 .053 .038 .035	.360 .254 .127 .125 .090 .087 .062 .059	.283 .201 .101 .100 .071 .069 .050 .051	.199 .142 .071 .070 .050 .049 .035 .033	.815 .575 .294 .284 .193 .204 .131 .124	4.073 3.529 3.134 3.126 3.044 3.069 3.009 3.038 3.006	744 276,396 131,072 65,536 65,536 32,768 32,768 16,384 16,384
12/37-12/38	1 t 2 E 8 S 8 R 16 S 16 R 3 2 S 3 2 R 1 2 8 S	.000 .073 .716 .731 .910 .863 1.009 1.020	7.187 6.919 3.008 2.776 2.219 2.145 1.801 1.832	1.307 1.307 1.307 1.305 1.307 1.303 1.307 1.318	.497 .351 .175 .176 .123 .129 .086 .098	.320 .236 .126 .126 .092 .096 .066 .074	.469 .344 .183 .183 .137 .095 .107	.380 .269 .134 .135 .094 .099 .066 .074	.245 .181 .097 .097 .070 .073 .051 .056	.179 .132 .070 .070 .051 .053 .036 .041	3.998 2.822 1.397 1.517 .979 1.179 .636 .822	44.628 23.675 7.977 8.203 5.385 5.657 3.950 3.857 3.136	780 303,810 131,072 65,536 65,536 32,768 32,768 16,384
12/38-12/39	1E 2E 8S 8R 16S 16R 32S 32R	.000 .098 .564 .561 .681 .672 .742 .773	2.830 2.623 1.553 1.654 1.329 1.391 1.229 1.239	.967 .967 .967 .970 .967 .974 .967 .995	.313 .221 .110 .111 .077 .079 .054 .056	.234 .168 .087 .087 .061 .063 .043 .049	.334 .241 .123 .124 .087 .089 .061 .063	.324 .229 .114 .114 .080 .081 .056 .056	.242 .174 .090 .090 .064 .064 .045 .050	.173 .125 .064 .064 .045 .045 .032 .032	1.140 .805 .390 .401 .259 .285 .201 .128	6.735 4.852 3.412 3.420 3.172 3.194 3.105 2.969 2.986	775 299,925 131,072 65,536 65,536 32,768 32,768 16,384
12/39-12/40	1E 2E 8S 8R 16S 16R 32S 32R	.000 .000 .509 .524 .572 .615 .710 .749	2.748 2.559 1.484 1.446 1.235 1.116 1.121	.901 .901 .901 .901 .904 .904 .916	.276 .195 .097 .097 .068 .069 .048 .049	.195 .144 .076 .076 .054 .054 .038 .040	.288 .210 .108 .108 .076 .077 .054 .055	.306 .216 .108 .108 .076 .076 .053 .054	.216 .160 .084 .084 .060 .060 .042 .044	.160 .117 .060 .060 .042 .043 .030 .030	.823 .581 .288 .361 .184 .256 .147 .171	8.584 5.770 3.650 3.681 3.312 3.286 3.178 3.088 2.994	778 302,253 131,072 65,536 65,536 32,768 32,768 16,384
12/40-12/41	1E 2E 8S 8R 16S 16R 32S 32R	.000 .000 .450 .452 .616 .635 .669 .707	2.941 2.689 1.493 1.560 1.306 1.259 1.160 1.132	.898 .898 .898 .898 .897 .897 .898 .902	.312 .221 .110 .109 .077 .076 .054 .052	.227 .166 .086 .085 .061 .060 .043 .041	.330 .239 .122 .122 .087 .085 .061 .058	.348 .246 .122 .121 .086 .085 .060 .057	.253 .184 .096 .095 .068 .067 .048 .046	.184 .133 .068 .068 .048 .048 .034 .032	1.101 .777 .386 .365 .263 .253 .182 .150	7.515 5.239 3.537 3.505 3.253 3.176 3.049 3.054 2.959	788 31C,078 131,072 65,536 65,536 32,768 32,768 16,384 16,384

TABLE A1, CONTINUED

FREQUENCY DISTRIBUTIONS OF MEALTH RATIOS FROM INVESTMENTS IN RANDOWLY SELECTED PORTFOLIOS CONTAINING SPECIFIED NUMBERS OF STOCKS LISTED ON THE NYSE, 1926-65

Number and	Size of Portfolio/				Centi	les of the	Frequency	Distributio	ns			
Length of Periods	Sampling Method	5th	10th	20th	30th	40th	(Median) 50th	60th	70th	80th	90th	95th
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
					ONE-YEA	R PERIODS	(CONTINU	ED)				
12/41-12/42		.844	.907	•986	1.054	1.120	1.190	1.273	1.380	1.527	1.766	2.106
	2E	.940	.990	1.060	1.119	1.175	1.234	1.299	1.376	1.484	1.686	1.965
	8S 8R	1.085	1.122 1.135	1.169 1.184	1.207	1.243 1.259	1.279 1.295	1.320 1.335	1.368	1.436 1.453	1.549 1.563	1.653 1.664
	165	1.141	1.170	1.208	1.238	1.266	1.294	1.325	1.360	1.407	1.479	1.543
	16R	1.170	1.200	1.240	1.271	1.299	1.327	1.359	1.395	1.441	1.511	1.575
	32S 32R	1.184	1.207	1.238	1.260 1.319	1.282	1.303	1.325	1.351 1.405	1.382	1.428 1.477	1.468 1.516
	128S	1.247	1.260	1.276	1.289	1.338 1.300	1.310	1.380 1.320	1.332	1.346	1.364	1.380
12/42-12/43	`1E	1.032	1.088	1.195	1.249	1.331	1.403	1.499	1.624	1.812	2.130	2.560
227 12 227 19	2 E	1.123	1.182	1.260	1.325	1.389	1.457 1.520	1.536 1.569	1.634	1.770 1.707	2.025	2.327 2.022
	8S 8R	1.287	1.330	1.387	1.433	1.476	1.520	1.569	1.627	1.707 1.719	1.852	2.022 2.037
	16S	1.301 1.352	1.342 1.386	1.398 1.432	1.444 1.468	1.486 1.503	1.531 1.537	1.581 1.574	1.639 1.619	1.680	1.868 1.779	1.872
	16R	1.378	1.413	1.459	1.496 1.498	1.532 1.524	1.566	1.604 1.578	1.651	1.715	1.817 1.715	1.906
	325	1.404		1.469	1.498	1.524	1.551		1.611	1.652 1.708	1.715	1.769
	32R 128S	1.454 1.483	1.482 1.499	1.519	1.547 1.534	1.575 1.548	1.602 1.561	1.632 1.575	1.666 1.589	1.607	1.769 1.632	1.826 1.653
12/43-12/44	. 1E	1.029	1.088	1.151	1.202	1.245	1.304	1.372	1.447	1.556	1.750	2.025
12,43 12,44	2E	1.097	1.143	1.200	1.246	1.289	1.334	1.383	1.442	1.526	1.684	1.853
	88	1.212	1.242	1.282	1.312	1.339	1.367	1.397	1.432	1.477	1.548	1.613
	8R 16S	1.215 1.255	1.245 1.279	1.284	1.313	1.340	1.367	1.396 1.396	1.429 1.421	1.472 1.453	1.538 1.500	1.599 1.542
	16R	1.262	1.284	1.309	1.333 1.335	1.353 1.355	1.374	1.396	1.421	1.455	1.488	1.527
	325	1.290	1.308	1.331	1.348	1.364	1.378	1.394	1.411	1.432	1.462	1.488
	32R 128S	1.303 1.336	1.321 1.346	1.341 1.359	1.356 1.368	1.370 1.375	1.385 1.382	1.399 1.390	1.414 1.398	1.433 1.407	1.460 1.421	1.484 1.432
1044 1046											2 074	2 220
12/44-12/45	1E 2E	1.179 1.264	1.238 1.312	1.316 1.377	1.372 1.430	1.427 1.482	1.500 1.535	1.578 1.596	1.672 1.670	1.805 1.772	2.074 1.941	2.329 2.122
	88	1.397	1.432	1.476	1.511	1.543	1.575	1.609	1.650	1.704	1.795	1.882
	8R	1.401	1.435	1.480	1.516	1.549	1.582	1.617	1.657	1.712	1.804	1.894
	16S 16R	1.447 1.457	1.475 1.486	1.510 1.521	1.537 1.549	1.562 1.573	1.586 1.600	1.613 1.627	1.642 1.657	1.680 1.694	1.738 1.752	1.789 1.803
	325	1.490	1.510	1.536	1.557	1.575	1.593	1.611	1.632	1.659	1.697	1.729
	32R 128S	1.513 1.544	1.533 1.555	1.560 1.569	1.581 1.580	1.598 1.589	1.617 1.597	1.635 1.606	1.656 1.616	1.679 1.627	1.714 1.643	1.746 1.657
	1283	1.544	1.000	1. 509	1.580	1.509	1.571	1.000	1.010	1.021	1.043	1.057
12/45-12/46	1E 2E	•529 •636	•592 •692	.698 .760	.785 .810	.847 .854	.898 .895	.944 .935	1.002 .980	1.083	1.187 1.115	1.308 1.190
	85	•766	.794	.830	.855	.877	.898	.920	.943	.971	1.010	1.045
	8R	.772	.799	-832	.856	.878 .885	.898 .900	.919	.941 .931	•968 •951	1.007	1.041
	16S 16R	.806 .813	.826 .832	.851 .855	.869 .871	.886	.900	.915 .914	.930	.949	•978 •975	1.002
	325	.833	.848	.866	.879	.891	.901	•912	•924	.937	. 956	•996 •973
	32R 128S	.842 .870	.854 .876	.870 .885	.882 .891	.892 .896	.901 .902	.910 .907	•920 •912	•932 •918	.949 .928	.964 .935
12/46-12/47		.617	.701	. 795	.857	.910	•962	1.025	1.091	1.175	1.312	1.456 1.322
	2E 8S	.727 .853	.780 .882	.845 .917	.893 .944	•935 •967	•977 •989	1.020 1.012	1.069 1.037	1.131 1.068	1.229 1.113	1.154
	8R	.852	.880 .914	•915	• 941	• 964	•987	1.009	1.035	1.066	1.110	1.149
	165	.894		.940	• 959	.976	•992	1.009	1.027	1.048	1.079	1.106
	16R 32S	.890 .922	•911 •937	•936 •955	•955 •969	.972 .981	.987 .992	1.003 1.004	1.020 1.016	1.041 1.031	1.071 1.052	1.097 1.070
	32R	•920	.933	•950	.963	.974	.984	• 995	1.006	1.019	1.038	1.054
	1285	.959	•966	•976	.983	•989	.994	1.000	1.006	1.013	1.023	1.030
12/47-12/48		•600	.703	.798	.858	.916	.967	1.007	1.050	1.107	1.212	1.321
	2E	.715	•772	• 8 4 0	•886 922	•924 •942	.959 .962	.994 .981	1.032	1.079	1.156	1.238
	8S 8R	.839 .842	•865 •868	.898 .900	•922 •923	.942	.962	.981	1.003	1.029	1.072	1.113
	165	.875	.894	.918	.935	•950	•964	•978	- 994	1.014	1.045	1.076
	16R	.879	.898	.920	.937	.951	• 965	.979	.995 .988	1.014 1.002	1.044	1.074 1.049
	32S 32R	•901 •904	•915 •917	.932 .933	.944 .944	•955 •954	•966 •964	•976 •974	.988 .985	.999	1.026	1.049
	1285	.934	•942	.951	.957	.963	.968	.974	.980	.999 .987	.998	1.007
12/48-12/49		.840	•906	•997	1.061	1.122	1.178	1.233	1.302	1.374	1.481	1.617
	2E	.930 1.055	.986 1.085	1.052 1.121	1.100 1.146	1.143 1.169	1.183 1.190	1.224 1.211	1.269 1.235	1.325 1.265	1.411 1.308	1.495 1.347
	8 S. 8 R	1.055	1.085	1.121	1.146	1.171	1.192	1.211	1.238	1.266	1.309	1.347
	165	L.094	1.115	1.141	1.161	1.177	1.192	1.208	1.225	1.246	1.276	1.302
	16R	1.103	1.122	1.148	1.166	1.181	1.196	1.212	1.229	1.249	1.279	1.304
	32S 32R	1.123	1.138 1.155	1.157 1.173	1.171 1.185	1.182 1.197	1.193 1.207	1.204 1.218	1.217 1.230	1.231 1.243	1.252 1.263	1.269 1.281
	1285	1.160	1.167	1.176	1.183	1.189	1.194	1.199	1.205	1.212	1.221	1.229

