Face Recognition using Eigenface, A Simple Example

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The size of image used in my Scilab program is 92 pixels wide x 112 pixels height. But in this example, assume that the size of image used is 2 pixels wide x 2 pixels height because I want to keep the calculations as simple as possible. We are going to find out who is this unknown face F_1 in the test dataset.

Training dataset	Test dataset
$T_1=\begin{bmatrix}3 & 1\\9 & 5\end{bmatrix}$ $T_2=\begin{bmatrix}2 & 1\\5 & 2\end{bmatrix}$ $T_3=\begin{bmatrix}7 & 4\\5 & 3\end{bmatrix}$	$F_1=\begin{bmatrix}3 & 2\\6 & 4\end{bmatrix}$

STEP 1: Convert the training faces T_1 , T_2 , and T_3 from image into a vector. The array of vectors Γ will store these face vectors.

$$\Gamma = \begin{bmatrix}T_1^T, & T_2^T, & T_3^T\end{bmatrix} = \begin{bmatrix}3 & 2 & 7\\1 & 1 & 4\\9 & 5 & 5\\5 & 2 & 3\end{bmatrix}$$

The reason why we need to convert image into vector because the Principle Component Analysis (PCA) does not work directly on image.

STEP 2: Find the common features of a human face by calculating the mean of the training faces.

$$\Psi = \frac{1}{M}\sum_{n=1}^{M}\Gamma_n = \frac{1}{3}\left(\begin{bmatrix}3\\1\\9\\5\end{bmatrix}+\begin{bmatrix}2\\1\\5\\2\end{bmatrix}+\begin{bmatrix}7\\4\\5\\3\end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix}12\\6\\19\\10\end{bmatrix} = \begin{bmatrix}4\\2\\19/3\\10/3\end{bmatrix}$$

STEP 3: Find the unique features of a human face by subtracting a face with common features.

Using the following equation to find the unique features:

$$\Phi_i = \Gamma_i - \Psi$$

$$\Phi_1 = \Gamma_1 - \Psi = \begin{bmatrix}3\\1\\9\\5\end{bmatrix} - \begin{bmatrix}4\\2\\19/3\\10/3\end{bmatrix} = \begin{bmatrix}-1\\-1\\8/3\\5/3\end{bmatrix}$$

$$\Phi_2 = \Gamma_2 - \Psi = \begin{bmatrix}2\\1\\5\\2\end{bmatrix} - \begin{bmatrix}4\\2\\19/3\\10/3\end{bmatrix} = \begin{bmatrix}-2\\-1\\-4/3\\-4/3\end{bmatrix}$$

$$\Phi_3 = \Gamma_3 - \Psi = \begin{bmatrix}7\\4\\5\\3\end{bmatrix} - \begin{bmatrix}4\\2\\19/3\\10/3\end{bmatrix} = \begin{bmatrix}3\\2\\-4/3\\-1/3\end{bmatrix}$$

The vector Φ_1 stores the unique feature of the face T_1, the vector Φ_2 stores the unique feature of the face T_2, and the vector Φ_3 stores the unique featore of the face T_3.

STEP 4: Construct a Covariance Matrix L with size $M \times M$, where M is the total number of faces in training dataset.

$$
\begin{aligned}
L &= \Phi^T \Phi \\
&= \begin{bmatrix} -1 & -1 & 8/3 & 5/3 \\ -2 & -1 & -4/3 & -4/3 \\ 3 & 2 & -4/3 & -1/3 \end{bmatrix} \begin{bmatrix} -1 & -2 & 3 \\ -1 & -1 & 2 \\ 8/3 & -4/3 & -4/3 \\ 5/3 & -4/3 & -1/3 \end{bmatrix} \\
&= \begin{bmatrix} 107/9 & -25/9 & -82/9 \\ -25/9 & 77/9 & -52/9 \\ -82/9 & -52/9 & 134/9 \end{bmatrix}
\end{aligned}
$$

We will find the eigenvalues and eigenvectors by using this matrix L. Those eigenvalues and eigenvectors are needed to find the Eigenface.

