# Plotting Shadow

Shaik Zeeshan Ali
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***Abstract*—This document is to explain how shadows cast onto a plane can be calculated.**

Download all python codes from

https://github.com/Zeeshan−IITH/IITH−EE5609/new/master/codes

and latex-tikz codes from

https://github.com/Zeeshan−IITH/IITH−EE5609

## 1 problem

Find the Shadow cast by a light parallel to a given direction vector onto the plane described by two orthonormal vectors.

## 2 construction

Let $\mathbf{u_1}$ and $\mathbf{u_2}$ be the two orthonormal vectors describing the plane.Let $\mathbf{m}$ be the direction vector of the light source.

The set of discrete points used for describing the object be denoted by a $3\times N$ where each column is point in the $\mathbf{R^3}$ – *plane*.

## 3 Explanation

Let $P$ be the point whose shadow is cast onto the plane by the light source.The shadow of the point can be imagined as a the point travelling along the direction vector $\mathbf{m}$ and finally lands on the plane described by vectors $\mathbf{u_1}$ and $\mathbf{u_2}$.

Let $x$ be the distance that the point $P$ needs to travel along the direction $\mathbf{m}$.So, the point on the plane can be

$$\hat{P} = P + x\mathbf{m} \tag{3.0.1}$$

let the normal vector to the plane be

$$\mathbf{n} = \mathbf{u_1}\times\mathbf{u_2} \tag{3.0.2}$$

Then the vector $\hat{\mathbf{P}}$ is normal to $\mathbf{n}$

$$\mathbf{n^T}\hat{\mathbf{P}} = 0 \tag{3.0.3}$$

$$\mathbf{n^T}\left(\mathbf{P} + x\mathbf{m}\right) = 0 \tag{3.0.4}$$

$$\mathbf{n^T P} + x\mathbf{n^T m} = 0 \tag{3.0.5}$$

$$\hat{x} = -\frac{\mathbf{n^T P}}{\mathbf{n^T m}} \tag{3.0.6}$$

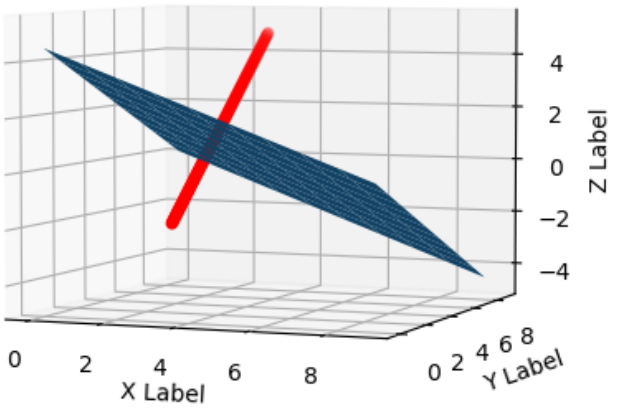


Fig. 1: projection of point along direction vector

So using the equation (3.0.6),we can calculate the points of the shadow on the plane as

$$\hat{P} = \mathbf{P} + \hat{x}\mathbf{m} \tag{3.0.7}$$

## 4 visualization

If we are viewing the shadow along the normal to the plane,then the plane itself can be thought of as $XY$-plane.So, to find the x,y coordinates we can do as following

$$\mathbf{A} = \begin{pmatrix} u_1 & u_2 \end{pmatrix} \tag{4.0.1}$$

The point can be calculated as

$$\hat{P} = xu_1 + yu_2 \tag{4.0.2}$$

$$\hat{P} = \begin{pmatrix} u_1 & u_2 & n \end{pmatrix}\begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \tag{4.0.3}$$

$$\begin{pmatrix} x \\ y \\ 0 \end{pmatrix} = \begin{pmatrix} u_1 & u_2 & n \end{pmatrix}^{-1}\hat{P} \tag{4.0.4}$$

By using equation (4.0.4),we find out the values of $x$ and $y$,which can be used to plot the shadow.