Circular Convolution

Shaik Zeeshan Ali
AI20MTECH11001

***Abstract*—This document tries to convert circular convolution in to matrix form**

Download all python codes from

https://github.com/Zeeshan−IITH/IITH−EE5609/new/master/codes

and latex-tikz codes from

https://github.com/Zeeshan−IITH/IITH−EE5609

1 PROBLEM

A finite-length discrete-time signal is basically a sequence, say, $(x_0, \dots, x_{m1})$ which can be written as an m-length vector $vecx \in R^m$.

Given two periodic signals $(x_0, \dots, x_n 1)$ and $(h_0, \dots, h_n 1)$, the circular convolution of the two signals is of length n, defined as

$$y(t) = (h \circledast x)_t = \sum_{\tau=0}^{\tau=n-1} x_\tau h_{(t-\tau) mod n} \tag{1.0.1}$$

$$0 \le t < n-1$$

2 CONSTRUCTION

Circular convolution is a special case of convolution where the summation is truncated to a nonzero periodic interval and the actual signal is just a periodic repetition of the circular convolution.

The time period of the two discrete signals be n,then the resultant convolution is also discrete periodic signal with a time period n.par The signal **Y** contains n elements.

$$Y = \begin{pmatrix} y_0 \\ y_1 \\ . \\ . \\ y_{n-1} \end{pmatrix} \tag{2.0.1}$$

where

$$y_t = \sum_{\tau=0}^{\tau=n-1} x_\tau h_{(t-\tau) mod n} \tag{2.0.2}$$

Therefore

$$Y = \begin{pmatrix} h_0 x_0 + h_{n-1} x_1 ... + h_1 x_{n-1} \\ h_1 x_0 + h_0 x_1 + ... + h_2 x_{n-1} \\ h_2 x_0 + h_1 x_1 + ... + h_3 x_{n-1} \\ . \\ . \\ h_{n-1} x_0 + h_{n-2} x_1 + ... h_0 x_{n-1} \end{pmatrix} \tag{2.0.3}$$

3 EXPLANATION

Simplifying

$$Y = \begin{pmatrix} h_0 & h_{n-1} & h_{n-2} & . & . & h_2 & h_1 \\ h_1 & h_0 & h_{n-1} & . & . & h_3 & h_2 \\ h_2 & h_1 & h_0 & . & . & h_4 & h_3 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ h_{n-1} & h_{n-2} & h_{n-3} & . & . & h_1 & h_0 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2. \\ . \\ . \\ x_{n-1} \end{pmatrix} \tag{3.0.1}$$

Therefore we can write equation (1.0.1) in matrix form as $\mathbf{Y} = \mathbf{HX}$ where

$$\mathbf{H} = \begin{pmatrix} h_0 & h_{n-1} & h_{n-2} & . & . & h_2 & h_1 \\ h_1 & h_0 & h_{n-1} & . & . & h_3 & h_2 \\ h_2 & h_1 & h_0 & . & . & h_4 & h_3 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ h_{n-1} & h_{n-2} & h_{n-3} & . & . & h_1 & h_0 \end{pmatrix} \tag{3.0.2}$$

H is a matrix of dimension $n \times n$.Since the actual signal is a periodic repetition of Each term of the signal **Y** can be written as

$$y_k = y_{(k \mod n)} \tag{3.0.3}$$

Where each term $y_0, y_1 ... y_{n-1}$ are derived from the circular convolution.