Linear Regression

By Dr. Nzamba Bignoumba

$y=-ax+b$

Average training duration: 4 hours 00 minute

Outline

Machine learning overview

AI

what can we do with it?

Natural Language Processing
Computer Vision
Signal Processing
⋮

Machine learning overview

Content summary of one or more documents
Translation from one language to another
Code generation
⋮

OpenAI-ChatGPT
DeepSeek
GitHub Copilot
Cursor | Codex

Visual content generation
Medical image classification
Agricultural Image Classification
Object detection
⋮

Midjourney
Canva
Aidoc
IA Agri
AgriHyphen AI

Natural Language Processing
Computer Vision
Signal Processing
⋮

Weather forecast
Disease and mortality forecasts/predictions
Stock market forecasts
Electricity consumption forecasts
Anomaly detection (cybersecurity)
⋮

AWS SageMaker – DeepAR
Nixtla-TimeGPT
Meta-Prophet
Zindi Africa
Amini

A. Vaswani, N. Shazeer, N. Parmar, J. Uszkoreit, L. Jones, A. N. Gomez,L. Kaiser, I. Polosukhin, Attention is all you need, Advances in neural information processing systems 30 (2017).
J. Redmon, S. Divvala, R. Girshick, A. Farhadi, You only look once: Unified, real-time object detection, in: Proceedings of the IEEE conference on computer vision and pattern recognition, 2016, pp. 779–788.
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Machine learning overview

AI

what can we do with it?

Natural Language Processing Computer Vision Signal Processing
⋮

With what type of algorithms?

Machine Learning

Deep Learning

Machine learning overview

K-NN

Logistic Regression

K-Means

Support Vector Machines

Decision Trees

Gradient Boosting Machines

Random Forest

Linear Regression

Principal Component Analysis

Machine Learning

...

Deep Learning

Generative Adversarial Neural Network

Varational Autoencoder

Feed Forward Neural Network

State Space Model

Word Embeddings

Autoencoder

Graph Neural Network

Neural Ordinary Differential Equations

Diffusion Model

Neural Radiance Field

Recurrent Neural Network

Transformer

Normalizing Flows

...

Machine learning overview

IA

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With what type of algorithms?

Machine Learning

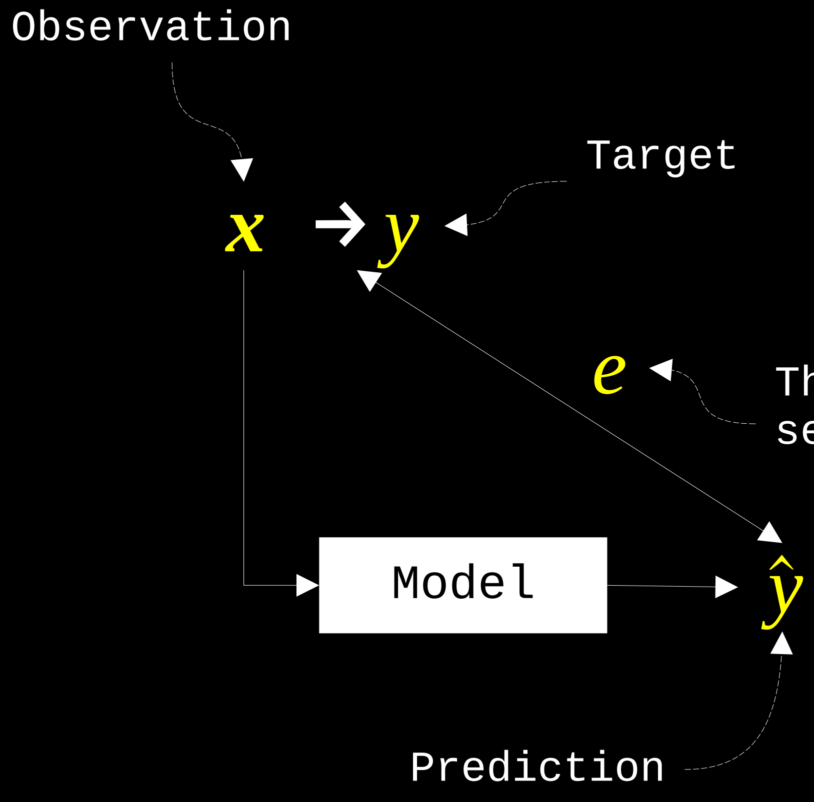
Deep Learning

How do they work ?

Supervised
Non-Supervised
Self-Supervised
⋮

Machine learning overview

Let’s have the dataset **Xs** and its corresponding target **ys**. The model will use **X** to predict **ŷ**, **modèle(X)=ŷ**. Then, **ŷ** will be compared to **y** to calculate the error **e** made by the model, **|y-ŷ|=e**. This error serves to **refine the model parameters** so that it can predict a value **ŷ** very close to **y**.

