Question 4

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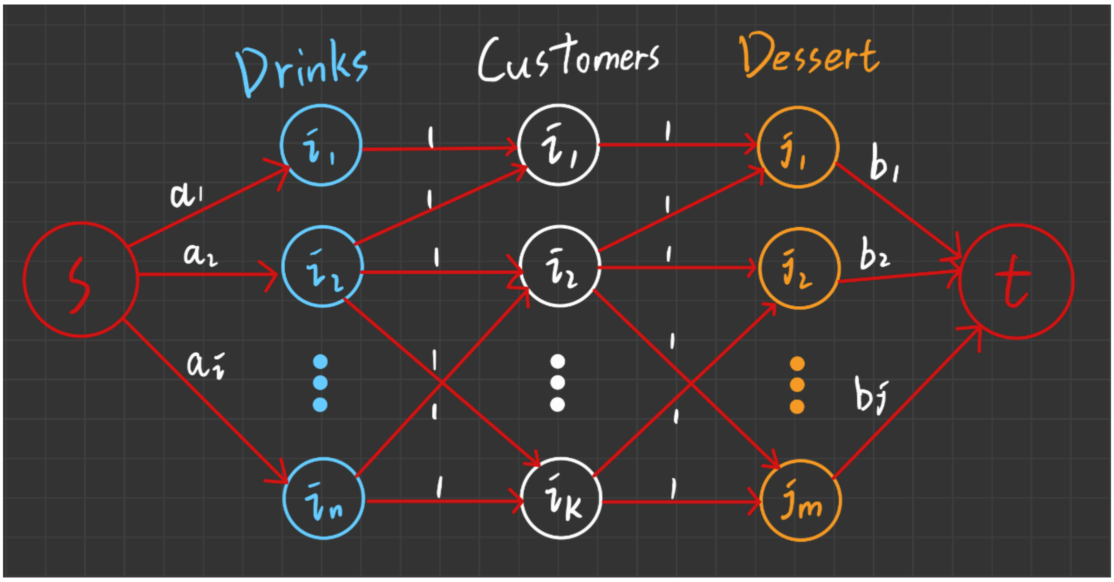
First, we need to transform the problem into a graph. We need to map all the different drinks and desserts and all the customers into vertices. Line up the vertices of all the drinks on the left, the vertices of the desserts on the right and the vertices of the customers between them.

Then source is placed on the far left, connecting the vertices of each drink, and the capacity of its edge is a_i, representing how many cups this drink can make. Then sink is placed on the far right, connecting all the desserts, and the capacity of its edge is b_j, representing how many desserts can make.

Next for each customer, simply connect the vertices of their favorite drinks and desserts according to the corresponding p_i and q_i. The capacity of these sides is 1, as a customer only needs one drink and one dessert.

We end up with a graph that looks like this.



Then we run the Edmonds Karp algorithm on this graph to find the maximum flow. We subtract the maximum flow from the number of customers $\boldsymbol{k}$ to get the smallest possible number of poor ratings that Alice can receive.

Transforming the problem into a graph takes $\boldsymbol{O}(\boldsymbol{nk}+\boldsymbol{mk})$ time complexity. Then the time complexity of running the Edmonds-Karp algorithm (Max bipartite matching) is $\boldsymbol{O}(|\boldsymbol{nk}+\boldsymbol{mk}| * |\boldsymbol{n}+\boldsymbol{m}+\boldsymbol{k}|)$. Therefore, the total time complexity of this algorithm is $\boldsymbol{O}(|\boldsymbol{nk}+\boldsymbol{mk}| * |\boldsymbol{n}+\boldsymbol{m}+\boldsymbol{k}|)$, which meets the polynomial time requirement of the question.