1 week: Conditional probability and Independence

During this week we discuss conditional probability and independence of events. Sometimes we can use this definition to find probabilities. Sometimes we check that this definition fulfills to assure whether events are independent. We discuss important law of total probability, which allows us to find probability of some event when we know its conditional probabilities provided some hypotheses and probabilities of the hypotheses. We also discuss Bayes's rule which allows us to find probability of hypothesis provided that some event occurred. We demonstrate how Python can be used for calculating conditional probabilities and checking independence of events.

Оглавление:

Example:

You run a shop and you have some client. You don’t know anything about your client when he came to your shop. But when toy see that client adds some dog food to rhe cart, you assume that this client probably has a dog. It means that it’s good idea to recommend client some stuff to the dog owners. For example, toys.

We can formalize it using notion of conditional probability

- select random person from the set of out potential customers
- A random event — this person wants to buy dog food
- B — this person wants to buy dog toy
- Assume every 10 client buys dog toys → P(B) = 1/10
- But if we know that this client already bought dog food (A occured), probability that they also buy toy will bw much larger that 1/10
- Thus events A and B **are not independent**
- We can use knowledge of A to predict that B occurs

Conditional probability example

Example:

Let us assume that we are performing a study that is interested in how marriage is related to happiness. To perform this study, we conducted some survey. And know we have some data set which contains information about “is a particular participant married and is he or she happy?”

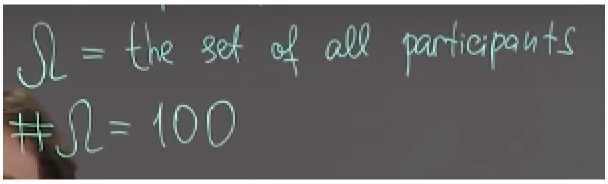
participant_id	married	happy
1	yes	no
2	yes	yes
3	no	yes
....	...	...

Contingency table:

happy	yes	no	**overall**
married			
yes	42	28	**70**
no	6	24	**30**
overall	**48**	**52**	100

Random experiment: pick random participant. We use equal probability for all participants.

Now the set of all outcomes is the same as the set of all participants



H - the chosen participant is happy

M - the chosen participant is happy married

P(H) = 48/100 = 0.48

P(M) = 70/100 = 0.7

P(H | M) = 42/70 = 0.6

P(H | M-) = 6/30

Definition of conditional probability

Definition of conditional prob:

A, B - events $\quad A \subset \Omega$ → all outcome have equal prob.

$B \subset \Omega$

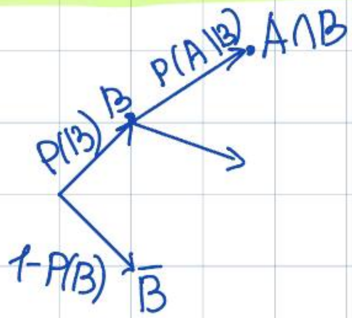
$$\boxed{P(A|B)} = \frac{\# A \cap B}{\# B} \quad \Big| : \# \Omega \Rightarrow$$

$$\Rightarrow \frac{\underbrace{\left(\frac{\# A \cap B}{\# \Omega}\right)}_{= P(A \cap B)}}{\underbrace{\left(\frac{\# B}{\Omega}\right)}_{= P(B)}} = \boxed{\frac{P(A \cap B)}{P(B)}}$$

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

Corollary:

$$\boxed{P(A \cap B) = P(A|B) \cdot P(B)}$$



(exist with only this condition)

Independent events. Example

Example of independent events:

„Is it true that a particular participant likes cheese or does he or she have a dog?"

likes cheese \ has dog	yes	NO	overall
yes	48	32	80
NO	12	8	20
overall	60	40	100

C – "chosen participant likes cheese"

D – "chosen participant has dog"

$$P(C|D) = \frac{48}{60} = 0.8$$

$$P(C) = 80/100 = 0.8$$

"In this case, we see that $P(C|D) = P(C)$. So the inf. that person has a dog doesn't give us any new information. It means that if we've interesting in predicting a cheese, presence or absence of dog does not give us any new clues. So we can safely ignore in our analysis.

C and D are independent events.

Independent events. Definition

Independent events. Definition

A, B - events $\quad P(A) > 0, \; P(B) > 0$

Definition: A and B are independent if:

1. $P(A|B) = P(A)$
   $\Updownarrow$
2. $P(B|A) = P(B)$
   $\Updownarrow$
3. $P(A \cap B) = P(A) \cdot P(B)$

достаточно одного факта для подтверждения независимости

Prove $1 \Rightarrow 3$: $P(A|B) = \boxed{\dfrac{P(A \cap B)}{P(B)} \overset{1)}{=} P(A)}$

$$\Downarrow$$

$3 \Rightarrow 1 \qquad P(A \cap B) = P(A) \cdot P(B)$

Experiment: toss dice and get number of points

$A =$ "number is even" $= \{2, 4, 6\}$

$B =$ "number ≥ 4" $= \{4, 5, 6\}$

$C =$ "number ≥ 5" $= \{5, 6\}$

$A \cap B = \{4, 6\}$

$P(A) = \dfrac{\#A}{\#\Omega} = \dfrac{3}{6} = \dfrac{1}{2}$; $\quad P(B) = \dfrac{\#B}{\#\Omega} = \dfrac{1}{2}$

$P(A \cap B) = \{4, 6\} = \dfrac{2}{6} = \dfrac{1}{3}$

Check A and B on independence:

$$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \neq P(A \cap B) = \frac{1}{3}$$

So they're dependent events.

$$P(B \mid A) = \begin{matrix} A\{2, 4, 6\} \\ B\{\underline{4}, 5, \underline{6}\} \end{matrix} = \frac{2}{3} \neq P(B)$$

"Noweledge that A occured changes the prob. of B"

$$P(C) = \frac{2}{6} = \frac{1}{3}$$

$$P(A \cap C) = \frac{1}{6}$$

$$P(A) \cdot P(C) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6} = P(A \cap C) = \frac{1}{6}$$

$$\Downarrow$$

A and C events are independent

Conditional probability practice

Question 1

A fair coin is tossed 2 times. Provided that at least 1 head occurred, what's the probability that no tails occurred at all? Enter the exact value below with two decimal places or as an ordinary irreducible fraction.

(for example, 0.12 or 13/28):

Solution:

All: {HH}, {TT}, {HT}, {TH}

at least 1 head occurred: {HH}, {HT}, {TH} no tails occurred at all: {HH}

Answer: 1/3

Question 2

Out of all 3-star hotels 25% cost more than $50 per night. Out of all hotels that cost more than $50 per night, 12.5% are 3-star hotels. If a random hotel is 3-star with probability p and its cost is more than $50 per night with probability q, what's the ratio p/q? Enter the exact value of p/q below with two decimal places or as an ordinary irreducible fraction.

(for example, 0.6, 0.33 or 2/9)

A = p – сл. отель имеет 3 звезды
B = q – стоимость > 50$ per night

$\frac{A}{B} = \frac{p}{q} = ?$

$P(q \mid p) = \frac{1}{4} = 0.25 = p(B/A)$

$P(p \mid q) = 12{,}5\% = 0.125 = p(A \mid B)$

$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(A \mid B) \cdot P(B)$

$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \Rightarrow P(A \cap B) = P(B/A) \cdot P(A)$

$P(A \mid B) \cdot P(B) = P(B/A) \cdot P(A)$

$P(B) = \frac{P(B/A) \cdot P(A)}{P(A \mid B)} = \frac{0.25 \cdot P(A)}{0.125} = 2P(A)$

(1) $P(\overset{q}{B}) = 2 \cdot P(\overset{p}{A})$ $\quad q = 2p$

(2) $P(\overset{p}{A}) = \frac{P(\overset{q}{B})}{2}$ $\quad p = \frac{q}{2}$

$p = \frac{q}{2} \quad \Big| : q \Rightarrow \frac{p}{q} = \frac{1}{2}$

Answer: $\frac{1}{2}$

Answer: 1/2

Coins, dices and conditional probability

Question 1

A fair coin is tossed 4 times. Provided that at least 2 heads occurred within the 4 tossings, what's the probability that at least one tail also occurred? Enter the exact value below with two decimal places or as an ordinary irreducible fraction.

