CP8. Formation and evolution of Gaseous Halos

8.1 Basic Fluid Dynamics and Radiative Processes

Approximation: ① gas as ideal fluid,

neglect heat conduction & viscous stress (粘滞)

② gas in local thermal equilibrium (LTE)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \qquad \text{mass conservation}$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\left(\nabla \Phi + \frac{\nabla P}{\rho}\right) \qquad \text{momentum}$$

$$\frac{\partial}{\partial t}\left[\rho\left(\frac{v^2}{2} + \mathcal{E}\right)\right] + \nabla \cdot \left[\rho\left(\frac{v^2}{2} + \frac{P}{\rho} + \mathcal{E}\right)\vec{v}\right] - \rho \vec{v} \cdot \nabla \Phi = \mathcal{H} - \mathcal{L} \qquad \text{energy}$$

$\mathcal{E}$: internal energy per unit mass

$\mathcal{H} / \mathcal{L}$: heating / cooling rates per unit volume

$$\nabla^2 \Phi = 4\pi G \rho_{tot} \qquad \text{gravity}$$

Heating and cooling processes

1. Compton cooling: $T_e \gg T_\gamma$, inverse Compton scatter

for primordial gas: $t_{comp} = \dfrac{3 m_e c}{4 \sigma_T a_r T_\gamma^4}$

For CMB cooling: $\dfrac{t_{comp}}{t_0} \approx 350\, \Omega_{m,0}^{\frac{1}{2}} h (1+z)^{-\frac{5}{2}}$

only important for high-Z

2. Radiative Cooling

for fully ionized high-T gas: <u>bremsstrahlung</u>

for low-T gas: + collisional ionization, recombination
collisional excitation

Calculate of $\Lambda(T)$:
ionization equations (equilibrium
↓
n_e, n_{H^+} ....
↓
cooling rate from different processes
↓
$\Lambda(T)$

cooling function:

$$\Lambda(T) = \frac{\mathcal{L}}{n_H^2}$$

Fig 8.1: $\Lambda(T)$ curve for gas with different metallicity

High-T: $\Lambda \propto T^{\frac{1}{2}}$ bremsstrahlung

$10^5 \sim 10^6$ K: peak for excited electronic levels of metal

below 10^4 K: molecular ....

3. Photoionization Heating

(can both cool and heat?)

$$\mathcal{H} = \sum_i n_i \varepsilon_i, \quad \varepsilon_i = \int_{\nu_i}^{\infty} \frac{4\pi J(\nu)}{h_p \nu} \sigma_{phot,i}(\nu) (h_p \nu - h_p \nu_i) \, d\nu$$

$\mathcal{H} - \mathcal{L}$: Fig 8.2 heating dominates $T < 10^4 K \sim 10^5 K$ for primordial gas

8.2 Hydrostatic Equilibrium

equilibrium: $\nabla P(\vec{r}) = -\rho(\vec{r}) \nabla \Phi(\vec{r})$

ideal gas:

① $PV = nRT$

$\Downarrow$

② $P = \frac{k_B T}{\mu m_p} \rho$

μ: mean molecular mass of a gas particle

③ specific internal energy

$\mathcal{E} = \frac{1}{\gamma - 1} \frac{k_B T}{\mu m_p}$

Assume ideal gas

$$\frac{dP}{dr} = \frac{d(k_B T \rho / \mu m_p)}{dr}, \quad \frac{d\Phi}{dr} = \frac{GM(r)}{r^2}$$

$$M(r) = -\frac{k_B T(r) r}{\mu m_p G} \left[\frac{d\ln\rho}{d\ln r} + \frac{d\ln T}{d\ln r} + \frac{P_{nt}}{P_{th}} \frac{d\ln P_{nt}}{d\ln r} \right]$$

- estimating mass of X-ray cluster
- P_{nt}: other contribution to pressure such as $\vec{B}$, CR
- significant uncertainty

