Pressure Drop in Heat Exchangers

Technical report by L. Querella, PhD, MSc, Data Scientist, Dec 2023

<u>Credentials</u>: This calculation was discussed and validated with Process engineers.

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1 Abstract

This document details the pressure drop calculation as implemented in TLE Fouling project at TotalEnergies (Data-driven asset performance).

2 References

- [1] *Measurement of Effective Thermal Conductivity of Coke,* A. KASAI et al., 1993
  - https://www.jstage.jst.go.jp/article/tetsutohagane1955/79/1/79_1_20/_article
- [2] *Perry's Chemical Engineers' Handbook*, 8th edition, Don W. Green & R. H. Perry, 2008

3 Theory

3.1 General equation for the pressure drop along the TLE

The general equation for the pressure drop is (at time t):

$$\Delta p = -\int_0^L \left(\frac{2f_F(z)}{d_i - 2\delta_c} + \beta\right)\rho(z)v^2(z)dz - \int_{v(0)}^{v(L)} \rho(z)v(z)dv$$

where the first and second integrals are the pressure drop due to friction (Darcy-Weisbach equation) and the pressure drop due to acceleration respectively. The β coefficient is the resistance coefficient for bends (by design of the TLE).

We can express the above equation in terms of velocity and solve the last integral:

$$\Delta p = -\Gamma\left[\int_0^L \left(\frac{2f_F(z)}{d_i - 2\delta_c} + \beta\right)v(z)dz + v(L) - v(0)\right]$$

where Γ is time- and fouling-dependent but does not vary along the TLE:

$$\Gamma = \frac{4\dot{m}_F}{\pi N(d_i - 2\delta_c)^2}$$

Pressure drop depends on the flow rates (operating conditions) and velocity within the TLE:

The speed of process stream is given by

$$v = \frac{4\dot{m}_F}{\pi\rho(z)N(d_i - 2\delta_c)^2}$$

3.2 Friction term

We were able to obtain a closed-form expression for the Fanning friction factor:

A closed-form solution can be derived from the Colebrook-White equation, neglecting the surface roughness of the tube, which amounts to considering the Prandtl-Karman equation for smooth pipe and turbulent fluid regime:

$$\frac{1}{\sqrt{f_F}} = 4\log(Re\sqrt{f_F}) - 0.4$$

where f_F is the Fanning friction factor and Re the Reynolds number, given at operating conditions (sor, temperature and pressure) by:

$$Re = \frac{4\dot{m}_F}{\pi\mu(z)N(d_i - 2\delta_c)}$$

Introducing the Lambert function $W(z)$ defined as the inverse function of $f(z) = ze^z; z \in \mathbb{C}$, that is $f^{-1}(ze^z) := W(ze^z)$, after some algebra one gets

$$f_F = \left[aW\left(\frac{1}{ab}\right)\right]^{-2}$$

where

- $a = 4/\ln 10 = 1.737...$
- $b = 10^{0.1}/Re$

Remarks:

- We restrict the support of the Lambert function W to the real numbers

- The Lambert function W in turn can be iteratively computed (cf. python function https://docs.scipy.org/doc/scipy-0.14.0/reference/generated/scipy.special.lambertw.html)
- See https://en.wikipedia.org/wiki/Darcy%E2%80%93Weisbach_equation#Smooth-pipe_regime for the Darcy-Weisbach equation (the Fanning factor is four times the friction factor)

Smooth-pipe regime [edit]

When the pipe surface is smooth (the "smooth pipe" curve in Figure 2), the fri
suitably adjusted

$$\frac{1}{\sqrt{f_D}} = 1.930 \log\left(\mathrm{Re}\sqrt{f_D}\right) - 0.537.$$

The numbers 1.930 and 0.537 are phenomenological; these specific values p
(dimensionless) parameter of the flow: at fixed values of $\mathrm{Re}\sqrt{f_D}$, the friction fa

In the Kármán–Prandtl resistance equation, f_D can be expressed in closed fo

$$\frac{1}{\sqrt{f_D}} = \frac{1.930}{\ln(10)} W\left(10^{\frac{-0.537}{1.930}} \frac{\ln(10)}{1.930} \mathrm{Re}\right) = 0.838\, W(0.629\, \mathrm{Re})$$

Expressed in terms of the Fanning factor, the coefficients in the right-hand side of the above expression are given by a and b in the above blue equation box

- The Reynolds number profile is:

The Reynolds number Re profile is given by:

$$Re(z) = \frac{\rho(z)v(z)(d_i - 2\delta_c)}{\mu(z)}$$

with ρ and μ the fluid density and viscosity respectively.

