Task 2

The main prize at some TV show is a car. At the beginning the car is placed behind one door randomly chosen out of three (with equal probabilities). Participant has to guess the door with a car. If the guess is correct, the participant wins the car, otherwise she wins nothing. After the participant picks the door and announces her choice, host opens one of the two remaining doors and shows that there are no car there. Then the participant can either switch her decision and pick the remaining closed door or keep the door she picked initially. After the final decision is made, the chosen door opens and the participant get her prize (if any). Assume that the host never opens a door that is picked by the participant initially and never opens a door with a car. If the host can chose between several doors, the choice is random (with equal probabilities). Let us enumerate doors in such a way that the door initially picked by the participant has number 1.

Consider events H_1: *the car is behind door number 1*, H_2: *the car is behind door number 2* and H_3: *the car is behind door number 3*. Consider also event A: *the host opened door number 2*.

1. What can you say about probability of H_3 before any door is opened?
2. Assume that the host opened door number 2. What would you say about probability of H_3 after you observe that?
3. Are events A and H_1 independent? A and H_2? A and H_3?
4. Use Bayes' rule to find $P(H_1 \mid A)$ and $P(H_3 \mid A)$ (find all necessary probabilities that are used in Bayes' rule first). Compare with your previous answers.
5. Should the participant change the initial decision to increase probability of winning?

1.First let's find probabilities of events H_1, H_2, H_3.
Before the games starts the car is placed behind one of them with equal probabilities, so we can say that $P(H_1) = P(H_2) = P(H_3) = 1/3$.
Probability of host opening any of the 3 doors is also equal 1/3 by default, so $P(A) = 1/3$.

2. Let us consider a situation in which host opened door number 2, and let's see what could lead to that.

We know that at the start participant of the show picked door 1. Initially, $P(H_1)$ is 1/3, so $P(H_2 \text{ } or \text{ } H_3) = 1 - P(H_1) = 2/3$. It means, that car is twice as likely to be behind door 2 or 3 as being behind door 1. After host opens door 2 this assumption doesn't change, car is still twice likely to be behind doors 2 or 3, but as we now know, it isn't behind door 2. This means, that probability of 2/3 is now being distributed not between doors 2 and 3, but it belongs exclusively to door 3