Number and Longth of Periods	Size of Portfolio and Sampling Method	Sample Minimum	Sample Maximum	Arithmetic Mean	Standard Deviation	Wean Deviation	Gini's Mean Difference	Coefficient of Variation	Relative Mean Deviation	Coefficient of Concentration	Skevness	Kurtosis	Number of Portfolios Examined
(14)	(15)	(16)	(17)	(18)	(19) ONE-YEAR	(20)	(21) (CONTINUE	(22) (0)	(23)	(24)	(25)	(26)	(27)
12/41-12/42	1E 2E 8S 8R 16S 16R 32S 32R 128S	.560 .584 .883 .913 .985 .979 1.055 1.112	5.907 5.268 2.496 2.638 2.155 2.148 1.840 1.778 1.517	1.311 1.311 1.312 1.327 1.312 1.344 1.312 1.366	.501 .354 .177 .176 .124 .125 .087 .084	.320 .242 .135 .133 .097 .098 .069 .078	.452 .344 .190 .191 .137 .138 .097 .094	.382 .270 .135 .133 .095 .093 .066 .062	.244 .184 .103 .100 .074 .073 .053 .057	.172 .131 .073 .072 .052 .051 .037 .034	3.358 2.370 1.148 1.120 .820 .765 .570 .491	21.015 11.947 4.989 5.007 4.007 3.954 3.458 3.452 2.995	797 317,206 131,072 65,536 65,536 32,768 32,768 16,384
12/42-12/43	1E 2E 8S 8R 16S 16R 32S 32R	.293 .459 1.063 1.035 1.169 1.182 1.258 1.311	7.469 7.459 3.513 3.128 2.609 2.570 2.158 2.343 1.792	1.564 1.564 1.565 1.577 1.564 1.594 1.564	.644 .455 .228 .228 .161 .163 .112 .114	.389 .299 .169 .168 .124 .124 .088 .095	.550 .424 .240 .240 .175 .178 .125 .127	.412 .291 .146 .145 .103 .102 .072 .070	.249 .191 .108 .106 .079 .078 .056 .059	.176 .136 .077 .076 .056 .056 .040 .039	4.134 2.918 1.454 1.425 1.039 .946 .673 .630	30.193 16.507 6.316 6.084 4.672 4.202 3.631 3.482 3.081	800 319,600 131,072 65,536 65,536 32,768 32,768 16,384
12/43-12/44	1E 2E 8S 8R 16S 16R 32S 32R	.417 .417 .992 .979 1.091 1.105 1.186 1.199	4.389 3.850 2.216 2.151 1.955 1.828 1.681 1.635 1.497	1.383 1.383 1.384 1.382 1.383 1.382 1.383 1.388 1.388	.353 .250 .125 .119 .088 .081 .061 .055	.237 .178 .096 .093 .069 .064 .048	.339 .255 .137 .131 .098 .090 .068 .062	.256 .181 .090 .086 .064 .059 .044 .039	.171 .129 .070 .067 .050 .046 .035 .032	.123 .092 .049 .047 .035 .033 .025 .022	2.451 1.730 .854 .783 .618 .524 .401 .335 .153	14.494 8.708 4.337 4.185 3.708 3.497 3.268 3.078 3.001	810 327,645 131,072 65,536 65,536 32,768 32,768 16,384
12/44-12/45	1E 2E 8S 8R 16S 16R 32S 32R	.649 .691 1.170 1.155 1.275 1.248 1.369 1.399	4.700 4.606 2.581 2.601 2.205 2.236 2.005 1.952	1.598 1.598 1.597 1.604 1.598 1.611 1.599 1.621 1.598	.422 .299 .149 .150 .105 .106 .074 .071	.283 .212 .114 .115 .082 .083 .058 .058	.400 .301 .162 .164 .116 .118 .082 .080	.264 .187 .093 .094 .065 .066 .046	.177 .133 .072 .072 .052 .052 .036 .036	.125 .094 .051 .051 .036 .037 .026 .025	2.807 1.982 .974 .934 .657 .622 .484 .395	16.262 9.587 4.518 4.343 3.678 3.587 3.351 3.176 3.028	826 340,725 131,072 65,536 65,536 32,768 32,768 16,384 16,384
12/45-12/46	1E 2E 8S 8R 16S 16R 32S 32S 32R 128S	.254 .271 .537 .604 .674 .679 .740 .762	2.230 2.113 1.345 1.329 1.189 1.160 1.078 1.087	.901 .901 .901 .901 .902 .902 .902	.242 .171 .085 .082 .059 .056 .042 .037	.184 .133 .067 .065 .047 .044 .033 .029	.266 .190 .096 .092 .067 .063 .047 .042	.268 .189 .094 .090 .066 .062 .047 .041	.204 .148 .075 .072 .052 .049 .037 .033	.147 .105 .053 .051 .037 .035 .026 .023	.609 .430 .225 .227 .159 .159 .093 .145	4.739 3.861 3.213 3.164 3.106 3.060 3.028 3.052 2.965	853 363,378 131,072 65,536 65,536 32,768 32,768 16,384 16,384
12/46-12/47	1E 2E 8S 8R 16S 16R 32S 32R	.348 .369 .663 .771 .767 .824 .831	2.577 2.342 1.592 1.455 1.298 1.297 1.235 1.174	.994 .994 .994 .992 .995 .989 .994	.260 .184 .092 .091 .064 .063 .045 .041	.195 .142 .072 .072 .051 .050 .036 .034	.280 .202 .103 .102 .073 .070 .051 .046	.262 .185 .092 .092 .065 .063 .045 .041	.196 .142 .073 .073 .052 .051 .036 .034	.141 .102 .052 .051 .036 .036 .026 .023	1.031 .728 .374 .342 .252 .213 .195 .155	5.812 4.395 3.364 3.287 3.141 3.102 3.082 3.006 2.988	904 408,156 131,072 65,536 65,536 32,768 32,768 16,384
12/47-12/48	1E 2E 8S 8R 16S 16R 32S 32R	.337 .342 .634 .594 .738 .749 .801 .815	4.544 3.691 1.776 1.675 1.411 1.400 1.202 1.194	.969 .969 .969 .969 .970 .969 .967	.259 .183 .092 .090 .064 .062 .045 .041	.169 .126 .067 .066 .048 .047 .035 .032	.250 .184 .097 .095 .070 .067 .050	.268 .189 .094 .093 .067 .064 .047 .043	.175 .130 .069 .068 .050 .048 .036 .033	.129 .095 .050 .049 .036 .035 .026 .024	3.571 2.521 1.258 1.280 .913 .817 .607 .484	44.575 23.673 8.080 8.075 5.552 5.088 4.046 3.737	939 440,391 131,072 65,536 65,536 32,768 32,768 16,384
12/48-12/49	1E 2E 8S 8R 16S 16R 32S 32R	.095 .236 .807 .851 .947 .966 1.033 1.054	2.885 2.845 1.737 1.666 1.539 1.524 1.391 1.413	1.194 1.194 1.194 1.196 1.194 1.199 1.194 1.208 1.194	.254 .180 .089 .088 .063 .062 .044 .042	.189 .136 .070 .069 .050 .049 .035 .035	.271 .196 .099 .099 .071 .069 .050 .047	.213 .150 .075 .074 .053 .051 .037	.158 .114 .059 .058 .042 .041 .029 .029	.114 .082 .042 .041 .030 .029 .021 .020	.967 .683 .345 .329 .231 .285 .170 .209	7.704 5.337 3.572 3.535 3.281 3.395 3.082 3.137 3.016	963 463,2C3 131,072 65,536 65,536 32,768 32,768 16,384 16,384

TABLE A1, CONTINUED

FREQUENCY DISTRIBUTIONS OF MEALTH RATIOS FROM INVESTMENTS IN RANDOWLY SELECTED PORTFOLIOS
CONTAINING SPECIFIED NUMBERS OF STOCKS LISTED ON THE NYSE, 1926-65

