The Minimum Number of Stocks Needed for Diversification

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■ A risk-averse investor seeking to improve investment performance by reading investment textbooks will learn the important distinction between diversifiable risk and non-diversifiable risk. The textbooks will then typically recommend that a portfolio of between *eight and twenty* stocks is the minimum necessary to eliminate diversifiable risk. (See Exhibit 1.)[1] In our view, this common recommendation, which originates from the work of Evans and Archer [3], seriously understates the minimum number of stocks that a risk-averse investor should aim for as a portfolio. This article argues that it may be desirable to have *substantially more than 20 stocks* in a portfolio to eliminate diversifiable risk.

The article first briefly reviews investment texts to show the pervasiveness of the standard recommendation to invest in eight to twenty stocks, and then argues that the standard recommendation is flawed — briefly, the standard recommendation is based upon the average risk that results from a very large number of equal-number stock portfolios. An individual investor, however, does not have a large number of such portfolios; the investor normally has one portfolio in any particular universe of stocks, and the risk outcome on this one portfolio could be other than the statistical average. Section II analyzes returns on the Standard & Poor's (S&P) 500 to demonstrate the variability of portfolio returns. The empirical results from analyzing these data lead to a recommendation that substantially more than 20 stocks may be needed to nearly eliminate diversifiable risk, depending on the weighing scheme used to construct portfolios and the personal risk preference. A final section provides a summary.

I. The Standard Recommendation

The risk on a portfolio is determined by the risks of the individual stocks, the relationships (covariances) between the risks of the individual stocks, and the percentage of the portfolio money invested in each of the stocks. The risk of a portfolio can be graphed against the number of stocks in the portfolio to show the reduction in risk due to diversification. To illustrate this, the S&P 500 was chosen as a possible universe of stocks from which to construct portfolios, and the market value of each company was used as the stock's weight in each portfolio. Then 1,000 portfolios in each category (i.e. 1,000 one-stock portfolios, 1,000 two-stock portfolios, etc.) were constructed over the period January 1988 to December 1990.[2] Using volatility of monthly returns, average risk on each sized portfolio was measured; Exhibit 2 shows the results. The steep fall of the curve shows *average* diversifiable risk being reduced as the number of stocks in the portfolio increases. Exhibit 2 is similar to that which appears in many textbooks, as listed in Exhibit 1.

Exhibit 1. Recommendations for Minimum Number of Stocks in a Portfolio

Source [a]	Exhibit 2 or Similar	# of Stocks
[12]	p. 100	8 - 16 (p. 99)
[6]	p. 698	8 - 20 (p. 697)
[4]	na[b]	10 - 15 (p. 231)
[8]	p. 14	10 - 15 (p. 144)
[7]	na	10 - 20 (p. 674)
[13]	na	12 - 15 (p. C16)
[9]	p. 288	12 - 18 (p. 288)
[1]	na	12 or more (p. 335)
[14]	p. 111	15 - 20 (p. 110)
[5]	p. 153	20 (p. 154)
[10]	na	20 (p. 218)
[2]	p. 139	20 (p. 139)

[a] See References.
[b] Not Available.

[1] The eight-to-twenty stock recommendation in the textbooks is based on naive or random diversification. Therefore we will not discuss other methods of achieving diversification, such as the Markowitz method.

[2] Since the estimation period was a fairly consistent bull market, our analysis would understate the volatility of portfolio risk.

Exhibit 2. Average Portfolio Risk

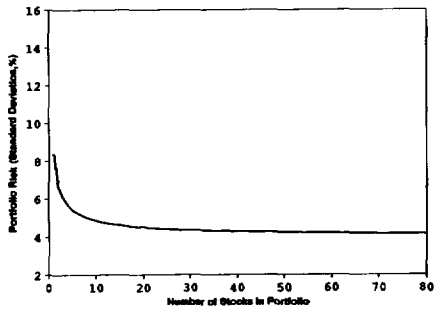


Exhibit 3. Upper and Lower Confidence Limits of Portfolio Risk

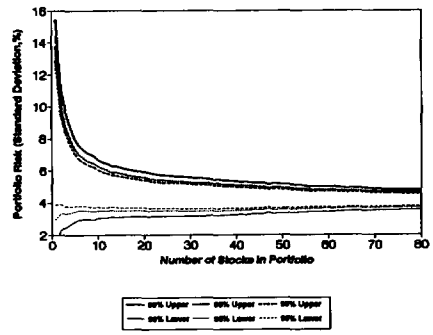


Exhibit 1 also cites the interpretations provided by the various authors of what this curve implies. The minimum number of stocks recommended by these authors for a risk-efficient portfolio varies from a low of eight to a high of twenty. Some authors imply that further diversification is unnecessary; others are emphatic: "Further spreading of the portfolio's assets is *superfluous diversification* (author's emphasis) and should be avoided" [4, p. 231]. Even without such clear guidance, our interpretation of these recommendations is that the investor with, say, $10,000, would be indifferent in terms of risk between buying, say, ten random S&P stocks and investing in an S&P index fund. (Note that the whole discussion is in terms of risk; returns are not being analyzed, so professional management fees and transaction costs are not being considered.)

However, the flaw in these recommendations lies in overlooking the fact that an investor typically has but one portfolio (in any particular universe of stocks), and so perhaps should be unwilling to jeopardize his/her funds on the basis of the *average* outcome of a large number of equal-size portfolios chosen from that same universe. Having but one portfolio from a particular universe, the investor has to face the fact that the risk on *his/her* portfolio can be slightly, or even substantially, above or below the average. Briefly, the standard recommendations hide the fact that a range of outcomes underlies every point on the curve in Exhibit 2.

