18-220 Ex 3 Review

Bode Plots

Transfer Function $H(j\omega) = A \dfrac{\prod_i (1+\frac{j\omega}{\omega_{i0}}) \prod_k (\frac{j\omega}{\omega_{k0}})}{\prod_a (1+\frac{j\omega}{\omega_{a0}}) \prod_b (\frac{j\omega}{\omega_{b0}})}$

We care about two things,

1. Magnitude, specifically in decibels (dB):

$$|H(j\omega)| \text{ in decibels} = 20\log_{10}(|H(j\omega)|)$$

$$\Rightarrow 20\left[\sum_i^1 (1+\tfrac{j\omega}{\omega_{i0}}) + \sum_k^1 (\tfrac{j\omega}{\omega_{k0}}) - \sum_a^1 (1+\tfrac{j\omega}{\omega_{a0}}) - \sum_b^1 (1+\tfrac{j\omega}{\omega_{b0}})\right]$$

notice logs turn products in $H(j\omega)$ into sums

2. Phase. Note that because

$$\begin{aligned}\angle(E_1 E_2) &= \angle(A_1 e^{j\theta_1} A_2 e^{j\theta_2}) \\ &= \angle(A_1 A_2 e^{j(\theta_1+\theta_2)}) \\ &= \theta_1+\theta_2 \\ &= \angle(E_1) + \angle(E_2)\end{aligned}$$

$$\angle(H(j\omega)) = \angle(A) + \sum^1 \angle(\text{terms in numerator}) - \sum^1 \angle(\text{terms in denominator})$$

Example

$$H(j\omega) = \frac{j\omega(1+\frac{j\omega}{10^4})}{(1+\frac{j\omega}{10})(1+\frac{j\omega}{10^5})}$$

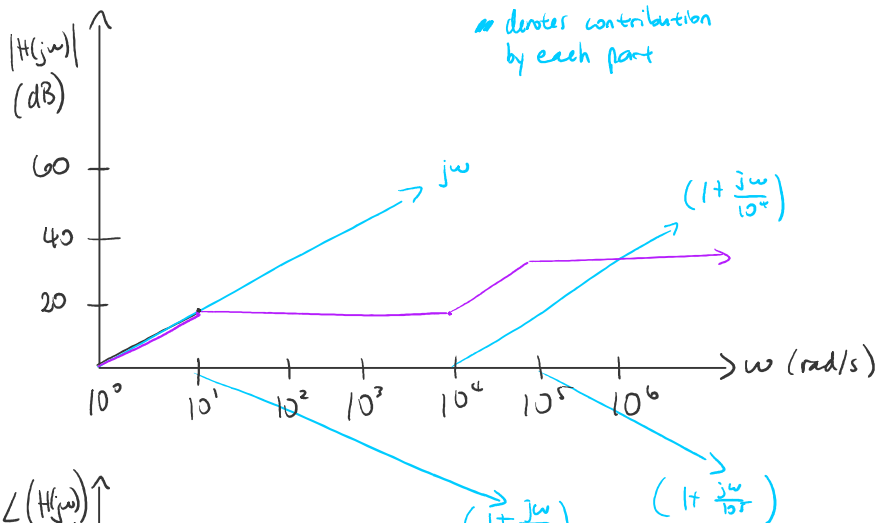
1. Draw your starting points, you can do this by evaluating ω @ $10^0=1$ and plugging into $H(j(1))$ and finding:

- $20\log_{10}(H(j))$ for mag
$= 20\log_{10}\left(\left|\frac{j\cdot 1\ (1+j/10^4)}{(1+j/10)(1+j/10^5)}\right|\right)$
$= 20\log_{10}(|j|) = 0$
- $\angle H(j\cdot 1) = \angle(j) = 90°$

notice that $|1+j/10^k, k\geq 1$ is considered ≈ 1 since $j/10$ is "small"