Number and	Size of Portfolio/				Cent	iles of the	Frequency	Distributio	ons			
Length of Periods	Sampling Method	5th	10th	20th	30th	40th	(Median) 50th	60th	70th	80th	90th	95th
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
					ONE-YEAR	R PERIODS	(CONTINU	ED)				
12/49-12/50	16	.904	•965	1.047	1.123	1.207	1.298	1.384	1.487	1.614	1.817	2.015
	2E 8S	1.000 1.161	1.057 1.198	1.138 1.246	1.203 1.283	1.263 1.315	1.322 1.347	1.385 1.380	1.457 1.417	1.548 1.463	1.694 1.532	1.836 1.595
	8 R	1.163	1.200	1.246	1.281	1.312	1.342	1.373	1.408	1.452	1.516	1.576
	16S 16R	1.215 1.212	1.243 1.238	1.278 1.272	1.305 1.296	1.329 1.317	1.352 1.338	1.376 1.359	1.403 1.382	1.435 1.410	1.481 1.452	1.523 1.489
	32S 32R	1.256	1.276 1.255	1.302 1.278	1.322	1.338	1.355 1.322	1.371 1.336	1.390 1.351	1.413	1.444 1.394	1.471 1.416
	1285	1.308	1.319	1.332	1.342	1.350	1.358	1.366	1.375	1.385	1.399	1.412
12/50-12/51	16	.832	•905	.981	1.039	1.081	1.122	1.172	1.228	1.294	1.419	1.554
	2E 8S	.909 1.022	.963 1.049	1.022 1.081	1.064 1.104	1.100 1.124	1.134 1.143	1.170 1.163	1.211 1.185	1.264 1.211	1.348 1.252	1.426 1.289
	8R	1.023	1.049	1.080	1.103	1.123	1.142	1.161	1.183	1.209	1.248	1.283
	16S 16R	1.057 1.058	1.076 1.076	1.099 1.097	1.116 1.114	1.131 1.128	1.145 1.142	1.159 1.156	1.175 1.171	1.194 1.189	1.223 1.216	1.250 1.239
	32S 32R	1.083	1.097 1.093	1.113 1.109	1.125 1.119	1.136 1.129	1.147 1.138	1.157 1.147	1.168 1.157	1.182 1.169	1.203 1.186	1.221 1.199
	1285	1.116	1.123	1.132	1.138	1.143	1.148	1.153	1.159	1.166	1.175	1.184
12/51-12/52	1E	•771	.841	•920	• 980	1.037	1.088	1.137	1.184	1.245	1.333	1.424
	2E 8S	.864 .973	.912 .999	.971 1.029	1.014 1.051	1.050 1.070	1.085 1.088	1.120 1.106	1.158 1.125	1.203 1.148	1.269 1.180	1.329 1.208
	8R 16S	.976 1.006	1.001 1.024	1.030	1.051 1.062	1.070 1.075	1.086 1.088	1.104 1.101	1.122 1.115	1.145 1.130	1.175 1.153	1.201 1.171
	16R	1.007	1.024	1.044	1.060	1.072	1.084	1.096	1.108	1.124	1.145	1.162
	32S 32R	1.031	1.043	1.059	1.070	1.079 1.068	1.088	1.097	1.107	1.118	1.134 1.118	1.147
	1285	1.060	1.066	1.073	1.079	1.084	1.088	1.093	1.097	1.103	1.110	1.117
12/52-12/53	1E 2F	.602	•704	.805	.864 .891	.918 .930	•969 •965	1.021	1.064	1.112	1.221 1.154	1.320 1.220
	8S	.727 .845	•781 •872	.845 .904	.928	-947	.966	•985	1.005	1.030	1.065	1.095
	8R 16S	.848 .881	.873 .900	•905 •922	•928 •939	.947 .953	•966 •967	•984 •980	1.005 .995	1.029 1.012	1.065 1.037	1.095 1.057
	16R	.883	•902	.924	. 940	. 954	•967	•981	.996	1.013	1.037	1.057
	32S 32R	.906 .918	.920 .931	•936 •946	.948 .957	•958 •967	•967 •975	•977 •984	•987 •994	.999 1.006	1.016 1.023	1.030 1.036
	1285	• 939	.945	•953	.959	.964	•968	.973	•978	•984	•992	.999
12/53-12/54	1 E 2 E	1.095 1.193	1.164 1.254	1.251 1.333	1.327 1.394	1.407 1.451	1.480 1.507	1.565 1.568	1.660 1.639	1.786 1.733	1.995 1.890	2.256 2.035
	88	1.349	1.386	1.434	1.470	1.503	1.535	1.568	1.605	1.651	1.722	1.791
	8R 16S	1.355 1.404	1.392 1.431	1.439 1.466	1.474 1.493	1.506 1.518	1.536 1.541	1.569 1.565	1.605 1.591	1.651 1.624	1.720 1.674	1.787 1.720
	16R	1.412	1.438	1.473 1.491	1.499 1.511	1.522 1.528	1.544 1.545	1.567 1.562	1.593 1.580	1.625 1.604	1.672 1.637	1.720 1.718 1.667
	32S 32R	1.457	1.465 1.476	1.500	1.519	1.535	1.551	1.566	1.585	1.607	1.639	1.666
	1285	1.496	1.507	1.520	1.530	1.539	1.547	1.556	1.565	1.575	1.590	1.604
12/54-12/55	1 E 2 E	.829 .932	•924 •983	1.003 1.042	1.049	1.093 1.123	1.144 1.161	1.193 1.204	1.261 1.256	1.354	1.516 1.432	1.687 1.544
	88	1.047	1.075	1.110	1.136	1.159	1.182	1.205	1.232	1.266	1.315	1.358
	8R 16S	1.047 1.086	1.075 1.107	1.108 1.133	1.134 1.153	1.156 1.170	1.179 1.186	1.203 1.203	1.229 1.222	1.262 1.244	1.311 1.278	1.305
	16R 32S	1.082	1.103 1.130	1.128 1.149	1.148 1.163	1.165 1.176	1.181 1.187	1.198 1.199	1.217 1.212	1.239 1.228	1.272 1.250	1.301 1.269
	32R	1.115 1.112	1.128	1.147	1.161	1.173	1.185	1.197	1.210	1.225	1.249	1.267
	1285	1.153	1.161	1.170	1.177	1.183	1.189	1.195	1.201	1.209	1.219	1.228
12/55-12/56	1E 2E	.711 .805	.789 .859	.880 .923	•941 •968	.989 1.007	1.026	1.082 1.087	1.141	1.236 1.197	1.378	1.518
	85	•926 •924	•954 •952	.989 .987	1.015	1.037	1.059	1.081	1.106	1.136 1.128	1.180	1.221
	8R 16S	965	.986	1.011	1.030	1.046	1.061	1.077	1.094	1.116	1.147	1.177
	16R	.960 .993	.980 1.008	1.004 1.027	1.022	1.037 1.051	1.051	1.066 1.074	1.083 1.086	1.101 1.101	1.130 1.124	1.155 1.145
	32S 32R	• 985	.999	1.015	1.027	1.037	1.047	1.057	1.068	1.080	1.099	1.115
	1285	1.031	1.037	1.047	1.054	1.060	1.065	1.071	1.077	1.084	1.095	1.104
12/56-12/57	1E 2E	•496 •597	•567 •652	.658 .721	•721 •773	•791 •817	.856 .859	•923 •901	•996 •947	1.060 1.001	1.149	1.228 1.139
	88	•729	.757	.793	.819 .822	.841 .843	.862 .863	.883 .883	.906 .906	.933 .932	.973 .971	1.008 1.005
	8R 16S	•739 •767	.765 .788	.798 .814	.832	.848	.863	.878	.894	.914	.941	965
	16R 32S	•780 •797	.799 .811	.822 .829	.838 .842	.853 .853	•867 •864	.881 .875	.896 .886	.915	.941 .919	•965 •935
	32R	.816	•829	.845	• 856	. 866	.875	.885	.896	.908	•926	.940 .898
	1285	.831	.839	.847	.853	.859	.864	.869	.875	.882	.891	. 898

Number and Length of Periods	Size of Portfolio and Sampling Method	Sample Minimum	Sample Maximum	Arithmetic Mean	Standard Deviation	Mean Deviation	Gini's Mean Difference	Coefficient of Variation	Relative Mean Deviation	Coefficient of Concentration	Skewness	Kurtosis	Number of Portfolios Examined
(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
					ONE-YEAR	PERIODS	(CONTINUE	D)					
12/49-12/50	1E 2E 8S 8R 16S 16R 32S 32R 128S	.645 .650 .936 .909 1.053 1.028 1.124 1.128	3.917 3.915 2.186 2.041 2.044 1.765 1.660 1.555	1.358 1.358 1.358 1.358 1.358 1.343 1.358 1.359	.378 .267 .133 .126 .094 .084 .066 .054	.283 .204 .105 .100 .075 .068 .052 .052	.397 .289 .148 .141 .106 .094 .074 .061	.279 .197 .098 .093 .069 .062 .048 .041	.208 .150 .077 .074 .055 .051 .039 .039	.146 .106 .055 .052 .039 .035 .027 .023	1.622 1.145 .555 .506 .401 .317 .283 .187	8.628 5.797 3.641 3.550 3.376 3.221 3.153 3.053 3.030	990 489,555 131.072 65,536 65,536 32,768 32,768 16,384
12/50-12/51	1E 2E 8S 8R 16S 16R 32S 32R 128S	.135 .311 .780 .826 .892 .962 .984 .996	4.047 3.253 1.765 1.815 1.574 1.496 1.415 1.335 1.246	1.149 1.149 1.148 1.147 1.149 1.145 1.149 1.139	.242 .171 .085 .083 .060 .057 .042 .037	.168 .124 .065 .064 .046 .044 .033 .030	.245 .179 .093 .091 .066 .063 .047 .041	.211 .149 .074 .072 .052 .049 .037 .032	.146 .108 .056 .055 .040 .039 .029 .026	.107 .078 .040 .040 .029 .027 .020 .018	2.249 1.588 .766 .734 .551 .462 .412 .261	24.699 13.792 5.465 5.462 4.258 4.059 3.632 3.525 3.075	1,010 509,545 131,072 65,536 65,536 32,768 32,768 16,384
12/51-12/52	1E 2E 8S 8R 16S 16R 32S 32R	.113 .291 .737 .769 .884 .912 .952 .938	1.866 1.832 1.401 1.416 1.334 1.285 1.261 1.211	1.089 1.089 1.089 1.087 1.088 1.084 1.088 1.076	.201 .142 .071 .069 .050 .047 .035 .032	.157 .112 .057 .054 .040 .038 .028 .027	.223 .159 .080 .077 .056 .053 .040 .036	.185 .131 .065 .063 .046 .043 .032 .029	.144 .103 .052 .050 .037 .035 .026 .025	.103 .073 .037 .036 .026 .024 .018 .017	.207 .146 .070 .053 .031 .052 .032 .017	3.915 3.452 3.082 3.131 3.038 3.037 2.999 2.960 2.994	1,029 528,906 131,072 65,536 65,536 32,768 32,768 16,384
12/52-12/53	1E 2E 8S 8R 16S 16R 32S 32R	.000 .174 .662 .650 .755 .776 .816 .848	2.135 1.990 1.384 1.382 1.195 1.234 1.124 1.122	.968 .968 .968 .968 .967 .969 .968 .976	.215 .152 .076 .075 .054 .053 .037 .036	.162 .117 .060 .059 .043 .042 .030 .029	.234 .168 .085 .084 .060 .060 .042 .040	.222 .157 .079 .078 .055 .055 .039 .037	.168 .121 .062 .061 .044 .044 .031 .030	.121 .087 .044 .044 .031 .031 .022 .021	•389 •274 •150 •170 •094 •117 •071 •123 •031	4.939 3.962 3.237 3.225 3.061 3.078 3.069 3.086 3.022	1,044 544,446 131,072 65,536 65,536 32,768 32,768 16,384
12/53-12/54	1E 2E 8S 8R 16S 16R 32S 32R	.608 .659 1.078 1.098 1.238 1.267 1.331 1.326	5.441 4.841 2.485 2.462 2.120 2.061 1.989 1.930 1.699	1.548 1.548 1.548 1.550 1.548 1.552 1.548 1.555 1.548	.392 .277 .138 .136 .097 .094 .068 .065	.279 .204 .106 .104 .076 .073 .054 .051	.397 .291 .151 .149 .108 .105 .076 .072	.253 .179 .089 .088 .063 .061 .044 .042	.180 .132 .069 .067 .049 .047 .035 .033	.128 .094 .049 .048 .035 .034 .025 .023	2.205 1.557 .770 .842 .557 .595 .368 .424	15.744 9.338 4.508 4.884 3.786 3.913 3.333 3.488 3.096	1,045 545,490 131,072 65,536 65,536 32,768 32,768 16,384
12/54-12/55	1E 2E 8S 8R 16S 16R 32S 32R	.163 .357 .830 .842 .946 .962 1.026 1.032	2.886 2.747 1.731 1.757 1.577 1.520 1.415 1.404	1.190 1.190 1.187 1.187 1.185 1.189 1.187 1.190	.270 .191 .095 .094 .067 .067 .047 .047	.194 .144 .075 .074 .053 .053 .053 .037	.280 .206 .106 .105 .075 .075 .053 .053	.227 .160 .080 .079 .056 .056 .039 .040	.163 .121 .063 .063 .045 .045 .031 .032	.118 .086 .045 .044 .032 .032 .022 .022	1.391 .982 .481 .490 .332 .328 .239 .208	7.370 5.172 3.485 3.536 3.263 3.221 3.139 3.098 2.973	1,052 552,826 131,072 65,536 65,536 32,768 32,768 16,384
12/55-12/56	1E 2E 8S 8R 16S 16R 32S 32R	•142 •221 •709 •741 •769 •807 •902 •902	4.282 3.510 1.852 1.730 1.452 1.398 1.303 1.271	1.065 1.065 1.065 1.060 1.065 1.054 1.065	.268 .189 .094 .091 .066 .060 .046 .040	.188 .138 .072 .070 .051 .049 .036 .035	.273 .199 .103 .099 .073 .067 .051 .044	.251 .178 .089 .086 .062 .057 .043 .038	.176 .130 .068 .066 .048 .046 .034 .033	.128 .093 .048 .047 .034 .032 .024 .021	2.342 1.654 .796 .738 .581 .409 .371 .253	24.249 13.570 5.413 5.372 4.296 3.966 3.535 3.420 3.096	1,055 555,985 131,072 65,536 65,536 32,768 32,768 16,384
12/56-12/57	1E 2E 8S 8R 16S 16R 32S 32R	.268 .274 .534 .563 .637 .658 .697 .761	2.266 2.242 1.313 1.379 1.179 1.137 1.050 1.027	.864 .864 .866 .866 .869 .865 .877	.242 .171 .085 .081 .060 .056 .042 .038	.191 .134 .067 .064 .048 .044 .033 .031	.268 .190 .095 .091 .068 .063 .047 .042	.280 .198 .098 .094 .069 .065 .049 .043	.221 .155 .078 .074 .055 .051 .039 .036	.155 .110 .055 .053 .039 .036 .027 .024	.638 .451 .226 .277 .158 .227 .114 .167	5.395 4.189 3.297 3.391 3.161 3.197 3.073 2.990 3.013	1,056 557,040 131,072 65,536 65,536 32,768 32,768 16,384

TABLE A1, CONTINUED

FREQUENCY DISTRIBUTIONS OF MEALTH RATIOS FROM INVESTMENTS IN RANDOMLY SELECTED PORTFOLIOS
CONTAINING SPECIFIED NUMBERS OF STOCKS LISTED ON THE MYSE, 1926-65

