

Continuity Activity

We will work on communicating clearly and precisely about the continuity of functions.

General Directions: Answer each question thoroughly. Incorrect answers with work shown may receive partial credit, but unsubstantiated answers will receive NO CREDIT. I do not want (decimal) approximations unless specifically asked for. I want the exact numbers. Justify all claims using calculus concepts (i.e., theorems, definitions, etc.). I am looking for mathematical logic and reasoning. Show all of your work!! Explain! Explain! Explain!

Problem 1 Given $f(x) = \begin{cases} \sqrt[5]{x} \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$, answer the following.

(a) *Explain what needs to be determined to show whether or not $f(x)$ is continuous at $x = 0$.*

(b) *Make an educated guess on whether or not $f(x)$ is continuous at $x = 0$. Explain your thinking.*

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(c) *If you think $f(x)$ is continuous at $x = 0$, then describe a plan to prove it. If you think it is not continuous, describe a plan to prove that.*

(d) *Carry out your plan in detail and explain your whole process.*

Problem 2 Given $f(x) = \begin{cases} \sqrt[5]{x} \cdot g(x) & x \neq 0 \\ 0 & x = 0 \end{cases}$, answer the following.

(a) Give examples of $g(x)$ such that f is continuous at $x = 0$.

(b) Give examples of $g(x)$ such that f is **not** continuous at $x = 0$.