The Intermediate Value Theorem

We will investigate and apply the Intermediate Value Theorem.

General Directions: Answer each question thoroughly. Incorrect answers with work shown may receive partial credit, but unsubstantiated answers will receive NO CREDIT. I do not want (decimal) approximations unless specifically asked for. I want the exact numbers. Justify all claims using calculus concepts (i.e., theorems, definitions, etc.). I am looking for mathematical logic and reasoning. Show all of your work!! Explain! Explain! Explain! **No graphing calculators are allowed for this activity.**

Theorem 1 (Intermediate Value Theorem)**.** *If f is a continuous function for all x in the closed interval $[a, b]$ and d is between $f(a)$ and $f(b)$, then there is a number c in (a, b) such that $f(c) = d$.*

1 Given $p(k) = k^3 - 3k + 1$, we want to use the Intermediate Value Theorem to estimate the zero of p on $[0, 1]$.

a First, we must establish the conditions of the Intermediate Value Theorem (IVT) hold for p on $[0, 1]$.

b What value should we choose for d in the theorem if we want to find a root of p? Is this choice for d between $p(0)$ and $p(1)$?

c Parts (a) and (b) allow us to use IVT to conclude that p **must** have a root on $[0, 1]$. To use IVT to find the root, we need to split the interval $[0, 1]$ into two intervals: $\left[0, \frac{1}{2}\right]$ and $\left[\frac{1}{2}, 1\right]$. Use IVT to determine which of these two intervals contains the root. Make sure to establish the conditions of the theorem. The conclusion alone is not sufficient.

d Repeat the steps in part (c) until you have estimated the root to at least one decimal place acuracy.