Face Recognition using Eigenface, A Simple Example

The size of face image used in this program is 92 pixels x 112 pixels. But in this example, assume that the size of face image is 2 x 2 pixels.

Training dataset	Test dataset
$T_1 = \begin{bmatrix} 3 & 1 \\ 9 & 5 \end{bmatrix}$ $T_2 = \begin{bmatrix} 2 & 1 \\ 5 & 2 \end{bmatrix}$ $T_3 = \begin{bmatrix} 7 & 4 \\ 5 & 3 \end{bmatrix}$	$F_1 = \begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}$

STEP 1: Convert the training faces 2D-matrix into vector. The matrix $\boxed{\Gamma}$ will store these vectors.

$$\boxed{\begin{aligned} \Gamma &= \left[T_1^T,\ T_2^T,\ T_3^T\right] \\ &= \begin{bmatrix} 3 & 2 & 7 \\ 1 & 1 & 4 \\ 9 & 5 & 5 \\ 5 & 2 & 3 \end{bmatrix} \end{aligned}}$$

STEP 2: Find $\boxed{\Psi}$, the mean face from training dataset.

$$\boxed{\Psi = \frac{1}{M}\sum_{n=1}^{M}\Gamma_n = \frac{1}{3}\left(\begin{bmatrix}3\\1\\9\\5\end{bmatrix}+\begin{bmatrix}2\\1\\5\\2\end{bmatrix}+\begin{bmatrix}7\\4\\5\\3\end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix}12\\6\\19\\10\end{bmatrix} = \begin{bmatrix}4\\2\\19/3\\10/3\end{bmatrix}}$$

STEP 3: Find Φ_i , the unique features of the training faces.

Using the following equation to find the unique features:

$$\Phi_i = \Gamma_i - \Psi$$

$$\Phi_1 = \Gamma_1 - \Psi = \begin{bmatrix} 3 \\ 1 \\ 9 \\ 5 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \\ 19/3 \\ 10/3 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 8/3 \\ 5/3 \end{bmatrix}$$

$$\Phi_2 = \Gamma_2 - \Psi = \begin{bmatrix} 2 \\ 1 \\ 5 \\ 2 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \\ 19/3 \\ 10/3 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ -4/3 \\ -4/3 \end{bmatrix}$$

$$\Phi_3 = \Gamma_3 - \Psi = \begin{bmatrix} 7 \\ 4 \\ 5 \\ 3 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \\ 19/3 \\ 10/3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -4/3 \\ -1/3 \end{bmatrix}$$

STEP 4: Construct a Covariance Matrix L with size $M \times M$, where M is the total number of faces in training dataset.

$$\begin{aligned} L &= \Phi^T \Phi \\ &= \begin{bmatrix} -1 & -1 & 8/3 & 5/3 \\ -2 & -1 & -4/3 & -4/3 \\ 3 & 2 & -4/3 & -1/3 \end{bmatrix} \begin{bmatrix} -1 & -2 & 3 \\ -1 & -1 & 2 \\ 8/3 & -4/3 & -4/3 \\ 5/3 & -4/3 & -1/3 \end{bmatrix} \\ &= \begin{bmatrix} 107/9 & -25/9 & -82/9 \\ -25/9 & 77/9 & -52/9 \\ -82/9 & -52/9 & 134/9 \end{bmatrix} \end{aligned}$$

STEP 5: Find eigenvalues of the Matrix $\boxed{L}$.

$$\det(\lambda I - L) = 0$$

$$\det\left(\lambda\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} - \begin{bmatrix}107/9 & -25/9 & -82/9\\-25/9 & 77/9 & -52/9\\-82/9 & -52/9 & 134/9\end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix}\lambda-107/9 & 25/9 & 82/9\\25/9 & \lambda-77/9 & 52/9\\82/9 & 52/9 & \lambda-134/9\end{bmatrix}\right) = 0$$

$$(\lambda-107/9)[(\lambda-77/9)(\lambda-134/9)-(52/9)(52/9)]-$$
$$25/9[(25/9)(\lambda-134/9)-(52/9)(82/9)]+$$
$$82/9[(25/9)(52/9)-(\lambda-77/9)(82/9)] = 0$$

