Plotting Shadow

Shaik Zeeshan Ali
AI20MTECH11001

***Abstract*—This document is to explain how shadows cast onto a plane can be calculated.**

Download all python codes from

https://github.com/Zeeshan−IITH/IITH−EE5609/new/master/codes

and latex-tikz codes from

https://github.com/Zeeshan−IITH/IITH−EE5609

1 Problem

Find the Shadow cast by a light parallel to a given direction vector onto the plane described by two orthonormal vectors.

2 Construction

Let $\mathbf{u_1}$ and $\mathbf{u_2}$ be the two orthonormal vectors describing the plane.Let $\mathbf{m}$ be the direction vector of the light source.

The set of discrete points used for describing the object be denoted by a $3 \times N$ where each column is point in the $\mathbf{R^3} - plane$.

3 Explanation

Let P be the shadow of a point on the object that is cast onto the plane by the light source.The shadow of the point can be imagined as a the point travelling along the direction vector $\mathbf{m}$ and finally lands on the plane described by vectors $\mathbf{u_1}$ and $\mathbf{u_2}$.

The point on the plane can be described as

$$\mathbf{P} = x\mathbf{u_1} + y\mathbf{u_2} \tag{3.0.1}$$

So the point on the plane needs to travel a distance z in opposite direction to the light source to end up on the object

$$\hat{P} = \mathbf{P} - z\mathbf{m} \tag{3.0.2}$$

$$\hat{P} = x\mathbf{u_1} + y\mathbf{u_2} - z\mathbf{m} \tag{3.0.3}$$

$$\hat{P} = \begin{pmatrix} \mathbf{u_1} & \mathbf{u_2} & -\mathbf{m} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{3.0.4}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \mathbf{u_1} & \mathbf{u_2} & -\mathbf{m} \end{pmatrix}^{-1} \hat{P} \tag{3.0.5}$$

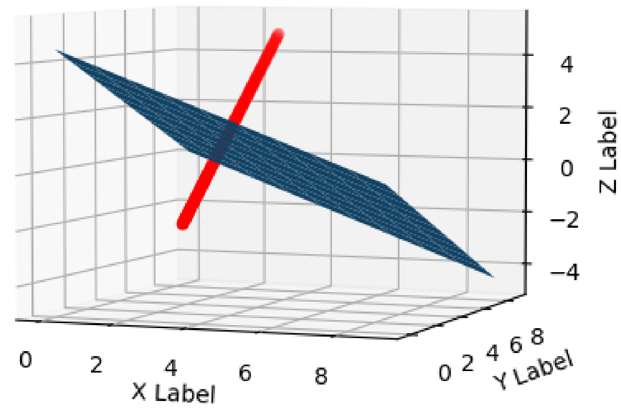

Fig. 1: projection of point along direction vector

where $\hat{P}$ is the point on the object.

By using the equation (3.0.5),we can find out the values of x, y and z

4 Calculation

Let O be a $3 \times N$ matrix describing the object and the corresponding shadow points be S of order $3 \times N$.

By using the equation (3.0.1),we get

$$S = \begin{pmatrix} \mathbf{u_1} & \mathbf{u_2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{4.0.1}$$

If we are viewing the shadow along the normal to the plane,then the plane itself can be thought of as XY-plane.So, by finding x, y we essentially find the shadow as observed by an observer whose line of sight is normal to the plane.