(for example, 0.12 or 13/28):

$\Omega = 2^4 = 16$ $\{* * * *\}$

A= "При усл., что за 4 подбрасывания выпало хотя бы 2 H

{HHTT}, {HTTH}, {HTHT}, {TTHH},

{HHHT}, {HHTH}, {HTHH}, {THHH}

{HHHH}, {THHT}, {THTH}

B×

A = 11

B - хотя бы одна T = 10

$P(B/A) = \frac{10}{11}$

Answer: $\frac{10}{11}$

Answer: 10/11

Question 2

Two fair dices are rolled simultaneously. Find the conditional probability of event "both dice got equal values" provided that the sum of values on dice is less than six. Enter the exact value below with two decimal places or as an ordinary irreducible fraction.

(for example, 0.12 or 13/28):

Solution:

A - "both dice got equal values"

B - the sum of values on dice is less than six.

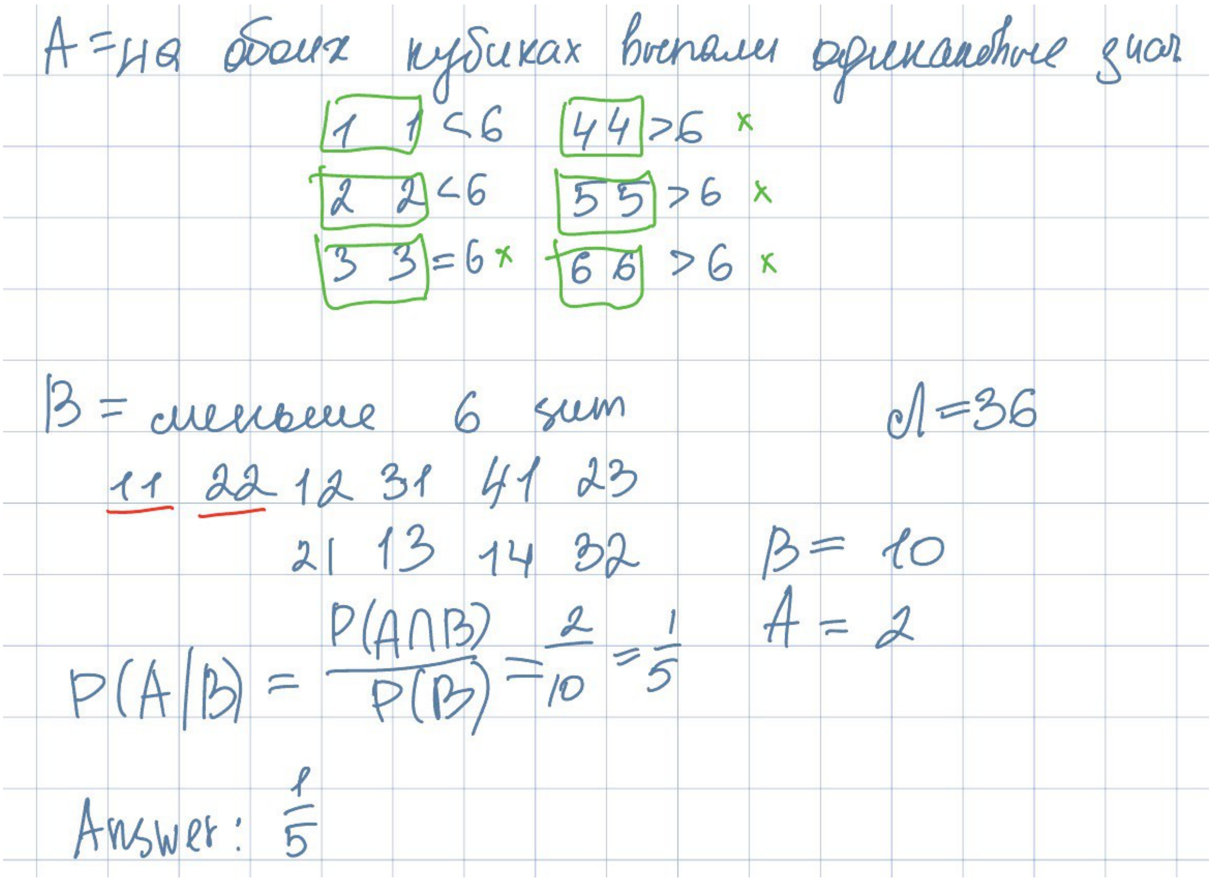
P(A|B)

B:

(1,1), (1,2), (1,3), (1,4), (2,1), **(2,2),** (2,3), (3,1), (3,2), (4,1)

B = 10

P(A|B) = 2/10 = 1/5



Answer: 1/5

Question 3

A coin is tossed until it gives either 10 heads or 10 tails. Player A bets on 10 heads and player B bets on 10 tails. The game is unexpectedly interrupted after

15 tossings with 8 heads and 7 tails observed. What would be the fair ratio to split the prize pool between player A and B? Consider it to be the ratio of winning probabilities of the players.

Монету подбрасывают до тех пор, пока не выпадет 10 орлов или 10 решек. Игрок А делает ставку на 10 орлов, а игрок Б ставит на 10 решек. Игра неожиданно прерывается после 15 бросков, при которых наблюдалось 8 орлов и 7 решек. Каково было бы справедливое соотношение распределения призового фонда между игроками А и Б? Считайте это соотношением вероятностей выигрыша игроков.

$A = 10\ H$

$B = 10\ T$

$n = 15$

$8H + 7T$

до выигрыша: 2, 3

15

$\underbrace{HHHHHHHH}_{8}\ \underbrace{TTTTTTT}_{7}$

* * * * *

<u>Победа для A (=H):</u>

5 * H : T

1) HHHHHHHH TTTTTTT HH 10 : 7

2) HHHHHHHH TTTTTTT HTH 11 : 8

3) HHHHHHHH TTTTTTT HTT 9 : 9 (дальше кто-то выиграет)

4) HHHHHHHH TTTTTTT HTTH 10 : 9

5) HHHHHHHH TTTTTTT THTH 10 : 9

6) HHHHHHHH TTTTTTT TTHH 10 : 9

1) $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ 4) $\frac{1}{16}$

2) $\frac{1}{8}$ 5) $\frac{1}{16}$

3) $\frac{1}{8}$ 6) $\frac{1}{16}$

шш $\Rightarrow +$

$$\frac{1}{4} + 2 \cdot \frac{1}{8} + 3 \cdot \frac{1}{16} = \frac{1}{4}^{\backslash 4} + \frac{2}{8}^{\backslash 2} + \frac{3}{16} = \frac{4}{16} + \frac{4}{16} + \frac{3}{16}$$

$= 11/16$

Победа для В: $\left(\overset{H:T}{\underbrace{8:7}_{15}}\right)$

1) HHHHHHHHTTTTTTT TTT 8 : 10

2) HHHHHHHHTTTTTTT HTTT 9 : 10

3) HHHHHHHHTTTTTTT THTT 9 : 10

4) HHHHHHHHTTTTTTT TTHT 9 : 10

1) $\frac{1}{8}$ 3) $\frac{1}{16}$

2) $\frac{1}{16}$ 4) $\frac{1}{16}$

$+ =$ шш

$$\frac{1}{8} + 3 \cdot \frac{1}{16} = \frac{1}{8}^{\backslash 2} + \frac{3}{16} = \frac{2}{16} + \frac{3}{16} = \frac{5}{16}$$

А теперь получим отношение вер-тей (А и В):

$$\frac{11}{16} : \frac{5}{16} = \frac{11}{16} \cdot \frac{16}{5} = \frac{11}{5}$$

Answer: $\frac{11}{5}$

Answer: 11/5

Question 4

In blackjack there's a 52-cards deck (4 suits of 13 cards each: 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, Ace). Digits are count as their face value, Aces count for 11 and pictures count for 10. Player wins if the total value of his cards is greater than the total value of croupier's cards, provided that the first value doesn't exceed 21. Croupier has two pictures. Player has an ace. What's the probability that player wins as the next card is dealt?

