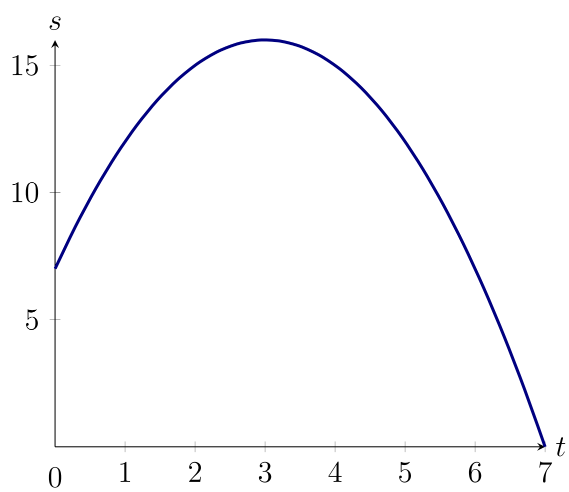
An Application of Limits Activity

General Directions: Answer each question thoroughly. Incorrect answers with work shown may receive partial credit, but unsubstantiated answers will receive NO CREDIT. I do not want (decimal) approximations unless specifically asked for. I want the exact numbers. Justify all claims using calculus concepts (i.e., theorems, definitions, etc.). I am looking for mathematical logic and reasoning. Show all of your work!! Explain! Explain! Explain!

Suppose the function $s(t) = 16 - (t-3)^2$ represents the height s of a ball (in feet) at time t in seconds. A graph of s is given below.



1. First, let's see what we can estimate directly from the graph.

   (a) From what height is the ball intially thrown?

   (b) At what time does the ball hit the ground?

   (c) At what time does the ball reach its maximum height?

   (d) What is the maximum height of the ball?

Reminder: $s(t) = 16 - (t-3)^2$

2. Compute the average velocity of the ball on the interval $[1, 2]$.

3. Compute the average velocity of the ball on the interval $[2, 3]$

4. Compute the average velocity of the ball on the interval $[2, 2+h]$ for $h > 0$. (Notice, when $h < 0$, this becomes the interval $[2+h, 2]$.)

Reminder: $s(t) = 16 - (t-3)^2$

5. Use your answer to number 4 to compute the average velocity on each of the following intervals.

   (a) What is the average velocity on $[2, 2.1]$?

   (b) What is the average velocity on $[1.9, 2]$?

   (c) What is the average velocity on $[2, 2.01]$?

   (d) What is the instantaneous velocity at $t = 2$?

Recall from your reading:

Definition 1. *The* ***derivative*** *of f at a, denoted $f'(a)$, is given by*

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a},$$

provided that the limit exists. We say that f is ***differentiable*** *at a if this limit exists. Otherwise, we say that f is* ***non-differentiable*** *at a.*

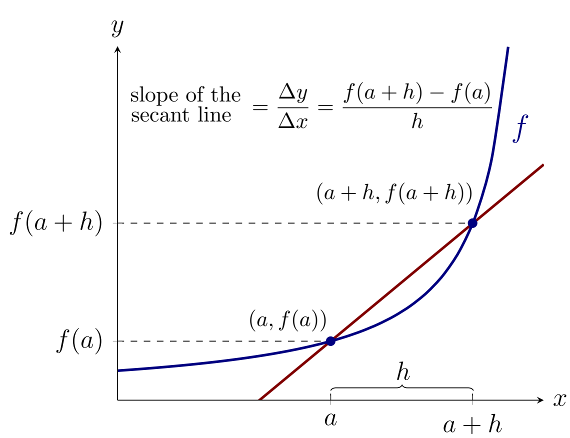
This definition makes it clear that the derivative of f at $x = a$ is the limit of the average rate of change (or the limit of the slope of the secant line between $(x, f(x))$ and $(a, f(a))$). However, using this definition to compute the derivative of f at a is often inconvenient. Therefore, we will use an alternative form of this definition.

Definition 2. *The* ***derivative*** *of f at $x = a$, denoted $f'(a)$, is given by*

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

provided that the limit exists.

The image below shows that this definition is equivalent to the original one. When computing derivatives, this second definition can be much easier to use.



6. Compute $s'(5)$.