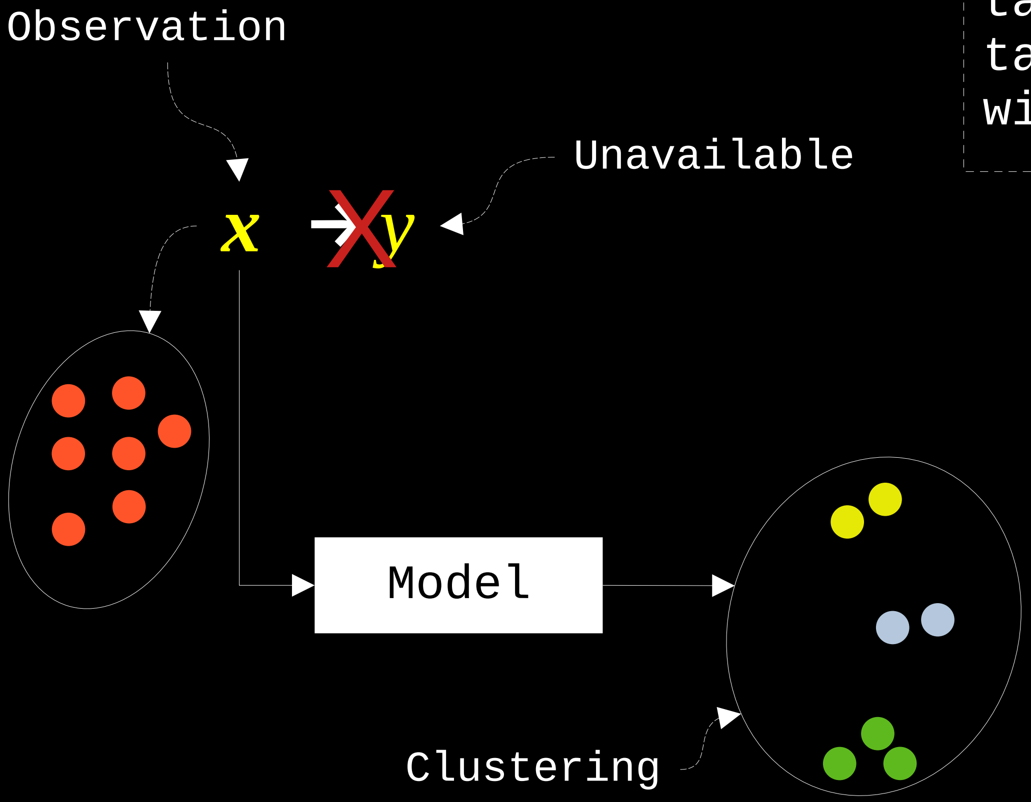


The error that the model seeks to minimize.

- Supervised
- Non-Supervised
- Self-Supervised
- ⋮

Machine learning overview

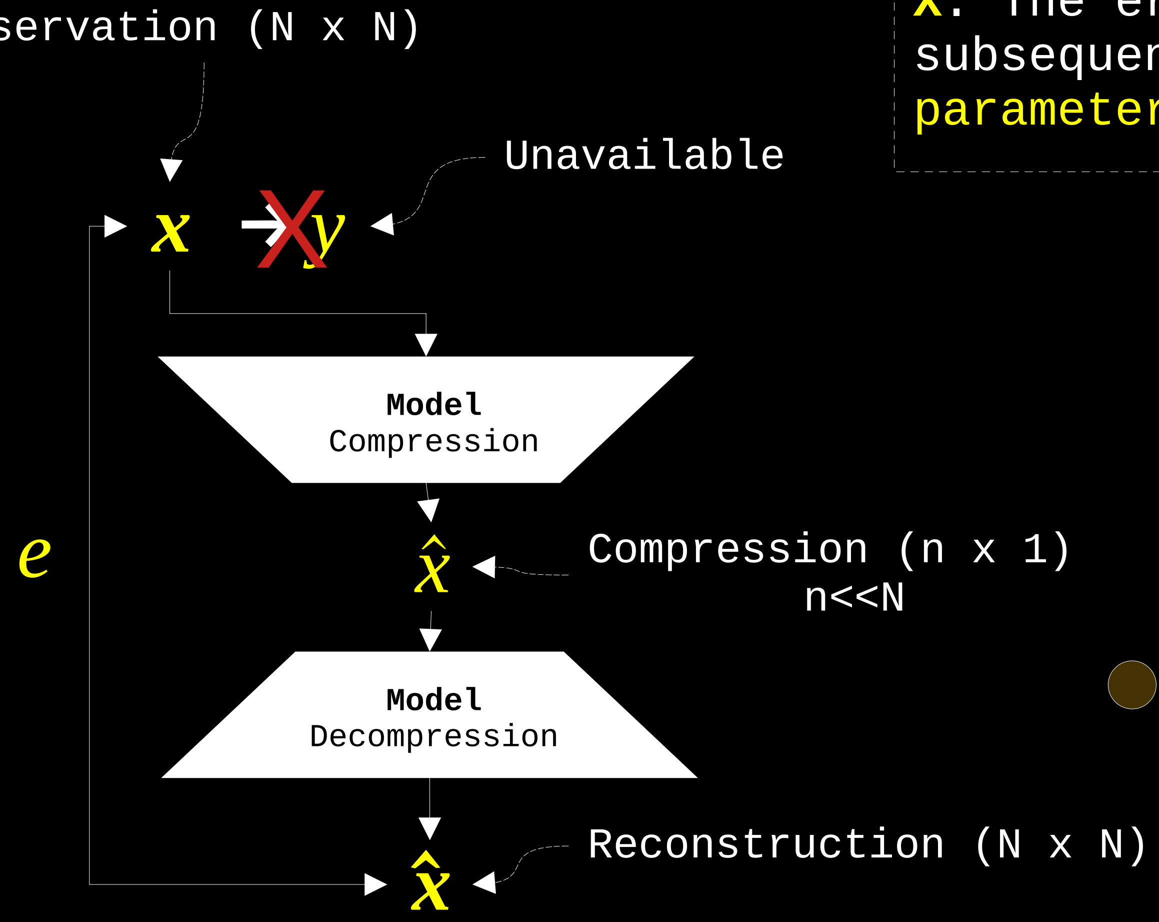
We only have **Xs** data. No matching **y** targets are available.The model will leverage data **similarities** and **co-occurrences** to perform the assigned task.For example, the clustering task, which consists of grouping data with similar patterns.