II. The Proposed Recommendation

To illustrate the range of risk that is hidden by Exhibit 2, Exhibit 3, with similar axes to Exhibit 2, shows the curves that would statistically accommodate 99 percent, 95 percent, and 90 percent confidence intervals of the 80,000 simulated portfolios. The effect of portfolio size on reducing diversifiable risk is still clear in the new exhibit — for any pair of curves does converge; but notice how risk reduction is delayed. Diversifiable risk is not effectively eliminated at eight, 10, 15, or even at 20 stocks. The upper and lower limit curves become virtually parallel only at a portfolio size much greater than 20 stocks.

Perhaps a more illustrative procedure than Exhibit 3 is to assign the mean risk on a portfolio a value of 100 and express possible portfolio risks as a percentage of this. (Some Exhibit 2s are in fact constructed this way.) Exhibit 4 gives some examples. Thus the one-stock portfolio risk, when compared with the average one-stock portfolio, has a 0.5 percent chance of being 185 percent greater than the average, a 2.5 percent chance of being 164 percent greater than the average, and a 5 percent chance of being 154 percent greater than the average. Perhaps the best way to read Exhibit 4 is to choose a probability level at which the typical investor would feel comfortable and then run down that column until the percentage risks are also acceptable. While this must essentially be a personal decision (different investors have different degrees of "risk-aversion"), it would seem that the usual recommendation of eight to 20 stocks is a severe underestimate of the minimum size needed to diversify with comfort. The actual minimum number of stocks for an investor would depend on the universe of stocks being analyzed, the personal risk preferences, and the desired confidence intervals.

Exhibit 4. Upper Risk Ranges of Mean Portfolio Risk

	Upper Confidence Limit			
# of Stocks	99%	95%	90%	Mean Risk [a]
1	185	164	154	100
8[b]	141	131	126	100
10[b]	137	128	124	100
12[b]	135	127	123	100
15[b]	133	125	121	100
20[b]	130	123	119	100
30	127	121	118	100
40	124	118	115	100
50	121	116	114	100
60	119	114	112	100
70	116	112	110	100
80	115	111	110	100

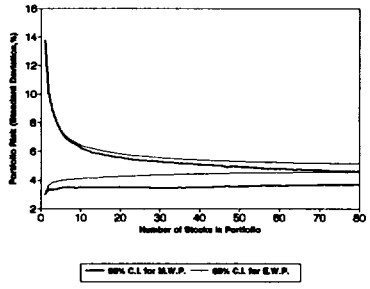
[a] The mean risk of each size portfolio is standardized at 100.

[b] These are the recommendations for the minimum number of stocks in a portfolio, in many textbooks. See Exhibit 1.

Two other points should be noted. Firstly, a conclusion that at least 30 stocks is the appropriate minimum has been arrived at previously, but by a different route. Statman [11] argued that diversification should be increased as long as the average marginal benefits (risk reduction) exceed the average marginal costs (transaction costs of buying more stocks) [p. 354]. Essentially, Statman was exploiting the slope in Exhibit 2 that continues between the usual recommendation (eight to 20 stocks) and the final leveling out of the *average* curve. As noted above, our argument that the minimum size is much greater than 20 stocks is based purely on the premise that an investor cannot afford to gamble on the probability of his or her being the average investor and should, if truly risk-averse, consider the range of risk outcomes that underlies the average.

Secondly, the 500 stocks used in our study had an equal chance of being selected for any portfolio, but once selected a stock was weighted in the portfolio according to the market value of the company. If investors weight their stock selections equally, it is possible that mean risk or convergence of risk range in Exhibit 3 might be different. The 80,000-portfolio simulation was rerun using equal probability of selection and equal weight once selected. The results are summarized in Exhibit 5. With equal weights, mean portfolio risk is higher but the deviation of risk is lower. However, Exhibit 5 does not appear to change our conclusion that the minimum number of stocks needed for diversification is substantially more than 20.

Exhibit 5. Market-Value Weighted versus Equal Weighted Portfolios



III. Summary

The recommendation that owning eight to 20 stocks achieves a risk-efficient portfolio pervades the investment literature. The risk-averse investor using this recommendation will find that in reality his/her portfolio is exposed to a range of risk which could be substantially different from the average. Probably there would be little consolation to the investor with an eight-stock portfolio experiencing a risk that was 141 percent of the risk on the average eight-stock portfolio if he/she was told that there was another investor at the other end of the distribution balancing him/her out -- yet this is what the standard recommendation is doing. To recognize that each individual investor is risk averse on *his/her own* individual portfolio outcome (and not merely risk averse on the *average* of all investors' outcomes), the standard diagram, Exhibit 2, should, in our view, be modified to conform to Exhibit 3. The empirical results from analyzing the S&P 500 data indicate that the recommendation would place the minimum number of stocks needed to achieve diversification much higher than 20 stocks. The actual number would depend upon the particular universe of stocks being analyzed, the weighting scheme used to construct portfolios, and the individual investor's desired confidence intervals and risk preference. ■

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We would like to thank the particpants at the UNLV Finance Research Seminar. We are also grateful to the anonymous referees and the editor, Jerry Stevens, for their helpful suggestions. The research grant from the First Interstate Bank Institute for Business Leadership at UNLV is gratefully acknowledged. Any error is our own responsibility.