

Measures of neutral real interest rate in Brazil

The economy's neutral real interest rate, a fundamental reference in the formulation of monetary policy, can be defined as the interest rate consistent, in the medium run, with inflation at the target and output growth equal to potential growth (Blinder, 1998). Thus, when the effective real interest rate is above the neutral rate, monetary policy is contractionary; when it is below, it is expansionary.

An inherent difficulty in using the neutral interest rate to guide monetary policy is that it is an unobservable variable. In addition, this rate varies over time in proportion to the evolution of its determinants, such as potential output growth rate; preferences of economic agents in terms of consumption, savings and investment; financial system efficiency; and the economy wide risk premium. Therefore, both the literature and the conduct of monetary policy consider a high degree of uncertainty to neutral interest rate estimates and prescribe that they be frequently reassessed. As an illustration of such uncertainty, this box presents several estimates of the neutral interest rate for the Brazilian economy¹, obtained by means of different methodologies, with the caveat that alternative approaches are present in the literature and may be considered by monetary authorities in their decision-making process. Therefore, this box is not an exhaustive analysis.

The inference on the neutral real interest rate combines, depending on the adopted methodology, the use of statistical filters and the support of economic theory at different levels. From this perspective, the methodologies presented in this box range from purely statistical approaches applied on ex-ante real interest rate measures to the analysis derived from Samba, the Banco Central do Brasil (BCB) DSGE model based on micro-foundations². The use of filters seek to remove high frequency movements from the data – which are not related with the dynamics of an object driven by structural low frequency movements. On the other hand, the use of economic theory seeks to impose restrictions on the evolution of the neutral interest rate over time that are consistent with information observed for other variables in the economy. Obviously, different theoretical models result in different restrictions and, therefore, different neutral interest rate trajectories.

It should be stressed that the starting point of the statistical approaches explored in this box are the observations of medium and long run interest rates. Intuitively, current interest rates lower than those expected in longer time horizons stimulate the demand and supply of credit, for both consumption and investment. Thus, long-term interest rates tend to be a good approximation to the neutral rate, even though, in principle, amenable to further improvement by removing residual cyclical effects still reflected in long-term rates. Despite being potentially informational, one difficulty of directly using long-term market rates is the presence of risk premia, such as the premium for investment horizon risk or exchange rate risk in foreign currency instruments. To circumvent this problem, this box makes extensive use of the Focus survey, either for directly calculating estimates of expected medium-run real interest rates, or to assist in the removal of risk premia from market rates in different time horizons.

1/ This topic was also covered in four other boxes of previous IR: "Equilibrium real interest rate" (September 2010); "Evolution of the equilibrium real interest rate in Brazil" (September 2012); "Neutral interest rate and the conduct of monetary policy in Brazil" (September 2017); and "Proxy of the neutral interest rate implied by expectations from the Focus survey" (December 2019).

2/ Model described in Fasolo *et al.* (2023).

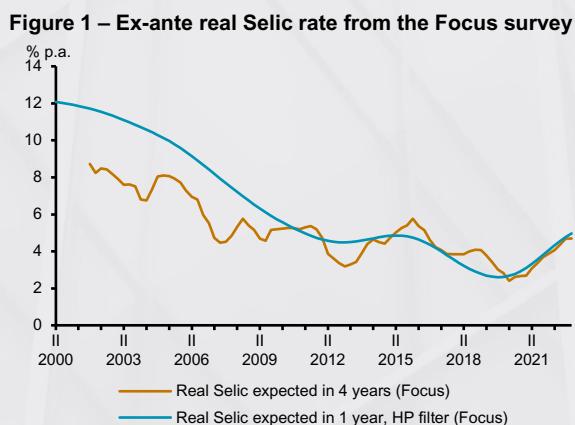
The following sections present the approaches adopted in the box, starting with the Focus survey expectations, moving to properly filtered market rates, and ending with structural models. The box also presents the most recent and the historical responses to the Pre-Copom Questionnaire (PCQ), in which the BCB with some frequency surveys neutral rate estimates from respondents in the same samples of the Focus survey.

1. Description of methodologies and results

1.1. Focus survey *ex-ante* real interest rate

This estimate is based on *ex-ante* real interest rates extracted from the Focus survey. With this in mind, the box considers the median of the distribution of nominal interest rates forecasts by respondents for both the short (1 year) and medium term (4 years). Next, the median of expected nominal interest rates are deflated by the median of expected inflation in each time horizon, thus obtaining the *ex-ante* real interest rates R_t^{focus1} and R_t^{focus4} , for 1 and 4 years, respectively.

