18220 Exam 2 Review

We got the POWER

Def. Power is defined as

$$P = \underbrace{I}_{\text{current}} \overbrace{V}^{\text{voltage}}$$

Cor: w/ $V=IR$ we can write:

$$P = IV = I^2R = \frac{V^2}{R}$$

Def: we define the capacitor equation as

$$i_c(t) = C\frac{dV_c(t)}{dt}$$

Cor: Using power, we have

$$P_c(t) = i_c(t)\cdot V_c(t)$$

Since energy = power · time,

$$E_c = \int P_c(t)dt = \int i_c(t)V_c(t)dt$$

$$= \int \frac{dV_c(t)}{dt} V_c(t)\,dt$$

$$= C\int V_c(t)\,dV_c(t)$$

$$= \frac{1}{2}CV_c^2$$

Def: The inductor equation is

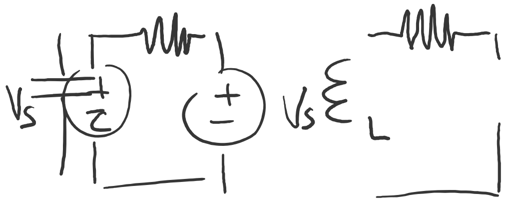
$$V_L(t) = L\frac{di_L(t)}{dt}$$

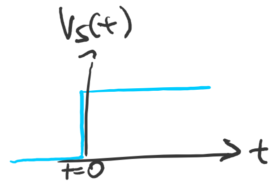
Cor: Similar to the deriv. for the cap, we can get

$$E_L = \frac{1}{2}LI_L^2$$

Long Time / Frequency Analysis of L/C

Consider





where C and L are not charged, then

	$t=0$	$t\to\infty$
Cap:	short	open
Ind:	open	short

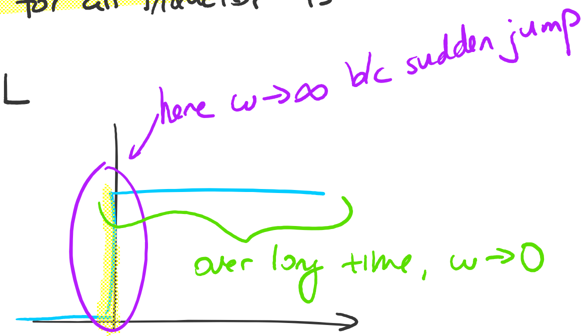
Def: The complex impedance for a capacitor is

$$Z_c = \frac{1}{j\omega C}$$

Def: The complex impedance for an inductor is

$$Z_L = j\omega L$$

	$\omega\to\infty$	$\omega=0$
Cap:	short	open
Ind:	open	short



Just an interesting coincidence...

Transforming the POWER

Def: We define $\underline{M}$ to be the mutual inductance. $M = k\sqrt{L_1L_2}$ (Henries)

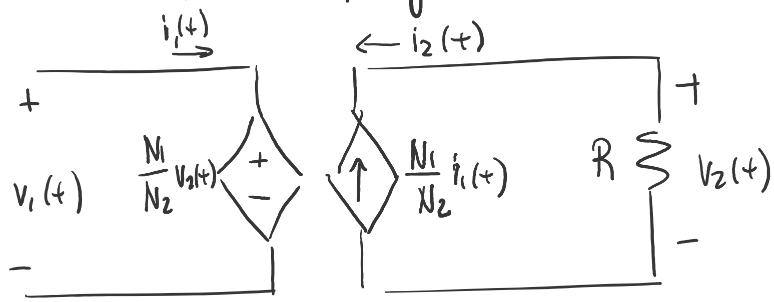
Def: k is the coupling coefficient. (unitless)

- $k=0$ if no coupling
- $k=1$ is a perfect transformer

Def: An ideal transformer is defined to be:

- $L, M \to \infty$ so little current flows through inductors and therefore the transformer stores no energy

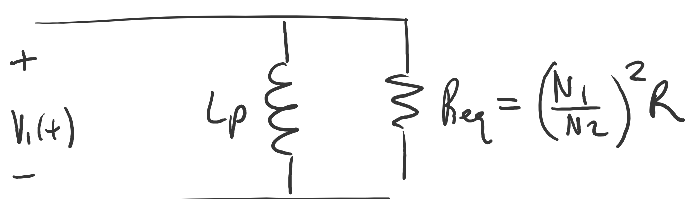
· $k=1$ perf. coupling



Cor: Using transformer properties, we can derive

primary

Def: A perfect transformer is a transformer w/ $k=1$ but M, L not nec. $\to \infty$.
This means we have a leakage inductance, so when we derive the equivalent resistance, we instead have:



where L_p is the leakage inductance, can be defined in several ways. One such way is:

$$L_p = k \cdot L_1$$

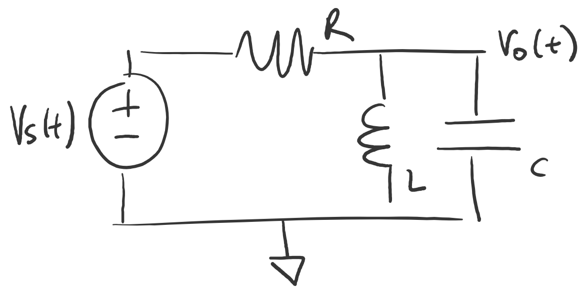
Def: A non-ideal transformer in general does not have any ideal assumptions. You have to write out the coupled differential equations to solve these problems.