8.2.1 Gas density profile

$$\boxed{\begin{cases} \nabla P(\vec{r}) = -\rho(\vec{r}) \nabla \Phi(\vec{r}) \\ \nabla^2 \Phi = 4\pi G (\rho_{dm} + \rho) \end{cases}} \quad \star$$

① Assume polytropic gas $P = A\rho^{\Gamma}$

$$k_B T(\vec{r}) = \frac{1-\Gamma}{\Gamma} \mu m_p \Phi(\vec{r})$$

② Assume an isothermal sphere $T(r) \equiv T$

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\Phi}{dr} \right) = 4\pi G \left[\rho_{dm}(r) + \rho_0 \exp\left(-\frac{\Phi}{\sigma_T^2} \right) \right] \qquad \sigma_T^2 \equiv \frac{k_B T}{\mu m_p}$$

1. When $\rho_{dm} = 0 \Rightarrow$ Lane-Emden equation

$\begin{cases} \text{assume } \rho(r) \propto \text{power law} \Rightarrow \rho(r) \propto r^{-2} \\ \text{boundary condition } \rho(r=0) = \rho_0 = \text{const}, \ \left. \frac{d\rho}{dr} \right|_{r=0} = 0 \end{cases}$

singular isothermal sphere

diverges at $r=0$

$\approx$ King Profile $(r \lesssim 2r_0)$

$$\rho(r) = \frac{\rho_0}{[1 + (r/r_0)^2]^{3/2}}, \quad r_0 = \frac{3\sigma_T}{\sqrt{4\pi G \rho_0}}$$

singular isothermal sphere $(r \gtrsim 10 r_0)$

2. consider $\rho_{dm} \neq 0$ and in equilibrium

① isotropic, isothermal ($\langle v_i v_j \rangle = \sigma^2 \delta_{ij}$, $\sigma^2, T = \text{const}$)

$$\rho_{gas}(r) \propto [\rho_{dm}(r)]^{\beta} \qquad \beta \equiv \frac{\mu m_p \sigma^2}{k_B T}$$

② Assume NFW DM profile, $\rho_g \ll \rho_{dm}$

$$\rho_{dm}(r) = \rho_{crit} \frac{\delta_{char}}{(r/r_s)(1+r/r_s)^2}$$

$$\rho(r) = \rho_0 e^{-b} \left(1+\frac{r}{r_s}\right)^{b r_s / r}$$

In reality, numerical for ☆.

8.2.2 convectively instability
(对流性)

Schwarzschild's criterion : $\frac{ds}{dz} < 0$ (for displace along $-z$)

ideal gas: $\frac{dT}{dz} < -\frac{g}{c_p}$

8.2.3 Virial Theorem Applied to a Gaseous Halo

$$\frac{1}{2}\frac{d^2 I}{dt^2} = 2K + W + \Sigma \Rightarrow 2K + W + \Sigma = 0 \quad \text{(virial theorem)}$$

$$\begin{cases} \frac{d^2 I}{dt^2} < 0 & \text{contracts} \\ \frac{d^2 I}{dt^2} > 0 & \text{expands} \end{cases}$$

(monatomic gas)

$$2 \times \frac{3 M_{gas} k_B T}{2 \mu m_p} - \frac{3 G M_{gas} M}{5 r_{cl}} - 4\pi r_{cl}^3 P_{ext} = 0$$

① $P_{ext} = 0$

$$M_J = \left(\frac{3}{4\pi \rho_{gas}}\right)^{\frac{1}{2}} \left(\frac{5 f_{gas} k_B T}{\mu m_p G}\right)^{\frac{3}{2}} \rightarrow \begin{cases} M_{gas} > M_J & \text{contracts} \\ M_{gas} < M_J & \text{expands} \end{cases}$$

for $T = \text{const}$ / polytropic
contracts can't stop
without P_{ext}

② $P_{ext} \neq 0$, compression criteria : ($f_{gas} = 1$)

