

## Question 5

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### Solution:

Since it is closely related to the all pairs shortest path problem, except that we are multiplying edge weights rather than adding them. Therefore, we can use the Floyd-Warshall algorithm and simply change plus sign for key comparison steps to multiplication sign to solve this problem.

### Subproblems:

Let  $MaxSafe(G)$  be the problem of determining  $safety(i, j)$ , count if the safety of a path from vertex  $i$  to  $j$  is greater than from  $i$  to  $k$  times from  $k$  to  $j$ .

### Recurrence:

For  $0 \leq k < n$ ,  $0 \leq i < n$  and  $0 \leq j < n$ :

$$safety(i, j) = \max(safety(i, j), safety(i, k) * safety(k, j)).$$

### Base cases:

For every edge in graph  $G$ , if there exists an edge from vertex  $i$  to  $j$ , set the weights into the corresponding  $safety(i, j)$ , if not let  $safety(i, j) = 0$ .

Because the algorithm uses three nested loops to traverse our directed graph  $G$ , the time complexity of the algorithm is  $O(n^3)$ .