STEP 5: Find eigenvalues of the Matrix $\boxed{L}$.

$$\det(\lambda I - L) = 0$$

$$\det\left(\lambda\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} - \begin{bmatrix}107/9 & -25/9 & -82/9\\ -25/9 & 77/9 & -52/9\\ -82/9 & -52/9 & 134/9\end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix}\lambda-107/9 & 25/9 & 82/9\\ 25/9 & \lambda-77/9 & 52/9\\ 82/9 & 52/9 & \lambda-134/9\end{bmatrix}\right) = 0$$

$$(\lambda-107/9)[(\lambda-77/9)(\lambda-134/9)-(52/9)(52/9)]-$$
$$25/9[(25/9)(\lambda-134/9)-(52/9)(82/9)]+$$
$$82/9[(25/9)(52/9)-(\lambda-77/9)(82/9)] = 0$$

$$(\lambda-107/9)[\lambda^2-211/9\lambda+94]-$$
$$25/9[25/9\lambda-94]+$$
$$82/9[-82/9\lambda+94] = 0$$

$$(\lambda^3-211/9\lambda^2+94\lambda-107/9\lambda^2+22577/81\lambda-10058/9)-$$
$$(625/81\lambda-2350/9)+$$
$$(-6724/81\lambda+7708/9) = 0$$

$$(\lambda^3-106/3\lambda^2+30191/81\lambda-10058/9)-$$
$$(625/81\lambda-2350/9)+$$
$$(-6724/81\lambda+7708/9) = 0$$

$$\lambda^3-106/3\lambda^2+30191/81\lambda-625/81\lambda-6724/81\lambda-$$
$$10058/9+2350/9+7708/9 = 0$$

$$\lambda^3-106/3\lambda^2+282\lambda+0 = 0$$

$$\lambda_1 = 23.1540$$
$$\lambda_2 = 0$$
$$\lambda_3 = 12.1793$$

Sort the eigenvalues in descending order

$$\lambda_1 = 23.1540$$
$$\lambda_2 = 12.1793$$
$$\lambda_3 = 0$$

STEP 6: Find the eigenvectors of the Matrix $\boxed{L}$ by using Gaussian Elimination method.

$$\boxed{\begin{aligned} B &= [\lambda I - L] \\ &= \begin{bmatrix} \lambda-107/9 & 25/9 & 82/9 \\ 25/9 & \lambda-77/9 & 52/9 \\ 82/9 & 52/9 & \lambda-134/9 \end{bmatrix} \end{aligned}}$$

To find the eigenvectors, we need to solve $\boxed{Bx = 0}$ for every eigenvalues $\boxed{\lambda = 23.1540}$, $\boxed{\lambda = 12.1793}$, and $\boxed{\lambda = 0}$.

Case 1: $\boxed{\lambda = 23.1540}$

$$Bx = 0$$

$$\begin{bmatrix} \lambda-107/9 & 25/9 & 82/9 \\ 25/9 & \lambda-77/9 & 52/9 \\ 82/9 & 52/9 & \lambda-134/9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 23.1540-107/9 & 25/9 & 82/9 \\ 25/9 & 23.1540-77/9 & 52/9 \\ 82/9 & 52/9 & 23.1540-134/9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 11.2651 & 25/9 & 82/9 \\ 25/9 & 14.5984 & 52/9 \\ 82/9 & 52/9 & 8.2651 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 11.2651 & 25/9 & 82/9 & 0 \\ 25/9 & 14.5984 & 52/9 & 0 \\ 82/9 & 52/9 & 8.2651 & 0 \end{array}\right]$$

$$\underline{R_1/11.2651} \left[\begin{array}{ccc|c} 1 & 0.2466 & 0.8088 & 0 \\ 25/9 & 14.5984 & 52/9 & 0 \\ 82/9 & 52/9 & 8.2651 & 0 \end{array}\right]$$

$$\underline{R_2-25/9\,R_1} \left[\begin{array}{ccc|c} 1 & 0.2466 & 0.8088 & 0 \\ 0 & 13.9134 & 3.5311 & 0 \\ 82/9 & 52/9 & 8.2651 & 0 \end{array}\right]$$

$$\underline{R_3-82/9\,R_1} \left[\begin{array}{ccc|c} 1 & 0.2466 & 0.8088 & 0 \\ 0 & 13.9134 & 3.5311 & 0 \\ 0 & 3.5310 & 0.8960 & 0 \end{array}\right]$$

$$\underline{R_2/13.9134} \left[\begin{array}{ccc|c} 1 & 0.2466 & 0.8088 & 0 \\ 0 & 1 & 0.2538 & 0 \\ 0 & 3.5310 & 0.8960 & 0 \end{array}\right]$$

