

Challenge 1

Shaik Zeeshan Ali

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Abstract—This document explains how to find points on two skew lines where the distance is shortest.

Download all python codes from

<https://github.com/Zeesan-IITH/IITH-EE5609/new/master/codes>

and latex-tikz codes from

<https://github.com/Zeesan-IITH/IITH-EE5609>

1 PROBLEM

Find the points on two skew lines where the distance between the lines is shortest

$$L_1: \mathbf{x} = x_1 + \lambda_1 \mathbf{v}_1 \quad (1)$$

$$L_2: \mathbf{x} = x_2 + \lambda_2 \mathbf{v}_2 \quad (2)$$

2 CONSTRUCTION

Let \mathbf{a}, \mathbf{b} be two points on the lines L_1, L_2 respectively, where the distance $\|\mathbf{a} - \mathbf{b}\|$ is shortest. The vector along the line $(\mathbf{a} - \mathbf{b})$ will be parallel to $\mathbf{v}_1 \times \mathbf{v}_2$

$$\mathbf{a} = x_1 + \lambda_1 \mathbf{v}_1 \quad (3)$$

$$\mathbf{b} = x_2 + \lambda_2 \mathbf{v}_2 \quad (4)$$

The vector along the line \mathbf{ab} will be

$$\begin{aligned} \mathbf{ab} &= x_1 + \lambda_1 \mathbf{v}_1 - x_2 - \lambda_2 \mathbf{v}_2 \\ \mathbf{ab} &= (x_1 \quad \mathbf{v}_1) \begin{pmatrix} 1 \\ \lambda_1 \end{pmatrix} - (x_2 \quad \mathbf{v}_2) \begin{pmatrix} 1 \\ \lambda_2 \end{pmatrix} \end{aligned} \quad (5)$$

3 EXPLANATION

The vectors $\mathbf{v}_1, \mathbf{v}_2$ are both perpendicular to the line \mathbf{ab} . So the dot product of $\mathbf{v}_1, \mathbf{v}_2$ with the line \mathbf{ab} is zero.

The dot product of \mathbf{v}_1 with the line \mathbf{ab} is

$$\mathbf{v}_1^T (x_1 \quad \mathbf{v}_1) \begin{pmatrix} 1 \\ \lambda_1 \end{pmatrix} - \mathbf{v}_1^T (x_2 \quad \mathbf{v}_2) \begin{pmatrix} 1 \\ \lambda_2 \end{pmatrix} = 0 \quad (6)$$

The dot product of \mathbf{v}_2 with the line \mathbf{ab} is

$$\mathbf{v}_2^T (x_1 \quad \mathbf{v}_1) \begin{pmatrix} 1 \\ \lambda_1 \end{pmatrix} - \mathbf{v}_2^T (x_2 \quad \mathbf{v}_2) \begin{pmatrix} 1 \\ \lambda_2 \end{pmatrix} = 0 \quad (7)$$

Rearranging the equations (5) and (6) in matrix form we get

$$\begin{pmatrix} \mathbf{v}_1^T x_1 & \mathbf{v}_1^T \mathbf{v}_1 & -\mathbf{v}_1^T x_2 & -\mathbf{v}_1^T \mathbf{v}_2 \\ \mathbf{v}_2^T x_1 & \mathbf{v}_2^T \mathbf{v}_1 & -\mathbf{v}_2^T x_2 & -\mathbf{v}_2^T \mathbf{v}_2 \end{pmatrix} \begin{pmatrix} 1 \\ \lambda_1 \\ 1 \\ \lambda_2 \end{pmatrix} = 0 \quad (8)$$

simplifying it further

$$\begin{pmatrix} \mathbf{v}_1^T \mathbf{v}_1 & -\mathbf{v}_1^T \mathbf{v}_2 \\ \mathbf{v}_2^T \mathbf{v}_1 & -\mathbf{v}_2^T \mathbf{v}_2 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} \mathbf{v}_1^T (x_2 - x_1) \\ \mathbf{v}_2^T (x_2 - x_1) \end{pmatrix} \quad (9)$$

Solving the equation (9) we get the values of λ_1 and λ_2 . Substituting the values of λ_1 and λ_2 in the equations (3) and (4) we get the values of the point \mathbf{a} and \mathbf{b} .