Supervised
Non-Supervised
Self-Supervised
⋮

Machine learning overview

We only have the training dataset **Xs**. The model takes an observation **X** as input, **model(X)=X̂**, and compares its prediction **X̂** to this same observation **X**. The error **|X-X̂|=e**, will subsequently be used to fit the model parameters.



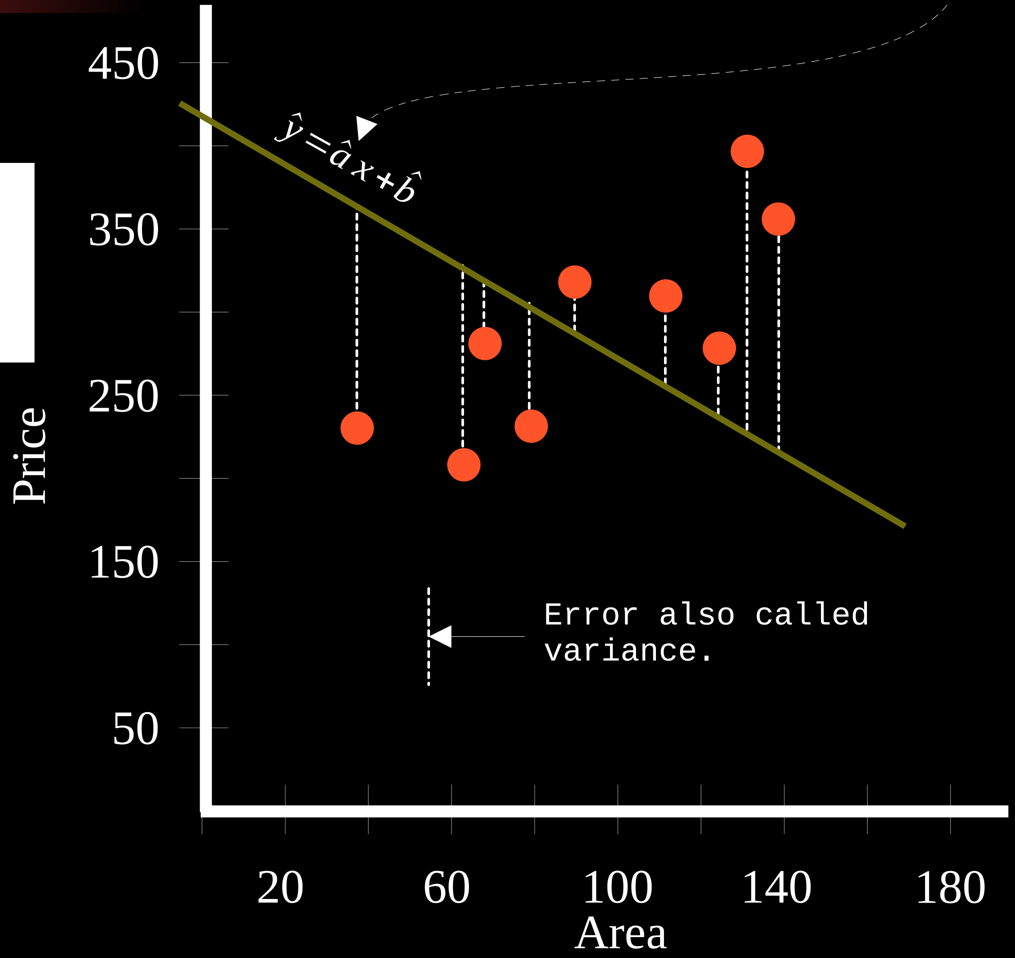
Supervised
Non-Supervised
Self-Supervised
⋮

Theory

Model→ Linear Regression= $\hat{y} = \hat{a}x + \hat{b}$

$\hat{a} < 0, \hat{b} \simeq 425$

Price chart (in XAF) of house locations based on their surface area (in m²).



The training consists of finding the values **â** and **b̂** that minimize the error (**E**) as much as possible.

