

Assignment 4

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Abstract—This document is about isosceles triangles having a common base.

Download all python codes from

<https://github.com/Zeehan-IITH/IITH-EE5609/new/master/codes>

and latex-tikz codes from

<https://github.com/Zeehan-IITH/IITH-EE5609>

1 PROBLEM

$\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC . Prove that $\angle ABD = \angle ACD$.

2 CONSTRUCTION

In an Isosceles triangle the angles opposite to sides of equal length are equal. Therefore the angles $\angle ABC = \angle ACB$ and $\angle DBC = \angle DCB$. Let the vertex B be at origin and not lose generality. Since the two triangles are isosceles, $\|B - D\| = \|C - D\|$ and $\|A - B\| = \|A - C\|$.

3 EXPLANATION

The triangles $\triangle ABC$ and $\triangle DBC$ are isosceles triangles, so

$$\|A - B\| = \|A - C\| \quad (3.0.1)$$

$$\|D - B\| = \|D - C\| \quad (3.0.2)$$

From equation (3.0.1), we get

$$\begin{aligned} (A - B)^T (A - B) &= (A - C)^T (A - C) \\ (A - B)^T (A - D + D - B) &= \\ (A - C)^T (A - D + D - C) &= \\ (A - B)^T (D - B) &= \\ (A - C)^T (D - C) + (B - C)^T (A - D) & \quad (3.0.3) \end{aligned}$$

Doing the below calculation, we get

$$\begin{aligned} (C - B)^T (A - B) - (C - B)^T (D - B) &= \\ (A - C)^T (B - C) - (D - C)^T (B - C) &= \\ (C - B)^T (A - D) &= (A - D)^T (B - C) \quad (3.0.4) \end{aligned}$$

Since $(A - D)^T (B - C) = (B - C)^T (A - D)$, the equation (3.0.4) can be written as

$$(B - C)^T (A - D) = (C - B)^T (A - D) \quad (3.0.5)$$

$$(B - C)^T (A - D) = -(B - C)^T (A - D) \quad (3.0.6)$$

$$2(B - C)^T (A - D) = 0 \quad (3.0.7)$$

$$(B - C)^T (A - D) = 0 \quad (3.0.8)$$

Taking the inner product of $A - B$, $B - D$ and $A - C$, $D - C$, we get

$$\cos \angle ABD = \frac{(A - B)^T (D - B)}{\|A - B\| \|D - B\|} \quad (3.0.9)$$

$$\cos \angle ACD = \frac{(A - C)^T (D - C)}{\|A - C\| \|D - C\|} \quad (3.0.10)$$

Subtracting the above equations we get

$$\cos \angle ABD - \cos \angle ACD \quad (3.0.11)$$

$$= \frac{(A - B)^T (D - B) - (A - C)^T (D - C)}{\|A - C\| \|D - C\|} \quad (3.0.12)$$

$$= \frac{(B - C)^T (A - D)}{\|A - C\| \|D - C\|} \quad (3.0.13)$$

Using the equation (3.0.8), we get

$$\cos \angle ABD - \cos \angle ACD = 0 \quad (3.0.14)$$

$$\cos \angle ABD = \cos \angle ACD \quad (3.0.15)$$

$$\angle ABD = \angle ACD \quad (3.0.16)$$

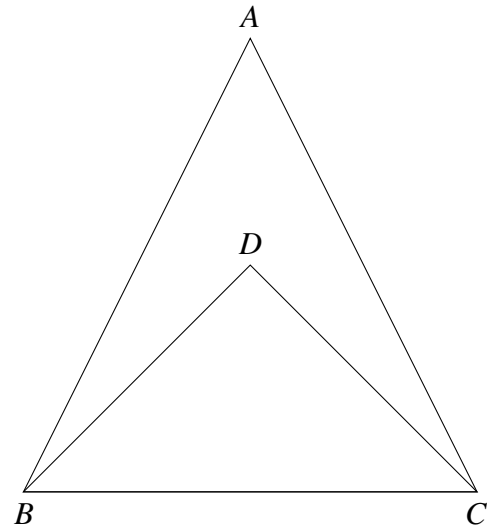


Fig. 1: Isosceles triangles with common base BC