Number and	Size of Portfolio	,			Cent	iles of the	Frequency	Distributi	ons			
Length of Periods	Sampling Method	5th	10th	20th	30th	40th	(Median) 50th	60th	70th	80th	90th	95 th
<u> </u>												
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
					ONE-YEA	R PERIODS	(CONTINU	ED)				
12/57 12/59										. 700		4
12/57-12/58	1E 2E	1.112 1.223	1.199 1.282	1.294 1.357	1.363 1.413	1.434 1.464	1.491 1.517	1.552 1.575	1.647 1.646	1.780 1.744	2.001 1.922	2.326 2.145
	8S 8R	1.373	1.407	1.453	1.488	1.521	1.554	1.590	1.633	1.690	1.786	1.875
	165	1.373 1.423	1.409 1.450	1.456 1.486	1.491 1.514	1.523 1.539	1.556 1.565	1.591 1.592	1.634 1.623	1.691 1.664	1.785 1.724	1.873 1.778
	16R	1.428	1.455	1.491	1.518	1.544	1.569	1.595	1.627	1.665	1.723	1.776
	32S 32R	1.464	1.485 1.490	1.513 1.516	1.534 1.536	1.553 1.553	1.572 1.570	1.592 1.589	1.614	1.641	1.680 1.669	1.716 1.699
	1285	1.520	1.532	1.547	1.558	1.568	1.577	1.586	1.597	1.609	1.626	1.641
12/58-12/59	16	.759	.845	•934	.988	1.028	1.080	1.149	1.214	1.323	1.527	1.715
12/30-12/37	2E	.862	•914	.977	1.022	1.064	1.107	1.155	1.213	1.290	1.418	1.552
	8S 8R	•986 •989	1.016 1.018	1.053	1.082	1.107	1.133	1.160	1.191	1.230	1.289	1.342 1.342
	165	1.030	1.051	1.054 1.079	1.083 1.101	1.109 1.120	1.134 1.139	1.161 1.158	1.191 1.180	1.230 1.207	1.290 1.246	1.279
	16R 32S	1.031 1.060	1.053 1.077	1.082	1.103	1.122	1.140	1.159	1.181	1.207	1.245	1.279 1.239
	32R	1.068	1.084	1.104	1.115 1.120	1.128 1.133	1.142 1.145	1.156 1.158	1.171 1.171	1.189 1.189	1.215 1.213	1.235
	1285	1.102	1.111	1.122	1.130	1.136	1.143	1.150	1.157	1.166	1.178	1.187
12/59-12/60	16	•591	•652	•749	.825	.887	•948	1.022	1.100	1.196	1.325	1.447
	2E	•692	•746	.816	.870	.919	•965	1.013	1.067	1.132	1.229	1.319
	8S 8R	.828 .835	.859 .864	.897 .901	•926 •929	•951 •953	•976 •976	1.000 1.000	1.027 1.025	1.059 1.057	1.107 1.103	1.148 1.143
	165	.871	A894	•923	• 943	.961	•978	4996	1.015	1.038	1.071	1.099
	16R 32S	.878 .902	.900 .919	•927 •940	•946 •955	•963 •968	.979 .980	•996 •992	1.014 1.005	1.036 1.021	1.069 1.043	1.095 1.062
	32R	.918	.934	.953	•967	.980 .975	• 992	1.003	1.016	1.031	1.052	1.070
	1285	•943	•951	• 961	.968	.975	•981	.987	•993	1.000	1.010	1.019
12/60-12/61	16	•852	•935	1.037	1.114	1.176	1.235	1.301	1.373	1.460	1.621	1.818
	2E 8S	•964 1•106	1.025 1.140	1.099 1.181	1.154 1.211	1.201 1.238	1.247 1.265	1.296 1.292	1.352 1.324	1.424 1.364	1.547 1.426	1.685 1.485
	8R	1.110	1.143	1.185	1.215	1.243	1.270	1.298	1.330	1.370	1.435	1.495
	16S 16R	1.152 1.162	1.176 1.187	1.207 1.219	1.230 1.242	1.250 1.263	1.269 1.283	1.290 1.304	1.313 1.327	1.341	1.384 1.400	1.422 1.440
	32 S	1.186	1.204	1.227	1.244	1.259	1.273	1.288	1.304	1.323	1.351	1.376
	32R 128S	1.214	1.232 1.241	1.255	1.273 1.261	1.288 1.268	1.303 1.275	1.318	1.334	1.355	1.384 1.311	1.411 1.322
	1200	1.1.51				14100		10101	20207	24270		
12/61-12/62	1E 2E	•544 •634	•616 •684	•696 •745	•761 •788	.813 .825	.856 .860	.909 .896	•965 •934	1.019 .980	1.114 1.048	1.207 1.110
	8S	•747	.773	.803	.826	.845	.863	.881	.901	•925	.958	.986
	8R	•748	•773	.803	. 825	.844 .851	.862 .864	.880 .877	.899 .891	.923 .908	.957 .931	•985 •951
	16S 16R	•781 •779	.800 .797	.821 .819	.837 .834	.848	.861	-874	.888	7.905	.928 .912	.947 .926
	32 S	.805	.818	.834	.845 .840	.855 .850	•864 •859	.874	.883 .878	.895 .889	.912	•926 •917
	32R 128S	.802 .835	.814 .842	.829 .850	.855	.860	.864	.868 .869	.874	.879	.904 .887	.894
									. 247	1 244	1.488	1 457
12/62-12/63	1E 2E	.808 .902	.894 .960	.985 1.026	1.041 1.072	1.089 1.111	1.137 1.150	1.186 1.192	1.247 1.241	1.344	1.417	1.657 1.539
	88	1.026	1.056	1.093	1.120	1.144	1.167	1,191	1.218	1.253	1.307	1.358
	8R 16S	1.027 1.067	1.058 1.089	1.094 1.116	1.121 1.136	1.146 1.154	1.169 1.171	1.194 1.189	1.222 1.209	1.257 1.233	1.312 1.269	1.363 1.302
	16R	1.072	1.093	1.121	1.142	1.159	1.177	1.195	1.216	1.240	1.277	1.308
	32S 32R	1.099	1.114 1.130	1.134 1.150	1.149 1.165	1.161 1.178	1.174 1.189	1.186 1.203	1.200 1.216	1.217 1.233	1.241 1.256	1.262 1.276
	1285	1.137	1.145	1.155	1.163	1.169	1.175	1.182	1.188	1.197	1.208	1.217
12/63-12/64	16	.765	.875	•978	1.049	1.099	1.142	1.194	1.248	1.323	1.466	1.622
12,05 12,01	2E	.883	.947	1.021	1.070	1.111	1.149	1.188	1.233	1.292	1.390	1.491
	8S 8R	1.018 1.021	1.048 1.051	1.085 1.088	1.112 1.115	1.136 1.138	1.158	1.180 1.184	1.205 1.210	1.236 1.240	1.283 1.287	1.324 1.328
	165	1.059	1.081	1.107	1.127	1.144	1.160	1.177	1.195	1.217	1.249	1.276
	16R 32S	1.065 1.090	1.088 1.105	1.115	1.135	1.152 1.150	1.169 1.162	1.185 1.174	1.204 1.186	1.226 1.202	1.257 1.223	1.285 1.241
	32R	1.112	1.128	1.147	1.138	1.173	1.184	1.196	1.209	1.223	1.244	1.261
	1285	1.126	1.134	1.144	1.151	1.157	1.163	1.169	1.175	1.182	1.192	1.200
12/64-12/65	16	. 856	.923	.995	1.056	1.116	1.196	1.275	1.387	1.510	1.732	1.963
	2E 8S	.937 1.081	.991 1.117	1.062 1.163	1.120 1.200	1.175 1.232	1.231 1.264	1.292 1.299	1.364	1.459	1.626 1.467	1.817 1.543
	8R	1.088	1.123	1.168	1.203	1.235	1.266	1.300	1.339	1.388	1.464	1.535
	165	1.134	1.161	1.197 1.202	1.224 1.228	1.248 1.251	1.273 1.274	1.297 1.298	1.326 1.325	1.360 1.359	1.413	1.464 1.455
	16R 32S	1.141 1.174	1.168 1.195	1.222	1.243	1.261	1.278	1.296	1.316	1.340	1.409 1.376	1.409
	32R 128S	1.180 1.227	1.199 1.239	1.223 1.253	1.241 1.263	1.257 1.272	1.272 1.281	1.288 1.290	1.305 1.300	1.326 1.311	1.357 1.328	1.385 1.341
	1202	1.221	10237	10233	10403	10212	10501	10270	1.500			

Number and Length of Periods	Size of Portfolio and Sampling Method	Sample Minimum	Sample Maximum	Arithmetic Mean	Standard Deviation	Mean Deviation	Gini's Mean Difference	Coefficient of Variation	Relative Mean Deviation	Concentration	Skewness	Kurtosis	Number of Portfolios Examined
(14)	(15)	(16)	(17)	(18)	(19) ONE-YEAR	(20) PERIODS	(21) (CONTINUE	(22) ED)	(23)	(24)	(25)	(26)	(27)
12/57-12/58	1E 2E 8S 8R 16S 16R 32S 32R	.803 .817 1.136 1.157 1.261 1.247 1.319 1.356	5.077 4.951 2.644 2.580 2.214 2.141 1.957 1.870	1.579 1.579 1.579 1.580 1.578 1.581 1.579 1.576	.440 .311 .155 .153 .109 .106 .077 .070	.285 .217 .119 .118 .086 .084 .061 .056	.412 .311 .168 .167 .121 .118 .086 .079	.279 .197 .098 .097 .069 .067 .049 .045	.181 .137 .075 .075 .054 .053 .039 .036	.131 .098 .053 .053 .038 .037 .027 .025	2.873 2.029 .983 .952 .713 .624 .493 .400	16.763 9.846 4.532 4.407 3.851 3.559 3.347 3.212 3.100	1,077 579,426 131,072 65,536 65,536 32,768 32,768 16,384
12/58-12/59	1E 2E 8S 8R 16S 16R 32S 32R	.428 .464 .794 .802 .888 .887 .947 .990	3.372 3.147 1.875 1.739 1.541 1.531 1.428 1.362	1.144 1.144 1.146 1.146 1.145 1.145 1.147	.310 .219 .109 .108 .076 .076 .054 .051	.219 .163 .086 .085 .061 .060 .043 .040	.314 .232 .121 .120 .086 .085 .061 .057	.271 .191 .096 .095 .067 .066 .047 .044	.191 .142 .075 .074 .053 .052 .038 .035	.137 .102 .053 .053 .037 .037 .027 .025	1.824 1.288 .639 .630 .442 .448 .309 .289	9.521 6.242 3.793 3.714 3.322 3.395 3.197 3.142 2.958	1,067 568,711 131,072 65,536 65,536 32,768 32,768 16,384
12/59-12/60	1E 2E 8S 8R 16S 16R 32S 32R	.253 .322 .631 .676 .702 .736 .803 .813	2.380 2.360 1.457 1.481 1.299 1.323 1.179 1.173	.981 .981 .980 .981 .981 .982 .981	.276 .195 .097 .094 .069 .066 .048 .046	.215 .153 .077 .075 .055 .052 .038 .038	.303 .217 .109 .106 .078 .074 .054 .052	.282 .199 .099 .096 .070 .067 .049 .046	.219 .156 .079 .076 .056 .053 .039 .038	.154 .110 .056 .054 .040 .038 .028 .026	.886 .626 .298 .328 .213 .252 .138 .135	5.001 3.993 3.203 3.262 3.110 3.141 2.971 3.000 2.907	1.088 591.328 131.072 65.536 65.536 32.768 32.768 10.384
12/60-12/61	1E 26 8S 8R 16S 16R 32S 32R	.000 .247 .845 .883 .997 .998 1.083 1.114	3.810 3.785 2.154 2.036 1.838 1.844 1.616 1.605	1.276 1.276 1.276 1.281 1.276 1.290 1.276 1.306 1.275	.330 .233 .116 .118 .083 .085 .058 .060	.229 .170 .090 .091 .065 .067 .046	.335 .246 .128 .131 .092 .095 .065 .067	.259 .183 .091 .092 .065 .066 .045	.180 .133 .071 .071 .051 .052 .036 .040	.131 .096 .050 .051 .036 .037 .025 .026	1.885 1.331 .660 .671 .477 .494 .322 .355	12.084 7.519 4.099 4.059 3.567 3.633 3.228 3.278 3.035	1.119 625.521 131.072 65.536 65.536 32.768 32.768 16.384
12/61-12/62	1E 2E 8S 8R 16S 16R 32S 32R 128S	.146 .203 .552 .535 .653 .670 .709 .711	1.741 1.737 1.232 1.228 1.090 1.096 1.022 1.004	.865 .865 .864 .865 .862 .865 .859	.206 .146 .073 .072 .051 .051 .036 .035	.159 .114 .058 .057 .041 .041 .029 .028	.228 .163 .082 .081 .058 .057 .041 .040	.239 .169 .084 .083 .059 .059 .042 .041	.184 .132 .067 .066 .047 .047 .033 .033	.132 .094 .047 .047 .033 .033 .024 .023	.364 .257 .121 .142 .094 .116 .076 .066	4.102 3.546 3.133 3.155 3.073 3.070 2.989 3.079 3.038	1,142 651,511 131,072 65,536 65,536 32,768 32,768 16,384
12/62-12/63	1E 2E 8S 8R 16S 16R 32S 32R 128S	.000 .175 .775 .734 .922 .936 1.002 1.000	3.214 3.177 1.807 1.872 1.569 1.537 1.426 1.449	1.176 1.176 1.178 1.176 1.182 1.176 1.192 1.176	.287 .203 .102 .102 .072 .072 .050 .049	.198 .148 .079 .080 .056 .057 .040 .041	.291 .215 .112 .114 .090 .081 .056 .055	.244 .173 .086 .087 .061 .061 .042 .041	.168 .126 .067 .068 .048 .048 .034 .034	.124 .091 .048 .048 .034 .034 .024 .023	1.680 1.187 .602 .571 .406 .387 .286 .242	10.576 6.769 3.966 3.815 3.433 3.358 3.191 3.164 3.005	1,162 674,541 131,072 65,536 65,536 32,768 32,768 16,384 16,384
12/63-12/64	1E 2E 8S 8R 16S 16R 32S 32H	.326 .347 .772 .786 .890 .894 .981 1.026	3.130 2.763 1.740 1.671 1.522 1.480 1.383 1.400	1.163 1.163 1.163 1.166 1.163 1.171 1.163 1.186 1.163	.265 .188 .094 .094 .066 .066 .046	.188 .140 .073 .074 .052 .053 .037 .040	.278 .203 .104 .105 .074 .075 .052 .051	.228 .161 .081 .081 .057 .057 .040 .038	.162 .120 .063 .063 .045 .045, .032 .034	.120 .087 .045 .045 .032 .032 .022 .021	1.043 .736 .369 .362 .250 .201 .170 .143	7.656 5.315 3.598 3.470 3.246 3.177 3.156 3.087 2.957	1,191 708,645 131,072 65,536 65,536 32,768 32,768 16,384 16,384
12/64-12/65	1E 2E 8S 8R 16S 16R 32S 32R	.289 .371 .876 .859 .963 .977 1.065 1.087	5.426 4.762 2.367 2.340 1.937 1.824 1.737 1.559	1.282 1.282 1.282 1.283 1.282 1.283 1.283 1.276	.410 .290 .144 .140 .102 .096 .071 .062	.283 .209 .111 .108 .080 .076 .056 .050	.401 .298 .158 .153 .113 .107 .080 .070	.320 .226 .112 .109 .079 .075 .056 .049	.221 .163 .087 .084 .062 .059 .044 .039	.156 .116 .061 .060 .044 .042 .031 .027	2.554 1.804 .887 .866 .656 .569 .445 .362	16.991 9.964 4.654 4.627 3.996 3.654 3.444 3.307 2.958	1,227 752,151 131,072 65,536 65,536 32,768 32,768 16,384

