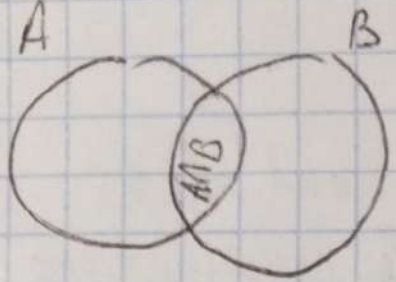
① $|A|=5$, $|B|=3$, $|A\cup B|=6$.

A configuration of sets like this is possible.

For example: $A=\{1,2,3,4,5\}$, $|A|=5$ | $\Rightarrow$ $A\cup B=\{1,2,3,4,5,6\}$

$B=\{4,5,6\}$, $|B|=3$ | $|A\cup B|=6$.

(2) $|A| = 5$; $|B| = 3$; $|A\cup B| = 6$; $|A\cap B| = 1$.



$$|A\cup B| = |A| + |B| - |A\cap B|$$

$$6 \neq 5+3-1=7 \Rightarrow$$

$\Rightarrow$ a configuration like this is impossible.

(3) $(A\cup B)\setminus C = (A\setminus C)\cup(B\setminus C)$

1) Suppose $x \in (A\cup B)\setminus C$.

By definition of set difference $x \in A\cup B$ and $x \notin C \Rightarrow$

$\Rightarrow (x\in A$ or $x \in B)$ and $x \notin C \Rightarrow$ $x\in A$, and $x\notin C$ or $x \in B$, and $x \notin C$ $\Rightarrow$

$\Rightarrow x \in A\setminus C$ or $x \in B\setminus C \Rightarrow x \in (A\setminus C)\cup(B\setminus C) \Rightarrow$

$\Rightarrow (A\cup B)\setminus C) \subset (A\setminus C)\cup(B\setminus C)$.

• Suppose $x \in (A\setminus C)\cup(B\setminus C)$.

By def. of $\cup$ $x \in A\setminus C$ or $x \in B\setminus C \Rightarrow$

$\Rightarrow$ $x \in A$ and $x \notin C$ or $x \in B$ and $x \notin C$ $\Rightarrow$ Either way $x \notin C$ and $x \in A$ or $x \in B \Rightarrow$

$\Rightarrow x \in (A\cup B)$ and $x \notin C \Rightarrow x \in (A\cup B)\setminus C \Rightarrow$

$\Rightarrow (A\setminus C)\cup(B\setminus C) \subset (A\cup B)\setminus C$

As both sets are subsets of each other, they are equivalent.

④ There are 6 mobile plans.

A client choses one favorite plan #1 from 6 avaliable and another favorite plan #2 from the 5 left, so there is $6 \cdot 5 = 30$ ways of chosing two favorite plans, as we can use rule of product to count number of such combinations.

Other than that, client also picks a plan he considers overpriced, which there are 6 options to chose from.

As client can pick any of 6 plans in combination with any pair of favorite plans (of which there are 30), we can use the rule of product again: $6 \cdot 30 = 180$ ways to select, so there are 180 outcomes.

(5) In the solution attempt each pair of houses is counted twice, as we count ordered pairs. But pairs like (house 1, house 2) and (house 2, house 1) are the same from the point of view on distance measering.

To correct the mistake we have to divide number of such pairs by 2, so the answer is $90:2=45$.

(6) There is only one way to write even numbers in ascending order: 0, 2, 4, 6, 8; same goes for positioning odd numbers in descending order: 9, 7, 5, 3, 1.

So all we need to do is to chose 5 positions out of 10 where we will put even (or odd) numbers. Rest of positions will be left for other numbers, for example:

$\underline{9}\ \underline{7}\ \_\ \_\ \underline{5}\ \underline{3}\ \underline{1}\ \_\ \_\ \_$

To count the number of ways to pick 5 positions out of 10 without repetitions we need to calculate $C_{10}^{5} = \frac{10!}{5!\,5!} =$

$$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 252.$$

There are 252 ways.

⑦ Yes, it is possible. Here is an example:
a fair dice is rolled.
Let event A be „number on dice is less than 4",
event B – „number on dice is greater than 4".
So $\Omega = \{1,2,3,4,5,6\}$, $|\Omega| = 6$

$A = \{1,2,3\}$, $|A| = 3$, $P(A) = \frac{|A|}{|\Omega|} = \frac{3}{6} = \frac{1}{2}$

$B = \{5,6\}$, $|B| = 2$, $P(B) = \frac{|B|}{|\Omega|} = \frac{2}{6} = \frac{1}{3}$

Events A and B can't occure simultaneously and $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$.

(8) There are 10 adults in the village.

(1) Council consists of three persons, so it can be represented with a triplet of adults.

Each adult can be represented with a number from 0 to 9. Then a Council is a triplet of numbers from 0 to 9, which can't repeat (as same person can't be elected twice).

So in this case the sample space Ω is $\{(x,y,z) \mid x,y,z \in \{0,1,\dots,9\}, x \neq y \neq z\}$

As there can be no repetitions while 3 persons are picked out of 10, the number of ways to do this is $|\Omega| = C_{10}^{3} = \frac{10!}{7!3!} =$

$= \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120.$

(2) A president is selected every 4 months, so there are 3 elections during a year.

The same person can be selected as a President repeatedly and it doesn't matter if he is also chosen into the Council. In other words, during a year 1 person out of 10 is selected to be a President, and this happens 3 times. Order of selection matters.

In this case sample space can be represented as a set of triplets, which consist of numbers from 0 to 9 that can be repeated.

$\Omega = \{0,1,2,3,4,5,6,7,8,9\}^3.$

$|\Omega| = 10^3 = 1000$.

⑨ Let's represent the result of rolling two dices as a pair ~~of~~ of numbers from 1 to 6. As the dices are indistinguishable, pairs of numbers like (1,2) and (2,1) are considered the same. At the same time we can see that pair (1,2) occures twice as often as pair (1,1) or (2,2) or any other pair with equal numbers. This happens as we can get (1,1) in only 1 way, but (1,2) in two ways: there can be 1 on one dice and 2 on the other, or vice versa.

From this observation we can conclude, that probability of a pair of different numbers is twice as big as the probability of a pair of equal numbers: $P(a,b) = 2 \cdot P(a,a)$, $a \neq b$.

The sample space is $\Omega = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,3), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (3,6), (4,4), (4,5), (4,6), (5,5), (5,6), (6,6)\}$, $|\Omega| = 21$.

There are 6 pairs of same numbers like (1,1), (2,2), ..., (6,6), which have same probabilities.
Let $P(a,a) = p$.
There are 15 pairs of different numbers like (1,2), (3,5), ..., which also have same probabilities, which are $2p$ (from the observation before).
Considering that sum of all probabilities should be equal to 1, we can calculate corresponding probabilities for all outcomes:
$6 \cdot P(a,a) + 15 \cdot P(a,b) = 6 \cdot p + 15 \cdot 2p = 36p = 1.$
$P(a,a) = p = 1/36;$
$P(a,b) = 2p = 1/18.$
- $P(1,1) = 1/36; \quad P(1,2) = \frac{1}{18}.$
- Sample space with non-equal probabilities of outcomes.

All of that could be avoided if only we could distinguish the indistinguishable dices...