02180: Introduction to Artificial Intelligence

Lecture 10: Logical Inference

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Outline

Propositional Theorem Proving

Resolution

Horn Clauses and Definite Clauses

Effective Propositional Model Checking

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- Propositional Theorem Proving
- Resolution
- Horn Clauses and Definite Clauses
- Effective Propositional Model Checking

Truth-table Method for Inference

The most intuitive way to check validity of inference by brut-force truth-tables.

- KB: Robert does well in the exam if and only if he is prepared or lucky. Robert does not do well in the exam.
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0	1	0	1	0	1	0
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Semantic vs Syntactic Approach

- (Semantic) model checking:

  enumerating models and showing it holds in all models.
- (Syntactic) theorem proving:

  applying rules of inference directly to the sentences in our knowledge base to construct a proof of the desired sentence without consulting models.

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  applying rules of inference directly to the sentences in our knowledge base to construct a proof of the desired sentence without consulting models.

Reason: If the number of models is large but the length of the proof is short, then theorem proving can be more efficient than model checking.

Some Crucial Concepts

- **Logical equivalence**

  two formulas φ and ψ are logically equivalent:

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  if each of them entails the other: $\varphi \equiv \psi$ if and only if $\varphi \models \psi$ and $\psi \models \varphi$.

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- **Satisfiability**

  φ is satisfiable if it is true in some model.

  The SAT problem, determining the satisfiability of sentences, was the first problem shown to be NP-complete.

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Theorem (Semantic Deduction Theorem)

For any sentences φ and ψ of propositional logic

$$\varphi \models \psi \text{ if and only if } \varphi \rightarrow \psi \text{ is valid.}$$

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(reductio ad absurdum, proof by refutation, proof by contradiction)

Inference and Proofs

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Examples of inference rules

$$\frac{\varphi \rightarrow \psi,\ \varphi}{\psi} \qquad \text{(Modus Ponens)}$$

$$\frac{\varphi \wedge \psi}{\varphi} \qquad \text{(And-Elimination)}$$

Using Search Algorithms to Find Proofs

A friendly statement of the theorem proving problem as a search problem:

- **Initial state**: the initial knowledge base.
- **Actions**: the inference rules.
- **Result**: the result of an action is to add the conclusion to KB.
- **Goal**: the sentence we are trying to prove.

Monotonicity

In **monotonic logics** the set of entailed sentences can only increase as information is added to the knowledge base.

For any sentences φ and ψ, if $KB \models \varphi$ then $KB \cup \psi \models \varphi$.

Note: **non-monotonic logics**, which violate the monotonicity property, capture a common property of human reasoning: changing one's mind.

Outline

Resolution: one rule to rule them all

$$\frac{\ell_1 \vee \ldots \vee \ell_k,\ m}{\ell_1 \vee \ldots \vee \ell_{i-1} \vee \ell_{i+1} \vee \ldots \vee \ell_k} \qquad \text{(Unit Resolution)}$$

where literals ℓ_i and m are complementary (i.e., one is negation of the other).

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Practical comment: mind the **Factoring** (removing duplicates)!

E.g., if we resolve $(A \vee B)$ with $(A \vee \neg B)$, we obtain $(A \vee A)$.
Single A is enough!

Conjunctive normal form (CNF)

- Resolution applies to clauses (disjunctions of literals).
- So, KBs should then be coded as conjunctions of clauses (CNFs).
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4. Distribute $\wedge$ over $\vee$: $(\neg r \vee p \vee s) \wedge (\neg p \vee r) \wedge (\neg s \vee r)$.

A Resolution Algorithm

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3. Each pair that contains complementary literals is resolved to produce a new clause, which is added to the set if it is not already present.
4. The process continues until one of two things happens:
   - A there are no new clauses that can be added, in which case KB does not entail φ; or,
   - B two clauses resolve to yield the empty clause, in which case KB entails φ.

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3. $clauses := (\neg r \lor p \lor s), (\neg p \lor r), (\neg s \lor r), \neg r, p$.
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6. Result: empty clause, so $KB \models \varphi$.

Resolution algorithm is good

1. Always terminates.
2. Is complete, by the **ground resolution theorem**: If a set of clauses is unsatisfiable, then the resolution closure of those clauses contains the empty clause.

Outline

Special kinds of clauses

Sometimes it's enough if we restrict the language to special types of clauses:

Definition
A **definite clause** is a clause of literals of which exactly one is positive.
For example: $(\neg p \vee \neg s \vee r)$ is a definite clause, while $(p \vee s \vee \neg r)$ is not.