В блэкджеке колода состоит из 52 карт (4 масти по 13 карт в каждой: 2, 3, 4, 5, 6, 7, 8, 9, 10, валет, дама, король, туз). Цифры учитываются по номиналу, тузы засчитываются за 11, а картинки засчитываются за 10.

Игрок выигрывает, если общая стоимость его карт превышает общую стоимость карт крупье, при условии, что первое значение не превышает 21.

У крупье есть две картинки.

У игрока есть туз.

Какова вероятность того, что игрок выиграет при раздаче следующей карты?

Solution:

1. **Переписываем условие для понимания:**

   13 разных карт: 2, 3, 4,   5, 6, 7,   8, 9, 10,   валет, дама, король,   туз

   4 масти

   туз = 11

   валет, дама, король = 10

   крупье:

   10 + 10 = 20

   игрока:

   11

   Надо добавить только одну карту!

2. **Что нужно игроку для выигрыша:**

- Игроку нужно, чтобы его следующая карта добавила к его тузу (11) значение, которое:
- Дополнит его руку до 21 или меньше (то есть карта должна быть от 2 до 10).

- Не позволит ему перебрать 21 (то есть карта не должна быть тузом или картинкой)
- У крупье уже две картинки (20 очков), поэтому игроку нужно получить больше 20 очков, но меньше 22.

3. **У нас осталось кар для выбора:**

52 - 3 карты(2 карты крупье и 1 карта для игрока) = 49 карт осталось

Сколькими способами можно получить 10?

Значит игроку подходит для выигрыша: нужна *карта 10* (т.к. 11+10 = 21, что не превышает 21 и при это больше 20 баллов, что у картье) или *любая из карт с картинкой из оставшихся:*

- валет, дама, король = 10 подходит = 3 * 4(масти) = 12 - 2 (уже взял игрок изначально) = 10 вариантов
- 4 карты(разные масти) по 10 = 4 вариантов

Итого: 10 + 4 = 14 вариантов нам подходит

4. **Находим вероятность:**

14/49 = 2/7

Answer: 2/7

Independence and intersection

Question 1

If two events have positive probability and are independent, then they:

- Might intersect or not
- Don't intersect
- **Intersect**

*Recall the definition of independence through the probability of intersection

Definition: A and B are independent if:

1. $P(A|B) = P(A)$
2. $P(B|A) = P(B)$
3. $P(A \cap B) = P(A) \cdot P(B)$

достатньо одного факта для підтвердження незалежності

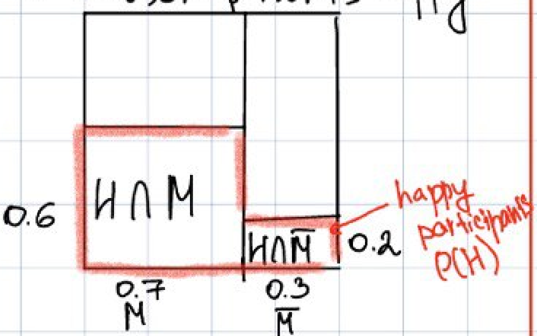
Mosaic Plot. Visualization of conditional probabilities and Independence

married \ happy	yes	NO	overall
yes	42	28	70
NO	6	24	30
overall	48	52	100

"Dependent events"

M - chosen person is married

H - chosen person is happy



the length of rectangle's sides is 1

$P(M) = \frac{70}{100} = 0.7$

$P(H/M) = \frac{42}{70} = 0.6$

$P(H|\bar{M}) = \frac{6}{30} = 0.2$

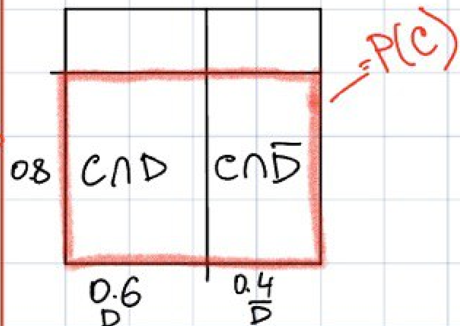
$H \cap M = \underbrace{P(H)}_{0.7} \cdot \underbrace{P(H|M)}_{0.6}$

likes cheese \ has dog	yes	NO	overall
yes	48	32	80
NO	12	8	20
overall	60	40	100

"Independent events"

C - likes cheese

D - has dog



$P(D) = 60/100 = 0.6$

$P(C|D) = \frac{48}{60} = 0.8$

$P(C|\bar{D}) = \frac{32}{40} = 0.8$

C is uniform distributed between D and $\bar{D}$

Independence:*

$P(C/D) = P(C/\bar{D}) = P(C)$

More independence and conditional probability practice

Question 1

In research on families with 1 kid, 20% of families showed that the kid and at least one of the parents were allergic. In 5% of cases at least one of the parents was allergic, but the kid wasn't, and in 10% of families kid was allergic despite that the parents weren't.

Are the events that the kid is allergic and that at least one of the parents is allergic independent or not?

Исследование семей с одним ребенком показало, что в 20% семей у ребенка и хотя бы у одного из родителей была аллергия.

В 5% случаев хотя бы у одного из родителей была аллергия, а у ребенка нет,

а в 10% семей у ребенка была аллергия, несмотря на то, что у родителей ее не было.

Являются ли события о том, что у ребенка аллергия и что хотя бы у одного из родителей аллергия, независимыми или нет?

Answer:

- **No, they are not**
- Yes, the events are independent

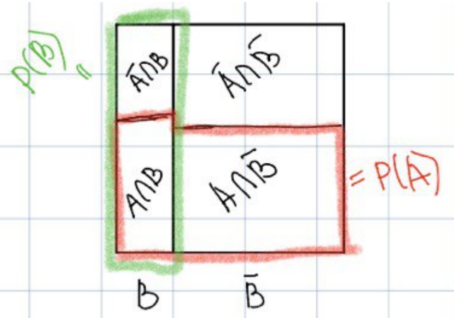
Question 2

If in this experiment a random kid turns out to be allergic, what's the probability that one of the parents also is?

Если в этом эксперименте у случайного ребенка окажется аллергия, какова вероятность того, что у одного из родителей тоже есть аллергия?

The correct answer is: 2/3

Solution for Question 1 and Question 2:



$(A|B)$ - ?

A - food allergies y parent
B - food allergies y children

$P(A\cap B) = 20\%$

$P(A\cap\bar{B}) = 5\%$

$P(\bar{A}\cap B) = 10\%$

$P(A\cap B) + P(A\cap\bar{B}) = P(A) = 20\% + 5\% = 25\%$

$P(A\cap B) + P(\bar{A}\cap B) = P(B) = 20\% + 10\% = 30\%$

$P(A\cap B) = P(A)\cdot P(A|B)$

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{20\%}{30\%} = \frac{0.2}{0.3} = \frac{2}{3}$$

$$P(A/\bar{B}) = \frac{P(A\cap\bar{B})}{P(\bar{B})} = \frac{5\%}{70\%} = \frac{5}{70}$$

$P(A/B) \neq P(A/\bar{B}) \neq P(A) \Rightarrow$ dependent events

Answer: Question 1: dependent events
Question 2: $P(A|B) = \frac{2}{3}$

Fair coin and independence

Question 1

A fair coin is tossed twice. Consider two events:

- There's at least one head.
- There's at least one tail.