TABLE A1, CONTINUED

FREQUENCY DISTRIBUTIONS OF MEALTH RATIOS FROM INVESTMENTS IN RANDOMLY SELECTED PORTFOLIOS
CONTAINING SPECIFIED NUMBERS OF STOCKS LISTED ON THE MYSE, 1926-65

Number and	Size of Portfolio	,			Cent	tiles of the	e Frequéncy	Distributi	ons			
Length of Periods	Sampling Method		10th	20th	30th	40th	(Median) 50th	60th	70th	80th	90th	95th
(1)	(2)	(3)	(4)	(5)	(6)	(7)		(9)			(12)	
(1)	(2)	(3)	(4)	(5)			(8)	(9)	(10)	(11)	(12)	(13)
					FIVE-YE	AR PERIOD	S					
1/26-12/30	1E 2E	.041 .159	.095 .254	.192	•340	-518	-682	.876	1.116	1.443	1.884	2.476 1.921
	8S	.469	.254 .542	•402 •642	.530 .718	•653 •786	.782 .853	.921 .923	1.083	1.299	1.630 1.241	1.921
	8R	.471	.545	.643	.718	.786	.854	.925	1.003	1.099	1,23	1.364
	16S 16R	.582 .584	.639 .639	•714 •709	•769 •763	.819 .812	.867 .859	.916 .908	.971 .961	1.037 1.026	1.131	1.210 1.200
	328	.667	•708	.762	. 802	.838	.872	•906	.944	.990	1.056	1.109
	32R 128S	.632 .779	.670 .798	.717 .825	.752 .844	.784 .862	.814 .877	.845 .892	.879 .908	.918 .931	.973 .955	1.019 .983
12/30-12/35	1E 2F	.098 .340	•227 •494	.417 .715	.660 .902	.897 1.086	1.147 1.283	1.412 1.518	1.766 1.811	2.293 2.225	3.296	4.601 3.854
	85	.813	-931	1.095	1.229	1.355	1.484	1.626	1.792	2.004	3.005 2.323	2.605
	8R	.826 1.000	.943 1.100	1.105	1.238	1.363 1.434	1.489	1.631	1.795 1.749	2.008	2.335	2.622
	16S 16R	1.016	1.115	1.233 1.251	1.338 1.359	1.455	1.530 1.552	1.633 1.651	1.763 1.700	1.894 1.907 1.795	2.101 2.121	2.288 2.312
	32S 32R	1.150 1.181	1.229 1.265	1.332	1.409	1.480 1.526	1.549 1.593	1.620 1.668	1.700 1.749	1.795 1.844	1.927	2.054
	1285	1.363	1.410	1.462	1.453 1.500	1.536	1.569	1.601	1.640	1.680	1.987 1.745	1.798
12/35-12/40	1E 2E	.119 .299	.218 .401	.364 .541	.539 .651	.673 .754	.832 .857	.959 .967	1.134 1.092	1.336 1.253	1.736 1.516	2.061 1.820
	88	•578	.643	.725	.789	.847	.903	.964	1.035	1.128	1.279	1.452
	8R 16S	.587 .673	.648 .723	.729 .785	.790 .834	.847 .877	.903 .920	•963 •966	1.034 1.018	1.129 1.088	1.286 1.206	1.465 1.335
	16R	•679	.726	.788	. 836	.879	•922	.969	1.024	1.095	1.219	1.352
	32S 32R	•744 •772	.781 .806	.829 .856	.866 .894	898 .929	•932 •964	•967 •999	1.006 1.042	1.057 1.097	1.140 1.187	1.213 1.261
	1285	.842	.865	.889	.911	.928	.945	.963	.983	1.005	1.040	1.069
						. 700						
12/40-12/45	1E 2E	1.459 1.883	1.721 2.116	2.084 2.470	2.408 2.794	2.708 3.124	3.155 3.501	3.688 3.982	4.335 4.584	5.556 5.382	7.601 6.974	10.036 9.008
	8S 8R	2.671	2.889	3.197	3.452	3.699	3.954	4.241	4.584	5.082	5.985	6.993
	16S	2.745 3.020	2.967 3.209	3.281 3.472	3.539 3.681	3.781 3.874	4.035 4.081	4.326 4.307	4.681 4.579	5.177 4.950	6.063 5.586	7.052 6.214
	16R	3.194	3.387	3.658	3.869	4.066	4.268	4.489	4.757	5.113	5.708	6.283
	32S 32R	3.308 3.680	3.466 3.845	3.674 4.051	3.845 4.212	4.002 4.361	4.159 4.510	4.327 4.672	4.524 4.850	4.783 5.080	5.193 5.428	5.547 5.727
	1285	3.761	3.866	3.987	4.080	4.167	4.250	4.335	4.432	4.549	4.714	4.843
12/45-12/50	16	.477	.627	.865	1.007	1.148	1.302	1.481	1.702	1.960	2.409	2.838
227 12 227 34	2E	.726	.846	1.008	1.135	1.254	1.375	1.505	1.657	1.851	2.159	2.450
	8S . 8R	1.052	1.126 1.126	1.225 1.224	1.299 1.297	1.366 1.365	1.432 1.430	1.501 1.498	1.578 1.576	1.671	1.812 1.813	1.935 1.937
	165	1.161	1.220	1.291	1.348	1.397	1.445	1.494	1.546	1.612	1.708	1.791
	16R 32S	1.159 1.238	1.215 1.285	1.286 1.341	1.340 1.380	1.389	1.436 1.450	1.484 1.485	1.537 1.522	1.602 1.569	1.696 1.633	1.779 1.682
	32R	1.241	1.284	1.333	1.369	1.402	1.434	1.465	1.502	1.541	1.603	1.658
	1285	1.346	1.364	1.400	1.419	1.435	1.452	1.469	1.492	1.514	1.537	1.575
12/50-12/55	16	.853	1.142	1.440	1.691	1.907	2.107	2.337	2.621	3.002	3.775	4.568
	2E 8S	1.230	1.420 1.826	1.662 1.973	1.848 2.086	2.017 2.189	2.187 2.288	2.374 2.396	2.600 2.521	2.908 2.674	3.425 2.907	3.954 3.118
	8R	1.710	1.825	1.968	2.081	2.183	2.280	2.387	2.509	2.661	2.893	3.099
	16S 16R	1.877	1.965 1.955	2.078 2.066	2.163 2.150	2.239 2.224	2.314 2.295	2.392 2.369	2.477 2.453	2.583 2.556	2.740 2.701	2.879 2.834
	32S	2.004	2.064	2.149	2.215	2.270	2.324	2.377	2.440	2.512	2.616	2.703
	32R 128S	2.000 2.159	2.060 2.202	2.141	2•204 2•276	2.254 2.306	2.305	2.355 2.359	2.410 2.385	2.475 2.429	2.574 2.474	2.662 2.517
	1203	2.137	2.202	2.23,	2.210	22300	24332	20337	24303	20127	20	
12/55-12/60	16	.532	.707	.939	1.130	1.292	1.477 1.533	1.656	1.883	2.181	2.708	3.396
	2E 8S	.798 1.168	.937 1.253	1.121 1.366	1.266 1.454	1.399	1.533	1.678 1.701	1.849 1.805	2.081 1.938	2.497 2.162	3.043 2.416
	8R	1.176	1.258	1.369	1.455	1.534	1.615	1.702 1.707	1.805 1.783	1.944	2.169	2.423
	16S 16R	1.292	1.360 1.371	1.449 1.458	1.517 1.524	1.579 1.586	1.641 1.648	1.707 1.714	1.783	1.884	2.050 2.071	2.241 2.286
	32 S	1.390	1.442	1.511	1.564	1.611	1.659	1.710		1.040	1.967	2.120
	32R 128S	1.426 1.527	1.480 1.555	1.546 1.596	1.598 1.627	1.645 1.654	1.693 1.680	1.745 1.712	1.808 1.746	1.894 1.796	2.091 1.885	2.674 1.961
12/60-12/65	1E 2E	.761 1.067	.977 1.222	1.256 1.421	1.440 1.576	1.605 1.720	1.778 1.870	1.979 2.040	2.241 2.255	2.595 2.568	3.402 3.160	4.445 3.826
	88	1.474	1.571	1.702	1.806	1.902	2.001	2.110	2.240	2.407	2.682	2.975
	8R 16S	1.483 1.619	1.584 1.698	1.717 1.801	1.823 1.884	1.920 1.960	2.020 2.037	2.130 2.117	2.261 2.210	2.432 2.331	2.710 2.524	3.022 2.724
	16R	1.653	1.737	1.846	1.930	2.007	2.082	2.167	2,265	2.392	2.610	2.831
	32S 32R	1.737	1.799 1.905	1.882 1.993	1.947 2.060	2.005 2.122	2.062 2.183	2.122 2.249	2.188 2.322	2.275 2.411	2.410 2.543	2.533 2.649
	1285	1.896	1.938	1.982	2.022	2.051	2.080	2.112	2.148	2.184	2.249	2.286

