

## Orders of Growth

(Limits used for succinctness, not correct.)

**Big Theta**, **Big Omega**, and **Big O**:

$$\begin{aligned}\theta(g(n)) &\iff \exists k_1, k_2 \in \mathbb{Z}^+ \exists n_0 \in \mathbb{R} \\ &\quad (\forall n > n_0 (k_1 \cdot g(n) \leq r(n) \leq k_2 \cdot g(n))) \\ O(g(n)) &\iff \exists k \in \mathbb{Z}^+ (\lim_{n \rightarrow \infty} (k \cdot g(n) \geq r(n))) \\ \Omega(g(n)) &\iff \exists k \in \mathbb{Z}^+ (\lim_{n \rightarrow \infty} (k \cdot g(n) \leq r(n)))\end{aligned}$$

**Order (smol to big):**  $1, \log n, n, n \log n, n^2, n^3, 2^n, 3^n, n^n$

Note:  $r(n)$  has OOGs  $\theta(r(n))$ ,  $O(r(n))$ , and  $\Omega(r(n))$ .

**Common Recurrence Relations**

$$\begin{aligned}T(n) &= O(1) + T(n-1) \implies O(n) \\ &= O(\log n) + T(n-1) \implies O(n \log n) \\ &= O(n) + T(n-1) \implies O(n^2) \\ &= O(1) + 2T(n-1) \implies O(2^n) \\ &= O(1) + T\left(\frac{n}{2}\right) \implies O(\log n) \\ &= O(n) + 2T\left(\frac{n}{2}\right) \implies O(n \log n) \\ &= O(n) + T\left(\frac{n}{2}\right) \implies O(n) \\ &= O(1) + 2T\left(\frac{n}{2}\right) \implies O(n)\end{aligned}$$

Generally,  $T(n) = O(n^k) + T(n-1) \implies O(n^{k+1})$

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## Lists

★ Check if x is in list xs

```
if (!is_null(member(x, xs))) { }
```

**Remove Duplicates**

```
function remove_duplicates(xs) {
  return accumulate(
    (curr, wish) =>
      pair(curr,
        filter(x => x !== curr, wish)),
    null, xs);
}
```

**Permutations**

```
function permutations(ys) {
  // list => list of lists
  return is_null(ys)
    ? list(null)
    : accumulate(append, null,
      map(x => map(p => pair(x, p),
        permutations(remove(x, ys))),
        ys));
}
```

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## Arrays

★ Reverse Index

```
A[len - i - 1];
```

**List-array conversion**

```
function list_to_array(xs) {
  const A = [];
  let i = 0;
  while (!is_null(xs)) {
    A[i] = head(xs);
    xs = tail(xs);
    i = i + 1;
  }

  return A;
}

function array_to_list(A) {
  function helper(i) {
    if (A[i] === undefined) {
      return null;
    } else {
      return pair(A[i], helper(i + 1));
    }
  }

  return helper(0);
}
```

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## Reminders :)

Return a block:

```
return {...; return x; };
```

`stream_tail` returns null if nothing left, `stream_ref` returns undefined.

On `equal()`:

```
const a = pair(null, null);
const b1 = pair(a, a);
const b2 = pair(a, a);

equal(b1, b2); # returns TRUE
```

## QRFs

- `integers_from(start)` - Stream.
- `for_each(f, xs)` - Maps functions in place.
- `build_stream/list(f, n)` - 0 to  $n-1$ .
- `enum_stream/list(start, end)` - Includes start and end.
- `eval_stream(s, n)` - Generates list of first  $n$  stream values.