Are these events independent?

- **No**
- Yes, they are independent

$\{TT\}, \{HH\}, \{TH\}, \{HT\} \qquad \Omega = 4$

$P(A) = \frac{3}{4} \Rightarrow \{HH\}, \{TH\}, \{HT\}$

$P(B) = \frac{3}{4} = \{TT\}, \{TH\}, \{HT\}$

$P(A \cap B) = \frac{2}{4} = \frac{1}{2}$

Independent condition:

$P(A \cap B) \neq P(A) \cdot P(B) = \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$

Answer: A and B are dependent

Question 2

A fair coin is tossed twice. Consider two events:

- There's exactly one head.
- There's exactly one tail.

Are these events independent?

- Yes, they are independent
- **No**

$\{TT\}, \{HH\}, \{TH\}, \{HT\}$

$\Omega = 4$

$A = \frac{2}{4} = \frac{1}{2} = \{\{TH\}, \{HT\}\}$

$B = \frac{2}{4} = \frac{1}{2} = \{\{TH\}, \{HT\}\}$

$P(A \cap B) = \frac{2}{4} = \frac{1}{2}$

$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \neq P(A \cap B)$

Answer: events are dependent

Question 3

A fair coin is tossed twice. Consider two events:

- The first tossing yields head
- The second tossing yields tail

Are these events independent?

- No
- **Yes, they are independent**

$\{HH\}, \{TT\}, \{HT\}, \{TH\}$ $\quad \Omega = 4$

$A = \{\{HH\}, \{HT\}\} = \frac{2}{4} = \frac{1}{2}$

$B = \{\{TT\}, \{HT\}\} = \frac{2}{4} = \frac{1}{2}$

$P(A \cap B) = \frac{1}{4}$

$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = P(A \cap B)$

Answer: A and B events are independent

Using independence to find probabilities. Examples

Example 1:

Let us consider a Professor who is going to the lecture. To go there, we have to drive through the city, and it's possible to stuck in traffic if there will be traffic jam. To avoid traffic jams, professor decides to depart earlier, and so he has to set up some alarm check. Unfortunately, an alarm clock can fail. We want find the probability with which Professor have to cancel their class.

A - alarm clock failed

T - Traffic jam occurred

C - class canceled

$C = A \cap T$ (C случается когда только одновременно происходят два события)

$P(A) = 0.1$ $\quad$ $P(T) = 0.3$ $\quad$ $P(C) =$

$\boxed{\text{A and T are independent (use sense)}}$

$\downarrow$

$P(C) = P(A \cap T) = P(A) \cdot P(T) = 0.1 * 0.3 = 0.03$

Example 2:

Assume that we're a company that have to make some decision. For ex: buy or not buy some startup. To make this decision, a company can ask expert: "Is it good idea to buy start up?" Unfortunately, every expert can be wrong. How can the company can decrease the probability that expert gives wrong advice! For ex: a company can ask several experts.

> **E1** - 1st expert is wrong
>
> **E2** - 2d expert is wrong
>
> **intersection of E1 and E2** - both experts are wrong

1. So let's assume that two experts make their decisions independently to each other:

- E1 and E2 are independent events → P(**intersection of E1 and E2**) = P(**E1**) * P(**E2**) < P(**E1**)

So if both experts give the same advice, the probability that they both wrong is less than P (E1). So, in a sense, committee of two experts is better than just one expert in terms of the error of mistake.

2. However, it's also possible that the decisions of these experts are not independent to each other. For ex: it's possible, in a sense, extreme case of dependence when 2d expert just copy the solution of 1t expert. It this case, they're both wrong or both right at the same time.

- E1 = E2 → P(**intersection of E1 and E2**) = P(**intersection of E1 and E1**) = P(**E1**)

So, in this case, using of the 2d expert gives us no new information and the probability of mistake is the same.
Conclusion: more independent events are better in this case.

We use the same idea in ML, if we have several predictive models that make independent predictions, we can unite these models and create a new model which called ensemble. If our models behave more or less independently to each other, the performance of ensemble will be better than performance of one model

Pairwise and mutual independence

However, when we have 3 or more events, things becomes a little bit more difficult. And we have different notions of independence in this case.

We can interpret this result as kind of dependence between C and both events (A and B). In this example, we have so called **pairwise independence**- all 3 events're pairwisly independent (A independent of C, B independent of C, A independent of B) . But if we consider all 3 events as a system, we see some dependencies between them.
In this case, these dependence can be understood as the fact that if A and B occurred then C occurred with probability 1. We don’t have
mutual independent(взаимная независимость) between 3 events.
A, B, C are
pairwise in dependent, but not mutually independent.

Definition:

A_1, A_2, A_3 – mutually independent if:

1. $P(A_i \cap A_j) = P(A_i) \cdot P(A_j)$ $i, j = 1..3$, $i \neq j$
2. $P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2) \cdot P(A_3)$

If we have more events, then we have to test this kind of equality (2) for any subset of a series of events.

Mutual independence conditions

Question 1

Can the condition of pairwise independence be omitted in the definition of three mutually independent events A1,A2,A3?

- Yes, omitting the condition of pairwise independence will not change anything.
- **No, events may be independent as a triplet, i.e., $P(A1 \cap A2 \cap A3) = P(A1) \times P(A2) \times P(A3)$, without being pairwise independent.**

Bernoulli Scheme

It's a probabilistic model that represents a series of independent trials. Each trial can result either in success or in failure. We will model this sequence by sequence of coin tossing.

However, now this is not a fair coin, but some unbalanced coin for which probability of H is different from probability of T. To find probability here we have to use independence condition

Bernoulli scheme: sequence of coin tossings

1) n = 2

H1 - 1st tossing gives H

H2 - 2d tossing gives H

In each the same coin in the tossings → P(H1) = P(H2)= p

{HH, TT, HT, TH}

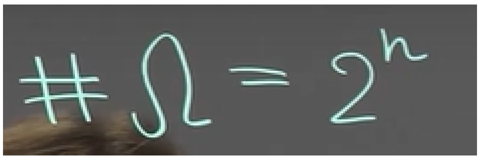
However, in this case, our coin is not necessarily fair, we cannot just say that probabilities of all these outcomes are equal and equal to 1/4. Because it's true, in a case, when probability of H is the same as a probability of T. However, we can use independence assumption to find probabilities of these outcomes.

H1 and H2 are independent because 2d tossing doesn't depend on 1st tossing and 1st tossing desn't depend on 2d tossing. Our coin doesn't have any memory

Probability space:

- P(HH) = P(H1∩H2) = P(H1) * P(H2) = p * p = p^2
- P(TT) = P(~H1∩~H2) = P(~H1) * P(~H2) = (1 - p) * (1 - p) = (1 - p)^2
- P(HT) = p(1-p)
- P(TH) = p(1-p)

2) Arbitrary n



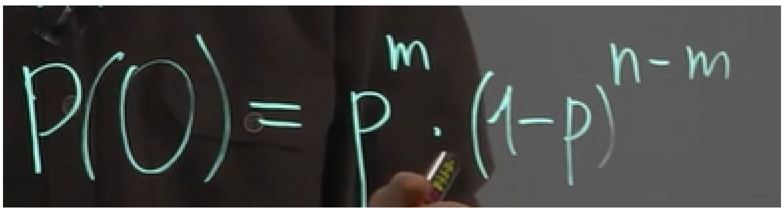
Example: n = 5

P(HHTHT) = p*p*(1-p)*p*(1-p) = p^3 *(1-p)^2

Mutually independent

Consider outcome O with m heads and n-m tails

P(O) = p



Note: this probability doesn't depend on the order with which H and T occur

Law of total probability

In real life, we have to make a lot of decisions. For example, we have several schools and we have to decide which school to visit. Or we have to choose which book to read at some particular moment of time. And these decisions can affect probabilities of various events, for ex: of getting a particular job. We can us ea probability theory to model to some extend this kind of relations between events like choosing of school and getting a job.