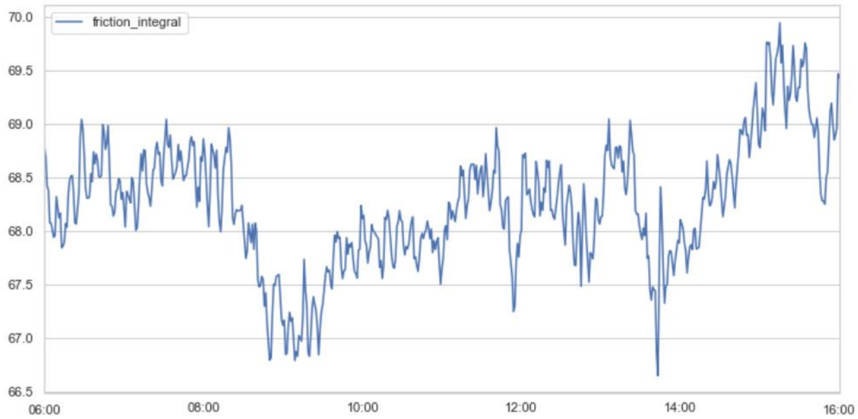
The speed of process stream is given by

$$v(z) = \frac{4\dot{m}_F}{\pi\rho(z)N(d_i - 2\delta_c)^2}$$

Hence, Re is given by:

$$Re(z) = \frac{4\dot{m}_F}{\pi\mu(z)N(d_i - 2\delta_c)}$$

- The z-profiles of thermal properties (heat capacity C_p, density rho, conductivity lambda, and viscosity mu) are determined at each time t by applying the polynomial regression models (cf. efficiency calculation) and assuming that the temperature profile is a logarithmic drop and pressure profile is a linear drop.
- Example of results for the friction integral at start-of-run conditions (median = 68.237)



3.3 Resistance coefficient for bends

The resistance coefficient for bends contributes to the pressure drop through the term

$$\Delta p_\beta = \int_0^L \beta\rho(z)v^2(z)dz$$

which can be computed at SOR conditions where the pressure drop is given by the process sheet:

$$\Delta p_\beta = \Delta p_{|\mathrm{sor}} = -0.06\,\mathrm{bar}$$

CAUTION: We should convert in SI units, i.e.

$$\Delta p_\beta = \Delta p_{|\mathrm{sor}} = -6.10^3\ \mathrm{Pa}$$

In order to obtain the coefficient beta, we integrate the pressure drop equation at start-of-run conditions.

4 Integration in clean conditions (SOR)

4.1 General equation at SOR

We first consider the general equation above at start-of-run (SOR) conditions, i.e. just after a mechanical cleaning.

At SOR conditions, the equation for the pressure drop reduces to:

$$\Delta p_{|\mathrm{sor}} = -\Gamma_{|\mathrm{sor}} \Big[\int_0^L \big(\frac{2 f_F(z)}{d_i} + \beta \big) v(z) dz + v(L) - v(0) \Big]_{|\mathrm{sor}}$$

where $\Gamma_{|\mathrm{sor}}$ is:

$$\Gamma_{|\mathrm{sor}} = \frac{4 \dot{m}_F}{\pi N_{\mathrm{clean}} d_i^2}$$

For the SOR period, we take 6 hours after the end of mechanical cleaning with an offset of 4 hours ("feed-in").

4.2 Number of tubes at SOR

On the process sheet, the number of tubes is 52.