Number and Length of Periods	Size of Portfolio and Sampling Method	Sample Minimum	Sample Maximum	Arithmetic Mean	Standard Deviation	Mean Deviation	Gini's Mean Difference	Coefficient of Variation	Relative Mean Deviation	Coefficient of Concentration	Skevness	Kurtosis	Number of Portfolios Examined
(14)	(15)	(16)	(17)	(18)	(19) FIVE-YE	(20) AR PERIODS	(21)	(22)	(23)	(24)	(25)	(26)	(27)
1/26-12/30	1E 2E 8S 8R 16S 16R 32S 32R 128S	.000 .000 .103 .122 .268 .278 .427 .428	4.487 4.445 2.467 2.259 1.857 1.801 1.483 1.354	.877 .877 .876 .877 .878 .871 .878 .818	.778 .550 .272 .272 .191 .187 .134 .116	.600 .432 .216 .217 .152 .150 .107 .106	.822 .602 .305 .305 .215 .210 .151 .131	.887 .627 .310 .310 .218 .215 .153 .142	.684 .492 .247 .247 .174 .172 .122 .130	.468 .343 .174 .174 .123 .121 .086 .080	1.412 .996 .484 .496 .334 .369 .252 .194	5.405 4.184 3.267 3.293 3.124 3.130 3.018 2.985 2.926	510 129,795 131,072 65,536 65,536 32,768 32,768 16,384 16,384
12/30-12/35	1E 2E 8S 8R 16S 16R 32S 32R	.000 .000 .203 .266 .486 .445 .745 .808	11.841 11.085 5.424 4.853 3.941 3.650 2.750 3.050 2.149	1.568 1.568 1.569 1.577 1.573 1.591 1.567 1.614 1.572	1.585 1.120 .557 .556 .396 .395 .274 .282	1.079 .820 .438 .436 .314 .311 .219 .225 .102	1.506 1.145 .614 .612 .442 .441 .308 .317	1.011 .714 .355 .353 .252 .248 .175 .175	.688 .523 .279 .276 .200 .196 .139 .065	.480 .365 .196 .194 .141 .139 .098 .098	2.463 1.738 .856 .868 .612 .588 .388 .415	11.345 7.140 4.029 4.036 3.505 3.435 3.126 3.265 3.082	737 271,216 131,072 65,536 65,536 32,768 32,768 16,384
12/35-12/40	1E 2E 8S 8R 16S 16R 32S 32R	.000 .000 .226 .263 .400 .411 .552 .572	10.457 9.666 3.512 3.312 2.406 2.352 1.656 1.752 1.249	.949 .949 .947 .950 .948 .954 .948 .982	.822 .581 .289 .292 .203 .207 .142 .149	.519 .384 .210 .211 .153 .156 .111 .116	.741 .552 .301 .303 .219 .223 .157 .165	.867 .613 .306 .307 .214 .217 .150 .151	.547 .405 .222 .222 .162 .163 .117 .118	.391 .291 .159 .159 .115 .117 .083 .084	4.460 3.147 1.557 1.603 1.068 1.112 .718 .685	41.754 22.236 7.697 7.606 5.072 5.186 3.817 3.657 3.019	719 258,121 131,072 65,536 65,536 32,768 32,768 16,384
12/40-12/45	1E 2E 8S 8R 16S 16R 32S 32S 128S	.000 .000 1.553 1.539 2.095 2.278 2.617 2.967	48.855 45.698 16.196 15.239 11.391 10.258 8.355 7.618 5.713	4.264 4.264 4.263 4.344 4.269 4.434 4.255 4.583 4.270	3.990 2.819 1.400 1.393 .991 .952 .686 .624	2.289 1.780 1.017 1.001 .752 .717 .538 .530 .259	3.150 2.486 1.436 1.434 1.062 1.031 .756 .695	.936 .661 .328 .321 .232 .215 .161 .136	.537 .418 .238 .230 .176 .162 .126 .116	.369 .292 .168 .165 .124 .116 .089 .076	5.010 3.536 1.734 1.681 1.225 1.078 .814 .660	41.665 22.205 7.502 7.202 5.219 4.642 3.931 3.591 3.017	788 310,078 131,072 65,536 65,536 32,768 32,768 16,384
12/45-12/50	1E 2L 8S 8R 16S 16R 32S 32R 128S	.063 .107 .553 .639 .846 .886 .910 1.022	6.514 6.024 3.177 2.884 2.697 2.333 2.128 1.910	1.455 1.455 1.454 1.453 1.455 1.455 1.455 1.439	.771 .545 .271 .270 .191 .188 .133 .123	.576 .418 .214 .213 .152 .150 .106 .099	.811 .591 .302 .301 .214 .211 .150 .138	.530 .374 .186 .186 .131 .130 .092 .085	.396 .287 .147 .147 .104 .104 .073 .069	.279 .203 .104 .104 .074 .073 .052 .048	1.525 1.076 .530 .520 .362 .371 .217 .235	7.489 5.228 3.559 3.432 3.268 3.219 3.113 3.012 3.004	853 363,378 131,072 65,536 65,536 32,768 32,768 16,384
12/50-12/55	1E 2E 8S 8R 16S 16R 32S 32R 128S	.113 .145 1.059 1.067 1.299 1.406 1.595 1.698	10.794 9.896 5.107 4.842 4.119 3.880 3.364 3.140 2.746	2.335 2.335 2.335 2.326 2.337 2.316 2.334 2.312 2.334	1.217 .860 .430 .423 .302 .292 .212 .196	.861 .643 .338 .334 .240 .233 .169 .157	1.240 .915 .477 .470 .339 .327 .238 .220	.521 .368 .184 .182 .129 .126 .091 .085	.369 .275 .145 .143 .103 .100 .072 .068	.266 .196 .102 .101 .072 .071 .051 .048	1.836 1.296 .644 .628 .466 .431 .323 .273	9.107 6.035 3.739 3.643 3.362 3.314 3.162 3.084	1,010 509,545 131,072 65,536 65,536 32,768 32,768 16,384
12/55-12/60	1E 2E 8S 8R 16S 16R 32S 32R 128S	.102 .107 .629 .666 .889 .896 1.066 1.115	35.876 24.524 7.417 7.554 5.052 4.934 3.480 3.471 2.314	1.701 1.701 1.701 1.709 1.701 1.722 1.702 1.773	1.508 1.066 .536 .555 .373 .413 .265 .335	.737 .562 .322 .326 .240 .251 .181 .213	1.067 .813 .467 .474 .348 .371 .262 .323	.886 .626 .315 .325 .219 .240 .156 .189	.433 .330 .189 .191 .141 .146 .106 .120	.314 .239 .137 .139 .102 .108 .077 .091	12.294 8.681 4.319 4.426 3.001 3.133 2.104 1.971 .883	257.373 129.571 33.403 33.331 17.842 16.777 9.975 6.963 3.833	1,055 555,985 131,072 65,536 65,536 32,768 32,768 16,384
12/60-12/65	1E 2E 8S 8R 16S 16R 32S 32R	.159 .187 .869 .878 1.209 1.205 1.383 1.495	18.598 16.508 6.516 5.599 4.381 4.128 3.507 3.293 2.650	2.086 2.086 2.085 2.106 2.083 2.137 2.088 2.205 2.086	1.382 .977 .486 .495 .342 .355 .241 .244	.851 .654 .361 .365 .262 .270 .189 .210	1.221 .931 .513 .523 .372 .388 .267 .275	.663 .468 .233 .235 .164 .166 .115 .111	.408 .314 .173 .173 .126 .126 .090 .095	.293 .223 .123 .124 .089 .091 .064 .062	4.087 2.886 1.444 1.410 .996 .914 .703 .419	34.270 18.560 6.908 6.435 4.792 4.182 3.883 3.071 3.120	1,119 625,521 131,072 65,536 65,536 32,768 16,384 16,384

