Question 3

Zeal Liang

Z5325156

Subproblems:

Let $E(n, k)$ be the problem of determining $even(n)$, count the number of n-digit

numbers in which the digit k appears an even number of times:

$$if\ k\ is\ not\ 0\ then\ even(n) = even(n-1) * 8 + odd(n-1),$$
$$if\ k\ is\ 0\ then\ even(n) = even(n-1) * 9.$$

Recurrence:

For $2 \leq i \leq n-1$:

$$even(i) = even(i-1) * 9 + odd(i-1),$$
$$odd(i) = odd(i-1) * 9 + even(i-1).$$

Base cases:

since after remove one k from 0 to 9 there still have 9 numbers left, so

$$even(1) = 9, odd(1) = 1$$

The time complexity of the algorithm is $O(n)$.