However, to be consistent with the efficiency calculation, we compute the optimal value of N at SOR giving a duty close to the computed duty (cf. efficiency calculation):

> One seeks the value of N that minimizes the error when computing the equation:

$$\frac{\mathrm{HTE\_num}}{F(t)} = U_{\mathrm{clean}}(N) A_{\mathrm{clean}}^{\mathrm{outer}}(N)$$

The lhs is given and the rhs varies with N through the Reynolds number (heat transfer resistance U) and available heat transfer area

With Reynolds number computed at bulk temperature and pressure.

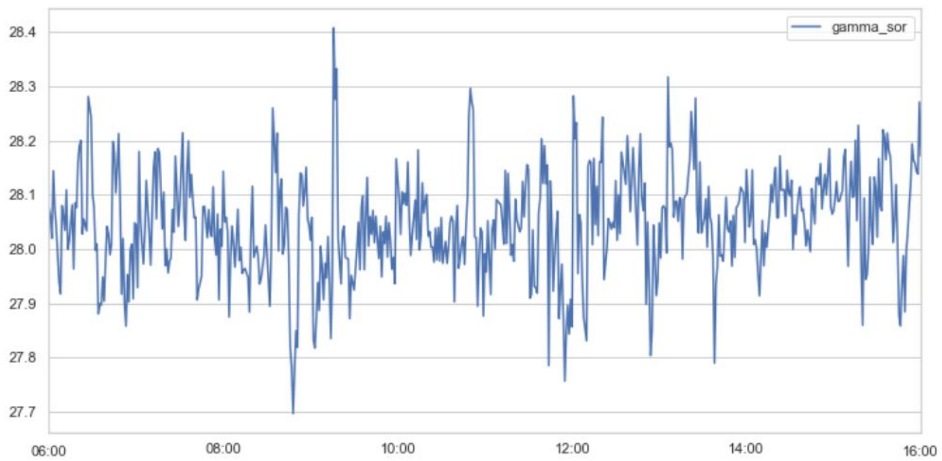
For all TLE's we get

N_sor = 50.625

4.3 Constant factor Gamma

Gamma_sor can be directly computed (cf. definition above).

For each TLE, its value is oscillating around 28.0 (example for Pass 1 – median = 28.048):



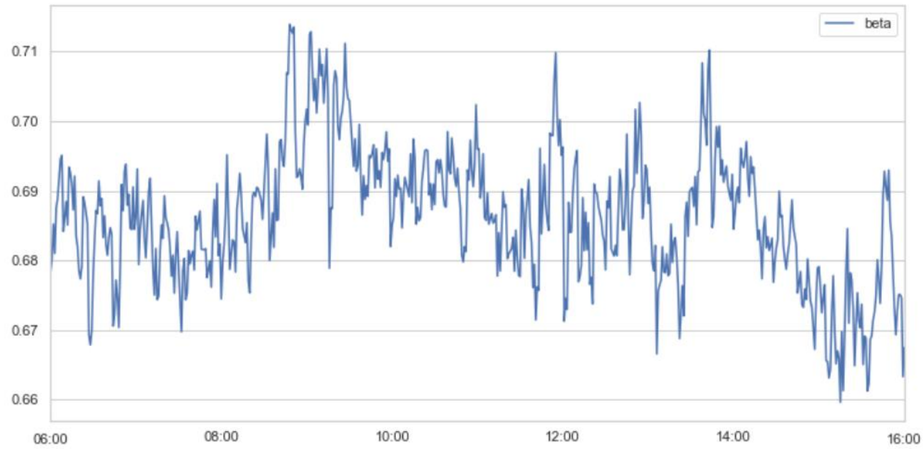
4.4 Friction term

Using the closed-from of the friction term (see above) we can integrate along the TLE (we denote by z the distance from inlet, z=0, to outlet, z=L=6.15m)

4.5 Resistance coefficient for bends

Computing the delta pressure at SOR with all contributing terms (friction factor, bends, acceleration term), one sets a median value for the beta coefficient, which varies around 0.68 for the 6 TLE's.