As argued in the introduction, long-term interest rates free of risk premia tend to be good estimates for the neutral rate. Thus, the first estimate for the neutral real interest rate to be considered is the *ex-ante* 4-year rate of the Focus survey³ itself, i.e., $R_t^{n|focus4} = R_t^{focus4}$. The second estimate based on the Focus survey is obtained by excluding the more cyclical movements of the expected 1-year rate with an HP filter. The results obtained with different approaches are presented in Figure 1. For 2023Q1, the median estimate is 5.0% for 1 year and 4.7% for 4 years.



In addition to the point estimate above, the frequency distribution of point estimates may be obtained from the Focus survey itself⁴. Figure 2 presents the result of this exercise considering the 4-year horizon. The figure shows at first a downward trend of the neutral interest rate period, reaching a trough in early 2020 due to the Covid-19 pandemic, after which it starts an upward trajectory. This overall trend is shared by several of the measures presented in this box. For 2023Q1, the median estimate is 4.8%, with an interval of 4.1% and 5.4% between the quartiles (Table 1).

3/ In the text, notations with the superscript “n” correspond to results of inferences about the neutral interest rate, to differentiate them from other interest rate measures observed in the economy.
 4/ While in the previous paragraph the medians for expectations of nominal interest rates and inflation were combined, here the expected real rate for each respondent of the Focus survey was calculated to obtain the desired percentiles, which can generate differences in the calculation of the median.

Figure 2 – Real Selic 4 years ahead - Focus percentiles

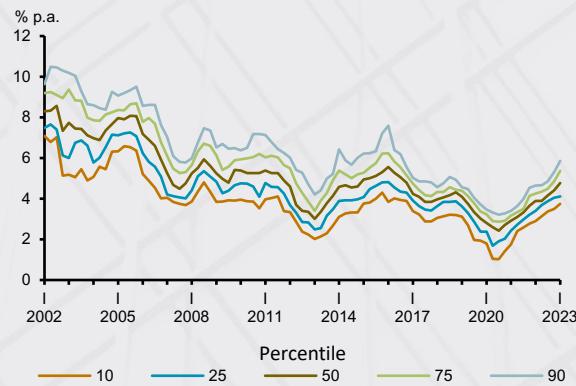


Table 1 – Ex-ante real interest rates from the Focus survey (4 years ahead)

2023Q1

Percentile	Rate (% p.a.)
10 th percentile	3.7
25 th percentile	4.1
50 th percentile	4.8
75 th percentile	5.4
95 th percentile	5.9

1.2. High and low frequency neutral interest rates

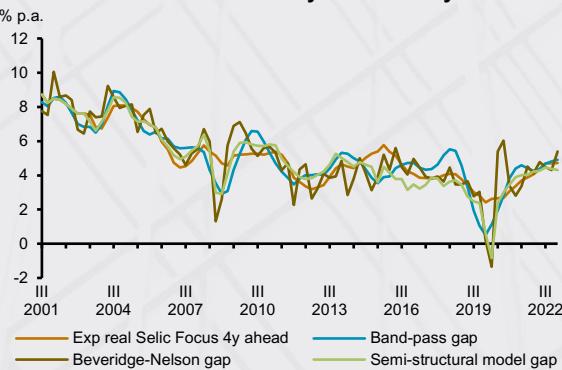
The following estimates are based on Roberts (2018). The author proposes to extract from long-term rates (relevant for the consumption and investment decisions) a “high frequency neutral rate”, defined as the rate that maintains the output gap at zero at each point in time. The author also proposes to smooth this measure to clear it from measurement errors and cyclical components, defining the smoothed measure as a “low frequency neutral rate”. The box applies this methodology, attempting to extract high and low frequency rates from a previously calculated long run rate, namely, the expected 4-year interest rate from the Focus survey, as described in the previous section.