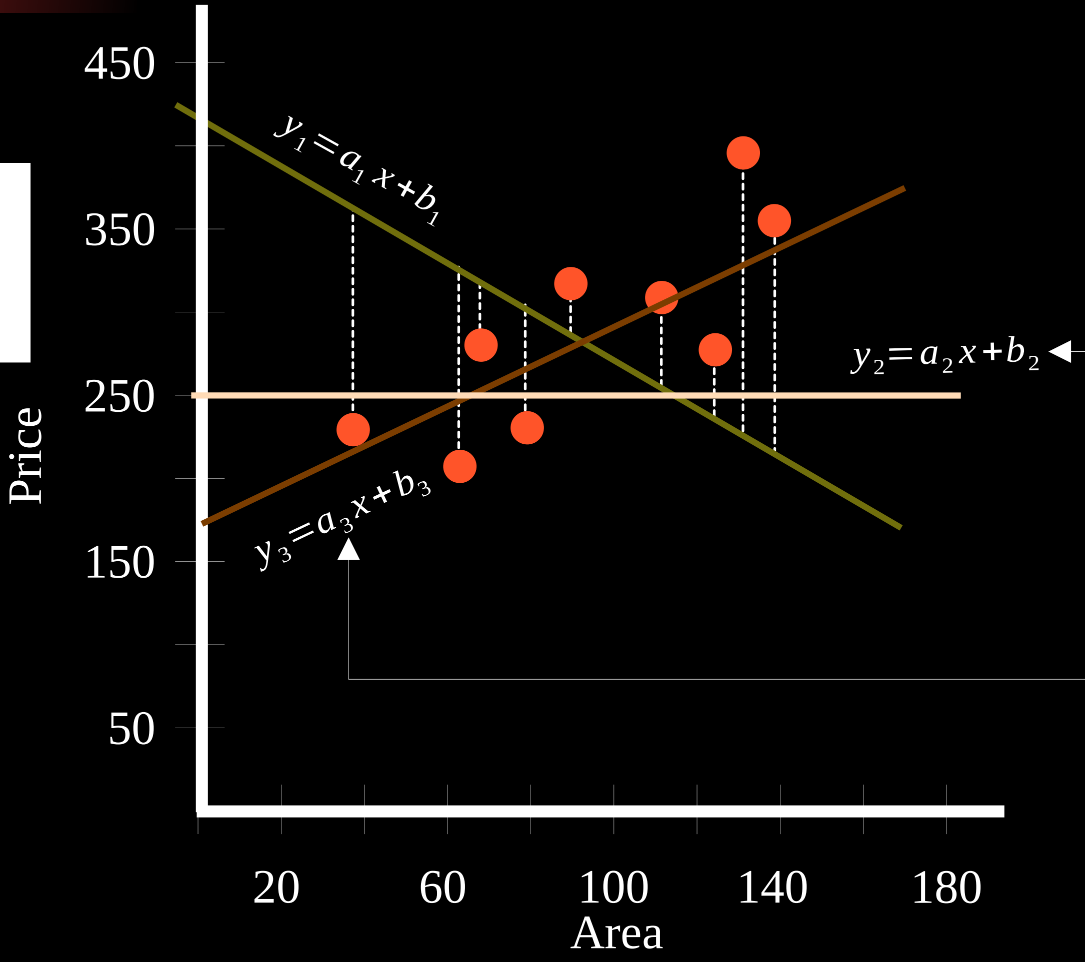
$$E = \quad + \quad + \quad + \cdots$$

$$E = (e_1 + e_2 + \ldots + e_9) / I$$

I=9, number of data points.

Theory

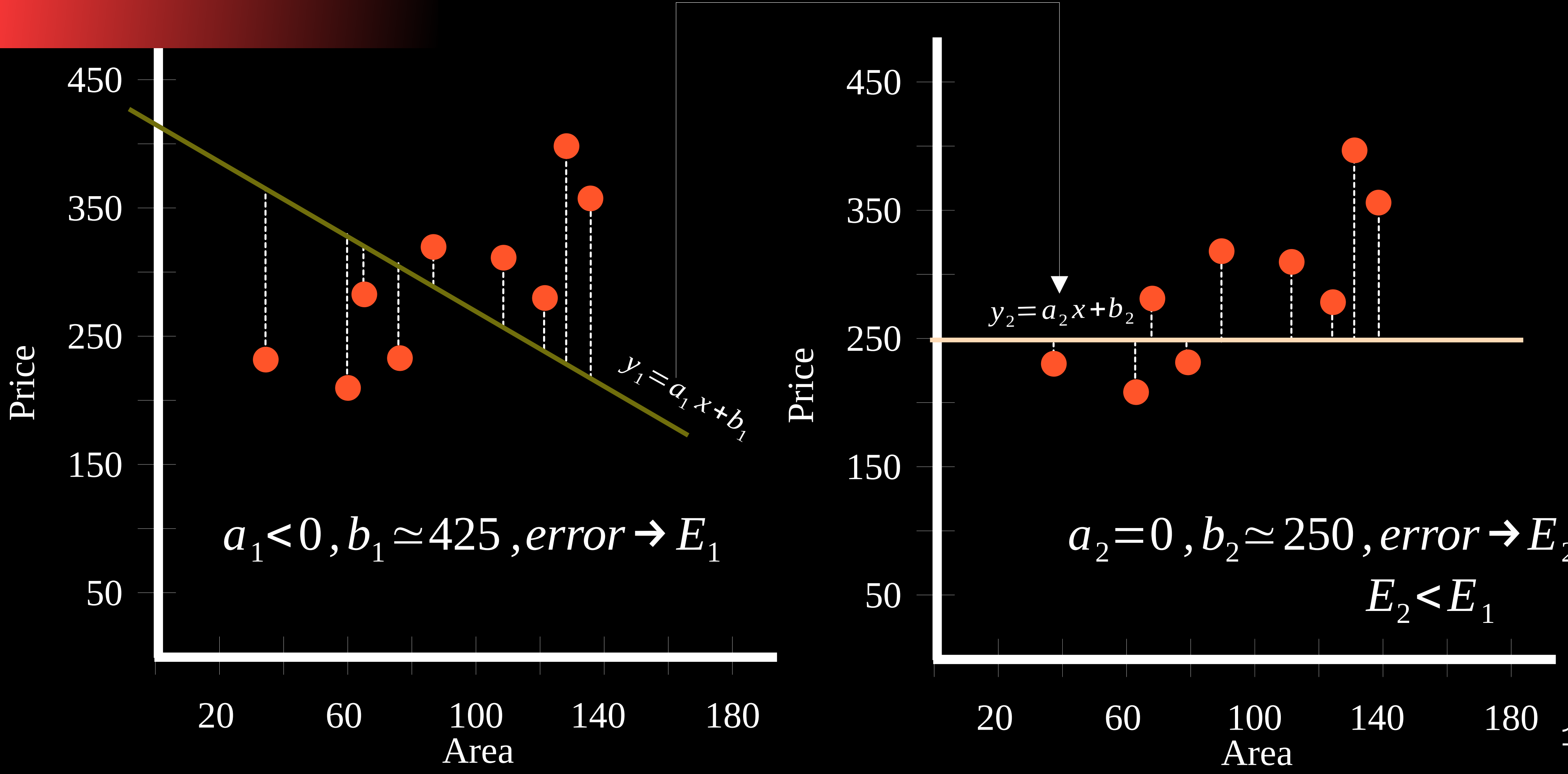
Price chart (in XAF) of house locations based on their surface area (in m²).