Let us consider one prominent example of this kind of reasoning which is called law of total probability.

Example:

We have a university with a 3 year educational program and we're interested in some ability of the students of this program. For ex: knowledge of calculus

Random experiment: pick random student

H1 - student on 1st year of education

H2 - student on 2d year of education

H3 - student on 3d year of education

C - student knows calculus

P(H1) = 0.5

P(H2) = 0.3

P(H3) = 0.2

A student can;t belongs to the different years at the same time →

$$H_i \cap H_j = \varnothing \quad i \neq j$$

$$H_1 \cup H_2 \cup H_3 = \Omega$$

3 events are **full probability space**. It means that these events is a **kind of alternative**. We are choosing between 1 of 3 and we have to choose at least one which means that we **have choose exactly one.**

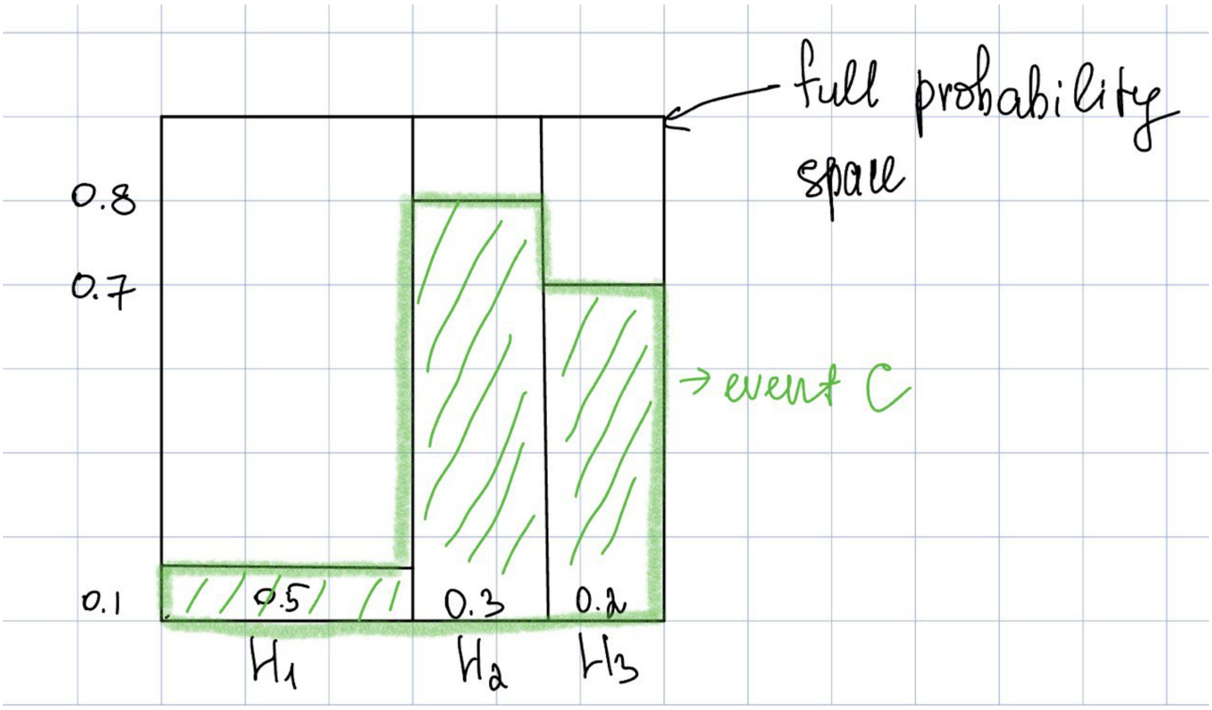
Now assume that we know conditional probabilities:

P(C | H1) = 0.1

P(C | H2) = 0.8

P(C | H3) = 0.7

Now, we need to know info that simply randomly chosen person (student) knows calculus. So we aree interested in probability of event C.



P(C) = 0.5*0.1 + 0.3*0.8 + 0.2*0.7 = 0.43

Law of total probability:

P(C) = P((C∩H1) U (C∩H2) U (C∩H1)) = P(C∩H1) + P(C∩H2) + P(C∩H3) = P(H1) * P(C | H1) + P(H2) * P(C | H2) + P(H3) * P(C | H3)=

$$\sum_{i=1}^{3} P(H_i)\cdot P(C|H_i)$$

These events are called hypothesis. **They generate some kind of partition of all probability space into some non overlapping areas**. In other words, we have alternative, we have to choose exactly one of these hypothesis.

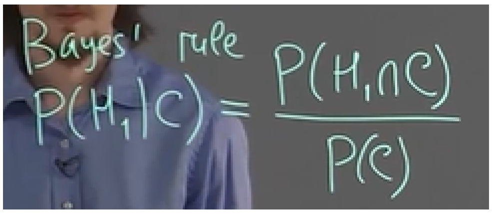
Assume that we know that event C occured. We know that randomly chosen person knows calculus. What is the probability that this person is from 1st year?

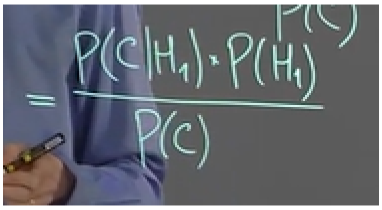
Bayes's rule

It helps as inverse this conditional probabilities

P(H1) - **prior probability**

P(H1 | C) - **posterior probability**





= (0.1*0.5) / 0.43 = 0.12

Дополнительные сведения по likelihood, posterior, prior:

6. **Bayes's Theorem**

$$P(E|F) = \frac{P(F|E) \times P(E)}{P(F)}$$

- $P(E|F)$: Posterior probability (what we want to find)
- $P(F|E)$: Likelihood of clicking on electronics email given they are interested in electronics (likelihood)
- $P(E)$: Prior probability of being interested in electronics
- $P(F)$: Total probability of clicking on electronics email

Law of total probability practice

Given that P(B)≠0, which is the correct inequality sign between the following expressions:

P(A|B) and 1−P(A⁻)/P(B)? Note that A⁻ represents the complement of event A

- ≥

- ≤
- =

Solution:

$P(B) \neq 0$

независимые (complement)

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \rightarrow \frac{P(A) \cdot \cancel{P(B)}}{\cancel{P(B)}}$$

$$\frac{1 - P(\overline{A})}{P(B)} = \frac{P(A)}{P(B)} \rightarrow \frac{P(A)}{P(B)}$$

$$P(A) \geqslant \frac{P(A)}{P(B)}$$

Bernoulli scheme skill test

Question 1

How many times (at least) a fair coin should be tossed to ensure that one or more heads are drawn with probability greater than 0.99?

Сколько раз (по крайней мере) следует подбросить честную монету, чтобы гарантировать, что один или несколько орлов выпадут с вероятностью больше 0,99?

- 8

- 9
- **7**
- 10

Solution:

1 - ни один из орлов не выпадет = выпадет один орёл или больше одного

1 - (1/2)^n > 0.99

- (1/2)^n > -0.01 |*(-1)

(1/2)^n < 0.01

(1/2)^6 = 0.015

(1/2)^7 = 0.0078 = 0.008

n = 7

Answer: 7

Question 2

Given a sequence of 4 independent experiments, in which the probability of achieving at least one success is 0.5904, and assuming the probability of success is equal in each experiment, what is the value of the probability of success in a single experiment?

