Definition of Limit Activity

This activity is intended to help students gain a better understanding of "closeness" in the intuitive definition of a limit.

General Directions: Answer each question thoroughly. Incorrect answers with work shown may receive partial credit, but unsubstantiated answers will receive NO CREDIT. I do not want (decimal) approximations unless specifically asked for. I want the exact numbers. Justify all claims using calculus concepts (i.e., theorems, definitions, etc.). I am looking for mathematical logic and reasoning. Show all of your work!! Explain! Explain! Explain!

Go to the In-Class Activities section of Blackboard and open the Definition of Limit activity.

1 Use the graph to estimate $\lim_{x \to 0} f(x)$. Then update the values of L and c in the tool.

   a Set Radius to 0.5. Use the a and b sliders (or the points on the x-axis) to find the largest interval, centered at $x = 0$, such that f maps any x-value in this interval to within 0.5 units of L.

   b Set Radius to 0.1. Use the a and b sliders (or the points on the x-axis) to find the largest interval, centered at $x = 0$, such that f maps any x-value in this interval to within 0.1 units of L.

   c Set Radius to 0.01. Use the a and b sliders (or the points on the x-axis) to find the largest interval, centered at $x = 0$, such that f maps any x-value in this interval to within 0.01 units of L.

2 Now set $L = 1$, but leave c set to 0.

a Set Radius to 3. Use the a and b sliders (or the points on the x-axis) to find the largest interval, centered at $x = 0$, such that f maps any x-value in this interval to within 3 units of L.

b Does this contradict that $\lim_{x \to 0} f(x) = 4$? Explain your answer.

3 In this problem we will investigate $\lim_{x\to 1} f(x)$. Notice, this limit doesn't exist since $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$. We will investigate three values for L: $2, 3$, and 2.5.

a Let $L = 2, c = 1$, and Radius $= 1.5$. Use the a and b sliders (or the points on the x-axis) to find the largest interval, centered at $x = 1$, such that f maps any x-value in this interval to within 1.5 units of L.

i. Does this contradict that $\lim_{x\to 1} f(x) = DNE$? Explain your answer.

ii. Change Radius to 0.5. Use the a and b sliders (or the points on the x-axis) to find the largest interval, centered at $x = 1$, such that f maps any x-value in this interval to within 0.5 units of L. What does this tell you? Explain.

b Let $L = 3, c = 1$, and Radius $= 1.5$.

i. Use the a and b sliders (or the points on the x-axis) to find the largest interval, centered at $x = 1$, such that f maps any x-value in this interval to within 1.5 units of L.

ii. Find a value for Radius such that there is no interval on the x-axis such that x-values get mapped to the interval created by Radius.

iii. Explain how this proves that $\lim_{x \to 1} f(x) \neq 3$.

c Let $L = 2.5$ and $c = 1$. Show that $\lim_{x \to 1} f(x) \neq 2.5$.

4 Explain how the intervals in this activity connect to the concept of "closeness" in the intuitive definition of a limit.