Quiz (Week 9)

Logic

Question 1

Which one (or more) of expressions listed below is a possible formalisation of the phrase: *Not all that glitters is gold.*

1. ✗ $\exists x.\ \ (\mathsf{Glitter}(x) \rightarrow \mathsf{Gold}(x))$
2. ✗ $\forall x.\ \ (\neg(\mathsf{Glitter}(x)) \rightarrow \mathsf{Gold}(x))$
3. ✓ $\neg\forall x.\,(\mathsf{Glitter}(x) \rightarrow \mathsf{Gold}(x))$
4. ✗ $\forall x.\ \ (\neg(\mathsf{Glitter}(x) \rightarrow \mathsf{Gold}(x)))$

Question 2

Which of the expressions listed below is a possible formalisation of the Abraham Lincoln quote: *You can fool all the people some of the time, and some of the people all the time, but you cannot fool all the people all the time.*

1. ✗ $\forall p.\,\forall t.\,(\mathsf{Fool}(p,t) \wedge \neg\mathsf{Fool}(p,t))$
2. ✗ $(\forall p.\,\exists t.\,\mathsf{Fool}(p,t)) \wedge (\exists p.\,\forall t.\,\mathsf{Fool}(p,t)) \rightarrow \exists p.\,\exists t.\,\neg\mathsf{Fool}(p,t)$
3. ✗ $(\forall p.\,\exists t.\,\mathsf{Fool}(p,t)) \wedge (\exists p.\,\forall t.\,\mathsf{Fool}(p,t)) \wedge \neg\forall p.\,\forall t.\,\neg\mathsf{Fool}(p,t)$
4. ✓ $(\forall p.\,\exists t.\,\mathsf{Fool}(p,t)) \wedge (\exists p.\,\forall t.\,\mathsf{Fool}(p,t)) \wedge \neg\forall p.\,\forall t.\,\mathsf{Fool}(p,t)$
5. ✗ $(\forall p.\,\exists t.\,\mathsf{Fool}(p,t)) \wedge (\exists p.\,\forall t.\,\mathsf{Fool}(p,t)) \wedge (\mathit{False} \rightarrow \forall p.\,\forall t.\,\mathsf{Fool}(p,t))$
6. ✓ $(\exists t.\,\forall p.\,\mathsf{Fool}(p,t)) \wedge (\forall t.\,\exists p.\,\mathsf{Fool}(p,t)) \wedge \neg\forall t.\,\forall p.\,\mathsf{Fool}(p,t)$

Question 3

Here is a proof of a logical statement in *natural deduction style*:

$$\dfrac{\dfrac{\dfrac{\overline{A \vee B}^{\alpha} \quad \dfrac{\dfrac{\overline{A \to C}^{\beta} \quad \overline{A}^{\delta_1}}{C}\textcircled{5}}{C \vee D}\textcircled{3} \quad \dfrac{\dfrac{\overline{B \to D}^{\gamma} \quad \overline{B}^{\delta_2}}{D}\textcircled{5}}{C \vee D}\textcircled{4}}{C \vee D}\textcircled{2}^{\delta_1;\delta_2}}{(B \to D) \to C \vee D}\textcircled{1}^{\gamma}}{(A \to C) \to (B \to D) \to C \vee D}\textcircled{1}^{\beta}$$

$$\dfrac{(A \to C) \to (B \to D) \to C \vee D}{A \vee B \to (A \to C) \to (B \to D) \to C \vee D}\textcircled{1}^{\alpha}$$

The names of the rules used have been replaced with circled numbers. What is the rule used in each position?

1. ✗ $\textcircled{1}$ is $\to$-E; $\textcircled{2}$ is $\vee$-I; $\textcircled{3}$ is $\vee$-E$_1$; $\textcircled{4}$ is $\vee$-E$_2$; $\textcircled{5}$ is $\to$-I
2. ✗ $\textcircled{1}$ is $\to$-I; $\textcircled{2}$ is $\vee$-E; $\textcircled{3}$ is $\vee$-I$_2$; $\textcircled{4}$ is $\vee$-I$_1$; $\textcircled{5}$ is $\to$-E
3. ✗ $\textcircled{1}$ is $\vee$-I; $\textcircled{2}$ is $\to$-E; $\textcircled{3}$ is $\to$-I$_2$; $\textcircled{4}$ is $\to$-I$_1$; $\textcircled{5}$ is $\vee$-E
4. ✗ $\textcircled{1}$ is $\vee$-I; $\textcircled{2}$ is $\to$-E; $\textcircled{3}$ is $\to$-I$_1$; $\textcircled{4}$ is $\to$-I$_2$; $\textcircled{5}$ is $\vee$-E
5. ✓ $\textcircled{1}$ is $\to$-I; $\textcircled{2}$ is $\vee$-E; $\textcircled{3}$ is $\vee$-I$_1$; $\textcircled{4}$ is $\vee$-I$_2$; $\textcircled{5}$ is $\to$-E

Curry-Howard Correspondence

Question 4

Selct all of the following types for which you can write a total, terminating Haskell function.