Oscillating w/ the POWER

Def: A RLC circuit has a resistor (usually in series)

Def: A RLC circuit has a resistor (usually in series) w/ a L, C either in parallel or series.

For our example, we'll do the one in parallel:



If we write the diff. eq for this circuit, we get

$$\frac{d^2 V_o(t)}{dt^2} + \frac{1}{RC}\frac{dV_o(t)}{dt} + \frac{1}{LC}V_o(t) = \frac{1}{RC}\frac{dV_s(t)}{dt}$$

To solve the homogeneous solution, we first make an ansatz of

$$V_o(t) = Ae^{st}$$

Plugging in gives us $s = -\frac{1}{2RC} \pm \sqrt{\frac{1}{(2RC)^2} - \frac{1}{LC}}$

$\Downarrow$

We like to define

- $\alpha = \frac{1}{2RC}$
- $\tau = \sqrt{LC}$
- $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\tau}$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

Now, before we write our general solutions, we need to consider 3 cases

- det. is < 0, then we have a complex root, i.e.

$$s = a \pm jb$$

so

$$A_1 e^{t(a+jb)} + A_2 e^{t(a-jb)}$$

$$\Longrightarrow A_1 e^{at}\cos(bt) + A_2 e^{at}\sin(bt) \quad \text{(spans same sol. space)}$$

This called underdamping

- $\det = 0$ Then we only have one sol. It turns out we need to make our general solution

$$A_1 e^{at} + A_2 t e^{at}$$

in this case.

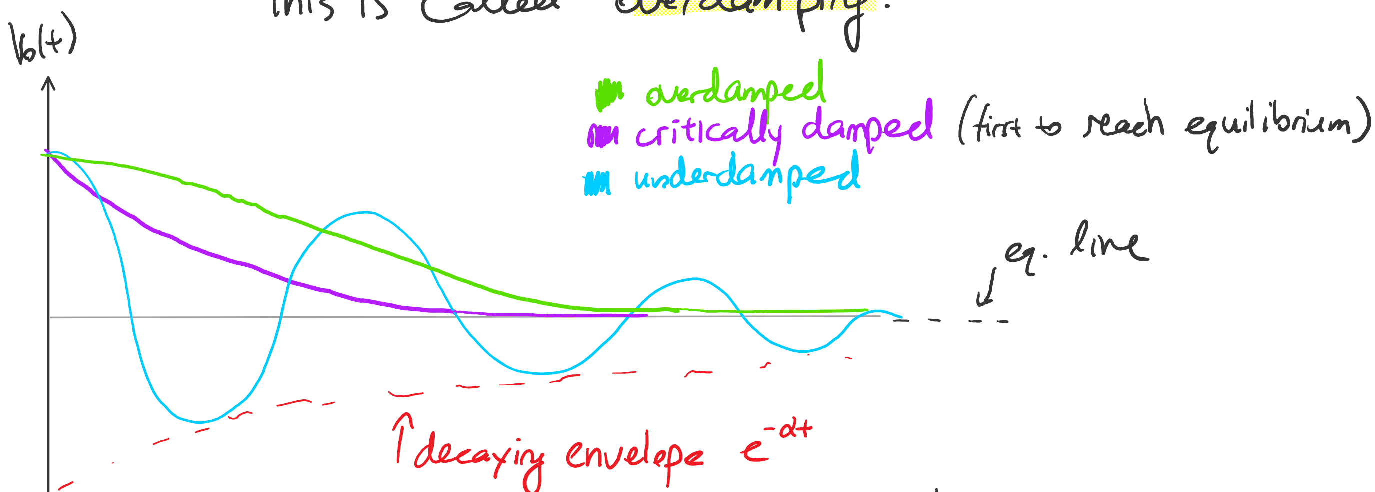
This is called critical damping.

