Task 1

Let $f: R_2 \to R_2$ be a linear operator with matrix A; $p = f(a)$, $q = f(b)$
vectors $a = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$, $b = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$, $p = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$, $q = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$ are vectors in standard basis $\mathbf{e}$.

- First we can find matrix A_e such that $(p, q) = A_e \cdot (a, b)$.

$$\begin{pmatrix} 2 & -1 \\ 4 & -2 \end{pmatrix} = A_e \cdot \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix} \Rightarrow A_e = \begin{pmatrix} 2 & -1 \\ 4 & -2 \end{pmatrix} \cdot \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 2 & -1 \\ 4 & -2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 6 & 8 \end{pmatrix}.$$

- Let $\mathbf{e}'$ be new basis $\{a, q\}$. Then transition matrix $T_{e \to \mathbf{e}'} = \begin{pmatrix} 2 & -1 \\ -1 & -2 \end{pmatrix}$.

$$T_{e' \to e} = T_{e \to e'}^{-1} = \begin{pmatrix} 2 & -1 \\ -1 & -2 \end{pmatrix}^{-1} = -\frac{1}{5} \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & -0.4 \end{pmatrix}$$

- Finally we can find matrix A in basis e'.

$$A_{e'} = T_{e' \to e} \cdot A_e \cdot T_{e \to e'} = \begin{pmatrix} 0.4 & -0.2 \\ -0.2 & -0.4 \end{pmatrix} \cdot \begin{pmatrix} 3 & 4 \\ 6 & 8 \end{pmatrix} \cdot \begin{pmatrix} 2 & -1 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ -2 & 11 \end{pmatrix}$$