Assignment 10

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AI20MTECH11001

***Abstract*—This document is about inverse of the given matrices.**

Download all python codes from

```
https://github.com/Zeeshan−IITH/IITH−EE5609/
    new/master/codes
```

and latex-tikz codes from

```
https://github.com/Zeeshan−IITH/IITH−EE5609
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1 PROBLEM

Let $\mathbf{A}$ and $\mathbf{B}$ be $n \times n$ matrices over the field F.Prove that if $(\mathbf{I} - \mathbf{AB})$ is invertibe

1) $(\mathbf{I} - \mathbf{BA})$ is invertible and
2) $(\mathbf{I} - \mathbf{BA})^{-1} = \mathbf{I} + \mathbf{B}(\mathbf{I} - \mathbf{AB})^{-1}\mathbf{A}$

2 PROOF

TABLE I: PROOF

Invertible	A matrix $\mathbf{M}$ is invertible if it is non-singular i.e. the null space of $\mathbf{M}$ contains only zero vector.If $\mathbf{x}$ is a vector such that $\mathbf{Mx} = 0 \implies \mathbf{x} = 0$
Proof for 1	Consider a vecor $\mathbf{y}$ such that $(\mathbf{I} - \mathbf{BA})\mathbf{y} = 0$ $(\mathbf{I} - \mathbf{BA})\mathbf{y} = 0 \implies \mathbf{y} = \mathbf{BAy}$ $\mathbf{Ay} = \mathbf{ABAy} \implies (\mathbf{I} - \mathbf{AB})\mathbf{Ay} = 0$ since the matrix $(\mathbf{I} - \mathbf{AB})$ is invertible,$\mathbf{Ay} = 0$ $\mathbf{y} = \mathbf{B}(\mathbf{Ay}) \implies \mathbf{y} = 0$ Hence the matrix $(\mathbf{I} - \mathbf{BA})$ is invertible.
Observation	Let $\mathbf{C} = (\mathbf{I} - \mathbf{AB})^{-1}$,then $(\mathbf{I} - \mathbf{BA})(\mathbf{BCA}) = \mathbf{BCA} - \mathbf{BABCA} = \mathbf{B}(\mathbf{I} - \mathbf{AB})\mathbf{CA} = \mathbf{BC^{-1}CA} = \mathbf{BA}$ $(\mathbf{I} - \mathbf{BA})(\mathbf{BCA}) = \mathbf{BA}$
Proof for 2	Let us consider the product $(\mathbf{I} - \mathbf{BA})(\mathbf{BCA} + \mathbf{I})$ $(\mathbf{I} - \mathbf{BA})(\mathbf{BCA} + \mathbf{I}) = (\mathbf{I} - \mathbf{BA})(\mathbf{BCA}) + (\mathbf{I} - \mathbf{BA})$ $= \mathbf{BA} + \mathbf{I} - \mathbf{BA} = \mathbf{I}$ $(\mathbf{I} - \mathbf{BA})^{-1} = \mathbf{I} + \mathbf{B}(\mathbf{I} - \mathbf{AB})^{\mathbf{-1}}\mathbf{A}$ Hence proved.