Assignment 10

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***Abstract*—This document is about inverse of the given matrices.**

Download all python codes from

```
https://github.com/Zeeshan−IITH/IITH−EE5609/
    new/master/codes
```

and latex-tikz codes from

```
https://github.com/Zeeshan−IITH/IITH−EE5609
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1 Problem

Let $\mathbf{A}$ and $\mathbf{B}$ be $n \times n$ matrices over the field F.Prove that if $(\mathbf{I}-\mathbf{AB})$ is invertibe

1) $(\mathbf{I}-\mathbf{BA})$ is invertible and
2) $(\mathbf{I}-\mathbf{BA})^{-1} = \mathbf{I}+\mathbf{B}(\mathbf{I}-\mathbf{AB})^{-1}\mathbf{A}$

2 Proof

Invertible	A matrix $\mathbf{M}$ is invertible if it is non-singular i.e. the null space of $\mathbf{M}$ contains only zero vector.If $\mathbf{x}$ is a vector such that $\mathbf{Mx} = 0 \implies \mathbf{x} = 0$
Proof for 1	Consider a vecor $\mathbf{y}$ such that $(\mathbf{I}-\mathbf{BA})\mathbf{y} = 0$ $(\mathbf{I}-\mathbf{BA})\mathbf{y} = 0 \implies \mathbf{y} = \mathbf{BAy}$ $\mathbf{Ay} = \mathbf{ABAy} \implies (\mathbf{I}-\mathbf{AB})\mathbf{Ay} = 0$ since the matrix $(\mathbf{I}-\mathbf{AB})$ is invertible,$\mathbf{Ay} = 0$ $\mathbf{y} = \mathbf{B}(\mathbf{Ay}) \implies \mathbf{y} = 0$ Hence the matrix $(\mathbf{I}-\mathbf{BA})$ is invertible.
Expansion of Inverse	$(\mathbf{I}-\mathbf{AB})^{-1} = \mathbf{I}+(\mathbf{AB})+(\mathbf{AB})^2+(\mathbf{AB})^3+...$ $(\mathbf{I}-\mathbf{BA})^{-1} = \mathbf{I}+(\mathbf{BA})+(\mathbf{BA})^2+(\mathbf{BA})^3+...$
Proof for 2	Right multiplying with $\mathbf{A}$ and left multiplying with $\mathbf{B}$ on both sides of $(\mathbf{I}-\mathbf{AB})^{-1}$ $\mathbf{B}(\mathbf{I}-\mathbf{AB})^{-1}\mathbf{A} = \mathbf{BA}+\mathbf{B}(\mathbf{AB})\mathbf{A}+\mathbf{B}(\mathbf{AB})^2\mathbf{A}+\mathbf{B}(\mathbf{AB})^3\mathbf{A}+...$ $= (\mathbf{BA})+(\mathbf{BA})^2+(\mathbf{BA})^3+...$ $= (\mathbf{I}-\mathbf{BA})^{-1}-\mathbf{I}$ Therefore we can say that $(\mathbf{I}-\mathbf{BA})^{-1} = \mathbf{I}+\mathbf{B}(\mathbf{I}-\mathbf{AB})^{-1}\mathbf{A}$

TABLE I: Proof