

Linear Convolution

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Abstract—This document converts convolution in to matrix form

Download all python codes from

<https://github.com/Zeesan-IITH/IITH-EE5609/new/master/codes>

and latex-tikz codes from

<https://github.com/Zeesan-IITH/IITH-EE5609>

Therefore

$$Y = \begin{pmatrix} h_0 x_0 \\ h_1 x_0 + h_0 x_1 \\ h_0 x_2 + h_1 x_1 + h_2 x_0 \\ \vdots \\ h_{n-1} x_0 + h_{n-2} x_1 + \dots h_0 x_{n-1} \\ \vdots \\ h_{m-1} x_0 + h_{m-2} x_1 + \dots h_{m-n} x_{n-1} \\ h_{m-1} x_1 + h_{m-2} x_2 + \dots h_{m-n-1} x_{n-1} \\ \vdots \\ h_{m-2} x_{n-1} + h_{m-1} x_{n-2} \\ h_{m-1} x_{n-1} \end{pmatrix} \quad (2.0.3)$$

1 PROBLEM

A finite-length discrete-time signal is basically a sequence, say, (x_0, \dots, x_{m-1}) which can be written as an m -length vector $\text{vec } x \in R^m$.

Given two signals (x_0, \dots, x_{n-1}) and (h_0, \dots, h_{m-1}) , the (linear) convolution of the two is an $m+n-1$ -length signal defined as

$$y(t) = (h * x)_t = \sum_{\tau=0}^{\tau=n-1} x_{\tau} h_{(t-\tau)} \quad (1.0.1)$$

$$0 \leq t < m + n - 1$$

2 CONSTRUCTION

The signal \mathbf{Y} contains $m + n - 1$ elements. Assuming $m > n$

$$Y = \begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_{m+n-2} \end{pmatrix} \quad (2.0.1)$$

where

$$y_{t_0} = \sum_{\tau=0}^{\tau=n-1} x_{\tau} h_{(t_0-\tau)} \quad (2.0.2)$$

Simplifying

$$Y = \begin{pmatrix} h_0 & 0 & 0 & \dots & 0 & 0 \\ h_1 & h_0 & 0 & \dots & 0 & 0 \\ h_2 & h_1 & h_0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ h_{m-1} & h_{m-2} & h_{m-3} & \dots & h_1 & h_0 \\ 0 & h_{m-1} & h_{m-2} & \dots & h_2 & h_1 \\ 0 & 0 & h_{m-1} & \dots & h_3 & h_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & h_{m-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_{n-1} \end{pmatrix} \quad (2.0.4)$$

Therefore we can write equation (1.0.1) in matrix form as $\mathbf{Y} = \mathbf{H}\mathbf{X}$ where

$$\mathbf{H} = \begin{pmatrix} h_0 & 0 & 0 & \dots & 0 & 0 \\ h_1 & h_0 & 0 & \dots & 0 & 0 \\ h_2 & h_1 & h_0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ h_{m-1} & h_{m-2} & h_{m-3} & \dots & h_1 & h_0 \\ 0 & h_{m-1} & h_{m-2} & \dots & h_2 & h_1 \\ 0 & 0 & h_{m-1} & \dots & h_3 & h_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & h_{m-1} \end{pmatrix} \quad (2.0.5)$$