Challenge 5

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***Abstract*—This document is to prove that the sides opposite to equal angles of a triangle are equal.**

Download all python codes from

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https://github.com/Zeeshan−IITH/IITH−EE5609/
    new/master/codes
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and latex-tikz codes from

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https://github.com/Zeeshan−IITH/IITH−EE5609
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1 Problem

Prove that sides opposite to equal angles of a triangle are equal.

2 construction

Consider the triangle in XY plane.So the points A, B, C are the three points of the triangle which has two angles equal.Let $\angle ABC{=}\angle ACB$.

Let θ be the angle made by $\angle ABC$ and $\angle ACB$ .The matrix which rotates the vector by an angle θ is

$$\mathbf{R}_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{2.0.1}$$

Let $\mathbf{m_{BC}}$ be the direction vector parallel to the line BC. So the equation of the line parallel to $\mathbf{AB}$ and $\mathbf{AC}$ will be

$$L_1: \ x = \mathbf{B} + \lambda_1 \mathbf{R}_\theta \mathbf{m_{BC}} \tag{2.0.2}$$

$$L_2: \ x = \mathbf{C} - \lambda_2 \mathbf{R}_{-\theta} \mathbf{m_{BC}} \tag{2.0.3}$$

The equation of a line that is perpendicular to the line BC and passing through the midpoint of B and C is

$$L_3: \ x = \frac{\mathbf{B}+\mathbf{C}}{2} + \lambda_3 \mathbf{R_{90}} \mathbf{m_{BC}} \tag{2.0.4}$$

Where

$$\mathbf{R_{90}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

If there exists a point that satisfies all the three equations then the sides opposite to equal angles will be equal because the perpendicular from that point will pass through the midpoint of B and C.Let A be a point which satisfies all the three equations (2.0.2),(2.0.3) and (2.0.4).

3 Explanation

The point A satisfies the equations (2.0.2), (2.0.3) and (2.0.4) we get

$$\mathbf{A} = \mathbf{B} + \lambda_1 \mathbf{R}_\theta \mathbf{m_{BC}} \tag{3.0.1}$$

$$\mathbf{A} = \mathbf{C} - \lambda_2 \mathbf{R}_{-\theta} \mathbf{m_{BC}} \tag{3.0.2}$$

$$\mathbf{A} = \frac{\mathbf{B}+\mathbf{C}}{2} + \lambda_3 \mathbf{R_{90}} \mathbf{m_{BC}} \tag{3.0.3}$$

Writing the above equations in matrix form we get

$$\begin{pmatrix} \mathbf{R}_\theta \mathbf{m_{BC}} & 0 & 0 \\ 0 & \mathbf{R}_{-\theta} \mathbf{m_{BC}} & 0 \\ 0 & 0 & \mathbf{R_{90}} \mathbf{m_{BC}} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \begin{pmatrix} \mathbf{A}-\mathbf{B} \\ \mathbf{C}-\mathbf{A} \\ \mathbf{A}-\frac{\mathbf{B}+\mathbf{C}}{2} \end{pmatrix} \tag{3.0.4}$$

Where

$$\mathbf{R} = \begin{pmatrix} \mathbf{R}_\theta \mathbf{m_{BC}} & 0 & 0 \\ 0 & \mathbf{R}_{-\theta} \mathbf{m_{BC}} & 0 \\ 0 & 0 & \mathbf{R_{90}} \mathbf{m_{BC}} \end{pmatrix} \tag{3.0.5}$$

If there exist a unique set of values for $\lambda_1, \lambda_2, \lambda_3$ for a given points A, B, C,Then the matrix R is non-singular. Let

$$\mathbf{m_{BC}} = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} \tag{3.0.6}$$

$$\mathbf{R}_\theta \mathbf{m_{BC}} = \begin{pmatrix} m_1 \cos\theta - m_2 \sin\theta \\ m_1 \sin\theta + m_2 \cos\theta \end{pmatrix} \tag{3.0.7}$$

The equation (3.0.7)=0 only when $m_1 = 0$ and $m_2 = 0$.So the matrix $\mathbf{R}$ is always non-singular when $\theta \neq 0$ and $\mathbf{m_{BC}} \neq 0$. Let the point M be the midpoint of the line $\mathbf{BC}$.Then the two triangles $\triangle AMB$ and $\triangle AMC$ both form a right angle at the point M,with the $\angle ABM = \angle ACM = \theta$.

Hence the sides opposite to equal angles of a triangle are equal.