Enter the exact value below with **one** decimal place or as an ordinary irreducible fraction.

Каково значение вероятности успеха в отдельном эксперименте, если принять последовательность из четырех независимых экспериментов, в которых вероятность достижения хотя бы одного успеха равна 0,5904, и предположить, что вероятность успеха в каждом эксперименте одинакова? Введите ниже точное значение с одним десятичным знаком или в виде обычной несократимой дроби.

$H_1 \quad H_2 \quad H_3 \quad H_4$ $\qquad p$ - успех

$q = (1-p)$ - не успех

1 - нет ни одного успеха = 0.5904

$$1-(1-p)^4 = 0.5904$$
$$-(1-p)^4 = -0.4096$$
$$(1-p)^4 = 0.4096$$
$$1-p = 0.8$$
$$-p = -0.2$$
$$p = 0.2 = \frac{2}{10} = \frac{1}{5}$$

Answer: 1/5

Balls in boxes

Question 1

Player picks a random ball from one of 3 boxes. It's known that for one box the probability of picking out white ball is 1/4, for another box it's 0 and for the last box it's 1/2. What's the probablity of player's picking a white ball if the choice of box is random.

Enter the exact value below with two decimal places or as an ordinary irreducible fraction.

Игрок выбирает случайный шар из одной из трех коробок. Известно, что для одного ящика вероятность вытащить белый шар равна 1/4, для другого — 0 и для последнего — 1/2. Какова вероятность того, что игрок выберет белый шар, если выбор коробки случайный. Введите ниже точное значение с двумя знаками после запятой или в виде обычной несократимой дроби.

$$\boxed{1K} = P(\text{Б} \mid 1K) = 1/4$$

$$\boxed{2K} = P(\text{Б} \mid 2K) = 0$$

$$\boxed{3K} = P(\text{Б} \mid 3K) = \frac{1}{2}$$

$$P(1K) = P(2K) = P(3K) = \frac{1}{3}$$

$$P(\text{Б}) = P(1K) \cdot P(\text{Б} \mid 1K) + P(2K) \cdot P(\text{Б} \mid 2K) + P(3K) \cdot P(\text{Б} \mid 3K)$$

$$= \frac{1}{4} \cdot \frac{1}{3} + 0 + \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{12} + \frac{1}{6} = \frac{3}{12} = \frac{1}{4}$$

Answer: 1/4

Question 2

There are 5 white balls and 4 black balls in a box. Three random balls are taken out. What's the probability that out of the 3 balls the number of black balls is odd provided that the last taken ball was black?

Enter the exact value below with two decimal places or as an ordinary irreducible fraction.

(for example, 0.12 or 13/28):

В коробке 5 белых и 4 черных шара. Вынимаются три случайных шара. Какова вероятность того, что среди трех шаров количество черных шаров будет нечетным, если последний взятый шар был черным? Введите ниже точное значение с двумя знаками после запятой или в виде обычной несократимой дроби. (например, 0,12 или 13/28):

Hint:

The straightforward approach to this problem is to enumerate all balls, then consider an ordered triples of them as outcomes. You can find the number of outcomes that satisfy conditions mentioned in the problem using combinatoric rules, then use the definition of conditional probability to obtain the answer.

Самый простой подход к этой задаче — пересчитать все шары, а затем рассматривать их упорядоченные тройки как результаты. Вы можете найти количество исходов, удовлетворяющих условиям, упомянутым в задаче, используя комбинаторные правила, а затем использовать определение условной вероятности для получения ответа.

Your sets do not consider the probability of them occurring which can be different for different sets.

Say the event of third ball being black is B and event of odd number of black balls is A.

Outcomes where third ball is black - $\{2B, B\}, \{2W, B\}, \{1B1W, B\}$

Outcomes where we have odd number of black balls and the last ball is black - $\{2B, B\}, \{2W, B\}$

So, $P(B) = \frac{4 \cdot 3 \cdot 2}{9 \cdot 8 \cdot 7} + \frac{5 \cdot 4 \cdot 4}{9 \cdot 8 \cdot 7} + 2 \cdot \frac{5 \cdot 4 \cdot 3}{9 \cdot 8 \cdot 7} = \frac{4}{9}$

$P(A \cap B) = \frac{4 \cdot 3 \cdot 2}{9 \cdot 8 \cdot 7} + \frac{5 \cdot 4 \cdot 4}{9 \cdot 8 \cdot 7} = \frac{13}{63}$

So, $P(A|B) = \frac{13}{28}$

Answer: 13/28

Call center total probability

Question 1

Support call center accepts calls and resolves clients' issues. There are 5 operators. Nina accepts half of all calls and successfully resolves 80% of issues, Tina and Dina accept 1/6 of calls each and resolve 75% and 90% of problems respectively. Mina and Lina accept 1/12 of calls each and resolve 50% and 25% of problems respectively. What's the probability that someone who calls to the call center will get their issue resolved? We suppose that the operators don't consult each other.

Enter the exact value below with two decimal places or as an ordinary irreducible fraction.

(for example, 0.12 or 13/28):

Колл-центр поддержки принимает звонки и решает вопросы клиентов. Есть 5 операторов. Нина принимает половину всех звонков и успешно решает 80% вопросов, Тина и Дина принимают по 1/6 звонков каждая и решают 75% и 90% проблем соответственно. Мина и Лина принимают по 1/12 звонков каждая и решают 50% и 25% проблем соответственно. Какова вероятность того, что тот, кто позвонит в колл-центр, решит свою проблему? Мы предполагаем, что операторы не консультируются друг с другом.

<u>Solution:</u>

1. Probability of a call being handled by each operator:

- Nina: 1/2
- Tina: 1/6
- Dina: 1/6
- Mina: 1/12
- Lina: 1/12

2. Probability of issue resolution by each operator:

- Nina: 80% = 0.8
- Tina: 75% = 0.75
- Dina: 90% = 0.9
- Mina: 50% = 0.5
- Lina: 25% = 0.25

Overall Probability = (1/2 *0.8) + (1/6* 0.75) + (1/6 *0.9) + (1/12* 0.5) + (1/12 * 0.25) = 0.4 + 0.125 + 0.15 + 0.041666666666666667 + 0.020833333333333333 = 0.7375 = **73.75%**.

Answer: 0.74

Bayes's rule practice

Question 1

Preliminary tests of students showed that 30% of them are very well prepared to the exam, 50% are prepared quite well and 20% are prepared somehow. For the first group of students (very well prepared) probability of getting the highest grade is 90%, for the second group of students (prepared quite well) it's 70% and for the last one (prepared somehow) — only 30%. A random student got the highest grade.

What's the probability that he was very well prepared?

Enter the exact value below with two decimal places or as an ordinary irreducible fraction.

(for example, 0.12 or 13/28):

Предварительное тестирование студентов показало, что 30% из них очень хорошо подготовлены к экзамену, 50% - достаточно хорошо и 20% - как-то подготовлены.

Для первой группы студентов (очень хорошо подготовленных) вероятность получить высшую оценку составляет 90%,

для второй группы студентов (достаточно хорошо подготовленных) — 70%

и для последней (как-то подготовленных) — всего 30%. Случайный студент получил высшую оценку. Какова вероятность того, что он был очень хорошо подготовлен?

Hint:

Use Bayes's rule carefully by answering the following questions. What are hypothesis here? What are their prior probabilities? What is the conditional probability of event you are interested in under condition of hypothesis? What is the total probability of this event?

Внимательно используйте правило Байеса, ответив на следующие вопросы. Какие здесь гипотезы? Какова их априорная вероятность? Какова условная вероятность интересующего вас события при условии выдвижения гипотезы? Какова общая вероятность этого события?