$$\underline{R_1-0.2466\,R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0.7462 & 0 \\ 0 & 1 & 0.2538 & 0 \\ 0 & 3.5310 & 0.8960 & 0 \end{array}\right]$$

$$\underline{R_3-3.5310R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0.7462 & 0 \\ 0 & 1 & 0.2538 & 0 \\ 0 & 0 & -0.0002 & 0 \end{array}\right]$$

$$\underline{R_3/-0.0002} \left[\begin{array}{ccc|c} 1 & 0 & 0.7462 & 0 \\ 0 & 1 & 0.2538 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{aligned} x_1 + 0.7462\,x_3 &= 0 \\ x_2 + 0.2538\,x_3 &= 0 \\ x_3 &= 1 \end{aligned}$$

$$\begin{aligned} x_1 &= -0.7462(1) \\ x_2 &= -0.2538(1) \\ x_3 &= 1 \end{aligned}$$

$$\begin{aligned} v_1 &= \frac{1}{\max|x_1, x_2, x_3|}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \\ &= \frac{1}{1}\begin{bmatrix} -0.7462 \\ -0.2538 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} -0.7462 \\ -0.2538 \\ 1 \end{bmatrix} \end{aligned}$$

Therefore, the eigenvector for eigenvalue $\boxed{\lambda = 23.1540}$ is

$$v_1 = \begin{bmatrix} -0.7462 \\ -0.2538 \\ 1.0000 \end{bmatrix}$$

Case 2: $\boxed{\lambda=12.1793}$

$$Bx = 0$$

$$\begin{bmatrix} \lambda-107/9 & 25/9 & 82/9 \\ 25/9 & \lambda-77/9 & 52/9 \\ 82/9 & 52/9 & \lambda-134/9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 12.1793-107/9 & 25/9 & 82/9 \\ 25/9 & 12.1793-77/9 & 52/9 \\ 82/9 & 52/9 & 12.1793-134/9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.2904 & 25/9 & 82/9 \\ 25/9 & 3.6237 & 52/9 \\ 82/9 & 52/9 & -2.7096 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 0.2904 & 25/9 & 82/9 & 0 \\ 25/9 & 3.6237 & 52/9 & 0 \\ 82/9 & 52/9 & -2.7096 & 0 \end{array}\right]$$

$$\underline{R_1/0.2904}\left[\begin{array}{ccc|c} 1 & 9.5654 & 31.3743 & 0 \\ 25/9 & 3.6237 & 52/9 & 0 \\ 82/9 & 52/9 & -2.7096 & 0 \end{array}\right]$$

$$\underline{R_2-25/9R_1}\left[\begin{array}{ccc|c} 1 & 9.5654 & 31.3743 & 0 \\ 0 & -22.9469 & -81.3731 & 0 \\ 82/9 & 52/9 & -2.7096 & 0 \end{array}\right]$$

$$\underline{R_3-82/9R_1}\left[\begin{array}{ccc|c} 1 & 9.5654 & 31.3743 & 0 \\ 0 & -22.9469 & -81.3731 & 0 \\ 0 & -81.3736 & -288.5643 & 0 \end{array}\right]$$

$$\underline{R_2/-22.9469}\left[\begin{array}{ccc|c} 1 & 9.5654 & 31.3743 & 0 \\ 0 & 1 & 3.5461 & 0 \\ 0 & -81.3736 & -288.5643 & 0 \end{array}\right]$$

$$\underline{R_1-9.5654R_2}\left[\begin{array}{ccc|c} 1 & 0 & -2.5456 & 0 \\ 0 & 1 & 3.5461 & 0 \\ 0 & -81.3736 & -288.5643 & 0 \end{array}\right]$$

$$\underline{R_3+81.3736R_2}\left[\begin{array}{ccc|c} 1 & 0 & -2.5456 & 0 \\ 0 & 1 & 3.5461 & 0 \\ 0 & 0 & -0.0054 & 0 \end{array}\right]$$

$$\underline{R_3/-0.0054}\left[\begin{array}{ccc|c} 1 & 0 & -2.5456 & 0 \\ 0 & 1 & 3.5461 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{aligned} x_1 - 2.5456\,x_3 &= 0 \\ x_2 + 3.5461\,x_3 &= 0 \\ x_3 &= 1 \end{aligned}$$

$$\begin{aligned} x_1 &= 2.5456(1) \\ x_2 &= -3.5461(1) \\ x_3 &= 1 \end{aligned}$$

$$\begin{aligned} v_2 &= \frac{1}{\max|x_1, x_2, x_3|}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \\ &= \frac{1}{3.5461}\begin{bmatrix} 2.5456 \\ -3.5461 \\ 1.0000 \end{bmatrix} \\ &= \begin{bmatrix} 0.7179 \\ -1.0000 \\ 0.2820 \end{bmatrix} \end{aligned}$$

