

Question 2

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Solution:

Using the dynamic programming method, the truth table of $(n+1) \times (m+1)$ can be obtained, and the answer is $\text{subset}(n, m)$

Subproblems:

For $\text{subset}(i, j) = \text{subset}(i-1, j) = \text{True}$, the element $S[i]$ is not in the subset s . For $\text{subset}(i, j) = \text{True}$ and $\text{subset}(i-1, j) = \text{False}$, the element $S[i]$ must be in the subset s . For $\text{subset}(i, j) = \text{True}$ and $\text{subset}(i-1, j) = \text{False}$, the element $S[i]$ must be in the subset s . In this case, $\text{subset}(i-1, j-S[i]) = \text{True}$, so that we can find the element in s by recursive method. For this problem, just start from $\text{subset}(n, M)$.

Recurrence:

For $1 \leq i \leq n$ and $1 \leq j \leq m$, Let $\text{subset}(i, j)$ denote the case where the subset of the first i elements in array A and less than j , then

If $A[i] > j$, then $A[i]$ is not in the subset s .

If $A[i] < j$, there are two cases:

one case is that $A[i]$ is not in subset s , then $\text{subset}(i, j) = \text{subset}(i-1, j)$;

one case is that $A[i]$ is in subset s , then $\text{subset}(i, j) = \text{subset}(i-1, j - S[i])$.

Base cases:

For $i=0,1,2,\dots,n$, we have $\text{subset}(i, 0)=\text{True}$, for $j=1,2,\dots,M$, we have $\text{subset}(0, j)=\text{False}$. if $\text{sum} = 0$ (return empty set).

Since i goes from 1 to N and j goes from 1 to M , the time complexity of the algorithm is $O(mn)$.