

PROBLEM 1

Problem 1 - Task 1

```
G = (V, E)
V = number of vertices, vertices numbered from 0..V-1
E = Array of edges in form of (start, end)

function getComponents(G)
  boolean checkedArray[0..V-1] <-- FALSE
  components <-- 0

  for i = 0 to numV - 1 do

    if checkedArray[i] == FALSE then
      components += 1
      getConnections(G, i, checkedArray)

  return components

function getConnections(G, i, checkedArray)
  checkedArray[i] <-- TRUE
  numE from E    // number of edges

  for i=0 to numE - 1 do
    if i in edgeStart or edgeEnd then
      NBR from edge *// Other side of edge, that isnt i*
      if checkedArray[NBR] == FALSE then
        getConnections(G, NBR, checkedArray)
```

Our worst case scenario is that we have 1 component, and that each vertice is connected to exactly one other vertice, a straight line of connected vertices. As we would then have to call getConnection on every vertice and check the entirety of the edge list for every iteration, giving us $O(V * \text{numE})$ time complexity

Problem 1 - Task 5

```

G = (V, E)
V = number of vertices, vertices numbered from 0..V-1
E = Array of edges in form of (start, end)

function getCritical(G)
    criticalServers[]

    components = getComponents(G, NULL)

    for i=0 to numV - 1 do
        curComponents = getComponents(G, i)

        if curComponents > components then
            append i to criticalServers

    return criticalServers

function getComponents(G)
    boolean checkedArray[0..V-1] <-- FALSE
    components <-- 0

    for i = 0 to numV - 1 do

        if checkedArray[i] == FALSE then
            components += 1
            getConnections(G, i, checkedArray)

    return components

function getConnections(G, i, checkedArray)
    checkedArray[i] <-- TRUE
    numE from E    // number of edges

    for i=0 to numE - 1 do
        if i in edgeStart or edgeEnd then
            NBR from edge // Other side of edge, that isnt i
            if checkedArray[NBR] == FALSE then
                getConnections(G, NBR, checkedArray)

```

This has a similar time complexity to Task 1, as it is just looping through and calling the algorithm created in Task 1 $\text{numVertices} + 1$ times, once for the initial check of how many components, and one for checking the number of components when each node is disconnected. So the worst case time complexity for this algorithm becomes $O((V + 1) * (V * \text{numE}))$

Problem 1 - Task 6

```

Needs Global access to these
G = (V,E), V = number of vertices, E = Adjacency Array
criticalNodes[0..V-1] <-- FALSE
pushOrder[0..V-1] <-- NULL
HRA[0..V-1] <-- INFINITY
parent[0..V-1] <-- NULL
visited[0..V-1] <-- FALSE
pushCount = 0

// This algorithm returns an Array criticalNodes which contains true or
// false values of if a node is critical or not

function getCritical()
    for i=0 to V-1 do
        if visited[i] == FALSE
            DFS(i)

    return criticalNodes

// DFS search from current Node
function DFS(curNode)
    pushCount++
    visited[curNode] == TRUE
    pushOrder[curNode] = HRA[curNode] = pushCount
    children = 0

    for i=0 to V-1 do
        if E[curNode][i] == 1
            if visited[i] == FALSE
                children = children + 1
                parent[i] = curNode
                DFS(i)

                HRA[curNode] = min(HRA[curNode], HRA[i])

                if parent[curNode] == NULL and children > 1 then
                    criticalNodes[curNode] = TRUE

                if parent[curNode] != NULL and HRA[i] >= pushOrder[curNode] then
                    criticalNodes[curNode] == TRUE

    else if parent[curNode] != i then
        HRA[curNode] = min(HRA[curNode], pushOrder[i])

```

This algorithm runs in $O(V + E)$ time as the algorithm only ever considers each edge and node once, and never has to double check a Node or edge due to using the back edge to check and store for ancestors in the graph. Therefore we get a worst and average time of $O(V + E)$.

PROBLEM 2

Problem 2 - A

Hash Table - Separate Chaining

The university wants a system that is fast for searching and to use Student ID's as keys. Using a large enough Hash Table with a sufficient Hash function, would give us average case $O(1)$, although worst case can be $O(n)$ given a poor hash function and a low amount of storage. Though in this case they can choose to upgrade storage and modify the Hash function in order to get less collisions. If student numbers were contiguous and non-finite then an Identity Hash would be a better option giving us average and worst case of $O(1)$.

Problem 2 - B

Sorted Array

The university wants a system that is fast for searching and has a lower memory cost, though they don't care too much about insertion and deletion speed. Due to this a sorted Array is the better option as it has $O(\log(n))$ search time and is more memory efficient when compared to a Hash Table that is too large for the dataset, as we don't end up with any empty index's. Although if we can assume that the student ID's are contiguous from 0 then an Identity Hash will be the better option.

Problem 2 - C

B-Tree

Due to the nature of allowing faster operation with a ram Cache, if we use a B-Tree, the tree will have less depth than a binary tree, and we can use the RAM Cache to store parallel record to the current search object to speed up search of these parallel records.

Problem 2 - D

Hash Table - Linear Probing

Based on the case of newly inputted records having a slower access time, despite the fact their storage is at 85%, I would assume they use a Hash Table with linear probing and a less than ideal Hash function. With this case newly inputted records have a higher chance of collision with records already in the Hash Table and hence being inserted further down the array, and thus when accessing the newer records they have to first get the Hash of the ID and then search down the array, hence making newer records take longer to search for.

Problem 3

Problem 3 - A

$\text{HASH}(A, j) = 1$

If the hash always returns 1, $\text{HASH}(A, j+1)$ is never less than $\text{HASH}(A, j)$, and hence no items ever get swapped and the original array remains unchanged

Problem 3 - B

$\text{HASH}(A, j) = -j$

If the hash returns the negative of the index, it ensures that a swap always occurs, hence running on worst case time complexity

Problem 3 - C

$\text{HASH}(A, j) = -A[j]$

If $\text{HASH}(A, j) = A[j]$ the pseudocode will run as a normal insertion sort, though by making the HASH return the $-A[j]$ we can swap which value is greater, and hence performing the sort in reverse.