Candidate models (linear functions) to minimize error.

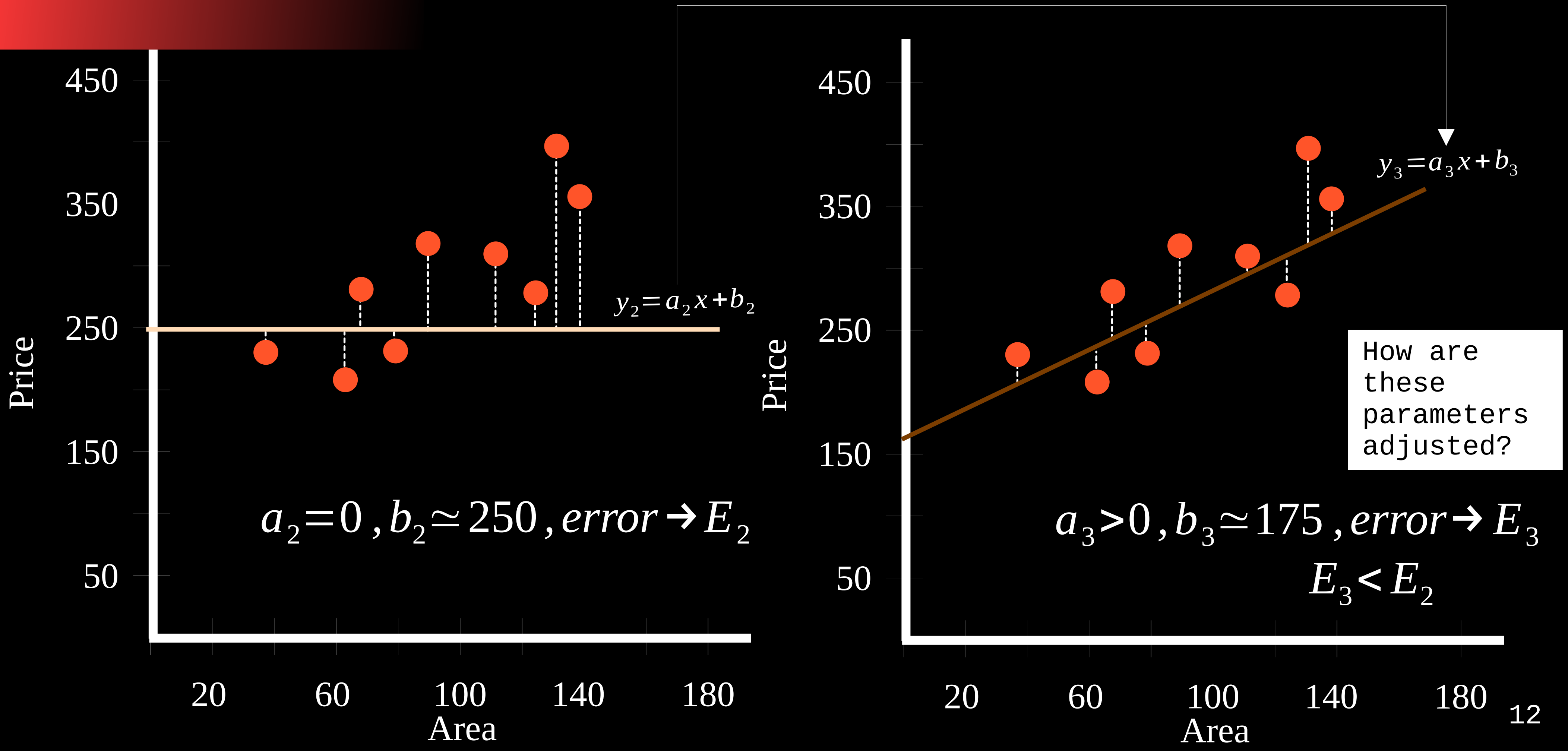
Theory

Model improvement after n iterations

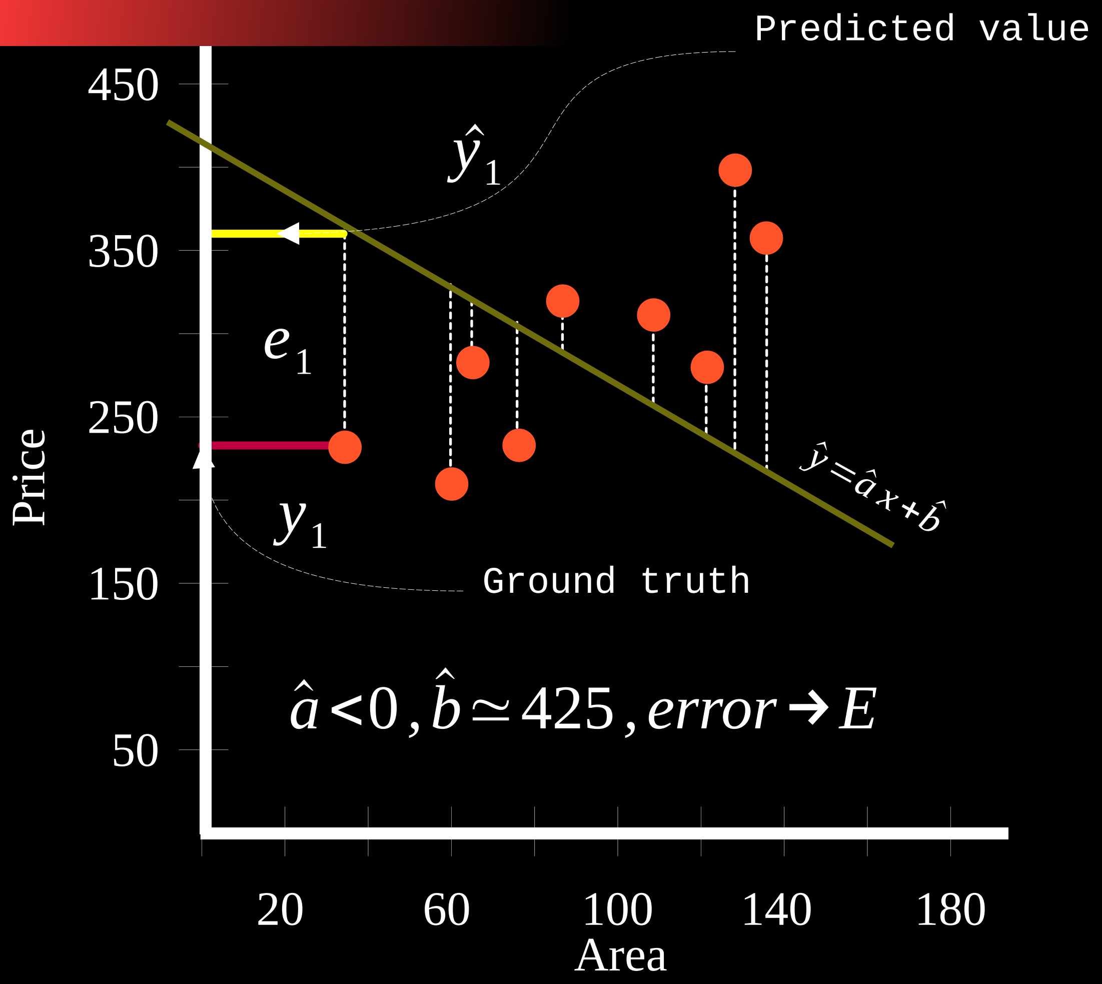


Theory

Model improvement after n+k (k>0) iterations



Theory



$$e_1=(y_1-\hat{y}_1)^2=[y_1-(\hat{a}x_1+\hat{b})]^2$$

$$E=(e_1+e_2+\ldots+e_9)/I=\frac{1}{I}\sum_{i=1}^{I} e_i$$

E is called **Mean Squared Error (MSE)** and I=9, is the number of data points.

$$E=\frac{1}{I}\sum_{i=1}^{I}[y_i-(\hat{a}x_i+\hat{b})]^2$$

How to minimize error E?
By finding the optimal values of â and ̂b that minimize this error.
Let’s differentiate E to find the optimal parameters.

