Definition of Derivative Activity

This activity is intended to help students gain a better understanding of what we mean in the intuitive definition of a derivative when we say: "$\lim_{x \to a} f(x) = L$, if the value of $f(x)$ is as close as one wishes to L for all x sufficiently close, but not equal to, a."

Directions: Go to the In-Class Activities section of Blackboard and open the Definition of Derivative activity.

1 Use the graph to estimate $\lim_{x \to 0} f(x)$. Then update the values of L and c in the tool.

  a Set Radius to 0.5. Use the a and b sliders (or the points on the x-axis) to find the largest interval, centered at $x = 0$, such that f maps any x-value in this interval to within 0.5 units of L.

  b Set Radius to 0.1. Use the a and b sliders (or the points on the x-axis) to find the largest interval, centered at $x = 0$, such that f maps any x-value in this interval to within 0.1 units of L.

  c Set Radius to 0.01. Use the a and b sliders (or the points on the x-axis) to find the largest interval, centered at $x = 0$, such that f maps any x-value in this interval to within 0.01 units of L.

2 Now set $L = 1$, but leave c set to 0.

  a Set Radius to 3. Use the a and b sliders (or the points on the x-axis) to find the largest interval, centered at $x = 0$, such that f maps any x-value in this interval to within 3 units of L.

  b Does this contradict that $\lim_{x \to 0} f(x) = 4$? Explain your answer.

3 In this problem we will investigate $\lim_{x \to 1} f(x)$. Notice, this limit doesn't exist since $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$. We will investigate three values for L: $2, 3$, and 2.5.

a Let $L = 2, c = 1$, and Radius $= 1.5$. Use the a and b sliders (or the points on the x-axis) to find the largest interval, centered at $x = 1$, such that f maps any x-value in this interval to within 1.5 units of L.

i. Does this contradict that $\lim_{x \to 1} f(x) = DNE$? Explain your answer.

ii. Change Radius to 0.5. Use the a and b sliders (or the points on the x-axis) to find the largest interval, centered at $x = 1$, such that f maps any x-value in this interval to within 0.5 units of L. What does this tell you? Explain.

b Let $L = 3, c = 1$, and Radius $= 1.5$.

i. Use the a and b sliders (or the points on the x-axis) to find the largest interval, centered at $x = 1$, such that f maps any x-value in this interval to within 1.5 units of L.

ii. Find a value for Radius such that there is no interval on the x-axis such that x-values get mapped to the interval created by Radius.

iii. Explain how this proves that $\lim_{x \to 1} f(x) \neq 3$.

c Let $L = 2.5$ and $c = 1$. Show that $\lim_{x \to 1} f(x) \neq 2.5$.

4 Explain how the intervals in this activity connect to the concept of "closeness" in the intuitive definition of a limit.