Advanced Algorithms & Data Structures
Assignment 1

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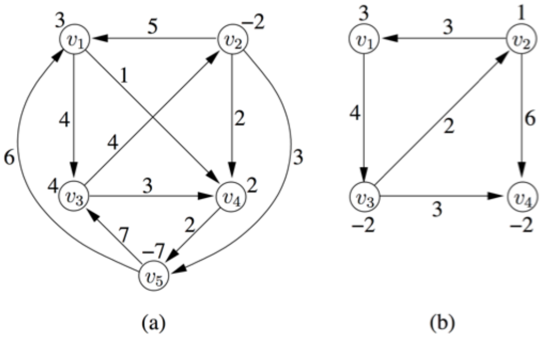
April 30, 2015

Contents

1 *b*-flow

Find for each of the two graphs in Figure 1, a b-flow for the graph or argue that the graph has no b-flow



For a directed graph $G = (V, E)$. For each vertex $v \in V$ let $\delta^+(v)$ be the set of outgiong edges from v and $\delta^-(v)$ be the set of incoming edges to v.
Given is that each a b-flow under the following constraints

$$\sum_{e \in \delta^-(v)} x_e - \sum_{e \in \delta^+(v)} x_e = b_v, \forall v \in V \tag{1}$$

$$0 \leq x_e \leq u_e, \forall e \in E \tag{2}$$

Given these constraints we find that the following b-flows exist for graph (a)

$$\delta(v_2 v_4) = 2$$
$$\delta(v_5 v_3) = 4$$
$$\delta(v_5 v_1) = 3$$

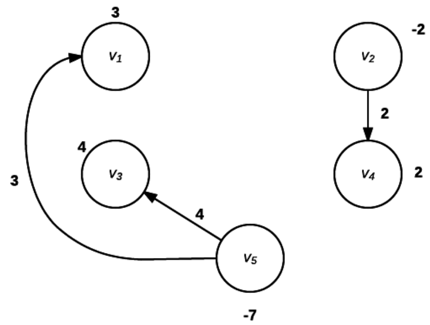


Figure 1: Graph 1A b-flow

We find that there is no b-flow in graph (b) as there is no outgoing edge from v_4 and we do not allow for negative flows. Thus only some of the nodes can satisfy equations (1) and (2) resulting in no existing b-flow.

2 Rectilinear Planar Embedding

2.1 Exercise 2.1

Table 1 shows the values for x_{vf} that correspond to Figure 3 from the Assignment Document.

x_{vf}	a	b	c	d	e
v_1	0	1	1	0	0
v_2	0	0	1	1	0
v_3	1	0	1	1	1
v_4	0	0	0	-1	1
v_5	1	0	0	0	-1
v_6	1	1	0	1	1
v_7	0	0	0	0	0

Table 1: x_{vf} values for Figure 3 in Assignment Document

Table 2 shows the values for z_{fg} that correspond to Figure 3 from the Assignment Document. There are 13 break-points in this graph.

z_{fg}	a	b	c	d	e
a	-	0	0	0	0
b	2	-	1	1	0
c	1	1	-	0	0
d	0	1	0	-	1
e	4	0	0	0	-

Table 2: z_{fg} values for Figure 3 in Assignment Document

We are asked to draw a rectilinear layout of graph 2(a) from the assignment description

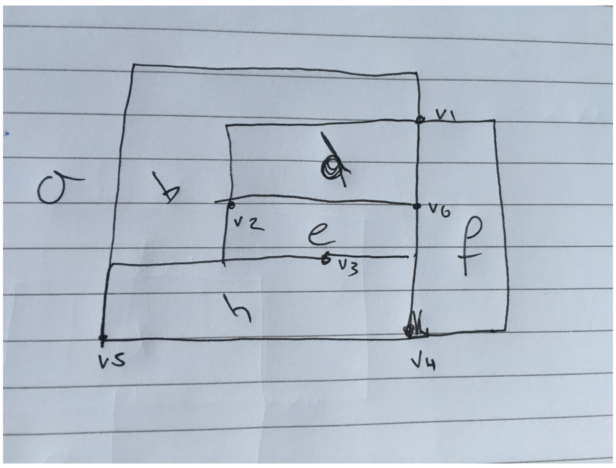


Figure 2: Rectilinear drawing of Graph 2(a) in assignment desc

2.2 Exercise 2.2

Let f_e be the external boundary cycle and F be the set of all boundary cycles. Given two boundary cycles x and y, inner turns (from x to y) are denoted by z_{xy} and outer turns (from y to x) are denoted by zyx.

$$\sum_v x_{vf_e} + \sum_{b \in F \backslash f_e} z_{f_e b} - z_{bf_e} = -4 \tag{3}$$

$$\forall b \in F \backslash f_e : \sum_v x_{vf} + \sum_{b \in F \backslash f} z_{fb} - z_{bf} = 4 \tag{4}$$

2.3 Exercise 2.3

Since G is a graph that contains only straight and horizontal edges, it is impossible for G to have more than 4 edges: a horizontal edge going left; a horizontal edge going right; a vertical edge going up; a vertical edge going down. To show that Equation 1 from the Assignment Document holds we will show that it is true for each of the 3 cases presented, ie v has degree $\in [2, 3, 4]$.

Firstly, the sum of the angles for a vertex v must equal $360°$. Secondly, as we only allow horizontal and vertical edges the only possible turn angles $\in [90, 180, 270]$ for an x_{vf}. We will use these fact to show that the conditions hold for each of the three cases presented.

