

Limits and Continuity

This will test students ability to interpret limits.

General Directions: Answer each question thoroughly. Incorrect answers with work shown may receive partial credit, but unsubstantiated answers will receive NO CREDIT. I do not want (decimal) approximations unless specifically asked for. I want the exact numbers. Justify all claims using calculus concepts (i.e., theorems, definitions, etc.). I am looking for mathematical logic and reasoning. Show all of your work!! Explain! Explain! Explain! Four points will be dedicated to how you perform as a group.

1. Let g be a function with the following properties. Determine if each statement below is always true, sometimes true, or never true. Justify your answers.

- $\lim_{x \rightarrow -\infty} g(x) = -5$, $\lim_{x \rightarrow -2} g(x) = 4$, $\lim_{x \rightarrow 0^-} g(x) = -1$, $\lim_{x \rightarrow 0^+} g(x) = 1$, and $\lim_{x \rightarrow 3} g(x) = -2$
- $g(-2) = 2$, $g(0) = 0$, $g(3) = -2$, $g(5) = 7$, and $g(-4)$ is undefined

(a) The graph of g has a vertical asymptote at $x = 5$

(b) The graph of g has a horizontal asymptote.

(c) g has a removable discontinuity.

Recall: Let g be a function with the following properties. Determine if each statement below is always true, sometimes true, or never true. Justify your answers.

- $\lim_{x \rightarrow -\infty} g(x) = -5$, $\lim_{x \rightarrow -2} g(x) = 4$, $\lim_{x \rightarrow 0^-} g(x) = -1$, $\lim_{x \rightarrow 0^+} g(x) = 1$, and $\lim_{x \rightarrow 3} g(x) = -2$
- $g(-2) = 2$, $g(0) = 0$, $g(3) = -2$, $g(5) = 7$, and $g(-4)$ is undefined

(d) g has a non-removable discontinuity.

(e) The graph of g has two distinct horizontal asymptotes.

(f) The domain of g is $(-\infty, \infty)$.