ECE 18-100 Study Guide

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Compiled notes of the main ideas for each lecture (not in any particular order, just what I feel is natural).

1 Basic Theory

Definition 1.1. A **node** is any continuous region of equal voltage.

This definition is very helpful when doing circuit analysis because you can identify the number of distinct "islands" of voltages. Since we assume wires are ideal, we usually are looking at islands of just wire. This process of finding node voltages is known as **nodal analysis**.

Definition 1.2. Ohm's Law gives us a *linear* relationship between voltage and current, related by a constant R, or resistance.

$$V = IR \tag{1}$$

The equation is often manipulated in equivalent forms such as $I = \frac{V}{R}$ and $R = \frac{V}{I}$.

Definition 1.3. Resistance

Definition 1.4. Impedance

Definition 1.5. Conductance

Definition 1.6. Admittance?

2 Kirchoff

Two laws that you can use to solve basically any circuit (there are obviously exceptions).

Definition 2.1. Kirchoff's Circuit Law (KCL) tells us that because charge cannot build up at any point in a circuit,

$$\sum i_{\text{in}} = \sum i_{\text{out}}, \tag{2}$$

which basically tells us that charge has to go distribute someplace.

This equation can be super confusing because there are multiple ways to define current directions. For example, a negative current defined in one direction is just a positive current defined in the opposite direction. As a result, you can also write

$$\sum i_{\text{out}} = 0 \tag{3}$$

or

$$\sum i_{\text{in}} = 0 \tag{4}$$

as long as you are *consistent* with your current directions. In other words, the way you draw your current direction is what you should take it to be, whether that results it in being negative or positive.

KCL is used in **nodal analysis** (see Section 4.1).

Definition 2.2. Kirchoff's Voltage Law (KVL) tells us that

$$\sum_{\text{loop}} V_i = 0 \tag{5}$$

The motivation for this equation is that if you start at a point in the circuit, and walk around and return to where you started, you must still be at the same potential that you started with, regardless of what happened in between. A good analogy to conceptualize this is if you start at your home, no matter what you do between walking out your house and coming back, whether it is hiking mountains, flying in a jet, scuba diving, caving, etc...when you step back in your house, you will be at the same height as you were when you first stood there.

KVL is used in **mesh analysis** (see Section 4.2).

3 Equivalence

Definition 3.1. Thévenin's Theorem states that any circuit with linear elements can be replaced at some nodes A and B with an equivalent voltage source V_{TH} in series with an equivalent resistance R_{eq}.

Definition 3.2. Norton's Theorem states that any circuit with linear elements can be replaced at some nodes A and B with an equivalent current source I_{NT} in parallel with an equivalent resistance R_{eq}.

In order to find these values for equivalence, we do the following:

- V_{TH} - we open the circuit between the two points where we want to find the equivalent voltage, and then find the voltage across these two points. This is why V_{TH} is also called V_{OC}, where OC means "open circuit."
- I_{NT} - we short the circuit between the two points where we want to find the equivalent current, and then find the current across these two points. This is why V_{TH} is also called I_{SC}, where SC means "short circuit."
- R_{eq} - we short all voltage sources, open all current sources, and figure out the total resistance as *seen by the "load"*, where the load includes everything between the two points where you want to find the equivalent resistance.

It is also important to note the following relationship between $V_{\text{TH}}, I_{\text{NT}}$, and R_{eq},

$$\boxed{V_{\text{TH}} = I_{\text{NT}} R_{\text{eq}}} \tag{6}$$

4 Some Circuit Solving Techniques

We are going to use the following circuit to illustrate the solving techniques in this section.