Case 1: v has degree 2.
Then there are exactly 2 boundary cycles that pass through v, say h and g. Then there are only two possible combinations of the inner and outer turns from the point of view from each boundary cycle. Either both are $180°$, and therefore $x_{vh} = 0$ and $x_{vg} = 0$. So $\sum_f x_{vf} = 0$. Or one boundary cycle has a turn of $90°$ (an inner turn, $x_{vf'} = 1$) and the other a turn of $270°$ (an outer turn, $x_{vf''} = -1$), then $\Sigma_f x_{vf} = 1 - 1 = 0$

Case 2: v has degree 3.
Then there are exactly 3 boundary cycles that pass through v, then since the angles around v must add to $360°$ and we can only have horizontal or vertical edges then there must be one angle of $180°$ and two $90°$ turns, thus there can only be two inner turns and $\sum_f x_{vf} = 0 + 1 + 1 = 2$

Case 3: v has degree 4.
Then there are exactly 4 boundary cycles that pass through v, as the angles around v must add to $360°$, we can only have four $90°$ turns, thus there can only be four inner turns and $\sum_f x_{vf} = 1 + 1 + 1 + 1 = 4$

2.4 Exercise 2.4

The objective function is given by

$$min \sum_{\forall f \in G, f \neq g} z_{fg}$$

given that the aim is to minimise the number of break points. The constraints that the problem is subjected to are:

$$z_{fg}, z_{gf} \geq 0 \tag{5}$$

$$\sum_v x_{vf_e} + \sum_{b \in F\backslash f_e} z_{f_e b} - z_{bf_e} = -4 \tag{6}$$

$$\forall b \in F\backslash f_e : \sum_v x_{vf} + \sum_{b \in F\backslash f} z_{fb} - z_{bf} = 4 \tag{7}$$

$$\sum_f x_{vf} = \begin{cases} 0 & \text{if } v \text{ has degree 2} \\ 2 & \text{if } v \text{ has degree 3} \\ 4 & \text{if } v \text{ has degree 4} \end{cases} \tag{8}$$

2.5 Exercise 2.5

Given a rectilinear graph $G = (V, E)$ and a MCFP $G' = (V', E')$. V' is the set of vertices which contains all vertices $v \in G$, and also a vertex for every face in G. E' is the set of edges which contains edges (f, g) for f,g faces in G , if f and g share a vertex, and edges (v, f) for $v \in V$ if v was a vertex in f. As we are dealing with turns and breakpoints, and these are relative to faces and vertices and faces and faces, there is no information stored on connections from vertices to vertices in G and therefore these connections are not present in G'. The z_{fg} variables.

The transformation from a rectilinear graph to a MCFP is given by the following conditions:

- (v, f) where vertex v lies on the boundary of face f in G. The capacity of these edges is lower bounded by 1 and upper bounded by 4. They have zero cost.
- (f, g) and (g, f) for every pair of adjacent faces, f and g in G. The capacity of these edges is lower bounded by 0 and upper bounded by infinity. They have unit cost.

(?)

Given the above our graph solution to this question is based on the assumption that two faces f and g are adjecent if they share a vertex v. This assumption yields the following sets of connected vertices:

$$a : \quad v_1, v_4, v_5, b, c, d, f, gh \tag{9}$$
$$b : \quad v_1, v_2, v_5, a, c, d, e, h \tag{10}$$
$$c : \quad v_2, v_3, v_5, a, b, d, e, h \tag{11}$$
$$d : \quad v_1, v_2, v_6, a, b, c, e, g, f \tag{12}$$
$$e : \quad v_2, v_3, v_6, b, c, d, f, g, h \tag{13}$$
$$f : \quad v_1, v_4, v_6, a, d, e, g, h \tag{14}$$
$$g : \quad v_3, v_4, v_6, a, d, e, f, h \tag{15}$$
$$h : \quad v_3, v_4, v_5, a, b, c, e, f, h \tag{16}$$

3 Exercise 3

3.1 Exercise 3.1

We start here with I_0, where there is the constraint

$$l_e \leq x_e \leq u_e, \forall e \in E, l_e \in \mathbb{R} \cup -\infty, u_e \in \mathbb{R} \cup +\infty$$

If we take an edge e from u to v,then if $x_e < 0$ we can say that flow is going in the other direction, i.e. along (v, u). As a flow is defined in a graph that does not have anti-parallel edges we cannot image flow going in both directions, and then we introduce a new vertex, say w. To ensure we are not altering the possible max-flow of the network, we say that b_w has value 0 and the cost from $(u, w)- > (w, v)$ will have the same cost as the edge (u, v). Now one of 3 possibilities exists.
First, the flow with value u_e cancels out the flow from l_e and l_e becomes zero. Second, the flow with value l_e cancels out the flow from u_e and u_e becomes zero. Finally, the forward flow and backward flow cancel each other out and then both become zero.

Doing this we ensure that there is at least one of l_e or u_e bounded and finite.

3.2 Exercise 3.2

3.3 Exercise 3.3

We start here with I_2, where there is the constraint

$$l_e \leq x_e \leq u_e, \forall e \in E$$

. $l_e \in \mathbb{R}$ and finite. $u_e \in \mathbb{R} \cup +\infty$.

Now we have two situations. Either $l_e < 0$ or $l_e > 0$. If $l_e < 0$: Then we can modify the the flow f in the original problem.