Problem 2 :

We can generalize the design problem in Problem 1 as follows.

$$y_i = \theta_0 + \theta_1 x_i + \theta_2 x_i^2 + \cdots + \theta_p x_i^p + \varepsilon_i, \quad i = 1, \cdots, n,$$

$$x_i \in [a, b], \quad \varepsilon_i \sim i.i.d\ N(0, \sigma^2).$$

[In problem 1: $p = 2$, $a = -1$, $b = 1$. ]

- Try $P = 3, 4, 5, 6, \cdots$

  and $[a, b] = [-2, 2]$, $[0, 1]$, $[0, 5]$, ...

- Changes in optimal design problem

  * Define $u_i = a + (b-a)(i-1)/(N-1)$, $i = 1, 2, \cdots, N$.

  * For $i = 1, \cdots, N$, let

$$A_i = \begin{pmatrix} 1 \\ u_i \\ u_i^2 \\ \vdots \\ u_i^p \end{pmatrix} \begin{pmatrix} 1 & u_i & u_i^2 & \cdots & u_i^p \end{pmatrix} \quad \left( A_i : \underset{\text{matrix}}{(p+1)\times(p+1)} \right)$$

  * Try $N = 21, 51, 1001, 2001, 5001, \cdots$

It is interesting to observe the computation times as $N$ increases.