

Assignment 10

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AI20MTECH11001

Abstract—This document is about inverse of the given matrices.

Download all python codes from

<https://github.com/ZeeShan-IITH/IITH-EE5609/new/master/codes>

and latex-tikz codes from

<https://github.com/ZeeShan-IITH/IITH-EE5609>

1 PROBLEM

Let \mathbf{A} and \mathbf{B} be $n \times n$ matrices over the field F . Prove that if $(\mathbf{I} - \mathbf{AB})$ is invertible

- 1) $(\mathbf{I} - \mathbf{BA})$ is invertible and
- 2) $(\mathbf{I} - \mathbf{BA})^{-1} = \mathbf{I} + \mathbf{B}(\mathbf{I} - \mathbf{AB})^{-1} \mathbf{A}$

2 PROOF

Invertible	A matrix \mathbf{M} is invertible if it is non-singular i.e. the null space of \mathbf{M} contains only zero vector. If \mathbf{x} is a vector such that $\mathbf{M}\mathbf{x} = 0 \implies \mathbf{x} = 0$
Proof for 1	<p>Consider a vector \mathbf{y} such that $(\mathbf{I} - \mathbf{BA})\mathbf{y} = 0$</p> $(\mathbf{I} - \mathbf{BA})\mathbf{y} = 0 \implies \mathbf{y} = \mathbf{BAy}$ $\mathbf{Ay} = \mathbf{ABAy} \implies (\mathbf{I} - \mathbf{AB})\mathbf{Ay} = 0$ <p>since the matrix $(\mathbf{I} - \mathbf{AB})$ is invertible, $\mathbf{Ay} = 0$</p> $\mathbf{y} = \mathbf{B}(\mathbf{Ay}) \implies \mathbf{y} = 0$ <p>Hence the matrix $(\mathbf{I} - \mathbf{BA})$ is invertible.</p>
Observation	<p>Let $\mathbf{C} = (\mathbf{I} - \mathbf{AB})^{-1}$, then</p> $(\mathbf{I} - \mathbf{BA})(\mathbf{BCA}) = \mathbf{BCA} - \mathbf{BABCA} = \mathbf{B}(\mathbf{I} - \mathbf{AB})\mathbf{CA} = \mathbf{BC}^{-1}\mathbf{CA} = \mathbf{BA}$ $(\mathbf{I} - \mathbf{BA})(\mathbf{BCA}) = \mathbf{BA}$
Proof for 2	<p>Let us consider the product $(\mathbf{I} - \mathbf{BA})(\mathbf{BCA} + \mathbf{I})$</p> $(\mathbf{I} - \mathbf{BA})(\mathbf{BCA} + \mathbf{I}) = (\mathbf{I} - \mathbf{BA})(\mathbf{BCA}) + (\mathbf{I} - \mathbf{BA})$ $= \mathbf{BA} + \mathbf{I} - \mathbf{BA} = \mathbf{I}$ $(\mathbf{I} - \mathbf{BA})^{-1} = \mathbf{I} + \mathbf{B}(\mathbf{I} - \mathbf{AB})^{-1} \mathbf{A}$ <p>Hence proved.</p>

TABLE I: Proof