

Assignment 1

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AI20MTECH11001

Abstract—This document is about inverse of the given matrices.

Download all python codes from

<https://github.com/ZeeShan-IITH/IITH-EE5609/new/master/codes>

and latex-tikz codes from

<https://github.com/ZeeShan-IITH/IITH-EE5609>

1 PROBLEM

Let \mathbf{V} be the vector space of $n \times n$ matrices over field \mathbf{F} . Let \mathbf{A} be a fixed $n \times n$ matrix. Let \mathbf{T} be the linear operator on \mathbf{V} defined by

$$\mathbf{T}(\mathbf{B}) = \mathbf{AB} \quad (1.0.1)$$

Show that the minimal polynomial for \mathbf{T} is the minimal polynomial for \mathbf{A} .

2 CONSTRUCTION

Given	\mathbf{A} is a fixed matrix from the vector space \mathbf{V} of $n \times n$ matrices. A linear operator on the finite dimensional vector space \mathbf{V} , \mathbf{T} is defined as $\mathbf{T}(\mathbf{B}) = \mathbf{AB}$.
Minimal polynomial	The minimal polynomial of a linear operator \mathbf{T} is a monic polynomial which annihilates \mathbf{T} .
Matrix representation of \mathbf{T}	<p>If we stack up the columns of the matrix \mathbf{B}, the linear operator \mathbf{T} can be represented in the equivalent form as</p> <p>If $\mathbf{B} = \begin{pmatrix} b_1 & b_2 & \dots & b_n \end{pmatrix}$, then the linear transformation of \mathbf{B} will be</p> $\mathbf{T}(\mathbf{B}) = \begin{pmatrix} \mathbf{A}b_1 & \mathbf{A}b_2 & \dots & \mathbf{A}b_n \end{pmatrix}$ $\mathbf{T}(\mathbf{B}) = \begin{pmatrix} \mathbf{T}(b_1) \\ \mathbf{T}(b_2) \\ \vdots \\ \mathbf{T}(b_n) \end{pmatrix} = \begin{pmatrix} \mathbf{A} & & & \\ & \mathbf{A} & & \\ & & \ddots & \\ & & & \mathbf{A} \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$ <p>here each element represents the elements of the matrix \mathbf{AB}</p>

	$\mathbf{T} = \begin{pmatrix} \mathbf{A} & & & \\ & \mathbf{A} & \mathbf{O} & \\ & & \ddots & \\ & \mathbf{O} & & \ddots & \\ & & & & \mathbf{A} \end{pmatrix}$
Properties of minimal polynomial	The roots of the characteristic polynomial, eigen values and the minimal polynomial are same, except for multiplicities. The roots of the minimal polynomial of \mathbf{A} are the roots of $\det(\mathbf{A} - \lambda \mathbf{I})$
The roots of minimal polynomial of \mathbf{T}	<p>The roots of the minimal polynomial of \mathbf{T} are the roots of $\det(\mathbf{T} - \lambda \mathbf{I})$</p> $\det(\mathbf{T} - \lambda \mathbf{I}) = \begin{vmatrix} (\mathbf{A} - \lambda \mathbf{I}) & & & \\ & (\mathbf{A} - \lambda \mathbf{I}) & \mathbf{O} & \\ & & \ddots & \\ & \mathbf{O} & & \ddots & \\ & & & & (\mathbf{A} - \lambda \mathbf{I}) \end{vmatrix}$ $= (\det(\mathbf{A} - \lambda \mathbf{I}))^n$ <p>Therefore we can see that the eigen values of \mathbf{A} are also the eigen values of the linear operator \mathbf{T}</p>
Minimal polynomial of \mathbf{T}	<p>The minimal polynomial of \mathbf{A} divides the characteristic polynomial of \mathbf{A} and \mathbf{T}. Let the minimal polynomial of \mathbf{A} is of degree $p \leq n$</p> $f(x) = a_0 + a_1x + a_2x^2 + \dots + a_px^p \text{ such that } f(\mathbf{A}) = \mathbf{O}$ $f(\mathbf{T}) = a_0\mathbf{I} + a_1\mathbf{T} + a_2\mathbf{T}^2 + \dots + a_p\mathbf{T}^p$ $f(\mathbf{T}) = \begin{pmatrix} f(\mathbf{A}) & & & \\ & f(\mathbf{A}) & \mathbf{O} & \\ & & \ddots & \\ & \mathbf{O} & & \ddots & \\ & & & & f(\mathbf{A}) \end{pmatrix} = \mathbf{O}_{n^2 \times n^2}$ <p>Therefore the minimal polynomial for \mathbf{T} is the minimal polynomial for \mathbf{A}.</p>