Task 2

Let X be uniform random variable on a segment $[0, 2]$. Consider random variable $Y = X^2$. Find CDF and PDF of Y. Is PDF a bounded function?

$X \sim Uniform(0; 2)$

$PDF_X(x) = \begin{cases} 0.5, & x \in [0; 2] \\ 0, & otherwise \end{cases}$ and $CDF_X(x) = \begin{cases} 0, & x < 0 \\ 0.5x, & x \in [0; 2] \\ 1, & x > 2 \end{cases}$

Let $Y = X^2$, $Y \in [0; 4]$.

Find CDF and PDF of Y.

$$P(Y \le y) = P(X^2 \le y) = P\left(-\sqrt{y} \le X \le \sqrt{y}\right) =$$
$$= P\left(0 \le X \le \sqrt{y}\right) = P\left(X \le \sqrt{y}\right) = 0.5\sqrt{y}$$

As $X \in [0; 2]$, it can't take on negative values

Now we can write down CDF_Y

$$CDF_Y(y) = \begin{cases} 0, & y < 0 \\ 0.5\sqrt{y}, & y \in [0; 4] \\ 1, & y > 4 \end{cases}$$

We can find PDF_Y as a derivative of CDF_Y on the segment $[0; 4]$.

$$PDF_Y(y) = \begin{cases} \dfrac{1}{4\sqrt{y}}, & y \in (0; 4] \\ 0, & otherwise \end{cases}$$

Is PDF a bounded function?

No, PDF_Y tends to $+\infty$ as y approaches 0, so it is not bounded.
It is bounded only from below by 0.