

Fibonacci numbers

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Abstract—This document depicts a way to setup a matrix equation to find the fibonacci sequence.

Download all python codes from

<https://github.com/Zeeshan-IITH/IITH-EE5609/new/master/codes>

and latex-tikz codes from

<https://github.com/Zeeshan-IITH/IITH-EE5609>

This matrix has the fibonacci numbers as its elements

$$\mathbf{F}^2 = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} \quad (3.0.2)$$

$$\mathbf{F}^3 = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix} \quad (3.0.3)$$

Therefore the n^{th} fibonacci number is the element in the first row,first column of \mathbf{F}^{n+1} , where $n \geq 2$

1 PROBLEM

Given a $k \times k$ matrix \mathbf{A} , find the powers of \mathbf{A}^n within $O(\log n)$ time.

2 CONSTRUCTION

For the sake of simplicity we will be calculating the powers given $n = 2^m$, where n is much larger than k . The required result will be of the form $\mathbf{A}^1, \mathbf{A}^2, \mathbf{A}^4, \mathbf{A}^8, \mathbf{A}^{16}..$

The first matrix multiplication will be

$$\mathbf{A}^2 = \mathbf{A}\mathbf{A} \quad (2.0.1)$$

Instead of using repeated multiplication by \mathbf{A} , we can use the previous result and square it to be closer to the result using less computations

$$\mathbf{A}^4 = \mathbf{A}^2\mathbf{A}^2 \quad (2.0.2)$$

$$\mathbf{A}^8 = \mathbf{A}^4\mathbf{A}^4 \quad (2.0.3)$$

$$\mathbf{A}^{16} = \mathbf{A}^8\mathbf{A}^8 \quad (2.0.4)$$

So \mathbf{A}^{2^m} need only m products of the resultant matrix from the previous computation. Since $m = \log_2(n)$, the result can be computed in $O(\log_2 n)$ time.

3 FIBONACCI

Consider the special matrix which begins with fibonacci numbers in it

$$\mathbf{F} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \quad (3.0.1)$$