Challenge 2

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***Abstract*—This document is to know the conditions where matrix multiplication can be commutative**

Download all python codes from

https://github.com/Zeeshan−IITH/IITH−EE5609/new/master/codes

and latex-tikz codes from

https://github.com/Zeeshan−IITH/IITH−EE5609

1 Problem

The conditions when matrix multiplication can be commutative,especially where both the matrices are simultaneously diagonalizable.

2 Explanation

Let $\boldsymbol{P}$ be an invertible matrix that can simultaneously diagonalize matrices $\boldsymbol{A}$ and $\boldsymbol{B}$.
Let $\boldsymbol{P}$=$\begin{pmatrix}\boldsymbol{X_1} & \boldsymbol{X_2}... & \boldsymbol{X_n}\end{pmatrix}$ where $\boldsymbol{X_1}$,$\boldsymbol{X_2}$ ..$\boldsymbol{X_n}$ are the eigen vectors.
Then

$$\boldsymbol{AX_1} = \lambda_{a1}\boldsymbol{X_1}, \boldsymbol{BX_1} = \lambda_{b1}\boldsymbol{X_1}$$
$$\boldsymbol{AX_2} = \lambda_{a2}\boldsymbol{X_2}, \boldsymbol{BX_2} = \lambda_{b2}\boldsymbol{X_2}$$

and so on

$$\boldsymbol{AX_n} = \lambda_{an}\boldsymbol{X_n}, \boldsymbol{BX_1} = \lambda_{bn}\boldsymbol{X_n} \tag{1}$$

But using the above equations we can write

$$\boldsymbol{A}(\boldsymbol{B}^{-1}\lambda_{b1}\boldsymbol{X_1}) = \lambda_{a1}\boldsymbol{X_1}$$
$$\boldsymbol{A}(\boldsymbol{B}^{-1}\lambda_{b2}\boldsymbol{X_2}) = \lambda_{a2}\boldsymbol{X_2}$$

and so on

$$\boldsymbol{A}(\boldsymbol{B}^{-1}\lambda_{bn}\boldsymbol{X_n}) = \lambda_{an}\boldsymbol{X_n} \tag{2}$$

Since $\boldsymbol{X_1}, \boldsymbol{X_2}..\boldsymbol{X_n}$ are not zero matrices
We can write

$$\lambda_{b1}\boldsymbol{A} = \lambda_{a1}\boldsymbol{B}$$
$$\lambda_{b2}\boldsymbol{A} = \lambda_{a2}\boldsymbol{B}$$

and so on

$$\lambda_{bn}\boldsymbol{A} = \lambda_{an}\boldsymbol{B} \tag{3}$$

Therefore this is possible only if

$$\frac{\lambda_{a1}}{\lambda_{b1}} = \frac{\lambda_{a2}}{\lambda_{b2}} = ... = \frac{\lambda_{an}}{\lambda_{bn}} \tag{4}$$

and

Matrix $\boldsymbol{A}$ is a scalar multiple of $\boldsymbol{B}$