More precisely, starting from the rate $R_t^{Focus^4}$ from the previous section, the “high frequency” neutral rate $R_t^{n,hf}$ is calculated as the solution to the following equation, which represents an stylized IS curve and thus captures the effect of the interest rate on the economy’s output gap:

$$gap_t = \eta gap_{t-1} - \sigma(R_t^{Focus^4} - R_t^{n,hf}) .$$

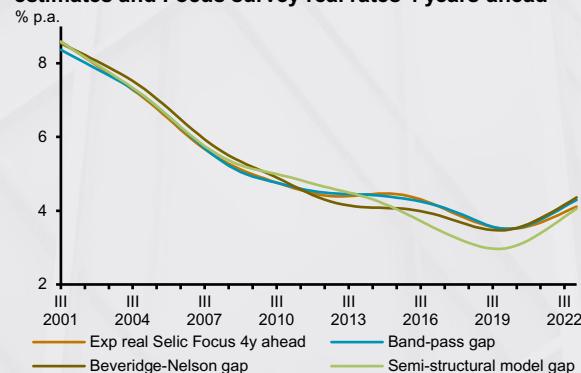
In this equation, the gap comes from the usual estimates in the literature, such as the Band-Pass and Beveridge-Nelson filters, or the output gap of the BCB’s semi-structural small-scale model. The parameter η measures the output gap persistence, and is calibrated at 0.84 based on the average persistence of output gaps considered in this exercise. The parameter σ measures the effect on economic activity of the degree of monetary policy tightening (assessed in terms of the medium-run rates), and is calibrated at 0.75 as suggested in Roberts (2018). It should be noticed that different output gap measures and parameters calibration would result in different estimates as well. In particular, for high values of output gap persistence or interest rate sensitivity, the calculated high-frequency neutral rate tends to be close to the rate $R_t^{Focus^4}$. Results for the calculation of the “high frequency neutral rate” are shown in Figure 3. Values for 2023Q1 are between 4.3% and 5.4%, which represent the values that would be necessary to immediately close the output gap in the stylized model above, relating the gap and expected real interest rate in a 4-year horizon.

Figure 3 – High frequency neutral rates from gap estimates and the Focus survey real rates 4 years ahead



Regarding the “low frequency” neutral rate $R_t^{n,lf}$, it is defined by Roberts (2018) as the result of smoothing the high frequency rate so as to remove measurement errors and economic cycle effects. Such smoothing is obtained with an HP filter applied to the high frequency neutral rate, i.e., $R_t^{n,lf} = \text{filter}(R_t^{n,hf})$. Figure 4 shows the results for the low frequency neutral interest rates. The values for 2023Q1 are between 4.1% and 4.4%, suggesting that in the most recent period the high frequency rate reflects, to a great extent, a relatively strong economic activity.

Figure 4 – Low frequency neutral rates from gap estimates and Focus survey real rates 4 years ahead



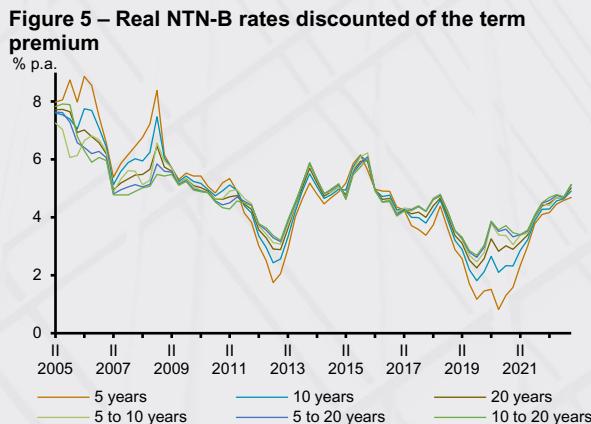
1.3. Market interest rates discounting the term premium

The following estimate is based on long-term market real interest rates, for which it is necessary to remove the term premium. The market based real interest rates used here are extracted from the National Treasury Notes Series B (NTN-B). NTN-B are securities issued by the National Treasury with nominal value updated by the IPCA variation and payment of an IPCA coupon. For longer terms, the IPCA coupons represent a quite reasonable approximation for the real interest rates of these securities⁵. The difference between the real interest rates of the market and the Focus survey at the same horizon is defined as the term premium⁶.

5/ See Araujo and Vicente (2017).

6/ It should be noticed that this methodology does not ensure the lack of arbitrage in the forward structure, which is argued to be a second-order factor for the calculation of the neutral interest rate. It should also be observed that four years is the maximum time horizon for which this method is feasible. This is the reason for using it as a parameter for the term premium in other time horizons, again with the purpose of making corrections with first-order effects on the estimated neutral rates.

The neutral rate estimate consists in the use of long-term market rates, discounted accordingly, as direct estimates for the neutral interest rate, i.e. $R_t^{n,market} = R_t^{market} - \text{term premium}$. Results are presented in Figure 5. Values for 2023Q1 are between 4.7% and 5.1%.



