02180 Intro to AI
Exercises for week 3
SOLUTIONS

Exercise 1

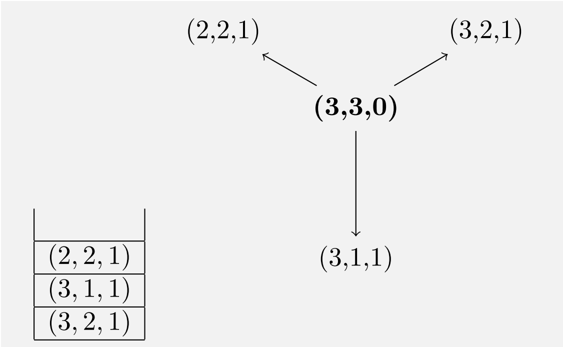
The following exercise is adapted from Exercise 9 in Chapter 3 of the textbook (3rd ed, Global Edition). The *missionaries and cannibals problem* is usually stated as follows. Three missionaries and three cannibals are on one side of a river, along with a boat that can hold one or two people. Find a way to get everyone to the other side without ever leaving a group of missionaries in one place outnumbered by the cannibals in that place.[1] This problem is famous in AI because it was the subject of the first paper that approached problem formulation from an analytical viewpoint (Amarel, 1968).

Solve the problem using the GRAPH-SEARCH algorithm. You can se the pseudocode from the book or the slides. Make sure to keep track of of your *frontier* and your *explored set*. Which *search strategy* are you using (in which order do you choose frontier nodes for expansion)? Does you search strategy guarantee that you find an optimal solution (a solution with fewest possible actions)?

SOLUTION. GRAPH-SEARCH automatically checks for repeated states, which is a good idea in this case, since a DFS tree search might get stuck in the oop (3, 3, 0) → (2, 2, 1) → (3, 2, 0) → (3, 1, 1) → (3, 3, 0). BFS tree search does not risk entering an infinite loop, but the search tree will grow exponentially with the depth, because the initial state will keep being re-expanded at every alternate level of the tree, and it has an outdegree of two (two outgoing edges).

GRAPH-SEARCH can be used with different search strategies. We will here choose DFS. This means that the frontier is a stack. Below we show the first 4 steps of the algorithm, where the boldface nodes are the explored ones, and the frontier is visualised as a stack to the left:

[1] Whenever the boat goes from one side to the other, everybody has to go out before the boat can go back again. The boat can not sail without anybody in it.


(2,2,1)
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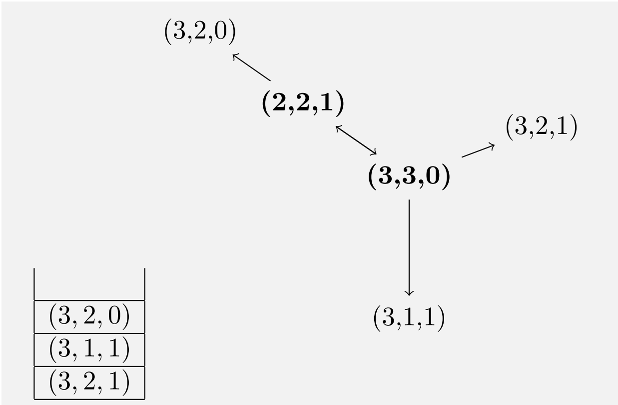
(3,2,0)
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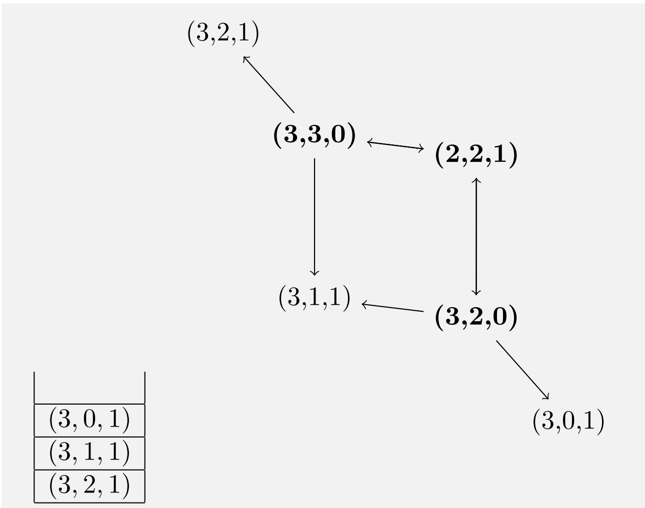
(3,2,1)
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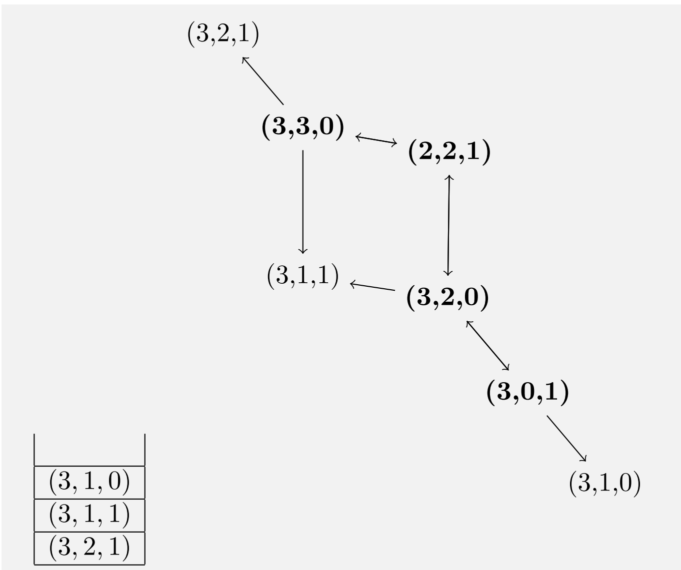