Example for Pass 1 – median value = 0.686:



5 Integration in fouling conditions

5.1 Important remarks

- The pressure drop at time t, in fouling mode, depends directly on the actual number of tubes N(t) and on the coke deposit thickness delta_c
- In the MVP release, we considered a constant number of tubes (N_sor) and computed the coke thickness (see below)
- Thermal properties and related profiles (e.g. density) along the TLE are predicted by the polynomial regressions assuming that pressure at TLE inlet is given by the COP tag value (which is not reliable in fouling mode) and pressure at TLE outlet is simply COP-0.1 bar.

5.2 Coke thickness

We assume that coke thickness is uniform along the tube.

Using the energy balance and expressing the overall heat transfer coefficient in service regime, $U(t)$, in terms of the reduced inner surface area of the tube, we are able to introduce the coke thickness in the heat transfer efficiency relationship:

$$F(t) = \frac{U(t)A(t)^{\text{inner}}}{U_{\text{clean}}(t)A_{\text{clean}}^{\text{outer}}} \rightarrow \frac{1}{U(t)} = \frac{A(t)^{\text{inner}}}{\text{HTE\_num}}$$

Under fouling conditions, we have:

$$A(t) = \pi(d_i - 2\delta_c)LN_{\text{tubes}}(t)$$
$$A_{\text{clean}} = \pi d_o LN_{\text{tubes, clean}} = 51.24[m^2]$$

where

- d_i is the inner design diameter of tubes
- $d_o = d_i + 2\delta_w$ is the outer diameter of tubes
- δ_c is the coke thickness
- L is the tube length
- N_{tubes} is the number of tubes at time t

We can write the explicit dependence of $U(t)$ on coke thickness as:

$$\frac{1}{U(t)} = \frac{1}{\alpha_i} + \frac{\delta_w}{\lambda_w}\frac{d_i}{M_{lm}(d_i, d_o)} + \frac{\delta_c}{\lambda_c}\frac{d_i}{M_{lm}(d_i - 2\delta_c, d_i)}$$

where

- α_i is the heat transfer coefficient of the fluid inside the tube
- λ_w is the tube material conductivity at average wall temperature T_{wa} between inner and outer (T_w)
- λ_c is the coke deposit conductivity (22.5 W/m/K)
- δ_w is the tube wall thickness (6.3mm)
- M_{lm} is the logarithmic mean

The latter term in the above equation accounts for additional heat transfer resistance due to coke deposit. It can be expanded using the definition of the logarithmic mean and, after some algebra, combining with equation for $F(t)$, and introducing the Lambert function $W(z)$, we get:

$$2\delta_c = d_i\left[1 - \frac{1}{2b\lambda_c}W\left(\frac{2b\lambda_c}{d_i}\exp(\frac{2a\lambda_c}{d_i})\right)\right]$$

where a and b are defined as:

$$a = \frac{1}{\alpha_i} + \frac{\delta_w}{\lambda_w}\frac{d_i}{M_{lm}(d_i, d_o)} + \frac{d_i \ln d_i}{2\lambda_c}$$
$$b = \frac{\pi LN_t}{F(t)U_{\text{clean}}(t)A_{\text{clean}}}$$

NB: From the definition of $F(t)$ we can write the denominator of b explicitly:

$$F(t)U_{\text{clean}}(t)A_{\text{clean}} = U(t)A(t) = \frac{\dot{m}_F\left[C_{p,\text{COT}}T_{\text{COT}} - C_{p,\text{TLE}}T_{\text{TLE}}\right]\ln\left(\frac{T_{\text{COT}}-T_w}{T_{\text{TLE}}-T_w}\right)}{T_{\text{COT}} - T_{\text{TLE}}} = \frac{Q(t)}{\Delta T_{\text{lm}}} = \text{HTE\_num}$$

NB: we take as the coke conductivity lambda_c = 2.87 instead of 22.5, according to the scientific literature [3][4]:

- [1]

Abstract

Effective thermal conductivity of coke was measured in the temperature range from 100°C to 1400°C by using the laser flash method. Effective thermal conductivity *k* of coke increased with a rise of temperature, and decreased with an increase of porosity of coke. The effect of radiation in the pore of coke on the value of effective thermal conductivity of coke was very small. The following equation was obtained for effective thermal conductivity *k* (W/m·k) as a function of temperature *T* (K) and porosity ε (-) of coke.