1.4. Uncovered interest parity

In an open economy with capital flows, the domestic and foreign interest rates are related by uncovered interest parity. Since this relationship should be present in the long run, it is reasonable to suppose that it also relates the natural interest rates of the respective economies. Therefore, the domestic neutral real interest rate can be estimated as the sum of the foreign neutral real interest rate and properly measured risk premia. Note that deflating nominal rates amounts imposes long run purchasing power parity. Among the relevant risk factors, this box considers sovereign and foreign exchange risks.

Taking the US economy as the reference foreign country, this exercise considers a nominal US rate deflated by inflation expectations, a US market real rate, and a US neutral interest rate estimated in the literature. More precisely, the nominal foreign interest rate considered here is the 1-year Treasury rate deflated by inflation expectations one year ahead, and smoothed with the HP filter⁷. In addition, the box also uses the 5-year real rate from Treasury Inflation-Protected Securities (TIPS) also smoothed by the HP filter, and the full sample smoothed neutral real interest rate estimated by the Federal Reserve Bank of New York (FED-NY)⁸.

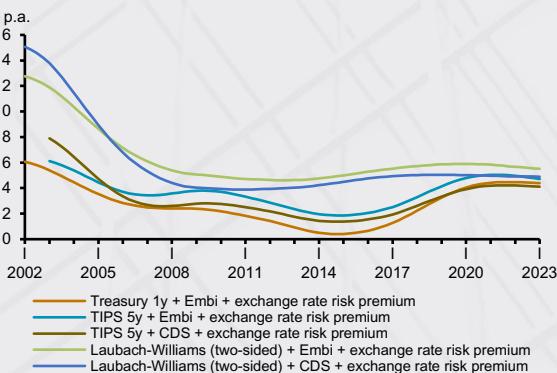
For the sovereign risk premia, this box uses the Embi+ and the 5-year CDS for Brazil, both of them HP filtered to obtain long run trends. To calculate the foreign exchange risk premium, the starting point is the parity relationship between the domestic interest rate in BRL (represented in the DI market) and the domestic interest rate in USD (represented in the exchange rate coupon market). According to this parity relationship, the difference between rates should correspond to the sum of the expected BRL depreciation against the USD and the foreign exchange risk premium. The Focus survey in the corresponding time horizon is used again to calculate the expected foreign exchange depreciation implicit in the interest rate differential and thus obtain the risk premium. Similarly to the sovereign risk indicators, an HP filter is applied to the foreign exchange risk premia.

The resulting neutral real interest rates obtained through this approach for the three real representative US interest rates can be seen in Figure 6. Values for 2023Q1 are between 4.1% and 5.5%.

7/ Expectations obtained from the Survey of Professional Forecasters, conducted by the Federal Reserve Bank of Philadelphia.

8/ Bilateral estimate using the Laubach-Williams model.

Figure 6 – Neutral rates based on foreign rates and risk premium



1.5. Neutral interest rate filtered by the semi-structural small-scale model

This approach combines several elements from the previous approaches. In particular, the aggregate model estimates the behavior of aggregate demand (IS curve), monetary authority (Taylor rule), price makers (Phillips curve), and investors arbitraging international interest rates (uncovered interest parity). All these elements were considered in isolation in the previous methods. In the aggregate model, however, all these elements are considered together and properly weighted in the filter that estimates the natural real interest rate trend.⁹ In an exercise that imposes a neutral rate of 4.5% in 2023Q3, the figures for 2023Q1 are around 4.8%.

1.6. Neutral interest rate in the Samba model

The Samba model approach will now be considered. Compared to the small-scale model of the previous sub-section, the Samba model encompasses a wider range of patterns in agent's behavior and considers long run trends consistent with economic theory.

Following the usual definition in the literature, the natural rate of the Samba model is the real interest rate in a counterfactual economy where there is no price and wage nominal rigidity.¹⁰ In other words, it is the real interest rate in an hypothetical economy where only real supply and demand shocks determine its evolution over time. For the purpose of comparison with neutral interest rate measures obtained by the other methods in this box, the presentation follows Del Negro *et al.* (2017) by computing the natural interest rate extracted from the Samba model as its expectation for 2 and 5 years ahead. By adopting this procedure, the presented values tend to isolate short-term components – similarly to what filters do in previous methodologies.