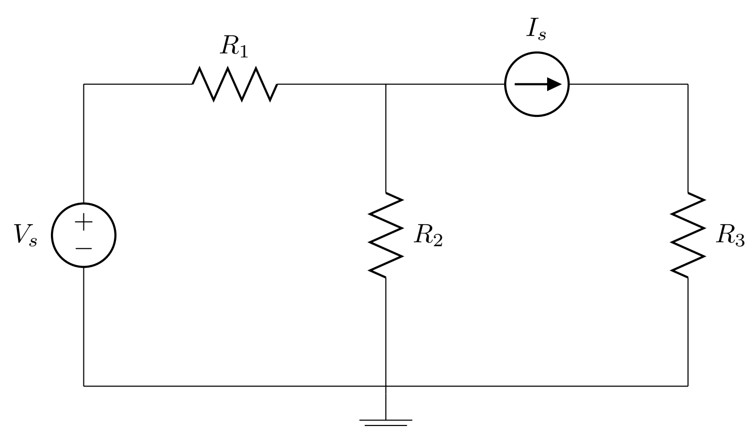
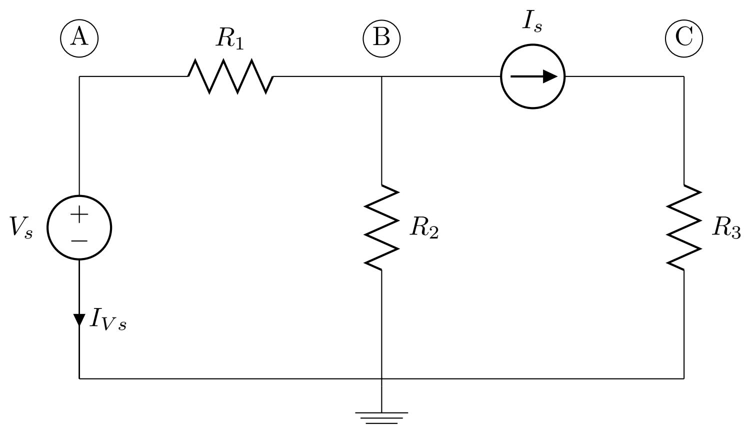


Figure 1: Problem for this section

4.1 Nodal Analysis

With nodal analysis, the idea is that once we find all the voltages of every node in a circuit, we can solve the entire circuit.

First, we first have to label our nodes.



Note that I did not label the very last node since it is connected to the ground and thus we already know it is defined to be 0 V.

Before we write our KCLs, it is good to figure some stuff out that is pretty obvious.

Since Node A is connected to the voltage source, which is directly connected to the ground, it must be true that $V_A = V_s$. In addition, we see that since there is a current of I_s going across R_3, the voltage drop from Node C to ground must be $I_s R_3$. Therefore, $V_C = I_s R_3$.

We now write our KCLs at each node: (I like to use $\sum i_{\text{out}} = 0$)

$$I_{Vs} + \frac{V_A - V_B}{R_1} = 0 \qquad \text{(A)}$$

$$\frac{V_B - V_A}{R_1} + \frac{V_B}{R_2} + I_s = 0 \qquad \text{(B)}$$

$$-I_s + \frac{V_C}{R_3} = 0 \qquad \text{(C)}$$

Note that I_{Vs} and V_B are the only two unknowns. Since we remember that in nodal analysis all we need to worry about is finding the voltage at each node, we just need to solve for V_B. The KCL at (B) is easiest for this, and gives

$$\begin{aligned}
\frac{V_B - V_A}{R_1} + \frac{V_B}{R_2} + I_s &= 0 \\
V_B\left(\frac{1}{R_1} + \frac{1}{R_2}\right) &= \frac{V_A}{R_1} - I_s \\
V_B &= \frac{R_1 R_2}{R_1 + R_2} \frac{V_s - R_1 I_s}{R_1} \\
V_B &= \frac{R_2(V_s - R_1 I_s)}{R_1 + R_2}
\end{aligned}$$

We are done solving the circuit, with nodal voltages:

$$\begin{aligned}
V_A &= V_s \\
V_B &= \frac{R_2(V_s - R_1 I_s)}{R_1 + R_2} \\
V_C &= I_s R_3
\end{aligned}$$

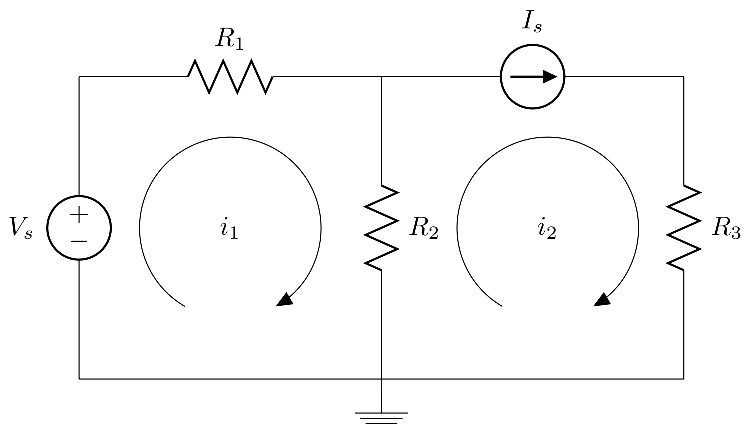
We can now solve the entire circuit. For example, if we wanted to find I_1, the current across R_1, we would just do

$$\begin{aligned} I_1 &= \frac{V_{R1}}{R_1} \\ &= \frac{V_A - V_B}{R_1} \end{aligned}$$

and we would plug in the value we found for the nodal voltages.

