Task 3

$$A=\begin{pmatrix}0&0&1&1\\0&1&0&1\\1&0&0&0\\1&0&1&1\end{pmatrix}$$ — adjacency matrix of a directed graph with $n=4$ vertices.

a) First we need to transpose matrix A.

$$A^T=\begin{pmatrix}0&0&1&1\\0&1&0&0\\1&0&0&1\\1&1&0&1\end{pmatrix}.$$

Next we divide each column of A^T by the sum of values in it.

$$P=\begin{pmatrix}0&0&1&1/3\\0&1/2&0&0\\1/2&0&0&1/3\\1/2&1/2&0&1/3\end{pmatrix}$$ — Markov transition matrix

b) Regularization with $\alpha=0.25$

$$P_\alpha=(1-\alpha)\cdot P+\alpha\cdot Q=\frac{3}{4}\cdot\begin{pmatrix}0&0&1&1/3\\0&1/2&0&0\\1/2&0&0&1/3\\1/2&1/2&0&1/3\end{pmatrix}+\frac{1}{4}\begin{pmatrix}1/4&1/4&1/4&1/4\\1/4&1/4&1/4&1/4\\1/4&1/4&1/4&1/4\\1/4&1/4&1/4&1/4\end{pmatrix}=$$

$$=\begin{pmatrix}1/16&1/16&13/16&5/16\\1/16&7/16&1/16&1/16\\7/16&1/16&1/16&5/16\\7/16&7/16&1/16&5/16\end{pmatrix}$$

c) Finding g

Let g be Perron vector, such that $P_\alpha\cdot g=g$, which means it's the eigenvector of matrix P_α, that corresponds to eigenvalue of 1. To find it we need to solve SLAE $(P_\alpha-I)\cdot g=0$

$$P_\alpha-I=\begin{pmatrix}-15/16&1/16&13/16&5/16\\1/16&-9/16&1/16&1/16\\7/16&1/16&-15/16&5/16\\7/16&7/16&1/16&-11/16\end{pmatrix}.$$

After solving this SLAE we obtain vector $x=\begin{pmatrix}322\\96\\253\\289\end{pmatrix}$.

By dividing it by the sum of its' elements we get Perron vector g.

$$g=\begin{pmatrix}322/960\\96/960\\253/960\\289/960\end{pmatrix}\approx\begin{pmatrix}0.3354\\0.1\\0.2635\\0.301\end{pmatrix}$$

From vector g we can see that largest value 0.3354 corresponds to the first vertex of the graph, so it is the most „influential" vertex.