

Assignment 14

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AI20MTECH11001

Download all python codes from

<https://github.com/Zeesan-IITH/IITH-EE5609/new/master/codes>

and latex-tikz codes from

<https://github.com/Zeesan-IITH/IITH-EE5609>

1 PROBLEM

Let $p_n(x) = x^n$ for $x \in \mathbb{R}$ and let $\mathcal{Q} = \text{span}\{p_0, p_1, p_2, \dots\}$. Then

- 1) \mathcal{Q} is a vector space of all real valued continuous functions on \mathbb{R} .
- 2) \mathcal{Q} is a subspace of all real valued continuous functions on \mathbb{R} .
- 3) $\{p_0, p_1, p_2, \dots\}$ is a linearly independent set in the vector space of all real valued continuous functions on \mathbb{R} .
- 4) Trigonometric functions belong to \mathcal{Q} .

2 CONSTRUCTION

Given	$p_n(x) = x^n$ for $x \in \mathbb{R}$ and $\mathcal{Q} = \text{span}\{p_0, p_1, p_2, \dots\}$.
Vector space of real continuous functions on \mathbb{R}	<p>The set S consisting of all real continuous functions on \mathbb{R} forms a vector space.</p> <p>Let f and g be two real continuous functions from the set S.</p> <p>Since the sum of two continuous function is a continuous function.</p> <p>i) Addition is commutative $f + g = g + f$</p> <p>ii) Addition is associative $f + (g + h) = (f + g) + h$</p> <p>iii) There is unique O, zero function which maps every element to 0.</p> <p>iv) Additive inverse. For each f in S, $-f$ is a function in S.</p> <p>v) Properties of scalar multiplication. For $c, c_1, c_2 \in \mathbb{R}$,</p> <p>a) $1f = f$ where the constant function 1 maps every element to 1.</p> <p>b) $(c_1c_2)f = c_1(c_2f)$</p> <p>c) $c(f + g) = cf + cg$</p> <p>d) $(c_1 + c_2)f = c_1f + c_2f$</p>
Option 1	\mathcal{Q} represents the vector space of polynomials. Polynomial functions are infinitely

	<p>continuously differentiable. So any function that is continuous but not differentiable can not be represented by polynomials.</p> <p>Example the function x is continuous but cannot be represented in polynomial basis. Therefore option 1 is incorrect.</p>
Option 2	<p>\mathcal{Q} forms a subspace of all real valued continuous function on \mathbb{R}</p> <p>Let α, β be two polynomial functions of order m and n, represented by the tuple of coefficients $(a_0, a_1, a_2, \dots, a_m)$ and $(b_0, b_1, b_2, \dots, b_n)$, then</p> <p>$c\alpha + \beta$ is also a polynomial function whose coefficients are $(ca_0 + b_0, ca_1 + b_1, ca_2 + b_2, \dots)$</p> <p>Therefore \mathcal{Q} is a subspace of all real valued continuous functions on \mathbb{R}.</p>
Option 3	<p>Consider the expression</p> $a_0p_0 + a_1p_1 + a_2p_2 + \dots = 0 \implies a_0 = a_1 = a_2 = \dots = 0$ <p>Hence $\{p_0, p_1, p_2, \dots\}$ are linearly independent set in the vector space of all real valued continuous functions on \mathbb{R}.</p>
Option 4	<p>The period of trigonometric functions is finite where as the period of polynomials is infinite. So, they cannot belong to the same class.</p>

TABLE 1: Answer