Therefore, the eigenvector for eigenvalue $\boxed{\lambda = 12.1793}$ is

$$v_2 = \begin{bmatrix} 0.7179 \\ -1.0000 \\ 0.2820 \end{bmatrix}$$

Case 3: $\boxed{\lambda = 0}$

Unfortunately, eigenvalue with value 0 or near to 0 will produce a useless eigenvector, so this case is discarded.

Therefore, the eigenvector of Matrix $\boxed{L}$ is:

$$\boxed{v = \begin{bmatrix} -0.7462 & 0.7179 \\ -0.2538 & -1.0000 \\ 1.0000 & 0.2820 \end{bmatrix}}$$

STEP 7: Find the Eigenface, $\boxed{u_l}$. We need eigenface value to extract useful features from a face.

Using the following equation to find the Eigenface:

$$\boxed{u_l = \sum_{k=1}^{M} v_{lk}\Phi_k}$$, where $\boxed{M = 3}$ the number of training faces.

$$
\begin{aligned}
u_1 &= (-0.7462)\begin{bmatrix}-1\\-1\\8/3\\5/3\end{bmatrix}+(-0.2538)\begin{bmatrix}-2\\-1\\-4/3\\-4/3\end{bmatrix}+(1.0000)\begin{bmatrix}3\\2\\-4/3\\-1/3\end{bmatrix}\\
&= \begin{bmatrix}0.7462\\0.7462\\-1.9899\\-1.2437\end{bmatrix}+\begin{bmatrix}0.5076\\0.2538\\0.3384\\0.3384\end{bmatrix}+\begin{bmatrix}3\\2\\-4/3\\-1/3\end{bmatrix}\\
&= \begin{bmatrix}4.2538\\3.0000\\-2.9848\\-1.2386\end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
u_2 &= (0.7179)\begin{bmatrix}-1\\-1\\8/3\\5/3\end{bmatrix}+(-1.0000)\begin{bmatrix}-2\\-1\\-4/3\\-4/3\end{bmatrix}+(0.2820)\begin{bmatrix}3\\2\\-4/3\\-1/3\end{bmatrix}\\
&= \begin{bmatrix}-0.7179\\-0.7179\\1.9144\\1.1965\end{bmatrix}+\begin{bmatrix}2\\1\\4/3\\4/3\end{bmatrix}+\begin{bmatrix}0.8460\\0.5640\\-0.3760\\-0.0940\end{bmatrix}\\
&= \begin{bmatrix}2.1281\\0.8461\\2.8717\\2.4358\end{bmatrix}
\end{aligned}
$$

Therefore, $$\boxed{u = \begin{bmatrix}4.2538 & 2.1281\\3.0000 & 0.8461\\-2.9848 & 2.8717\\-1.2386 & 2.4358\end{bmatrix}}$$

STEP 8: Find the weight values $\boxed{\Omega_k}$ for each training faces.

Using the following equations to find the weight:

$$\boxed{\Omega^T = [w_1, \quad w_2, \quad w_3 \quad \ldots \quad w_M]} \quad ,$$

where $\boxed{M}$ number of faces in the training dataset.

$$\boxed{w_k = u_k^T(\Gamma - \Psi)} \quad ,$$

where $\boxed{\Gamma}$ is a face vector from training dataset.