4.2 Mesh Analysis

The idea of mesh analysis is that in any *mesh*, which is a closed loop, we can assign a current that runs through the loop and use KVL to write equations. In any given circuit, it is always possible to write n equations for the n meshes there are, but there are only $n-1$ linearly independent ones. For example, in the circuit below, we only need to do two meshes (the big outer mesh is a linear combination of the two meshes shown so we don't need the extraneous equation).



Once we have the meshes chosen, we just do KVL on each. For the i_1 loop,

$$\begin{aligned} V_s - i_1R_1 - (i_1 - i_2)R_2 &= 0 \\ i_1(R_1 + R_2) - i_2R_2 &= V_s. \end{aligned}$$

For the i_2 loop, we could write,

$$\begin{aligned} V_I - i_2R_3 - (i_2 - i_1)R_2 &= 0 \\ -i_1R_2 + i_2(R_2 + R_3) &= V_I, \end{aligned}$$

but this is unnecessary since we see that $I_s = i_2$ in the circuit. We substitute i_2 back into the first mesh, and solve for i_1,

$$i_1 = \frac{V_s + I_sR_2}{R_1 + R_2}$$

With the two mesh currents, we can solve the rest of the circuit. Suppose we wanted to find the voltage drop across R_1. This would just be

$$\begin{aligned} V_{R_1} &= I_1R_1 \\ &= i_1R_1 \\ &= \frac{R_1(V_s + I_sR_2)}{R_1 + R_2} \end{aligned}$$

4.3 Superposition

The idea of superposition is sometimes, a circuit can look crazy when there are several voltage and current sources. Fortunately, superposition allows us to look at the contribution of one source at a time, to make our calculations easier.

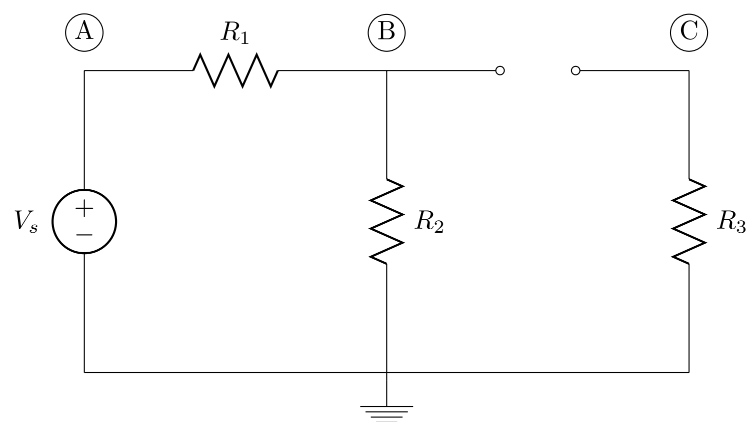
The basis for superposition comes from the fact that since we are dealing with *linear* elements, contributions from individual voltage and current sources (independent) sum up to give the total contribution from all of them together. When we use superposition, we first get rid of all sources except one, and solve the circuit. We do this for each source in the circuit, and once we have all of the solutions, we add up the circuits to find the final answer.

When we get rid of a source we do one of two things:

1. To get rid of a voltage source, make it a short (because it would be 0 V across and thus would act like the voltage source wasn't there)

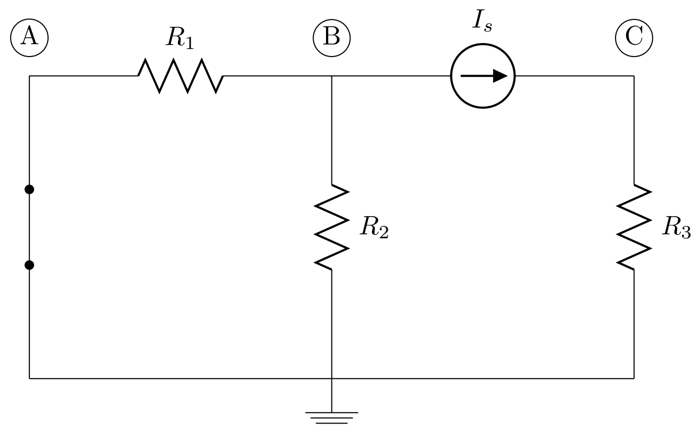
2. To get rid of a current source, make it open (because there would be no current flowing across it and it would act like the current source wasn't there)

We begin by removing the current source by replacing it with an open.