- $\det > 0$ Both characteristic roots are real, so

$$A_1 e^{t(a+b)} + A_2 e^{t(a-b)}$$

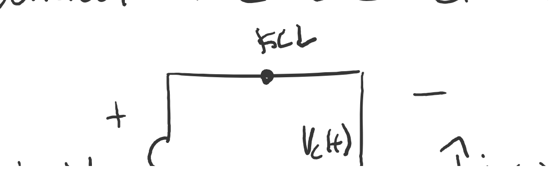
is our general solution

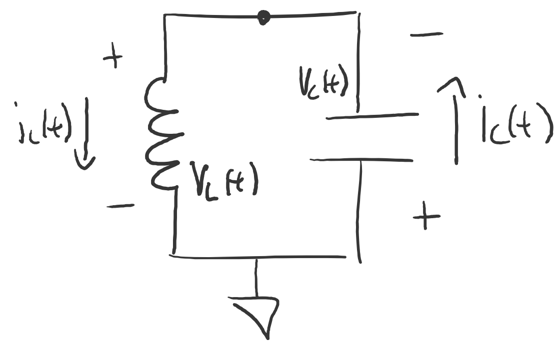
This is called overdamping.



Resonating w/ POWER

Consider the LC circuit





KVL gives us:

$$0 - V_C(t) - V_L(t) = 0$$

$$V_C(t) + V_L(t) = 0 \Rightarrow V_C(t) + L\frac{di_L(t)}{dt} = 0$$

KCL gives us:

$$-j_C(t) + i_L(t) = 0$$

$$\Rightarrow C\frac{dV_C(t)}{dt} = i_L(t)$$

$$V_C(t) + L\frac{d}{dt}\left(C\frac{dV_C(t)}{dt}\right) = 0$$

$$\frac{d^2V_C(t)}{dt^2} + \frac{1}{LC}V_C(t) = 0$$

solving w/ Ae^{st} gives

$$s^2 = -\frac{1}{LC} \Rightarrow s = \pm j\left(\frac{1}{\sqrt{LC}}\right)$$

from $\omega_0 = 1/\sqrt{LC}$

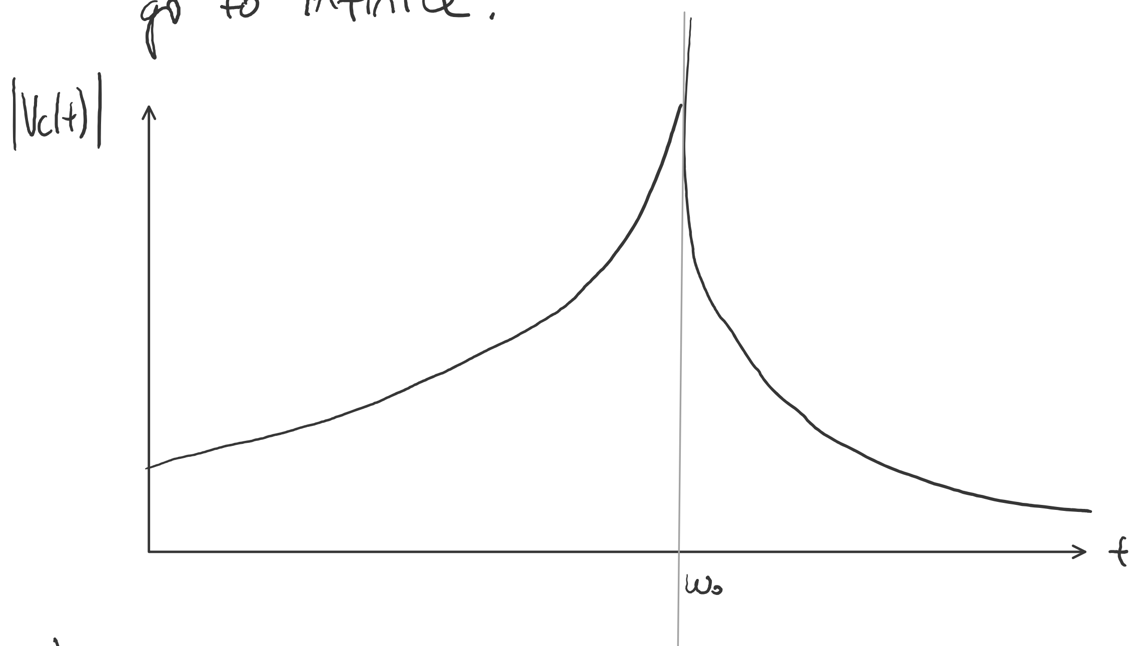
our solution is in general

$$V_C(t) = A_1\cos(\omega_0 t) + A_2\sin(\omega_0 t)$$

$$= K\cos(\omega_0 t + \phi)$$

for this circuit, resonance occurs at

$\omega_0 = \frac{1}{\sqrt{LC}}$, which means if you applied a driving source w/ $f = \omega_0$ then the oscillation would essentially go to infinite.



<u>Def:</u> Since the amplitude at resonance is not ∞ in real life, we define our quality factor as $\propto$ height of resonance, which is

$$Q = \frac{\omega_0}{2\alpha}$$

Notice that since we did not have a resistor in our LC circuit, there was no damping. In general, the damping caused by the resistor depends on the circuit, i.e.

	RLC series	RLC parallel
α	$\frac{1}{2}\frac{R}{L}$	$\frac{1}{2RC}$