Find the weights for training face 1 $\boxed{\Gamma_1}$:

$$
\begin{aligned}
\Omega_1^T &= [w_1, \quad w_2] \\
w_1 &= u_1^T(\Gamma_1 - \Psi) \\
&= [4.2538, \quad 3.0000, \ -2.9848, \quad -1.2386] \left(\begin{bmatrix} 3 \\ 1 \\ 9 \\ 5 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \\ 19/3 \\ 10/3 \end{bmatrix} \right) \\
&= [4.2538, \quad 3.0000, \ -2.9848, \quad -1.2386] \left(\begin{bmatrix} -1 \\ -1 \\ 8/3 \\ 5/3 \end{bmatrix} \right) \\
&= -17.2778 \\
w_2 &= u_2^T(\Gamma_1 - \Psi) \\
&= [2.1281, \quad 0.8461, \ -2.8717, \quad -2.4358] \left(\begin{bmatrix} 3 \\ 1 \\ 9 \\ 5 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \\ 19/3 \\ 10/3 \end{bmatrix} \right) \\
&= [2.1281, \quad 0.8461, \ -2.8717, \quad -2.4358] \left(\begin{bmatrix} -1 \\ -1 \\ 8/3 \\ 5/3 \end{bmatrix} \right) \\
&= -14.6918
\end{aligned}
$$

Therefore,

$$\boxed{\Omega_1^T = [-17.2778, \quad -14.6918]}$$

Find the weights for training face 2 $\boxed{\Gamma_2}$:

$$
\begin{aligned}
\Omega_2^T &= [w_1, \quad w_2] \\
w_1 &= u_1^T(\Gamma_2 - \Psi) \\
&= [4.2538, \quad 3.0000, \quad -2.9848, \quad -1.2386] \left(\begin{bmatrix} 2 \\ 1 \\ 5 \\ 2 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \\ 19/3 \\ 10/3 \end{bmatrix} \right) \\
&= [4.2538, \quad 3.0000, \quad -2.9848, \quad -1.2386] \left(\begin{bmatrix} -2 \\ -1 \\ -4/3 \\ -4/3 \end{bmatrix} \right) \\
&= -5.8764 \\
w_2 &= u_2^T(\Gamma_2 - \Psi) \\
&= [2.1281, \quad 0.8461, \quad -2.8717, \quad -2.4358] \left(\begin{bmatrix} 2 \\ 1 \\ 5 \\ 2 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \\ 19/3 \\ 10/3 \end{bmatrix} \right) \\
&= [2.1281, \quad 0.8461, \quad -2.8717, \quad -2.4358] \left(\begin{bmatrix} -2 \\ -1 \\ -4/3 \\ -4/3 \end{bmatrix} \right) \\
&= 1.9744
\end{aligned}
$$

Therefore,

$$\boxed{\Omega_2^T = [-5.8764, \quad -1.9744]}$$

Find the weights for training face 3 $\boxed{\Gamma_3}$:

$$
\begin{aligned}
\Omega_3^T &= [w_1, \quad w_2] \\
w_1 &= u_1^T(\Gamma_3 - \Psi) \\
&= [4.2538, \quad 3.0000, \; -2.9848, \quad -1.2386] \left(\begin{bmatrix} 7 \\ 4 \\ 5 \\ 3 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \\ 19/3 \\ 10/3 \end{bmatrix} \right) \\
&= [4.2538, \quad 3.0000, \; -2.9848, \quad -1.2386] \left(\begin{bmatrix} 3 \\ 2 \\ -4/3 \\ -1/3 \end{bmatrix} \right) \\
&= 23.1540 \\
w_2 &= u_2^T(\Gamma_3 - \Psi) \\
&= [2.1281, \quad 0.8461, \; -2.8717, \quad -2.4358] \left(\begin{bmatrix} 7 \\ 4 \\ 5 \\ 3 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \\ 19/3 \\ 10/3 \end{bmatrix} \right) \\
&= [2.1281, \quad 0.8461, \; -2.8717, \quad -2.4358] \left(\begin{bmatrix} 3 \\ 2 \\ -4/3 \\ -1/3 \end{bmatrix} \right) \\
&= 12.7174
\end{aligned}
$$

Therefore,

$$\Omega_3^T = [23.1540, \quad 12.7174]$$

Thus, the weight for training faces in the training dataset is:

$$\Omega_k = \begin{bmatrix} -17.2778 & -5.8764 & 23.1540 \\ -14.6918 & -1.9744 & 12.7174 \end{bmatrix}$$

STEP 9: Find the weight of the unknown face $\boxed{F_1}$ from the training dataset.