(3,2,1)
(3,3,0)
(2,2,1)
(3,1,1)
(3,2,0)
(3,0,1)
(3,1,0)
(3, 1, 0)
(3, 1, 1)
(3, 2, 1)

Exercise 2

Your goal is to navigate a robot out of a maze. The robot starts in the center of the maze facing north. You can turn the robot to face north, east, south, or west. You can direct the robot to move forward a certain distance, although it will stop before hitting a wall. When thinking about the size of the state space etc. it can be an advantage to look at concrete mazes. Consider for instance the maze and video shown here:

http://ing.dk/video/video-robotmus-overvinder-labyrint-pa-3921-sekunder-124487

a. How can the states of this problem be represented? How large is then the state space, measured as a function of the number of locations in the maze?

SOLUTION. The states of the problem can be represented as triples (x, y, d) consisting of the x- and y-coordinates of the agent as well as its direction d. The size of the state space is the number of possible triples (x, y, d), which is 4 times the number of locations in the maze.

b. Define ACTIONS and RESULTS.

SOLUTION. The actions can be taken to be $\mathit{Turn}(d)$, where $d = W, E, N, S$ and $\mathit{Move}(l)$, where l is a natural number (the length/distance). All actions are applicable in any state, so

$$\textsc{Actions}(s) = \{\mathit{Turn}(d) \mid d \in W, E, N, S\} \cup \{\mathit{Move}(l) \mid l > 0\}$$

The transition model is:

$$\begin{aligned} &\textsc{Result}((x, y, d), \mathit{Turn}(d')) = (x, y, d') \\ &\textsc{Result}((x, y, d), \mathit{Move}(l)) = (x + x', y + y', d), \end{aligned}$$

where if $d = E$ then x' is the maximal number $\leq l$ such that there is no wall in any of the locations $(x + 1, y), (x + 2, y), \ldots, (x + x', y)$. Similarly for $d = W, N, E$.

c. In navigating a maze, the only places we need to turn is at dead ends or intersections of two or more corridors (why?). Hence, from each point of the maze, we can move in any of the four directions until we reach a turning point, and this is the only action we need to do. Reformulate the problem using this observation, that is, reformulate ACTIONS and RESULTS. How large is the state space now?

