

Face Recognition using Eigenface, A Simple Example

The size of face image used in this program is 92 pixels x 112 pixels. But in this example, assume that the size of face image is 4 x 4 pixels.

Training dataset

$$Face_1 = \begin{bmatrix} 3 & 1 \\ 9 & 5 \end{bmatrix} \quad Face_2 = \begin{bmatrix} 2 & 1 \\ 5 & 2 \end{bmatrix} \quad Face_3 = \begin{bmatrix} 7 & 4 \\ 5 & 3 \end{bmatrix}$$

Test dataset

$$UnknownFace_1 = \begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}$$

STEP 1: Convert the face images from training dataset into face vectors

Let matrix $A =$

$$\begin{matrix} & \begin{matrix} Face_1 & Face_2 & Face_3 \end{matrix} \\ \begin{matrix} \swarrow \\ \downarrow \\ \swarrow \end{matrix} & \begin{bmatrix} 3 & 2 & 7 \\ 1 & 1 & 4 \\ 9 & 5 & 5 \\ 5 & 2 & 3 \end{bmatrix} \end{matrix}$$

STEP 2: Find Ψ , the mean face from training dataset

$$\Psi = \begin{bmatrix} (3+2+7)/3 \\ (1+1+4)/3 \\ (9+5+5)/3 \\ (5+2+3)/3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 19/3 \\ 10/3 \end{bmatrix} \xrightarrow{\text{Duplicate columns}} \begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \\ 19/3 & 19/3 & 19/3 \\ 10/3 & 10/3 & 10/3 \end{bmatrix}$$

$$\Psi = \begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \\ 19/3 & 19/3 & 19/3 \\ 10/3 & 10/3 & 10/3 \end{bmatrix}$$

STEP 3: Find Φ , the unique features of the training dataset faces

$$\begin{aligned} \Phi &= A - \Psi \\ &= \begin{bmatrix} 3 & 2 & 7 \\ 1 & 1 & 4 \\ 9 & 5 & 5 \\ 5 & 2 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \\ 19/3 & 19/3 & 19/3 \\ 10/3 & 10/3 & 10/3 \end{bmatrix} \\ &= \begin{bmatrix} -1 & -2 & 3 \\ -1 & -1 & 2 \\ 8/3 & -4/3 & -4/3 \\ 5/3 & -4/3 & -1/3 \end{bmatrix} \end{aligned}$$

STEP 4: Construct Matrix L with size $M \times M$, where M is the total number of faces in training dataset

$$\begin{aligned} L &= \Phi^T \Phi \\ &= \begin{bmatrix} -1 & -1 & 8/3 & 5/3 \\ -2 & -1 & -4/3 & -4/3 \\ 3 & 2 & -4/3 & -1/3 \end{bmatrix} \begin{bmatrix} -1 & -2 & 3 \\ -1 & -1 & 2 \\ 8/3 & -4/3 & -4/3 \\ 5/3 & -4/3 & -1/3 \end{bmatrix} \\ &= \begin{bmatrix} 107/9 & -25/9 & -82/9 \\ -25/9 & 77/9 & -52/9 \\ -82/9 & -52/9 & 134/9 \end{bmatrix} \end{aligned}$$

STEP 5: Find eigenvalues of the Matrix L

Determinant matrix



$$\det(\lambda I - L) = 0$$

$$\det \left(\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 107/9 & -25/9 & -82/9 \\ -25/9 & 77/9 & -52/9 \\ -82/9 & -52/9 & 134/9 \end{bmatrix} \right) = 0$$