Theory

$$E=\frac{1}{I}\sum_{i=1}^{I}[y_i-(\hat{a}x_i+\hat{b})]^2$$

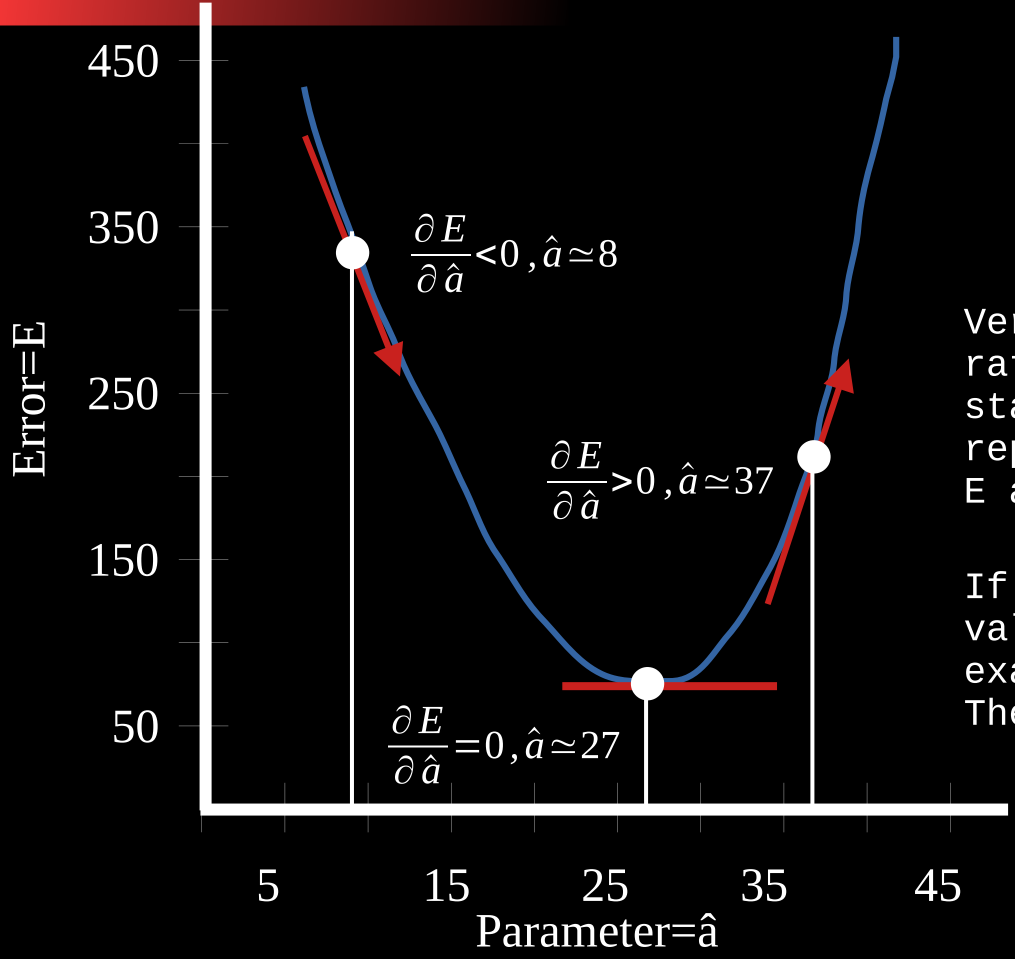
$$[(u-v)^2]'=2(u-v)(u'-v'),u=y_i \text{ and } v=(\hat{a}x_i+\hat{b})$$

$$\frac{\partial E}{\partial\hat{a}}=\frac{1}{I}\sum_{i=1}^{I}2[y_i-(\hat{a}x_i+\hat{b})](-x_i)=\frac{-2}{I}\sum_{i=1}^{I}[y_i-(\hat{a}x_i+\hat{b})](x_i)$$

$$\frac{\partial E}{\partial\hat{b}}=\frac{1}{I}\sum_{i=1}^{I}2[y_i-(\hat{a}x_i+\hat{b})](-1)=\frac{-2}{I}\sum_{i=1}^{I}[y_i-(\hat{a}x_i+\hat{b})]$$

Theory

Reminder on the derivatives of functions.



Suppose we want to calculate the derivative of E at the point â=x, formally we have:

$$E'(\hat{a}=x)=\frac{\partial E}{\partial \hat{a}}$$

Verbally, the above function means: what is the rate of change of E (decrease, increase, stagnation), when we are at point â. This rate is represented by a vector tangent to the curve of E at â.

If we want to minimize E, we need to find the value â that leads to this minimization In our example, this value is approximately equal to 27. The same process is carried out for b̂.

Let’s calculate the derivative of E as a function of â and b̂.

Linear algebra reminder.

Addition of vectors

$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_k \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_k \end{bmatrix} = \begin{bmatrix} a_1+b_1 \\ a_2+b_2 \\ \vdots \\ a_k+b_k \end{bmatrix} \text{ Exemple } \rightarrow \begin{bmatrix} 2.5 \\ 18.0 \\ \vdots \\ 7.5 \end{bmatrix} + \begin{bmatrix} 3.9 \\ 8.1 \\ \vdots \\ 4.2 \end{bmatrix} = \begin{bmatrix} 6.4 \\ 26.1 \\ \vdots \\ 11.7 \end{bmatrix}$$

Product of a scalar with a vector

$$X \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_k \end{bmatrix} = \begin{bmatrix} Xa_1 \\ Xa_2 \\ \vdots \\ Xa_k \end{bmatrix} \text{ Exemple } \rightarrow 2 \begin{bmatrix} 3.9 \\ 8.1 \\ \vdots \\ 4.2 \end{bmatrix} = \begin{bmatrix} 7.8 \\ 16.2 \\ \vdots \\ 8.4 \end{bmatrix}$$

Linear algebra reminder.

To multiply two matrices, the number of columns in the first matrix must be equal to the number of rows in the second.