SOLUTION. The idea is to remove the Turn action, and only have actions $\mathit{Move}(d)$ where $d = W, E, N, S$. The transition model is:

$$\textsc{Result}((x, y, d), \mathit{Move}(d')) = (x + x', y + y', d'),$$

where if $d' = E$ then x' is the maximal number such that $(x+1, y), (x+2, y), \ldots, (x+x'-1, y)$ are not goal locations, not walls and not intersections, and $(x+x', y)$ is not a wall. Note that the parameter d in the argument of RESULT is not used in defining the value of RESULT. This means that we can omit d from the state description and simply describe states as pairs (x, y). The transition model then becomes:

$$\textsc{Result}((x, y), \mathit{Move}(d')) = (x + x', y + y'),$$

where x' and y' are defined in terms of d' as above. The size of the state space is clearly now only the number of locations. We are abstracting away from the direction the robot is facing. In fact, the state space is only the number of intersections and dead ends plus possible the initial state and the goal (that might neither be intersections nor dead ends).

d. In our initial description of the problem we already abstracted from the real world, restricting actions and removing details. List three such simplifications we made.

SOLUTION. (1) The robot can only be in a discrete location, not between locations (compare with the robot mouse video). (2) The robot can only face one of 4 discrete directions, N, S, E and W. (3) The robot is assumed to be able to move precisely a given number of cells. In real life, the robot would probably be a bit imprecise and would need sensors to tell whether a wall has been hit (even if it is assumed to have a full internal representation of the maze).

e. Can you say something about the search strategy that the mouse in the video appears to be using? You are not expected to be able to give a very precise answer to this question.

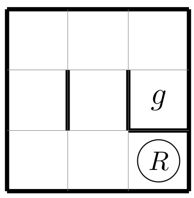
SOLUTION. It appears to be a kind of depth-first (online) search. It could be a greedy-best first search with some heuristics guiding it to the center of the maze, but this is not quite clear. Not also that the mouse does not initially know the maze, so it is really a search problem under *partial observability*, which is different from what we have considered so far.

f. Is finding your way through a maze like this a difficult problem in AI?

SOLUTION. No, in general it is a very simple problem, since it is a rather basic search problem. It can still be a challenge in very big and complex mazes, but computers would generally be much better than humans at solving mazes. They can search a bigger space much more quickly, and there is no deep intuition that a human can use to solve mazes more efficiently than using a simple search algorithm as the computer does (unlike the situation in e.g. chess or Go or natural language understanding).

Exercise 3

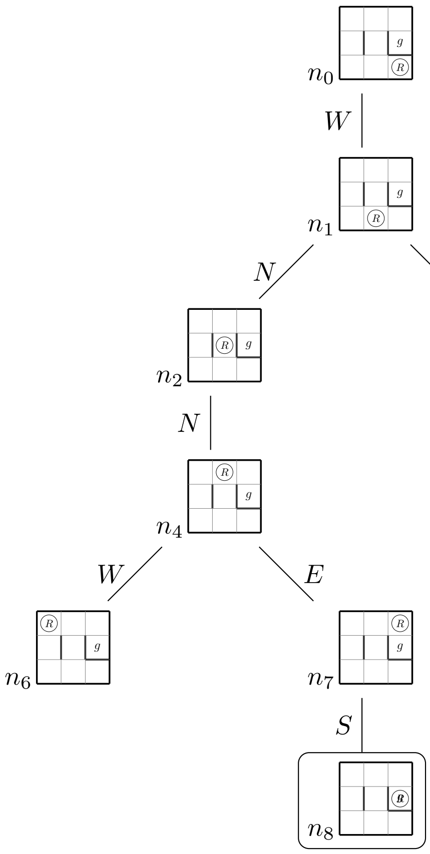
In the previous exercise, we considered a robot navigating a maze. Consider the following slight variation of the problem. The robot, R, is characterised by its position only, it does not have a direction. In each state, the robot can move one cell north (N), west (W), south (S) or east (E) (unless, of course, blocked by a wall in that direction). The goal of the robot is to reach the cell marked with a g. Consider the following instance of the problem:



You should assume that the robot—and the algorithms considered below—always explore the possible moves in the following order: N, W, S, E.

a. Illustrate how BFS graph search would solve this instance of the problem. You should: 1) draw the graph generated by BFS; 2) put numbers on the nodes of the graph according to the order in which they are generated, e.g. use names $n_0, n_1, n_2, \ldots$; 3) for each iteration of the search loop, show the content of the FIFO queue used for the frontier.

