

# Assignment 3

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**Abstract**—This document depicts a way to setup a matrix equation to find the fibonacci sequence.

Download all python codes from

<https://github.com/Zeehan-IITH/IITH-EE5609/new/master/codes>

and latex-tikz codes from

<https://github.com/Zeehan-IITH/IITH-EE5609>

## 1 PROBLEM

For the matrix  $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}$ . Show that

$$A^3 - 6A^2 + 5A + 11I = 0 \quad (1.0.1)$$

and hence find  $A^{-1}$ .

## 2 CONSTRUCTION

Given matrix is

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix} \quad (2.0.1)$$

$$(2.0.2)$$

The characteristic polynomial of the matrix will be

$$[A - \lambda I] = 0 \quad (2.0.3)$$

$$= \begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 2-\lambda & -3 \\ 2 & -1 & 3-\lambda \end{vmatrix} = 0 \quad (2.0.4)$$

$$= (1-\lambda)[(2-\lambda)(3-\lambda)-3] - 1[(3-\lambda)+6] + 1[-1-2(2-\lambda)] = 0 \quad (2.0.5)$$

$$= \lambda^3 - 6\lambda^2 + 5\lambda + 11 = 0 \quad (2.0.6)$$

According to Cayley-Hamilton theorem every matrix satisfies its own characteristic equation. So the matrix  $A$  satisfies the equation (1.0.1)

$$A^3 - 6A^2 + 5A + 11I = 0 \quad (2.0.7)$$

$$(2.0.8)$$

## 3 INVERSE

The matrix  $A^2 = AA$

$$A^2 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{pmatrix}$$

The matrix  $A$  satisfies the characteristic equation, so

$$A^3 - 6A^2 + 5A + 11I = 0$$

Multiplying with  $A^{-1}$  we get

$$A^2 - 6A + 5I + 11A^{-1} = 0$$

$$A^{-1} = \frac{1}{11} (6A - A^2 - 5I)$$

$$A^{-1} = \frac{1}{11} \left( 6 \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix} - \begin{pmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{pmatrix} - 5 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right)$$

$$A^{-1} = \frac{1}{11} \begin{pmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{pmatrix} \quad (3.0.1)$$

Therefore

$$A^{-1} = \begin{pmatrix} \frac{-3}{11} & \frac{4}{11} & \frac{5}{11} \\ \frac{9}{11} & \frac{-1}{11} & \frac{-4}{11} \\ \frac{5}{11} & \frac{-3}{11} & \frac{-1}{11} \end{pmatrix} \quad (3.0.2)$$