The evolution of the natural rate of the Samba model largely tracks the expected variation in productivity within the model. In fact, productivity is a key element in DSGE models for estimating the natural rate, in as far as it links present consumption decisions with the future possibility of higher consumption just by raising productivity.¹¹ The natural rate reached its lowest values during the 2014-2016 crisis, when the average growth of the Brazilian economy showed a strong deceleration, remaining at a very low level

9/ Further details on the model are available in the box "Revision of the small-scale aggregate model", of the December 2021 IR.

10/ See Del Negro *et al.* (2017) on the formal definition of the natural interest rate in a more recent context, and Zhang *et al.* (2021) on the calculation of natural rates for small open economies.

11/ In technical terms, productivity affects the expected variation for consumption in the Euler equation that determines intertemporal choice of household consumption. The discount factor for this intertemporal choice – the real interest rate – considers the expected productivity of the economy.

(Figures 7 and 8). During the pandemic, the natural interest rate assumes negative values, reflecting the strong deceleration of aggregate demand in the period, with the consequent need for stimuli to reach the “natural” output level. With the end of the pandemic and economic activity recovery, the rate rose again. For 2023Q1, the estimated neutral interest rate lies between 4.5% and 4.8%.

It should be highlighted that the rate presented here is conditional on the DSGE model's structure defined *a priori*. Therefore, on the one hand, the choices about the value of the stationary state in the model justify the fluctuations of the natural interest rate around 4% p.a. in the entire sample. On the other hand, missing elements in the model, such as the financial accelerator, capable of relating economic activity to risk premium measures, may affect the estimation of the natural rate.

Figure 7 – Natural interest rate – 2 years
Samba model

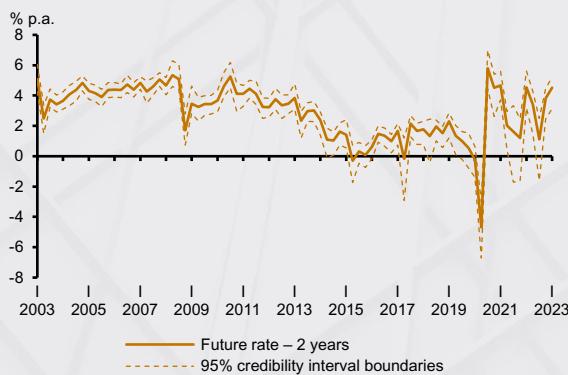
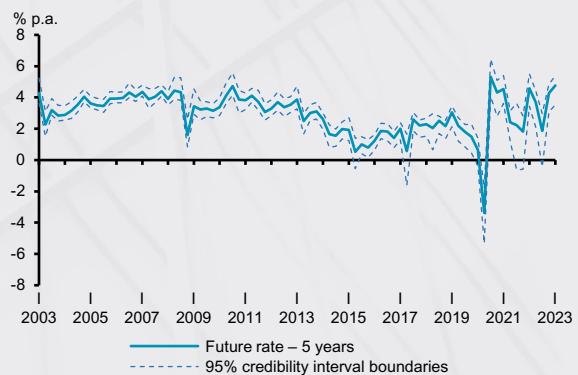


Figure 8 – Natural interest rate – 5 years
Samba model



2. Neutral real interest rates estimated by economic analysts consulted by the BCB

On several occasions over the past few years, in order to consult economic analysts' estimates for the Brazilian economy's neutral interest rate, the BCB has sent the PCQ¹² to participants in the Market Expectations System.¹³ Tables 2 and 4 summarize the survey results, which help to contextualize the analysis made by the BCB throughout the box.

Considering the median response (Figure 9), one can see that between 2017 and 2020 the analysts' estimate for the neutral real interest rate decreased for all time horizons¹⁴. Between June 2020 and August 2021, when the current domestic monetary tightening cycle had already begun, estimates of the short-term and two-year ahead neutral rate rose, but long-term (“five years from now”) estimates remained stable. From August 2021 to the most recent consultation, estimates for all time horizons have risen. The sharpest increase is between August and December 2021, with subsequent adjustments being more moderate.

Also according to the same data, one notes that in 2017, analysts' estimate for the neutral rate was higher at the near-term, at 5%, than at the more distant horizons, at 4.5% in two years and 4% in five years. The opposite is seen in June 2020 (around the most acute moment of the global economic crisis associated with the Covid-19 pandemic), when estimates of the short-term neutral real interest rate, at 2%, were at a lower level than estimates for more distant time horizons, at 2.5% in two years and 3% in five years. In the

12/ Further information about this survey may be obtained at <https://www.bcb.gov.br/controleinflacao/precopm>.