$$(\lambda-107/9)[\lambda^2-211/9\lambda+94]-$$
$$25/9[25/9\lambda-94]+$$
$$82/9[-82/9\lambda+94] = 0$$

$$(\lambda^3-211/9\lambda^2+94\lambda-107/9\lambda^2+22577/81\lambda-10058/9)-$$
$$(625/81\lambda-2350/9)+$$
$$(-6724/81\lambda+7708/9) = 0$$

$$(\lambda^3-106/3\lambda^2+30191/81\lambda-10058/9)-$$
$$(625/81\lambda-2350/9)+$$
$$(-6724/81\lambda+7708/9) = 0$$

$$\lambda^3-106/3\lambda^2+30191/81\lambda-625/81\lambda-6724/81\lambda-$$
$$10058/9+2350/9+7708/9 = 0$$

$$\lambda^3-106/3\lambda^2+282\lambda+0 = 0$$

$$\lambda_1 = 23.1540$$
$$\lambda_2 = 0$$
$$\lambda_3 = 12.1793$$

Sort the eigenvalues in descending order

$$\lambda_1 = 23.1540$$
$$\lambda_2 = 12.1793$$
$$\lambda_3 = 0$$

STEP 6: Use Gaussian Elimination method to find $\boxed{v}$, the eigenvectors of the Matrix $\boxed{L}$.

$$\boxed{\begin{aligned} B &= \left[\lambda I - L\right] \\ &= \begin{bmatrix} \lambda-107/9 & 25/9 & 82/9 \\ 25/9 & \lambda-77/9 & 52/9 \\ 82/9 & 52/9 & \lambda-134/9 \end{bmatrix} \end{aligned}}$$

To find the eigenvectors, we need to solve $\boxed{Bx = 0}$ for every eigenvalues.

Case 1: $\boxed{\lambda = 23.1540}$

$$Bx = 0$$

$$\begin{bmatrix} \lambda-107/9 & 25/9 & 82/9 \\ 25/9 & \lambda-77/9 & 52/9 \\ 82/9 & 52/9 & \lambda-134/9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 23.1540-107/9 & 25/9 & 82/9 \\ 25/9 & 23.1540-77/9 & 52/9 \\ 82/9 & 52/9 & 23.1540-134/9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 11.2651 & 25/9 & 82/9 \\ 25/9 & 14.5984 & 52/9 \\ 82/9 & 52/9 & 8.2651 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 11.2651 & 25/9 & 82/9 & 0 \\ 25/9 & 14.5984 & 52/9 & 0 \\ 82/9 & 52/9 & 8.2651 & 0 \end{array}\right]$$

$$\underline{R_1/11.2651}\left[\begin{array}{ccc|c} 1 & 0.2466 & 0.8088 & 0 \\ 25/9 & 14.5984 & 52/9 & 0 \\ 82/9 & 52/9 & 8.2651 & 0 \end{array}\right]$$

$$\underline{R_2-25/9R_1}\left[\begin{array}{ccc|c} 1 & 0.2466 & 0.8088 & 0 \\ 0 & 13.9134 & 3.5311 & 0 \\ 82/9 & 52/9 & 8.2651 & 0 \end{array}\right]$$

$$\underline{R_3-82/9R_1}\left[\begin{array}{ccc|c} 1 & 0.2466 & 0.8088 & 0 \\ 0 & 13.9134 & 3.5311 & 0 \\ 0 & 3.5310 & 0.8960 & 0 \end{array}\right]$$

$$\underline{R_2/13.9134}\left[\begin{array}{ccc|c} 1 & 0.2466 & 0.8088 & 0 \\ 0 & 1 & 0.2538 & 0 \\ 0 & 3.5310 & 0.8960 & 0 \end{array}\right]$$

$$\underline{R_1-0.2466R_2}\left[\begin{array}{ccc|c} 1 & 0 & 0.7462 & 0 \\ 0 & 1 & 0.2538 & 0 \\ 0 & 3.5310 & 0.8960 & 0 \end{array}\right]$$