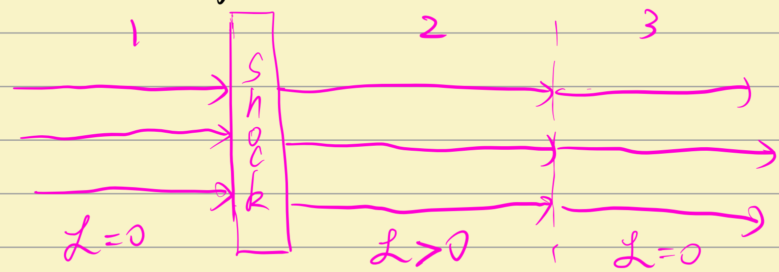
$$P_{ext} > P_{crit}(r) = \frac{3 M_{gas}}{4\pi r^3} \left(\frac{k_B T}{\mu m_p} - \frac{G M_{gas}}{5 r}\right)$$

P_{ext} source : ① turbulent motion
② magnetic field
③ shallow DM halo profile ($\alpha < 2$)

8.3 The Formation of Hot Gaseous Halos

8.3.1 Accretion Shocks

(a) Shock front



(in frame of shock wave)

$$\hat{\mathcal{M}}_1^2 = \frac{v_1^2}{c_{s,1}^2} = \frac{v_1^2 \rho_1}{\gamma P_1} \overset{\text{ideal gas}}{=} \frac{v_1^2}{\gamma(\gamma-1)\mathcal{E}_1}$$

$1\to2$ With continuity, Euler, energy equation + ideal gas assumption

$$\begin{cases} \rho_1 v_1 = \rho_2 v_2 \\ \rho_1 v_1^2 + P_1 = \rho_2 v_2^2 + P_2 \\ \frac{1}{2}v_1^2 + \frac{P_1}{\rho_1} + \mathcal{E}_1 = \frac{1}{2}v_2^2 + \frac{P_2}{\rho_2} + \mathcal{E}_2 \\ P = \rho(\gamma-1)\mathcal{E} \end{cases} \Rightarrow \frac{\rho_1}{\rho_2}, \frac{P_1}{P_2}, \frac{T_1}{T_2} \cdots \sim f(\hat{\mathcal{M}}_1^2)$$

($\hat{\mathcal{M}}_1$: mach number, $\hat{\mathcal{M}}_1 \equiv \frac{v_1}{c_{s,1}}$)

$$\Downarrow$$

$\hat{\mathcal{M}}_1 > 1$, gas compressed, decelerated, heated by shock

$2\to3$ gas cool for a new equilibrium

$$\begin{cases} \rho v = \rho_2 v_2 \\ \rho v^2 + P = \rho_2 v_2^2 + P_2 \\ \frac{1}{\gamma-1}[c_s^2 - v^2]\frac{dv}{dx} = -\mathcal{L}(\rho, T) \end{cases}$$

(for $v < c_s, \mathcal{L} > 0, \frac{dv}{dx} < 0$
$\Rightarrow \frac{d\rho}{dx} > 0, \frac{dP}{dx} > 0, \frac{dT}{dx} < 0$)
(?)

(b) heating by accretion shocks

(shell crossing is not allowed for gas)

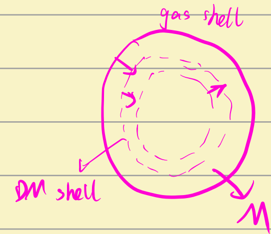
Assuming: radiative cooling neglected

$v_1 = v_{in} + v_{sh}, \quad v_2 = v_{sh}$

if $v_1^2 >> \frac{k_B T}{\mu m_p}$: $T_2 = (\gamma-1) T_{vir} \left(\frac{v_{in}}{V_c}\right)^2_2 \quad \frac{\rho_2}{\rho_1} = \frac{\gamma+1}{\gamma-1}$

$$\frac{1}{2} v_{in}^2 = \frac{1}{2} v_{ff}^2 + \Delta W - \frac{c_{s,1}^2}{\gamma-1}\left[1 - \left(\frac{\rho_{ta}}{\rho_{sh}}\right)^{\gamma-1}\right]$$

$$\frac{1}{2} v_{ff}^2 \equiv \frac{GM}{r_{sh}} - \frac{GM}{r_{ta}}$$



$\Delta W \neq 0$: for shell-crossing of DM (M can change)

