Assignment 4

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***Abstract*—This document is about isosceles triangles having a common base.**

Download all python codes from

```
https://github.com/Zeeshan−IITH/IITH−EE5609/
    new/master/codes
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and latex-tikz codes from

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https://github.com/Zeeshan−IITH/IITH−EE5609
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1 Problem

$\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC.Prove that $\angle ABD = \angle ACD$.

2 Construction

In an Isosceles triangle the angles opposite to sides of equal length are equal.Therefore the angles $\angle ABC = \angle ACB$ and $\angle DBC = \angle DCB$.Let the vertex B be at origin and not lose generality.Since the two triangles are isosceles,$\|\mathbf{B}-\mathbf{D}\| = \|\mathbf{C}-\mathbf{D}\|$ and $\|\mathbf{A}-\mathbf{B}\| = \|\mathbf{A}-\mathbf{C}\|$.

3 Explanation

The triangles $\triangle ABC$ and $\triangle DBC$ are isosceles triangles,so

$$\|\mathbf{A}-\mathbf{B}\| = \|\mathbf{A}-\mathbf{C}\| \tag{3.0.1}$$
$$\|\mathbf{D}-\mathbf{B}\| = \|\mathbf{D}-\mathbf{C}\| \tag{3.0.2}$$

From equation (3.0.1),we get

$$(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{B}) = (\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{C}) \tag{3.0.3}$$
$$(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{D}+\mathbf{D}-\mathbf{B}) = (\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{D}+\mathbf{D}-\mathbf{C}) \tag{3.0.4}$$
$$(\mathbf{A}-\mathbf{B})^T(\mathbf{D}-\mathbf{B}) = (\mathbf{A}-\mathbf{C})^T(\mathbf{D}-\mathbf{C}) + (\mathbf{B}-\mathbf{C})^T(\mathbf{A}-\mathbf{D}) \tag{3.0.5}$$

Doing the below calculation,we get

$$\begin{aligned}(\mathbf{C}-\mathbf{B})^T(\mathbf{A}-\mathbf{B}) - (\mathbf{C}-\mathbf{B})^T(\mathbf{D}-\mathbf{B}) = \\ (\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) - (\mathbf{D}-\mathbf{C})^T(\mathbf{B}-\mathbf{C})\end{aligned} \tag{3.0.6}$$
$$(\mathbf{C}-\mathbf{B})^T(\mathbf{A}-\mathbf{D}) = (\mathbf{A}-\mathbf{D})^T(\mathbf{B}-\mathbf{C}) \tag{3.0.7}$$

Since $(\mathbf{A}-\mathbf{D})^T(\mathbf{B}-\mathbf{C})$=$(\mathbf{B}-\mathbf{C})^T(\mathbf{A}-\mathbf{D})$, the equation (3.0.7) can be written as

$$(\mathbf{B}-\mathbf{C})^T(\mathbf{A}-\mathbf{D}) = (\mathbf{C}-\mathbf{B})^T(\mathbf{A}-\mathbf{D}) \tag{3.0.8}$$
$$(\mathbf{B}-\mathbf{C})^T(\mathbf{A}-\mathbf{D}) = -(\mathbf{B}-\mathbf{C})^T(\mathbf{A}-\mathbf{D}) \tag{3.0.9}$$
$$2(\mathbf{B}-\mathbf{C})^T(\mathbf{A}-\mathbf{D}) = 0 \tag{3.0.10}$$
$$(\mathbf{B}-\mathbf{C})^T(\mathbf{A}-\mathbf{D}) = 0 \tag{3.0.11}$$

Taking the inner product of $A-B, B-D$ and $A-C, D-C$,we get

$$\cos\angle ABD = \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{D}-\mathbf{B})}{\|\mathbf{A}-\mathbf{B}\|\|\mathbf{D}-\mathbf{B}\|} \tag{3.0.12}$$
$$\cos\angle ACD = \frac{(\mathbf{A}-\mathbf{C})^T(\mathbf{D}-\mathbf{C})}{\|\mathbf{A}-\mathbf{C}\|\|\mathbf{D}-\mathbf{C}\|} \tag{3.0.13}$$

Subtracting the above equations we get

$$\cos\angle ABD - \cos\angle ACD \tag{3.0.14}$$
$$= \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{D}-\mathbf{B}) - (\mathbf{A}-\mathbf{C})^T(\mathbf{D}-\mathbf{C})}{\|\mathbf{A}-\mathbf{C}\|\|\mathbf{D}-\mathbf{C}\|} \tag{3.0.15}$$
$$= \frac{(\mathbf{B}-\mathbf{C})^T(\mathbf{A}-\mathbf{D})}{\|\mathbf{A}-\mathbf{C}\|\|\mathbf{D}-\mathbf{C}\|} \tag{3.0.16}$$

Using the equation (3.0.11),we get

$$\cos\angle ABD - \cos\angle ACD = 0 \tag{3.0.17}$$
$$\cos\angle ABD = \cos\angle ACD \tag{3.0.18}$$
$$\angle ABD = \angle ACD \tag{3.0.19}$$

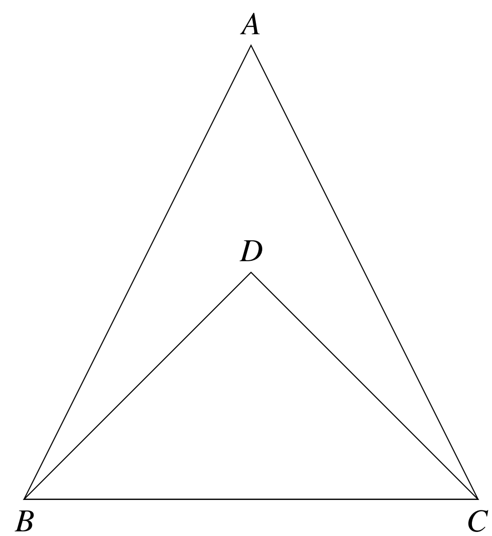

Fig. 1: Isosceles triangles with common base BC