$$\underline{R_3-3.5310R_2}\left[\begin{array}{ccc|c} 1 & 0 & 0.7462 & 0 \\ 0 & 1 & 0.2538 & 0 \\ 0 & 0 & -0.0002 & 0 \end{array}\right]$$

$$\underline{R_3/-0.0002}\left[\begin{array}{ccc|c} 1 & 0 & 0.7462 & 0 \\ 0 & 1 & 0.2538 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{aligned} x_1+0.7462\,x_3 &= 0 \\ x_2+0.2538\,x_3 &= 0 \\ x_3 &= 1 \end{aligned}$$

$$\begin{aligned} x_1 &= -0.7462(1) \\ x_2 &= -0.2538(1) \\ x_3 &= 1 \end{aligned}$$

$$\begin{aligned} v_1 &= \frac{1}{\max|x_1, x_2, x_3|}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \\ &= \frac{1}{1}\begin{bmatrix} -0.7462 \\ -0.2538 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} -0.7462 \\ -0.2538 \\ 1 \end{bmatrix} \end{aligned}$$

Therefore, the eigenvector for eigenvalue $\boxed{\lambda = 23.1540}$ is

$$\boxed{v_1 = \begin{bmatrix} -0.7462 \\ -0.2538 \\ 1.0000 \end{bmatrix}}$$

Case 2: $\boxed{\lambda=12.1793}$

$$Bx = 0$$

$$\begin{bmatrix} \lambda-107/9 & 25/9 & 82/9 \\ 25/9 & \lambda-77/9 & 52/9 \\ 82/9 & 52/9 & \lambda-134/9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 12.1793-107/9 & 25/9 & 82/9 \\ 25/9 & 12.1793-77/9 & 52/9 \\ 82/9 & 52/9 & 12.1793-134/9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.2904 & 25/9 & 82/9 \\ 25/9 & 3.6237 & 52/9 \\ 82/9 & 52/9 & -2.7096 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 0.2904 & 25/9 & 82/9 & 0 \\ 25/9 & 3.6237 & 52/9 & 0 \\ 82/9 & 52/9 & -2.7096 & 0 \end{array}\right]$$

$$\underline{R_1/0.2904}\left[\begin{array}{ccc|c} 1 & 9.5654 & 31.3743 & 0 \\ 25/9 & 3.6237 & 52/9 & 0 \\ 82/9 & 52/9 & -2.7096 & 0 \end{array}\right]$$

$$\underline{R_2-25/9R_1}\left[\begin{array}{ccc|c} 1 & 9.5654 & 31.3743 & 0 \\ 0 & -22.9469 & -81.3731 & 0 \\ 82/9 & 52/9 & -2.7096 & 0 \end{array}\right]$$

$$\underline{R_3-82/9R_1}\left[\begin{array}{ccc|c} 1 & 9.5654 & 31.3743 & 0 \\ 0 & -22.9469 & -81.3731 & 0 \\ 0 & -81.3736 & -288.5643 & 0 \end{array}\right]$$

$$\underline{R_2/-22.9469}\left[\begin{array}{ccc|c} 1 & 9.5654 & 31.3743 & 0 \\ 0 & 1 & 3.5461 & 0 \\ 0 & -81.3736 & -288.5643 & 0 \end{array}\right]$$

$$\underline{R_1-9.5654R_2}\left[\begin{array}{ccc|c} 1 & 0 & -2.5456 & 0 \\ 0 & 1 & 3.5461 & 0 \\ 0 & -81.3736 & -288.5643 & 0 \end{array}\right]$$

$$\underline{R_3+81.3736R_2}\left[\begin{array}{ccc|c} 1 & 0 & -2.5456 & 0 \\ 0 & 1 & 3.5461 & 0 \\ 0 & 0 & -0.0054 & 0 \end{array}\right]$$

$$\underline{R_3/-0.0054}\left[\begin{array}{ccc|c} 1 & 0 & -2.5456 & 0 \\ 0 & 1 & 3.5461 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{aligned} x_1 - 2.5456\,x_3 &= 0 \\ x_2 + 3.5461\,x_3 &= 0 \\ x_3 &= 1 \end{aligned}$$

$$\begin{aligned} x_1 &= 2.5456(1) \\ x_2 &= -3.5461(1) \\ x_3 &= 1 \end{aligned}$$

$$\begin{aligned} v_2 &= \frac{1}{\max|x_1, x_2, x_3|}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \\ &= \frac{1}{3.5461}\begin{bmatrix} 2.5456 \\ -3.5461 \\ 1.0000 \end{bmatrix} \\ &= \begin{bmatrix} 0.7179 \\ -1.0000 \\ 0.2820 \end{bmatrix} \end{aligned}$$