Solution:

H1 - студенты очень хорошо подготовлены к экзамену

H2 - студенты достаточно хорошо подготовлены к экзамену

H3 - студенты как-то подготовлены подготовлены к экзамену

P(H1) = 30%

P(H2) = 50%

P(H3) = 20%

M - student got the highest grade

P(M | H1) = 90%

P(M | H2) = 70%

P(M | H3) = 30%

Какова их априорная вероятность? P(M)

Какова условная вероятность интересующего вас события при условии выдвижения гипотезы? P(H1|M)

P(M) = P(M | H1) * P(H1) + P(M | H2) * P(H2) + P(M | H3) * P(H3) = 0.3*0.9 + 0.5*0.7 + 0.2*0.3 = 0.27 + 0.35 + 0.06 = 0.68

P(H1|M) = P(M | H1) * P(H1) / P(M) = 0.27/0.68 = 27/68

Answer: 27/68

Question 2

Under the same conditions, what's the probability that the student was quite well prepared?

Enter the exact value below with two decimal places or as an ordinary irreducible fraction.

Какова вероятность того, что при тех же условиях студент окажется достаточно хорошо подготовленным?

Solution:

P(H2|M) = P(M | H2) * P(H2) / P(M) = 0.35/0.68 = 35/68

Answer: 35/68

Question 3

Under the same conditions, what's the probability that the student was prepared just somehow?

Какова вероятность того, что при тех же условиях студент был как-то подготовлен?

Solution:

P(H3|M) = P(M | H3) * P(H3) / P(M) = 0.06/0.68 = 6/68 = 3/34

Answer: 3/34

Bayes's taxi companies (Не решена)

Question 1

There are 2 taxi companies. Company A has 40% of German cars and company B has 10% of German cars. All other cars are Korean. Taxi aggregator assigns orders to A or B with probabilities 1/3 and 2/3 respectively. If German car arrives, what's the probability that it's from company B?

Есть 2 компании такси. Компания А имеет 40% немецких автомобилей, а компания Б — 10% немецких автомобилей. Все остальные машины корейские. Агрегатор такси назначает заказы А или Б с вероятностью 1/3 и 2/3 соответственно. Если прибудет немецкая машина, какова вероятность того, что она от компании Б?

Solution:

A - taxi is from company A
B - taxi is from company B
G - taxi is German

$P(B|G) - ?$

$P(A) - 1/3$

$P(B) = 2/3$

$P(G|A) = 0.4$

$P(G|B) = 0.1$

$$P(B|G) = \frac{P(G|B) \cdot P(B)}{P(G)}$$

$$P(B|G) = \frac{P(B \cap G)}{P(G)}$$

$$P(G) = P(A) \cdot P(G|A) + \overbrace{P(B) \cdot P(G|B)}^{P(G \cap B)} = \frac{1}{3} \cdot \frac{4}{10} + \frac{1}{10} \cdot \frac{2}{3} =$$

$$= \frac{4}{30} + \frac{2}{30} = \frac{6}{30} = \frac{2}{10} = \frac{1}{5}$$

$$P(B|G) = \frac{2/30}{\frac{1}{5}} = \frac{2}{\cancel{30}_{6}} \cdot \frac{\cancel{5}}{1} = \frac{2}{6} = \frac{1}{3}$$

Answer: $\frac{1}{3}$

The correct answer is: 1/3

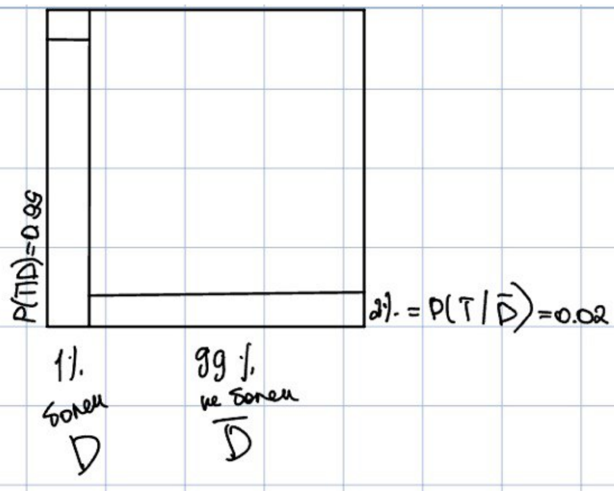
Rare disease paradox

Question 1

There's a rare disease that occurs in 1% of the population. There's a test that yields positive result on 99% of ill people, and false positive on 2% of healthy people. The test shows positive result on a patient. What's the probability that the patient does not have the disease? Enter the exact value below with two decimal places (use a dot as a separator) or as a regular irreducible fraction (for example, 0.12 or 13/28):

Редкое заболевание, которое встречается у 1% населения. Есть тест, который дает положительный результат у 99% больных и ложноположительный у 2% здоровых. **Тест показывает положительный результат у пациента. Какова вероятность того, что у пациента нет заболевания?** Введите ниже точное значение с двумя знаками после запятой (используйте точку в качестве разделителя) или в виде обычной несократимой дроби (например, 0,12 или 13/28):

Solution:



D - болен

T - тест прав

$P(D)=0.01$

$P(T|D)=0.99$

$P(T|\bar{D})=0.02$

$P(\bar{D}|T)$ - ?

$$P(T|D)=\frac{P(T\cap D)}{P(D)} \Rightarrow P(T\cap D)=P(T|D)\cdot P(D)$$

$$P(T\cap D)=0.99\cdot 0.01=0.0099$$

$$P(T\cap \bar{D})=0.99\cdot 0.02=0.0198$$

$$P(T\cap \bar{D})+P(T\cap D)=P(T)=0.0099+0.0198=$$
$$=0.0297$$

$$P(\bar{D}|T)=\frac{P(\bar{D}\cap T)}{P(T)}=\frac{0.0198}{0.0297}=\frac{198}{297}=\frac{66}{99}=\frac{22}{33}=\frac{2}{3}$$

Answer: $\frac{2}{3}$

Answer: 2/3

Python for conditional probabilities

```
from itertools import product #cartesian product

list(product([1, 2], ['a', 'b', 'c']))
```

```
#[(1, 'a'), (1, 'b'), (1, 'c'), (2, 'a'), (2, 'b'), (2, 'c')]

n = 3 #number of coin tossings
omega = set(product(['H', 'T'], repeat= n)) #sample space
'''
{('H', 'H', 'H'),
 ('H', 'H', 'T'),
 ('H', 'T', 'H'),
 ('H', 'T', 'T'),
 ('T', 'H', 'H'),
 ('T', 'H', 'T'),
 ('T', 'T', 'H'),
 ('T', 'T', 'T')}
'''

len(omega) #8

A = {om for om in omega if om[0]== 'T'} #1sr tossing is tail
#{('T', 'H', 'H'), ('T', 'H', 'T'), ('T', 'T', 'H'), ('T', 'T
B = {om for om in omega if om.count('T') == 2}
#{('H', 'T', 'T'), ('T', 'H', 'T'), ('T', 'T', 'H')}

def prob(X):
    return len(X) / len(omega)

def cond_prob(X, Y):
    return len(X & Y) / len(Y)



prob(A) #0.5
prob(B) #0.375

cond_prob(A, B) #0.6666666666666666
cond_prob(B, A) #0.5
prob(A & B) #0.25
prob(A) * prob(B) #0.1875 --> A and B are not independent
```

```
def are_indep(X, Y):
    return prob(X & Y) == prob(X) * prob(Y)

are_indep(A, B) #False

C = {om for om in omega if om[1]== 'H'}
are_indep(A, C) #True
```

Choosing the right strategy based on probabilities (Не решила потом вернуться)

Player got 10 blue and 10 red chips, and he was offered a game where he should distribute those chips into 2 similar boxes. (All chips should be placed in one of the boxes.) Then, another person randomly choose a box and then takes out one random chip from this box (unless the box is empty). The player wins if a red chip is drawn and lose otherwise. What's the maximum probability of the player's winning outcome assuming that he distributes chips the right way?

