

Task 1

Consider two random events A and B defined for the same random experiment.

1. Is it possible that A and B are independent and mutually exclusive (disjoint) at the same time? Explain your answer.
2. Does the answer change if given that $P(A) > 0$ and $P(B) > 0$? Explain your answer.

1. If A and B are independent events, then by definition $P(A \cap B) = P(A) \cdot P(B)$. But if A and B are mutually exclusive, then $A \cap B = \emptyset$, hence $P(A \cap B) = 0$. This implies that A and B can be independent and mutually exclusive only if $P(A) \cdot P(B) = 0$ or in other words if $P(A) = 0$ or $P(B) = 0$ (or both).

2. On the other hand, if A and B are independent events and $P(A) > 0$, $P(B) > 0$, then $P(A \cap B) = P(A) \cdot P(B) \rightarrow P(A \cap B) > 0$. This implies, that $A \cap B$ isn't an empty set, thus events A and B can happen together and are not mutually exclusive.