Therefore, the eigenvector for eigenvalue $\boxed{\lambda = 12.1793}$ is

$$\boxed{v_2 = \begin{bmatrix} 0.7179 \\ -1.0000 \\ 0.2820 \end{bmatrix}}$$

Case 3: $\boxed{\lambda=0}$

Eigenvalue with value 0 or very near to 0 will produce a useless eigenvector, so this case is discarded.

Therefore, the eigenvector of Matrix $\boxed{L}$ is:

$$\boxed{v = \begin{bmatrix} -0.7462 & 0.7179 \\ -0.2538 & -1.0000 \\ 1.0000 & 0.2820 \end{bmatrix}}$$

STEP 7: Find the Eigenface, $\boxed{u}$.

Using the following equation to find the Eigenface:

$$\boxed{u_1 = \sum_{k=1}^{M} v_{1k}\Phi_k}$$, where $\boxed{M = 3}$, the number of training faces.

$$u_1 = (-0.7462)\begin{bmatrix} -1 \\ -1 \\ 8/3 \\ 5/3 \end{bmatrix} + (-0.2538)\begin{bmatrix} -2 \\ -1 \\ -4/3 \\ -4/3 \end{bmatrix} + (1.0000)\begin{bmatrix} 3 \\ 2 \\ -4/3 \\ -1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7462 \\ 0.7462 \\ -1.9899 \\ -1.2437 \end{bmatrix} + \begin{bmatrix} 0.5076 \\ 0.2538 \\ 0.3384 \\ 0.3384 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \\ -4/3 \\ -1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 4.2538 \\ 3.0000 \\ -2.9848 \\ -1.2386 \end{bmatrix}$$

$$u_2 = (0.7179)\begin{bmatrix} -1 \\ -1 \\ 8/3 \\ 5/3 \end{bmatrix} + (-1.0000)\begin{bmatrix} -2 \\ -1 \\ -4/3 \\ -4/3 \end{bmatrix} + (0.2820)\begin{bmatrix} 3 \\ 2 \\ -4/3 \\ -1/3 \end{bmatrix}$$

$$= \begin{bmatrix} -0.7179 \\ -0.7179 \\ 1.9144 \\ 1.1965 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ 4/3 \\ 4/3 \end{bmatrix} + \begin{bmatrix} 0.8460 \\ 0.5640 \\ -0.3760 \\ -0.0940 \end{bmatrix}$$

$$= \begin{bmatrix} 2.1281 \\ 0.8461 \\ 2.8717 \\ 2.4358 \end{bmatrix}$$

Therefore, $$\boxed{u = \begin{bmatrix} 4.2538 & 2.1281 \\ 3.0000 & 0.8461 \\ -2.9848 & 2.8717 \\ -1.2386 & 2.4358 \end{bmatrix}}$$

STEP 8: Find training weights $\boxed{\Omega_k}$.

Using the following equations to find the weight:

$$\boxed{\Omega^T = [w_1, \quad w_2, \quad w_3 \quad \ldots \quad w_M]} \quad ,$$

where $\boxed{M}$ number of faces in the training dataset.

$$\boxed{w_k = u_k^T(\Gamma - \Psi)} \quad ,$$

where $\boxed{\Gamma}$ is a face vector from training dataset.

