Introduction to AI
Exercises at week 8

Ex. 1 True or false?

1. $\top \models \bot$
2. $\bot \models \top$
3. $p \wedge q \models p \leftrightarrow q$
4. $p \leftrightarrow q \models p \wedge q$
5. $p \leftrightarrow q \models \neg p \wedge q$
6. $(p \vee q) \wedge (\neg r \vee \neg s \vee t) \models (p \vee q \vee r) \wedge (q \vee \neg r \vee s \rightarrow t)$
7. $(p \vee q) \wedge \neg(p \rightarrow q)$ is satisfiable
8. $(p \leftrightarrow q) \wedge (\neg p \vee q)$ is satisfiable
9. $(p \leftrightarrow q) \leftrightarrow r$ has the same number of models as $(p \leftrightarrow q)$ for any fixed set of proposition symbols that includes p, q, r.

Ex. 2 How many models are there for the following sentences (assuming we only have p, q, r, s in the vocabulary)?

1. $q \vee r$
2. $\neg p \vee \neg q \vee \neg r \vee \neg s$
3. $(p \rightarrow q) \wedge p \wedge \neg q \wedge r \wedge s$

Ex. 3 We have defined four binary logical connectives.

1. Are there others that can be useful?
2. How many logical connectives can there be?
3. Why are some of them not very useful?

Ex. 4 Decide for each of the following, is it valid, unsatisfiable or neither?

1. $Smoke \rightarrow Smoke$
2. $Smoke \rightarrow Fire$
3. $(Smoke \rightarrow Fire) \rightarrow (\neg Smoke \rightarrow \neg Fire)$
4. $Smoke \vee Fire \vee \neg Fire$
5. $Big \vee Long \vee (Big \rightarrow Long)$
6. $(Big \wedge Long) \vee \neg Long$

Ex. 5 (EXTRA exercise, to recall truth-tables, if needed) For each of the following formulas decide: are they tautologies? are they satisfiable?

1. $p \vee \neg p$
2. $p \wedge \neg p$
3. $(p \wedge q) \rightarrow p$
4. $(p \wedge q) \rightarrow \neg p$
5. $(p_1 \wedge p_2) \rightarrow p_3) \rightarrow (p_2 \rightarrow (p_1 \rightarrow p_3))$
6. $(((p \wedge q) \rightarrow s) \wedge ((p \wedge q) \rightarrow t)) \rightarrow ((p \wedge q) \rightarrow (s \wedge t))$