

Assignment 3

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Abstract—This document depicts a way to setup a matrix equation to find the fibonacci sequence.

Download all python codes from

<https://github.com/Zeehan-IITH/IITH-EE5609/new/master/codes>

and latex-tikz codes from

<https://github.com/Zeehan-IITH/IITH-EE5609>

1 PROBLEM

For the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}$. Show that $A^3 - 6A^2 + 9A - 4I = O$ and hence find A^{-1} .

2 CONSTRUCTION

Given matrix is

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix} \quad (2.0.1)$$

$$(2.0.2)$$

The characteristic polynomial of the matrix will be

$$[A - \lambda I] = 0 \quad (2.0.3)$$

$$= \begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 2-\lambda & -3 \\ 2 & -1 & 3-\lambda \end{vmatrix} = 0 \quad (2.0.4)$$

$$= (1-\lambda)[(2-\lambda)(3-\lambda) - 3] - 1[(3-\lambda) + 6] + 1[-1 - 2(2-\lambda)] = 0 \quad (2.0.5)$$

$$= \lambda^3 - 6\lambda^2 + 5\lambda + 11 = 0 \quad (2.0.6)$$

According to cayley-hamilton theorem every matrix satisfies it's own characteristic equation. So

$$A^3 - 6A^2 + 5A + 11I = 0 \quad (2.0.7)$$

$$A^3 - 6A^2 = -(5A + 11I) \quad (2.0.8)$$

Therefore the given equation implies

$$\begin{aligned} A^3 - 6A^2 + 9A - 4I &= -(5A + 11I) + 9A - 4I \\ &= 4A - 15I \end{aligned} \quad (2.0.9)$$

3 EXPLANATION

$$\begin{aligned} 4A - 15I &= 4 \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix} - 15 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} -11 & 4 & 4 \\ 4 & -7 & -12 \\ 8 & -4 & -3 \end{pmatrix} \\ &\neq 0 \end{aligned} \quad (3.0.1)$$

4 INVERSE

The augmented matrix of A with I will be

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & -3 & 0 & 1 & 0 \\ 2 & -1 & 3 & 0 & 0 & 1 \end{pmatrix} \quad (4.0.1)$$

using Gauss jordan elimination method we get

$$\xleftrightarrow{R_2=R_2-R_1, R_3=R_3-2R_1} \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -4 & -1 & 1 & 0 \\ 0 & -3 & 1 & -2 & 0 & 1 \end{pmatrix} \quad (4.0.2)$$

$$\xleftrightarrow{R_1=R_1-R_2, R_3=R_3+3R_2} \begin{pmatrix} 1 & 0 & 5 & 2 & -1 & 0 \\ 0 & 1 & -4 & -1 & 1 & 0 \\ 0 & 0 & -11 & -5 & 3 & 1 \end{pmatrix} \quad (4.0.3)$$

$$\xleftrightarrow{R_1=R_1+\frac{5}{11}R_3, R_2=R_2-\frac{4}{11}R_3} \begin{pmatrix} 1 & 0 & 0 & -\frac{3}{11} & \frac{4}{11} & \frac{5}{11} \\ 0 & 1 & 0 & \frac{9}{11} & -\frac{1}{11} & -\frac{4}{11} \\ 0 & 0 & -11 & -5 & 3 & 1 \end{pmatrix} \quad (4.0.4)$$

$$\xleftrightarrow{R_3=-\frac{1}{11}R_3} \begin{pmatrix} 1 & 0 & 0 & -\frac{3}{11} & \frac{4}{11} & \frac{5}{11} \\ 0 & 1 & 0 & \frac{9}{11} & -\frac{1}{11} & -\frac{4}{11} \\ 0 & 0 & 1 & \frac{5}{11} & -\frac{3}{11} & -\frac{1}{11} \end{pmatrix} \quad (4.0.5)$$

Therefore

$$A^{-1} = \begin{pmatrix} -\frac{3}{11} & \frac{4}{11} & \frac{5}{11} \\ \frac{9}{11} & -\frac{1}{11} & -\frac{4}{11} \\ \frac{5}{11} & -\frac{3}{11} & -\frac{1}{11} \end{pmatrix} \quad (4.0.6)$$