Assignment 3

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***Abstract*—This document depicts a way to setup a matrix equation to find the fibonacci sequence.**

Download all python codes from

https://github.com/Zeeshan−IITH/IITH−EE5609/new/master/codes

and latex-tikz codes from

https://github.com/Zeeshan−IITH/IITH−EE5609

1 Problem

For the matrix A=$\begin{pmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{pmatrix}$.Show that

$$A^3 - 6A^2 + 9A - 4I = O \tag{1.0.1}$$

and hence find A^{-1}.

2 construction

Given matrix is

$$A = \begin{pmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{pmatrix} \tag{2.0.1}$$

$$\tag{2.0.2}$$

The characteristic polynomial of the matrix will be

$$\left[A - \lambda I\right] = 0 \tag{2.0.3}$$

$$= \begin{bmatrix}1-\lambda & 1 & 1\\ 1 & 2-\lambda & -3\\ 2 & -1 & 3-\lambda\end{bmatrix} = 0 \tag{2.0.4}$$

$$= (1-\lambda)[(2-\lambda)(3-\lambda) - 3] - 1[(3-\lambda) + 6])$$

$$+ 1[-1 - 2(2-\lambda)] = 0 \tag{2.0.5}$$

$$= \lambda^3 - 6\lambda^2 + 5\lambda + 11 = 0 \tag{2.0.6}$$

According to cayley-hamilton theorem every matrix satisfies it's own characteristic equation. So

$$A^3 - 6A^2 + 5A + 11I = 0 \tag{2.0.7}$$

$$A^3 - 6A^2 = -(5A + 11I) \tag{2.0.8}$$

Therefore the equation (1.0.1) can be simplified using (2.0.8) as

$$A^3 - 6A^2 + 9A - 4I = -(5A + 11I) + 9A - 4I$$

$$= 4A - 15I \tag{2.0.9}$$

3 Explanation

The equation (2.0.9) will be

$$4A - 15I = 4\begin{pmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{pmatrix} - 15\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$= \begin{pmatrix}-11 & 4 & 4\\ 4 & -7 & -12\\ 8 & -4 & -3\end{pmatrix}$$

$$\neq 0 \tag{3.0.1}$$

4 Inverse

The matrix A^2=AA

$$A^2 = \begin{pmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{pmatrix}\begin{pmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{pmatrix}$$

$$= \begin{pmatrix}4 & 2 & 1\\ -3 & 8 & -14\\ 7 & -3 & 14\end{pmatrix}$$

The matrix A satisifes the characteristic equation,so

$$A^3 - 6A^2 + 5A + 11I = 0$$

Multiplying with A^{-1} we get

$$A^2 - 6A + 5I + 11A^{-1} = 0$$

$$A^{-1} = \frac{1}{11}\left(6A - A^2 - 5I\right)$$

$$A^{-1} = \frac{1}{11}\left(6\begin{pmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{pmatrix} - \begin{pmatrix}4 & 2 & 1\\ -3 & 8 & -14\\ 7 & -3 & 14\end{pmatrix} - 5\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\right)$$

$$A^{-1} = \frac{1}{11}\begin{pmatrix}-3 & 4 & 5\\ 9 & -1 & -4\\ 5 & -3 & -1\end{pmatrix} \tag{4.0.1}$$

We can also find the inverse of a matrix using Gauss Jordan elimination method.The augmented matrix of A with I will be

$$\begin{pmatrix}1 & 1 & 1 & 1 & 0 & 0\\ 1 & 2 & -3 & 0 & 1 & 0\\ 2 & -1 & 3 & 0 & 0 & 1\end{pmatrix} \tag{4.0.2}$$

using Gauss jordan elimination method we get

$$\xleftrightarrow{R_2=R_2-R_1,R_3=R_3-2R_1}\begin{pmatrix}1 & 1 & 1 & 1 & 0 & 0\\0 & 1 & -4 & -1 & 1 & 0\\0 & -3 & 1 & -2 & 0 & 1\end{pmatrix} \tag{4.0.3}$$

$$\xleftrightarrow{R_1=R_1-R_2,R_3=R_3+3R_2}\begin{pmatrix}1 & 0 & 5 & 2 & -1 & 0\\0 & 1 & -4 & -1 & 1 & 0\\0 & 0 & -11 & -5 & 3 & 1\end{pmatrix} \tag{4.0.4}$$

$$\xleftrightarrow{R_1=R_1+\frac{5}{11}R_3,R_2=R_2-\frac{4}{11}R_3}\begin{pmatrix}1 & 0 & 0 & -\frac{3}{11} & \frac{4}{11} & \frac{5}{11}\\0 & 1 & 0 & \frac{9}{11} & -\frac{1}{11} & -\frac{4}{11}\\0 & 0 & -11 & -5 & 3 & 1\end{pmatrix} \tag{4.0.5}$$

$$\xleftrightarrow{R_3=-\frac{1}{11}R_3}\begin{pmatrix}1 & 0 & 0 & -\frac{3}{11} & \frac{4}{11} & \frac{5}{11}\\0 & 1 & 0 & \frac{9}{11} & -\frac{1}{11} & -\frac{4}{11}\\0 & 0 & 1 & \frac{5}{11} & -\frac{3}{11} & -\frac{1}{11}\end{pmatrix} \tag{4.0.6}$$

Therefore

$$A^{-1} = \begin{pmatrix}-\frac{3}{11} & \frac{4}{11} & \frac{5}{11}\\\frac{9}{11} & -\frac{1}{11} & -\frac{4}{11}\\\frac{5}{11} & -\frac{3}{11} & -\frac{1}{11}\end{pmatrix} \tag{4.0.7}$$