Product of matrices

2 x 2   2 x 3   2 x 3

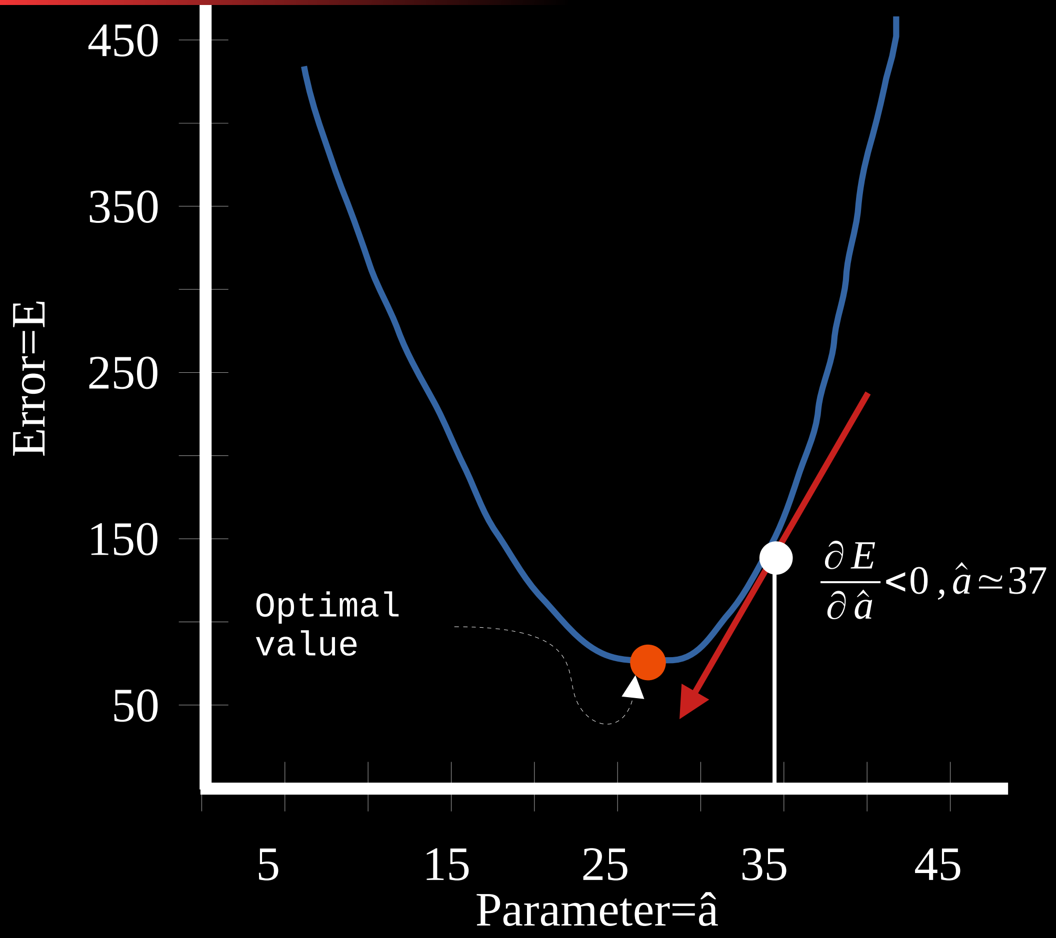
$$\begin{bmatrix} a_1^1 & a_2^1 \\ a_1^2 & a_2^2 \end{bmatrix} \begin{bmatrix} b_1^1 & b_2^1 & b_3^1 \\ b_1^2 & b_2^2 & b_3^2 \end{bmatrix} = \begin{bmatrix} a_1^1 b_1^1 + a_2^1 b_1^2 & a_1^1 b_2^1 + a_2^1 b_2^2 & a_1^1 b_3^1 + a_2^1 b_3^2 \\ a_1^1 b_1^1 + a_2^1 b_1^2 & a_1^1 b_2^1 + a_2^1 b_2^2 & a_1^1 b_3^1 + a_2^1 b_3^2 \end{bmatrix}$$

Exemple

$$\begin{bmatrix} 2{,}1 & 4{,}3 \\ 1{,}7 & 7{,}0 \end{bmatrix} \begin{bmatrix} 2{,}5 & 10{,}0 & 13{,}2 \\ 5{,}4 & 4{,}4 & 6{,}5 \end{bmatrix} = \begin{bmatrix} 2{,}1*2{,}5+4{,}3*5{,}4 & 2{,}1*10{,}0+4{,}3*4{,}4 & 2{,}1*13{,}2+4{,}3*6{,}5 \\ 1{,}7*2{,}5+7{,}0*5{,}4 & 1{,}7*10{,}0+7{,}0*4{,}4 & 1{,}7*13{,}2+7{,}0*6{,}5 \end{bmatrix}$$

$$\begin{bmatrix} 2{,}1 & 4{,}3 \\ 1{,}7 & 7{,}0 \end{bmatrix} \begin{bmatrix} 2{,}5 & 10{,}0 & 13{,}2 \\ 5{,}4 & 4{,}4 & 6{,}5 \end{bmatrix} = \begin{bmatrix} 28{,}47 & 39{,}92 & 55{,}67 \\ 42{,}05 & 47{,}8 & 67{,}94 \end{bmatrix}$$

Theory



The new value of â will be:

$$\hat{a} = \hat{a} - \epsilon \frac{\partial E}{\partial \hat{a}}$$

$\epsilon > 0$ Is the **learning rate**. It is often set at 0.001.

This adjustment process called **gradient descent** is repeated n times until the optimal value of â (as well as b̂)that minimizes E is found.

What happens if we have several features?

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