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Horn clauses are closed under resolution:
if you resolve two Horn clauses, you get back a Horn clause.

The Importance of Horn Clauses

Why are Horn clauses interesting?

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- Definite clause can be written as an implication whose premise is a conjunction of positive literals and whose conclusion is a single positive literal. E.g., $(\neg p \vee \neg s \vee r)$ is equivalent to $(p \wedge s) \rightarrow r$. The premise is called the **body** and the conclusion is called the **head**. A sentence consisting of a single positive literal, such as r, is called a **fact** $(\top \rightarrow r)$.

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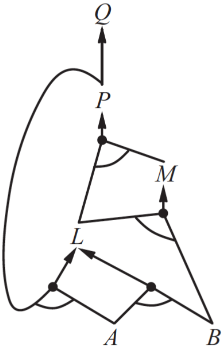
- Inference with Horn clauses can be done by **forward**- and **backward-chaining**.
- Deciding entailment with Horn clauses is **linear** in the size of the knowledge base.

Forward- and Backward-chaining on AND-OR Graphs

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$



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General Model-Checking Algorithms for Propositional Inference

- the algorithms checking satisfiability: the SAT problem
- testing entailment $\varphi \models \psi$, is done by testing unsatisfiability of $\varphi \wedge \neg\psi$

Davis-Putnam Algorithm (DPLL Algorithm)

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Improvements over TT:

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2. **Pure symbol heuristic**, e.g., in $(A \vee \neg B)$, $(\neg B \vee \neg C)$, $(C \vee A)$, the symbol A is pure. If a sentence has a model, then it has a model with the pure symbols assigned so as to make their literals true, because doing so can never make a clause false.

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3. **Unit clause heuristic**, e.g., if the model contains $B = \top$, then $(\neg B \vee \neg C)$ simplifies to $\neg C$, which is a unit clause. Assigning one unit clause can create another unit clause, such 'cascade' of forced assignments is called **unit propagation**.

Davis-Putnam Algorithm (DPLL Algorithm)

```
function DPLL-SATISFIABLE?(s) returns true or false
  inputs: s, a sentence in propositional logic

  clauses ← the set of clauses in the CNF representation of s
  symbols ← a list of the proposition symbols in s
  return DPLL(clauses, symbols, { })
```

```
function DPLL(clauses, symbols, model) returns true or false

  if every clause in clauses is true in model then return true
  if some clause in clauses is false in model then return false
  P, value ← FIND-PURE-SYMBOL(symbols, clauses, model)
  if P is non-null then return DPLL(clauses, symbols – P, model ∪ {P=value})
  P, value ← FIND-UNIT-CLAUSE(clauses, model)
  if P is non-null then return DPLL(clauses, symbols – P, model ∪ {P=value})
  P ← FIRST(symbols); rest ← REST(symbols)
  return DPLL(clauses, rest, model ∪ {P=true}) or
         DPLL(clauses, rest, model ∪ {P=false}))
```

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- We flip the truth value of one symbol at a time and escape local minima with randomness.

- Local search algorithms can be applied to SAT problem given the right evaluation function.
- The goal is to find an assignment that satisfies every clause, so
- evaluation function counts the number of unsatisfied clauses.
- We flip the truth value of one symbol at a time and escape local minima with randomness.

WalkSat: On every iteration, the algorithm picks an unsatisfied clause and picks a symbol in the clause to flip. It chooses randomly between two ways to pick which symbol to flip:

1. a **min-conflicts** step that minimises the number of unsatisfied clauses in the new state, and
2. a **random walk** step that picks the symbol randomly.

WalkSat Algorithm

function WALKSAT(*clauses*, *p*, *max_flips*) **returns** a satisfying model or *failure*
  inputs: *clauses*, a set of clauses in propositional logic
    p, the probability of choosing to do a "random walk" move, typically around 0.5
    max_flips, number of flips allowed before giving up

  model ← a random assignment of *true*/*false* to the symbols in *clauses*
  for $i = 1$ **to** *max_flips* **do**
    if *model* satisfies *clauses* **then return** *model*
    clause ← a randomly selected clause from *clauses* that is false in *model*
    with probability *p* flip the value in *model* of a randomly selected symbol from *clause*
    else flip whichever symbol in *clause* maximizes the number of satisfied clauses
  return *failure*

End of Lecture 10