1. ✓ `(a -> b) -> (b -> c) -> (a -> c)`
2. ✗ `((a, b) -> c) -> (a -> c)`
3. ✓ `(a -> c) -> ((a, b) -> c)`
4. ✗ `((a -> c) -> c) -> a`

Question 5

What is the computational interpretation of the theorem $(A \to (B \to C)) \to ((A \wedge B) \to C)$?

1. ✓ The function that transforms a *curried* function to an *uncurried* one.
2. ✗ The function that transforms an *uncurried* function to a *curried* one.
3. ✗ The function that creates a tuple of the two given A values and B values.
4. ✗ There is no computational interpretation of this logical formula.

Question 6

Which of the following Haskell programs constitutes a valid proof of the theorem given in Question 3?

1. ✓

```
proof (Left a) f g = Left (f a)
proof (Right b) f g = Right (g b)
```

2. ✗

```
proof (a, b) f g = (f a, g b)
```

3. ✗

```
proof x f g = if x then f x else g x
```

4. ✗

```
proof x f g = x (f x) (g x)
```

5. ✗

```
proof (Left a) f g = Left (g a)
proof (Right b) f g = Right (f b)
```

Question 7

Below is a complicated proof that assuming A and B, we can derive $A \wedge B$:

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{B \wedge A}^{\delta}}{A}\wedge\text{-E}_2 \quad \dfrac{\overline{B \wedge A}^{\delta}}{B}\wedge\text{-E}_1}{A \wedge B}\wedge\text{-I}}{(B \wedge A) \rightarrow (A \wedge B)}\rightarrow\text{-I}^{\delta} \quad \dfrac{B \quad A}{B \wedge A}\wedge\text{-I}}{A \wedge B}\rightarrow\text{-E}$$

What is the equivalent program to this proof, in typed lambda calculus (using Haskell-style syntax for pairs)? Assume $a : A$ and $b : B$.

1. ✗ (a, b)
2. ✓ $(\lambda x.\ (\mathsf{snd}\ x, \mathsf{fst}\ x))\ (b, a)$
3. ✗ $(\mathsf{snd}\ (b, a), \mathsf{fst}\ (b, a))$
4. ✗ $(\mathsf{fst}\ (a, b), \mathsf{snd}\ (a, b))$
5. ✗ $(\lambda x.\ (\mathsf{fst}\ x, \mathsf{snd}\ x))\ (a, b)$

Question 8

What proof results from applying *proof simplification* as much as possible to the proof from Question 7?

1. ✗

$$\dfrac{\overline{A} \qquad \dfrac{\dfrac{B \quad A}{B \wedge A}\wedge\text{-I}}{B}\wedge\text{-E}_1}{A \wedge B}\wedge\text{-I}$$

2. ✗

$$\dfrac{\dfrac{\dfrac{B \quad A}{B \wedge A}\wedge\text{-I}}{A}\wedge\text{-E}_2 \qquad \overline{B}}{A \wedge B}\wedge\text{-I}$$

3. ✓

$$\dfrac{\overline{A} \quad \overline{B}}{A \wedge B}\wedge\text{-I}$$

4. ✗

$$\dfrac{\dfrac{\dfrac{B \quad A}{B \wedge A}\wedge\text{-I}}{A}\wedge\text{-E}_2 \qquad \dfrac{\dfrac{B \quad A}{B \wedge A}\wedge\text{-I}}{B}\wedge\text{-E}_1}{A \wedge B}\wedge\text{-I}$$

5. ✗

$$\dfrac{\dfrac{\dfrac{A \quad B}{A \wedge B}\wedge\text{-I}}{A}\wedge\text{-E}_1 \qquad \dfrac{\dfrac{A \quad B}{A \wedge B}\wedge\text{-I}}{B}\wedge\text{-E}_2}{A \wedge B}\wedge\text{-I}$$

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