Asmyptote Activity

This activity is intended to increase students understanding of the connection between asymptotes and limits.

Directions: Answer each question thoroughly. Incorrect answers with work shown may receive partial credit, but unsubstantiated answers will receive NO CREDIT. I do not want (decimal) approximations unless specifically asked for. I want the exact numbers. Justify all claims using calculus concepts (i.e., theorems, definitions, etc.). I am looking for mathematical logic and reasoning. Show all of your work!! Explain! Explain! Explain! Four points will be dedicated to how you perform as a group.

1 Take 5 minutes and discuss with your group everything you remember about asymptotes from precalculus. Write a brief summary of this discussion here.

2 Copy each definition completely from section 6.2 or section 6.3:

- Vertical asymptote:

- Horizontal asymptote:

Definition 1. *We say that a function g* ***dominates*** *function f provided that* $\lim_{x \to a} \frac{f(x)}{g(x)} = 0$ *or* $\lim_{x \to a} \frac{g(x)}{f(x)} = \pm\infty$ *where a can also be $\pm\infty$.*

3 Given $r(x) = 5x^2 - 3x + 7$ and $k(x) = -2x^3 + 4x^2 - 7x + 5$, use the definition of dominance to determine which function dominates the other.

4 Given $n(t) = 4t^6 - 3t^2 + 18t - 7$ and $m(t) = 5t^4 - 6t$, use the definition of dominance to determine which function dominates the other.

5 Given $p(z) = -3z^2 - 2z + 7$ and $q(z) = 2z^2 - 8$, use the definition of dominance to determine which function dominates the other. What is $\lim_{z \to \infty} \frac{p(z)}{q(z)}$?

6 Based on these examples, make a conjecture about how the degree of the polynomials in the numerator and denominator of a rational expression relate to the horizontal asymptotes of a rational function. Can you prove your conjecture?

7 Use dominance to quickly determine the value of each limit. There is no need to show work.

a $\displaystyle\lim_{r\to-\infty} \frac{-2r^2+5r-7}{3r^2-7} =$

b $\displaystyle\lim_{n\to\infty} \frac{-8n^3-4n}{n^4+5n^2-9} =$

c $\displaystyle\lim_{k\to-\infty} \frac{-8k^4-2k+12}{2k^2+7} =$

8 Given $f(x) = \sin(x)$ and $g(x) = x^2$, guess which function is dominant. Why? Explain!

9 Find all asymptotes of $Q(r) = \dfrac{r^2 + r - 6}{\sqrt{r^4 - 16}}$. Remember to support your answers with appropriate calculus calculations and explanations.