Question 5

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(a)

We can derive the relationship between $\boldsymbol{f(n)}$ and $\boldsymbol{g(n)}$ by computing $\boldsymbol{\lim_{n\to\infty}\left(\frac{f(n)}{g(n)}\right)}$.

Since $\boldsymbol{f(n) = n^{1+\log n}}$, $\boldsymbol{g(n) = n \log n}$, and $\boldsymbol{\log_e n = \ln n}$.

Then we have

$$\boldsymbol{\lim_{n\to\infty}\left(\frac{n^{1+\ln n}}{n \ln n}\right)}$$

$$\boldsymbol{= \lim_{n\to\infty}\left(\frac{n * n^{\ln n}}{n \ln n}\right)}$$

$$\boldsymbol{= \lim_{n\to\infty}\left(\frac{n^{\ln n}}{\ln n}\right)}$$

$$\boldsymbol{= \infty.}$$

Thus, the growth rate of $\boldsymbol{f(n)}$ is greater than that of $\boldsymbol{g(n)}$, which means

$\boldsymbol{f(n) = \Omega(g(n))}$.

So (II) fits this pair of functions.

(b)

We can derive the relationship between $\boldsymbol{f(n)}$ and $\boldsymbol{g(n)}$ by computing $\boldsymbol{\lim_{n \to \infty} \left(\frac{f(n)}{g(n)}\right)}$.

Since $\boldsymbol{f(n) = n^{1+\frac{1}{2}\cos(\pi n)}}$, and $\boldsymbol{g(n) = n.}$

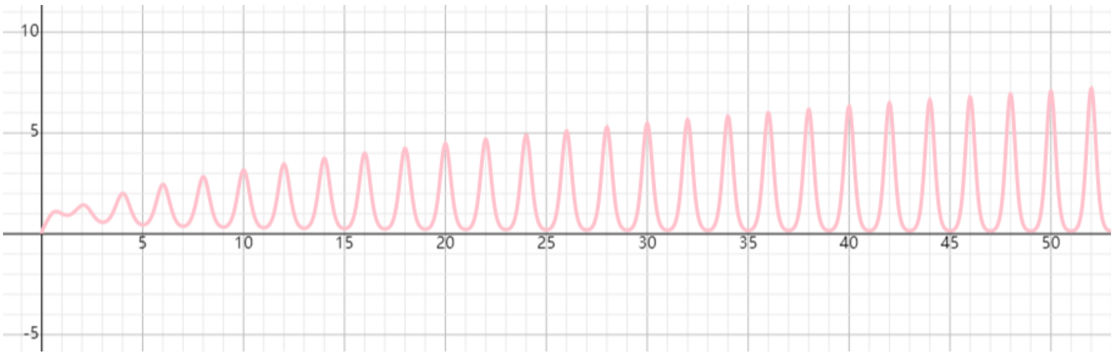
Then we have

$$\lim_{n \to \infty} \left(\frac{n^{1+\frac{1}{2}\cos(\pi n)}}{n} \right)$$

$$= \lim_{n \to \infty} \left(\frac{n * n^{\frac{1}{2}\cos(\pi n)}}{n} \right)$$

$$= \lim_{n \to \infty} \left(n^{\frac{\cos(\pi n)}{2}} \right)$$

= 0 . . ∞ (diverges)



When we look at the graph of this equation, we see that:

When $\boldsymbol{n}$ is odd, the limit tends to 0, and at this point $\boldsymbol{g(n)}$ is the upper bound of $\boldsymbol{f(n)}$.

When $\boldsymbol{n}$ is even, the limit tends to infinity, and at this point $\boldsymbol{f(n)}$ is the upper bound of $\boldsymbol{g(n)}$.

So, the relationship between this pair of functions is unstable, then only (IV) fits this scenario.

(c)

We can derive the relationship between $\boldsymbol{f(n)}$ and $\boldsymbol{g(n)}$ by computing $\boldsymbol{\lim_{n\to\infty}\left(\frac{f(n)}{g(n)}\right)}$.

Since $\boldsymbol{f(n) = \log_2 n^{\log(n\log n)}}$, $\boldsymbol{g(n) = (\log n)^2}$, and $\boldsymbol{log_e n = ln\, n}$.

Then we have

$$\boldsymbol{\lim_{n\to\infty}\left(\frac{\log_2 n^{\ln(n\ln n)}}{(\ln n)^2}\right)}$$

$$\boldsymbol{= \lim_{n\to\infty}\left(\frac{\ln(n\ln n)\log_2 n}{(\ln n)^2}\right)}$$

$$\boldsymbol{= \lim_{n\to\infty}\left(\ln(n\ln n)\frac{\log_2 n}{(\ln n)^2}\right)}$$

$$\boldsymbol{= \lim_{n\to\infty}\left(\ln(n\ln n) * \lim_{n\to\infty}\left(\frac{\log_2 n}{(\ln n)^2}\right)\right)}.$$

Since

$$\boldsymbol{\lim_{n\to\infty}(\ln(n\ln n)) = \infty},$$

$$\boldsymbol{\lim_{n\to\infty}\left(\frac{\log_2 n}{(\ln n)^2}\right) = \frac{1}{2\ln(2)}}.$$

Thus

$$\boldsymbol{\lim_{n\to\infty}\left(\frac{f(n)}{g(n)}\right) = \infty * \frac{1}{2\ln(2)} = \infty}.$$

Because $\boldsymbol{\frac{1}{2\ln(2)} < 1}$, there will be a positive constant $\boldsymbol{c}$ and $\boldsymbol{n}$ make $\boldsymbol{\forall n \geq N, c * g(n) \leq f(n)}$. Therefore, we can tell that $\boldsymbol{f(n) = O(g(n))}$.

Moreover, then $\boldsymbol{n}$ approach to infinity, the value of the above polynomial is close to $\boldsymbol{\frac{1}{2\ln(2)}}$. In this case, we can tell that $\boldsymbol{f(n) = \Omega(g(n))}$.

Thus, only (III) fits this scenario, and the relationship between this pair of functions is $\boldsymbol{f(n) = \Theta(g(n))}$.