$k = \{0.973 + 6.34 \times 10^{-3} (T\text{-}273)\} (1\text{-}\varepsilon^{2/3})$

Thermal conductivity is varying from 1.18 to 5.50 when temperature is ranging from 350 to 950°C and porosity from 10% to 50%. Indicative value is chosen as is 2.87.

- [2]:

TRANSPORT PROPERTIES 2-459

TABLE 2-326 Thermal Conductivities of Some Building and Insulating Materials*

k = Btu/(h·ft²)(°F/ft)

Material	Apparent density ρ, lb/ft³ at room temperature	t, °C	k
Aerogel, silica, opacified	8.5	120	0.013
		290	.026
Asbestos-cement boards	120	20	.43
Asbestos sheets	55.5	51	.096
Asbestos slate	112	0	.087
	112	60	.114
Asbestos	29.3	−200	.043
	29.3	0	.090
	36	0	.087
	36	100	.111
	36	200	.120
	36	400	.129
	43.5	−200	.090
	43.5	0	.135
Aluminum foil (7 air spaces per 2.5 in.)	0.2	38	.025
		177	.038
Ashes, wood		0–100	.041
Asphalt	132	20	.43
Boiler scale (Note 1)			
Bricks:			
Alumina (92–99% Al_2O_3 by wt.) fused		427	1.8
Alumina (64–65% Al_2O_3 by wt.)		1315	2.7
(See also Bricks, fire clay)	115	800	0.62
	115	1100	.63
Building brick work		20	.4
Carbon	96.7		3.0
Chrome brick (32% Cr_2O_3 by wt.)	200	200	.67
	200	650	.85
	200	1315	1.0
Diatomaceous earth, natural, across strata (Note 2)	27.7	204	0.051
	27.7	871	.077
Diatomaceous, natural, parallel to strata (Note 2)	27.7	204	.081
	27.7	871	.106
Diatomaceous earth, molded and fired (Note 2)	38	204	.14
	38	871	.18
Diatomaceous earth and clay, molded and fired (Note 2)	42.3	204	.14
	42.3	871	.19
Diatomaceous earth, high burn, large pores (Note 3)	37	200	.13
	37	1000	.34
Fire clay (Missouri)		200	.58
		600	.85
		1000	.95
		1400	1.02
Kaolin insulating brick (Note 3)	27	500	0.15
	27	1150	.26
Kaolin insulating firebrick (Note 4)	19	200	.050
	19	760	.113
Magnesite (86.8% MgO, 6.3% Fe_2O_3, 3%			

