Assignment 6

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Abstract—This document is about tracing a parabola

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1 PROBLEM

Trace the following parabola

$$4x^2 - 4xy + y^2 - 12x + 6y + 9 = 0 \qquad (1.0.1)$$

2 CONSTRUCTION

The given quadratic equation can be written in the matrix form as

$$\mathbf{x}^T \begin{pmatrix} 4 & -2 \\ -2 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -6 & 3 \end{pmatrix} \mathbf{x} + 9 = 0 \qquad (2.0.1)$$

Calculating the parameters

$$|\mathbf{V}| = \begin{vmatrix} 4 & -2 \\ -2 & 1 \end{vmatrix} = 0 \qquad (2.0.2)$$

$$\begin{vmatrix} \mathbf{V} & \mathbf{u} \\ \mathbf{u}^T & f \end{vmatrix} = \begin{vmatrix} 4 & -2 & -6 \\ -2 & 1 & 3 \\ -6 & 3 & 9 \end{vmatrix} = 0 \qquad (2.0.3)$$

Therefore the given parabola equation is a degenerate.

The characteristic equation of $\mathbf{V}$ will be

$$|\mathbf{V} - \lambda \mathbf{I}| = \begin{vmatrix} 4 - \lambda & -2 \\ -2 & 1 - \lambda \end{vmatrix} \qquad (2.0.4)$$

$$= \lambda^2 - 5\lambda \qquad (2.0.5)$$

$$\lambda_1 = 0, \lambda_2 = 5 \qquad (2.0.6)$$

3 EQUATION OF THE COINCIDENT LINE

Let the equation of the coincident line be $mx - y + c = 0$. The quadratic equation will be

$$(mx - y + c)^2 = 0 \qquad (3.0.1)$$

$$m^2 x^2 - 2mxy + y^2 + 2cmx - 2cy + c^2 = 0 \qquad (3.0.2)$$

$$\mathbf{x}^T \begin{pmatrix} m^2 & -m \\ -m & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} cm & -c \end{pmatrix} \mathbf{x} + c^2 = 0 \qquad (3.0.3)$$

Comparing the equations (2.0.1) and (3.0.3), we get

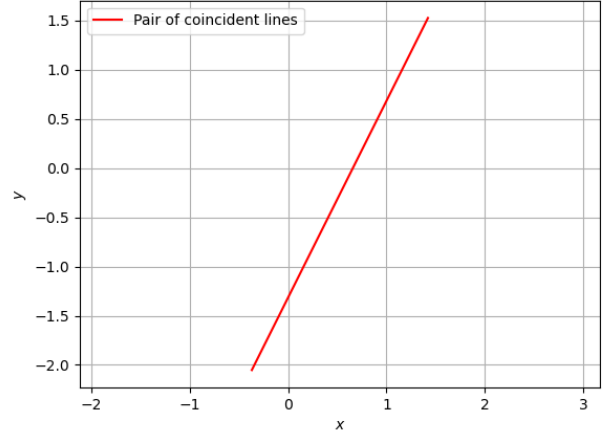$$m = 2, c = -3 \qquad (3.0.4)$$

Therefore the equation of coincident lines is $(2x - y - 3)^2 = 0$.

Fig. 1: Pair of coincident lines