

Assignment 4

COMP3121/9101 21T3

Released November 3, due November 17

In this assignment we apply maximum flow. There are *five problems*, for a total of 100 marks.

Your solutions must be typed, machine readable PDF files. *All submissions will be checked for plagiarism!*

For each question requiring you to design an algorithm, you *must* justify the correctness of your algorithm. If a time bound is specified in the question, you also *must* argue that your algorithm meets this time bound.

Partial credit will be awarded for progress towards a solution.

1. (20 points) A school is doing a fire drill. The school consists of n rooms and m corridors, where each corridor connects two rooms. There are x students who must be moved from room 1 (their classroom, with teacher A) to room n (a safe room, where teacher B is waiting).

The teachers decided to divide the class of x students into several waves. Each wave will be released from room 1 by teacher A , and make their way through the corridors. To prevent overcrowding, each corridor has a limit l_i , which is the maximum number of students in a single wave who can use this corridor. Once all students in this wave have reached room n , teacher B will call teacher A and instruct them to send the next wave.

Design a polynomial time algorithm which determines the minimum number of waves that must be formed and the number of students in the largest wave.

2. (20 points) There are some stones in a rectangular grid with r rows and c columns. The stone in row i and column j has height h_{ij} and holds a_{ij} lizards. Both h_{ij} and a_{ij} are non-negative integers. If h_{ij} is zero, this denotes that there is no stone at (i, j) and hence a_{ij} is guaranteed to also be zero.

Each lizard can jump between two stones if they are separated a distance of at most d , i.e. it can jump from a stone at (i_1, j_1) to an unoccupied stone at (i_2, j_2) if

$$\sqrt{(i_1 - i_2)^2 + (j_1 - j_2)^2} \leq d.$$

However, the stones are not stable, so whenever a lizard leaves a stone, the height of the stone is decreased by 1. If the new height of the stone is zero, the stone has disappeared below the surface. Any remaining lizards on this stone will drown, and lizards will no longer be able to jump onto this stone.

We want to help as many lizards as possible to escape the grid. A lizard escapes if a jump of distance k takes them beyond the boundary of the grid.

Design a polynomial time algorithm to find the maximum number of lizards that can escape from the grid.

3. (20 points) You are the head of n spies, who are all wandering in a city. On one day you received a secret message that the bad guys in this city are going to arrest all your spies, so you'll have to arrange for your spies to run away and hide in strongholds.

You have T minutes before the bad guys arrive. Your n spies are currently located at

$$(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n),$$

and your m strongholds are located at

$$(a_1, b_1), (a_2, b_2), \dots, (a_m, b_m).$$

The i th spy can move v_i units per minute, and each stronghold can hold only one spy.

Design a polynomial time algorithm which determines which spies should be sent to which strongholds so that you have the maximum number of spies hiding from the bad guys.

4. (20 points) Alice is the manager of a café which supplies n different kinds of drink and m different kinds of dessert.

One day the materials are in short supply, so she can only make a_i cups of each drink type i and b_j servings of each dessert type j .

On this day, k customers come to the café and the i th of them wants to order one cup of their favourite drink c_i and one serving of their favourite dessert d_i . If Alice refuses to serve them, or if either their favourite drink or their favourite dessert is unavailable, the customer will instead leave the café and provide a poor rating.

Alice wants to save the restaurant's rating. From her extensive experience with these k customers, she has listed out the favourite drink and dessert of each customer, and she wants your help to decide which customers' orders should be fulfilled.

Design a polynomial time algorithm which determines the smallest possible number of poor ratings that Alice can receive.

5. (20 points) You will be in charge of a delivery network consisting of n warehouses for d days. During this time, your job is to redistribute items between these warehouses – specifically, the i th warehouse starts with A_i items and must have at least B_i items by the end of the d th day. All items are identical, so this requirement can be fulfilled using items from any warehouse.

You are also given a schedule of m deliveries between warehouses that you can use to redistribute items. The k th delivery leaves warehouse w_k on the evening of day t_k , carrying at most c_k items, and drops them all off at warehouse w'_k on the morning of day t'_k . You can also keep an unlimited number of items at each warehouse overnight.

Design a polynomial time algorithm which determines whether it is possible to have at least B_i items present at each warehouse i at the end of the d th day.