

Face Recognition using Eigenface, A Simple Example

The size of face image used in this program is 92 pixels x 112 pixels. But in this example, assume that the size of face image is 4 x 4 pixels.

Training dataset

$$Face_1 = \begin{bmatrix} 3 & 1 \\ 9 & 5 \end{bmatrix} \quad Face_2 = \begin{bmatrix} 2 & 1 \\ 5 & 2 \end{bmatrix} \quad Face_3 = \begin{bmatrix} 7 & 4 \\ 5 & 3 \end{bmatrix}$$

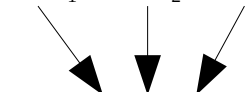
Test dataset

$$UnknownFace_1 = \begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}$$

STEP 1: Convert the face images from training dataset into face vectors

Let matrix $A =$

$Face_1$ $Face_2$ $Face_3$


$$A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 1 & 4 \\ 9 & 5 & 5 \\ 5 & 2 & 3 \end{bmatrix}$$

STEP 2: Find Ψ , the mean face from training dataset

$$\Psi = \begin{bmatrix} (3+2+7)/3 \\ (1+1+4)/3 \\ (9+5+5)/3 \\ (5+2+3)/3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 19/3 \\ 10/3 \end{bmatrix} \xrightarrow{\text{Duplicate columns}} \begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \\ 19/3 & 19/3 & 19/3 \\ 10/3 & 10/3 & 10/3 \end{bmatrix}$$

$$\Psi = \begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \\ 19/3 & 19/3 & 19/3 \\ 10/3 & 10/3 & 10/3 \end{bmatrix}$$

STEP 3: Find Φ , the unique features of the training dataset faces

$$\Phi = A - \Psi$$

$$= \begin{bmatrix} 3 & 2 & 7 \\ 1 & 1 & 4 \\ 9 & 5 & 5 \\ 5 & 2 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \\ 19/3 & 19/3 & 19/3 \\ 10/3 & 10/3 & 10/3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -2 & 3 \\ -1 & -1 & 2 \\ 8/3 & -4/3 & -4/3 \\ 5/3 & -4/3 & -1/3 \end{bmatrix}$$

STEP 4: Construct Matrix L with size $M \times M$, where M is the total number of faces in training dataset

$$L = \Phi^T \Phi$$

$$= \begin{bmatrix} -1 & -1 & 8/3 & 5/3 \\ -2 & -1 & -4/3 & -4/3 \\ 3 & 2 & -4/3 & -1/3 \end{bmatrix} \begin{bmatrix} -1 & -2 & 3 \\ -1 & -1 & 2 \\ 8/3 & -4/3 & -4/3 \\ 5/3 & -4/3 & -1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 107/9 & -25/9 & -82/9 \\ -25/9 & 77/9 & -52/9 \\ -82/9 & -52/9 & 134/9 \end{bmatrix}$$

STEP 5: Find eigenvalues of the Matrix L

$$\det(\lambda I - L) = 0$$

$$\det \left(\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 107/9 & -25/9 & -82/9 \\ -25/9 & 77/9 & -52/9 \\ -82/9 & -52/9 & 134/9 \end{bmatrix} \right) = 0$$

$$\det \begin{bmatrix} \lambda - 107/9 & 25/9 & 82/9 \\ 25/9 & \lambda - 77/9 & 52/9 \\ 82/9 & 52/9 & \lambda - 134/9 \end{bmatrix} = 0$$

$$(\lambda - 107/9) [(\lambda - 77/9)(\lambda - 134/9) - (52/9)(52/9)] - 25/9 [(25/9)(\lambda - 134/9) - (52/9)(82/9)] + 82/9 [(25/9)(52/9) - (\lambda - 77/9)(82/9)] = 0$$

$$(\lambda - 107/9) [\lambda^2 - 211/9 \lambda + 94] - 25/9 [25/9 \lambda - 94] + 82/9 [-82/9 \lambda + 94] = 0$$

$$(\lambda^3 - 211/9 \lambda^2 + 94 \lambda - 107/9 \lambda^2 + 22577/81 \lambda - 10058/9) - (625/81 \lambda - 2350/9) + (-6724/81 \lambda + 7708/9) = 0$$

$$(\lambda^3 - 106/3 \lambda^2 + 30191/81 \lambda - 10058/9) - (625/81 \lambda - 2350/9) + (-6724/81 \lambda + 7708/9) = 0$$

$$\lambda^3 - 106/3 \lambda^2 + 30191/81 \lambda - 625/81 \lambda - 6724/81 \lambda - 10058/9 + 2350/9 + 7708/9 = 0$$

$$\lambda^3 - 106/3 \lambda^2 + 282 \lambda + 0 = 0$$

λ_1	=	23.1540	Sort the eigenvalues in descending order	λ_1	=	23.1540
λ_2	=	0		λ_2	=	12.1793
λ_3	=	12.1793		λ_3	=	0

STEP 6: Use Gaussian Elimination method to find the eigenvectors of the Matrix L

Case 1: $\lambda = 23.1540$

$$\text{Let } B = \begin{bmatrix} \lambda - 107/9 & 25/9 & 82/9 \\ 25/9 & \lambda - 77/9 & 52/9 \\ 82/9 & 52/9 & \lambda - 134/9 \end{bmatrix}$$

To find the eigenvectors, solve $Bx = 0$

$$Bx = 0$$

$$\begin{bmatrix} 23.1540 - 107/9 & 25/9 & 82/9 \\ 25/9 & 23.1540 - 77/9 & 52/9 \\ 82/9 & 52/9 & 23.1540 - 134/9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 11.2651 & 25/9 & 82/9 \\ 25/9 & 14.5984 & 52/9 \\ 82/9 & 52/9 & 8.2651 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 11.2651 & 25/9 & 82/9 & 0 \\ 25/9 & 14.5984 & 52/9 & 0 \\ 82/9 & 52/9 & 8.2651 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 11.2651 & 25/9 & 82/9 & 0 \\ 25/9 & 14.5984 & 52/9 & 0 \\ 82/9 & 52/9 & 8.2651 & 0 \end{array} \right] \xrightarrow{R_1/11.2651} \left[\begin{array}{ccc|c} 1 & 0.2466 & 0.8088 & 0 \\ 25/9 & 14.5984 & 52/9 & 0 \\ 82/9 & 52/9 & 8.2651 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0.2466 & 0.8088 & 0 \\ 25/9 & 14.5984 & 52/9 & 0 \\ 82/9 & 52/9 & 8.2651 & 0 \end{array} \right] \xrightarrow{R_2 - 25/9 R_1} \left[\begin{array}{ccc|c} 1 & 0.2466 & 0.8088 & 0 \\ 0 & 13.9134 & 3.5311 & 0 \\ 82/9 & 52/9 & 8.2651 & 0 \end{array} \right]$$