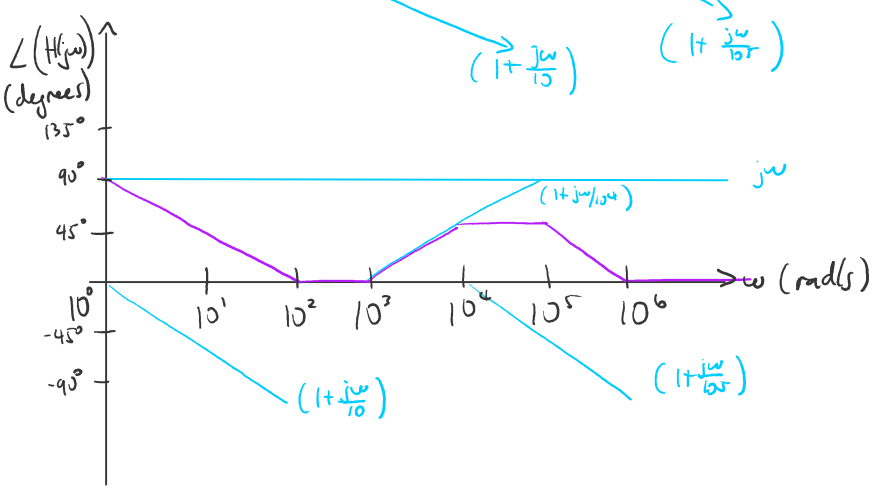
2. Sweep ω from 1 to 10^k where k captures the asymptotic behavior of the transfer function.

MAG



∞ denotes contribution by each part

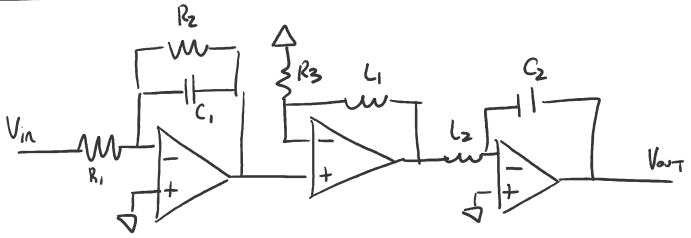
PHASE



- $1 + j\omega/10^k$ for $\omega \le 10^k$ is insignificant
- $1 + j\omega/10^k$ for $\omega > 10^k$ is significant

- $1 + j\omega/10^k$ for $\omega \le 10^{k-1}$ is ≈ 1, $\angle(1) = 0°$
- $1 + j\omega/10^k$ for $\omega = 10^k$ $\approx 1+j$, $\angle(1+j) = 45°$ ← *notice phase contributes at a smaller ω than mag
- $1 + j\omega/10^k$ for $\omega > 10^k$ is $\approx H(\text{big \#}) \cdot j \Rightarrow \angle(\quad) = 90°$

Exercise 1

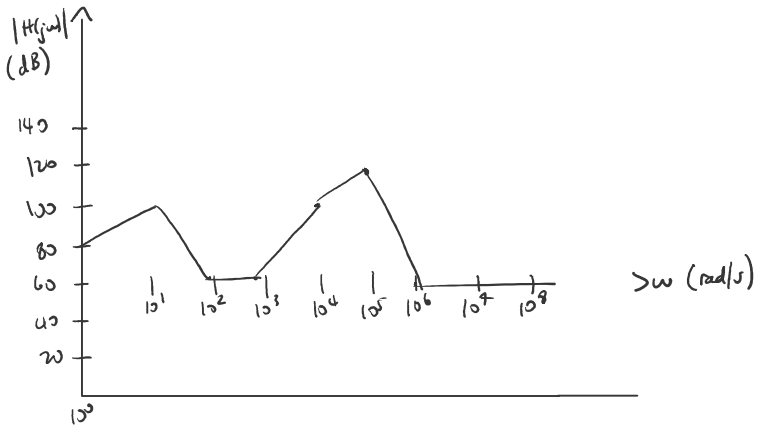


$R_1 = 200\,k\Omega$
$R_2 = 2\,k\Omega$
$R_3 = 30\,k\Omega$
$L_1 = 300\,H$
$L_2 = 1\,H$
$C_1 = 5\,nF$
$C_2 = 10\,\mu F$

Assuming ideal op-amps, find $H(j\omega) = \frac{V_{OUT}}{V_{in}}$, and

1. State the # of poles and zeros
2. State the type of filter
3. Plot the Bode Plot for mag and phase

Exercise 2



Find a transfer func. that represents this Bode plot
Find the Bode plot of the phase of the func. you found