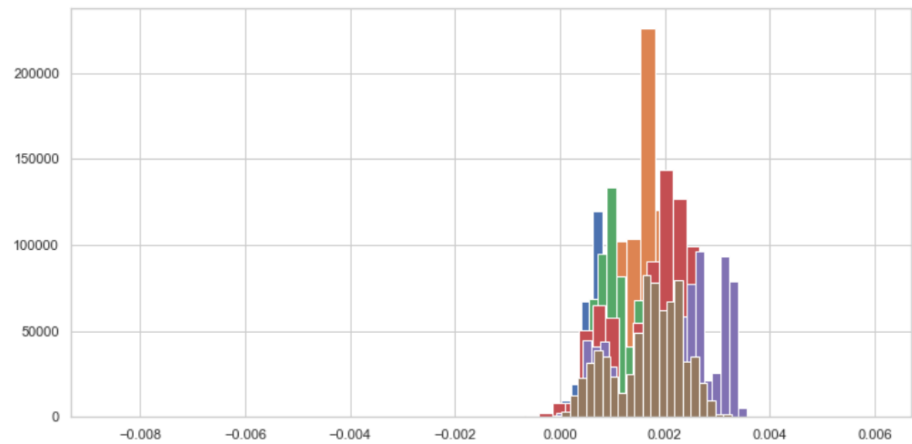
Material	Apparent density ρ, lb/ft³ at room temperature	t, °C	k
Cotton wool	5	30	0.024
Cork board	10	30	.025
Cork (regranulated)	8.1	30	.026
(ground)	9.4	30	.025
Diatomaceous earth powder, coarse	20.0	38	.036
(Note 2)	20.0	871	.082
fine (Note 2)	17.2	204	.040
	17.2	871	.074
molded pipe covering (Note 2)	26.0	204	.051
	26.0	871	.088
4 vol. calcined earth and 1 vol. cement, poured and fired (Note 2)	61.8	204	.16
	61.8	871	.23
Dolomite	167	50	1.0
Ebonite			0.10
Enamel, silicate	38		0.5–0.75
Felt, wool	20.6	30	0.03
Fiber insulating board	14.8	21	.028
Fiber, red	80.5	20	.27
(with binder, baked)		20–97	.097
Gas carbon		0–100	2.0
Glass			0.2–0.73
Borosilicate type	139	30–75	0.63
Window glass			0.3–0.61
Soda glass			0.3–0.44
Granite			1.0–2.3
Graphite, longitudinal		20	95
powdered, through 100 mesh	30	40	0.104
Gypsum (molded and dry)	78	20	.25
Hair felt (perpendicular to fibers)	17	30	.021
Ice	57.5	0	1.3
Infusorial earth, see diatomaceous earth			
Kapok	0.88	20	0.020
Lampblack	10	40	.038
Lava			.49
Leather, sole	62.4		.092
Limestone (15.3 vol. % H_2O)	103	24	.54
Linen		30	.05
Magnesia (powdered)	49.7	47	.35
Magnesia (light carbonate)	13	21	0.034
Magnesium oxide (compressed)	49.9	20	.32
Marble			1.2–1.7
Mica (perpendicular to planes)		50	0.25
Mill shavings			0.033–0.05
Mineral wool	9.4	30	0.0225
	19.7	30	.024
Paper			.075
Paraffin wax		0	.14
Petroleum coke		100	3.4
		500	2.9
Porcelain		200	0.88
Portland cement, see concrete		90	.17

Conversion table:

TABLE 1-4 Conversion Factors: U.S. Customary and Commonly Used Units to SI Units *(Continued)*