SOLUTION. Following the pseudocode from R&N figure 3.11, we get the search tree:



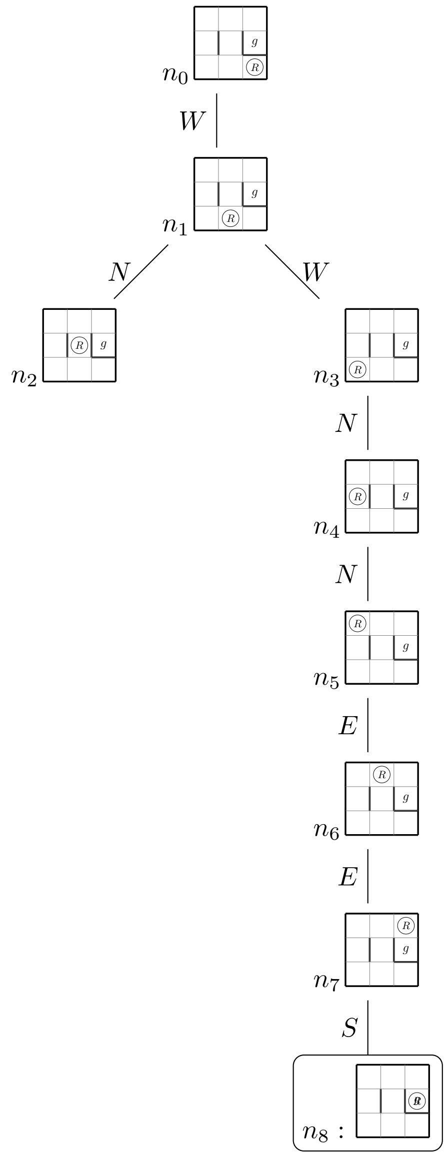
The frontier content is shown as it looks in the beginning of each loop iteration:

Iteration	**Frontier**
1	$[n_0]$
2	$[n_1]$
3	$[n_2, n_3]$
4	$[n_3, n_4]$
5	$[n_4, n_5]$
6	$[n_5, n_6, n_7]$
7	$[n_6, n_7]$
8	$[n_7]$

Because the BFS algorithm checks for goal states before adding states to the frontier, we will never see n_8 in the frontier and the algorithm terminates in the 8th iteration. The solution found is $WNNES$ of length 5. It is optimal (since we're using BFS).

b. Illustrate how DFS graph search would solve this instance of the problem. You should: 1) draw the graph generated by DFS; 2) put numbers on the nodes of the graph according to the order in which they are visited, e.g. use names $n_0, n_1, n_2, \ldots$; 3) for each iteration of the search loop, show the content of the LIFO queue (stack) used for the frontier.

SOLUTION. Following the pseudocode in R&N figure 3.7 for Graph-Search with a stack for the frontier, we get the search tree:



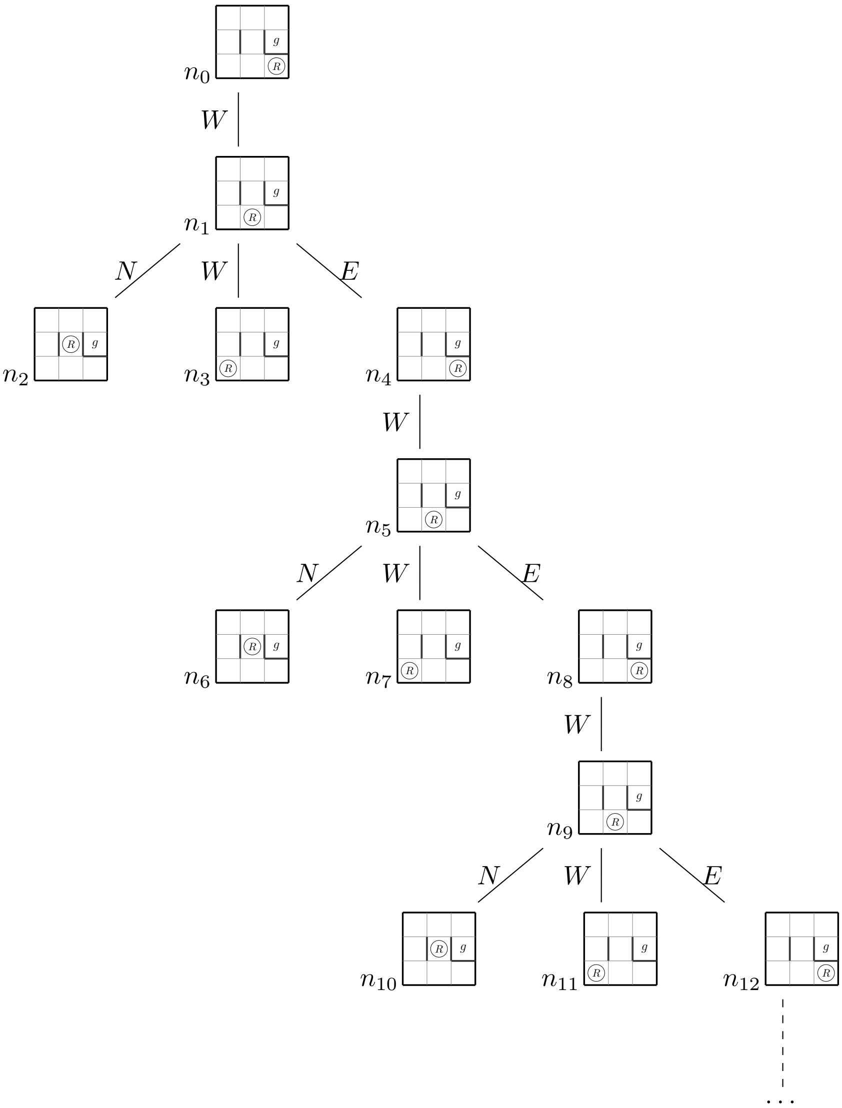
The frontier content is shown as it looks in the beginning of each loop iteration:

Iteration	**Frontier**
1	$[n_0]$
2	$[n_1]$
3	$[n_2, n_3]$
4	$[n_2, n_4]$
5	$[n_2, n_5]$
6	$[n_2, n_6]$
7	$[n_2, n_7]$
8	$[n_2, n_8]$

This time we see the goal state n_8 in the frontier, and terminate when we pop it in the 8th iteration. The solution found is $WWNNEES$ of length 7. It is not optimal.

c. Illustrate how DFS tree search would solve this instance of the problem. You should: 1) draw the tree generated by DFS; 2) put numbers on the nodes of the tree according to the order in which they are visited, e.g. use names $n_0, n_1, n_2, \ldots$; 3) for each iteration of the search loop, show the content of the LIFO queue (stack) used for the frontier.

SOLUTION. Following the pseudocode in R&N figure 3.7 for Tree-Search with a stack for the frontier, we get the search tree:



The frontier content is shown as it looks in the beginning of each loop iteration:

Iteration	**Frontier**
1	$[n_0]$
2	$[n_1]$
3	$[n_2, n_3, n_4]$
4	$[n_2, n_3, n_5]$
5	$[n_2, n_3, n_6, n_7, n_8]$
6	$[n_2, n_3, n_6, n_7, n_9]$
7	$[n_2, n_3, n_6, n_7, n_{10}, n_{11}, n_{12}]$
...	...

The search is stuck in a loop of exploring the same sequence of non-goal states over and over, and will never terminate.

d. *OPTIONAL.* Do the same as in (c) for iterative deepening DFS. This time you don't have to show each single node explored and each single configuration of the LIFO queue, but you should try to at least informally go through each step of the algorithm.

e. Could a different order of exploring the moves change the solution found by DFS? And the solution found by BFS?

SOLUTION. There is only one optimal solution, so any order of exploring possible moves will result in BFS finding the same (optimal) solution. Different orders of possible moves can lead DFS to find different solutions, e.g. the order W, S, E, N will result in DFS finding the optimal solution with both Graph-Search and Tree-Search. In general, there may not be any single fixed order leading DFS to find optimal solutions (or any solutions at all in the case of the Tree-Search variant).

f. Provide a table with an overview of the performance of the algorithms considered above in terms of: 1) number of nodes generated; 2) cost of the solution found (number of moves to get to the goal).

SOLUTION. The performance of each algorithm is as follows:

Algorithm	**Nodes generated**	**Solution length**
BFS	9	5
Graph-Search DFS	9	7
Tree-Search DFS	∞	N/A