TABLE A1, CONTINUED

FREQUENCY DISTRIBUTIONS OF MEALTH RATIOS FROM INVESTMENTS IN RANDOMLY SELECTED PORTFOLIOS

CONTAINING SPECIFIED NUMBERS OF STOCKS LISTED ON THE NYSE, 1926-65

Number and	Size of Portfolio Sampling Method	Size of Portfolio/ Centiles of the Frequency Distributions											
Length of Periods			10th	20th	30th	40th	(Median) 50th	60th	70th	80th	90th	95th	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	
					TEN-YEA	AR PERIODS	.						
1/26-12/35	16	.016	.048	•152	•256	•461	-688	1.007	1.368	1.850	2.730	4.297	
	2E 8S	•115 •488	•196 •590	.365 .736	•540 •859	.716 .979	.908 1.104	1.106 1.246	1.373	1.790 1.641	2.593 2.002	3.633 2.353	
	8R	.502	.607	.754	.875	.992	1.116	1.259	1.428	1.647	2.009	2.370	
	165	.657	•749	.870	.971	1.065	1.162	1.268	1.390	1.545	1.790	2.102	
	16R	.672	.763	.888	• 985	1.080	1.174	1.276	1.395	1.551	1.816	2.169	
	32S 32R	•796 •743	.871	.972	1.050	1.123	1.194	1.271	1.357	1.471	1.667	1.859 1.705	
	1285	1.008	.813 1.043	.898 1.109	.964 1.147	1.026 1.185	1.087 1.223	1.157 1.264	1.234 1.306	1.334 1.351	1.520 1.434	1.476	
12/35-12/45	16	-614	1.056	1.592	1.912	2.210	2.557	2.999	3.493	4.216	5.500	7.664	
	2E	1.221	1.515	1.890	2.186	2.464	2.751	3.068	3.462	4.030	5.194	6.442	
	88	1.987	2.174	2.419	2.616	2.464 2.797	2.987	3.193	3.434	3.760	4.317	4.996	
	8R	2.024	2.211	2.457	2.657	2.843	3.035	3.248	3.495	3.819	4.370	5.105	
	16S 16R	2.257 2.365	2.414 2.502	2.614 2.714	2.767 2.884	2.914 3.029	3.057 3.181	3.217 3.342	3.387 3.538	3.638 3.787	4.054 4.268	4.599 4.886	
	325	2.467	2.608	2.755	2.886	2.994	3.103	3.233	3.363	3.568	3.911	4.454	
	32R	2.755	2.895	3.073	3.205	3.326	3.454	3.590	3.742	3.991	4.961	5.642	
	1285	2.805	2.873	2.952	3.030	3.109	3.190	3.270	3.351	3.481	3.672	3.852	
12/45-12/55	16	.621	.982	1.505	1.895	2.302	2.750	3.270	4.059	5.100	7.169	9.001	
	2E 8S	1.211	1.512	1.943	2.315	2.690	3.089	3.550	4.121	4.908	6.125	7.333	
	85 8R	2.128 2.142	2.367 2.377	2.682 2.686	2.939 2.935	3.175 3.161	3.410 3.389	3.659 3.634	3.947 3.914	4.303 4.271	4.827 4.780	5.329 5.250	
	165	2.482	2.678	2.935	3.135	3.306	3.479	3.654	3.858	4.093	4.450	4.768	
	16R	2.479	2.668	2.912	3.104	3.267	3.431	3.603	3.795	4.021	4.362	4.676	
	32 S	2.747	2.903	3.111	3.244	3.376	3.502	3.627	3.769	3.941	4.194	4.382	
	32R 128S	2.786 3.129	2.915 3.185	3.106 3.295	3.221 3.400	3.337 3.463	3.453 3.525	3.571 3.588	3.689 3.650	3.861 3.723	4.042 3.897	4.272 3.984	
12/55-12/65	16	.836	1.157	1.654	2.035	2.427	2.814	3.208	3.681	4.289	5.473	7.075	
12/33 12/03	2E	1.382	1.669	2.054	2.361	2.640	2.923	3.232	3.602	4.112	5.036	6.036	
	88	2.158	2.336	2.576	2.759	2.933	3.103	3.295	3.518	3.814	4.336	4.885	
	8R	2.170	2.350	2.591	2.774	2.949	3.122	3.315	3.544	3.849	4.372	4.901	
	16S 16R	2.414 2.432	2.565 2.598	2.747 2.782	2.899 2.934	3.029 3.071	3.165 3.211	3.309 3.353	3.479 3.534	3.687 3.747	4.025 4.075	4.346 4.377	
	325	2.624	2.723	2.885	2.993	3.101	3.208	3.314	3.434	3.595	3.819	4.003	
	32R	2.684	2.831	2.962	3.089	3.195	3.298	3.403	3.538	3.674	3.916	4.072	
	1285	2.879	2.934	3.043	3.128	3.182	3.235	3.289	3.342	3.398	3.560	3.642	
					20-YEAR	PERIODS							
1/26-12/45	1E	•000	•052	•324	.772	1.273	1.864	2.772	3.914	5.133	7.395	11.389	
	2E 8S	.249 1.319	.523 1.608	.982 2.010	1.496 2.345	2.010 2.662	2.527 3.006	3.072 3.396	3.770 3.848	4.797 4.532	7.002 5.659	9.909 6.734	
	8R	1.366	1.660	2.061	2.393	2.717	3.063	3.470	3.941	4.593	5.751	6.813	
	165	1.783	2.012	2.360	2.645	2.911	3.177	3.502	3.835	4.337	5.062	5.592	
	16R	1.854	2.100	2.437	2.728	2.992	3.267	3.587	3.928	4.419	5.149	5.676	
	32S 32R	2.185 2.095	2.350 2.293	2.665 2.569	2.867 2.787	3.070 2.980	3.281 3.172	3.514 3.393	3.746 3.620	4.078 3.848	4.520 4.338	4.956 4.597	
	1285	2.706	2.791	2.962	3.132	3.272	3.389	3.506	3.623	3.739	3.856	4.215	
12/45-12/65	16	•912	1.886	3.357	4.549	6.269	8.242	10.111	12.529	16.068	21.992	30.115	
	2E	2.648	3.608	5.140	6.452	7.741	9.138	10.688	12.552	15.132	19.496	24.671	
	88	5.790	6.623	7.674	8.562	9.372	10.231	11.115	12.220	13.514	15.838	18.168	
	8R	5.805	6.646	7.673 8.493	8.552 9.174	9.358 9.844	10.210 10.489	11.088 11.135	12.175 11.939	13.466 12.984	15.769 14.497	18.061 15.874	
	16S 16R	6.947 6.994	7.590 7.634	8.515	9.174	9.856	10.489	11.105	11.871	12.924	14.378	15.811	
	325	7.992	8.383	9.058	9.706	10.174	10.642	11.110	11.629	12.523	13.417	14.352	
	32R	8.128	8.468	9.147	9.764	10.206	10.649	11.092	11.547	12.449	13.352	14.072	
	1285	8.903	9.662	9.924	10.185	10.447	10.709	10.971	11.232	11.494	12.574	13.164	
					40-YEAR	PERIOD							
1/26-12/65	1E 2E	.000 1.041	.258 2.663	1.283 6.631	3.724 10.964	8.257 15.669	14.323 21.557	21.581 28.267	33.613 37.228	50.787 50.393	82.532 76.499	127.554 107.820	
	88	10.323	12.954	17.309	21.097	24.948	28.812	33.363	39.326	45.335	60.472	76.912	
	8R	10.505	13.251	17.570	21.354	25.136 27.547	28.917	33.404	39.255	45.106	60.046	75.798	
	165	15.068	17.211	21.496	24.556	27.547	30.538	34.621	39.153	43.685	56.084	64.379	
	16R 32S	15.190 18.151	17.236 21.735	21.328 24.177	24.320 26.618	27.194 29.060	30.068 31.551	33.820 35.280	38.339 39.010	42.857 42.740	54.034 54.865	63.398 72.821	
	323 32R	16.895	19.399	22.798	24.802	26.806	28.810	30.814	34.606	39.154	43.701	57.493	
	1285	23.045	24.410	27.140	29.870	32.555	35.182	37.809	40.436	43.063	48.040	56.520	

Number and Length of Periods	Size of Portfolio and Sampling Method	Sample Minimum	Sample Maximum	Arithmetic Mean	Standard Deviation	Mean Deviation	Gini's Mean Difference	Coefficient of Variation	Relative Mean Deviation	Coefficient of Concentration	Skerness	Kurtosis	Number of Portfolios Examined
(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
					TEN-YEAR	PERIODS							
1/26-12/35	1E 2E 8S 8R 16S 16R 32S 32R	.000 .000 .049 .079 .192 .228 .475 .476	24.679 17.915 7.590 6.002 4.421 3.756 2.862 2.468 1.807	1.238 1.238 1.236 1.251 1.237 1.253 1.237 1.131	1.852 1.308 .651 .656 .456 .462 .319 .285	1.086 .836 .471 .468 .342 .341 .246 .249	1.480 1.166 .664 .665 .485 .488 .349 .311	1.496 1.057 .526 .524 .368 .369 .258 .258 .252	.877 .675 .381 .374 .276 .272 .199 .220	.598 .471 .268 .266 .196 .195 .141 .137	5.481 3.864 1.894 1.947 1.305 1.351 .882 .943	56.199 29.331 9.102 9.257 5.767 5.738 4.104 4.175 2.962	510 129,795 131,072 65,536 65,536 32,768 32,768 16,384
12/35-12/45	1E 2E 8S 8R 16S 16R 32S 32R 128S	.000 .000 .822 1.017 1.4.0 1.621 1.864 2.069 2.476	74.724 51.189 16.967 16.527 10.241 9.974 6.930 7.813 4.637	3.226 3.226 3.217 3.286 3.221 3.379 3.223 3.650 3.225	3.675 2.597 1.274 1.356 .900 1.010 .635 .811	1.708 1.329 .773 .787 .582 .616 .440 .580	2.459 1.912 1.112 1.157 .841 .928 .634 .813	1.139 .805 .396 .413 .279 .299 .197 .222	.529 .412 .240 .240 .181 .182 .137 .159	.381 .296 .173 .176 .131 .137 .098 .111	11.440 8.073 3.958 4.018 2.769 2.709 1.887 1.633	205.680 103.621 27.222 26.253 14.393 12.609 7.961 5.430 3.224	719 258,121 131,072 65,536 65,536 32,768 32,768 16,384
12/45-12/55	1E 2E 8S 8R 16S 16R 32S 32R	.047 .064 .947 1.035 1.451 1.523 2.034 1.912 2.753	21.753 20.431 9.541 9.523 7.562 6.869 5.787 5.420 4.409	3.526 3.526 3.522 3.501 3.531 3.481 3.523 3.473 3.524	2.766 1.955 .976 .952 .687 .653 .483 .429	2.012 1.489 .771 .754 .545 .524 .385 .348 .183	2.779 2.074 1.084 1.058 .770 .733 .543 .484 .259	.785 .554 .277 .272 .195 .188 .137 .123	.571 .422 .219 .215 .154 .151 .109 .100	.394. .294 .154 .151 .109 .105 .077 .070	1.958 1.382 .693 .683 .477 .451 .324 .256	8.668 5.814 3.692 3.662 3.347 3.236 3.112 2.979 2.963	853 363,378 131,072 65,536 65,536 32,768 32,768 16,384
12/55-12/65	1E 2E 8S 8R 16S 16R 32S 32R	.084 .130 .837 1.085 1.519 1.597 1.999 2.089 2.600	22.340 21.851 9.564 9.047 7.398 6.935 5.491 5.244 4.055	3.241 3.241 3.240 3.260 3.239 3.282 3.241 3.329 3.235	2.350 1.661 .829 .832 .581 .590 .409 .408 .192	1.506 1.137 .627 .630 .453 .457 .325 .327 .154 PERIODS	2.166 1.630 .892 .897 .640 .651 .458 .458	.725 .513 .256 .255 .180 .180 .126 .123	.465 .351 .194 .193 .140 .139 .100 .098	.334 .252 .138 .138 .099 .071 .069	3.278 2.315 1.145 1.102 .807 .764 .552 .455 .219	20.969 11.938 5.171 4.917 4.073 3.945 3.434 3.193 3.020	1.055 555.985 131.072 65.536 65.536 32.768 32.768 16.384
1/26-12/45	1E 2E 8S 8R 16S 16R 32S 32R 128S	.000 .000 .172 .154 .545 .646 1.208 1.190 2.213	40.763 39.315 18.387 18.345 12.660 10.138 7.561 6.953 4.861	3.361 3.361 3.362 3.419 3.367 3.450 3.361 3.234	4.759 3.362 1.674 1.669 1.177 1.167 811 .744 .363	2.943 2.240 1.274 1.269 .924 .914 .644 .610	4.018 3.150 1.784 1.788 1.295 1.290 .906 .834	1.416 1.000 .498 .488 .350 .338 .241 .230	.876 .666 .379 .371 .274 .265 .192 .189	.598 .469 .265 .261 .192 .187 .135 .129	3.718 2.622 1.294 1.234 .896 .828 .604 .523	22.395 12.596 5.274 5.007 4.089 3.820 3.417 3.393 2.992	510 129,795 131,072 65,536 65,536 32,768 32,768 16,384
12/45-12/65	1E 2E 8S 8R 16S 16R 32S 32R 128S	.116 .119 2.018 2.095 3.560 3.340 5.425 5.838 7.893	110.916 94.155 41.127 36.448 28.019 24.726 21.171 19.714	10.766 10.766 10.779 10.753 10.760 10.741 10.755 10.769 10.761	10.593 7.486 3.731 3.698 2.629 2.568 1.838 1.744 .860	7.083 5.299 2.858 2.834 2.057 2.010 1.456 1.383 .686	9.866 7.469 4.043 4.005 2.904 2.839 2.058 1.955	.984 .695 .346 .344 .244 .239 .171 .162	.658 .492 .265 .264 .191 .187 .135 .128	.458 .347 .188 .186 .135 .132 .096 .091	3.111 2.196 1.079 1.083 .766 .764 .523 .507	19.804 11.349 4.989 4.961 4.013 3.906 3.444 3.335 3.012	853 363,378 131,072 65,536 65,536 32,768 32,768 16,384
1/26-12/65	1E 2E 8S 8R 16S 16R 32S 32R 128S	.000 .527 .611 3.781 5.401	192.144 187.310 121.271	35.124 35.124 35.210 35.280 35.143 34.451 35.138 31.215	89.807 63.440 31.657 31.870 22.158 21.616 15.524 12.718 6.933	36.247 28.714 17.366 17.214 13.321 12.928 10.304 9.591 5.666	48.377 39.764 24.888 24.720 19.228 18.429 14.839 11.730 7.734	2.557 1.806 .899 .903 .631 .627 .442 .407	1.032 .818 .493 .488 .379 .375 .293 .307	.689 .566 .353 .350 .274 .267 .211 .188	13.439 9.475 4.645 4.727 3.211 3.355 2.145 2.633 .670	242.255 121.439 30.636 31.201 15.717 16.712 8.076 11.739 2.638	510 129.795 131,072 65.536 65,536 32,768 32,768 16,384

TABLE A2
MINIMUM AND MAXIMUM MEALTH RATIOS FROM INVESTMENTS IN INFINITE NUMBERS
OF RANDOMLY SELECTED PORTFOLIOS CONTAINING SPECIFIED NUMBERS
OF STOCKS LISTED ON THE NYSE, 1926-65²⁴