Find the weights for training face 1:

$$
\begin{aligned}
\Omega_1^T &= [w_1, \quad w_2] \\
w_1 &= u_1^T(\Gamma_1 - \Psi) \\
&= [4.2538, \quad 3.0000, \ -2.9848, \quad -1.2386] \left(\begin{bmatrix} 3 \\ 1 \\ 9 \\ 5 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \\ 19/3 \\ 10/3 \end{bmatrix} \right) \\
&= [4.2538, \quad 3.0000, \ -2.9848, \quad -1.2386] \left(\begin{bmatrix} -1 \\ -1 \\ 8/3 \\ 5/3 \end{bmatrix} \right) \\
&= -17.2778 \\
w_2 &= u_2^T(\Gamma_1 - \Psi) \\
&= [2.1281, \quad 0.8461, \ -2.8717, \quad -2.4358] \left(\begin{bmatrix} 3 \\ 1 \\ 9 \\ 5 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \\ 19/3 \\ 10/3 \end{bmatrix} \right) \\
&= [2.1281, \quad 0.8461, \ -2.8717, \quad -2.4358] \left(\begin{bmatrix} -1 \\ -1 \\ 8/3 \\ 5/3 \end{bmatrix} \right) \\
&= -14.6918
\end{aligned}
$$

Therefore,

$$
\boxed{\Omega_1^T = [-17.2778, \quad -14.6918]}
$$

Find the weights for training face 2:

$$
\begin{aligned}
\Omega_2^T &= \begin{bmatrix} w_1, & w_2 \end{bmatrix} \\
w_1 &= u_1^T(\Gamma_2 - \Psi) \\
&= \begin{bmatrix} 4.2538, & 3.0000, & -2.9848, & -1.2386 \end{bmatrix} \left(\begin{bmatrix} 2 \\ 1 \\ 5 \\ 2 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \\ 19/3 \\ 10/3 \end{bmatrix} \right) \\
&= \begin{bmatrix} 4.2538, & 3.0000, & -2.9848, & -1.2386 \end{bmatrix} \left(\begin{bmatrix} -2 \\ -1 \\ -4/3 \\ -4/3 \end{bmatrix} \right) \\
&= -5.8764 \\
w_2 &= u_2^T(\Gamma_2 - \Psi) \\
&= \begin{bmatrix} 2.1281, & 0.8461, & -2.8717, & -2.4358 \end{bmatrix} \left(\begin{bmatrix} 2 \\ 1 \\ 5 \\ 2 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \\ 19/3 \\ 10/3 \end{bmatrix} \right) \\
&= \begin{bmatrix} 2.1281, & 0.8461, & -2.8717, & -2.4358 \end{bmatrix} \left(\begin{bmatrix} -2 \\ -1 \\ -4/3 \\ -4/3 \end{bmatrix} \right) \\
&= 1.9744
\end{aligned}
$$

Therefore,

$$\boxed{\Omega_2^T = \begin{bmatrix} -5.8764, & -1.9744 \end{bmatrix}}$$

Find the weights for training face 3:

$$
\begin{aligned}
\Omega_3^T &= [w_1, \quad w_2] \\
w_1 &= u_1^T(\Gamma_3 - \Psi) \\
&= [4.2538, \quad 3.0000, \quad -2.9848, \quad -1.2386] \left(\begin{bmatrix} 7 \\ 4 \\ 5 \\ 3 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \\ 19/3 \\ 10/3 \end{bmatrix} \right) \\
&= [4.2538, \quad 3.0000, \quad -2.9848, \quad -1.2386] \left(\begin{bmatrix} 3 \\ 2 \\ -4/3 \\ -1/3 \end{bmatrix} \right) \\
&= 23.1540 \\
w_2 &= u_2^T(\Gamma_3 - \Psi) \\
&= [2.1281, \quad 0.8461, \quad -2.8717, \quad -2.4358] \left(\begin{bmatrix} 7 \\ 4 \\ 5 \\ 3 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \\ 19/3 \\ 10/3 \end{bmatrix} \right) \\
&= [2.1281, \quad 0.8461, \quad -2.8717, \quad -2.4358] \left(\begin{bmatrix} 3 \\ 2 \\ -4/3 \\ -1/3 \end{bmatrix} \right) \\
&= 12.7174
\end{aligned}
$$