Quantity	Customary or commonly used unit	SI unit	Alternate SI unit	Conversion factor; multiply customary unit by factor to obtain SI unit
Velocity (linear), speed	knot	km/h		1.852* E + 00
	mi/h	km/h		1.609 344* E + 00
	ft/s	m/s		3.048* E − 01
		cm/s		3.048* E + 01
	ft/min	m/s		5.08* E − 03
	ft/h	mm/s		8.466 667 E − 02
	ft/day	mm/s		3.527 778 E − 03
		m/d		3.048* E − 01
	in/s	mm/s		2.54* E + 01
	in/min	mm/s		4.233 333 E − 01
Corrosion rate	in/year (ipy)	mm/a		2.54* E + 01
	mil/year	mm/a		2.54* E − 02
Rotational frequency	r/min	r/s		1.666 667 E − 02
		rad/s		1.047 198 E − 01
Acceleration (linear)	ft/s^2	m/s^2		3.048* E − 01
		cm/s^2		3.048* E + 01
Acceleration (rotational)	rpm/s	rad/s^2		1.047 198 E − 01
Momentum	(lbm·ft)/s	(kg·m)/s		1.382 550 E − 01
Force	U.K. tonf	kN		9.964 016 E + 00
	U.S. tonf	kN		8.896 443 E + 00
	kgf (kp)	N		9.806 650* E + 00
	lbf	N		4.448 222 E + 00
	dyn	mN		1.0 E − 02
Bending moment, torque	U.S. tonf·ft	kN·m		2.711 636 E + 00
	kgf·m	N·m		9.806 650* E + 00
	lbf·ft	N·m		1.355 818 E + 00
	lbf·in	N·m		1.129 848 E − 01
Bending moment/length	(lbf·ft)/in	(N·m)/m		5.337 866 E + 01
	(lbf·in)/in	(N·m)/m		4.448 222 E + 00
Moment of inertia	lbm·ft^2	kg·m^2		4.214 011 E − 02
Stress	U.S. tonf/in^2	MPa	N/mm^2	1.378 951 E + 01
	kgf/mm^2	MPa	N/mm^2	9.806 650* E + 00
	U.S. tonf/ft^2	MPa	N/mm^2	9.576 052 E − 02
	lbf/in^2 (psi)	MPa	N/mm^2	6.894 757 E − 03
	lbf/ft^2 (psf)	kPa		4.788 026 E − 02
	dyn/cm^2	Pa		1.0* E − 01
Mass/length	lbm/ft	kg/m		1.488 164 E + 00
Mass/area structural loading, bearing capacity (mass basis)	U.S. ton/ft^2	Mg/m^2		9.764 855 E + 00
	lbm/ft^2	kg/m^2		4.882 428 E + 00
Miscellaneous transport properties				
Diffusivity	ft^2/s	m^2/s		9.290 304* E − 02
	m^2/s	mm^2/s		1.0* E + 06
	ft^2/h	m^2/s		2.580 64* E − 05
Thermal resistance	(°C·m^2·h)/kcal	(K·m^2)/kW		8.604 208 E + 02
	(°F·ft^2·h)/Btu	(K·m^2)/kW		1.761 102 E + 02
Heat flux	Btu/(h·ft^2)	kW/m^2		3.154 591 E − 03
Thermal conductivity	(cal·cm)/(s·cm^2·°C)	W/(m·K)		4.184* E + 02
	(Btu·ft)/(h·ft^2·°F)	W/(m·K)		1.730 735 E + 00
		(kJ·m)/(h·m^2·K)		6.230 646 E + 00
	(kcal·m)/(h·m^2·°C)	W/(m·K)		1.162 222 E + 00
	(Btu·in)/(h·ft^2·°F)	W/(m·K)		1.442 279 E − 01
	(cal·cm)/(h·cm^2·°C)	W/(m·K)		1.162 222 E − 01

As we should start at SOR with a thickness close to zero, we calibrate our calculations by rescaling the result of the above computation and obtain:



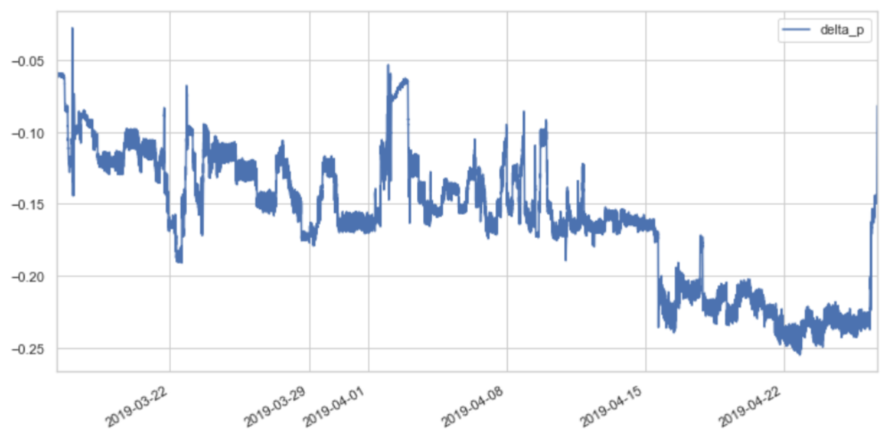
With colors respective to the six passes.

Trend during the first run cycle after MC for Pass 1:

1.1 Pressure drop

We compute each integral of the general equation also considering the coke thickness at time t and using the beta coefficient for bends calculated above.

For the first run cycle after the last MC and Pass 1:



2 Releases

- Number of tubes at time t
  - Determined from the number of spallings that occurred since the last decoke (for each spalling one computes the number of tube loss from the efficiency loss due to spalling – one tube corresponds to ~2% of efficiency)
  - As it is not possible to get the efficiency recovery for every decoke, one is assuming that just after a decoke, we start in SOR conditions for the number of tubes, i.e. N_sor = 50.625