Question 4

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From the question we know that the ends of the line are x_n and x_{-n} respectively. We can summarize the location x of the food into three cases.

The first one is at the left end of the line, which can be expressed as

$$x = x_{-n}.$$

Then, the total length of the ant's walk in this case would be $\sum_{i=1}^{x_n - x_{-n}} i$.

The second one is at the right end of the line, which can be expressed as

$$x = x_n.$$

Then, the total length of the ant's walk in this case would be $\sum_{i=1}^{|x_{-n} - x_n|} i$.

The third one is at the middle of the line, which can be expressed as

$$x_{-n} < x < x_n.$$

Then, the total length of the ant's walk in this case would be $\sum_{i=1}^{x_n - x} 2i$.

We can substitute the numbers to calculate, for example, when we have five ants the total distance for different cases will be:

Case 1, $\sum_{i=1}^{4} i \ = 1 + 2 + 3 + 4 = 10.$

Case 2, $\sum_{i=1}^{4} i \ = 1 + 2 + 3 + 4 = 10.$

Case 3, $\sum_{i=1}^{2} 2i = 2 * 1 + 2 * 2 \ = 4.$

In summary, we can clearly see that only in the third case, the ants move the shortest distance.

So, the value of x which minimises the total distance walked by all ants is 0 which is right in the middle of the line.