Question 3

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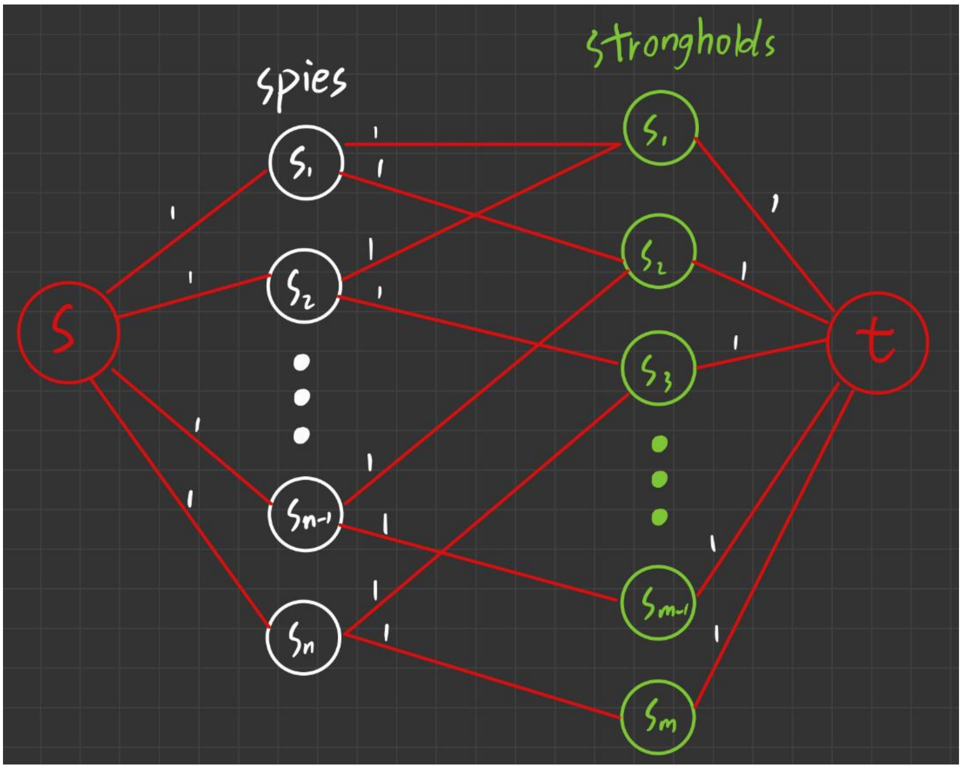
This is a typical bipartite problem, so we need to transform the problem into a graph. First, we separate all the spies on one side and connect them all to source, and all the strongholds on the other side and connect them all to sink. the capacity of these newly created edges is 1, because a stronghold can only hide one spy.

Then the case of an edges between spies and strongholds is by calculation if a spy can move from its current position (x, y) to a stronghold position (a, b) in t minutes, i.e., if the distance it moves is less than or equal to the speed of a spy $v * t$, then add an edge to them, also with a capacity of 1.

We end up with a graph that looks like this.



Then we run the Edmonds-Karp algorithm on this graph to find the maximum flow. The maximum flow obtained is the maximum number of spies hiding from the bad guys. Then the flow process corresponding to max flow can determines which spies should be sent to which strongholds.

Transforming the problem into a graph takes $O(nm)$ time complexity.

Then given that the number of vertices is $n + m + 2$ and the number of edges is at most $nm + n + m$, the time complexity of running the Edmonds-Karp algorithm (Max bipartite matching) is $O(|nm| * |n + m|)$. Therefore, the total time complexity of this algorithm is $O(|nm| * |n + m|)$, which meets the polynomial time requirement of the question.