Note that by getting rid of our current source, this problem becomes easier to find the nodal voltages. Specifically, since there is no current going through the Node C branch, we know that $V_C = 0\,\text{V}$, since it is at the same voltage as the ground. V_A we still know is V_s, and for V_B, we see that it is just a voltage divider, so our three nodal voltages are

$$V_A = V_s$$
$$V_B = \frac{R_2}{R_1 + R_2} V_s$$
$$V_C = 0\,\text{V}$$



In this second case, we also see that our problem is easier than the original. We can immediately find $V_A = 0\,\mathrm{V}$ because it is part of the same node as ground. We can find V_C using the voltage drop across R_3, which is $I_s R_3$. For V_B, we write the KCL at B.

$$\begin{aligned}\frac{V_B - V_A}{R_1} + \frac{V_B}{R_2} + I_s &= 0\\ V_B\left(\frac{1}{R_1} + \frac{1}{R_2}\right) &= -I_s\\ V_B &= -\frac{R_1 R_2}{R_1 + R_2} I_s\end{aligned}$$

So our three voltages in this case are

$$\begin{aligned}V_A &= 0\,\mathrm{V}\\ V_B &= -\frac{R_1 R_2}{R_1 + R_2} I_s\\ V_C &= I_s R_3\end{aligned}$$

To find the nodal voltages in the original circuit, we just sum up the voltages we got in each case. Therefore, our final nodal voltages are

$$\begin{aligned}V_A &= V_s + 0\,\mathrm{V} = V_s\\ V_B &= \frac{R_2}{R_1 + R_2} V_s - \frac{R_1 R_2}{R_1 + R_2} I_s = \frac{R_2(V_s - R_1 I_s)}{R_1 + R_2}\\ V_C &= 0\,\mathrm{V} + I_s R_3 = I_s R_3\end{aligned}$$

Superposition gives us insight into the individual contributions from each source that, for example, the nodal analysis in Section 4.1 (although yielding the same answer) did not show us. We were also able to deal with greatly simplified scenarios that were easy to solve, which is useful in very complicated circuits where not using superposition can result in lots of mistakes being made.

4.4 What to use?

Of course, with all these circuit-solving techniques, you're probably wondering: "*what technique should I be using?*" Long answer short—whatever you're most comfortable with, and whatever the situation calls for. *However*, as circuits become more complicated, the only technique that runs into problems is mesh analysis. The reason is that really complicated circuits might not have well-defined loops, which make writing the equations very difficult or impossible. Thus, if I had any recommendation for what to learn for solving circuits, I would say nodal analysis, since it generalizes nicely and is what most simulators use.

5 Transient Analysis

Since capacitors and inductors have similar equations, it is useful to analyze them side by side.

Capacitor	**Inductor**

Add stuff about inductor direction, why it's important to define V_L a certain direction.

6 Sinusoidal Steady State (SSS)

It turns out that all linear circuit elements (we will work with resistors, capacitors and inductors) can be treated like a resistor of a complex impedance.

Definition 6.1. The impedance of a capacitor with capacitance C is

$$Z_C = \frac{1}{j\omega C} \tag{7}$$

Definition 6.2. The impedance of a inductor with inductance L is

$$Z_L = j\omega L \tag{8}$$

7 Diodes

7.1 Theory

Diodes are made of N and P-type *things* that cause polarization and thus control the direction that current can flow.

Definition 7.1. A typical diode is denoted by the symbol: and has a V_{ON} (usually 0.6 V) for silicon-based diodes.

- If $V_D > V_{\mathrm{ON}}$, then the diode is on, and $I_D > 0$.
- Otherwise, the diode is off and $I_D = 0$.

Definition 7.2. A **zener diode** is denoted by the symbol: and has a V_{ON} and a V_Z.

- If $V_D > V_{\mathrm{ON}}$, the diode is on, and $I_D > 0$.
- If $-V_Z < V_D < V_{\mathrm{ON}}$, the diode is off and $I_D = 0$.
- If $V_D < -V_Z$, the diode is reverse on, and $I_D < 0$.

One thing to keep in mind is that at the switching points, it doesn't really matter if there is equality, because the two states are almost identical at the points where they meet. Therefore, we don't have to worry about defining what happens with equality at the changing points.

8 Transistors