Circular Convolution

Shaik Zeeshan Ali
AI20MTECH11001

***Abstract*—This document tries to convert circular convolution in to matrix form**

Download all python codes from

https://github.com/Zeeshan−IITH/IITH−EE5609/new/master/codes

and latex-tikz codes from

https://github.com/Zeeshan−IITH/IITH−EE5609

1 Problem

A finite-length discrete-time signal is basically a sequence, say, $(x_0, \ldots, x_{m1})$ which can be written as an m-length vector $vecx \in R^m$.

Given two periodic signals $(x_0, \ldots, x_n 1)$ and $(h_0, \ldots, h_n 1)$, the circular convolution of the two signals is of length $2n1$, defined as

$$y(t) = (h \circledast x)_t = \sum_{\tau=0}^{\tau=n-1} x_\tau h_{(t-\tau) mod n} \tag{1.0.1}$$
$$0 \le t < 2n-1$$

2 Construction

Circular convolution is a special case of convolution where the summation is truncated to a non-zero periodic interval and the actual signal is just a periodic repetition of the circular convolution.

The time period of the two discrete signals be n,then the resultant convolution is also discrete periodic signal with a time period $2n-1$.par The signal $\mathbf{Y}$ contains $2n-1$ elements.

$$Y = \begin{pmatrix} y_0 \\ y_1 \\ . \\ . \\ y_{2n-2} \end{pmatrix} \tag{2.0.1}$$

where

$$y_{t_0} = \sum_{\tau=0}^{\tau=n-1} x_\tau h_{(t_0-\tau)} \tag{2.0.2}$$

Therefore

$$Y = \begin{pmatrix} h_0x_0 \\ h_1x_0 + h_0x_1 \\ h_0x_2 + h_1x_1 + h_2x_0 \\ . \\ . \\ h_{n-1}x_0 + h_{n-2}x_1 + ...h_0x_{n-1} \\ . \\ h_{n-1}x_0 + h_{n-2}x_1 + ...h_0x_{n-1} \\ h_{n-1}x_1 + h_{n-2}x_2 + ...h_1x_{n-1} \\ . \\ . \\ h_{n-2}x_{n-1} + h_{n-1}x_{n-2} \\ h_{n-1}x_{n-1} \end{pmatrix} \tag{2.0.3}$$

3 Explanation

Simplifying

$$Y = \begin{pmatrix} h_0 & 0 & 0 & . & . & 0 & 0 \\ h_1 & h_0 & 0 & . & . & 0 & 0 \\ h_2 & h_1 & h_0 & . & . & 0 & 0 \\ . & . & . & . & . & . & . \\ h_{n-1} & h_{n-2} & h_{n-3} & . & . & h_1 & h_0 \\ 0 & h_{n-1} & h_{n-2} & . & . & h_2 & h_1 \\ 0 & 0 & h_{n-1} & . & . & h_3 & h_2 \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & 0 & h_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2. \\ . \\ . \\ x_{n-1} \end{pmatrix} \tag{3.0.1}$$

Therefore we can write equation (1.0.1) in matrix form as $\mathbf{Y} = \mathbf{HX}$ where

$$\mathbf{H} = \begin{pmatrix} h_0 & 0 & 0 & . & . & 0 & 0 \\ h_1 & h_0 & 0 & . & . & 0 & 0 \\ h_2 & h_1 & h_0 & . & . & 0 & 0 \\ . & . & . & . & . & . & . \\ h_{n-1} & h_{n-2} & h_{n-3} & . & . & h_1 & h_0 \\ 0 & h_{n-1} & h_{n-2} & . & . & h_2 & h_1 \\ 0 & 0 & h_{n-1} & . & . & h_3 & h_2 \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & 0 & h_{n-1} \end{pmatrix} \tag{3.0.2}$$

$\mathbf{H}$ is a matrix of dimension $(2n-1) \times n$.Since the actual signal is a periodic repitition of Each term of the signal $\mathbf{Y}$ can be written as

$$y_k = y_{(k \mod 2n-1)} \tag{3.0.3}$$

Where each term $y_0, y_1...y_{2n-2}$ are derived from the circular convolution.