Therefore,

$$
\Omega_3^T = [23.1540, \quad 12.7174]
$$

Thus, the weight for training faces in the training dataset is:

$$
\Omega_k = \begin{bmatrix} -17.2778 & -5.8764 & 23.1540 \\ -14.6918 & -1.9744 & 12.7174 \end{bmatrix}
$$

STEP 9: Find the weight of unknown face from the training dataset.

$$
\begin{aligned}
\Omega^T &= [w_1, \quad w_2] \\
w_1 &= u_1^T(F_1 - \Psi) \\
&= [4.2538, \quad 3.0000, \; -2.9848, \quad -1.2386] \left(\begin{bmatrix} 3 \\ 2 \\ 6 \\ 4 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \\ 19/3 \\ 10/3 \end{bmatrix} \right) \\
&= [4.2538, \quad 3.0000, \; -2.9848, \quad -1.2386] \left(\begin{bmatrix} -1 \\ 0 \\ -1/3 \\ 2/3 \end{bmatrix} \right) \\
&= -4.0846 \\
w_2 &= u_2^T(F_1 - \Psi) \\
&= [2.1281, \quad 0.8461, \; -2.8717, \quad -2.4358] \left(\begin{bmatrix} 3 \\ 2 \\ 6 \\ 4 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \\ 19/3 \\ 10/3 \end{bmatrix} \right) \\
&= [2.1281, \quad 0.8461, \; -2.8717, \quad -2.4358] \left(\begin{bmatrix} -1 \\ 0 \\ -1/3 \\ 2/3 \end{bmatrix} \right) \\
&= -2.7947
\end{aligned}
$$

Therefore,

$$\boxed{\Omega^T = [-4.0846, \quad -2.7947]}$$

Thus, the weight for unknown face in the test dataset is:

$$\boxed{\Omega = \begin{bmatrix} -4.0846 \\ -2.7947 \end{bmatrix}}$$

STEP 10: Recognize the unknown face from the test dataset.

To recognize an unknown face, we need to find the Euclidean Distance between training weights and unknown face weight.

Using the following Euclidean Distance equation:

$$\boxed{\epsilon_k^2 = \|(\Omega - \Omega_k)\|^2}$$

$$\begin{aligned}
\epsilon_1^2 &= \|(\Omega - \Omega_1)\|^2 \\
&= \left\|\left(\begin{bmatrix}-4.0846\\-2.7947\end{bmatrix} - \begin{bmatrix}-17.2778\\-14.6918\end{bmatrix}\right)\right\|^2 \\
&= \left\|\begin{bmatrix}13.1932\\11.8971\end{bmatrix}\right\|^2 \\
&= \left(\sqrt{(13.1932)^2 + (11.8971)^2}\right)^2 \\
&= (13.1932)^2 + (11.8971)^2 \\
&= 315.60151
\end{aligned}$$

$$\begin{aligned}
\epsilon_2^2 &= \|(\Omega - \Omega_2)\|^2 \\
&= \left\|\left(\begin{bmatrix}-4.0846\\-2.7947\end{bmatrix} - \begin{bmatrix}-5.8764\\-1.9744\end{bmatrix}\right)\right\|^2 \\
&= \left\|\begin{bmatrix}1.7918\\-0.8203\end{bmatrix}\right\|^2 \\
&= \left(\sqrt{(1.7918)^2 + (-0.8203)^2}\right)^2 \\
&= (1.7918)^2 + (-0.8203)^2 \\
&= 3.8834
\end{aligned}$$

$$\begin{aligned}
\epsilon_3^2 &= \|(\Omega - \Omega_3)\|^2 \\
&= \left\|\left(\begin{bmatrix}-4.0846\\-2.7947\end{bmatrix} - \begin{bmatrix}23.1540\\12.7174\end{bmatrix}\right)\right\|^2 \\
&= \left\|\begin{bmatrix}-27.2386\\-15.5121\end{bmatrix}\right\|^2 \\
&= \left(\sqrt{(-27.2386)^2 + (-15.5121)^2}\right)^2 \\
&= (-27.2386)^2 + (-15.5121)^2 \\
&= 982.56658
\end{aligned}$$

Therefore, the unknown face belongs to face 2.