$$
\begin{aligned}
\Omega^T &= \begin{bmatrix} w_1, & w_2 \end{bmatrix} \\
w_1 &= u_1^T(F_1 - \Psi) \\
&= \begin{bmatrix} 4.2538, & 3.0000, & -2.9848, & -1.2386 \end{bmatrix} \left(\begin{bmatrix} 3 \\ 2 \\ 6 \\ 4 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \\ 19/3 \\ 10/3 \end{bmatrix} \right) \\
&= \begin{bmatrix} 4.2538, & 3.0000, & -2.9848, & -1.2386 \end{bmatrix} \left(\begin{bmatrix} -1 \\ 0 \\ -1/3 \\ 2/3 \end{bmatrix} \right) \\
&= -4.0846 \\
w_2 &= u_2^T(F_1 - \Psi) \\
&= \begin{bmatrix} 2.1281, & 0.8461, & -2.8717, & -2.4358 \end{bmatrix} \left(\begin{bmatrix} 3 \\ 2 \\ 6 \\ 4 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \\ 19/3 \\ 10/3 \end{bmatrix} \right) \\
&= \begin{bmatrix} 2.1281, & 0.8461, & -2.8717, & -2.4358 \end{bmatrix} \left(\begin{bmatrix} -1 \\ 0 \\ -1/3 \\ 2/3 \end{bmatrix} \right) \\
&= -2.7947
\end{aligned}
$$

Therefore,

$$\Omega^T = \begin{bmatrix} -4.0846, & -2.7947 \end{bmatrix}$$

Thus, the weight for unknown face $\boxed{F_1}$ in the test dataset is:

$$\Omega = \begin{bmatrix} -4.0846 \\ -2.7947 \end{bmatrix}$$

STEP 10: Recognize the unknown face $\boxed{F_1}$ from the test dataset.

To recognize an unknown face, we need to find the Euclidean Distance between the training face weights and the unknown face weight. We will compare the distance of weight value between $\boxed{\Gamma_1}$ with $\boxed{F_1}$, $\boxed{\Gamma_2}$ with $\boxed{F_1}$, and $\boxed{\Gamma_3}$ with $\boxed{F_1}$ to find out which one has the closest distance.

Using the following Euclidean Distance equation:

$$\boxed{\epsilon_k^2 = \|(\Omega - \Omega_k)\|^2}$$

$$
\begin{aligned}
\epsilon_1^2 &= \|(\Omega-\Omega_1)\|^2 \\
&= \left\|\left(\begin{bmatrix}-4.0846\\-2.7947\end{bmatrix}-\begin{bmatrix}-17.2778\\-14.6918\end{bmatrix}\right)\right\|^2 \\
&= \left\|\begin{bmatrix}13.1932\\11.8971\end{bmatrix}\right\|^2 \\
&= \left(\sqrt{(13.1932)^2+(11.8971)^2}\right)^2 \\
&= (13.1932)^2+(11.8971)^2 \\
&= 315.60151
\end{aligned}
$$

$$
\begin{aligned}
\epsilon_2^2 &= \|(\Omega-\Omega_2)\|^2 \\
&= \left\|\left(\begin{bmatrix}-4.0846\\-2.7947\end{bmatrix}-\begin{bmatrix}-5.8764\\-1.9744\end{bmatrix}\right)\right\|^2 \\
&= \left\|\begin{bmatrix}1.7918\\-0.8203\end{bmatrix}\right\|^2 \\
&= \left(\sqrt{(1.7918)^2+(-0.8203)^2}\right)^2 \\
&= (1.7918)^2+(-0.8203)^2 \\
&= 3.8834
\end{aligned}
$$

$$
\begin{aligned}
\epsilon_3^2 &= \|(\Omega-\Omega_3)\|^2 \\
&= \left\|\left(\begin{bmatrix}-4.0846\\-2.7947\end{bmatrix}-\begin{bmatrix}23.1540\\12.7174\end{bmatrix}\right)\right\|^2 \\
&= \left\|\begin{bmatrix}-27.2386\\-15.5121\end{bmatrix}\right\|^2 \\
&= \left(\sqrt{(-27.2386)^2+(-15.5121)^2}\right)^2 \\
&= (-27.2386)^2+(-15.5121)^2 \\
&= 982.56658
\end{aligned}
$$

Therefore, the unknown face $\boxed{F_1}$ belongs to face 2 $\boxed{\Gamma_2}$ because their euclidean distance $\boxed{\epsilon_2^2 = 3.8834}$ is the smallest.

References

[1] Eigenfaces for recognition - Turk, Pentland - 1991