Size of Portfolio/								
Sampling Meth	od 192	26	192	27	192	28	192	29
8S 8R 16S 10R 32S 32R 128S	MIN 0.279 0.293 0.328 0.370 0.383 0.523 0.589	MAX 2.447 2.447 2.114 2.046 1.811 1.630 1.383	MIN 0.188 0.251 0.355 0.418 0.487 0.638 0.737	MAX 3.943 3.839 3.308 3.136 2.807 2.434 2.000	MIN 0.494 0.495 0.543 0.568 0.616 0.721 0.820	MAX 6.760 6.760 5.267 4.928 4.150 3.354 2.536	MIN 0.074 0.077 0.106 0.113 0.138 0.254 0.269	MAX 1.568 1.522 1.482 1.397 1.193 1.153
1203	0.367	1.303	0.737	2.000	0.620	2.730	0.207	1.175
	19	30	19	31	19:	32	193	33
85 8R .65 16R 32S 32R 128S	MIN 0.102 0.104 0.122 0.126 0.148 0.203 0.245	MAX 1.610 1.616 1.460 1.412 1.309 1.201 1.061	MIN 0.005 0.005 0.038 0.041 0.076 0.132 0.185	MAX 1.720 1.591 1.508 1.419 1.336 1.175 1.007	MIN 0.028 0.046 0.103 0.140 0.185 0.283 0.386	MAX 2.867 2.649 2.529 2.263 2.181 1.786 1.589	MIN 0.071 0.109 0.249 0.305 0.415 0.587 0.775	MAX 9.188 9.169 7.532 7.246 6.204 5.234 4.077
	19	34	19	35	19:	36	19	37
8S 8R 16S 16R 32S 32R 128S	MIN 0.359 0.375 0.418 0.464 0.475 0.567	MAX 4.260 3.798 3.407 3.009 2.796 2.340 1.963	MIN 0.233 0.353 0.380 0.476 0.522 0.695 0.795	MAX 4.534 4.438 4.045 3.817 3.555 3.042 2.611	MIN 0.472 0.484 0.600 0.622 0.705 0.795 0.891	MAX 5.803 5.727 4.568 4.423 3.706 3.299 2.513	MIN 0.172 0.178 0.197 0.205 0.226 0.253 0.293	MAX 1.254 1.239 1.151 1.094 1.043 0.914 0.858
	19	3.8	19:	39	194	40	194	1
85 8R 16S 16R 32S 32R 128S	MIN 0.208 0.270 0.320 0.427 0.427 0.641 0.729	MAX 3.942 3.942 3.156 3.118 2.641 2.512 2.022	MIN 0.259 0.307 0.340 0.394 0.411 0.506 0.571	MAX 2.330 2.270 2.122 2.007 1.854 1.696 1.462	MIN 0.080 0.173 0.173 0.301 0.293 0.458 0.504	MAX 2.136 2.053 1.841 1.746 1.619 1.491 1.308	MIN 0.127 0.191 0.205 0.297 0.298 0.425 0.476	MAX 2.260 2.165 2.042 1.880 1.793 1.597 1.385
	19	42	19	43	19	44	194	4 5
8S 8R .6S 16R 32S 32R 128S	MIN 0.690 0.691 0.739 0.748 0.778 0.345	MAX 4.299 4.096 3.759 3.247 3.115 2.531 2.141	MIN 0.724 0.764 0.807 0.888 0.885 1.023	MAX 5.808 5.704 4.723 4.423 3.851 3.255 2.609	MIN 0.627 0.757 0.762 0.898 0.874 1.021	MAX 3.254 3.080 2.921 2.656 2.590 2.219 1.991	MIN 0.955 0.962 1.027 1.051 1.085 1.156	MAX 4.100 3.596 3.530 3.109 3.044 2.609 2.336

^{*}The corresponding minima and maxima for portfolios of one and two stocks are shown in Table 5.

TABLE A2, CONTINUED

MINIMUM AND MAXIMUM WEALTH RATIOS FROW INVESTMENTS IN INFINITE MUMBERS OF NANDOWLY SELECTED PORTOLOGO COUNTLINKS SEPECITED INVESTED SELECTED ON THE MYSE, 1926-654

MINIMUM AND MAXIMUM WEALTH RATIOS FROM INVESTHENTS IN INFINITE NUMBERS OF RANDOMLY SELECTED POWERSOLOS CONTAINING SEPECITED NUMBERS OF STOCKS LISTED ON THE NYES, 1965-65* TABLE A2, CONTINUED

1941-45	MAX 31.511 26.887 24.365 20.627 18.623 14.447	1961-65	MAX 11.463 9.420 9.273 7.693 7.610 5.795	1956-65	MAX 19.032 16.832 15.819 13.017 12.382 9.761 7.800			Table 5.	
194]	MIN 0.756 0.958 1.003 1.224 1.175 1.663	1961	MIN 0.308 0.351 0.387 0.467 0.678 0.753	195	MIN 0.200 0.214 0.301 0.370 0.456 0.732			re showm in	
0 0	MAX 5.817 5.789 4.484 4.220 3.421 3.020 2.136	09-	MAX 12.182 11.794 8.973 8.221 6.790 5.677 3.987	-55	MAX 16.431 16.215 14.600 13.488 12.608 9.511 8.765	-65	MAX 483.576 459.290 332.143 307.518 234.007 178.207	The corresponding minima and maxima for portfolios of one and two stocks are shown in Table	
1936-40	MIN 0.004 0.021 0.021 0.057 0.057 0.148	1956-60	MIN 0.180 0.224 0.242 0.304 0.312 0.499	1946-55	MIN 0.155 0.158 0.221 0.254 0.316 0.603	1926-65	MIN 0. 4 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.	of one and t	
-35	MAX 9.572 9.407 8.352 7.958 7.027 5.657	-55	MAX 8.402 8.057 7.533 6.905 6.550 5.376 4.742	-45	MAX 25.166 24.508 18.215 17.543 13.577 12.58 7.541	-65	MAX 73.778 73.778 59.439 55.418 48.120 39.077	portfolios	
1931-35	MIN 0. 0. 0.013 0.020 0.119	1951–55	MIN 0.281 0.312 0.375 0.431 0.497 0.751	1936-45	MIN 0.067 0.078 0.291 0.291 0.291 0.888	1946-65	MIN 0.197 0.202 0.275 0.340 0.417 1.211	maxima for	
-30	MAX 3.719 3.591 3.345 3.135 2.922 2.175 1.965	-50	MAX 4.928 4.752 4.328 4.105 3.801 3.221 2.836	-35	MAX 10.651 9.855 8.559 7.041 6.500 4.320 3.347	-45	MAX 29.113 28.161 23.406 20.750 17.654 12.785 8.998	minima and	
, 1926–30	MIN 0.006 0.013 0.016 0.031 0.028 0.126	1946-50	MIN 0.232 0.232 0.282 0.315 0.348 0.482	1926-	MIN 0. 0. 0.004 0.005 0.007 0.010	1926-45	MIN 0. 0. 0.002 0.001 0.142	rresponding	
Size of Portfolio/ Sampling Method	85 87 168 168 325 328 1285		85 86 165 165 325 328 1285		85 88 165 168 325 325 1285		85 84 168 168 328 328	*The co	
1949	HAX 2.313 2.272 2.118 2.033 1.928 1.795	53	MAX 1.798 1.775 1.684 1.632 1.561 1.453	57	MAX 1.855 1.855 1.655 1.629 1.501 1.410	51	3.093 3.029 2.65 2.687 2.452 2.261	55	3.665 3.414 3.232 2.901 2.866 2.300
19	MIN 0.481 0.591 0.632 0.683 0.683 0.683	1953	MIN 0.378 0.403 0.444 0.473 0.496 0.608	1957	MIN 0.329 0.339 0.350 0.395 0.395 0.493	196	MIN 0.522 0.526 0.611 0.636 0.637 0.776	1965	MIN 0.509 0.517 0.582 0.593 0.653 0.123
1948	MAX 2.400 2.307 2.012 1.897 1.747 1.568	52	MAX 1.733 1.690 1.684 1.596 1.612 1.612 1.465	99	2.376 2.274 2.092 1.929 1.638 1.638	09	MAX 2.168 2.093 1.974 1.872 1.792 1.618	49	2
19	MIN 0.381 0.416 0.425 0.479 0.469 0.583	1952	MIN 0.505 0.506 0.587 0.600 0.600 0.697	1956	MIN 0.421 0.427 0.492 0.525 0.529 0.653	1960	MIN 0.408 0.408 0.450 0.460 0.488 0.587	1964	MIN 0.424 0.467 0.501 0.556 0.575 0.720
146	MAX 2.077 1.994 1.904 1.787 1.757 1.498 1.453	156	2.328 2.241 2.057 1.983 1.873 1.691 1.575	55	MAX 2.437 2.437 2.274 2.232 2.084 1.889 1.722	65	2.709 2.694 2.482 2.308 2.232 1.924 1.770	696	MAX 2.731 2.653 2.448 2.335 2.208 1.957 1.752
19	MIN 0.437 0.437 0.478 0.521 0.521 0.554	19	MIN 0.507 0.510 0.581 0.599 0.650 0.741	195	MIN 0.548 0.556 0.628 0.651 0.701 0.766	19	MIN 0.527 0.558 0.572 0.630 0.630 0.729	15	MIN 0.449 0.532 0.551 0.607 0.719
9,	MAX 1.821 1.742 1.671 1.575 1.533 1.338 1.292	1950	MAX 3.196 2.866 2.867 2.450 2.578 2.578 2.086	54	MAX 3.609 3.465 3.149 3.015 2.853 2.530 2.304	58	MAX 4.306 3.942 3.864 3.355 3.339 2.734 2.458	62	MAX 1.629 1.573 1.516 1.461 1.420 1.305
/ hod 1946	MIN 0.353 0.409 0.394 0.468 0.468 0.455	19	MIN 0.703 0.725 0.752 0.787 0.787 0.870	19	MIN 0.732 0.823 0.824 0.920 1.016	1958	MIN 0.879 0.884 0.921 0.942 0.942 1.054 1.054	1961	MIN 0.283 0.286 0.350 0.364 0.409 0.474
Size of Portfolio/ Sampling Method	85 8R 165 168 328 328 1285		85 4R 1.65 16R 32S 32S 32R		85 8R 165 168 328 325 328		85 88 1668 168 325 328 1285		85 87 164 164 328 1285

"The corresponding minima and maxima for portfolios of one and two stocks are shown in Table 5.

APPENDIX B

INDUSTRY GROUPS USED IN THE RESTRICTED SAMPLING PROCESS

Samples shown with an R in tables 5, A1, and A2 were random samples subject to the restriction that no more than one stock in a given industry be included in any given portfolio. This procedure, as well as simple random sampling, was employed for portfolios of eight, sixteen, and thirty-two stocks.

We classified the companies listed on the New York Stock Exchange into thirty-six industry groups. In defining an index group for this purpose, we used the Securities and Exchange Commission (SEC) two-digit groupings¹⁷ subject to the restriction that there be at least one stock in each group at the beginning of each period. This restriction made it necessary to aggregate several two-digit groups in a number of instances. Table B1 contains the list of industry groups we used.

¹⁷ The SEC two-digit groups correspond closely to the Standard Industrial Classification (SIC) groups.

TABLE B1
LIST OF INDUSTRY GROUPS USED IN RESTRICTED RANDOM SAMPLES

Industry Group	Description	Industry Group	Description
10	Metal mining	38	Instruments and related products
11–12	Coal mining	39	Miscellaneous manufacturing industries
13	Crude petroleum	40, 47	Railroads; miscellaneous transportation
20	Food and kindred products	·	services
21	Tobacco manufacturing	41–42	Local and highway transportation and
22	Textile-mill products		public warehousing
23	Apparel and other finished textiles	44	Water transportation
24-25	Lumber and wood products; furniture	48	Wire and radio communication
	and fixtures	49	Electric, gas, and water utilities
26	Paper and allied products	53	Department stores, mail order houses
27	Printing, publishing, and allied industries		and vending-machine operators
28	Chemical and allied products	54	Food stores
29	Products of petroleum and coal	56	Retail clothing and shoe stores
30	Rubber products	58	Restaurants
31*	Leather and leather products	50-52, 55,	
32*	Stone, clay, and glass products	57, 59	Other wholesale and retail trade
33*	Primary metal industries	60-63	Banks, savings and loan associations,
34	Fabricated metal products		finance companies, and insurance
35	Machinery except electrical	67	Investment companies
36	Electrical machinery	70–79	Services
37	Transportation equipment	All other	

^{*} Because of a programming error, these industry groups were combined.