Assignment 14

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Download all python codes from

https://github.com/Zeeshan−IITH/IITH−EE5609/new/master/codes

and latex-tikz codes from

https://github.com/Zeeshan−IITH/IITH−EE5609

1 PROBLEM

Let $p_n(x) = x^n$ for $x \in \mathbb{R}$ and let $\varrho = span\{p_0, p_1, p_2, ...\}$. Then

1) ϱ is a vector space of all real valued continuous functions on $\mathbb{R}$.
2) ϱ is a subspace of all real valued continuous functions on $\mathbb{R}$.
3) $\{p_0, p_1, p_2, ...\}$ is a linearly independent set in the vector space of all real valued continuous functions on $\mathbb{R}$.
4) Trigonometric functions belong to ϱ.

2 CONSTRUCTION

Given	$p_n(x) = x^n$ for $x \in \mathbb{R}$ and $\varrho = span\{p_0, p_1, p_2, ...\}$.
Vector space of real continuous functions on $\mathbb{R}$	The set S consisting of all real continuous functions on $\mathbb{R}$ forms a vector space. Let f and g be two real continuous functions from the set S. Since the sum of two continuous function is a continuous function. $i)$ Addition is commutative $f + g = g + f$ $ii)$ Addition is associative$f + (g + h) = (f + g) + h$ $iii)$There is unique O, zero function which maps every element to 0. $iv)$Additive inverse.For each f in S, $-f$ is a function in S. $v)$Properties of scalar multiplication.For $c, c_1, c_2 \in \mathbb{R}$, $a)$ $1f = f$ where the constant function 1 maps every element to 1. $b)$ $(c_1c_2)f = c_1(c_2f)$ $c)$ $c(f + g) = cf + cg$ $d)$ $c_1 + c_2)f = c_1f + c_2f$ Hence the set S forms a vector space.

Option 1	ϱ represents the vector space of polynomials. Polynomial functions are infintely continuously differentiable.So any function that is continuous but not differentiable can not be represented by polynomials. Example the function $\|\mathbf{x}\|$ is continous but cannot be represented in polynomial basis.Therefore option 1 is incorrect.
Option 2	ϱ forms a subspace of all real valued continuous function on $\mathbb{R}$ Let α,β be two polynomial functions of order m and n, represented by the tuple of coefficients $(a_0, a_2, a_2..a_m)$ and $(b_0, b_1, b_2...b_n)$,then $c\alpha + \beta$ is also a polynomial function whose coefficients are $(ca_0 + b_0, ca_1 + b_1, ca_2 + b_2...)$ Therefore ϱ is a subspace of all real valued continuous functions on $\mathbb{R}$. For example consider two functions $f = \{2, 0, 4\}$ and $g = \{0, 2, 1, 5\}$,then $2f + g$ will be $2f + g = 2\left(2 + 4x^2\right) + \left(2x + x^2 + 5x^3\right) = 4 + 2x + 9x^2 + 5x^3 = \{4, 2, 9, 5\}$.
Option 3	Consider the expression $a_0p_0 + a_1p_1 + a_2p_2 + ... = O \implies a_0 = a_1 = a_2 = ... = 0$ Hence $\{p_0, p_1, p_2, ..\}$ are linearly independent set in the vector space of all real valued continuous functions on $\mathbb{R}$.
Option 4	The fundamental period of trigonometric functions is finite, where as polynomials are aperiodic. So, they cannot belong to the same class. For example $\sin x$ has a fundamental period of 2π. $\tan x$ is continuous in the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$, but is not defined at $k\frac{\pi}{2}$ where $k \in odd(\mathbb{N})$.

TABLE 1: Answer