Self-similar solution

$Q(r,t) = Q_{ch}\, \mathcal{Q}(r/R)$

↓ length scale

8.3.2 Self-similar Collapse of Collisional Gas

Assuming: ① EdS universe, $a \propto t^{2/3}$ ② $P_{init}(k) \propto k^2$

③ gas initially cold and radiative cooling is negligible

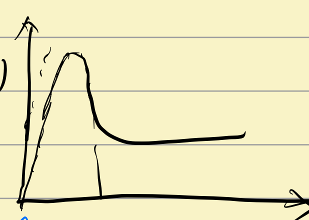
$\boxed{\text{Self similar only for EdS}}$

dimensionless: $\lambda \equiv \frac{r}{r_{ta}}$, self-similar profiles: $\rho \quad P \quad T \quad M$: $\mathcal{D}(\lambda), \mathcal{P}(\lambda), \mathcal{T}(\lambda), \mathcal{M}(\lambda)$

mass shell motion:

$r/r_{ta} \quad - \ln(t/t_{ta})$

Fig 8.5 left



8.3.3 The Impact of a Collisionless Component

Fig 8.5 right

8.3.4 More General Models of Spherical Collapse

$$\begin{cases} S(M_{gas}) = \dfrac{P(M_{gas})}{\rho^{\gamma}(M_{gas})} = \dfrac{k_B}{\mu m_p} \dfrac{T(M_{gas})}{\rho^{\gamma-1}(M_{gas})} \\ \dfrac{dP}{dM_{gas}} = \dfrac{GM_{tot}}{4\pi r^4}, \quad \dfrac{dr}{dM_{gas}} = \dfrac{1}{4\pi \rho_{gas} r^2} \end{cases}$$

8.4 Radiative cooling

8.4.1 t_{cool} for uniform clouds

For a uniform cloud in virial equilibrium (no P_{ext})

fully ionized primordial gas:

$$M_{gas} \simeq 6.1 \times 10^{3} T_6^{3/2} f_{gas} (1+\delta)^{-\frac{1}{2}} (\Omega_{m,0} h^2)^{-\frac{1}{2}} (1+z)^{-\frac{3}{2}} M_\odot$$

$$\boxed{t_{cool} = \frac{\rho \mathcal{E}}{\mathcal{C}} = \frac{\frac{3}{2} n k_B T}{n_H^2 \Lambda(T)}} \propto (1+z)^{-3}$$

ideal gas, $\gamma = \frac{5}{3}$

free-fall

$$t_{ff} = \sqrt{\frac{3\pi}{32 G \rho}} = \sqrt{\frac{3\pi f_{gas}}{32 G n \mu m_p}} \propto (1+z)^{-\frac{3}{2}}$$

(alternative : t_{lmm} last major merger

$t_{cool} >> t_{ff}$: cooling effective

☆ Fig 8.6

8.4.2 Evolution of the Cooling radius

In a non-uniform system:

$$t_{cool}(r) = \frac{3n(r)k_B T(r)}{2n_H^2(r)\Lambda(T)}$$

For power-law profile and $\Lambda(T)$:

$$t_{cool} = t_0 \left(\frac{r}{r_0}\right)^{\frac{1}{\tau}}, \quad r_{cool}(t) = r_0 \left(\frac{t}{t_0}\right)^{\tau}$$

8.4.3 Self-similar Cooling Waves

Similar solution length scale: r_{cool}

8.4.4 Spherical Collapse with Cooling

In reality, r_{cool}, r_{sh} both evolve with time.