13/ In April 2017 and June 2020, the question was “What is your estimate for the neutral interest rate of the Brazilian economy, in the short term, two years from now, and five years from now?” On the other occasions, the question became explicit about the neutral real interest rate and the exact wording was “What is your estimate for the neutral real interest rate of the Brazilian economy, in the short term, two years from now, and five years from now?”

14/ These data were published on the PCQ page or in previous IR, except for the June 2020 data that had not been released so far.

following surveys, analysts' expectation was for a relatively stable neutral real interest rate between the short term and the next five years.

Finally, data from the most recent survey show less dispersion between point estimates of the neutral rate in the short term, when the range between percentiles 25 and 75 is 4.5% to 5%, than in the medium to long term, when the range is 4% to 5% (see Tables 2 to 4 and Figure 10).

Figure 9 – Median of the neutral interest rate for different time horizons according to the PCQ



Figure 10 – Quartiles of the neutral interest rate two years ahead according to the PCQ



Table 2 – Quartiles of the neutral real interest rate for the short-term according to the PCQ

	25th p	med	75th p
Apr 2017	4.8	5.0	5.5
Jun 2020	1.0	2.0	2.5
Aug 2021	3.0	3.0	3.5
Dec 2021	3.2	4.0	4.2
Dec 2022	4.0	4.5	5.0
Jun 2023	4.5	4.8	5.0

Table 3 – Quartiles of the neutral real interest rate for 2 years ahead according to the PCQ

	25th p	med	75th p
Apr 2017	4.0	4.5	5.0
Jun 2020	2.0	2.5	3.0
Aug 2021	3.0	3.0	3.5
Dec 2021	3.5	4.0	4.0
Dec 2022	4.0	4.5	5.0
Jun 2023	4.0	4.8	5.0

Table 4 – Quartiles of the neutral real interest rate for 5 years ahead according to the PCQ

	25th p	med	75th p
Apr 2017	3.5	4.0	4.5
Jun 2020	2.5	3.0	3.3
Aug 2021	3.0	3.0	3.5
Dec 2021	3.2	4.0	4.0
Dec 2022	4.0	4.3	4.9
Jun 2023	4.0	4.5	5.0

3. Conclusion

Table 5 summarizes the different measures for 2023Q1, including the more recent PCQ presented in the previous section. The median and average of the neutral real interest rate obtained by the different methodologies are both at 4.8%, with an interval from 4.5% to 5.0% considering the 25th and 75th percentiles.

The neutral real interest rate is an unobservable variable whose estimation is subject to high uncertainty. In general, estimates pointed to a downward trend of the neutral interest rate, which reaches a trough in early 2020, due to the Covid-19 pandemic, and then starts an upward trajectory.

Table 5 – Neutral real interest rate in Brazil
Estimates for 2023Q1

Methodology	Rate (% p.a.)
Ex-ante real Selic rate from the Focus survey	
Real Selic expected in 4 years	4.7
Real Selic expected in 1 year, HP filter	5.0
High frequency neutral rates	
Band-Pass gap	4.9
Beveridge-Nelson gap	5.4
Semi-structural model gap	4.3
Low frequency neutral rates	
Band-Pass gap	4.3
Beveridge-Nelson gap	4.4
Semi-structural model gap	4.1
Real market rates discounted of the term premium	
5 years	4.7
10 years	4.9
20 years	5.0
5 to 10 years	5.1
5 to 20 years	5.1
10 to 20 years	5.1
Uncovered interest rate parity	
Treasury 1y + Embi + exchange rate risk premium	4.4
TIPS 5y + Embi + exchange rate risk premium	4.7
TIPS 5y + CDS + exchange rate risk premium	4.1
Laubach-Williams (two-sided) + Embi + exchange rate risk premium	5.5
Laubach-Williams (two-sided) + CDS + exchange rate risk premium	4.9
Natural interest rate from the Samba model	
Two years future rate	4.5
Five years future rate	4.8
Neutral real interest rate from the PCQ ¹	
Short-term median	4.8
2 years median	4.8
5 years median	4.5
Summary	
Mean	4.8
Median	4.8
25th percentile	4.5
75th percentile	5.0

1/ Survey of June 2023.

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