Note: This is a tough problem. It requires a general formula for winning probability and some analysis, brute force or insight. Don't worry if you cannot solve it on the first try. You may also find Python useful here.

Enter the exact value below with two decimal places or as an ordinary irreducible fraction.

Игрок получил 10 синих и 10 красных фишек, и ему была предложена игра, в которой он должен распределить эти фишки по 2 одинаковым коробкам. (Все фишки должны быть помещены в одну из коробок.) Затем другой человек случайным образом выбирает коробку и затем достает из этой коробки одну случайную фишку (если коробка не пуста). Игрок выигрывает, если вытягивается красная фишка, и проигрывает в противном случае. Какова максимальная вероятность выигрыша игрока при условии, что он правильно распределит фишки? Примечание. Это сложная проблема. Для этого требуется общая формула вероятности выигрыша и некоторый анализ, грубая сила или понимание.

Не волнуйтесь, если вам не удастся решить задачу с первой попытки. Здесь вы также можете найти Python полезным. Введите ниже точное значение с двумя знаками после запятой или в виде обычной несократимой дроби.

Hint:

You can try to derive the general formula for winning probability using conditional probabilities and conduct its analysis. Another way is to play with numbers a bit and get some intuition. For example, does it make sense to leave one box empty? Does it make sense to put all chips of one color to one box? How can you guarantee that the probability of winning is at least 1/2?

Можно попытаться вывести общую формулу вероятности выигрыша с помощью условных вероятностей и провести ее анализ. Другой способ — немного поиграть с числами и пробудить интуицию. Например, имеет ли смысл оставлять одну коробку пустой? Имеет ли смысл складывать все фишки одного цвета в одну коробку? Как можно гарантировать, что вероятность выигрыша будет не менее 1/2?

The correct answer is: 14/19

Week one skill test

Question 1

Two competing clinics A and B treat one disease. After the treatment round in clinic A, 80% of patients recover. In clinic B, 70% of patients recover after one treatment round. Patients are directed for treatment by an insurance company to either clinic A or B, wherein clinic A is chosen twice as often as clinic B.

What's the probability that a client of the insurance company, who needs treatment will get to clinic A and recover after the treatment round?

Enter the exact value below with two decimal places or as a regular irreducible fraction (for example, 0.12 or 13/28)

Две конкурирующие клиники А и Б лечат одно заболевание. После курса лечения в клинике А выздоравливают 80% пациентов. В клинике Б 70% пациентов выздоравливают после одного курса лечения. Пациенты направляются на лечение страховой компанией либо в клинику А, либо в клинику Б, причем клинику А выбирают в два раза чаще, чем клинику Б. Какова вероятность того, что клиент страховой компании, нуждающийся в лечении, попадет в клинику А и выздоровеет после курса лечения?

Solution:

P(A): Probability of being directed to clinic A = 2/3 (twice as often as clinic B)

P(B): Probability of being directed to clinic B = 1/3

P(R|A) : Probability of recovery given treatment in clinic A = 0.8

P(R|B): Probability of recovery given treatment in clinic B = 0.7

$P(A \cap R) = P(R|A) * P(A)$

$P(A \cap R) = (0.8) * (2/3) = 16/30 = 8/15$

The correct answer is: 8/15

Question 2

Under the same conditions, What's the probability that a client of the insurance company, who needs treatment will get to clinic A and **not** recover after the treatment round?
Какова при тех же условиях вероятность того, что клиент страховой компании, нуждающийся в лечении,
попадет в клинику А и не выздоровеет после курса лечения?

Solution:

$P(B \cap \sim R)$ - ?

Можно отходить от того, что клиники не зависимые.

$P(R|A) + P(R^{-}|A) = 1$

$P(R^{-}|A) = 1 - P(R|A) = 1 - 0.8 = 0.2$

$P(A \cap R^{-}) = P(R^{-}|A) * P(A) = 0.2\ (2/3) = 4/30 = 2/15$

The correct answer is: 2/15

Question 3

Under the same conditions, what's the probability that a client of the insurance company, who needs treatment will get to clinic B and recover after the treatment round?
Какова вероятность того, что при тех же условиях клиент страховой компании, нуждающийся в лечении,
попадет в клинику Б и выздоровеет после курса лечения?

Solution:

$P(B \cap R)$ - ?

$P(B \cap R^{-}) = P(R^{-}|B) * P(B) = 0.7\ (1/3) = 7/30$

The correct answer is: 7/30

Question 4

Under the same conditions, What's the probability that a client of the insurance company, who needs treatment will get to clinic B and **not** recover after the treatment round?
Какова при тех же условиях вероятность того, что клиент страховой компании, нуждающийся в лечении,
попадет в клинику Б и не выздоровеет после курса лечения?

Solution:

$P(B \cap \sim R)$ - ?

$P(R|A) + P(R^{-}|A) = 1$

$P(R^{-}|A) = 1 - P(R|A) = 1 - 0.7 = 0.3$

$P(A \cap R^{-}) = P(R^{-}|A) * P(A) = 0.3\ (1/3) = 3/10 * 1/3 = 1/10$

The correct answer is: 1/10

Question 5

Under the same conditions, what's the probability that a client of the insurance company, who needs treatment will get recovered after one treatment round?

Какова вероятность того, что при тех же условиях клиент страховой компании, нуждающийся в лечении, выздоровеет после одного курса лечения?

Solution:

$P(R)$ - ?

$P(R) = (0.8\ 2/3) + (0.7\ 1/3) = 16/30 + 7/30 = 23/30$

The correct answer is: 23/30

Question 6

Under the same conditions, what's the probability that a client of the insurance company, who recovered after one treatment round, was treated in clinic A?

Какова вероятность того, что при тех же условиях клиент страховой компании, выздоровевший после одного курса лечения, проходил лечение в клинике А?

Solution:

P(A|R) - ?

use Bayes's rule:

P(A|R) = (0.8 *2/3) / (23/30) = 16/30 * 30/23 = 16/23

Python skill test

Question 1

A fair coin is tossed 20 times. Consider events A: *number of heads is even* and B:

less than 4 tails occurred. Use Python to find P(A ∣ B). Enter answer with 5 digits after decimal point.

Solution:

```
from itertools import product #cartesian product

n = 20 #number of coin tossings
omega = set(product(['H', 'T'], repeat= n)) #sample space

A = {om for om in omega if om.count('H') % 2 == 0}
B = {om for om in omega if om.count('T') < 4}

def cond_prob(X, Y):
    return len(X & Y) / len(Y)

cond_prob(A, B) #0.14137675795706883
```

The correct answer is: 0.14137

Question 2

A fair dice is rolled 5 times. Consider events$A$: *sum of all points is divisible by 3* and B: *product of all points is larger than 500*. For example, if the result of rolling is (5, 3, 2, 6, 3), then event A did not occur (because 5+3+2+6+3=19, not divisible by 3) and event B occurred (because 5×3×2×6×3=540>500). Use Python to find P(B ∣ A). Enter answer with 3 digits after decimal point.

Solution:

```
from itertools import product #cartesian product

n = 5 #number of coin tossings
omega = set(product([1, 2, 3, 4, 5, 6], repeat= n)) #sample s

A = {om for om in omega if sum(om) % 3 == 0}

import math
B = {om for om in omega if math.prod(om) > 500}

def are_indep(X, Y):
    return prob(X & Y) == prob(X) * prob(Y)

cond_prob(B, A) #0.30709876543209874
```

The correct answer is: 0.307