t_{crit}: $r_{cool} = r_{sh}$ (for isothermal sphere $t_{crit} \propto \Lambda(T)/V_c^2$)

$t \gg t_{crit}$ ($r_{cool} \ll r_{sh}$): self-similar

$t \ll t_{crit}$ ① gas can cool as soon as its shocked,

cold mode { ② the accreted gas will be in free-fall and no accretion shock
③ gas reach center via cold flow }

hot mode: gas heated due to accretion shock

stability criteria:

$$\gamma_{eff} = \frac{\dot{P}}{P}\frac{\rho}{\dot{\rho}} > \frac{2\gamma}{\gamma + \frac{2}{3}}$$

$$\Lambda_{crit} \sim T_{vir}\,\delta_{vir}^{\frac{1}{2}}\,(\Omega_{b,0}h^2)^{-1}\,h\,(1+z)^{-\frac{3}{2}}$$

$$T_{vir} \sim M_{vir}^{\frac{2}{3}}\,\delta_{vir}^{\frac{1}{3}}\,(1+z)$$

8.5 Thermal and Hydrodynamical Instabilities of Cooling Gas

8.5.1 Thermal Instability

cooling-heating balance : $\mathcal{L} \equiv (\mathcal{C}-\mathcal{H})/\rho = 0$

(1) the shape of balance curve : (Fig 8.8)

① $T \nearrow 10^4 K$: collisional excitation begin → radiative de-excitation cooling ↓

ρ large change for balance ← cooling rate rapids up

② $T > \nearrow 10^4 K$: no new excitation, large change T for balance ρ

until ($10^6 K$) excitation for heavy elements

(2) isobaric perturbation

We can assume pressure equilibrium → $\rho T =$ constant

→ two cases $\begin{cases} \left(\frac{\partial \ln T}{\partial \ln \rho}\right)_{\mathcal{L}=0} > -1 \; (P_2) : \text{stable} \\ \left(\frac{\partial \ln T}{\partial \ln \rho}\right)_{\mathcal{L}=0} < -1 \; (P_1) : \text{unstable} \end{cases}$

thermal instability : $\left(\frac{\partial \mathcal{L}}{\partial T}\right)_P < 0$

$$\frac{d(\delta S)}{dt} = -\delta\left(\frac{\mathcal{L}}{T}\right) \Rightarrow \left[\frac{\partial}{\partial S}\left(\frac{\mathcal{L}}{T}\right)\right]_A < 0$$

Field criteria ($\mathcal{L}=0$) : $\left(\frac{\partial \mathcal{L}}{\partial S}\right)_A < 0$

⇓

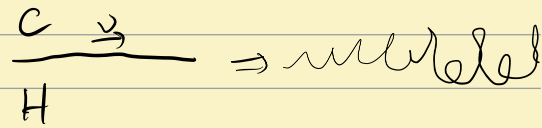
gas multi-phase medium : $T \sim 10^6 K, 10^4 K, 10^2 K$

8.5.2 Hydrodynamical Instabilitis

(a) Kelvin-Helmhotz Instability

the interface between hot and cold gas with velocity difference (no ⊥ gravity)

$$w = \frac{(\rho_h \rho_c)^{\frac{1}{2}} v}{\rho_c + \rho_h} k$$



$k = R_c$, $\tau_{KH} \equiv \frac{2\pi}{w} = \frac{R_c}{v} \frac{2+\delta}{\sqrt{1+\delta}}$

