Assignment 3

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AI20MTECH11001

***Abstract*—This document depicts a way to setup a matrix equation to find the fibonacci sequence.**

Download all python codes from

```
https://github.com/Zeeshan−IITH/IITH−EE5609/
    new/master/codes
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and latex-tikz codes from

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https://github.com/Zeeshan−IITH/IITH−EE5609
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1 Problem

For the matrix A=$\begin{pmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{pmatrix}$.Show that

$$A^3 - 6A^2 + 5A + 11I = 0 \tag{1.0.1}$$

and hence find A^{-1}.

2 construction

Given matrix is

$$A = \begin{pmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{pmatrix} \tag{2.0.1}$$

The characteristic polynomial of the matrix will be

$$\left[A - \lambda I\right] = 0 \tag{2.0.2}$$

$$= \begin{bmatrix}1-\lambda & 1 & 1\\ 1 & 2-\lambda & -3\\ 2 & -1 & 3-\lambda\end{bmatrix} \tag{2.0.3}$$

$$= (1-\lambda)[(2-\lambda)(3-\lambda) - 3] - 1[(3-\lambda) +6]) + 1[-1 - 2(2-\lambda)] \tag{2.0.4}$$

$$= \lambda^3 - 6\lambda^2 + 5\lambda + 11 = 0 \tag{2.0.5}$$

According to cayley-hamilton theorem every matrix satisfies it's own characteristic equation. So the matrix A satifies the equation (1.0.1)

$$A^3 - 6A^2 + 5A + 11I = 0 \tag{2.0.6}$$

3 Inverse

The matrix A^2=AA

$$A^2 = \begin{pmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{pmatrix}\begin{pmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{pmatrix}$$

$$= \begin{pmatrix}4 & 2 & 1\\ -3 & 8 & -14\\ 7 & -3 & 14\end{pmatrix} \tag{3.0.1}$$

The matrix A satisifes the characteristic equation,so

$$A^3 - 6A^2 + 5A + 11I = 0$$

Multiplying with A^{-1} we get

$$A^2 - 6A + 5I + 11A^{-1} = 0$$

$$A^{-1} = \frac{1}{11}\left(6A - A^2 - 5I\right) \tag{3.0.2}$$

$$A^{-1} = \frac{1}{11}\left(6\begin{pmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{pmatrix} - \begin{pmatrix}4 & 2 & 1\\ -3 & 8 & -14\\ 7 & -3 & 14\end{pmatrix} - 5\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\right. \tag{3.0.3}$$

$$A^{-1} = \frac{1}{11}\begin{pmatrix}-3 & 4 & 5\\ 9 & -1 & -4\\ 5 & -3 & -1\end{pmatrix} \tag{3.0.4}$$

Therefore

$$A^{-1} = \begin{pmatrix}\frac{-3}{11} & \frac{4}{11} & \frac{5}{11}\\ \frac{9}{11} & \frac{-1}{11} & \frac{-4}{11}\\ \frac{5}{11} & \frac{-3}{11} & \frac{-1}{11}\end{pmatrix} \tag{3.0.5}$$