Question 2

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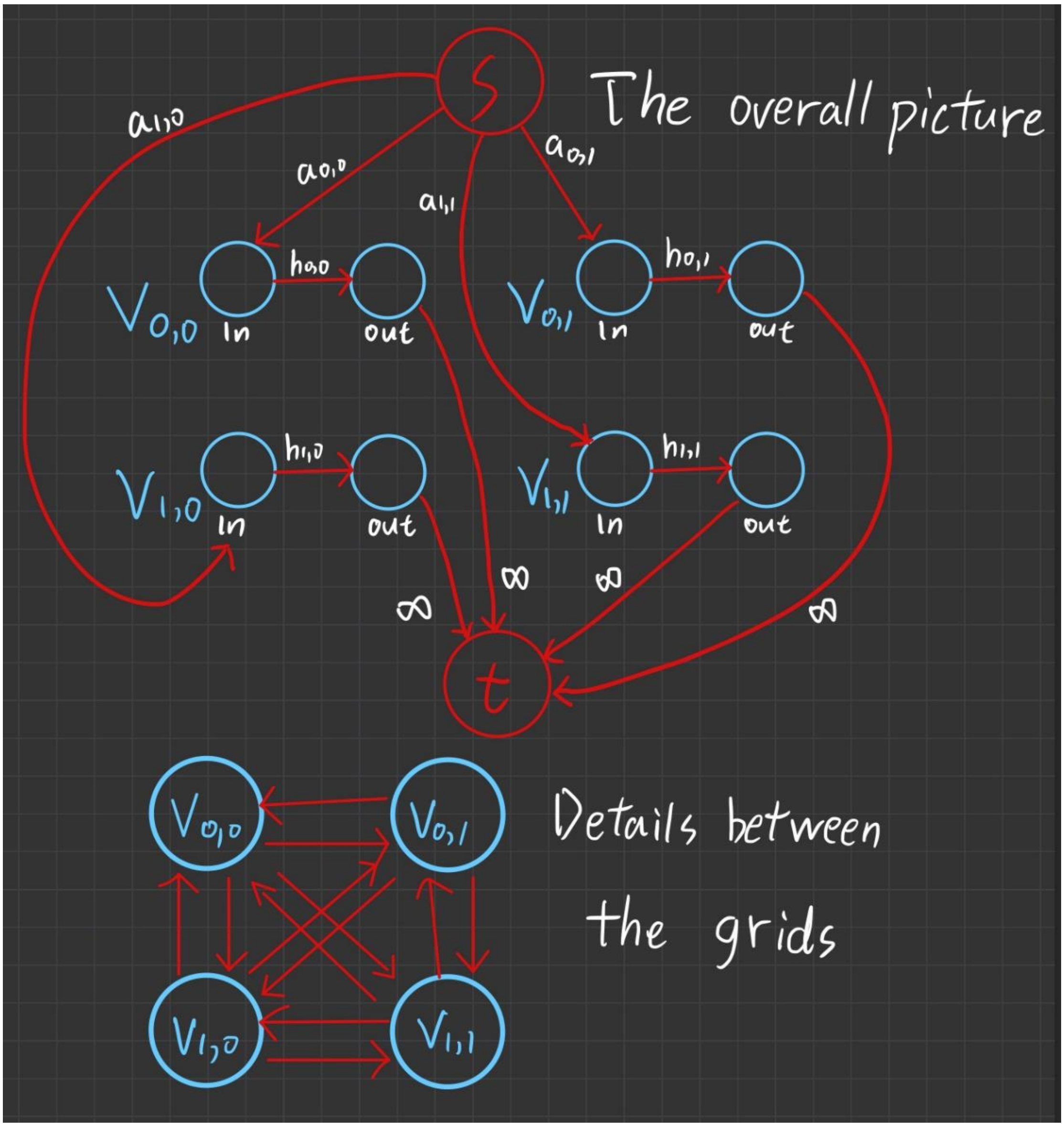
First, we transform the problem into a graph, which can be obtained by mapping each of the stones into vertices. In this step we need to split each stone into two vertices, one is v_{in} and one is v_{out} and the capacity of the edge between these two vertices is the height of the stone h_{ij}, which is the maximum number of lizards that can pass through this stone.

Then, the number of lizards on each stone is mapped to the edge connected from source to v_{in} of this stone, and the capacity of this edge is the number of lizards a_{ij}.

Next connect all stones that can jump out from its v_{out} to sink. If d is 1, for example, it is a circle of stones at the outermost edge of the grid that are connected to sink, and there is no restriction on the capacity of these edges.

Finally, the case of all edges between stones is that if it is possible to jump from one stone to another, i.e., if the distance between any two stones is less than or equal to d, then a two-way edge is added to them. How the distance is calculated can be derived using the formula provided in the question, and there is no limit to the capacity of these edges.

If r and c were equal to 2 we would get a graph like this. (Split into two separate layers because there are too many edges to draw)



Then we run the Edmonds-Karp algorithm on this graph to find the maximum flow. The maximum flow obtained is the maximum number of Lizards that can escape from the grid.

Transforming the problem into a graph takes $\boldsymbol{O}((\boldsymbol{rc})^{4})$ time complexity. Then given that the number of vertices is $\boldsymbol{2rc+2}$ and the number of edges is at most $(\boldsymbol{rc}+\mathbf{2})^{2}+\boldsymbol{rc}$, the time complexity of running the Edmonds-Karp algorithm is $\boldsymbol{O}(|\boldsymbol{rc}| * |\boldsymbol{rc}|^{4})$. Therefore, the total time complexity of this algorithm is $\boldsymbol{O}(|\boldsymbol{rc}| * |\boldsymbol{rc}|^{4})$, which meets the polynomial time requirement of the question.