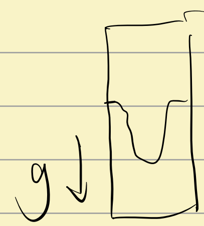
($\delta = \frac{\rho_c}{\rho_h} - 1$

(b) Rayleigh-Taylor Instability

when a heavy fluid rests on top of a light fluid

in an effective gravitational field

$$w^2 = \left(\frac{\rho_c - \rho_h}{\rho_c + \rho_h} \right) g k = g k \frac{\delta}{2+\delta}$$



$$\tau_{RT} \sim \frac{R_c}{v} \sqrt{\frac{(2+\delta)(1+\delta)}{\delta}}$$

8.6 Evolution of Gaseous Halos with Energy Sources

conservation equations with source:

$$
\begin{cases}
\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = S_m \\
\frac{\partial(\rho\vec{v})}{\partial t} + \sum_i \nabla_i(\rho v_i \vec{v}) + \nabla P + \rho\nabla\Phi = \vec{S}_{mom} \\
\frac{\partial}{\partial t}\left[\rho\left(\frac{v^2}{2}+\varepsilon\right)\right] + \nabla\cdot\left[\rho\left(\frac{v^2}{2}+\frac{P}{\rho}+\varepsilon\right)\vec{v}\right] - \rho\vec{v}\cdot\nabla\Phi = S_e
\end{cases}
$$

$$S_e = \mathcal{H} - \mathcal{C} + S_m\left(\Phi + \frac{3}{2}\theta_{inj} + \frac{1}{2}v_{inj}^2\right)$$

8.6.1 Blast wave

a large amount energy ε_0 release during short time

(a) Self-similar model (when ε_0 large enough)

length scale: $r_{sh}(t)$

time: t, age of explosion

$$\rho(r,t) = \rho_0 \mathcal{D}(\lambda), \quad v(r,t) = \frac{r_{sh}}{t}\mathcal{V}(\lambda), \quad P(r,t) = \rho_0\left(\frac{r_{sh}}{t}\right)^2 \mathcal{P}(\lambda)$$

By dimension analysis:

$$r_{sh}(t) = A\left(\frac{\varepsilon_0}{\rho_0}\right)^{\frac{1}{5}} t^{\frac{2}{5}}$$

Specify A: $\int_0^{r_{sh}} \left(\frac{P}{\gamma-1} + \frac{1}{2}\rho v^2\right)$

(b) Applications to Supernova Remnants

4-phase of SN blast waves:

① free-expansion

$\downarrow$ $M_{sh} = M_{ejecta}$

② self-similar

$\downarrow$ $f_{loss}(t_{rad}) \approx \frac{1}{4}$

③ radiative loss : $t > t_{rad}$

$t = t_{sp} \gg t_{rad}$: snowplow $M_{sh} v_{sh}$ = constant

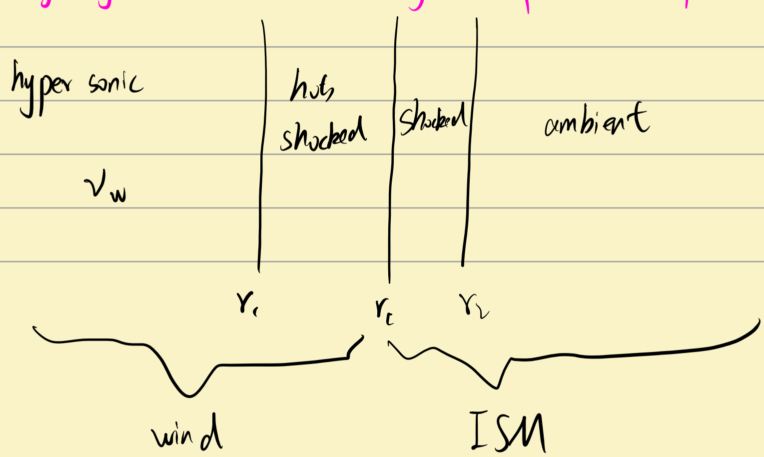
$\downarrow$ P

④ fade in ISM

8.6.2 Winds and Wind-driven Bubbles

a long period of energy injection

(e.g.. galactic wind driven by multiple SN explosions ~ extended periods of SF



r_c : $P \to 0$

r_1, r_2 : 2 shocks