Introduction to AI: Solutions to Logic Exercises week 08

March 23rd, 2021

Ex. 1 True or false?

1. $\top \models \bot$
2. $\bot \models \top$
3. $p \wedge q \models p \leftrightarrow q$
4. $p \leftrightarrow q \models p \wedge q$
5. $p \leftrightarrow q \models \neg p \wedge q$
6. $(p \vee q) \wedge (\neg r \vee \neg s \vee t) \models (p \vee q \vee r) \wedge (q \vee \neg r \vee s \rightarrow t)$
7. $(p \vee q) \wedge \neg(p \rightarrow q)$ is satisfiable
8. $(p \leftrightarrow q) \wedge (\neg p \vee q)$ is satisfiable
9. $(p \leftrightarrow q) \leftrightarrow r$ has the same number of models as $(p \leftrightarrow q)$ for any fixed set of proposition symbols that includes p, q, r.

Answers:

1. false
2. true
3. true
4. false
5. false
6. false
7. true
8. true
9. true

Ex. 2 How many models are there for the following sentences (assuming we only have p, q, r, s in the vocabulary)?

1. $q \vee r$
2. $\neg p \vee \neg q \vee \neg r \vee \neg s$
3. $(p \rightarrow q) \wedge p \wedge \neg q \wedge r \wedge s$

Answers:

1. 12
2. 15
3. 0

Ex. 3 We have defined four binary logical connectives.

1. Are there others that can be useful?
2. How many logical connectives can there be?
3. Why are some of the not very useful?

Answers:

1. Yes, for instance the familiar connective XOR (exclusive or). It has multiple uses in computer science and in particular in AI.
2. As many as there are boolean functions of two arguments, i.e., $2^{2^2} = 16$.
3. It depends on the context of course. For instance, the NAND operator (Sheffer stroke, $p|q$, meaning not-conjunction) is functionally complete, i.e., it's sole use allows expressing any other possible connective. But then the formulas can get very long and unintuitive.

Ex. 4 Decide for each of the following, is it valid, unsatisfiable or neither?

1. $Smoke \rightarrow Smoke$
2. $Smoke \rightarrow Fire$
3. $(Smoke \rightarrow Fire) \rightarrow (\neg Smoke \rightarrow \neg Fire)$
4. $Smoke \vee Fire \vee \neg Fire$
5. $Big \vee Long \vee (Big \rightarrow Long)$
6. $(Big \wedge Long) \vee \neg Long$

Answers:

1. valid
2. neither
3. neither
4. valid
5. valid
6. neither