Advanced Algorithms & Data Structures
Assignment 1

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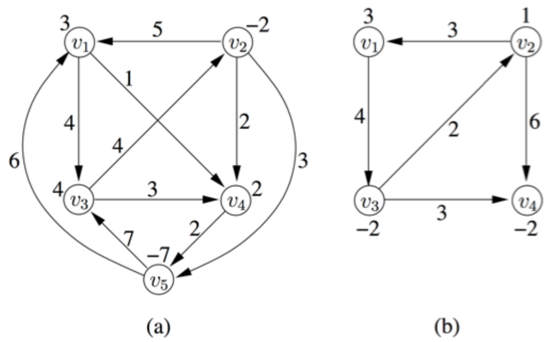
Contents

1 *b*-flow

Find for each of the two graphs in Figure 1, a b-flow for the graph or argue that the graph has no b-flow



For a directed graph $G = (V, E)$. For each vertex $v \in V$ let $\delta^+(v)$ be the set of outgiong edges from v and $\delta^-(v)$ be the set of incoming edges to v.
Given is that each a b-flow under the following constraints

$$\sum_{e \in \delta^-(v)} x_e - \sum_{e \in \delta^+(v)} x_e = b_v, \forall v \in V \tag{1}$$

$$0 \leq x_e \leq u_e, \forall e \in E \tag{2}$$

Given these constraints we find that the following b-flows exist for graph (a)

$$\delta(v_2 v_4) = 2$$
$$\delta(v_5 v_3) = 4$$
$$\delta(v_5 v_1) = 3$$

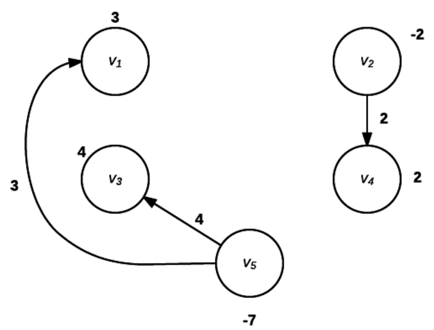


Figure 1: Graph 1A b-flow

We find that there is no b-flow in graph (b) as there is no outgoing edge from v_4 and we do not allow for negative flows. Thus only some of the nodes can satisfy equations (1) and (2) resulting in no existing b-flow.

2 Rectilinear Planar Embedding

2.1 Exercise 2.1

Table 1 shows the values for x_{vf} that correspond to Figure 3 from the Assignment Document.

x_{vf}	a	b	c	d	e
v_1	0	1	1	0	0
v_2	0	0	1	1	0
v_3	1	0	1	1	1
v_4	0	0	0	-1	1
v_5	1	0	0	0	-1
v_6	1	1	0	1	1
v_7	0	0	0	0	0

Table 1: x_{vf} values for Figure 3 in Assignment Document

Table 2 shows the values for z_{fg} that correspond to Figure 3 from the Assignment Document. There are 13 break-points in this graph.

z_{fg}	a	b	c	d	e
a	-	0	0	0	0
b	2	-	1	1	0
c	1	1	-	0	0
d	0	1	0	-	1
e	4	0	0	0	-

Table 2: z_{fg} values for Figure 3 in Assignment Document

We are asked to draw a rectilinear layout of graph 2(a) from the assignment description

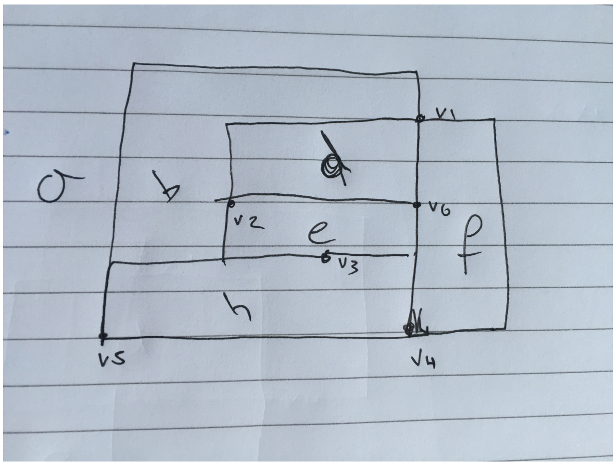


Figure 2: Rectilinear drawing of Graph 2(a) in assignment desc

2.2 Exercise 2.2

Let f_e be the external boundary cycle and F be the set of all boundary cycles. Given two boundary cycles x and y, inner turns (from x to y) are denoted by z_{xy} and outer turns (from y to x) are denoted by zyx.

$$\sum_v x_{vf_e} + \sum_{b \in F \backslash f_e} z_{f_e b} - z_{b f_e} = -4 \tag{3}$$

$$\forall b \in F \backslash f_e : \sum_v x_{vf} + \sum_{b \in F \backslash f} z_{fb} - z_{bf} = 4 \tag{4}$$

2.3 Exercise 2.4

The objective function is given by

$$min \sum_{\forall f \in G, f \neq g} z_{fg}$$

given that the aim is to minimise the number of break points. The constraints that the problem is subjected to are:

$$z_{fg}, z_{gf} \geq 0 \tag{5}$$

$$\sum_v x_{vf_e} + \sum_{b \in F \backslash f_e} z_{f_e b} - z_{b f_e} = -4 \tag{6}$$

$$\forall b \in F \backslash f_e : \sum_v x_{vf} + \sum_{b \in F \backslash f} z_{fb} - z_{bf} = 4 \tag{7}$$

$$\sum_f x_{vf} = \begin{cases} 0 & \text{if } v \text{ has degree 2} \\ 2 & \text{if } v \text{ has degree 3} \\ 4 & \text{if } v \text{ has degree 4} \end{cases} \tag{8}$$

2.4 Exercise 2.5

The transformation from a rectilinear graph to a MCFG is given by the following conditions:

- (v, f) where vertex v lies on the boundary of face f in G. The capacity of these edges is lower bounded by 1 and upper bounded by 4. They have zero cost.
- (f, g) and (g, f) for every pair of adjacent faces, f and g in G. The capacity of these edges is lower bounded by 0 and upper bounded by infinity. They have unit cost.

(**?**)

Given the above our graph solution to this question is based on the assumption that two faces f and g are adjecent if they share a vertex v. This assumption yields the following sets of connected vertices:

$$
\begin{aligned}
a &: && v_1, v_4, v_5, b, c, d, f, gh && (9)\\
b &: && v_1, v_2, v_5, a, c, d, e, h && (10)\\
c &: && v_2, v_3, v_5, a, b, d, e, h && (11)\\
d &: && v_1, v_2, v_6, a, b, c, e, g, f && (12)\\
e &: && v_2, v_3, v_6, b, c, d, f, g, h && (13)\\
f &: && v_1, v_4, v_6, a, d, e, g, h && (14)\\
g &: && v_3, v_4, v_6, a, d, e, f, h && (15)\\
h &: && v_3, v_4, v_5, a, b, c, e, f, h && (16)
\end{aligned}
$$