Linear Convolution

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***Abstract*—This document converts convolution in to matrix form**

Download all python codes from

https://github.com/Zeeshan−IITH/IITH−EE5609/new/master/codes

and latex-tikz codes from

https://github.com/Zeeshan−IITH/IITH−EE5609

1 Problem

A finite-length discrete-time signal is basically a sequence, say, $(x_0, \ldots, x_m 1)$ which can be written as an m-length vector $vecx \in R^m$.

Given two signals $(x_0, \ldots, x_n 1)$ and $(h_0, \ldots, h_m 1)$, the (linear) convolution of the two is an m+n1-length signal defined as

$$y(t) = (h * x)_t = \sum_{\tau=0}^{\tau=n-1} x_\tau h_{(t-\tau)} \tag{1.0.1}$$
$$0 \le t < m + n - 1$$

2 Construction

The signal $\mathbf{Y}$ contains $m + n - 1$ elements.Assuming $m > n$

$$Y = \begin{pmatrix} y_0 \\ y_1 \\ . \\ . \\ y_{m+n-2} \end{pmatrix} \tag{2.0.1}$$

where

$$y_{t_0} = \sum_{\tau=0}^{\tau=n-1} x_\tau h_{(t_0-\tau)} \tag{2.0.2}$$

Therefore

$$Y = \begin{pmatrix} h_0 x_0 \\ h_1 x_0 + h_0 x_1 \\ h_0 x_2 + h_1 x_1 + h_2 x_0 \\ . \\ . \\ h_{n-1} x_0 + h_{n-2} x_1 + ...h_0 x_{n-1} \\ . \\ h_{m-1} x_0 + h_{m-2} x_1 + ...h_{m-n} x_{n-1} \\ h_{m-1} x_1 + h_{m-2} x_2 + ...h_{m-n-1} x_{n-1} \\ . \\ . \\ h_{m-2} x_{n-1} + h_{m-1} x_{n-2} \\ h_{m-1} x_{n-1} \end{pmatrix} \tag{2.0.3}$$

Simplifying

$$Y = \begin{pmatrix} h_0 & 0 & 0 & . & . & 0 & 0 \\ h_1 & h_0 & 0 & . & . & 0 & 0 \\ h_2 & h_1 & h_0 & . & . & 0 & 0 \\ . & . & . & . & . & . & . \\ h_{n-1} & h_{n-2} & n_{n-3} & . & . & h_1 & h_0 \\ . & . & . & . & . & . & . \\ h_{m-1} & h_{m-2} & h_{m-3} & . & . & h_{m-n+1} & h_{m-n} \\ 0 & h_{m-1} & h_{m-2} & . & . & h_{m-n+2} & h_{m-n+1} \\ 0 & 0 & h_{m-1} & . & . & h_{m-n+3} & h_{m-n+2} \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & 0 & h_{m-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2. \\ . \\ . \\ x_{n-1} \end{pmatrix} \tag{2.0.4}$$

Therefore we can write equation (1.0.1) in matrix form as $\mathbf{Y} = \mathbf{HX}$ where

$$\mathbf{H} = \begin{pmatrix} h_0 & 0 & 0 & . & . & 0 & 0 \\ h_1 & h_0 & 0 & . & . & 0 & 0 \\ h_2 & h_1 & h_0 & . & . & 0 & 0 \\ . & . & . & . & . & . & . \\ h_{n-1} & h_{n-2} & n_{n-3} & . & . & h_1 & h_0 \\ . & . & . & . & . & . & . \\ h_{m-1} & h_{m-2} & h_{m-3} & . & . & h_{m-n+1} & h_{m-n} \\ 0 & h_{m-1} & h_{m-2} & . & . & h_{m-n+2} & h_{m-n+1} \\ 0 & 0 & h_{m-1} & . & . & h_{m-n+3} & h_{m-n+2} \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & 0 & h_{m-1} \end{pmatrix} \tag{2.0.5}$$