Question 5

Zeal Liang

Z5325156

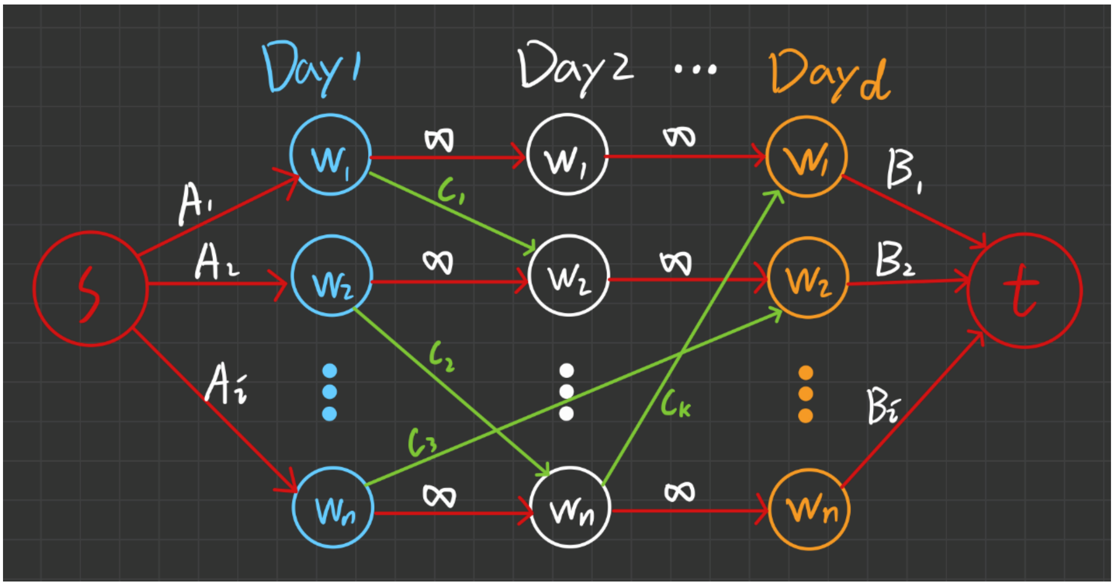
First, we need to transform the problem into a graph. We need to map all the warehouses into vertices. Because the deliveries occur on different days, we need to make d copies of the warehouse's vertices. Arrange them in order. For example, Day1: W1, W2, W3 as one column, Day2: W1, W2, W3 as another column, etc.

Then connect the vertices of the warehouses of the first day with source. The capacity of these edges is the number of items A_i that each warehouse has at the beginning. Then connect the vertices of each warehouse of the dth day to sink. The capacity of these edges is the B_i items that needed by the end of the dth day.

Moreover, the vertices of the same warehouse on adjacent days are also to be connected to each other, with no limit on the capacity of their edges.

Next, consider the case of deliveries: if a delivery is sent from W_k to W_k' on day t_k and arrives on day t_k'. Let the vertex of W_k on day t_k and the vertex of W_k' on day t_k' be connected by an edge, whose capacity is c_k.

We end up with a graph that looks like this.



Then we run the Edmonds Karp algorithm on this graph to find the maximum flow. If the maximum flow obtained is equal to the sum of all the B_i together it is possible to have at least B_i items present at each warehouse i at the end of the dth day, if not then it is impossible.

Transforming the problem into a graph takes $O(nd + m)$ time complexity.

Then the time complexity of running the Edmonds-Karp algorithm is $O(|nd| * |nd + m|^2)$. Therefore, the total time complexity of this algorithm is $O(|nd| * |nd + m|^2)$, which meets the polynomial time requirement of the question.