Advanced Algorithms & Data Structures
Assignment 1

Waseem, Zeerak - csp265
McGillon, Sarah - qfm201

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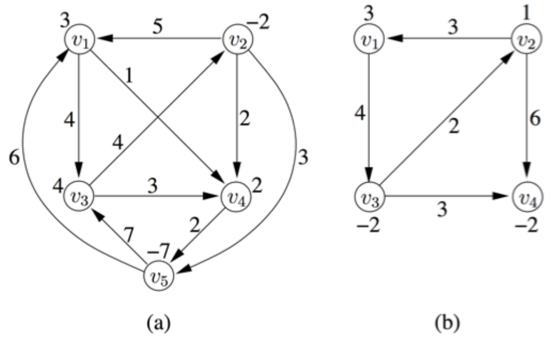
Contents

1 b-flow

Find for each of the two graphs in Figure ??, a b-flow for the graph or argue that the graph has no b-flow



For a directed graph $G = (V, E)$. For each vertex $v \in V$ let $\delta^+(v)$ be the set of outgiong edges from v and $\delta^-(v)$ be the set of incoming edges to v.
Given is that each a b-flow under the following constraints

$$\sum_{e \in \delta^-(v)} x_e - \sum_{e \in \delta^+(v)} x_e = b_v, \forall v \in V \tag{1}$$

$$0 \leq x_e \leq u_e, \forall e \in E \tag{2}$$

Given these constraints we find that the following b-flows exist for graph (a)

$$\delta(v_2 v_4) = 2$$
$$\delta(v_5 v_3) = 4$$
$$\delta(v_5 v_1) = 3$$

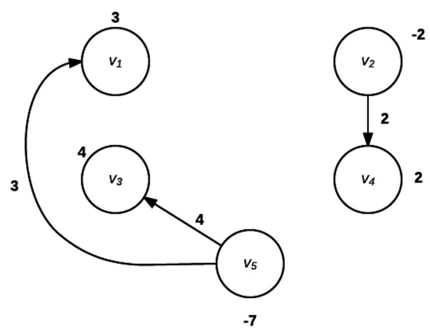


Figure 1: Graph 1A b-flow

We find that there is no b-flow in graph (b) as there is no outgoing edge from v_4 and we do not allow for negative flows. Thus only some of the nodes can satisfy equations (??) and (??) resulting in no existing b-flow.

2 Rectilinear Planar Embedding

2.1 Exercise 2.1

Table **??** shows the values for x_{vf} that correspond to Figure 3 from the Assignment Document.

x_{vf}	a	b	c	d	e
v_1	0	1	1	0	0
v_2	0	0	1	1	0
v_3	1	0	1	1	1
v_4	0	0	0	-1	1
v_5	1	0	0	0	-1
v_6	1	1	0	1	1
v_7	0	0	0	0	0

Table 1: x_{vf} values for Figure 3 in Assignment Document

Table **??** shows the values for z_{fg} that correspond to Figure 3 from the Assignment Document. There are 13 break-points in this graph.

z_{fg}	a	b	c	d	e
a	-	0	0	0	0
b	2	-	1	1	0
c	1	1	-	0	0
d	0	1	0	-	
e	4	0	0	0	-

Table 2: z_{fg} values for Figure 3 in Assignment Document

We are asked to draw a rectilinear layout of graph 2(a) from the assignment description

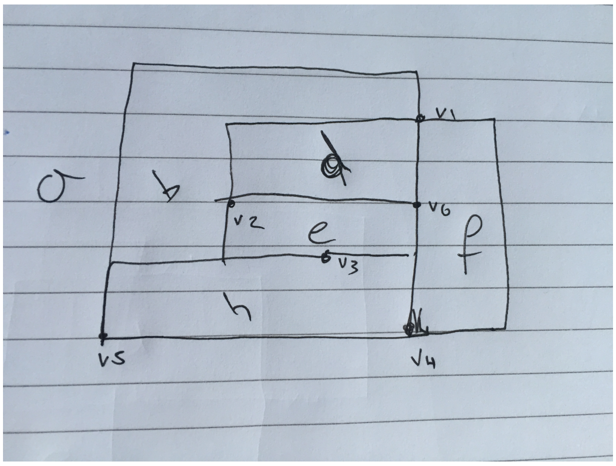


Figure 2: Rectilinear drawing of Graph 2(a) in assignment desc

2.2 Exercise 2.2

Let f_e be the external boundary cycle and B be the set of all boundaries. Inner turns (from x to y) are denoted by z_{xy} and outer turns (from y to x) are denoted by zyx.

$$\forall f \in B\ f_e : \sum_v x_{vf_e} + \sum_{b \in B\ f_e} f_e - \ldots = -4 \tag{3}$$

$$\sum_v x_{vf} + \sum_{b \in B\ f_e} f - \ldots = 4 \tag{4}$$

2.3 Exercise 2.3

Since G is a graph that contains only straight and horizontal edges, it is impossible for G to have more than 4 edges: a horizontal edge going left; a horizontal edge going right; a vertical edge going up; a vertical edge going down. To show that Equation 1 from the Assignment Document holds we will show that it is true for each of the 3 cases presented, ie v has degree $\in [2, 3, 4]$.

Firstly, the sum of the angles for a vertex v must equal $360°$. Secondly, as we only allow horizontal and vertical edges the only possible turn angles $\in [90, 180, 270]$ for an x_{vf}. We will use these fact to show that the conditions hold for each of the three cases presented.

Case 1: v has degree 2.
Then there are exactly 2 boundary cycles that pass through v, say h and g. Then there are only two possible combinations of the inner and outer turns from the point of view from each boundary cycle. Either both are $180°$, and therefore $x_{vh} = 0$ and $x_{vg} = 0$. So $\sum_f x_{vf} = 0$. Or one boundary cycle has a turn of $90°$ (an inner turn, $x_{vf'} = 1$) and the other a turn of $270°$ (an outer turn, $x_{vf''} = -1$), then $\Sigma_f x_{vf} = 1 - 1 = 0$

Case 2: v has degree 3.
Then there are exactly 3 boundary cycles that pass through v, then since the angles around v must add to $360°$ and we can only have horizontal or vertical edges then there must be one angle of $180°$ and two $90°$ turns, thus there can only be two inner turns and $\sum_f x_{vf} = 0 + 1 + 1 = 2$

Case 3: v has degree 4.
Then there are exactly 4 boundary cycles that pass through v, as the angles around v must add to $360°$, we can only have four $90°$ turns, thus there can only be four inner turns and $\sum_f x_{vf} = 1 + 1 + 1 + 1 = 4$